Seeking long-lived SEC-violating string solutions

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based on hep-th/2506.19914, with G. Shiu and H.V. Tran

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PIANO NAZIONALE DI RIPRESA E RESILIENZA

What do DESI data say about string theory?

- DESI data profoundly challenge the standard ΛCDM model of the late universe
  - is dark energy (DE) really a cosmological constant?
  - does DE fulfill the null energy condition ($w_{\mathrm{DE}} \geq -1$)?

DESI DR2 Results I [astro-ph.CO/2503.14739]
DESI DR2 Results II [astro-ph.CO/2503.14738]

- how well do string compactifications and DESI fit together?
  - since long before DESI, several arguments and circumstantial evidence have been raising questions about dS vacua in string EFT corners under perturbative control

    for reviews, see e.g. Danielsson, Van Riet [hep-th/1804.01120]
    Cicoli, de Alwis, Maharana, Muia, Quevedo [hep-th/1808.08967]
    Cicoli, Conlon, Maharana, Parameswaran, Quevedo, Zavala [hep-th/2303.04819]

  - quintessence-like solutions are also challenging to build: in particular, there is a generic tension in constructing cosmic horizons (prior to even trying to match data)

    see e.g. Andriot, Tsimpis, Wrase [hep-th/2309.03938]
    Hassfeld, Hebecker, Schiller [hep-th/2505.07934]

    see also review talk by G. Shiu
    and talks by I. Ruiz, S. Parameswaran and M. Montero

- independently of whether long-lived string-theoretic dS solutions are consistent, it is an exciting time to push for string-theoretic realizations of time-dependent DE

  in this talk we will be concerned with one fundamental question:

  by exploiting couplings of DE with dark matter (DM), is it possible to violate the strong energy condition (SEC) for a long-lasting epoch, in string compactifications?

1. Exponential field couplings and spatial curvature

in several models, the linearly-stable solutions, also late-time attractors, are known exactly

- e.g. field content: canonical scalar ϕ, with exponential potential $V = \Lambda\, \mathrm{e}^{-\gamma\phi}$

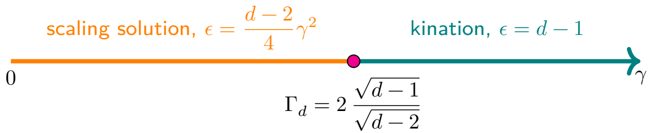
  arguably the most generic string model in moduli-space asymptotic corners
  because of string-coupling and volume expansions

- geometry: FLRW-metric $ds^2_{1,d-1} = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}l^2_{d-1}$ with spatial curvature $k = 0, -\frac{1}{\ell^2}$

note:

- we will focus on the ϵ-parameter $\epsilon = -\frac{\dot{H}}{H^2}$, with $H = \frac{\dot{a}}{a}$
- we may interchangeably talk about $w = -1 + \frac{2\epsilon}{d-1}$
  - accelerated expansion if $\epsilon < 1$, or equivalently if $w \leq -\frac{d-3}{d-1}$, i.e. $\ddot{a} > 0$

- linearly-stable solutions for $k = 0$:



  - strong dS and asymptotic TCC conjectures: $\frac{1}{V}\left|\frac{\partial V}{\partial\phi}\right| = \gamma \geq \frac{2}{\sqrt{d-2}}$, whence $\epsilon \geq 1$

Rudelius [hep-th/2101.11617]
Bedroya, Vafa [hep-th/1909.11063]

note: in principle, semi-eternal acceleration, and yet no cosmic horizons, for $\epsilon = 1$

- linearly-stable solutions for $k = -\frac{1}{\ell^2}$:

Marconnet, Tsimpis [hep-th/2210.10813]
Andriot, Tsimpis, Wrase [hep-th/2309.03938]

phenomenological challenges discussed e.g. in Andriot, Parameswaran, Tsimpis, Wrase, Zavala [hep-th/2405.09323]
Alestas, Delgado, Ruiz, Akrami, Montero, Nesseris [hep-th/2406.09212]

not easier with kinetic couplings: see e.g. Cicoli, Dibitetto, Pedro [hep-th/2002.02695]
Brinkmann, Cicoli, Dibitetto, Pedro [hep-th/2206.10649]

2. DM/DE couplings

violating the null energy condition (NEC, $w < -1$) leads to a growing energy density as the universe expands: how?

a possible explanation:

- if a fluid decays into another one, one density component may grow over time
- for example, we can consider a scalar ϕ and non-relativistic DM with field-dependent DM mass $m_{\rm DM} = m_{\rm DM}(\phi)$, with
  - the DM energy density $\rho_{\rm DM}$ and DE-component term ρ_ϕ can be repackaged as

$$\rho_\phi + \underbrace{n_{\rm DM,0}\left(\frac{a_0}{a}\right)^{d-1} m_{\rm DM}}_{\rho_{\rm DM}} = \underbrace{\rho_\phi + \rho_{\rm DM}\left(1 - \frac{m_{\rm DM,0}}{m_{\rm DM}}\right)}_{\rho_\phi^{\rm eff}} + \underbrace{n_{\rm DM,0}\left(\frac{a_0}{a}\right)^{d-1} m_{\rm DM,0}}_{\rho_{\rm DM}^{\rm eff}}$$

  - if $m_{\rm DM,0} > m_{\rm DM}$, one can have $w_\phi^{\rm eff} = \dfrac{w_\phi}{1 + \dfrac{\rho_{\rm DM}}{\rho_\phi}\left(1 - \dfrac{m_{\rm DM,0}}{m_{\rm DM}}\right)} < -1$

Das, Corasaniti, Khoury [astro-ph/0510628]

- we may interpret the DESI evidence for phantom regime as evidence for DM/DE coupling

see Bedroya, Obied, Vafa, Wu [astro-ph.CO/2507.03090]

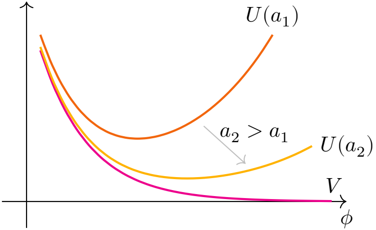
- to have $\rho_{\rm DM} = n_{\rm DM,0}\left(\frac{a_0}{a}\right)^{d-1} m_{\rm DM}$, the continuity equation must be

$$\dot\rho_{\rm DM} + (d-1)H\rho_{\rm DM} = \frac{\rho_{\rm DM}}{m_{\rm DM}}\frac{\partial m_{\rm DM}}{\partial\phi}\,\dot\phi$$

- Hubble conservation then also requires

$$\ddot\phi + (d-1)H\dot\phi + \frac{\partial V}{\partial\phi} = -\frac{\rho_{\rm DM}}{m_{\rm DM}}\frac{\partial m_{\rm DM}}{\partial\phi}$$

- practically, we have an effective potential $U(\phi) = V(\phi) + n_{\rm DM,0}\left(\frac{a_0}{a}\right)^{d-1} m_{\rm DM}(\phi)$



e.g. $\text{sgn}\,(V') < 0$
$\text{sgn}\,(m') > 0$

Das, Corasaniti, Khoury [astro-ph/0510628]
recently, this has been revived in view of DESI data:
see e.g. Chakraborty, Chanda, Das, Dutta [astro-ph.CO/2503.10806]
Khoury, Lin, Trodden [astro-ph.CO/2503.16415]
Andriot [hep-th/2505.10410]
Bedroya, Obied, Vafa, Wu [astro-ph.CO/2507.03090]
for recent earlier studies, see e.g. Agrawal, Obied, Vafa [astro-ph.CO/1906.08261]
Amendola, Tsujikawa [gr-qc/2003.02686]
Liu, Tsujikawa, Ichiki [astro-ph.CO/2309.13946]

- DM/DE-couplings can help achieve a violation of SEC
  - how does this fit into the quest for long-lived SEC-violating solutions from string compactifications?
  - is there an upper bound on the duration of the phase of cosmic acceleration?

- remaining (ideally) within the domain of string-compactification frameworks evolving towards the field-space boundary, we are seeking long-lived cosmic acceleration
  - the study of interacting DM/DE models has a long history, for a variety of couplings and/or short-lived transient solutions

e.g. "chameleon fields", in Brax, van de Bruck, Davis, Khoury, Weltman [astro-ph/0410103]
Das, Corasaniti, Khoury [astro-ph/0510628]

see also Khoury, Weltman [astro-ph/0309300]
Khoury, Weltman [astro-ph/0309411]

e.g. "fading dark sector" models, in Bedroya, Obied, Vafa, Wu [astro-ph.CO/2507.03090]
see also Agrawal, Obied, Vafa [astro-ph.CO/1906.08261]

see also Gomes, Hardy, Parameswaran [hep-ph/2311.08888]
Casas, Montero, Ruiz [hep-th/2406.07614]

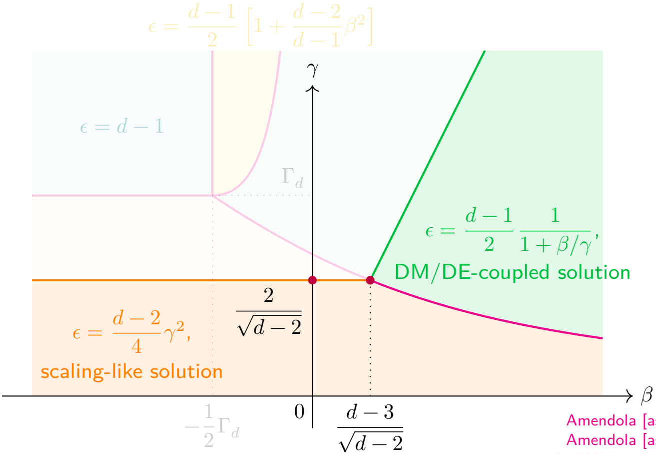
see also talks by P. Brax and I. Ruiz

  - we will focus on linearly-stable solutions with exponential couplings

- we consider exponential couplings:

$$m_{\mathrm{DM}} = \mu\, \mathrm{e}^{\beta\phi}$$
$$V = \Lambda\, \mathrm{e}^{-\gamma\phi}$$

- linearly-stable solutions are completely classified: only two classes allow for $\epsilon \leq 1$



Amendola [astro-ph/9904120]
Amendola [astro-ph/9908023]
see also Wetterich [hep-th/9408025]
Shiu, FT, Tran #5 [hep-th/2506.19914]

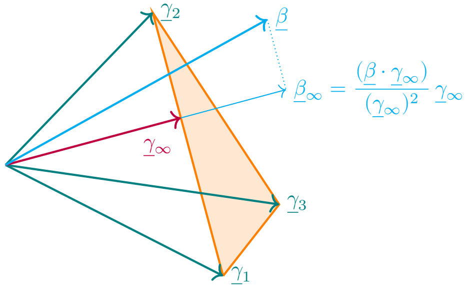
▶ as we would like $\gamma > \frac{2}{\sqrt{d-2}}$, we will focus on the DM/DE-coupled solution

- we also know what happens in multi-field models, such as

$$V = \sum_i \Lambda_i \, \mathrm{e}^{-\underline{\gamma}_i \cdot \underline{\phi}}$$
$$m_{\mathrm{DM}} = \mu \, \mathrm{e}^{\underline{\beta} \cdot \underline{\phi}}$$

- there exists a solution that is equivalent to projecting the multi-field problem onto $\underline{\gamma}_\infty$

$$\epsilon = \frac{d-1}{2} \frac{1}{1 + \frac{\underline{\beta} \cdot \underline{\gamma}_\infty}{(\underline{\gamma}_\infty)^2}}$$



- the linearly-stable solution is the critical-point solution with the smallest ϵ
- if $\underline{\beta} \cdot \underline{\gamma}_\infty > 0$, and if $|\underline{\beta}_\infty|$ is large enough, the DM/DE-coupled solution is linearly-stable

Shiu, FT, Tran #5 [hep-th/2506.19914]

A UV-embedding with $\epsilon \leq 1$?, pt. 1

- to have $\epsilon = \frac{d-1}{2}\frac{1}{1+\beta/\gamma} \leq 1$, we need $\beta > 0$, with $m_{\rm DM} = m_{\rm DM,0}\left(\frac{t}{t_0}\right)^{\frac{2\beta}{\gamma}}$
  - how does this fit in a UV-complete scenario?

- as we are treating DM as a cosmological fluid made out of non-relativistic matter constituents, it is not necessary for the DM mass to be below the cutoff
  - the DM fluid can be treated as a classical source of energy density, which does not need a microscopic field to describe DM
  - example: dark macroscopic objects made out of dark particles

- also:
  - the DM Compton wavelength falls, compared to the horizon size: $\frac{H}{m_{\rm DM}} = \left(\frac{H_0}{m_{\rm DM,0}}\right)\left(\frac{t_0}{t}\right)^{1+\frac{2\beta}{\gamma}}$
  - although the DM mass grows, the universe expansion is still sufficient to dilute the DM energy density away over time: $\rho_{\rm DM} = \rho_{\rm DM,0}\left(\frac{t_0}{t}\right)^2$

- however, DM may become so heavy that the Schwarzschild radius outgrows the Hubble radius:
  - for the DM Schwarzschild radius to always fit in the horizon, we need $m_{\rm DM}^{\frac{1}{d-3}} \leq \frac{1}{H}$
  - this is fulfilled as long as $\frac{2\beta}{\gamma} \leq d-3$, which implies $\epsilon \geq 1$

A UV-embedding with $\epsilon \leq 1$?, pt. 2

in the asymptotics of string compactifications ($\phi = \infty$):

- the potential slope is expected to be bounded as $\gamma \geq \dfrac{2}{\sqrt{d-2}}$
- distance conjecture: towers of states become light with mass gap $m_{\rm DC}(\phi) = \mu_{\rm DC}\, {\rm e}^{-\alpha\phi}$

Ooguri, Vafa [hep-th/0605264]

  - evidence for $\alpha \geq \dfrac{1}{\sqrt{d-2}}$

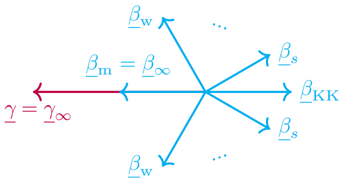
Etheredge, Heidenreich, Kaya, Qiu, Rudelius [hep-th/2206.04063]
see also Lee, Lerche, Weigand [hep-th/1904.06344]
see also Lee, Lerche, Weigand [hep-th/1910.01135]

  - due to string dualities, we expect dual towers in the opposite direction, with $\alpha \mapsto -\beta$

see also Calderón-Infante, Uranga, Valenzuela [hep-th/2012.00034]
Etheredge, Heidenreich, McNamara, Rudelius, Ruiz, Valenzuela [hep-th/2306.16440]
Etheredge, Heidenreich, Rudelius, Ruiz, Valenzuela [hep-th/2405.20332]

ex. 1: saturating values $(\gamma, \beta) = \left(\dfrac{2}{\sqrt{d-2}}, \dfrac{1}{\sqrt{d-2}}\right)$ give $\epsilon = \dfrac{d-1}{3}$

ex. 2: curvature potential and KK monopole in 4d, with isotropy, can give $\epsilon \leq 1$



Viable transient solutions?

- *fading dark sector* model:

Bedroya, Obied, Vafa, Wu [astro-ph.CO/2507.03090]
see also Agrawal, Obied, Vafa [astro-ph.CO/1906.08261]

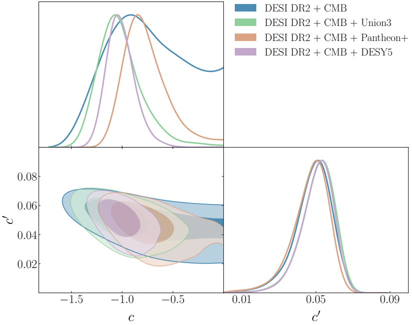
  - dark dimension scenario, with EFT as a linearization for small field displacement

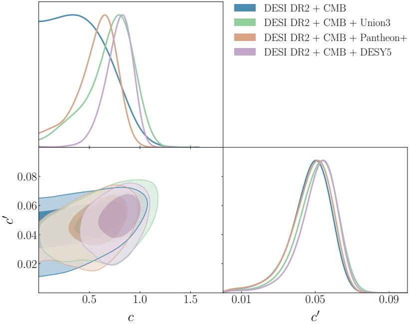
$$m_{\rm DM} = m_{\rm DM,0}\, e^{-c'\delta\varphi}$$
$$V = V_0\, e^{-c\,\delta\varphi}$$

see Montero, Vafa, Valenzuela [hep-th/2205.12293]
Gonzalo, Montero, Obied, Vafa [hep-th/2209.09249]

  - fits with DESI DR2 (DM-dominated era, c'), SN datasets (DE-dominated era, c), and CMB data

- evidence for $c' = |\beta| \simeq 0.05$ and $|c| = |\gamma| < \sqrt{2}$, with mild preference for $c < 0$ (i.e. $\gamma, \beta > 0$)





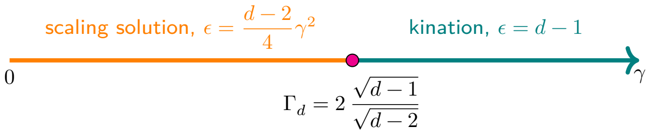
- DM/DE couplings can induce a long-lived SEC violation: is this a possibility to overcome the apparent limitations to long-lived SEC violations in the string field-space asymptotics?
  - the fundamental requirement is for the DM mass and DE potential to have opposite slopes
    see also "fading dark sector" models in Bedroya, Obied, Vafa, Wu [astro-ph.CO/2507.03090]
    see also Agrawal, Obied, Vafa [astro-ph.CO/1906.08261]
  - DM may be simply treated as a classical source, circumventing EFT-cutoff bounds
- the DM/DE-coupled solution is linearly stable, and hence attainable without fine-tuned initial conditions and (in a mathematical sense) arbitrarily long-lived
- (very) long-lived epochs of cosmic acceleration face obstructions due to BH arguments
  - novel data points where cosmic horizons are obstructed

Thank you!

Extra material

3.1. Infinitely-long epochs of cosmic acceleration and cosmic horizons

linearly-stable solutions for $k = 0$:



- strong dS and asymptotic TCC conjectures: $\frac{1}{V}\left|\frac{\partial V}{\partial \phi}\right| = \gamma \geq \frac{2}{\sqrt{d-2}}$, whence $\epsilon \geq 1$

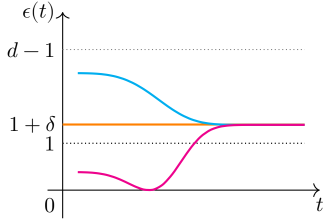
Bedroya, Vafa [hep-th/1909.11063]
Rudelius [hep-th/2101.11617]

see also Obied, Ooguri, Spodyneiko, Vafa [hep-th/1806.05506]
Ooguri, Palti, Shiu, Vafa [hep-th/1810.05506]

note: in principle, semi-eternal acceleration, and yet no cosmic horizons, for $\epsilon = 1$

- one may approach $\epsilon = 1$ from below, with saturating value $\gamma = \frac{2}{\sqrt{d-2}}$
- however, no example known in fully-fledged compactifications

see also a general argument in
Bedroya, Lee, Steinhardt [hep-th/2504.13260]



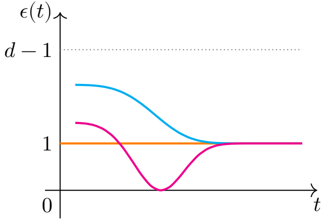
- interior TCC: $\gamma \geq \frac{2}{\sqrt{d-1}\sqrt{d-2}}$, possible cosmic acceleration

Single-field exponentials: hyperbolic spatial slices

linearly-stable solutions for $k = -\frac{1}{\ell^2}$:

- semi-eternal cosmic acceleration is possible!
- yet, no cosmic horizon

see also Friedrich, Hebecker, Schiller [hep-th/2505.07934]



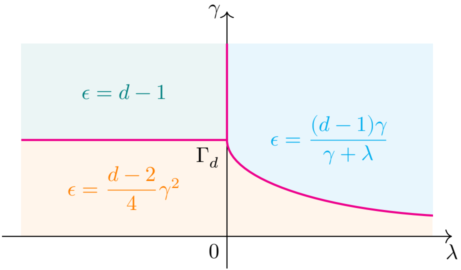
Marconnet, Tsimpis [hep-th/2210.10813]
Andriot, Tsimpis, Wrase [hep-th/2309.03938]

phenomenological challenges discussed e.g. in Andriot, Parameswaran, Tsimpis, Wrase, Zavala [hep-th/2405.09323]
Alestas, Delgado, Ruiz, Akrami, Montero, Nesseris [hep-th/2406.09212]
Akrami, Alestas, Nesseris [astro-ph.CO/2504.04226]

scalar with exponential potential and kinetically-coupled axion:

$$T - V = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\,\mathrm{e}^{-\lambda\phi}\,\dot{\zeta}^2 - \Lambda\,\mathrm{e}^{-\gamma\phi}$$

- ϵ-parameter for the linearly-stable attractor:



Sonner, Townsend [hep-th/0608068]
Russo, Townsend [hep-th/2203.09398]

phenomenological challenges discussed in Cicoli, Dibitetto, Pedro [hep-th/2002.02695]
Cicoli, Dibitetto, Pedro [hep-th/2007.11011]

see also Revello [hep-th/2311.12429]
Grimm, van de Heisteeg, Revello [hep-th/2510.12879]

- generalizations to higher-dimensional field spaces also possible

Shiu, FT, Tran #4 [hep-th/2406.17030]

3.2. Multi-field models

Multi-field interactions to the rescue, with scaling solutions?

- could a more general multi-field multi-exponential potential

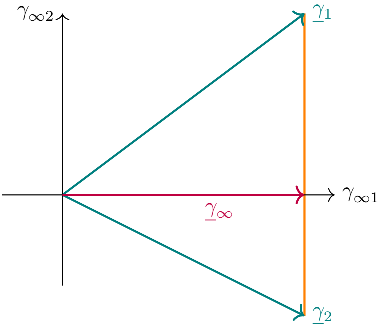
$$V = \sum_{i=1}^{m} \Lambda_i \, \mathrm{e}^{-\underline{\gamma}_i \cdot \underline{\phi}}$$

provide a way out for long-lived phases with $\epsilon < 1$?

- in string compactifications, probably not!

A simple way to handle multi-exponential potentials

- take e.g. $V = \sum_{i=1}^{2} \Lambda_i \, \mathrm{e}^{-\underline{\gamma_i} \cdot \underline{\phi}}$, and draw vectors $\underline{\gamma_i}$
- there is always an optimal basis such that $V = \left[\Lambda_1 \, \mathrm{e}^{|\gamma_{12}|\phi^2} + \Lambda_2 \, \mathrm{e}^{-|\gamma_{22}|\phi^2}\right] \mathrm{e}^{-\gamma_\infty \phi^1}$



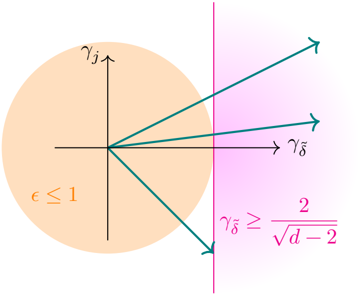
Shiu, FT, Tran #2 [hep-th/2306.07327]
see also Collinucci, Nielsen, Van Riet [hep-th/0407047]
Hartong, Ploegh, Van Riet, Westra [gr-qc/0602077]
Calderón-Infante, Ruiz, Valenzuela [hep-th/2209.11821]
Marconnet, Tsimpis [hep-th/2505.03449]

- ▶ asymptotically, all fields but ϕ^1 get stabilized
- ▶ the effective 1-field 1-term potential $V_\infty = \Lambda_\infty \, \mathrm{e}^{-\gamma_\infty \phi^1}$ gives $\epsilon = \frac{d-2}{4} (\gamma_\infty)^2$

- since the length of γ_∞ can be shorter than the individual γ_is, is our task easier now?
  - ▶ unfortunately, no: the d-dim. dilaton always gives too large of a γ_∞ in any case!

in Einstein frame, the canonical d-dim. dilaton has a universal coupling $\gamma_{\tilde{\delta}} = \frac{d}{\sqrt{d-2}} - \frac{1}{2}\chi_{\mathrm{E}}\sqrt{d-2}$

- because $\chi_{\mathrm{E}} \leq \chi_{\mathrm{E}}(\mathrm{S}^2) = 2$, we have the lower bound $\gamma_{\tilde{\delta}} \geq \frac{2}{\sqrt{d-2}}$
- a universal lower bound on ϵ is implied, i.e. $\epsilon = \frac{d-2}{4}(\gamma_\infty)^2 \geq \frac{d-2}{4}\gamma_{\tilde{\delta}}^2 \geq 1$



consistent with Bedroya, Vafa [hep-th/1909.11063]
Rudelius [hep-th/2101.11617]

possible ways out (besides field-space curvature):

- theory not at weak string coupling
- stabilized dilaton
- presence of negative-definite potential terms:
  bound takes a different form, less obvious but still restrictive

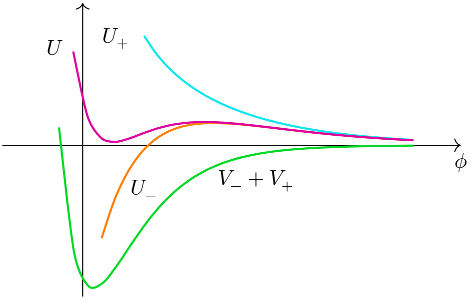
complete analysis in Shiu, FT, Tran #2 [hep-th/2306.07327]
see also Van Riet [hep-th/2308.15035]
Marconnet, Tsimpis [hep-th/2505.03449]

3.3. Field-space boundaries with negative potentials

- because of the string-coupling and volume expansions, negative potentials $V_- = -K\,\mathrm{e}^{-\gamma_- \phi}$ may push the EFT away from perturbative control
  - a coupling to a light tower of states can however allow for a dynamical evolution towards controlled regions:

$$U_-(\phi) = -K\,\mathrm{e}^{-\gamma_-\phi} + n_{\mathrm{DM},0}\left(\frac{a_0}{a}\right)^{d-1}\mu\,\mathrm{e}^{-\alpha\phi}$$

  - as a is not fixed, the positive term falls down much more quickly than with α, and even transient epochs can never have $\epsilon \leq 1$
  - to host $\epsilon \leq 1$, one may just include a positive potential $V_+ = \Lambda\,\mathrm{e}^{-\gamma_+\phi}$, with $\gamma_+ > \gamma_-$:



- note: without the DM/DE coupling, the potential would not be a runaway towards $\phi = +\infty$!

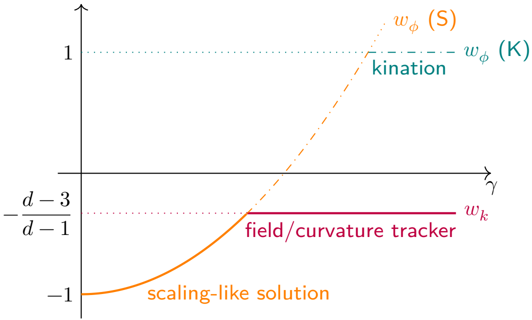
3.4. DM/DE coupling: complete classification of solutions

e.g. single-field exponential potentials

for a fluid with constant w-parameter, energy density is $\rho = \rho_0 \left(\frac{a_0}{a}\right)^{(d-1)(1+w)}$
and the lowest-w fluid eventually dominates

- scalars as a fluid with field-dependent $w_\phi = \frac{\dot\phi^2 - 2V}{\dot\phi^2 + 2V}$, but seemingly two preferred values:
  - $w_\phi = -1 + \frac{1}{2}\frac{d-2}{d-1}\gamma^2$, scaling-like solution
  - $w_\phi = 1$, kination

- spatial-curvature terms ($k = -1/\ell^2$) in the Friedmann equations
act like an on-shell fluid saturating the strong energy condition (SEC): $w_k = -\frac{d-3}{d-1}$

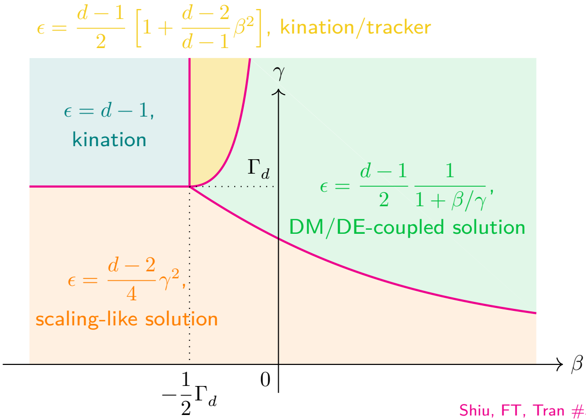


see also Shiu, FT, Tran #4 [hep-th/2406.17030]

- we consider exponential couplings:

$$m_{\rm DM} = \mu\, e^{\beta\phi}$$
$$V = \Lambda\, e^{-\gamma\phi}$$

- linearly-stable solutions are completely classified:



Shiu, FT, Tran #5 [hep-th/2506.19914]
see also Wetterich [hep-th/9408025]
Amendola [astro-ph/9904120]
Amendola [astro-ph/9908023]