Strings & Cosmology Meeting
December 12th, 2025

Lucas Pinol
CNRS researcher, LPENS, Paris

the primordial curvature perturbation

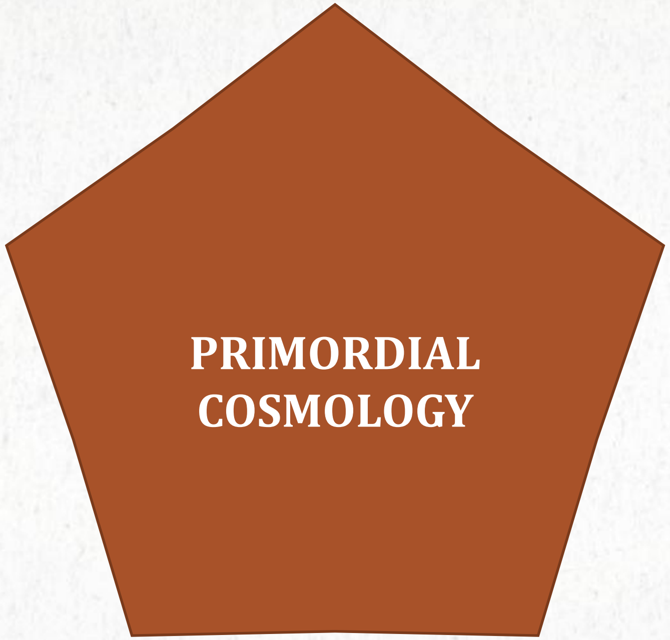
ONE-LOOP FREEZING OF ζ

AND RENORMALIZATION OF THE EFT OF INFLATIONARY FLUCTUATIONS

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]
[Braglia, Cespedes, LP 26xx.xxx]
...

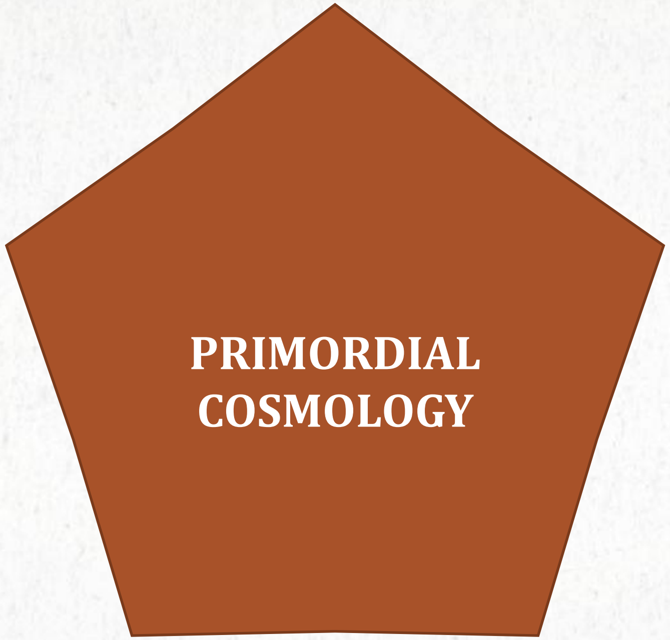
How primordial is early cosmology to fundamental physics?

Disclaimer: this is nothing bug a very biased list of interesting topics related to inflation

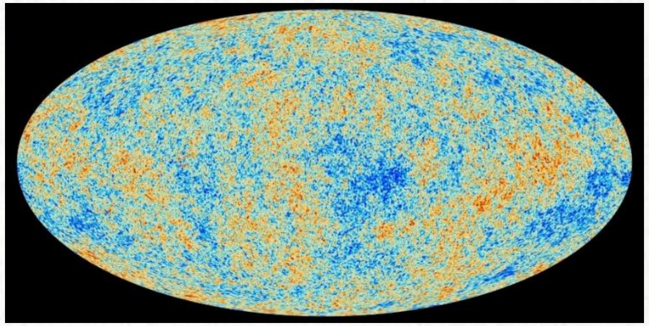


How primordial is early cosmology to fundamental physics?

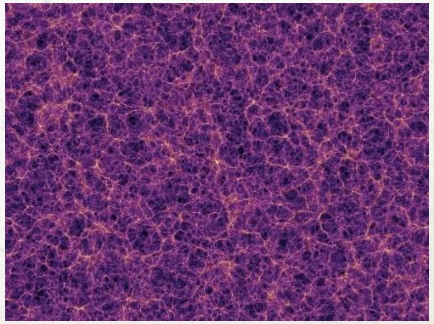
Gravity



Primordial fluctuations undergo gravitational collapse into structures



Cosmic Microwave Background



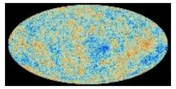
Large-scale distribution of galaxies

How primordial is early cosmology to fundamental physics?

Gravity

PRIMORDIAL COSMOLOGY

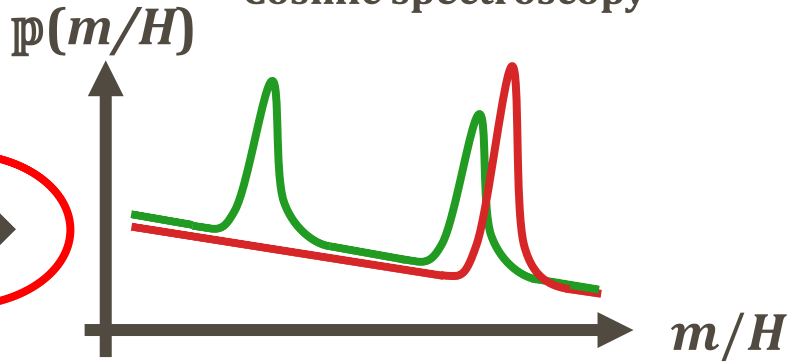
Particle physics

⟨  ^{3}⟩ + ⟨  ^{3}⟩ ⇒

ζ is the primordial curvature perturbation

$\langle \zeta^3 \rangle$ ⇒

Cosmic spectroscopy



cosmological observations

initial conditions

particle content in the early universe

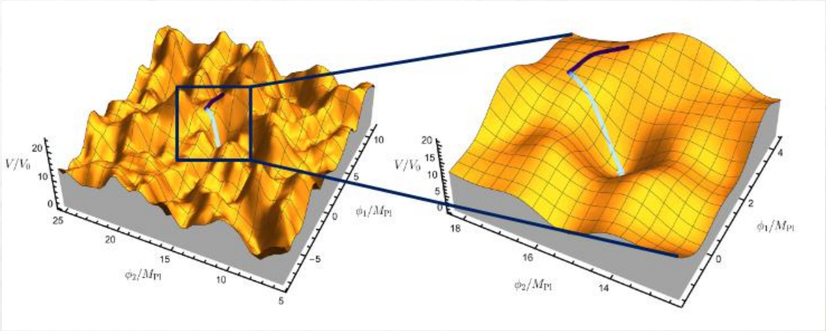
How primordial is early cosmology to fundamental physics?

Gravity

PRIMORDIAL COSMOLOGY

Particle physics

String theory



General Non-Linear Sigma Model: curved field space and potential

$$\mathcal{L} = -\frac{1}{2}\sum_{A,B} g^{\mu\nu}\boldsymbol{G_{AB}}(\vec{\boldsymbol{\phi}})\partial_\mu\phi^A\partial_\nu\phi^B - \boldsymbol{V}(\vec{\boldsymbol{\phi}})$$

Axions, dilatons, moduli, …

How primordial is early cosmology to fundamental physics?

Gravity

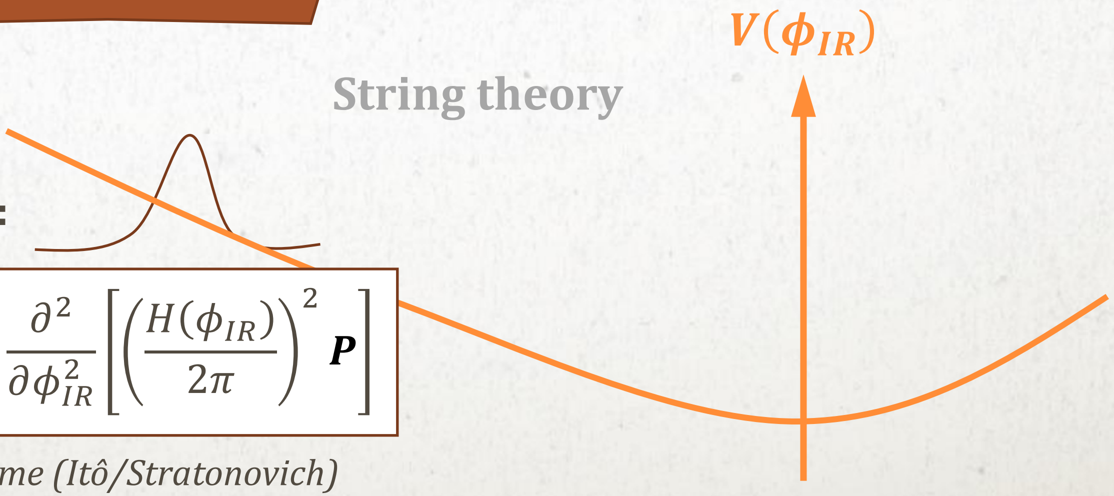
PRIMORDIAL COSMOLOGY

Particle physics

Statistical physics

String theory

Stochastic inflation (coarse-graining):



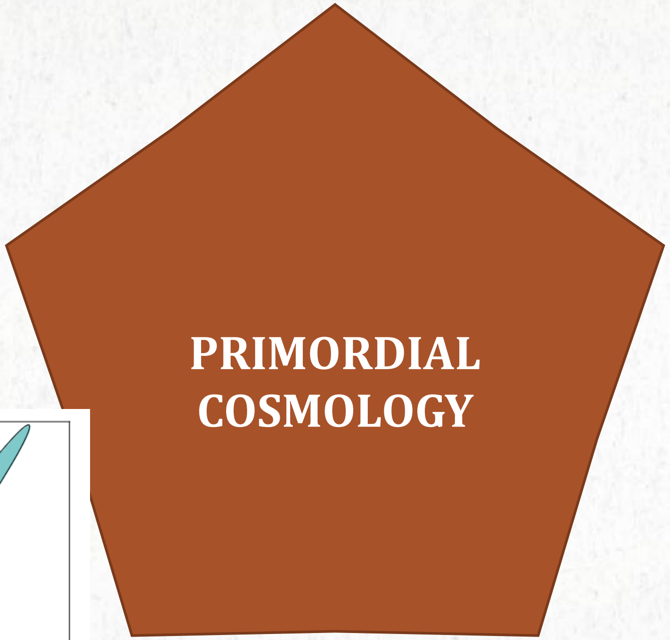
$$\frac{\partial \boldsymbol{P}}{\partial N} = \frac{\partial}{\partial \phi_{IR}}\left[\left(\frac{V_{,\phi}(\phi_{IR})}{3H(\phi_{IR})^2} - \boldsymbol{\alpha}\,\frac{H(\phi_{IR})}{2\pi}\frac{\partial}{\partial \phi_{IR}}\frac{H(\phi_{IR})}{2\pi}\right)\boldsymbol{P}\right] + \frac{1}{2}\frac{\partial^2}{\partial \phi_{IR}^2}\left[\left(\frac{H(\phi_{IR})}{2\pi}\right)^2 \boldsymbol{P}\right]$$

$\boldsymbol{\alpha}$ represents the discretization scheme (Itô/Stratonovich)

How primordial is early cosmology to fundamental physics?

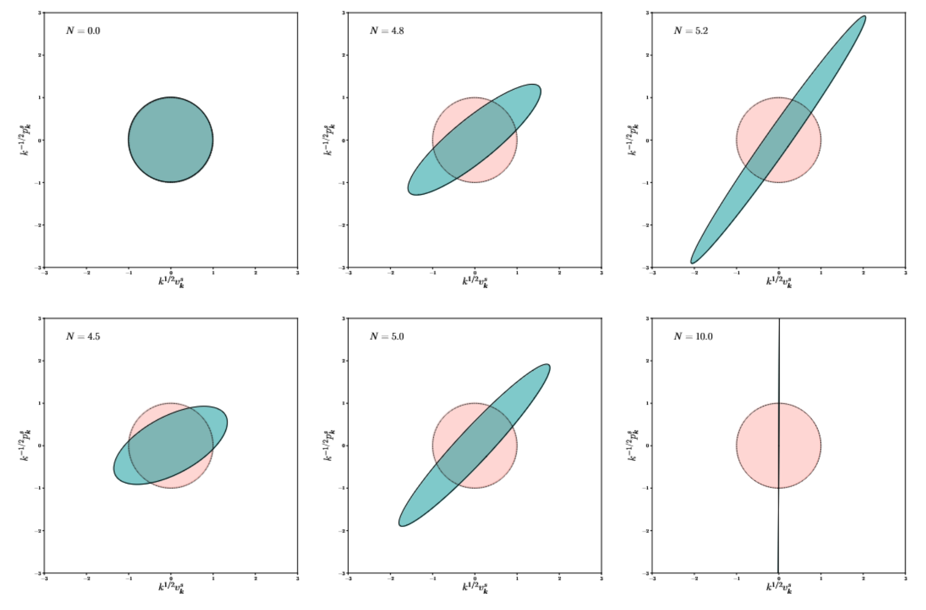
Gravity

Quantum physics



Particle physics

Inflationary two-mode squeezed states:



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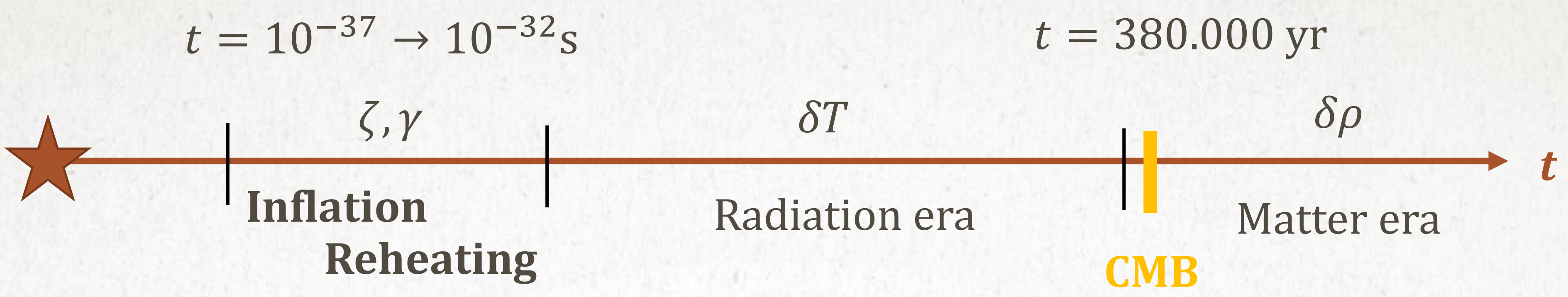
String theory

[Martin, Micheli, Vennin 2022]

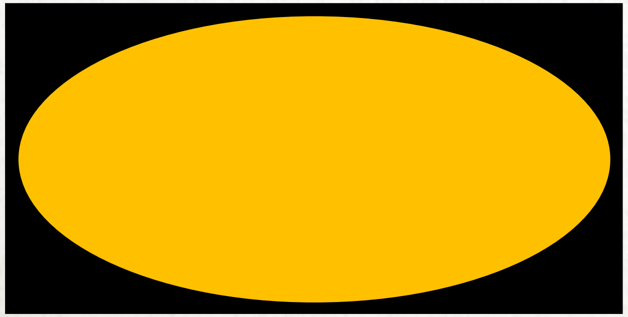
Early universe: the cosmological context



Scalar fluctuations ~ density fluctuations → $\zeta(t, \vec{x})$

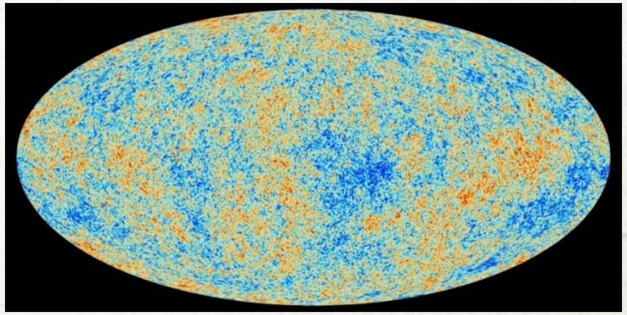
Tensor fluctuations ~ gravitational waves → $\gamma(t, \vec{x})$

Cosmic Microwave Background (CMB)



Penzias-Wilson (1964)

motivates → **Inflation** → predicts



Planck (2013)

Early universe: the high-energy viewpoint

Unique framework: general relativity + quantum field theory + precision data

CMB → $\sqrt{\langle\zeta^2\rangle} = (4.57 \pm \mathbf{0.02}) \times 10^{-5}$

$\log(E/\mathrm{GeV})$ Natural units: $\hbar = c = 1$ and the only dimension is energy

19 — M_{Pl} Planck mass
M_{s} String scale

16 — M_{GUT} Grand Unified Theories

14 —

??? — **H** **Inflation**

Scale of inflation is unknown
Particle content is unknown

3 — Standard Model, LHC

} Accessible experiments on Earth

Inflation sensitive to high energies

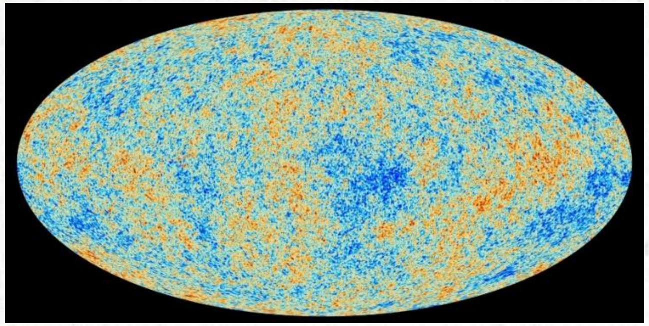
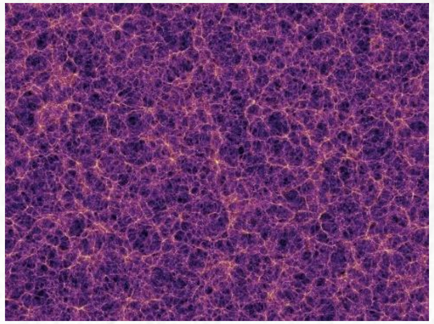
+

Precision data (current and future)

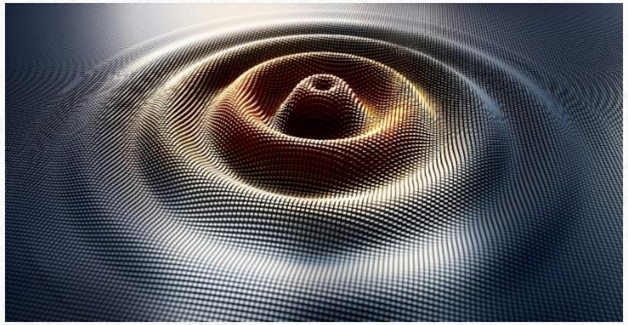
=

Formidable opportunity to test high-energy physics beyond the reach of terrestrial experiments

Early universe: the observational probes

Current $=$  $+$  **(scalars)**

Cosmic Microwave Background **Large-Scale Structures**

Future $=$  **(tensors) ?**

Primordial Gravitational Waves

Objects of study $=$ **primordial correlations functions**

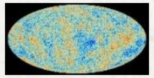
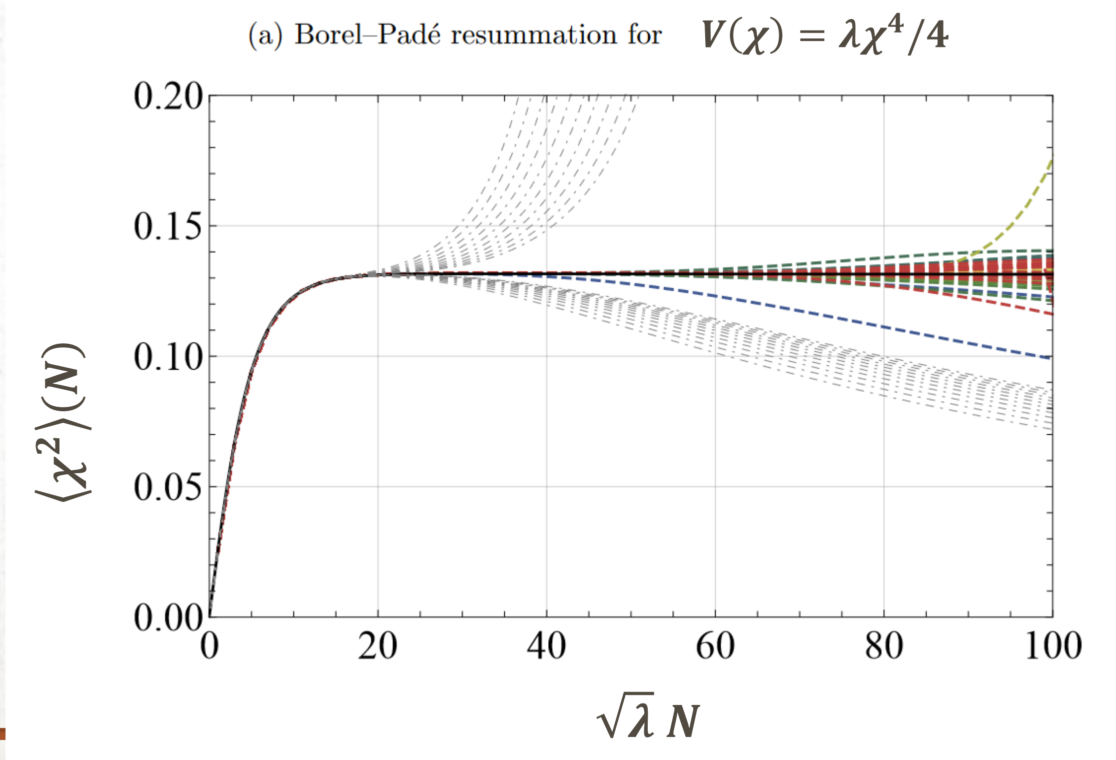
e.g. $\langle \zeta^2 \rangle \rightarrow \Big\langle$  $^2 \Big\rangle$; $\langle \gamma^2 \rangle \rightarrow \Big\langle$  $^2 \Big\rangle$; $\langle \zeta^3 \rangle \rightarrow \Big\langle$  $^3 \Big\rangle$; etc.

TABLE OF CONTENTS

I. FROM TEST FIELDS TO ζ

Secular divergences
Stochastic inflation
Robustness of inflation



YET ANOTHER WEINBERG THEOREM

[Weinberg 2005]

- In-in formalism at any order in the interactions:

$$\langle Q(t) \rangle = \sum_{N=0}^{\infty} i^N \int_{-\infty}^{t} dt_N \int_{-\infty}^{t_N} dt_{N-1} \cdots \int_{-\infty}^{t_2} dt_1 \times \left\langle \left[H_I(t_1), \left[H_I(t_2), \cdots \left[H_I(t_N), Q^I(t) \right] \cdots \right] \right] \right\rangle , \qquad (2)$$

[Weinberg 2006]

- Given the mode functions of the linear perturbation theory, and their commutators, he proves:

Weinberg theorem

The late-time, $t \to +\infty$, behavior of any correlation function, at any order in the interaction Hamiltonian, is proportional to the scale factor $a(t)$ at a power of <u>at most 0</u>.

SECULAR DIVERGENCES

What is left? $\rightarrow$ Logarithmic time divergences $\lim_{t\to+\infty} \langle Q(t) \rangle \ni \log a(t), \log a(t)^2, \ldots$

Are secular divergences common?

SECULAR DIVERGENCES

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Are secular divergences common?

- Massive mode functions decay strictly faster than $a(t)^0$ → **no secular divergence**
- Derivative interactions introduce additional negative factors of $a(t)$ → **no secular divergence**

→ **Mostly relevant to massless fields with non-derivative (e.g. test field) or mixed interactions (e.g. gravity)**

Are they physical?

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Are they physical?

It will be interesting to see if the power series in $\log a(t)$ encountered in calculating cosmological correlation functions at time t [...], can be summed [...]. [Weinberg 2006]

STOCHASTIC INFLATION FOR TEST FIELDS

An example where the secular growth is physical and indeed can be summed

- Consider a test massless interacting scalar field minimally coupled to gravity: $\mathcal{L} = \frac{M_{\mathrm{Pl}}^2}{2} R - \frac{1}{2}(\partial\chi)^2 - \frac{\lambda}{4}\chi^4$
- Suppose background spacetime is de Sitter: $\mathrm{d}s^2 = -\mathrm{d}t^2 + e^{2Ht}\mathrm{d}\vec{x}^2$

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- Two-point function at one loop reads: $\langle\chi^2\rangle(t) = \left(\frac{H}{2\pi}\right)^2 \log a(t)\left[1 - \frac{\lambda}{6\pi^2}\log a(t)^2 + \cdots\right]$

[Tsamis, Woodard 2005]

- Stochastic inflation is an open EFT for the super-horizon modes, predicting:

$$\langle\chi^2\rangle(t) = \left(\frac{H}{2\pi}\right)^2 \log a(t)\left[1 - \frac{\lambda}{6\pi^2}\log a(t)^2 + \frac{\lambda^2}{20\pi^4}\log a(t)^4 + \cdots\right] \underset{t\to\infty}{\to} \sqrt{\frac{3}{2\pi^2}}\frac{\Gamma(3/4)}{\Gamma(1/4)}\frac{H^2}{\sqrt{\lambda}}$$

[Starobinsky, Yokohama 1994]

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[Starobinsky, Yokohama 1994]

You can get any order, first closed formula in [Honda, Jinno, LP, Tokeshi 2023]

But... The series is divergent! $a_k \sim k!$

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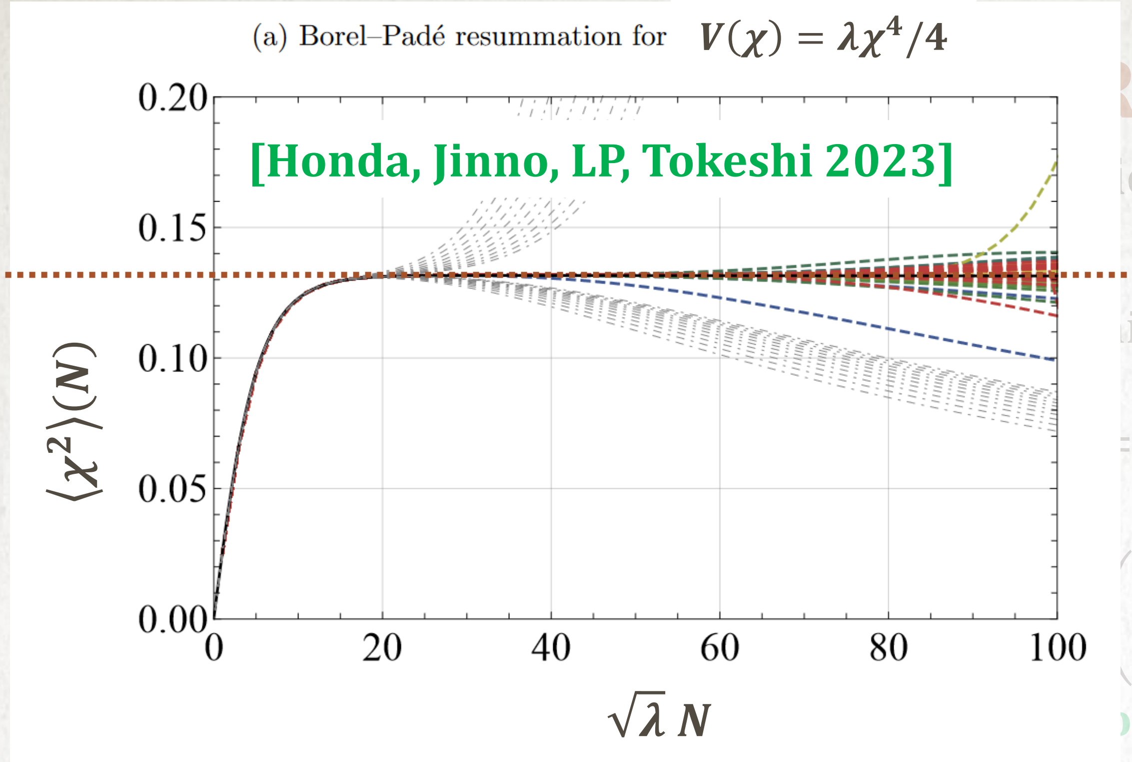
The divergent series can be summed via - the equilibrium PDF [Starobinsky, Yokohama 1994]

[Honda, Jinno, LP, Tokeshi 2023]



...R TEST FIELDS

...ical and indeed can be summed

...imally coupled to gravity: $\mathcal{L} = \frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial\chi)^2 - \frac{\lambda}{4}\chi^4$

$= -dt^2 + e^{2Ht}d\vec{x}^2$

$\left(\frac{H}{2\pi}\right)^2 \log a(t)\left[1 - \frac{\lambda}{6\pi^2}\log a(t)^2 + \cdots\right]$

...oodard 2005]

0.13176 ...

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The divergent series can be summed via - the equilibrium PDF **[Starobinsky, Yokohama 1994]**
- Borel summation **[Honda, Jinno, LP, Tokeshi 2023]**

transition regime included!

ROBUSTNESS OF INFLATION

Does the curvature perturbation develop secular divergences?

- Inflationary spacetime is classically and linearly stable
- First non-linear classical correction → primordial non-Gaussianities

[Maldacena 2003]

$\boldsymbol{B^0_\zeta(k_1, k_2, k_3)}$ *No secular divergences*

$\boldsymbol{t \to \infty}$

$$\left\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \right\rangle (t) = (2\pi)^3 \delta^{(3)}\left(\vec{k}_1 + \vec{k}_2 + \vec{k}_3\right) B_\zeta(k_1, k_2, k_3; t)$$

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- First non-linear quantum correction: $\left\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \right\rangle (t) = (2\pi)^3 \delta^{(3)}\left(\vec{k}_1 + \vec{k}_2\right) \left[P_\zeta^{\text{tree}}(k_1; t) + P_\zeta^{1-\text{loop}}(k_1; t) + \cdots \right]$

P_ζ^{tree}: $t \to \infty$ → $\boldsymbol{P_\zeta^0(k_1)}$ — *No secular divergences*

$P_\zeta^{1-\text{loop}}$ and higher: $t \to \infty$ → ??? — *Are there secular divergences?*

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- Imagine $P_\zeta \;(k_1; t) \ni \log a(t)^N$ →
  - ❖ Inflationary spacetime secularly unstable
    could actually provide a natural end
  - ❖ Breakdown of perturbation theory?
    the framework to make calculations becomes wrong
  - ❖ Loss of predictivity through reheating and Big Bang cosmology
    absence of adiabatic limit implies model dependence

THE CURVATURE PERTURBATION AT ONE LOOP

Conceptual and technical difficulties

- Gravity is (perturbatively) dynamical →
  - ❖ Gauge ambiguity
    gauge-invariant predictions must be obtained
  - ❖ GR constraints must be solved
    metric components are non-dynamical but non-zero
  - ❖ Gravitational interactions are mixed
    (anti-)derivative / non-derivative, e.g. $\zeta\dot{\zeta}^2$, $\zeta\partial_i\zeta\partial_i\partial^{-2}\zeta$, etc.

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- Backreaction on the background must be (perturbatively) accounted for
- Three kinds of divergences:
  - ❖ Secular divergences: $\log a(t)$
  - ❖ UV divergences: $\Lambda_{\mathrm{UV}}^2, \Lambda_{\mathrm{UV}}, \log \Lambda_{\mathrm{UV}}$
  - ❖ IR divergences: $\log \Lambda_{\mathrm{IR}}$

$$\int \mathrm{d}t_1 \int \mathrm{d}t_2 \, [H(t_2), [H(t_1), Q(t)]]$$

$$\int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \, \boldsymbol{f(k)} \quad \xrightarrow{\boldsymbol{k \to \infty}} \quad \frac{\mathbf{1}}{\boldsymbol{k}} + \frac{\mathbf{1}}{\boldsymbol{k^2}} + \frac{\mathbf{1}}{\boldsymbol{k^3}} + \cdots$$

$$\int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \, \boldsymbol{f(k)} \quad \xrightarrow{\boldsymbol{k \to 0}} \quad \frac{\mathbf{1}}{\boldsymbol{k^3}} + \frac{\mathbf{1}}{\boldsymbol{k^2}} + \cdots$$

THE CURVATURE PERTURBATION AT ONE LOOP

Conceptual and technical difficulties

- Gravity is (perturbatively) dynamical → ❖ Gauge ambiguity
  gauge-invariant predictions must be obtained
- Backre
- Three l

❖ IR divergences: $\log \Lambda_{\mathrm{IR}}$

How to address all that at once?

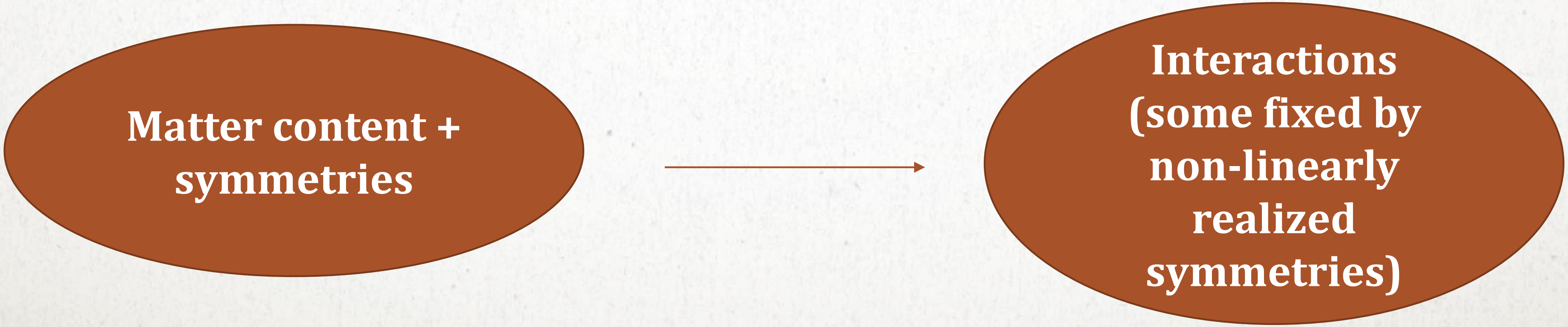
<u>The effective field theory of inflationary fluctuations is an exceptional framework</u>

II. THE EFT OF INFLATIONARY FLUCTUATIONS

Model-(in)dependence

Free theory

Leading cubic and quartic gravitational interactions



EFT FOR THE ADIABATIC FLUCTUATION

The unitary gauge

- Inflation ↔ time-translation symmetry breaking: $H_0 \to H(t)$

We can expect a (pseudo) Nambu-Goldstone boson

extrinsic spatial curvature

We build an EFT from Lorentz-violating blocks: $g^{\mu\nu}, g^{0\mu}, g^{00}, \boldsymbol{K_{ij}}, \boldsymbol{R_{ij}^{(3)}}, \nabla_\mu, f(t)$

intrinsic spatial curvature

This is called the **unitary gauge**: the NG boson is hidden in the metric

EFT FOR THE ADIABATIC FLUCTUATION

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- Most general unitary gauge Lagrangian compatible with an FLRW background:
  [Creminelli et al. 2006] [Cheung et al. 2008]

$$S = \int \mathrm{d}^4x \sqrt{-g}\,\mathcal{L}\,, \quad \text{with}$$

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\left(\dot{H} + 3H^2\right)\right] + M_{\rm Pl}^2 \dot{H} g^{00} + F^{(2)}\left(\delta g^{00}, \delta K_{ij}, \delta^{(3)}R_{ij}; \nabla_\mu; t\right)$$

EFT FOR THE ADIABATIC FLUCTUATION

The derivative expansion

- Higher number of derivatives brings higher powers of H/Λ in the observables
- Given a sought precision, one can truncate the EFT to some derivative order
- Non-derivative order: $F^{(2)} \supset \sum_{n=2}^{\infty} M_n^4 \frac{(\delta g^{00})^n}{n!}$
- Higher orders: $F^{(2)} \supset -\bar{M}_1^3 \delta g^{00} \delta K - \bar{M}_2^2 \delta K_{ij} \delta K^{ij} - \bar{M}_3^2 \delta K^2 + \bar{m}_1^2 (\nabla^0 \delta g^{00})^2 + \bar{m}_1^2 \delta g^{00} \delta R^{(3)} + \cdots$

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- Higher orders: $F^{(2)} \supset -\bar{M}_1^3 \delta g^{00} \delta K - \bar{M}_2^2 \delta K_{ij} \delta K^{ij} - \bar{M}_3^2 \delta K^2 + \bar{m}_1^2 (\nabla^0 \delta g^{00})^2 + \bar{m}_1^2 \delta g^{00} \delta R^{(3)} + \cdots$

<u>Example</u>: Suppose there is a unique scale Λ at which new physics enter (background excitation, new particle, etc.)

$$\delta K \sim H \times \delta g, \delta R^{(3)} \sim H^2 \times \delta g \quad \rightarrow \quad M_2^4 (\delta g^{00})^2 \sim \Lambda^4 \times \delta g^2$$

$$\bar{M}_1^3 \delta g^{00}\, \delta K \sim \Lambda^3 H \times \delta g^2$$

$$\bar{M}_3^2 \delta K^2 \sim \Lambda^2 H^2 \times \delta g^2$$

smaller

EFT FOR THE ADIABATIC FLUCTUATION

The derivative expansion

- Higher number of derivatives brings higher powers of H/Λ in the observables
- Given a sought precision, one can truncate the EFT to some derivative order
- Non-derivative order: $\boldsymbol{F^{(2)} \supset \sum_{n=2}^{\infty \to 2} M_n^4 \frac{(\delta g^{00})^n}{n!}}$
- Higher orders: ~~$F^{(2)} \supset -\bar{M}_1^3 \delta g^{00} \delta K - \bar{M}_2^2 \delta K_{ij} \delta K^{ij} - \bar{M}_3^2 \delta K^2 + \bar{m}_1^2 (\nabla^0 \delta g^{00})^2 + \bar{m}_1^2 \delta g^{00} \delta R^{(3)} + \cdots$~~

- Example: **Suppose there is a unique scale $\boldsymbol{\Lambda}$** at which new physics enter (background excitation, new particle, etc.)

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$\bar{M}_1^3 \delta g^{00}\, \delta K \sim \Lambda^3 H \times \delta g^2$

$\bar{M}_3^2 \delta K^2 \sim \Lambda^2 H^2 \times \delta g^2$

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

In these works, (for now) we
- **truncate** (theory assumption) at leading non-derivative order
- **fine-tune** (model dependence) $M_3 = 0 = M_4$

EFT FOR THE ADIABATIC FLUCTUATION

The Stückelberg trick

$$M_2(t)^4 \frac{(\delta g^{00})^2}{2}$$

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\left(\dot{H} + 3H^2\right)\right] + M_{\rm Pl}^2 \dot{H} g^{00} + \overbrace{F^{(2)}\left(\delta g^{00}, \delta K_{ij}, \delta^{(3)} R_{ij}; \nabla_\mu; t\right)}$$

Stückelberg trick = introduce explicitly the Goldstone boson π to restore diffeomorphism invariance

- Define $\pi(t, \vec{x})$ such that $t + \pi(t, \vec{x})$ is invariant under full diffeomorphisms

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EFT FOR THE ADIABATIC FLUCTUATION

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- Still a gauge freedom → flat gauge → $\zeta = -H\pi + (\dot{H}\pi^2)/2 - (\ddot{H}\pi^3)/6 + \cdots$
- $t \to t + \pi(t,\vec{x}) \;\Rightarrow\; \delta g^{00} \to \delta g^{00}_{\rm flat} + 2\delta g^{0i}_{\rm flat}\partial_i \pi - 2\dot{\pi} - \dot{\pi}^2 + (\partial\pi)^2/a^2$

  lapse δN *shift δN^i*

  non-linearly realized symmetry (diffeo. inv.)
- <u>Example</u>: $H \to H\big(t + \pi(t,\vec{x})\big) = H + \dot{H}\,\pi(t,\vec{x}) + \ddot{H}\,\pi(t,\vec{x})^2/2 + \cdots$

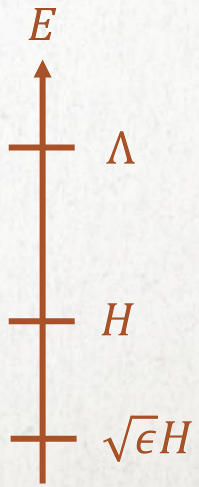
EFT FOR THE ADIABATIC FLUCTUATION

The decoupling limit

- Lapse and shift are non-dynamical (no $\delta \dot{g}^{00}_{\text{flat}}$, no $\delta \dot{g}^{0i}_{\text{flat}}$) but non-zero
- In principle, constraints must be solved → $\delta g^{00}_{\text{flat}} = 2\epsilon H \pi$
- I want to neglect those, e.g. in $(\delta g^{00})^2 \to \left(-2\dot{\pi} + \delta g^{00}_{\text{flat}}\right)^2$ + cubic
- $\dot{\pi} \sim \omega \times \pi \quad \to \quad \boldsymbol{\omega^2 \gg \epsilon H^2}$

At high enough energies, the NG boson alone describes the whole dynamics

the decoupling limit



- <u>Remarks</u>: using the decoupling limit around horizon crossing $\omega \sim H \quad \to \quad \epsilon \ll 1$
  no constraints on η and its time derivatives
- On super-horizon scales $\omega \ll H$ and the decoupling ceases to be valid

Not a problem if an adiabatic limit is reached and ζ has become constant

EFT FOR THE ADIABATIC FLUCTUATION

Summary

$$\epsilon = -\dot{H}/H^2$$
$$\eta = \dot{\epsilon}/(\epsilon H)$$
$$\eta_2 = \dot{\eta}/(\eta H)$$

- Starting point EFT: most general Lagrangian compatible with symmetries and no matter fields
- Assumption about the UV physics: a single high-energy scale Λ for new physics
- Fine-tuning (technical assumption, not required): $M_3 = 0 = M_4$
- Decoupling limit (technical assumption, not required): $\delta g^{00}_{\text{flat}} \ll \dot{\pi}, \ldots$

$$\mathcal{L}^{\pi,(2)}_{\text{decoup.}} = \frac{\epsilon H^2 M_{\text{Pl}}^2}{c_s^2}\left[\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2}\right]$$

[Braglia, LP 2504.07926]

$$\mathcal{L}^{\pi,(3)}_{\text{decoup.}} = -\frac{\epsilon H^3 M_{\text{Pl}}^2}{c_s^2}\left[\frac{f_0 c_s^2}{H}\dot{\pi}\frac{(\partial_i \pi)^2}{a^2} + \frac{f_1}{H}\dot{\pi}^3 + f_2 \dot{\pi}^2 \pi + f_3 c_s^2 \pi \frac{(\partial_i \pi)^2}{a^2}\right],$$

with $f_0 = \frac{1}{c_s^2} - 1\,, \quad f_1 = c_s^2 - 1\,, \quad f_2 = -\eta - s(1 - c_s^2)\,, \quad$ and $\quad f_3 = \eta$

Matches:

- $P(X, \phi)$ Lagrangians

[Burrage, Ribeiro, Seery, 2011]

- Multi-field Lagrangians after integrating out isocurvature fluctuations

[Garcia-Saenz, LP, Renaux-Petel, 2019]

[LP 2020]

EFT FOR THE ADIABATIC FLUCTUATION

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$$\mathcal{L}^{\pi,(4)}_{\text{decoup.}} = \frac{\epsilon H^4 M_{\text{Pl}}^2}{c_s^2}\left\{\frac{g_0 c_s^2}{H^2}\left[\frac{(\partial_i\pi)^2}{a^2}\right]^2 + \frac{g_1 c_s^2}{H^2}\dot{\pi}^2\frac{(\partial_i\pi)^2}{a^2} + \frac{g_2}{H^2}\dot{\pi}^4 + \frac{g_3}{H}\pi\dot{\pi}^3 + \frac{g_4 c_s^2}{H}\pi\dot{\pi}\frac{(\partial_i\pi)^2}{a^2} + g_5\pi^2\dot{\pi}^2 + g_6 c_s^2\pi^2\frac{(\partial_i\pi)^2}{a^2}\right\},$$

[Braglia, LP 2504.07926]

with
$$g_0 = \frac{1}{4}\left(\frac{1}{c_s^2}-1\right), \quad g_1 = -\frac{1}{2}\left(\frac{1}{c_s^2}-1\right), \quad g_2 = \frac{1-c_s^2}{4}, \quad g_3 = (1-c_s^2)(\eta+s),$$
$$g_4 = -\left(\frac{1}{c_s^2}-1\right)(\eta+s), \quad g_5 = \frac{\eta(\eta+\eta_2)}{2} + (1-c_s^2)s\,(\eta+s+s_2), \quad g_6 = -\frac{\eta(\eta+\eta_2)}{2}$$

EFT FOR THE ADIABATIC FLUCTUATION

Free theory and interactions → "Feynman-like" rules

$$\epsilon = -\dot{H}/H^2$$
$$\eta = \dot{\epsilon}/(\epsilon H)$$
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$$\mathcal{L}^{\pi,(2)}_{\text{decoup.}} = \frac{\epsilon H^2 M_{\text{Pl}}^2}{c_s^2}\left[\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2}\right] \text{ with } \frac{1}{c_s^2} - 1 = \frac{2M_2^4}{\epsilon H^2 M_{\text{Pl}}^2}$$

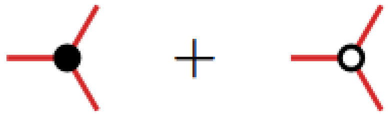
Speed of sound $0 < c_s^2 \leq 1$

$$\pi_k^I(\tau) = \frac{1}{\sqrt{4\epsilon c_s k^3} M_{\text{Pl}}} (1 + i c_s k \tau)\, e^{-i c_s k \tau}$$

Mode functions / propagators

$$a\mathcal{H}^{(3)}_{\text{int}} = -a^2 \epsilon \eta H^3 M_{\text{Pl}}^2 \left[\frac{1}{c_s^2}\pi \pi'^2 - \pi(\partial_i \pi)^2\right]$$

$$= -\mathcal{L}^{(3)}$$



Leading cubic gravitational interactions

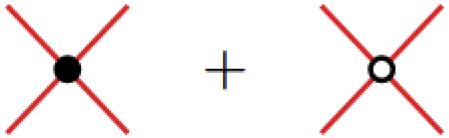
those surviving in the limit $c_s^2 \to 1,\ \epsilon \ll \eta$

$$a\mathcal{H}^{(4)}_{\text{int}} = \frac{a^2}{2}\epsilon \eta H^4 M_{\text{Pl}}^2 \left[\frac{\eta - \eta_2}{c_s^2}\pi^2 \pi'^2 + (\eta + \eta_2)\pi^2(\partial_i \pi)^2\right]$$

⚠ $\neq -\mathcal{L}^{(4)}$

Leading quartic gravitational interactions

those surviving in the limit $c_s^2 \to 1,\ \epsilon \ll \eta, \eta_2$



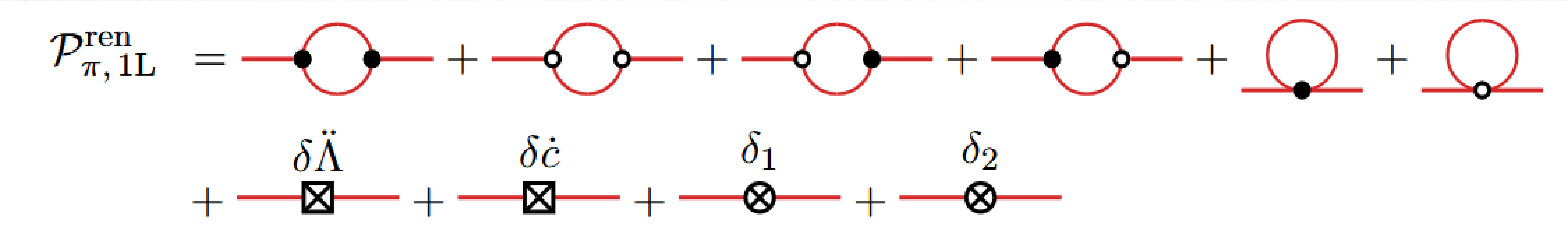
[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

III. LOOP CALCULATION

Bare contributions

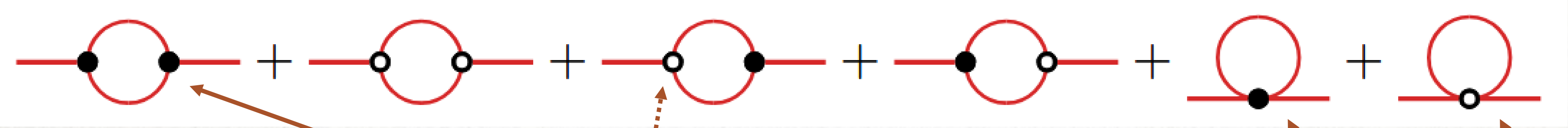
Renormalization: tadpoles and UV divergences

Discussion



BARE CONTRIBUTIONS

UV divergences and dimensional regularization



$$a\mathcal{H}_{\rm int}^{(3)} = -a^2\epsilon\eta H^3 M_{\rm Pl}^2\left[\frac{1}{c_s^2}\pi\pi'^2 - \pi(\partial_i\pi)^2\right]$$

$$a\mathcal{H}_{\rm int}^{(4)} = \frac{a^2}{2}\epsilon\eta H^4 M_{\rm Pl}^2\left[\frac{\eta-\eta_2}{c_s^2}\pi^2\pi'^2 + (\eta+\eta_2)\pi^2(\partial_i\pi)^2\right]$$

All these diagrams are IR-divergent + UV-divergent + secular-divergent

To manipulate finite quantities, we **regulate** them →

- Comoving IR cutoff in Fourier space $k > \Lambda_{\rm IR}$
- Dimensional regularization $d = 3 + \delta$ spatial dimensions
- Finite external super-horizon time $x = -p\,\tau$

BARE CONTRIBUTIONS

UV divergences and dimensional regularization

Modifies both the mode functions and the Fourier integrals

$$\pi_k(\tau) = \frac{\sqrt{\pi} e^{i\pi\delta/4} c_s^{-(1+\delta)/2}}{2\sqrt{2\epsilon}} \frac{1}{M_{\text{Pl}}} \left(\frac{H}{\mu}\right)^{\delta/2} \frac{(-c_s k\tau)^{(3+\delta)/2}}{k^{(3+\delta)/2}} H^{(1)}_{(3+\delta)/2}(-c_s k\tau)$$

$$\int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} f(\vec{k}) \mapsto \int \frac{\mathrm{d}^d k}{(2\pi)^d} f(\vec{k}) = \Omega_{d-3} \int_0^\infty \mathrm{d}k\, k^{d-1} \int_0^\pi \mathrm{d}\theta \sin^{d-2}\theta \int_0^\pi \mathrm{d}\varphi \sin^{d-3}\varphi\, f(k,\theta,\varphi)$$

All these diagrams are IR-divergent + UV-divergent + secular-divergent

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- Comoving IR cutoff in Fourier space $k > \Lambda_{\text{IR}}$
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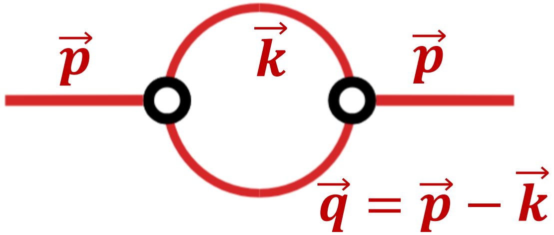
BARE CONTRIBUTIONS

Strategy and results



- Use dimensionless variables, expand the mode function to linear order in δ, perform time and angular integrals, **[Senatore, Zaldarriaga 2009]**

  introduce a fake UV cutoff to expand the integrand at infinity and extract the pole, introduce a comoving IR cutoff **[Ballesteros, Gambín Egea, Riccardi 2024]**

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

BARE CONTRIBUTIONS

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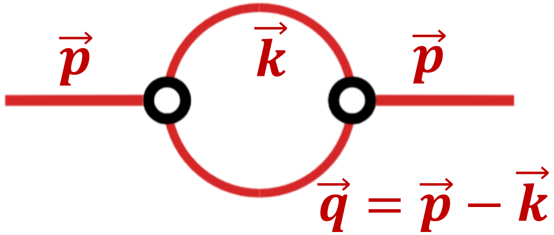

$$x \equiv -p\tau$$

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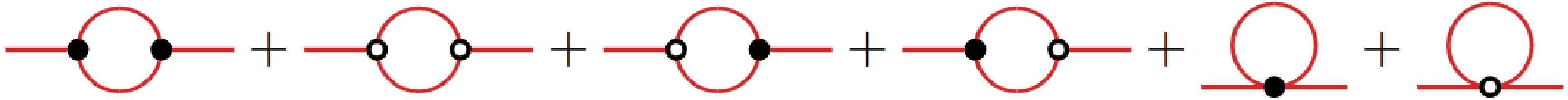
- $\mathcal{P}^{\text{bare}}_{\pi,1\text{L}}(x) =$ 

$$= \mathcal{P}^{\text{b, UV}}_{\pi,\,1\text{L}}(x) + \mathcal{P}^{\text{b, IR}}_{\pi,\,1\text{L}}(x) + \mathcal{P}^{\text{b, fin}}_{\pi,\,1\text{L}}(x)$$

[Braglia, LP 2504.07926]
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BARE CONTRIBUTIONS

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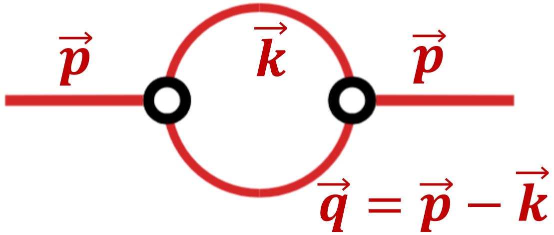

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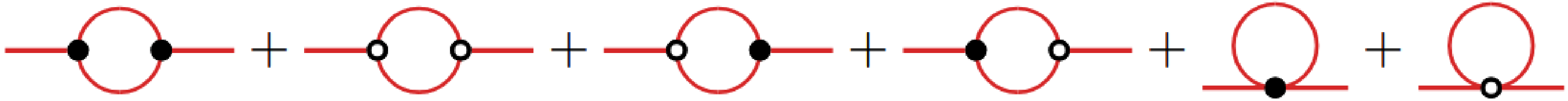

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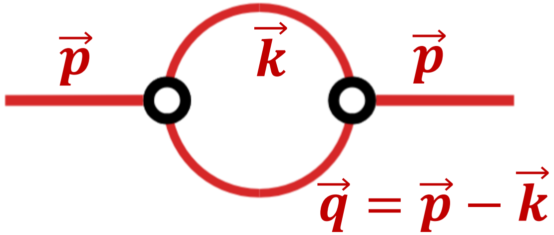
$$\mathcal{P}^{\text{b, UV}}_{\pi,\,1\text{L}}(x) = -\left(1 + c_s^2 x^2\right)\mathcal{P}^{\text{tree}^2}_{\pi,\,0} H^2 \frac{\eta(\eta - 2\eta_2)}{4}\left[\frac{1}{\delta} + 2\log\left(\frac{H}{\mu}\sqrt{\pi e^{-\gamma_{\text{E}}}}\right)\right]$$

$\left(1 + c_s^2 x^2\right)/\delta \rightarrow$ The UV divergence is time-dependent!

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

BARE CONTRIBUTIONS

Strategy and results



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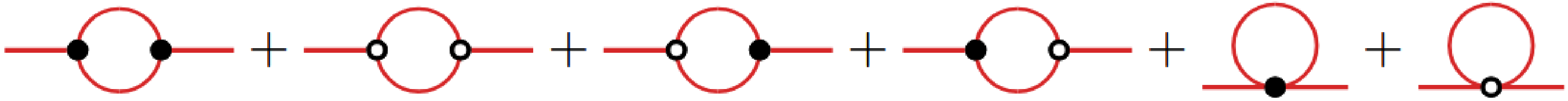

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UV divergence $(1 + c_s^2 x^2)/\delta$
Running log (H/μ)

$$\lim_{x\to 0} \mathcal{P}^{\text{b, fin}}_{\pi,\,1\text{L}}(x) \sim -\mathcal{P}^{\text{tree}^2}_{\pi,0} H^2 \frac{1}{2}\eta(2\eta - \eta_2)\log x.$$

Sum of individual secular divergences not vanishing!

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

RENORMALIZATION

Cancelling UV divergences

- We need quadratic counterterms to absorb the **time-dependent** UV divergences

At lowest order in derivatives we can renormalize the speed of sound $\left(\frac{1}{c_s^2}-1\right)\dot{\pi}^2=\left(\frac{1}{c_{s,\mathrm{ren}}^2}-1\right)\dot{\pi}^2+\delta_{c_s^2}\dot{\pi}^2$

The higher-order derivatives are degenerate, a minimal set is [Bordin, Cabass, Creminelli, Vernizzi 2017]

$$\mathcal{L}\supset-\frac{\bar{M}_3^2}{2}\delta K^2+\frac{m_3^2}{2}h^{ij}\left(\nabla_i\delta g^{00}\right)\left(\nabla_j\delta g^{00}\right)\rightarrow-\delta_1\frac{(\partial^2\pi)^2}{a^4}-\delta_2\frac{(\partial_i\dot{\pi})^2}{a^2}+\dots$$

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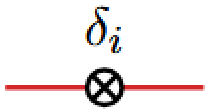
We can tune the δ_i so as to cancel UV divergences **at all times**:

- Counterterms contribute as: $\mathcal{P}_{\pi,\,1\mathrm{L}}^{\mathrm{c.t.},(\delta_i)}(x) = \sum_{i=c_s^2,\,1,\,2}$  $= \sum_i \delta_i\, f_i(x)$

We can tune the δ_i so as to cancel UV divergences **at all times**:

$$\delta_1 = c_s^2\delta_2, \qquad \delta_2 = -\frac{1}{\delta}\frac{H^2}{M_{\mathrm{Pl}}^2}\frac{\eta(\eta-2\eta_2)}{8\pi^2 c_s}$$

$$\delta_{c_s^2} = 0$$

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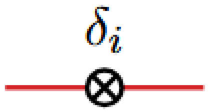
- Counterterms contribute as: $\mathcal{P}_{\pi,\,1\mathrm{L}}^{\mathrm{c.t.},(\delta_i)}(x) = \sum_{i=c_s^2,1,2}$ 

$= \sum_i \delta_i\, f_i(x)$

We can tune the δ_i so as to cancel UV divergences at all times: $\delta_1 = c_s^2 \delta_2, \qquad \delta_2 = -\frac{1}{\delta}\frac{H^2}{M_{\mathrm{Pl}}^2}\frac{\eta(\eta-2\eta_2)}{8\pi^2 c_s}$

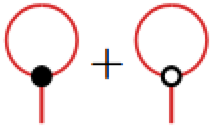
- **No additional secular divergence…** $\delta_{c_s^2} = 0$

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

RENORMALIZATION

Tadpole cancellation must be enforced

- We forgot a set of diagrams! **[Pimentel, Senatore, Zaldarriaga 2012]**

$\langle \pi_{\vec{p}}(\tau) \rangle'_{\mathrm{bare}} =$  $\neq 0$

- We introduce one-point counterterms to cancel them, in the unitary gauge so they respect the EFT symmetries

$$\mathcal{L} \supset -g^{00}\delta c(t) - M_{\mathrm{Pl}}^2 \delta\Lambda(t)$$

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

RENORMALIZATION

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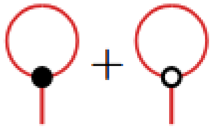
$\langle \pi_{\vec{p}}(\tau) \rangle'_{\text{bare}} =$  $\neq 0$

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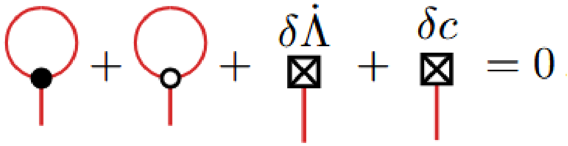
$$a\mathcal{H}^{(1)}_{\text{c.t.}} = a^4 M_{\text{Pl}}^2\, \delta\dot{\Lambda}\, \pi - 2a^4\, \delta c\, \dot{\pi}$$



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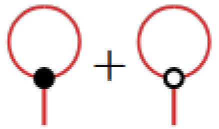
$\langle \pi_{\vec{p}}(\tau) \rangle'_{\text{bare}} =$  $\neq 0$

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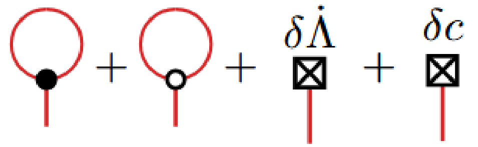

$= 0$

$$a\mathcal{H}^{(2)}_{\text{c.t.}} = \frac{a^4 M_{\text{Pl}}^2}{2}\, \delta\ddot{\Lambda}\, \pi^2 - 2a^4\, (\delta\dot{c} - \eta H\, \delta c)\, \pi\dot{\pi}$$

Non-linearly realized symmetries imply quadratic counterterms are necessarily induced !

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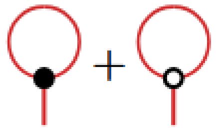
$\langle \pi_{\vec{p}}(\tau) \rangle'_{\text{bare}} =$  $\neq 0$

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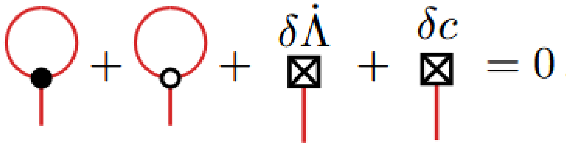

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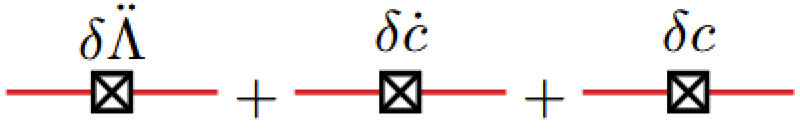
Non-linearly realized symmetries imply quadratic counterterms are necessarily induced !

- New contributions to the power spectrum $\mathcal{P}^{\text{c.t.},(\Lambda,c)}_{\pi,\,1\text{L}}(x) =$ 

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

$$\lim_{x\to 0} \mathcal{P}^{\text{c.t.},(\Lambda,c)}_{\pi,\,1\text{L}}(x) \sim \mathcal{P}^{\text{tree}^2}_{\pi,0} H^2 \frac{1}{2}\eta(2\eta - \eta_2)\log x.$$

REMEMBER

$$\lim_{x\to 0} \mathcal{P}^{\text{bare}}_{\pi,\,1\text{L}}(x) \sim -\mathcal{P}^{\text{tree}^2}_{\pi,\,0} H^2 \frac{1}{2}\eta(2\eta - \eta_2)\log x.$$

NOW

$$\lim_{x\to 0} \mathcal{P}^{\text{c.t.},(\Lambda,\,c)}_{\pi,\,1\text{L}}(x) \sim \mathcal{P}^{\text{tree}^2}_{\pi,\,0} H^2 \frac{1}{2}\eta(2\eta - \eta_2)\log x.$$

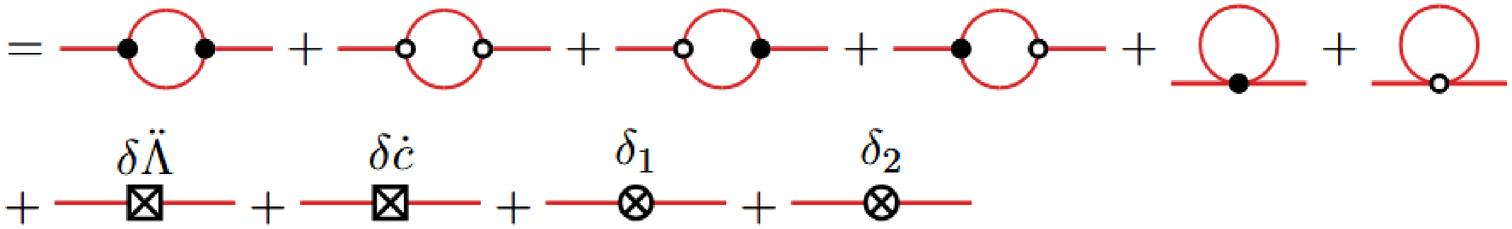
THEIR SUM IS LATE-TIME CONVERGENT!

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

RENORMALIZATION

Total result

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]



- Secular divergences cancel in the renormalized power spectrum
- The conservation of $\pi(t, \vec{x})$ at one loop is enforced by non-linearly realized symmetries

RENORMALIZATION

Total result

[Braglia, LP 2504.07926]
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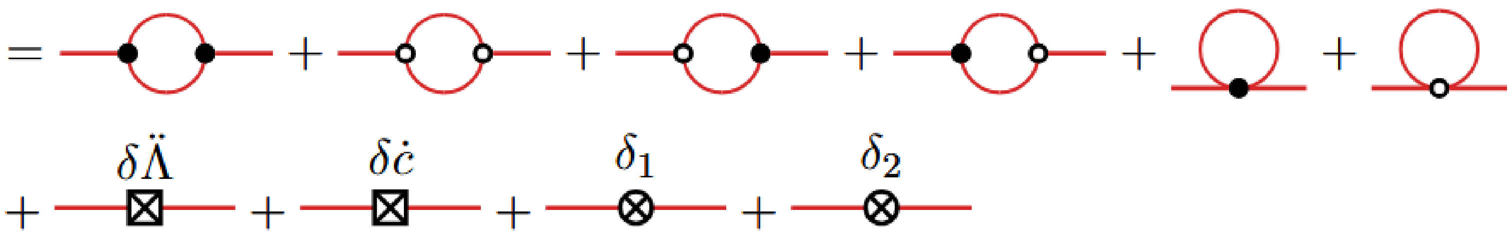

- Secular divergences cancel in the renormalized power spectrum

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- The result on super-horizon scales is: ($x = 0$)

$$\frac{\mathcal{P}^{\rm ren}_{\pi,1\rm L,0}}{\mathcal{P}^{\rm tree}_{\pi,0}} = \frac{1}{8\pi^2}\left(\frac{H}{\Lambda_\eta}\right)^2\left[\frac{269}{160} - \frac{29}{16}\log(2) - \frac{1}{4}\log\left(\frac{H}{\mu}\sqrt{\frac{\pi}{4c_s^2 e^{\gamma_E}}}t_{\rm IR}^2\right)\right] + \frac{1}{8\pi^2}\left(\frac{H}{\Lambda_{\eta_2}}\right)^2\left[\frac{11}{24} + \frac{1}{2}\log\left(\frac{H}{\mu}\sqrt{\frac{\pi}{4c_s^2 e^{\gamma_E}}}t_{\rm IR}\right)\right],$$

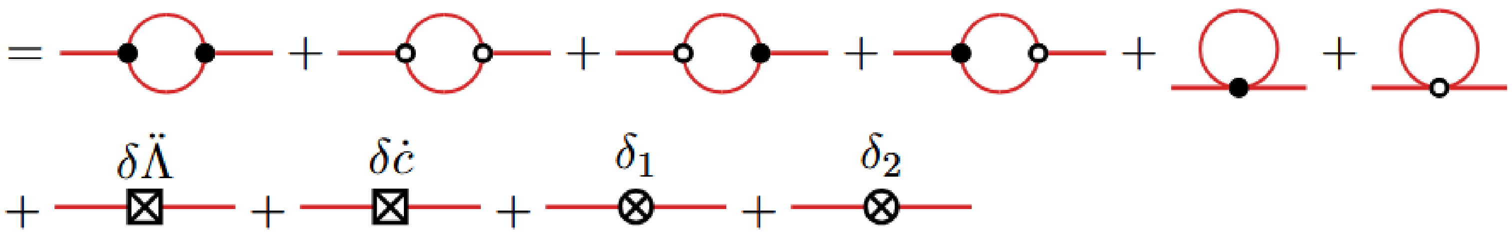
with the strong coupling scales $\Lambda_\eta^2 = M_{\rm Pl}^2\frac{\epsilon c_s}{\eta^2}$, and $\Lambda_{\eta_2}^2 = M_{\rm Pl}^2\frac{\epsilon c_s}{\eta\eta_2}$, also $t_{IR} \simeq p/\Lambda_{\rm IR}$

- We can safely match $\pi(t,\vec{x})$ to $\zeta(t,\vec{x})$ on super-horizon scales

RENORMALIZATION

Total result

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]



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…non-linearly realized symmetries

Successful perturbative renormalization of the EFT of inflation at order H^2/Λ^2

- …is:

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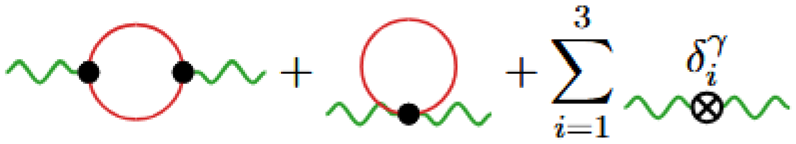
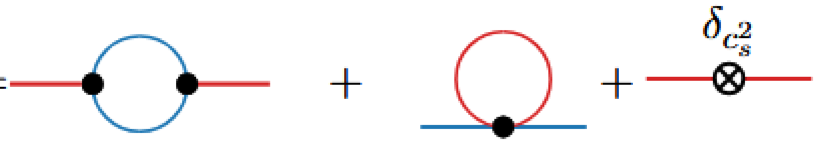
DISCUSSION

What’s done, what’s not done

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

- Scalar-induced tensor two-point function: $\mathcal{P}^{\mathrm{ren}}_{\gamma,\,1\mathrm{L}}(x) =$ 

- Massive (conformal) scalar field correction to the one-loop curvature perturbation:

$\mathcal{P}^{\mathrm{ren}}_{\pi,\,1\mathrm{L\ from\ }\mathcal{S}}(x) =$ 

- All cases are scale-invariant scenarios: running $\log(H/\mu)$ is not observable

→ Strongly scale-dependent cases with striking small-scale phenomenology remain to be thoroughly studied

- Relax technical assumption of the decoupling limit → anti-derivative interactions $\zeta\partial_i\zeta\partial_i\partial^{-2}\zeta$

- Explore different scenarios: heavy fields with any mass, different dispersion relations, excited initial states, etc.

DISCUSSION

What's done, what's not done

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

- Scalar-induced tensor two-point function:
- Massive (conformal) scalar field correction to the one-loop curvature perturbation:
- **All cases are scale-invariant scenarios: running $\log(H/\mu)$ is not observable**

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[Braglia, Cespedes, LP 26xx.xxx]

- Relax technical assumption of the decoupling limit → anti-derivative interactions $\zeta\partial_i\zeta\partial_i\partial^{-2}\zeta$
- Explore different scenarios: heavy fields with any mass, different dispersion relations, excited initial states, etc.

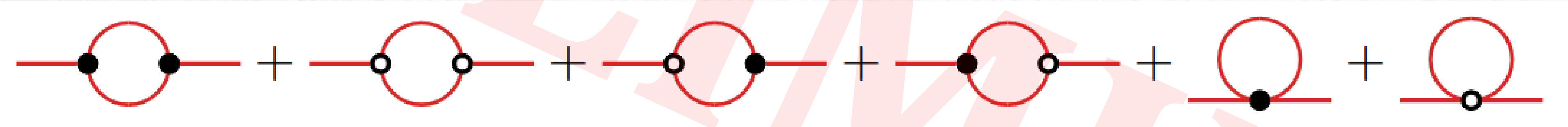
$$x \equiv -p\tau$$

BONUS

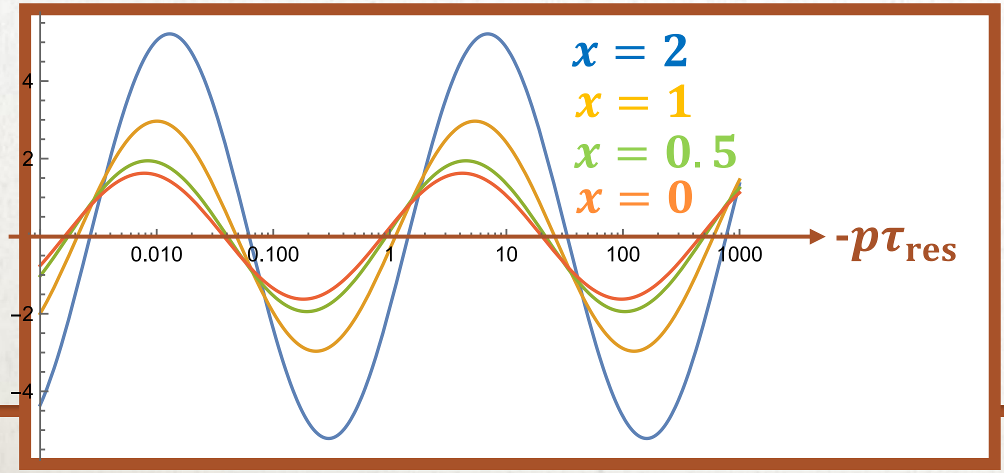
First scale-dependent renormalization of inflation

[Braglia, Cespedes, LP 26xx.xxx]

Repeat the analysis with interactions that violate slow-roll $\epsilon(\tau) \sim \sin(w \log(-\tau/\tau_{\text{res}}))$ → ***resonant features***



$$\mathcal{P}_{\pi,1\text{L}}^{\text{bare,UV}}(p, x) = f(p, x)\left(\frac{1}{\delta} + \log\left(\frac{H}{\mu}\right)\right)$$



- The UV divergences are scale-dependent (p) and time-dependent (x)
- No secular divergences
- With only 4 counterterms we can cancel them for any pair (p, x)
- The ***β*-function** is now scale-dependent
- Yet, is the loop effect "observable"?

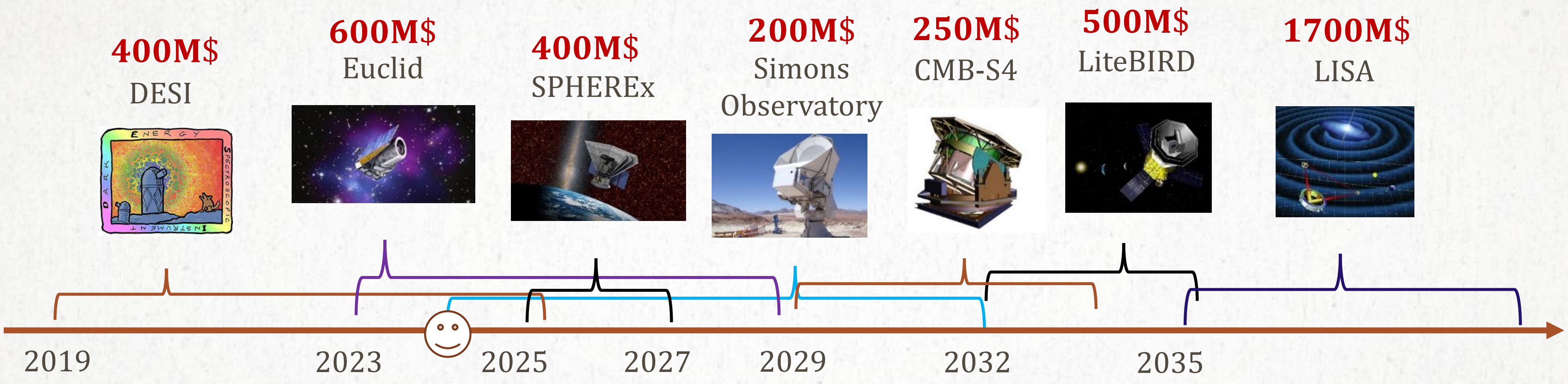
BACKUP SLIDES

A BRIGHT FUTURE

Approximate budgets

Total ≃ 4 billions $

Towards a standard model of inflation



Exciting era for primordial cosmology

But discoveries = data + interpretation

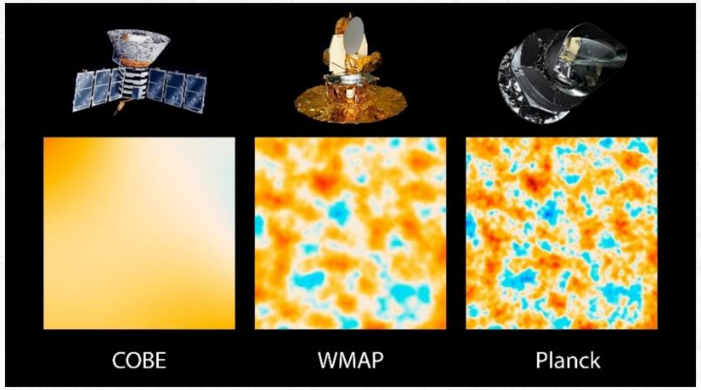
Cosmic Microwave Background

CMB

1964

Penzias-
Wilson

1992 2006 2018

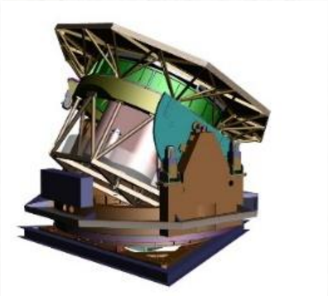


[credits: NASA]

Simons Observatory (2024-?)

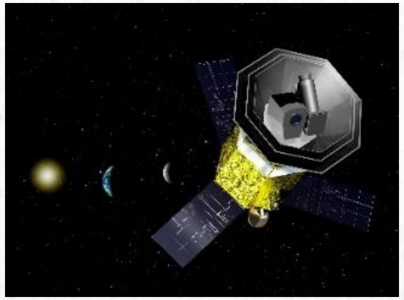
The coming one
(Chile)

CMB-S4 (2029-?)

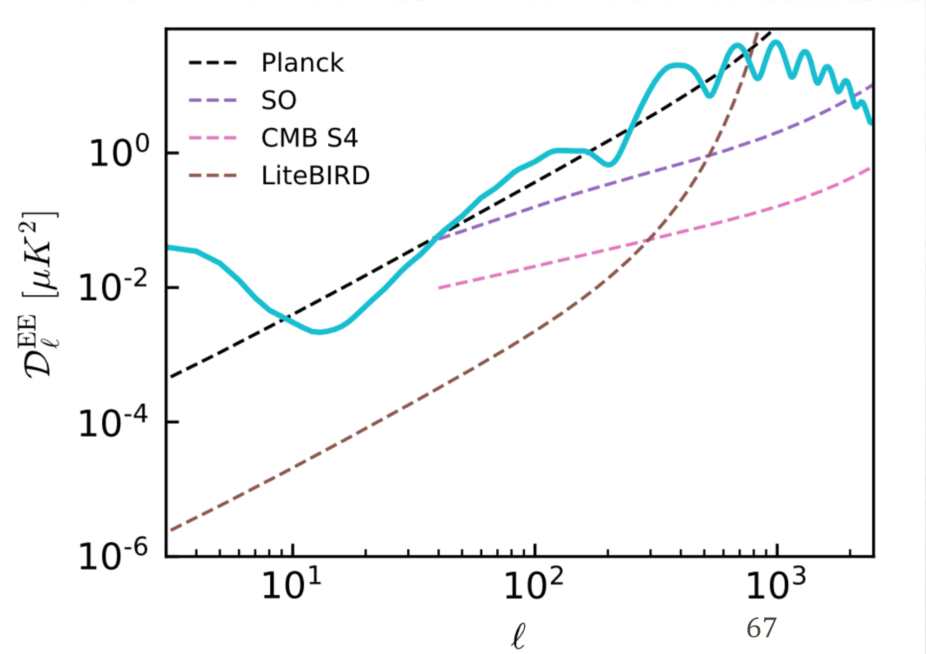


Multiple telescopes
Better foreground removal

LiteBIRD (2032-?)

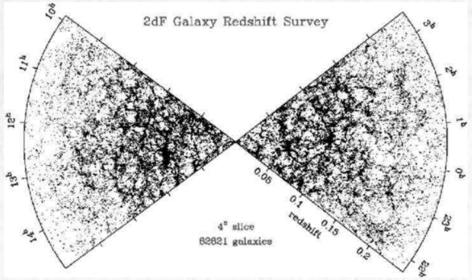


Satellite
Much better for low-ℓ

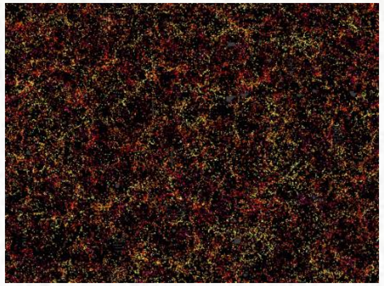


Large-Scale Structures

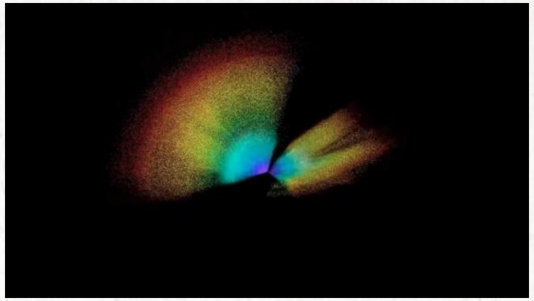
2-dFGRS (~2000)



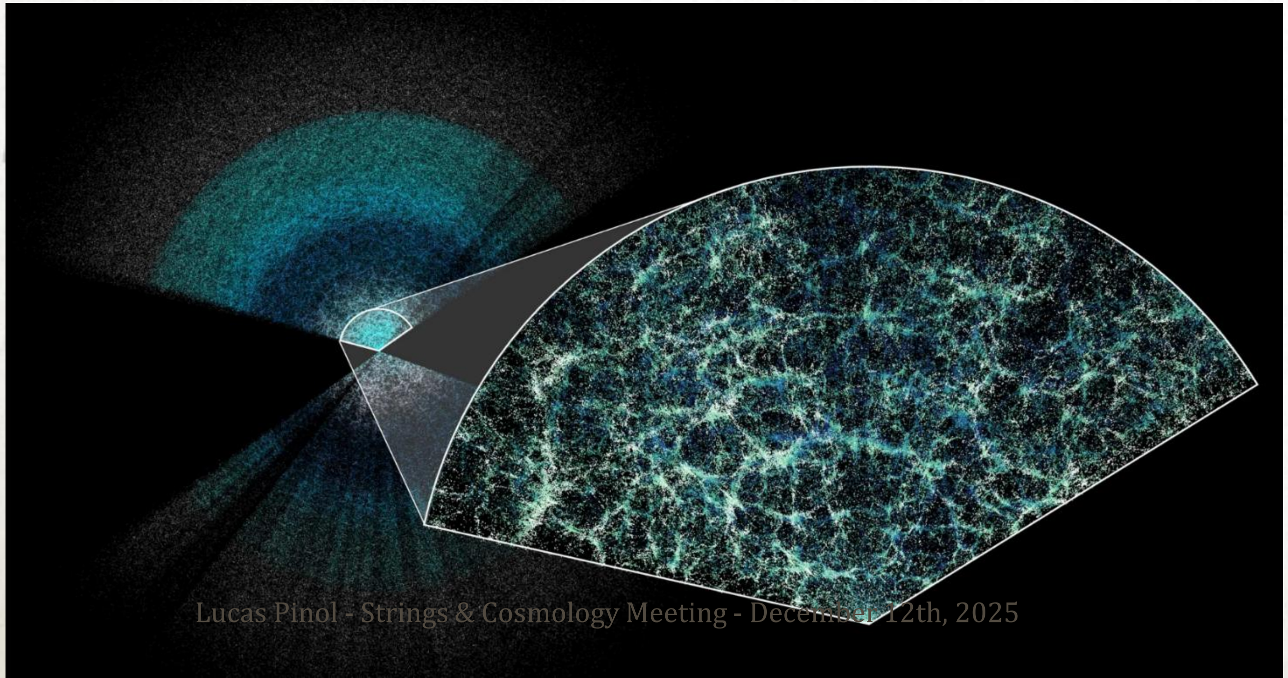
BOSS (2009-2014)



e-BOSS (2014-2020)



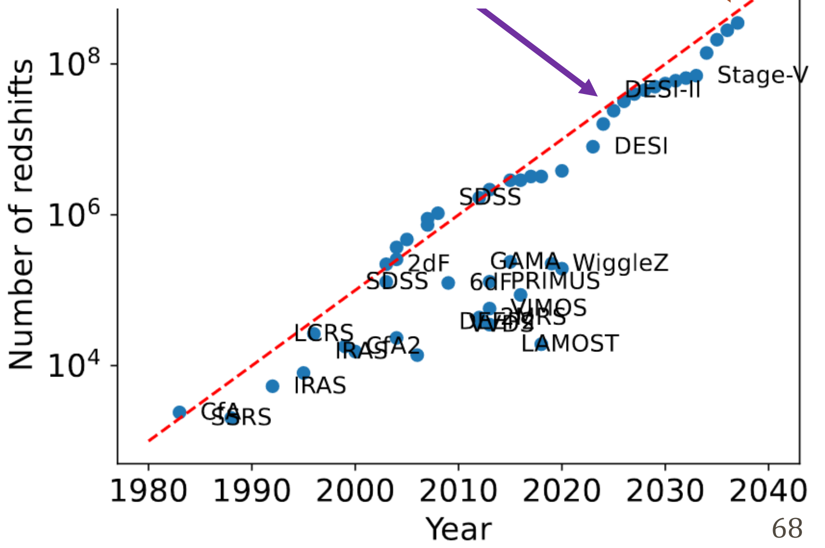
DESI (2020-?)



Euclid (2023-?)

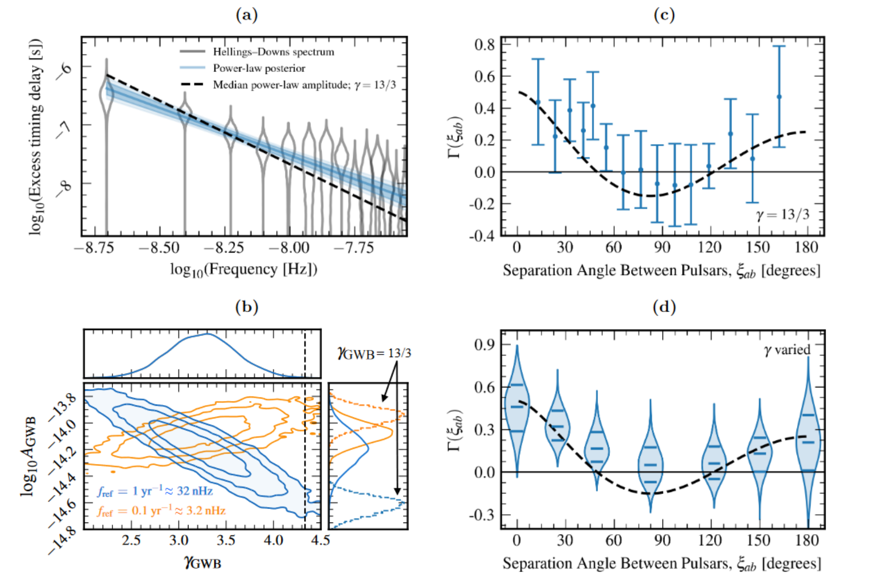
PUMA, MegaMapper, …

Schlegel, Ferraro et al. 2022

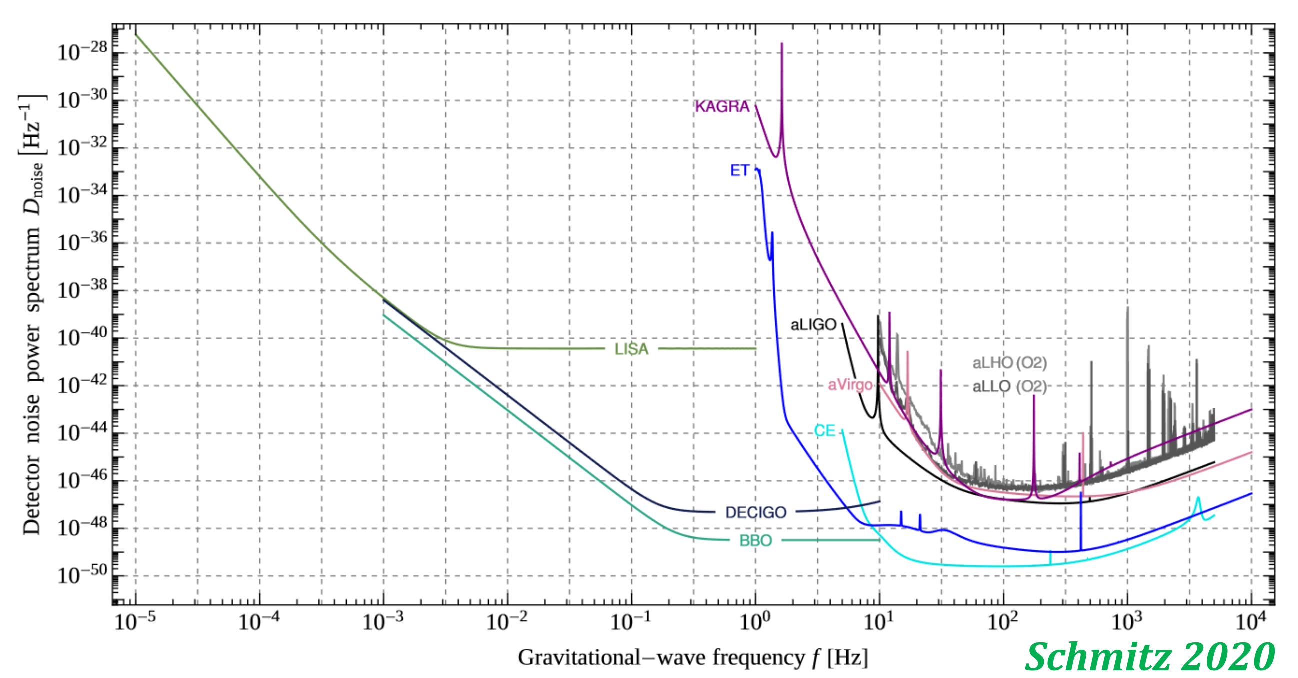


Gravitational-Wave Backgrounds

IPTA (2012-?)

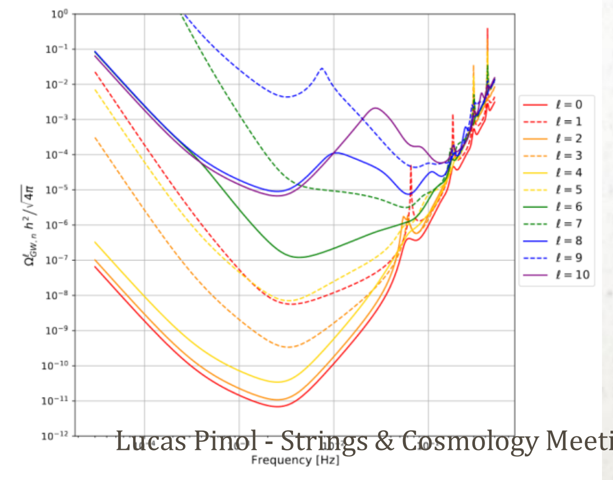


"No GW background" hypothesis is excluded at 3σ

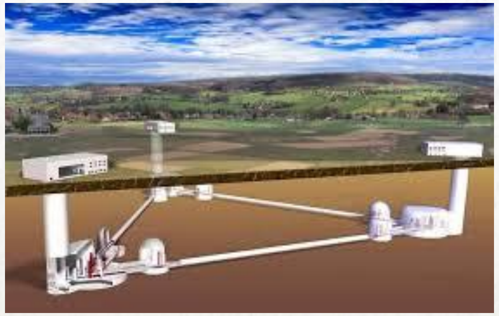


Schmitz 2020

Anisotropies in *Europe* **LISA (2034-?)**

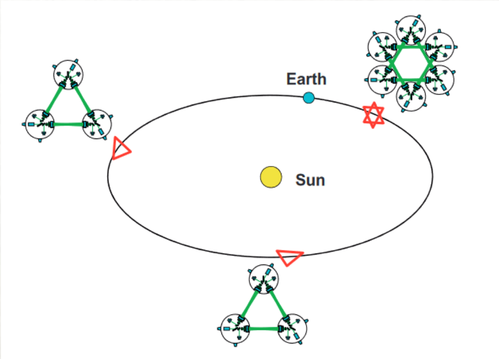


Europe
Einstein Telescope



USA
Cosmic Explorer

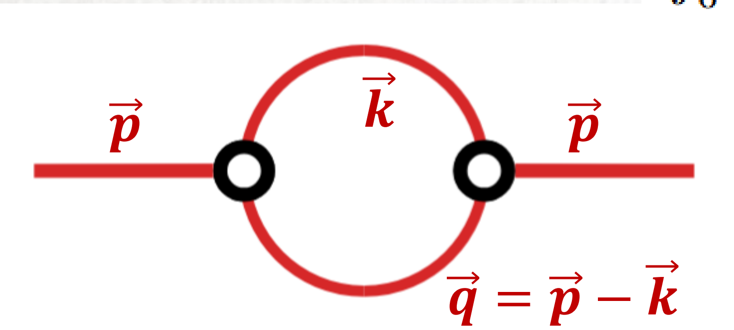
Japan *USA+Europe*
DECIGO, BBO



BARE CONTRIBUTIONS

Strategy

- use dimensionless variables $$\int_0^\infty \mathrm{d}t\, t^\delta \int_{-1}^{1} \mathrm{d}s \int \mathrm{d}\tau_1 \int \mathrm{d}\tau_2\, f(\delta, t, s, \tau_1, \tau_2)$$



$$v = \frac{k}{p}, \qquad u = \frac{q}{p}$$

$$v = \frac{t - s + 1}{2}, \qquad u = \frac{t + s + 1}{2}$$

- worst divergence is $1/\delta$ so we expand the mode function to linear order in δ **[Senatore, Zaldarriaga 2012]**
- perform time (τ_1, τ_2) and angular (s) integrals $$\int_0^\infty \mathrm{d}t\, t^\delta\, [F_0(t) + \delta \cdot F_1(t)]$$
- introduce a fake cutoff $t_{\mathrm{UV}} \to \infty$ to expand $F_1(t)$ in $[t_{\mathrm{UV}}, \infty[$ and extract the pole **[Ballesteros, Gambín Egea, Riccardi 2024]**
- introduce a comoving IR cutoff t_{IR} $$\left(\int_0^{t_{\mathrm{IR}}} + \int_{t_{\mathrm{IR}}}^{t_{\mathrm{UV}}} + \int_{t_{\mathrm{UV}}}^{\infty}\right) \mathrm{d}t\, t^\delta\, [F_0(t) + \delta \cdot F_1(t)]$$