University of Sheffield

Axion-dilaton interactions in the dark sector

Adam Smith

Adam Smith,[a] Maria Mylova,[b,c,d] Philippe Brax,[e] Carsten van de Bruck,[a] C.P. Burgess[d,f,g] and Anne-Christine Davis[h,i]

+ Eleonora Di Valentino

Scalar-tensor interactions

Scalar tensor theory in the low-energy semiclassical limit is at heart a derivative expansion

$$-\frac{\mathcal{L}_{\rm eff}}{\sqrt{-g}} = v^4 V(\theta) + \frac{M_p^2}{2} g^{\mu\nu}\Big[W(\theta)\, R_{\mu\nu} + G_{ij}(\theta)\, \partial_\mu\theta^i \partial_\nu\theta^j\Big]$$

$$+A(\theta)(\partial\theta)^4 + B(\theta)\, R^2 + C(\theta)\, R\, (\partial\theta)^2 + \frac{E(\theta)}{M^2}\, (\partial\theta)^6 + \frac{F(\theta)}{M^2}\, R^3 + \cdots$$

Scalar-tensor interactions

Scalar tensor theory in the low-energy semiclassical limit is at heart a derivative expansion

$$-\frac{\mathcal{L}_{\text{eff}}}{\sqrt{-g}} = \boxed{v^4 V(\theta)} + \frac{M_p^2}{2} g^{\mu\nu} \left[W(\theta)\, R_{\mu\nu} + G_{ij}(\theta)\, \partial_\mu \theta^i \partial_\nu \theta^j \right]$$

$$+ A(\theta)(\partial\theta)^4 + B(\theta)\, R^2 + C(\theta)\, R\, (\partial\theta)^2 + \frac{E(\theta)}{M^2} (\partial\theta)^6 + \frac{F(\theta)}{M^2} R^3 + \cdots$$

- Potential interactions have to be suppressed by symmetries

Scalar-tensor interactions

Scalar tensor theory in the low-energy semiclassical limit is at heart a derivative expansion

Typically well constrained locally

$$-\frac{\mathcal{L}_{\text{eff}}}{\sqrt{-g}} = \boxed{v^4 V(\theta)} + \frac{M_p^2}{2} g^{\mu\nu} \left[W(\theta)\, R_{\mu\nu} + G_{ij}(\theta)\, \partial_\mu \theta^i \partial_\nu \theta^j \right]$$

$$+ A(\theta)(\partial\theta)^4 + B(\theta)\, R^2 + C(\theta)\, R\, (\partial\theta)^2 + \frac{E(\theta)}{M^2} (\partial\theta)^6 + \frac{F(\theta)}{M^2} R^3 + \cdots$$

- Potential interactions have to be suppressed by symmetries

Scalar-tensor interactions

Scalar tensor theory in the low-energy semiclassical limit is at heart a derivative expansion

Typically well constrained locally

$$-\frac{\mathcal{L}_{\rm eff}}{\sqrt{-g}} = \boxed{v^4 V(\theta)} + \frac{M_p^2}{2} g^{\mu\nu}\left[W(\theta)\, R_{\mu\nu} + \boxed{G_{ij}(\theta)\, \partial_\mu\theta^i \partial_\nu\theta^j}\right]$$

$$+A(\theta)(\partial\theta)^4 + B(\theta)\, R^2 + C(\theta)\, R\, (\partial\theta)^2 + \frac{E(\theta)}{M^2}\, (\partial\theta)^6 + \frac{F(\theta)}{M^2}\, R^3 + \cdots$$

- Potential interactions have to be suppressed by symmetries

Axion-dilaton Class

Consequence of the axion's shift symmetry

$$\mathcal{L}_{axio-dilaton} = -\frac{1}{2}M_p^2\sqrt{-g}\left[(\partial\chi)^2 + W^2(\chi)(\partial\mathfrak{a})^2\right]$$

Axion-dilaton Class

$$\mathcal{L}_{axio-dilaton} = -\frac{1}{2}M_p^2\sqrt{-g}\left[(\partial\chi)^2 + W^2(\chi)(\partial\mathfrak{a})^2\right]$$

Fundamental theory

- Combine into complex axion-dilaton fields in extra-dimensional UV completions

$$\Phi = \frac{1}{2}\left(e^{\zeta\chi} + i\mathfrak{a}\right)$$

Dilaton ≪=≫ Volume modulus

Axion-dilaton Class

$$\mathcal{L}_{axio-dilaton} = -\frac{1}{2}M_p^2\sqrt{-g}\left[(\partial\chi)^2 + W^2(\chi)(\partial\mathfrak{a})^2\right]$$

Fundamental theory

- Combine into complex axion-dilaton fields in extra-dimensional UV completions

$$\Phi = \frac{1}{2}\left(e^{\zeta\chi} + i\mathfrak{a}\right)$$

Dilaton ≪=≫ Volume modulus

$$W = W_0 e^{-\zeta\chi}$$

Axion-dilaton Class

$$\mathcal{L}_{axio-dilaton} = -\frac{1}{2}M_p^2\sqrt{-g}\left[(\partial\chi)^2 + W^2(\chi)(\partial\mathfrak{a})^2\right]$$

Fundamental theory

- Combine into complex axion-dilaton fields in extra-dimensional UV completions

$$\Phi = \frac{1}{2}\left(e^{\zeta\chi} + i\mathfrak{a}\right)$$

Dilaton ≪=≫ Volume modulus

$$W = W_0 e^{-\zeta\chi}$$

Cosmology

- Dilaton is naturally light, DE scalar candidate

$$V = U e^{-\lambda\chi}$$

Axion-dilaton Class

$$\mathcal{L}_{axio-dilaton} = -\frac{1}{2}M_p^2\sqrt{-g}\left[(\partial\chi)^2 + W^2(\chi)(\partial\mathfrak{a})^2\right]$$

Fundamental theory

- Combine into complex axion-dilaton fields in extra-dimensional UV completions

$$\Phi = \frac{1}{2}\left(e^{\zeta\chi} + i\mathfrak{a}\right)$$

Dilaton ⋘=⋙ Volume modulus

$$W = W_0 e^{-\zeta\chi}$$

Cosmology

- Dilaton is naturally light, DE scalar candidate

$$V = Ue^{-\lambda\chi}$$

- Axions have wide range of uses, e.g. CDM

$$V(a) = \frac{m_{\mathfrak{a}}^2}{2}(\mathfrak{a} - \mathfrak{a}_+)^2 + \dots$$

The Full Action

$$S = -\int d^4x\sqrt{-g}\left\{\frac{1}{2}M_p^2\left[R + \underbrace{\partial_\mu\chi\,\partial^\mu\chi}_{\text{Dilaton Kinetic Term}} + \underbrace{W^2(\chi)\,\partial_\mu\mathfrak{a}\,\partial^\mu\mathfrak{a}}_{\text{Axion Kinetic Term}}\right] + \underbrace{V(\chi,\mathfrak{a})}_{\text{Axio-dilaton Potential}}\right\} + \mathcal{L}_m(\Psi,\tilde{g})$$

The dilaton couples to matter as a pseudo-Brans-Dicke scalar:

$$\tilde{g}_{\mu\nu} := C^2(\chi)\,g_{\mu\nu}$$

$$C(\chi) = e^{\mathbf{g}\chi}$$

The Full Action

$$S = -\int d^4x\sqrt{-g}\left\{\frac{1}{2}M_p^2\left[R + \underbrace{\partial_\mu\chi\,\partial^\mu\chi}_{\text{Dilaton Kinetic Term}} + \underbrace{W^2(\chi)\,\partial_\mu\mathfrak{a}\,\partial^\mu\mathfrak{a}}_{\text{Axion Kinetic Term}}\right] + \underbrace{V(\chi,\mathfrak{a})}_{\text{Axio-dilaton Potential}}\right\} + \mathcal{L}_m(\Psi,\tilde{g})$$

The dilaton couples to matter as a pseudo-Brans-Dicke scalar:

$$\tilde{g}_{\mu\nu} := C^2(\chi)\,g_{\mu\nu} \qquad \frac{1}{\sqrt{-g}}\frac{\partial\mathcal{L}_m}{\partial\chi} = \mathbf{g}\rho_m$$

$$C(\chi) = e^{\mathbf{g}\chi} \qquad \boxed{m_B(\chi) = m e^{\mathbf{g}\chi}}$$

The Full Action

$$S = -\int d^4x\sqrt{-g}\left\{\frac{1}{2}M_p^2\left[R + \underbrace{\partial_\mu\chi\,\partial^\mu\chi}_{\text{Dilaton Kinetic Term}} + \underbrace{W^2(\chi)\,\partial_\mu\mathfrak{a}\,\partial^\mu\mathfrak{a}}_{\text{Axion Kinetic Term}}\right] + \underbrace{V(\chi,\mathfrak{a})}_{\text{Axio-dilaton Potential}}\right\} + \mathcal{L}_m(\Psi,\tilde{g})$$

Everything dilaton related is a runaway exponential

$$m_B(\chi) = me^{\mathbf{g}\chi}$$

$$V(a) = \frac{m_{\mathfrak{a}}^2}{2}(\mathfrak{a}-\mathfrak{a}_+)^2 + \ldots$$

$$W = W_0e^{-\zeta\chi}$$

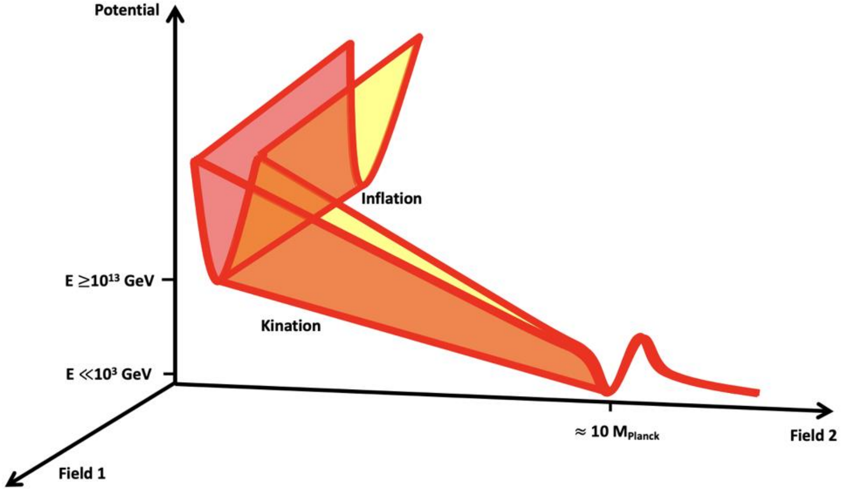
$$V = U(\chi)e^{-\lambda\chi} + V(\mathfrak{a})$$

$$U(\chi) = U_0\left[1 - u_1\chi + \frac{u_2}{2}\chi^2\right]$$

Potential Motivations

$$V = U(\chi)e^{-\lambda\chi} + V(\mathfrak{a}) \qquad U(\chi) = U_0\left[1 - u_1\chi + \frac{u_2}{2}\chi^2\right]$$

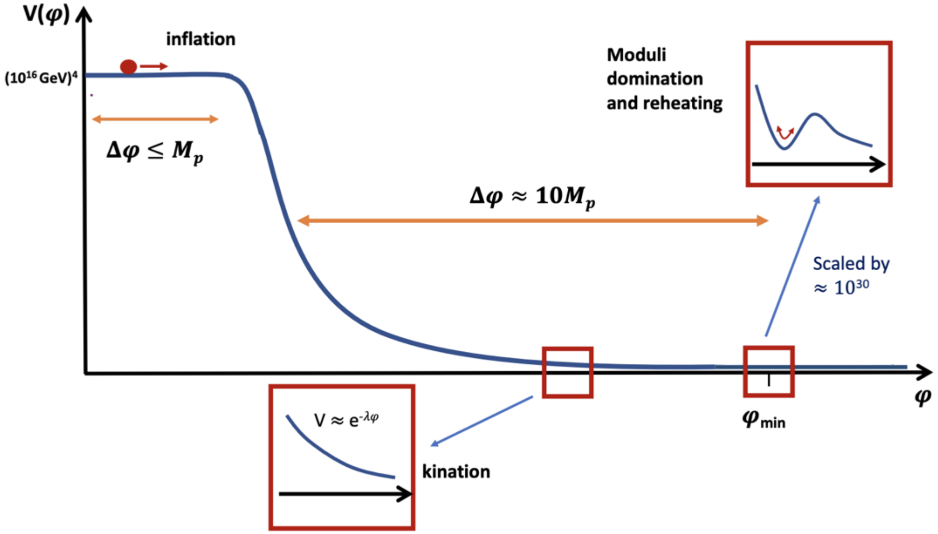
In the String phenomenology literature



Cicoli et al (2023) [2303.04819]

String Cosmology: from the Early Universe to Today

Michele Cicoli, Joseph P. Conlon, Anshuman Maharana, Susha Parameswaran, Fernando Quevedo, Ivonne Zavala



Apers et al (2024) [2401.04064]

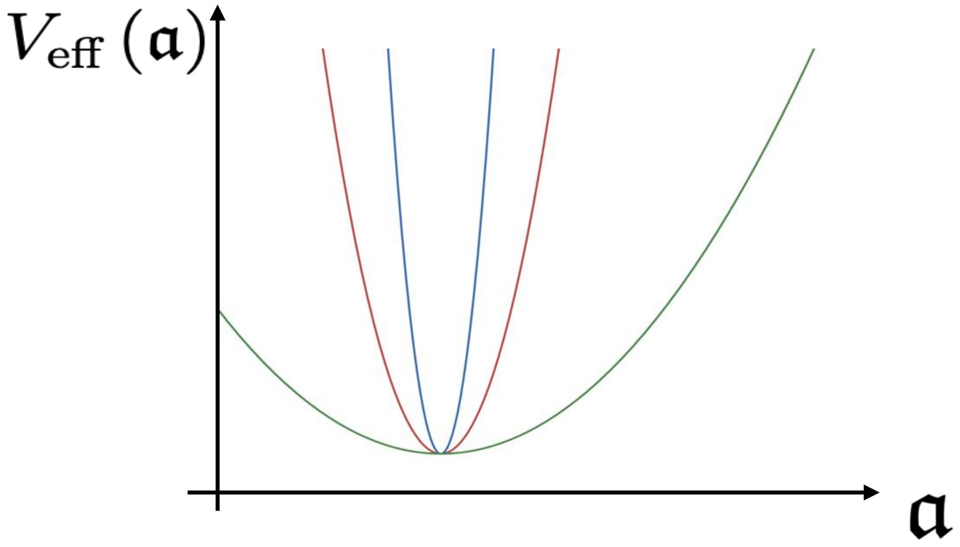
String Theory and the First Half of the Universe

Fien Apers, Joseph P. Conlon, Edmund J. Copeland, Martin Mosny, Filippo Revello

Axion Phenomenology

$$V(a) = \frac{m_{\mathfrak{a}}^2}{2}(\mathfrak{a} - \mathfrak{a}_+)^2 + \dots$$

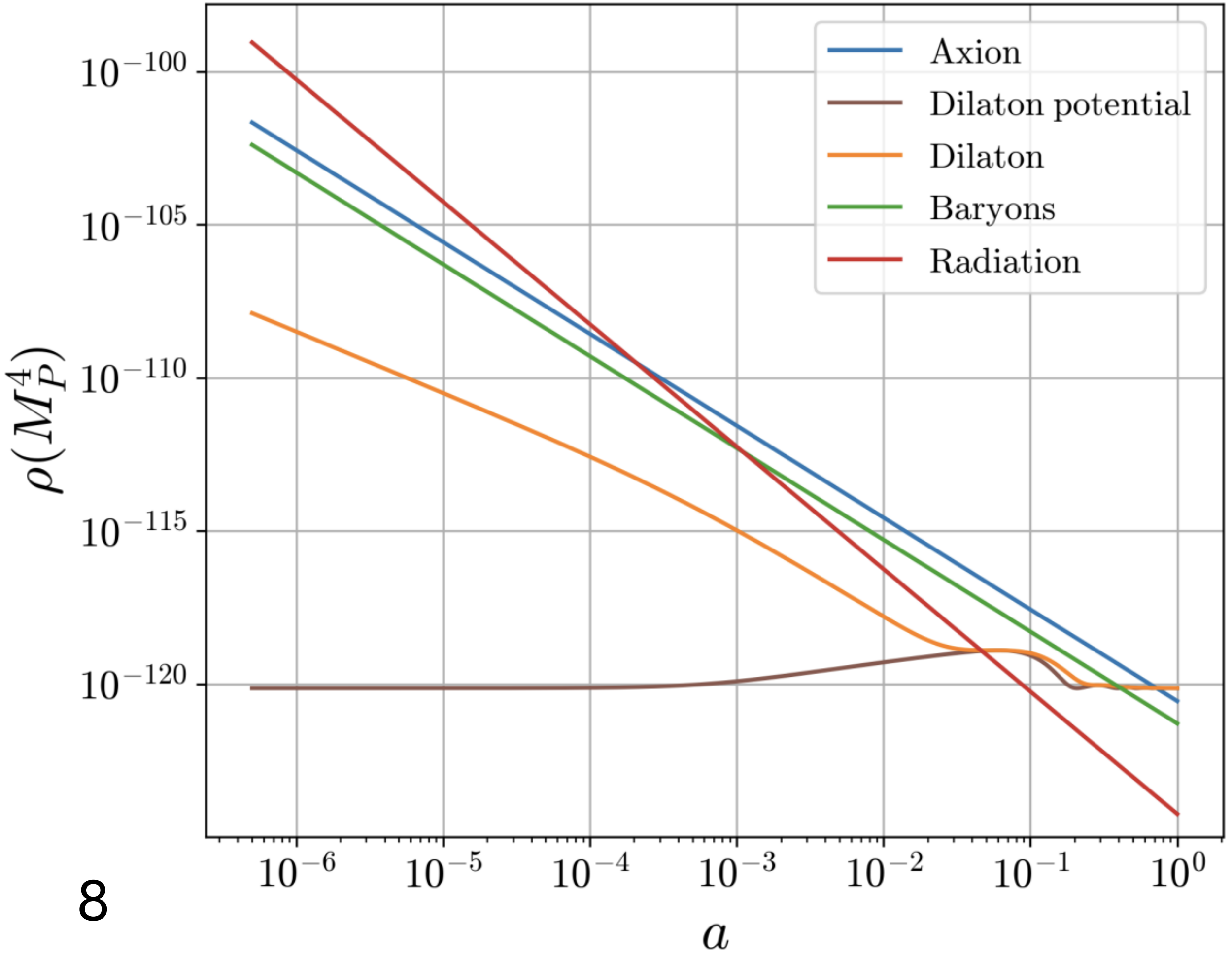
$$\bar{\rho}_{\rm ax} = W^2(\bar{\chi})\bar{\rho}_{\mathfrak{a}} = \frac{C\mathfrak{m}(t)}{a^3}$$

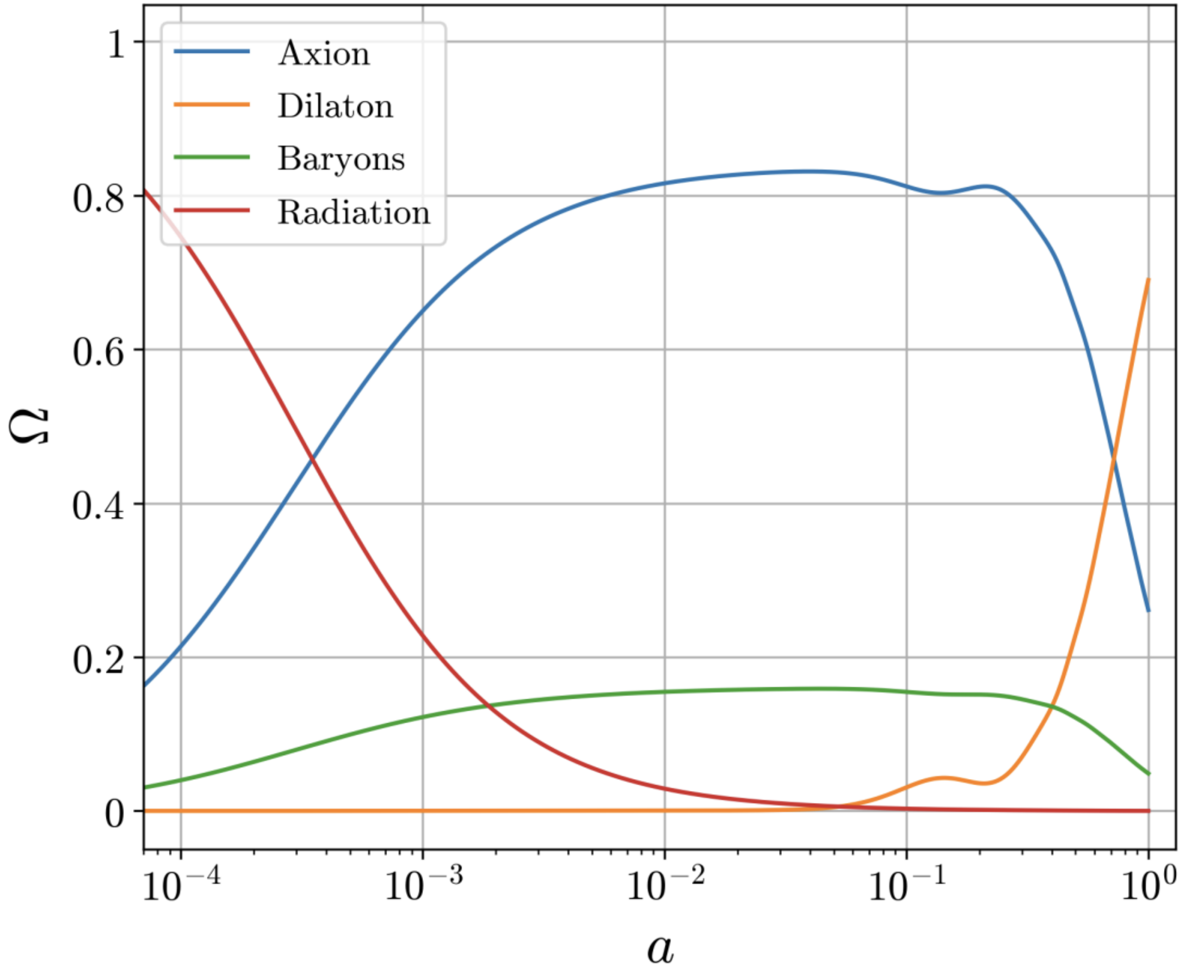


$$\mathfrak{a} = \bar{\mathfrak{a}}(\bar{\rho}) + \frac{1}{\sqrt{2}}\left(e^{-i\int_0^t dt\,\mathfrak{m}(t)}\psi + e^{i\int_0^t dt\,\mathfrak{m}(t)}\psi^\star\right), \quad \text{where} \quad \mathfrak{m}^2(t) = \frac{m_{\mathfrak{a}}^2}{W^2(\bar{\chi})}$$

Cosmology

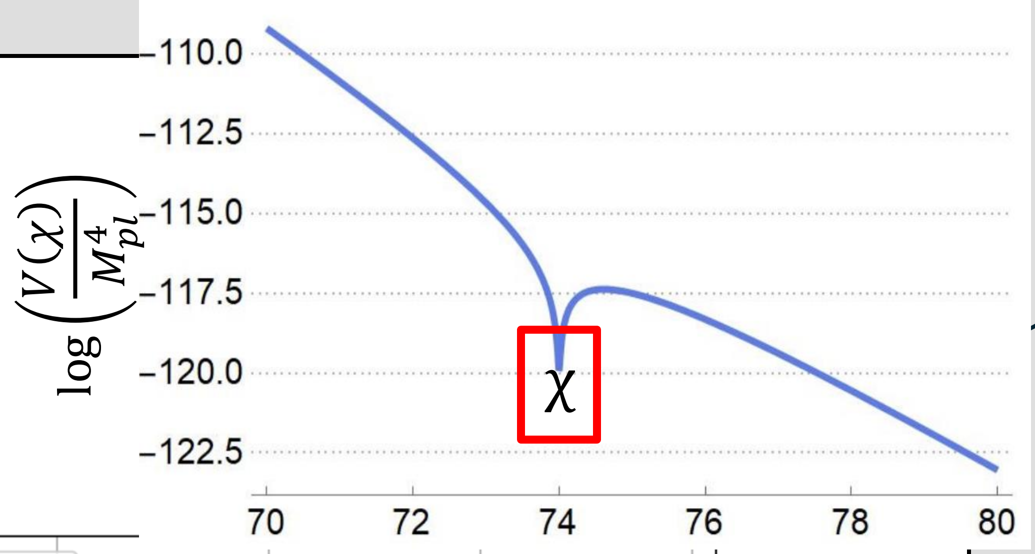
$\zeta = 0.1 \quad \text{and} \quad \mathbf{g} = -10^{-3}$

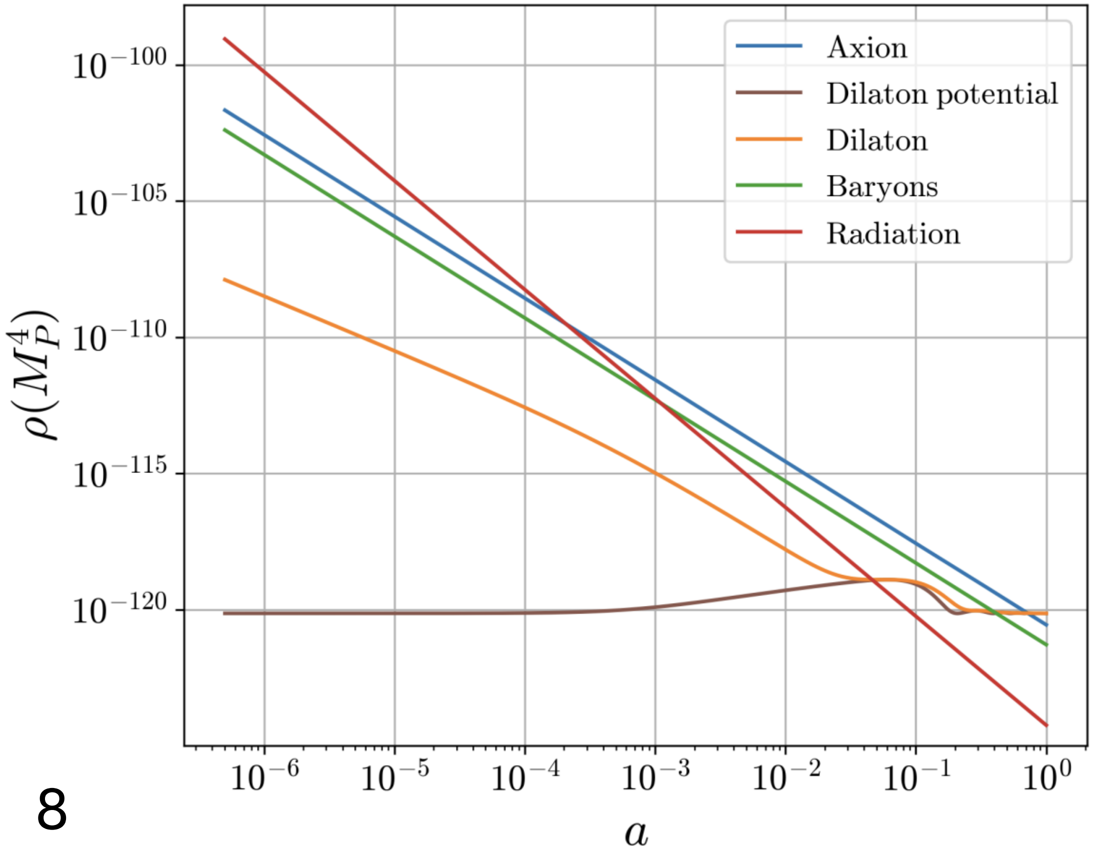


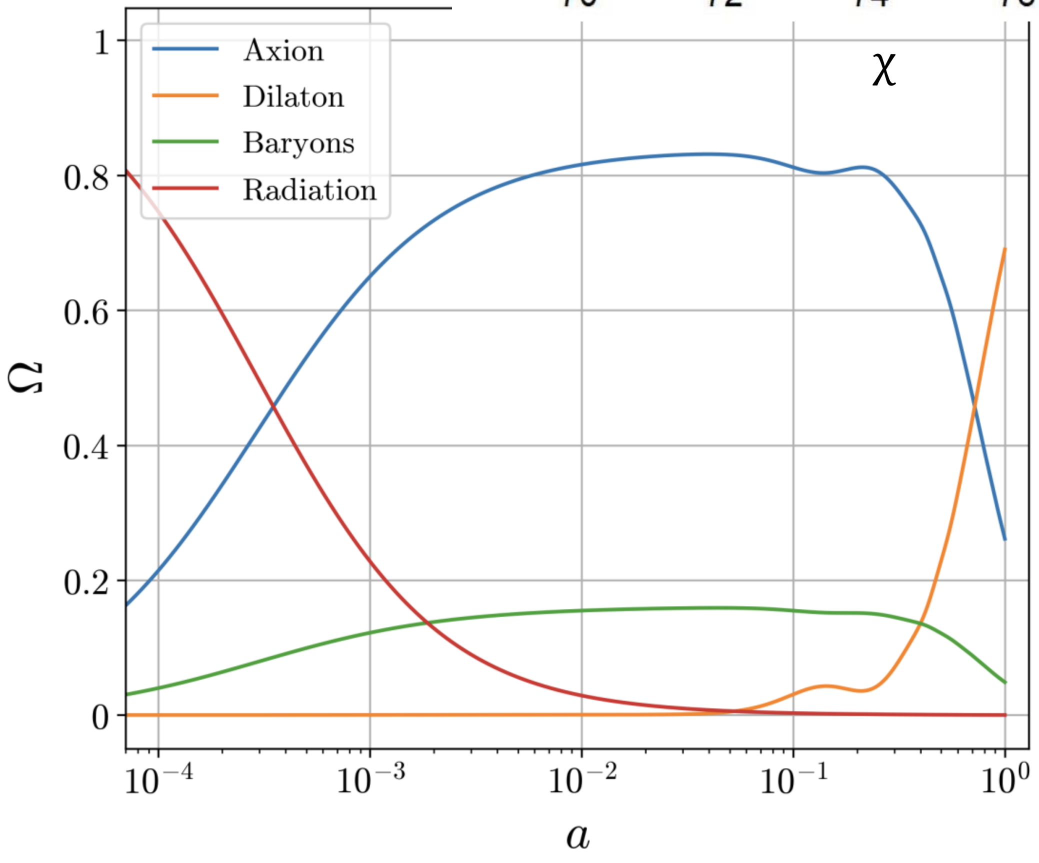


Cosmology

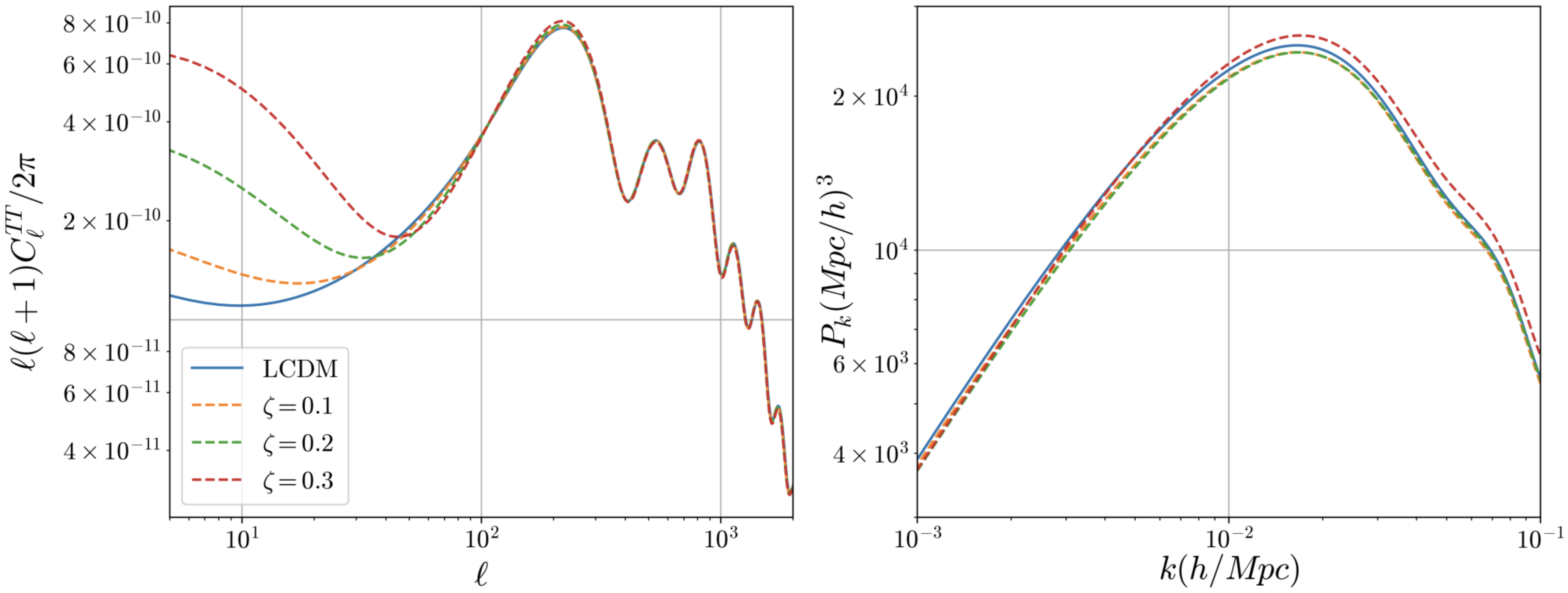
$\zeta = 0.1 \quad \text{and} \quad \mathbf{g} = -10^{-3}$







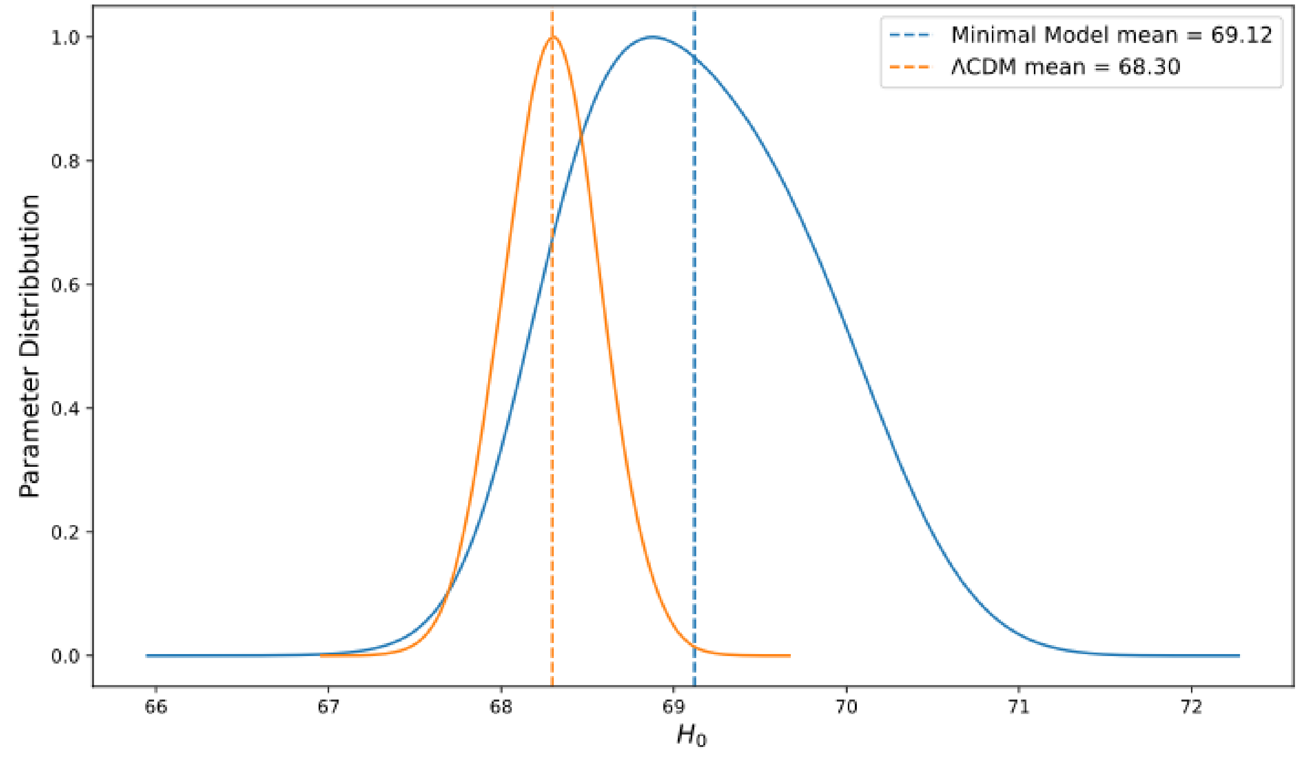
Power Spectra of the CMB

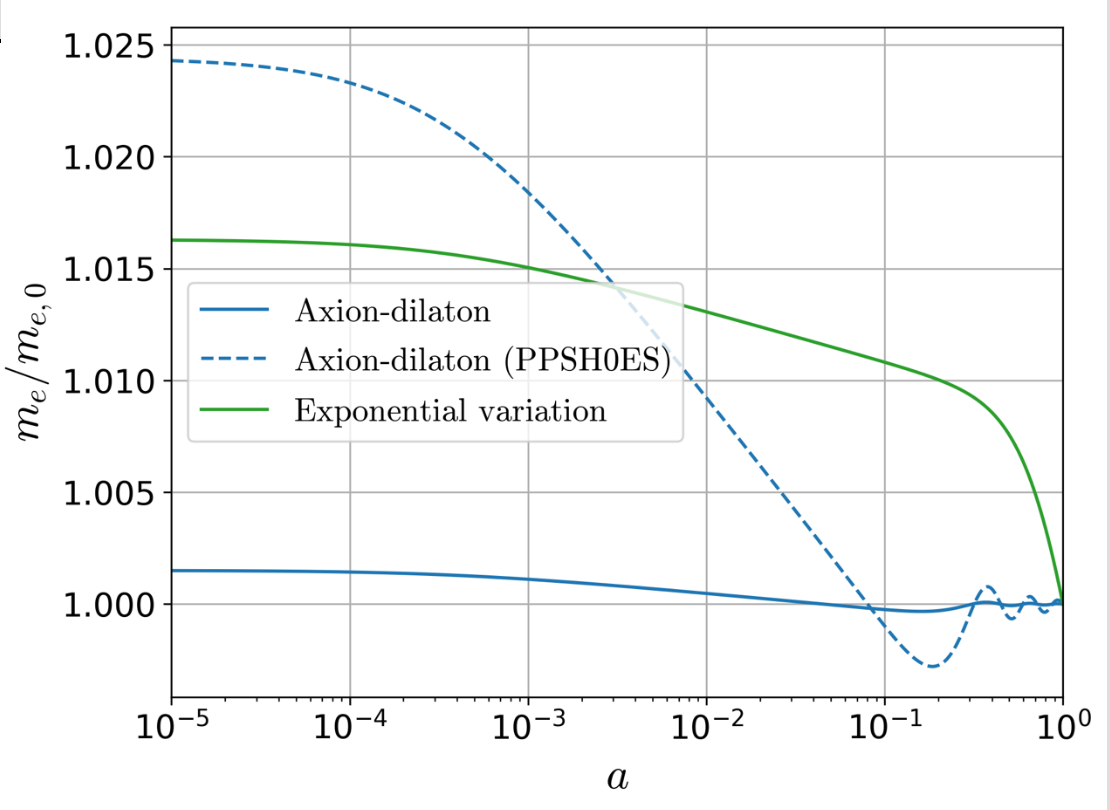


Hubble Tension

MCMC results using:

- Planck 2018 + ACT Dr6 lensing
- Desi Dr2 BAO
- Pantheon +





$$\chi_i$$

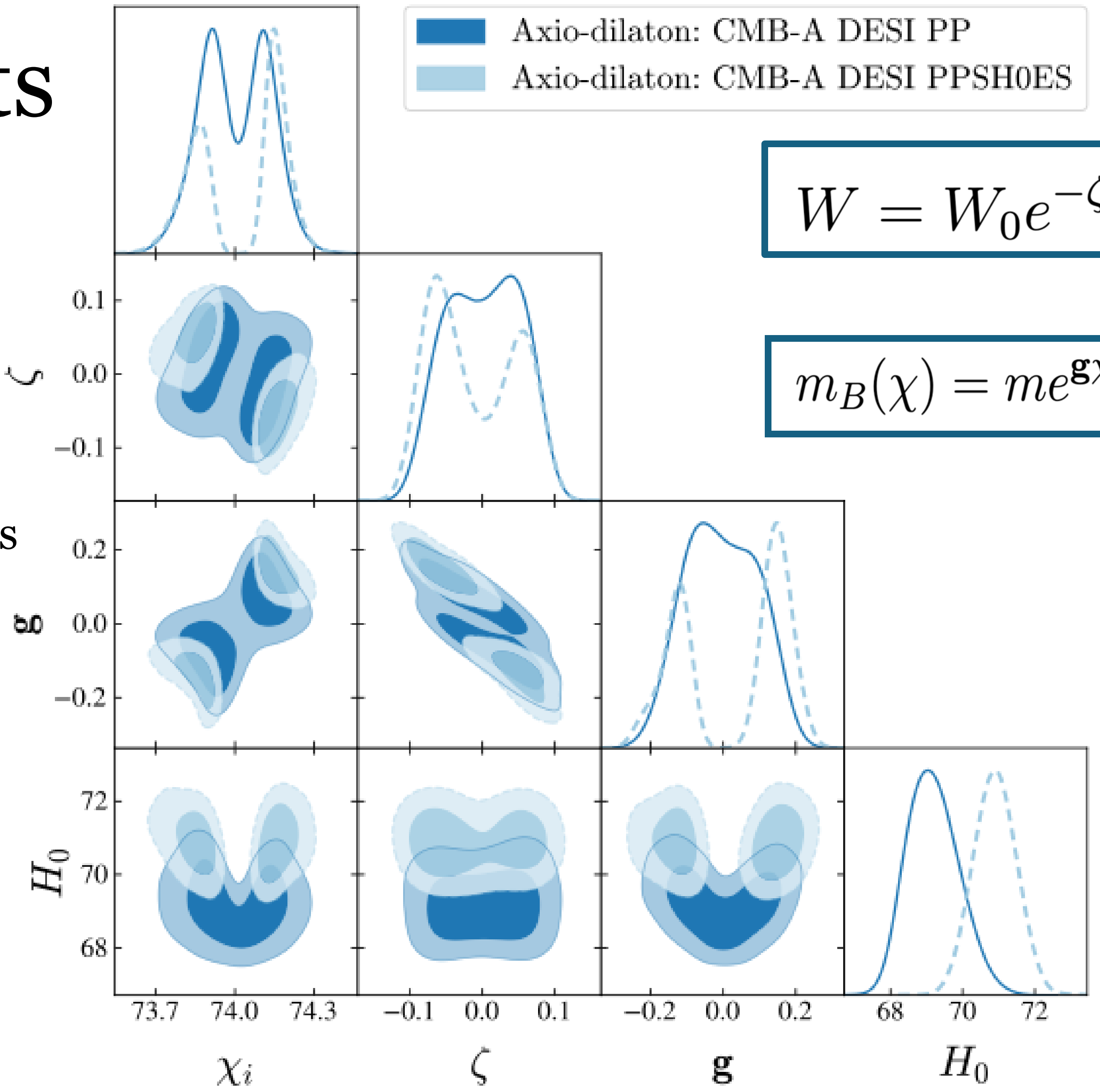
$$W = W_0 e^{-\zeta\chi}$$

$$m_B(\chi) = m e^{\mathbf{g}\chi}$$

Coupling constraints

- If the Hubble tension is real, couplings preferred non-zero

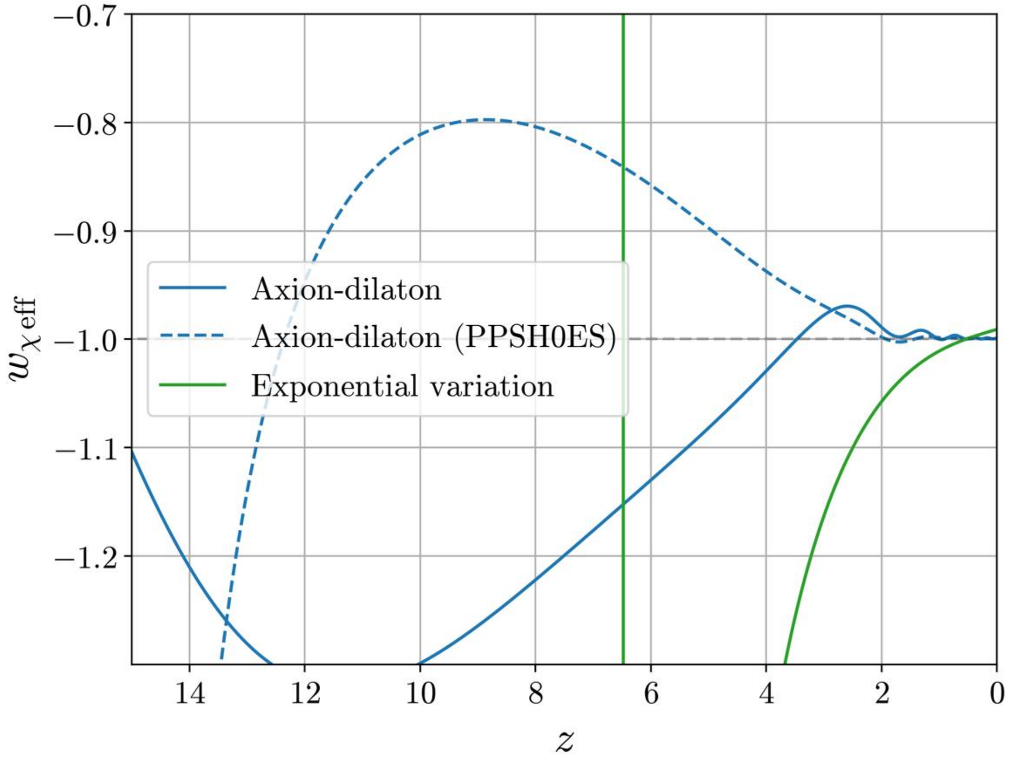
- Grossly violates local physics constraints (unless screened)



$$W = W_0 e^{-\zeta\chi}$$

$$m_B(\chi) = m e^{\mathbf{g}\chi}$$

The effective equation of state



- The preference for a phantom equation of state from DESI results assumes matter species evolve $\propto 1/a^3$
- This preference remains robust in the coupled quintessence cases studied here.

$$\rho_\chi^{\mathrm{eff}} = \rho_\chi + \left[\frac{W(\chi_0)}{W(\chi)} - 1\right]\frac{\rho_{\mathrm{ax0}}}{a^3}$$

$$\omega_{\chi\,\mathrm{eff}} = \frac{\omega_\chi(\chi)}{1 + \left[e^{\zeta(\chi-\chi_0)} - 1\right]\frac{\rho_{\mathrm{ax0}}}{a^3\rho_\chi}}$$

Fitness Tests (With SHoES calibration)

Model	H_0	g	ζ	$\Delta\chi^2$
Yoga VI	70.85 ± 0.58 (71.11)	$0.03^{+0.17}_{-0.20}$ (-0.214)	-0.012 ± 0.060 (0.082)	-19.7
EXP	70.79 ± 0.59 (71.03)	0.000 ± 0.160 (0.161)	-0.003 ± 0.062 (-0.058)	-18.9
Λ CDM+m_e	70.97 ± 0.57 (71.34)	–	–	-19.2
$w_0 w_a$+m_e	70.51 ± 0.73 (70.37)	–	–	-19.4

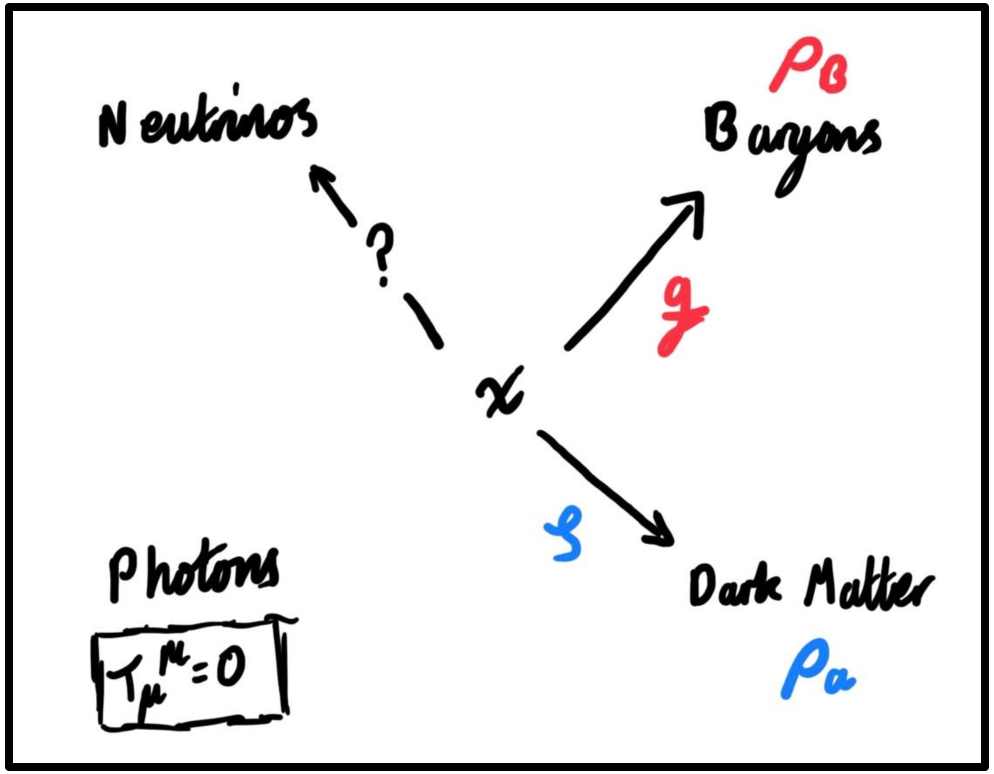
- Any model with varying electron mass gets massively preferred
- Not possible to distinguish between them with current data

Conclusion

- Axion can play the role of dark matter
- Dilaton can play the role of dark energy

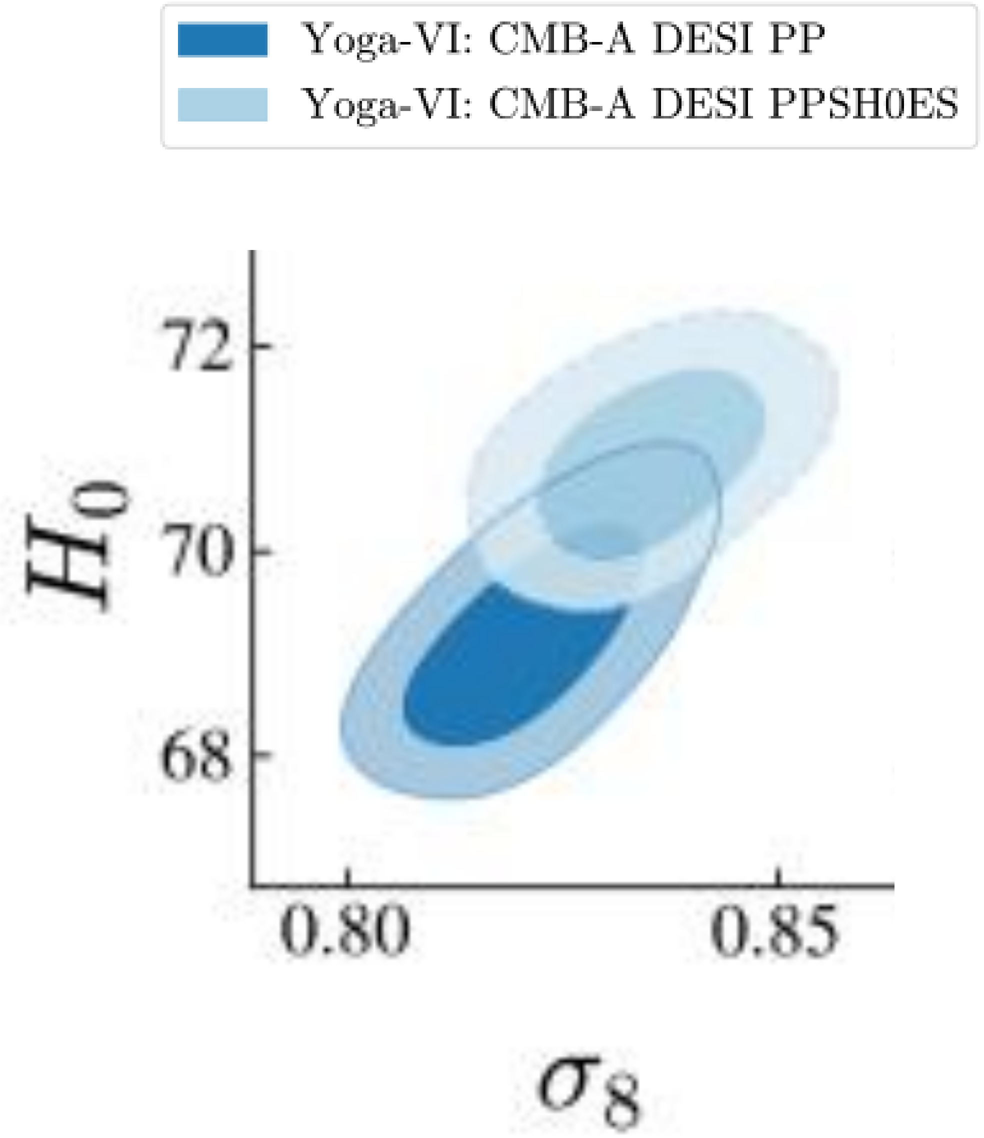
- Interactions between them can give:

i. Oscillations in structure growth
ii. Oscillations in particle masses
iii. Deviations in the CMB



- The models can fit cosmological data very well, but are un-screened and violate solar system gravity tests

Structure Growth



SPT data Included

Model	Dataset	H_0	$\mathbf{g}$	ζ	χ_i	$R-1$	$\Delta\chi^2$
Yoga-VI	CMB-B DESI PP	69.19 ± 0.70 (69.13)	0.003 ± 0.095 (-0.052)	0.002 ± 0.050 (-0.003)	74.020 ± 0.150 (73.844)	0.075	-7.2
	CMB-B DESI	69.42 ± 0.71 (69.50)	-0.012 ± 0.098 (-0.037)	-0.001 ± 0.048 (-0.032)	$73.98^{+0.21}_{-0.18}$ (73.814)	0.228	-6.8
EXP	CMB-B DESI PP	69.16 ± 0.68 (69.99)	0.002 ± 0.093 (0.104)	-0.001 ± 0.044 (-0.022)	–	0.165	-7.3
	CMB-B DESI	69.38 ± 0.71 (70.16)	$-0.013^{+0.090}_{-0.10}$ (-0.164)	0.001 ± 0.045 (0.071)	–	0.596	-6.2
	CMB-B	$67.47^{+0.68}_{-1.2}$ (67.78)	-0.004 ± 0.074 (0.041)	0.001 ± 0.048 (-0.002)	–	0.039	–
w0-wa + me	CMB-B DESI PP	68.40 ± 0.84 (68.51)	–	–	–	0.007	-12.3
	CMB-B DESI	$64.2^{+1.9}_{-2.6}$ (63.78)	–	–	–	0.041	-10.5
w0-wa	CMB-B DESI PP	67.65 ± 0.59 (67.52)	–	–	–	0.010	-9.6
Λ CDM	CMB-B DESI PP	68.04 ± 0.26 (68.25)	–	–	–	0.007	0.0
	CMB-B DESI	68.13 ± 0.26 (68.23)	–	–	–	0.006	0.0

TABLE IV: Posterior means with quoted 1σ marginal uncertainties and best-fit values in parentheses for all models fit to the CMB-B dataset combinations including SPT-3G. Columns list the inferred Hubble constant, the dilaton coupling $\mathbf{g}$, the axion CDM kinetic coupling ζ, the initial dilaton value χ_i when present, and the change in best-fit $\Delta\chi^2$ relative to the corresponding ΛCDM run.

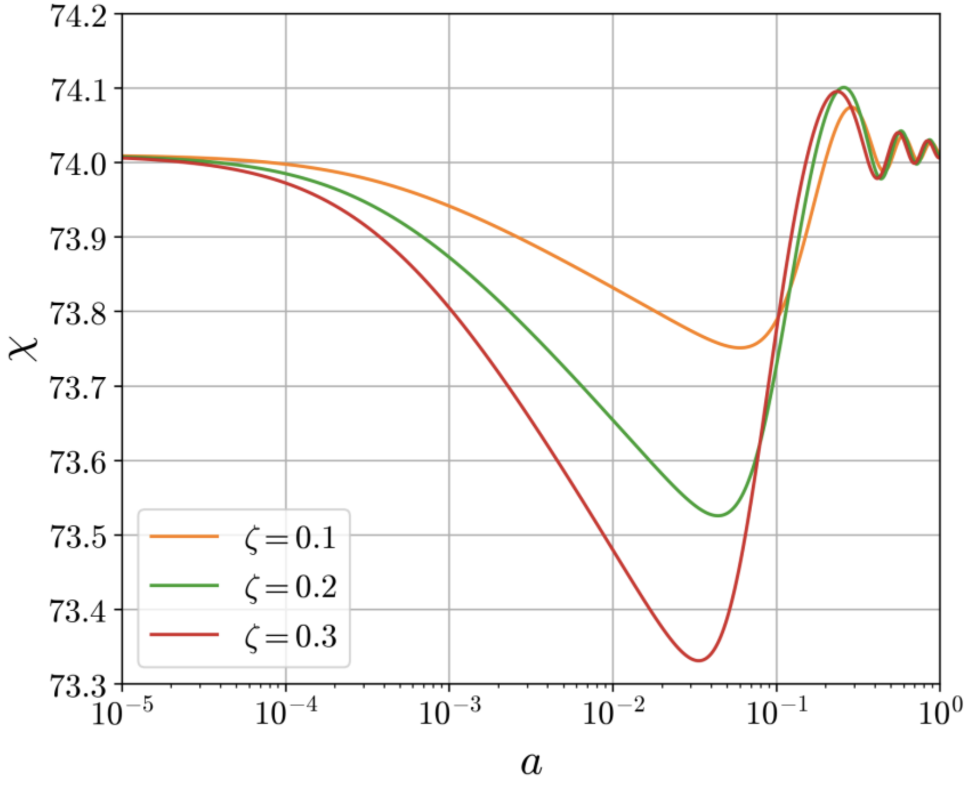
Coupling just base datasets

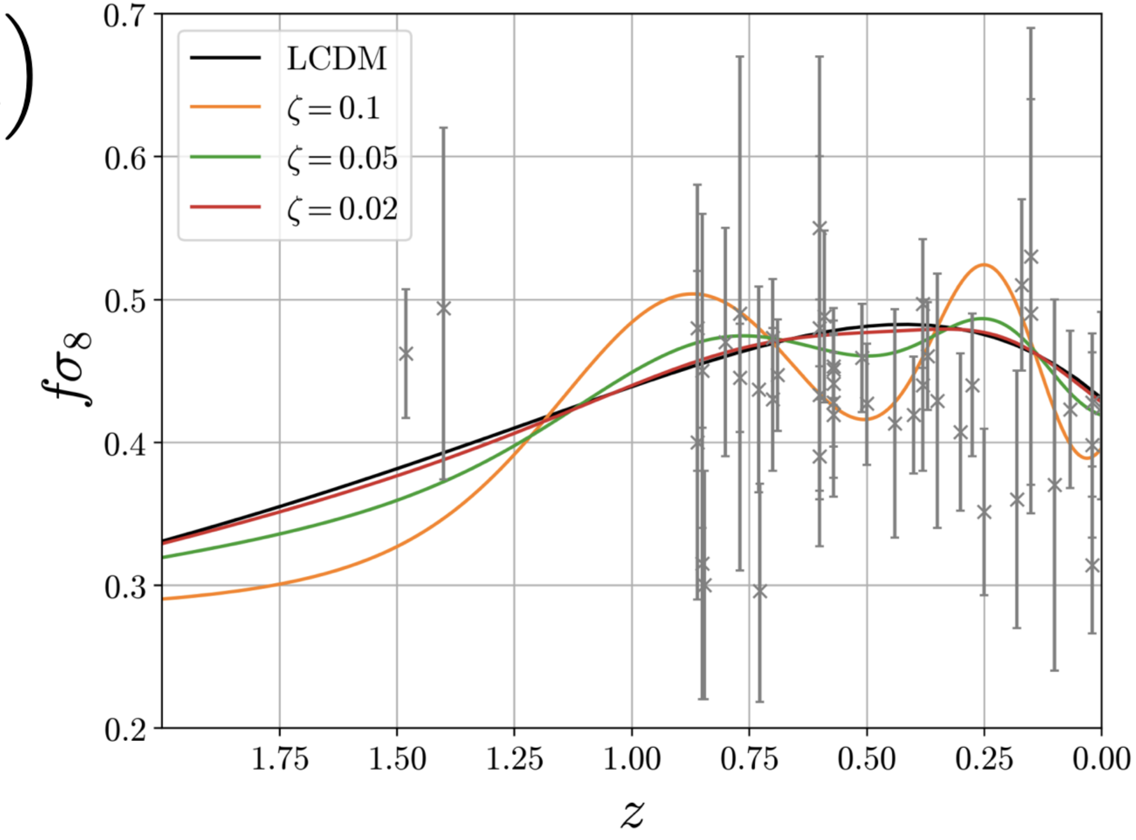
Model	Dataset	H_0 [km s^{-1} Mpc^{-1}]	**g**	ζ	χ_i	$R-1$	$\Delta\chi^2$
Yoga-VI	CMB-A DESI PP	$69.18^{+0.63}_{-0.81}$ (69.71)	0.00 ± 0.10 (0.14)	0.002 ± 0.052 (-0.061)	74.00 ± 0.13 (74.10)	0.071	-2.1
	CMB-A DESI	$69.38^{+0.68}_{-0.83}$ (69.61)	0.00 ± 0.10 (0.15)	0.000 ± 0.052 (-0.060)	74.00 ± 0.14 (74.09)	0.034	-3.5
(no-m_e)	CMB-A DESI PP	68.50 ± 0.35 (68.51)	0.006 ± 0.083 (-0.009)	-0.001 ± 0.054 (-0.050)	$74.01^{+0.18}_{-0.16}$ (73.88)	0.016	-2.1
Yoga	CMB-A DESI PP	68.42 ± 0.35 (68.33)	-0.038 ± 0.086 (-0.055)	$0.020^{+0.077}_{-0.094}$ (0.032)	–	0.010	-0.7
	CMB-A DESI	68.53 ± 0.36 (68.54)	-0.041 ± 0.088 (-0.078)	$0.023^{+0.076}_{-0.097}$ (0.050)	–	0.025	-1.8
EXP	CMB-A DESI PP	$69.10^{+0.64}_{-0.76}$ (68.81)	-0.003 ± 0.099 (0.127)	0.005 ± 0.049 (-0.058)	–	0.082	-2.7
	CMB-A DESI	$69.35^{+0.69}_{-0.78}$ (69.96)	$0.004^{+0.12}_{-0.098}$ (-0.167)	-0.001 ± 0.049 (0.077)	–	0.057	-3.8
	CMB-A	$68.03^{+0.78}_{-1.6}$ (67.43)	0.000 ± 0.089 (-0.013)	0.001 ± 0.051 (0.036)	–	0.051	-1.1
w0-wa + me	CMB-A DESI PP	68.26 ± 0.87 (68.29)	–	–	–	0.008	-8.2
	CMB-A DESI	$63.9^{+1.9}_{-2.7}$ (64.78)	–	–	–	0.006	-8.6
w0-wa	CMB-A DESI PP	67.64 ± 0.60 (67.71)	–	–	–	0.012	-6.9
Λ CDM+me	CMB-A DESI PP	69.62 ± 0.69 (69.91)	–	–	–	0.007	-2.6
	CMB-A DESI	69.85 ± 0.69 (70.12)	–	–	–	0.006	-4.2
Λ CDM	CMB-A DESI PP	68.30 ± 0.28 (68.03)	–	–	–	0.021	0.0
	CMB-A DESI	68.39 ± 0.29 (68.53)	–	–	–	0.009	0.0
	CMB-A	67.30 ± 0.54 (67.27)	–	–	–	0.007	0.0

TABLE III: Posterior means with 1σ confidence intervals, best-fit values in parentheses, and $\Delta\chi^2$ relative to the corresponding ΛCDM run for each dataset combination. The table summarises how the different axio–dilaton (Yoga and EXP), w_0–w_a, and varying-m_e models shift H_0, the scalar couplings $(\mathbf{g}, \zeta)$, and the overall goodness of fit relative to ΛCDM across the CMB-A, CMB-A DESI, and CMB-A DESI PP datasets.

Minimal Dark Sector

$$\bar{\chi}'' + 2\mathcal{H}\bar{\chi}' + \frac{a^2}{M_p^2}V_{,\chi} = \frac{a^2}{M_p^2}\left(-\mathbf{g}\bar{\rho}_B - \zeta\bar{\rho}_{\mathbf{ax}}\right)$$





$$f\sigma_8 = \frac{\sigma_8(z, k_{\sigma 8})}{\mathcal{H}}\frac{\delta_m'(z, k_{\sigma 8})}{\delta_m(z, k_{\sigma 8})}$$

$$\mathbf{g} = -10^{-3}$$

Data points catalogued from SDSS redshift space distortions