Geometry of Kinematic Flow

Differentiation for Cosmological Correlators

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Based on 2504.14890
with Daniel Baumann, Austin Joyce, Hayden Lee,
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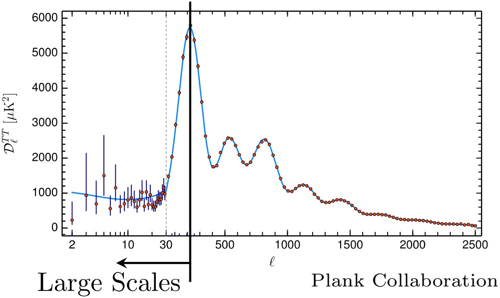
Why do we care about Inflation?

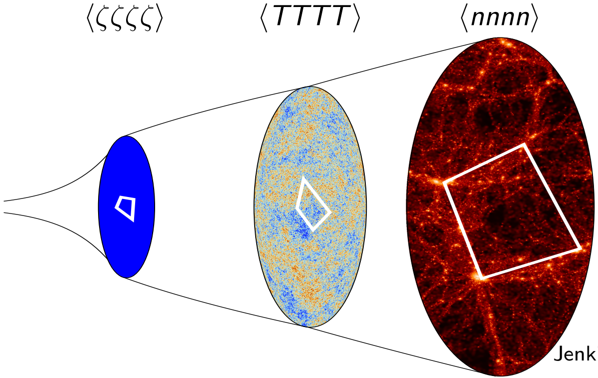
- We know that physics depends on energy scales
- The universe began much hotter and denser than it is today
- Perturbations in the universe today are primordial



Plank Collaboration [2018]

What do we measure?

Correlation Functions



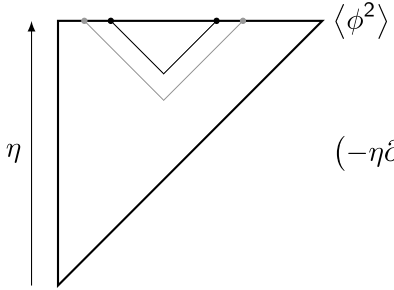
Jenkins et al. [1998]

Plank Collaboration [2014]

The Cosmological Bootstrap

Avoiding the time integrals

- We only measure correlation functions at the end of inflation
- The properties in the bulk are encoded through kinematics
- This encoding is the goal of the **cosmological bootstrap**
- We will look for differential equations



$$\left(-\eta\partial_\eta + 3 + k^i\partial_{k_i}\right)\langle \phi^2 \rangle = 0$$

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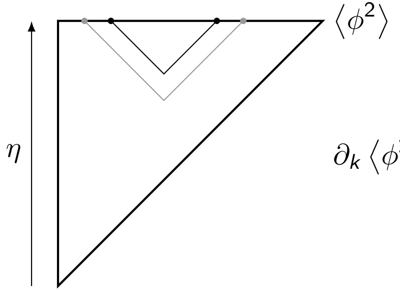


$$\partial_k \langle \phi^2 \rangle = \frac{(2\Delta - 3)}{k} \langle \phi^2 \rangle$$
$$\Rightarrow \langle \phi^2 \rangle \propto k^{2\Delta - 3}$$

Cosmological Correlators

Toy Model

Simplification in the pursuit of structure

- We assume a power law FRW background

$$ds^2 = \left(\frac{\eta}{\eta_0}\right)^{-2(1+\epsilon)} \left(-d\eta^2 + dx_i dx^i\right)$$

- Plus an interacting conformally coupled scalar

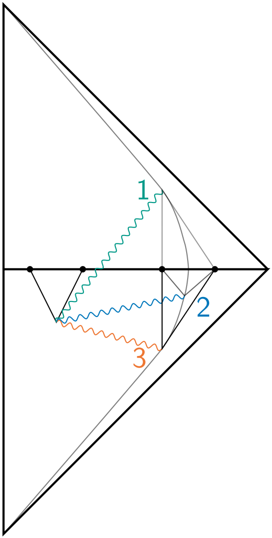
$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{12}R\phi^2 - \frac{\lambda}{3!}\phi^3\right]$$

The Bulk Calculation

Integration is hard

- Even at tree level we have integrals to do
- These integrals split into three cases
    1. Annihilation to vacuum
    2. Particle decay
    3. Vacuum production

$$\int_{-\infty}^{0}\int_{-\infty}^{\eta_1} \frac{d\eta_1 d\eta_2}{(\eta_1\eta_2)^{1+\epsilon}} e^{iY(\eta_2-\eta_1)} e^{iX_1\eta_1+iX_2\eta_2}$$



Energy Integrals

Removing Time

- To eliminate the dependence on time we perform a Fourier transform

$$(-\eta)^{-1-\epsilon} \propto \int \mathrm{d}\omega \omega^{\epsilon} e^{i\omega\eta}$$

- This allows us to perform the time integrals

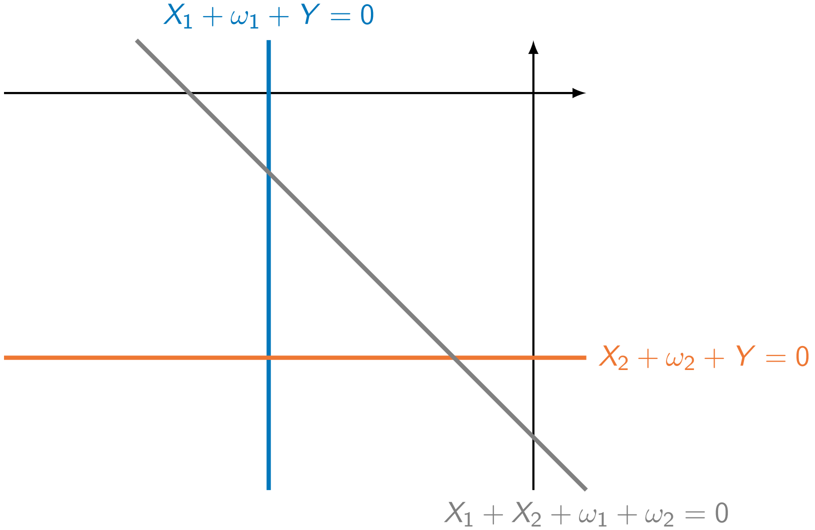
$$\begin{aligned}\langle \phi^4 \rangle_3 &= \int \mathrm{d}\omega_1 \mathrm{d}\omega_2 \omega_1^{\epsilon}\omega_2^{\epsilon} \int_{-\infty}^{\eta_1} \mathrm{d}\eta_1 \mathrm{d}\eta_2 e^{i(X_1+\omega_1-Y)\eta_1 + i(X_2+\omega_2+Y)\eta_2} \\ &= \int \mathrm{d}\omega_1 \mathrm{d}\omega_2 \frac{\omega_1^{\epsilon}\omega_2^{\epsilon}}{(X_2+\omega_2+Y)(X_1+X_2+\omega_1+\omega_2)}\end{aligned}$$

- These integrals can be analysed using **twisted cohomology**

Planes and Tubes

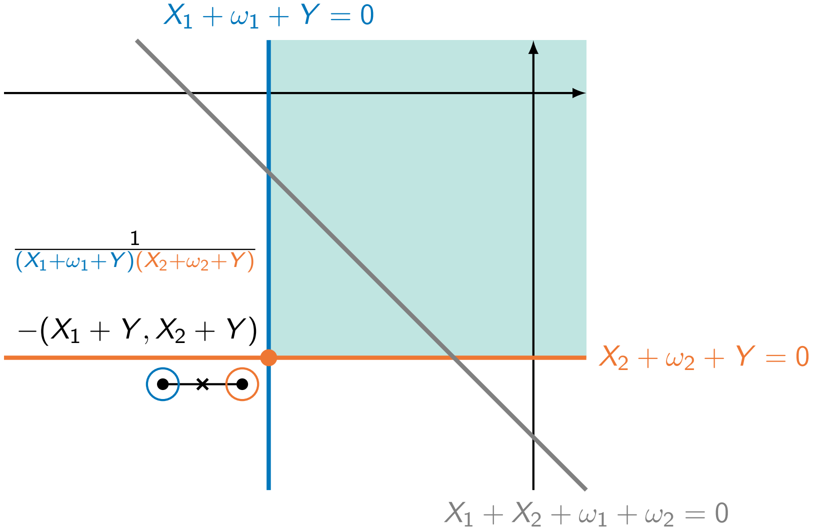
Energy Planes

Unbounded Canonical Forms



Energy Planes

Unbounded Canonical Forms



Energy Planes

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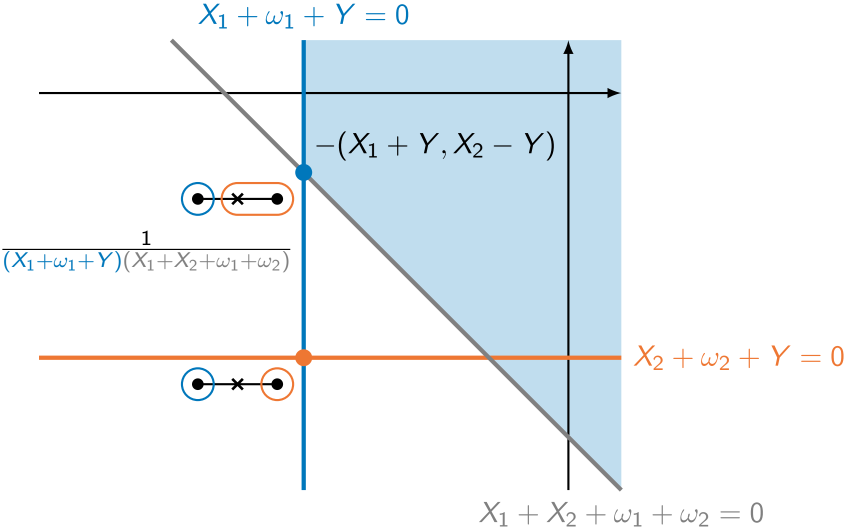

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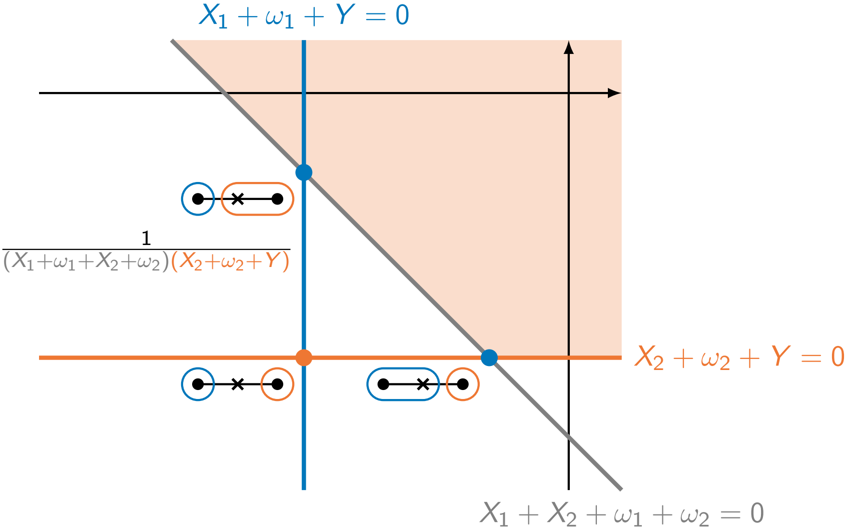

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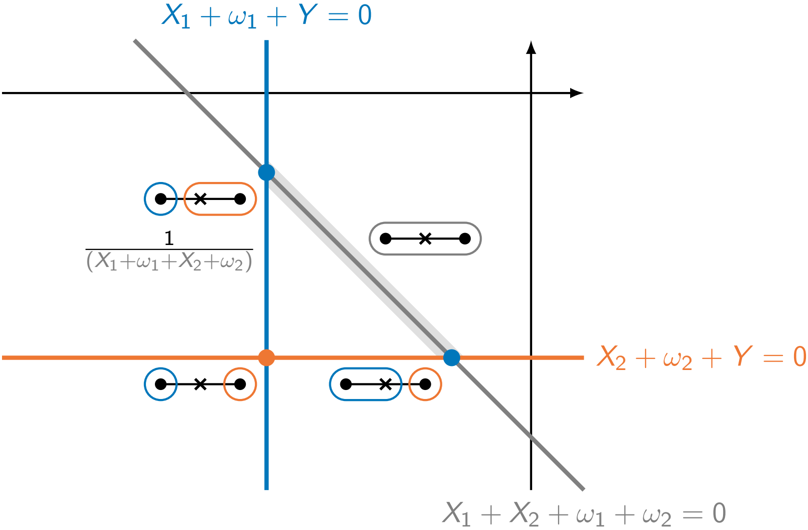


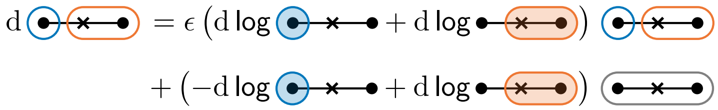
Kinematic Flow

Rule 1: Activation

Each tube in the tubing gets “activated” and becomes a letter in the differential equation.

Rule 2: Merger

Each tube containing a cross merges with adjacent tubes and the resulting tubing appears as a source

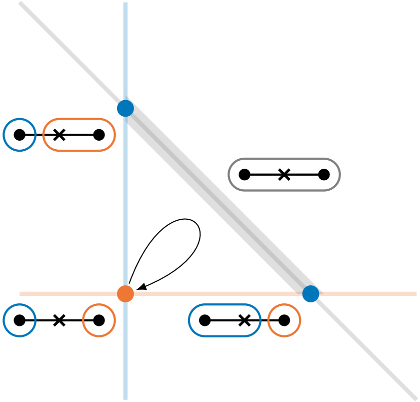


This new function permits no further mergers so the system must close

Geometric Structure

The Geometric Picture

Derivatives from planes



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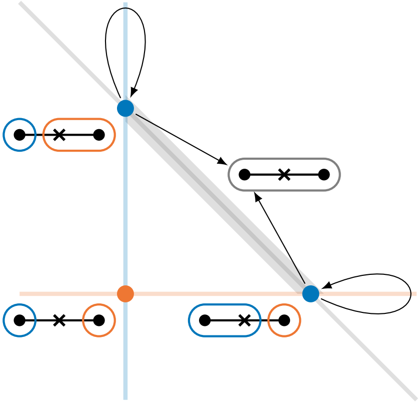
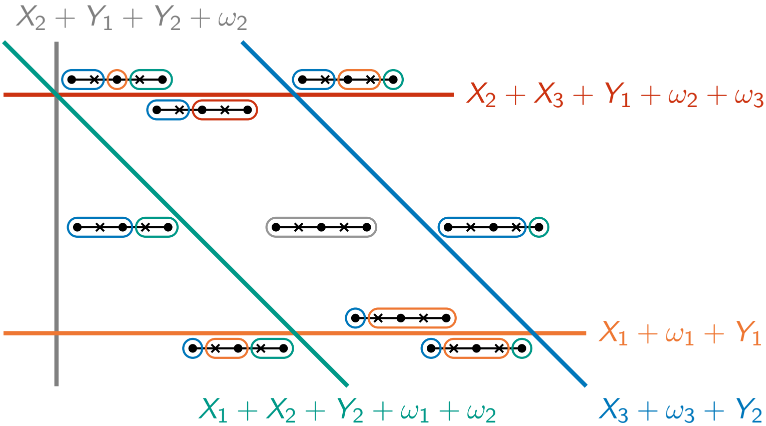


More Complex Graphs

5-point function



Summary

- There is a simple set of rules that describe the kinematic dependence of cosmological correlators
- These rules arrange the correlators into simple subsystems with a recursive structure
- This structure becomes clear when we consider the arrangement of hyperplanes in the energy space
- There is much to explore at both a fundamental and technical level:
    - Massive Fields
    - Loops
    - Spin
    - Connecting together diagrams

Thank you for listening

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