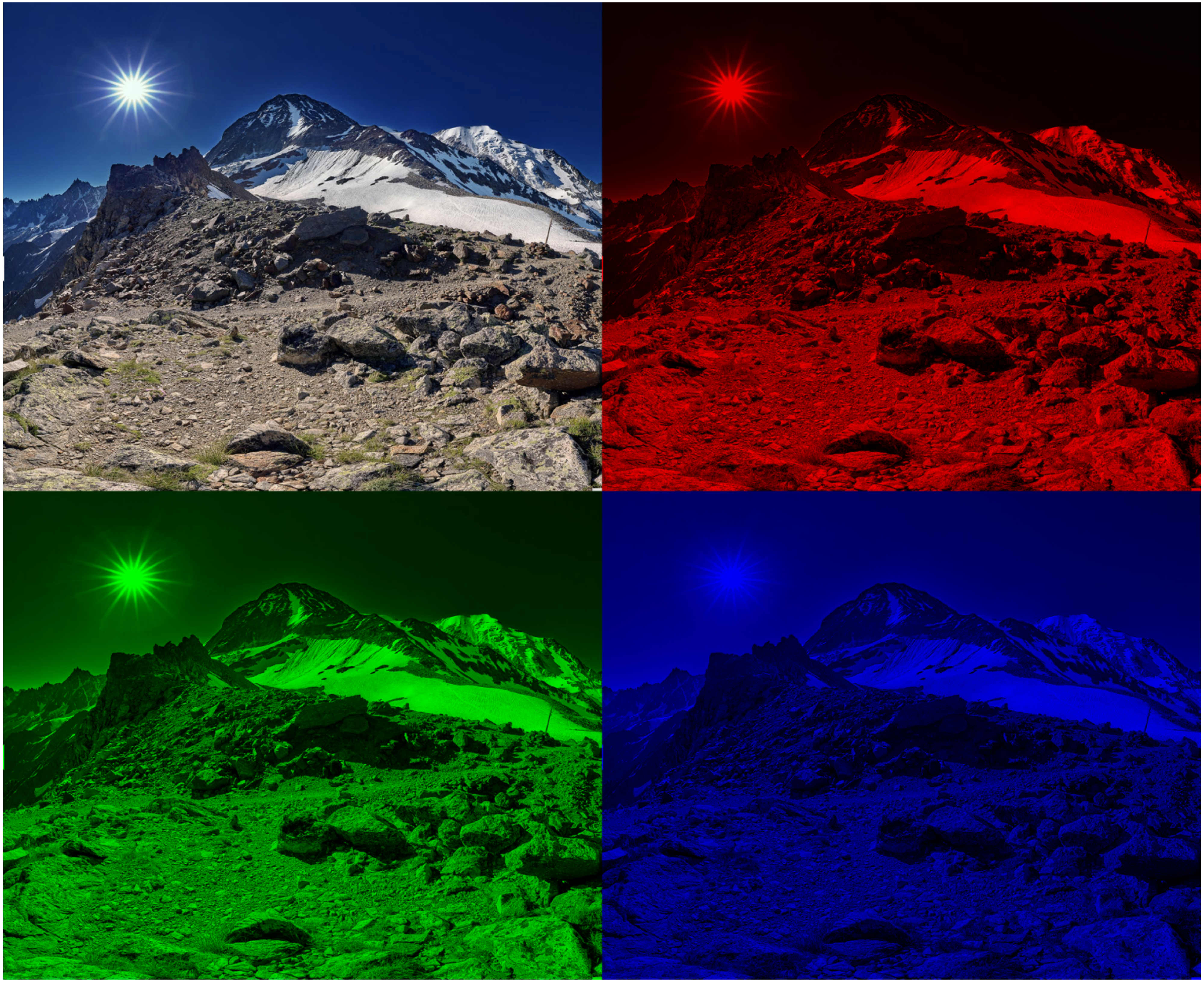
Spectral Purity and Purification in Imaging



Albert Stebbins
Fermilab

2025-06-02
Line Intensity Mapping 2025
Annecy, France

Line Intensity Mapping (LIM)

- LIM is a technique whereby one estimates the 3D spatial distribution of galaxies by from spectral/angular imaging of intensity - in the limit where individual galaxies are not resolved in angle.

- The 3rd dimension is redshift - which can be inferred from the spectrum if one can isolate the emission of one or more spectral lines.

- There can be confusion with other spectral lines corresponding to different redshifts as well as contamination by continuum emission.

    - Hydrogen intensity mapping (HIM) has nearly no line confusion but the continuum contamination is extremely large.

- Isolation of specific line emission can be differentiated from continuum emission because it produces sharp spectral features not produced by continuum emission.

Beam Chromaticity

- Differentiating the spectral pattern of emission from the angular pattern is essential for LIM to work well.

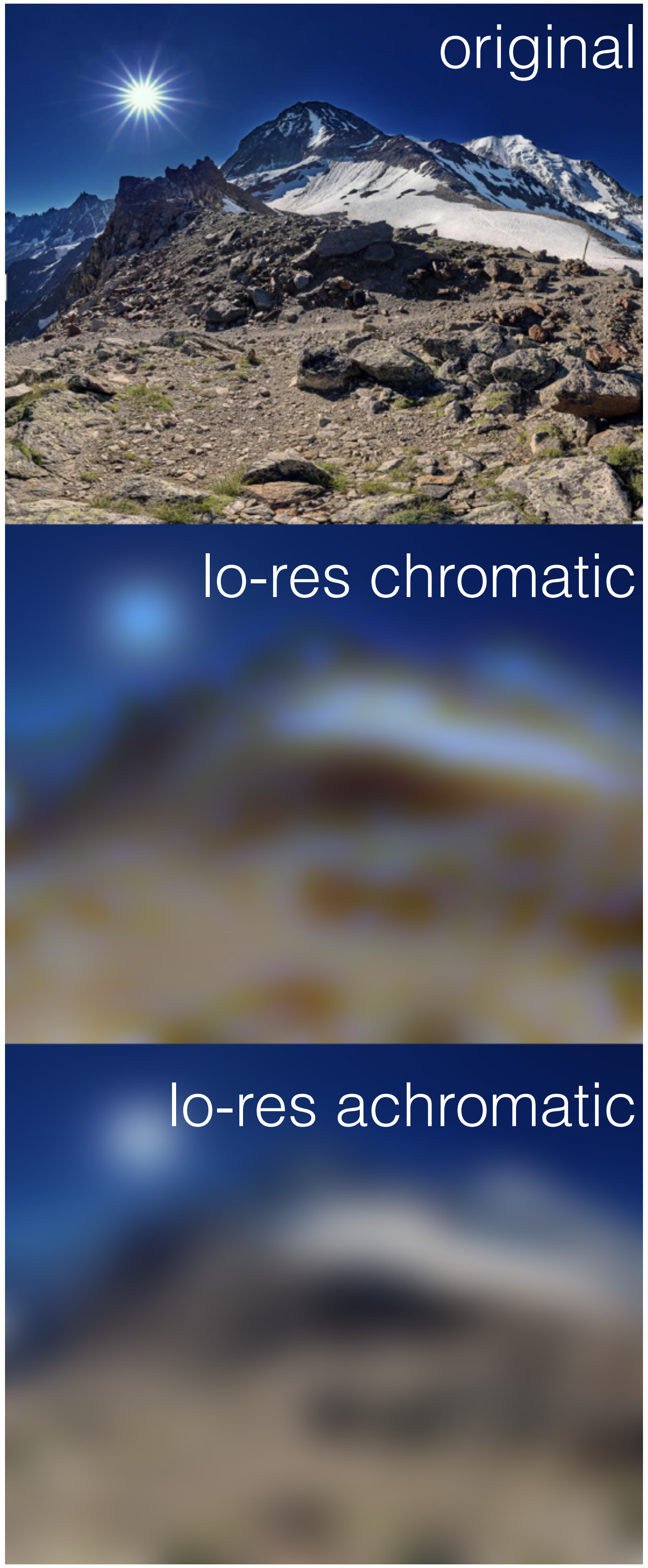
- Typically all spectral channels use the same imaging optics which is usually diffraction limited. Diffraction causes the beam size to grow with wavelength.

    - More generally any wavelength dependence of the angular beam shape is known a *chromaticity*. This is always present at some level.

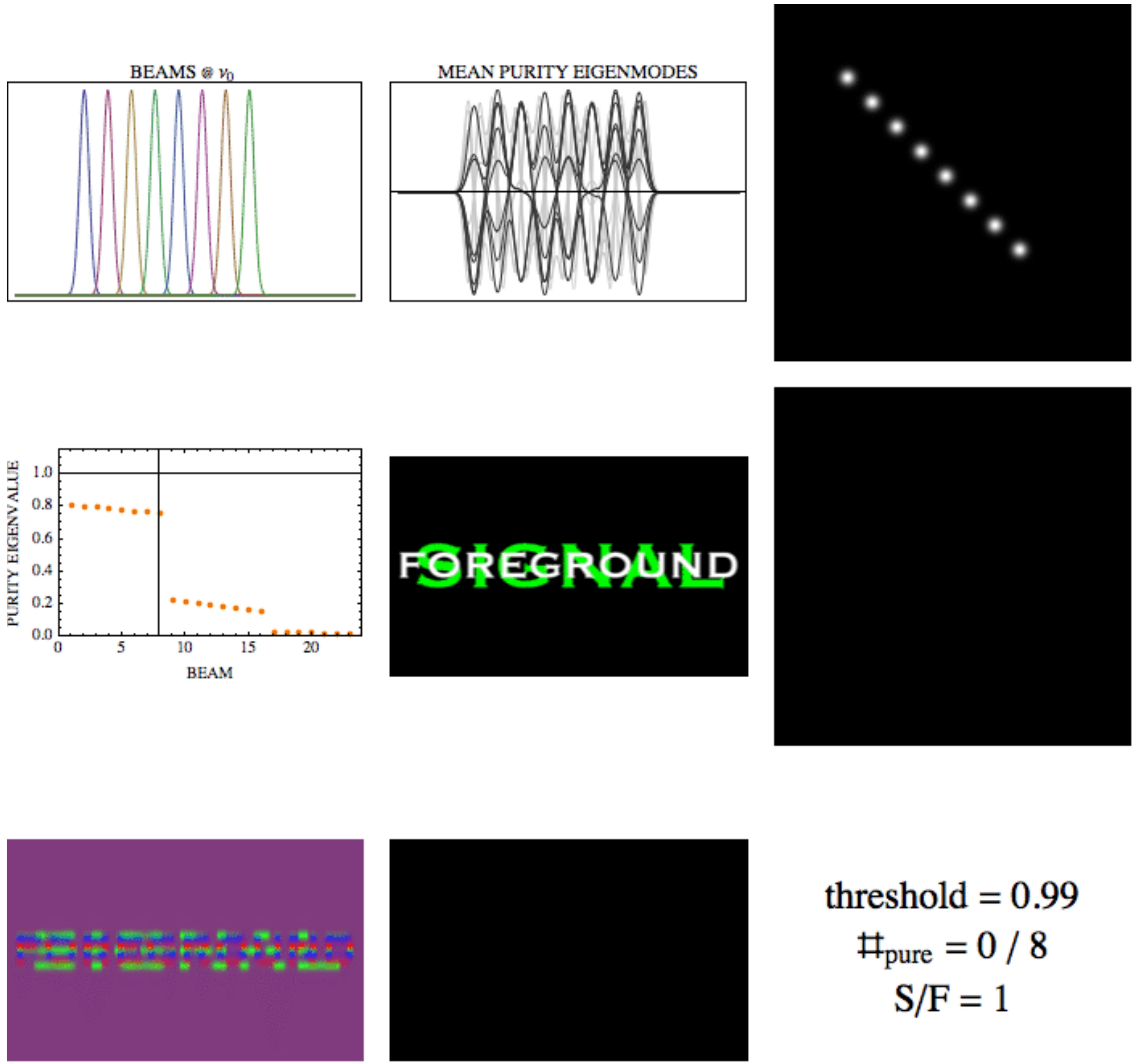
    - HIM imaging is often done with interferometers with which one can synthesize beams localized on the sky but these synthetic beams suffer from large achromaticity.

Spectral / Angular Aliasing

- Ignoring chromaticity leads to spatial structure being aliased into spectral structure and vice versa.
- imaging data generally does not have sufficient information to completely remove aliasing.
- One can optimally synthesize beams (linear superpositions) which are less chromatic.
- In addition: optimizing the optical design may significantly reduce the chromaticity of the optimal synthetic beams.



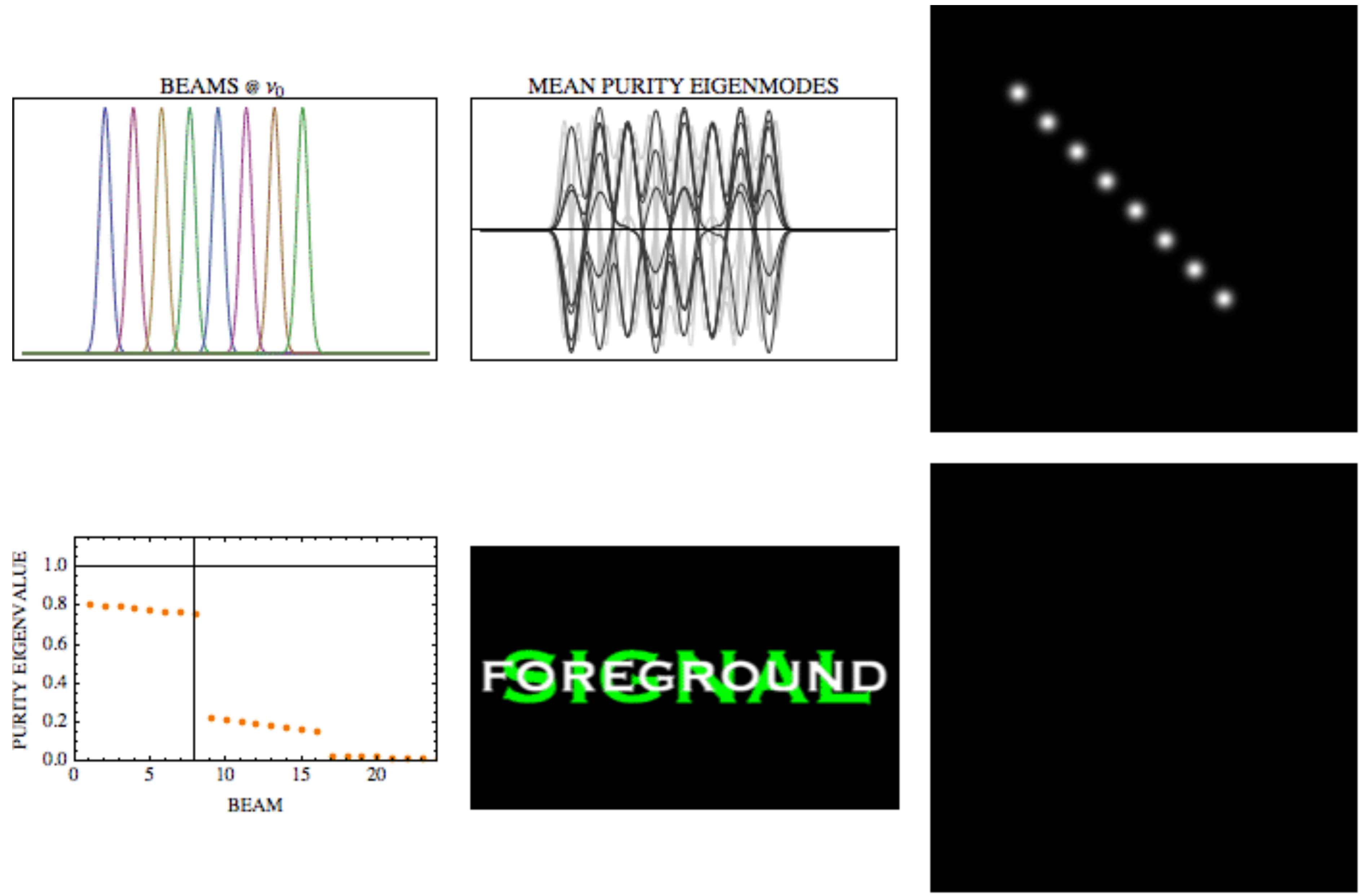
Spectral Purification: Signal/Foreground=1

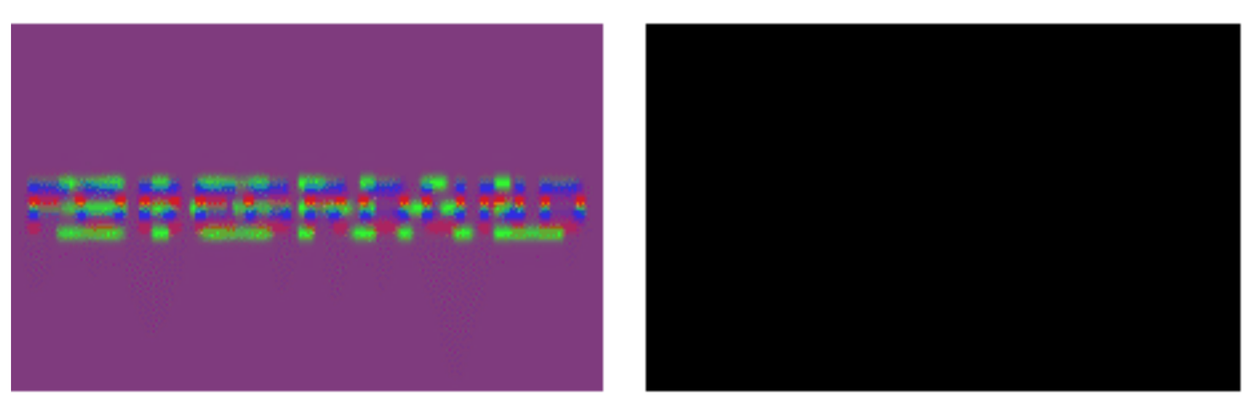


Zhang, Ansari++ 2016

non-overlapping beam ↔ high chromaticity: overlapping beam ↔ achromaticity

Spectral Purification: Signal/Foreground=1





threshold = 0.99

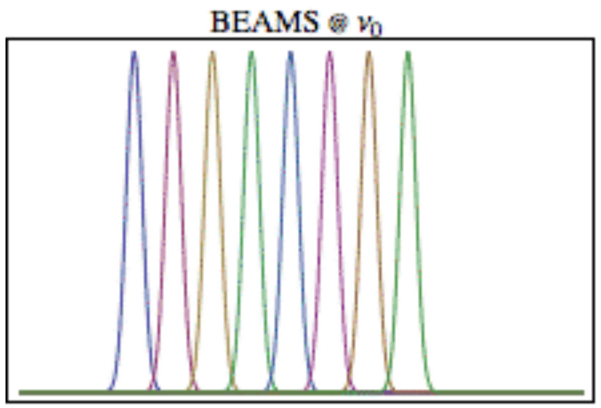
$\#_{pure} = 0 / 8$

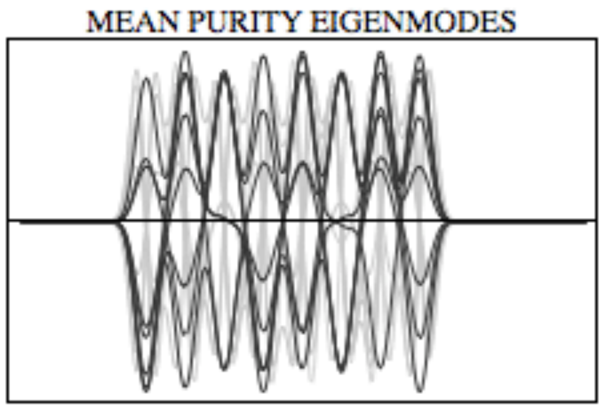
S/F = 1

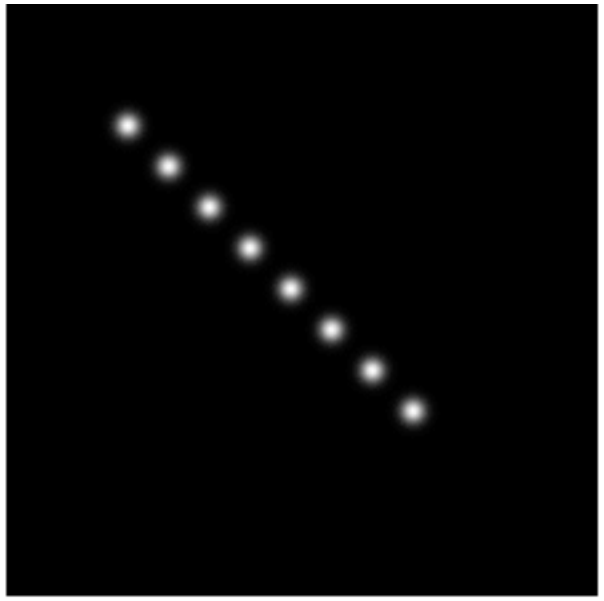
Zhang, Ansari++ 2016

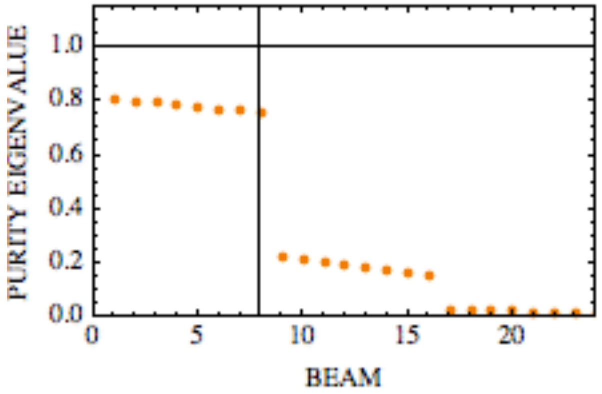
non-overlapping beam ↔ high chromaticity: overlapping beam ↔ achromaticity

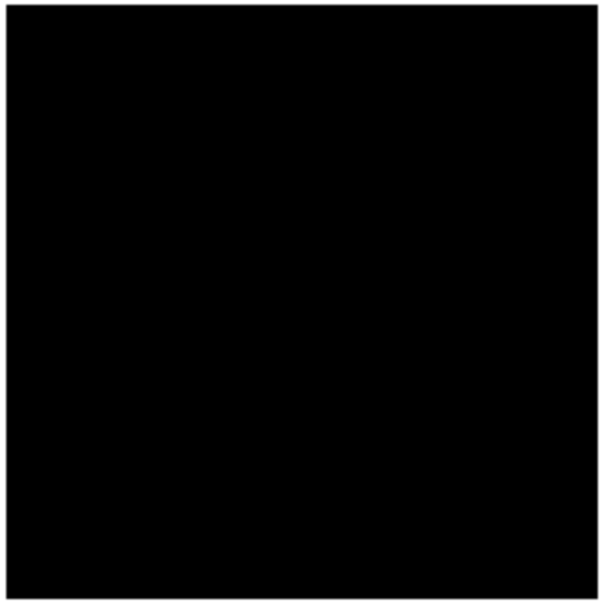
Spectral Purification: Signal/Foreground=0.1

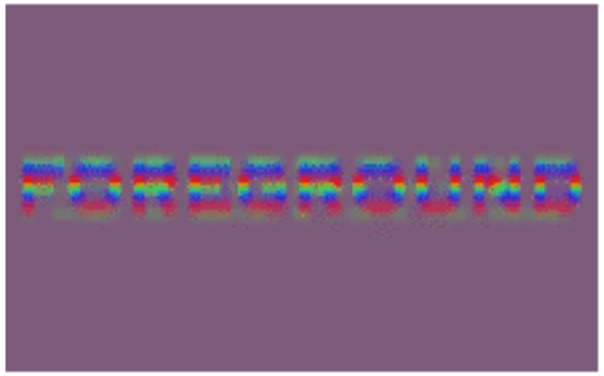










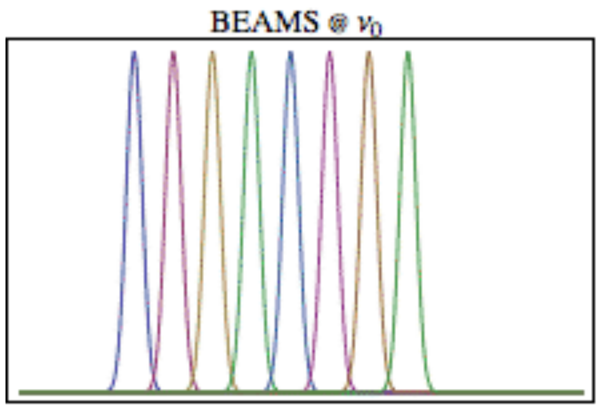


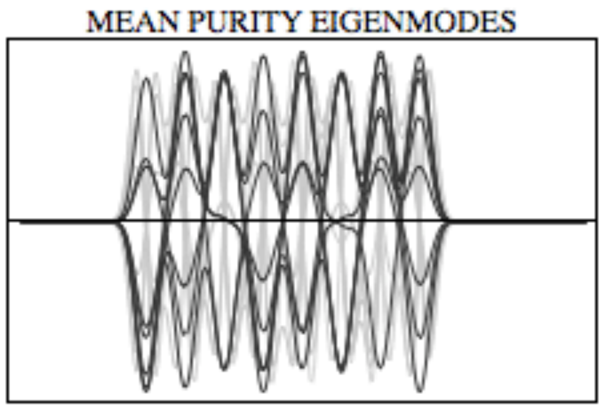
threshold = 0.99

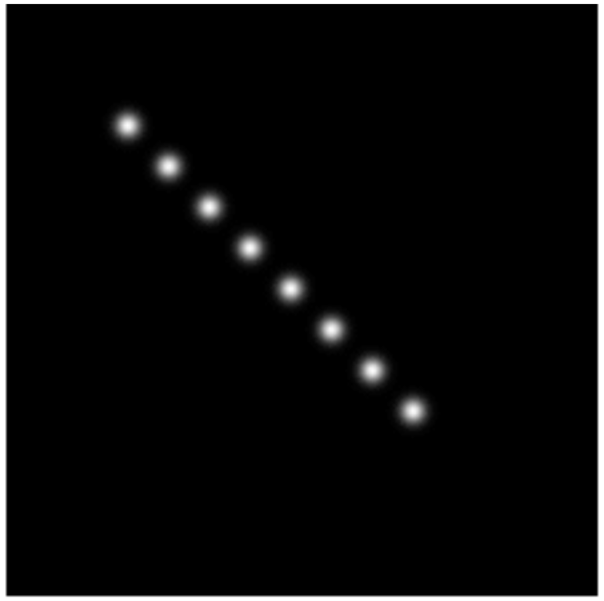
$\sharp_{\text{pure}} = 0 / 8$

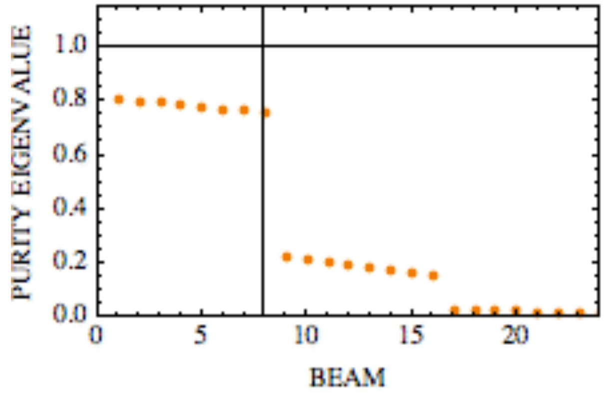
S/F = 0.1

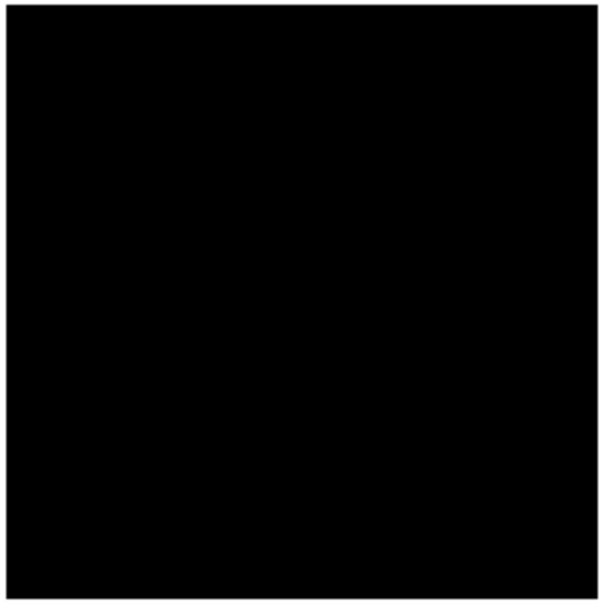
Spectral Purification: Signal/Foreground=0.1

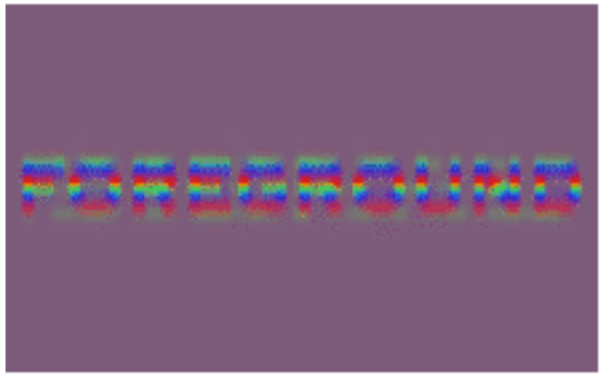










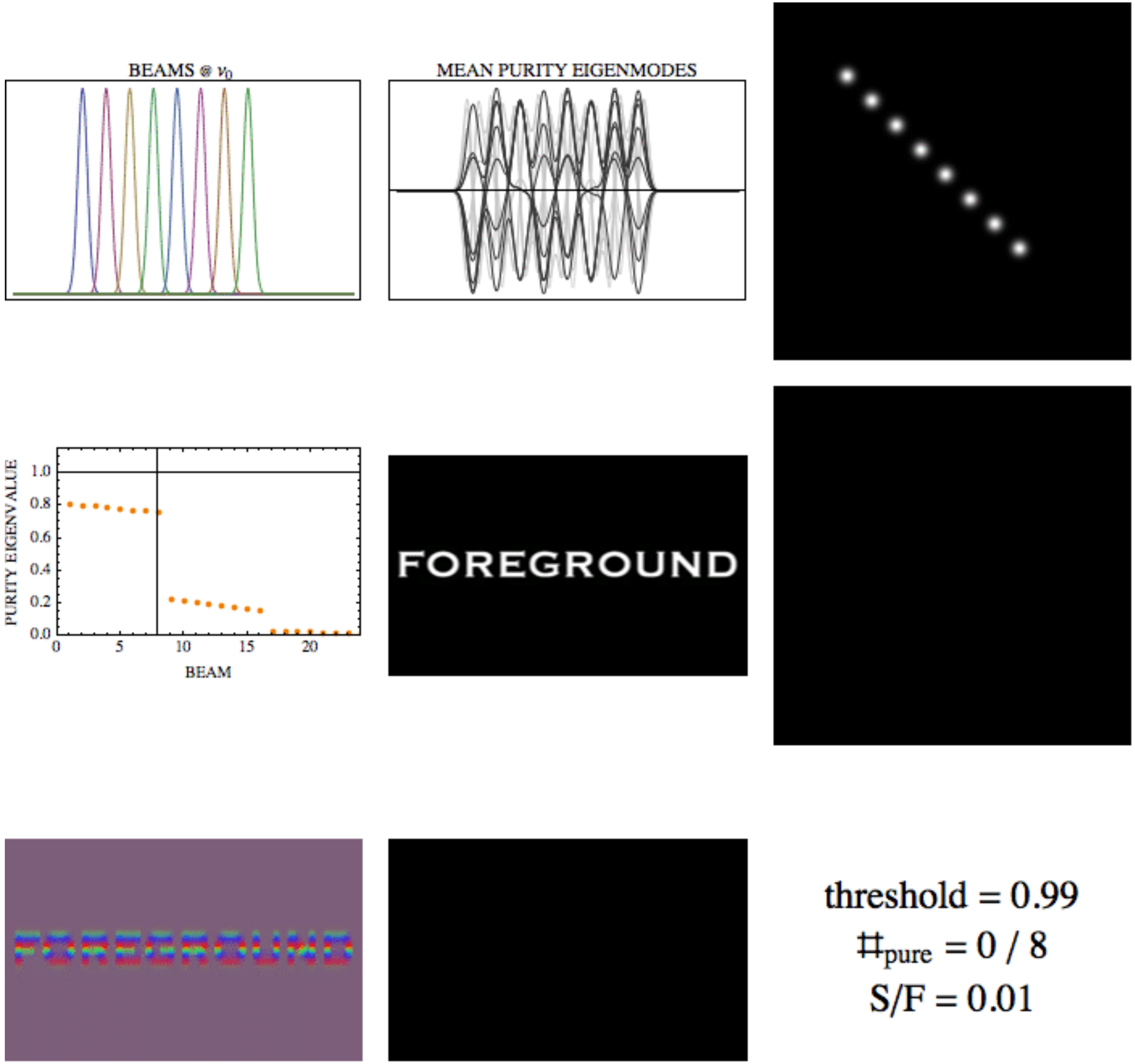


threshold = 0.99

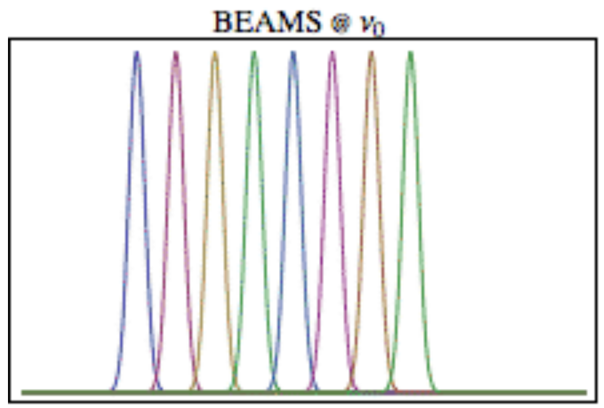
$\sharp_{pure} = 0 / 8$

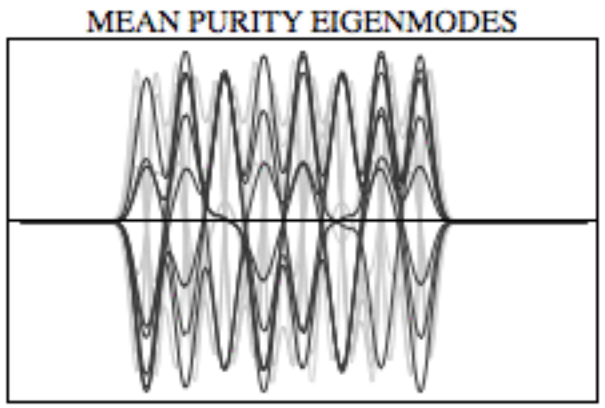
S/F = 0.1

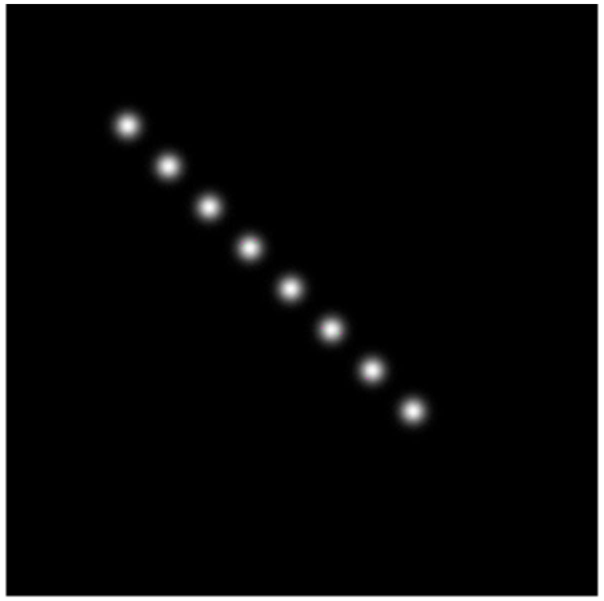
Spectral Purification: Signal/Foreground=0.01

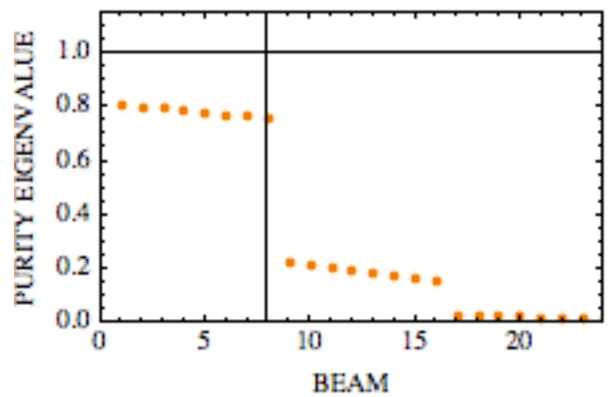


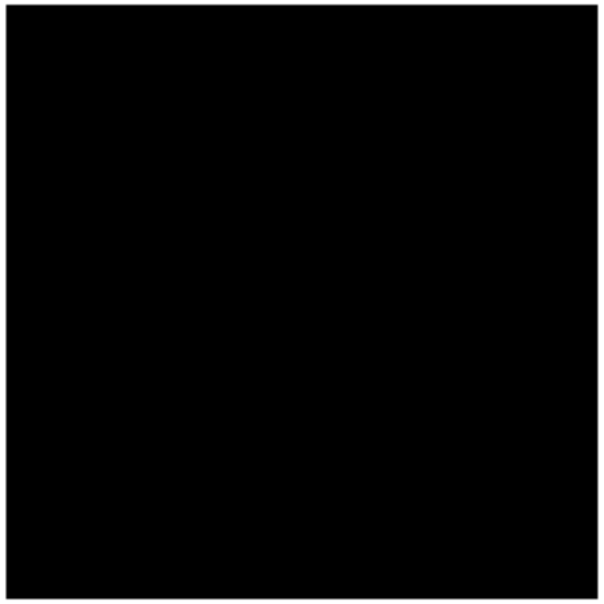
Spectral Purification: Signal/Foreground=0.01

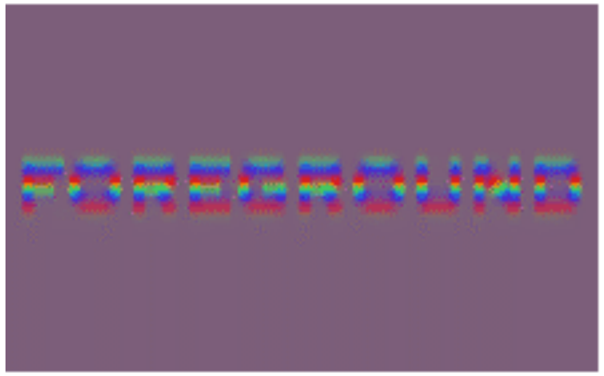












threshold = 0.99

$\sharp_{pure} = 0\ /\ 8$

S/F = 0.01

Mode Mixing: Abstract

- Any finite "camera" will only "see" a finite number of "beams" on the sky.
- The Hilbert space of all linear combinations of beams is the "space of beams".
- This space of beams varies with frequency. This frequency dependence of the space of beams is "mode mixing". It irreducibly mixes frequency dependence and angle dependence.
- Without mode mixing one could precisely measure the (spatially averaged) frequency spectrum with no contamination from angular structure. Generally this is not possible.
- Hi-Pass filtering out smooth spectrum foregrounds works better when the amount of mode mixing is minimized.
- Goal: to "purify" the spectrum from mode mixing contamination.
- Purification can be used to:
    - optimally analyze data from a given camera
    - optimize optical design of cameras

Re-Imaging / Beam Projection

- Given a camera with a finite set of beams (a.k.a. pixels) whose datum of output is

$$d_{i,\alpha} = \int d^2\hat{\mathbf{n}} \int d\nu \, B_{i,\alpha}[\hat{\mathbf{n}},\nu] \, I_\nu[\hat{\mathbf{n}}] + N_{i,\alpha}$$

  where i is the beam number and α the frequency channel.

  - $N_{i,\alpha}$ is the noise which we henceforth ignore.
  - The space-of-beams has dimensions given by the number of $B_{i,\alpha}[\hat{\mathbf{n}},\nu]$,
    - i.e. $n_{\mathrm{beam}} \times n_{\mathrm{ch}}$ (number of angular beams times number of frequency channels)

- Define a metric, $\circ$, on sky pattern

$$(f \circ g) \equiv \int d\nu \int d\nu' \int d^2\hat{\mathbf{n}} \int d^2\hat{\mathbf{n}}' \, K[\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}';\nu,\nu'] \, f[\hat{\mathbf{n}},\nu] \, g[\hat{\mathbf{n}}',\nu']$$

  - the kernel K is used to weight angular scales and frequencies according to one's needs.
  - e.g. $K[\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}';\nu,\nu'] = \delta^{(2)}[\hat{\mathbf{n}},\hat{\mathbf{n}}']$ weights all angular scales frequencies equally

- The "re-imaged" or "beam projected"data is

$$\hat{I}_\nu[\hat{\mathbf{n}}] \equiv \sum_{i,\alpha} \sum_{j,\beta} B_{i,\alpha}[\hat{\mathbf{n}},\nu] \, (B_{i,\alpha} \circ B_{j\beta})^{-1} \, d_{j,\beta}$$

  - $\hat{I}_\nu[\hat{\mathbf{n}}]$ is in the space of beams. Everything else is set to zero.
  - $\hat{I}_\nu[\hat{\mathbf{n}}]$ will have mode mixing if the space-of-beams does.

Spectral Purity

- Consider the simplest case of factorizable beams with uniform frequency channels ($i = 1,\ldots,n_{\rm beam}$ and $\alpha = 1,\ldots,n_{\rm ch}$)

  $$B_{i,\alpha}[\hat{\mathbf{n}},\nu] = b_{i,\alpha}[\hat{\mathbf{n}}]\,\beta_\alpha[\nu] \quad \text{where} \quad \int d\nu\,\beta_\alpha[\nu]\,\beta_\beta[\nu] = \delta_{\alpha,\beta}$$

  and scale-free weighting

  $$K[\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}';\nu,\nu'] = \delta^{(2)}[\hat{\mathbf{n}},\hat{\mathbf{n}}']\,\delta[\nu-\nu']$$

- One can construct a complete orthonormal bases ($a = 1,\ldots,n_{\rm beam}$ and $\alpha = 1,\ldots,n_{\rm ch}$)

  $$\hat{B}_{a,\alpha}[\hat{\mathbf{n}},\nu] = \hat{b}_{a,\alpha}[\hat{\mathbf{n}}]\,\beta_\alpha[\nu] \text{ where } \int d^2\hat{\mathbf{n}}\,\hat{b}_{a,\alpha}[\hat{\mathbf{n}}]\,\hat{b}_{b,\alpha}[\hat{\mathbf{n}}] = \delta_{a,b} \text{ so } \hat{B}_{a,\alpha}\circ\hat{B}_{b,\beta} = \delta_{a,b}\,\delta_{\alpha,\beta}$$

  where the α dependence of $\hat{b}_{a,\alpha}[\hat{\mathbf{n}}]$ gives the chromaticity of spectrograph a.

  - An achromatic spectrograph would have no α dependence which is generally not possible.
  - One can quantify the achromaticity or spectral purity of a spectrograph a by

    $$p_a \equiv \int d^2\hat{\mathbf{n}}\,\bar{b}_a[\hat{\mathbf{n}}]^2 \quad \text{where} \quad \bar{b}_a[\hat{\mathbf{n}}] \equiv \frac{1}{n_{\rm ch}}\sum_{\alpha=1}^{n_{\rm ch}}\hat{b}_{a,\alpha}[\hat{\mathbf{n}}]$$

  - $p_a \in [0,1]$ $\quad \sum_{a=1}^{n_{\rm ch}} p_a = n_{\rm beam}$ $\quad p_a = 1$ only for an achromatic spectrograph

- .One can define the purity orthonormal basis where $\hat{b}_{a,\alpha}[\hat{\mathbf{n}}] = \hat{p}_{a,\alpha}[\hat{\mathbf{n}}]$ and p_1 has the largest possible value, p_2 has the largest possible value in the subspace orthogonal to $\hat{p}_{1,\alpha}[\hat{\mathbf{n}}]$, p_3 has the largest value in the subspace orthogonal to $\hat{p}_{1,\alpha}[\hat{\mathbf{n}}]$ and $\hat{b}_{2,\alpha}[\hat{\mathbf{n}}]$, etc

- This is a type of Karhunen-Loeve decomposition similar to that developed in Shaw et al. (2013 & 2014) *m*-mode analysis.

Purification

- The re-imaging / beam projection algorithm in the simple case described in the previous slide is

$$\hat{I}_\nu[\hat{\mathbf{n}}] = \int d^2\hat{\mathbf{n}}' \int d\nu' \, \mathscr{B}[\hat{\mathbf{n}}, \hat{\mathbf{n}}'; \nu, \nu'] \, I_{\nu'}[\hat{\mathbf{n}}']$$

which uses the beam projection kernel:

$$\mathscr{B}[\hat{\mathbf{n}}, \hat{\mathbf{n}}'; \nu, \nu'] \equiv \sum_{i,\alpha} \sum_{j,\beta} (B_{i,\alpha} \circ B_{j\,\beta})^{-1} \, B_{i,\alpha}[\hat{\mathbf{n}}, \nu] \, B_{j,\alpha}[\hat{\mathbf{n}}', \nu'] = \sum_{a=1}^{n_{\rm beam}} \sum_{\alpha=1}^{n_{\rm ch}} \sum_{b=1}^{n_{\rm beam}} \sum_{\beta=1}^{n_{\rm ch}} \hat{p}_{a,\alpha}[\hat{\mathbf{n}}] \, \beta_\alpha[\nu] \, \hat{p}_{b,\beta}[\hat{\mathbf{n}}'] \, \beta_\alpha[\nu']$$

- One can modify this algorithm to give a "purified image" which has less chromaticity / mode mixing by choosing a minimal threshold, $p_{\rm min}$, which only uses the purity basis up to $a \leq n_{\rm pure}$ the achromaticity parameter

$$\hat{I}_\nu[\hat{\mathbf{n}}] = \int d^2\hat{\mathbf{n}}' \int d\nu' \, \mathscr{P}[\hat{\mathbf{n}}, \hat{\mathbf{n}}'; \nu, \nu'] \, I_{\nu'}[\hat{\mathbf{n}}']$$

which uses the purification kernel:

$$\mathscr{P}[\hat{\mathbf{n}}, \hat{\mathbf{n}}'; \nu, \nu'] \equiv \sum_{a=1}^{n_{\rm pure}} \sum_{\alpha=1}^{n_{\rm ch}} \sum_{b=1}^{n_{\rm pure}} \sum_{\beta=1}^{n_{\rm ch}} \hat{p}_{a,\alpha}[\hat{\mathbf{n}}] \, \beta_\alpha[\nu] \, \hat{p}_{b,\beta}[\hat{\mathbf{n}}'] \, \beta_\alpha[\nu']$$

- If $p_{\rm min}$ is very close to unity or the space-of-beams has a lot of mode-mixing then the purified image has thrown out a lot of data.

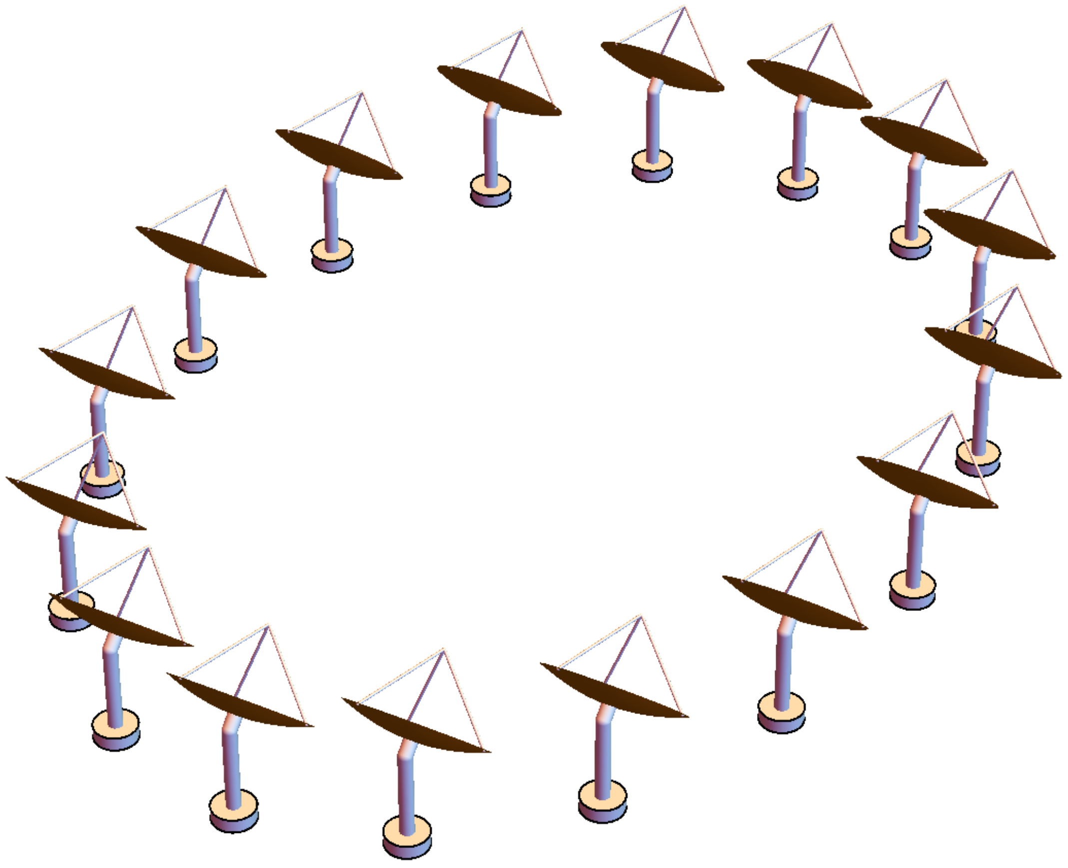
- In the Shaw et al. papers it was found in the case where continuum contamination was large that very little useful information was contained modes without very high purity.

- This purification algorithm was used in the examples in this talk

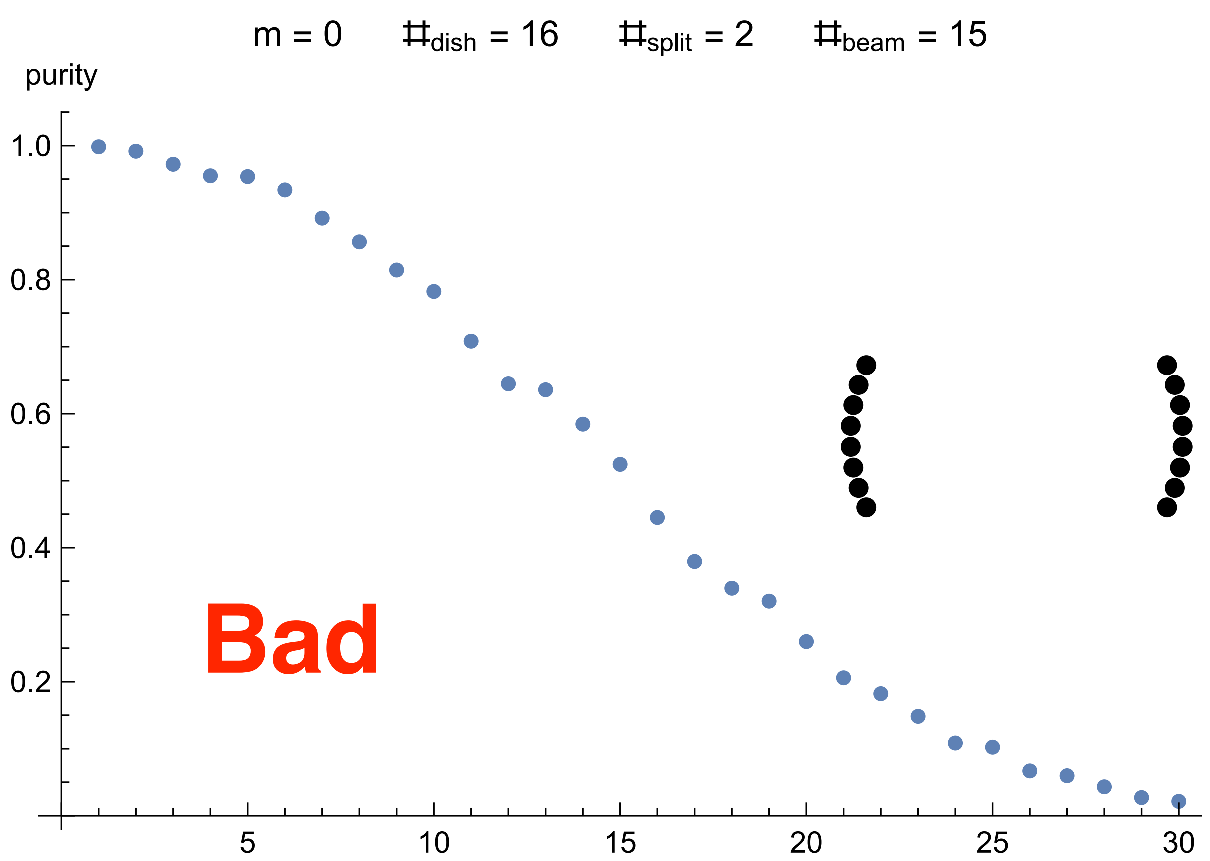
Example: Polarscope Dish Interferometer

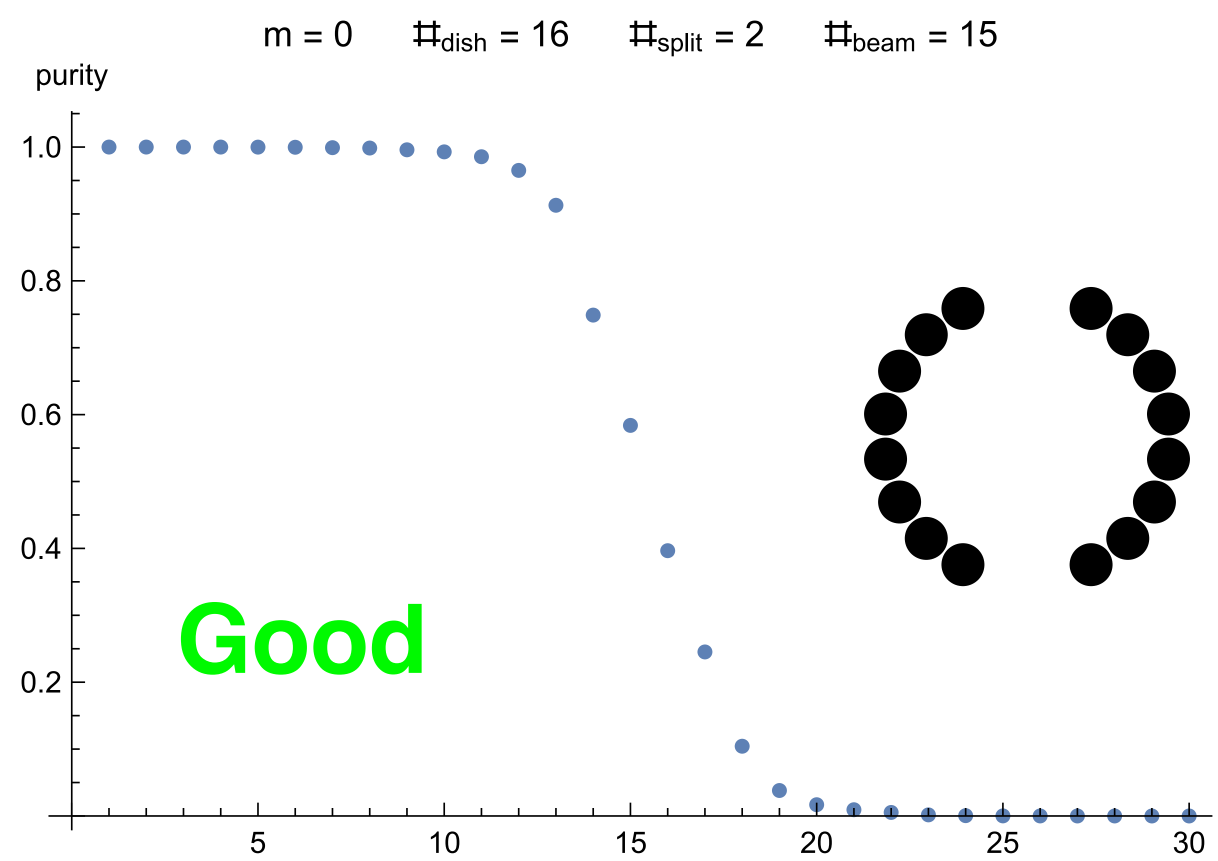
The Tianlai project has a 16 dish interferometer has long integrations staring at the North Celestial Pole for Hydrogen Intensity Mapping (HIM) using Earth rotation to create synthetic beam using *m*-mode analysis.

One can use the formalism outlines above to determine the dish configuration which minimizes the chromaticity of the synthetic beams.

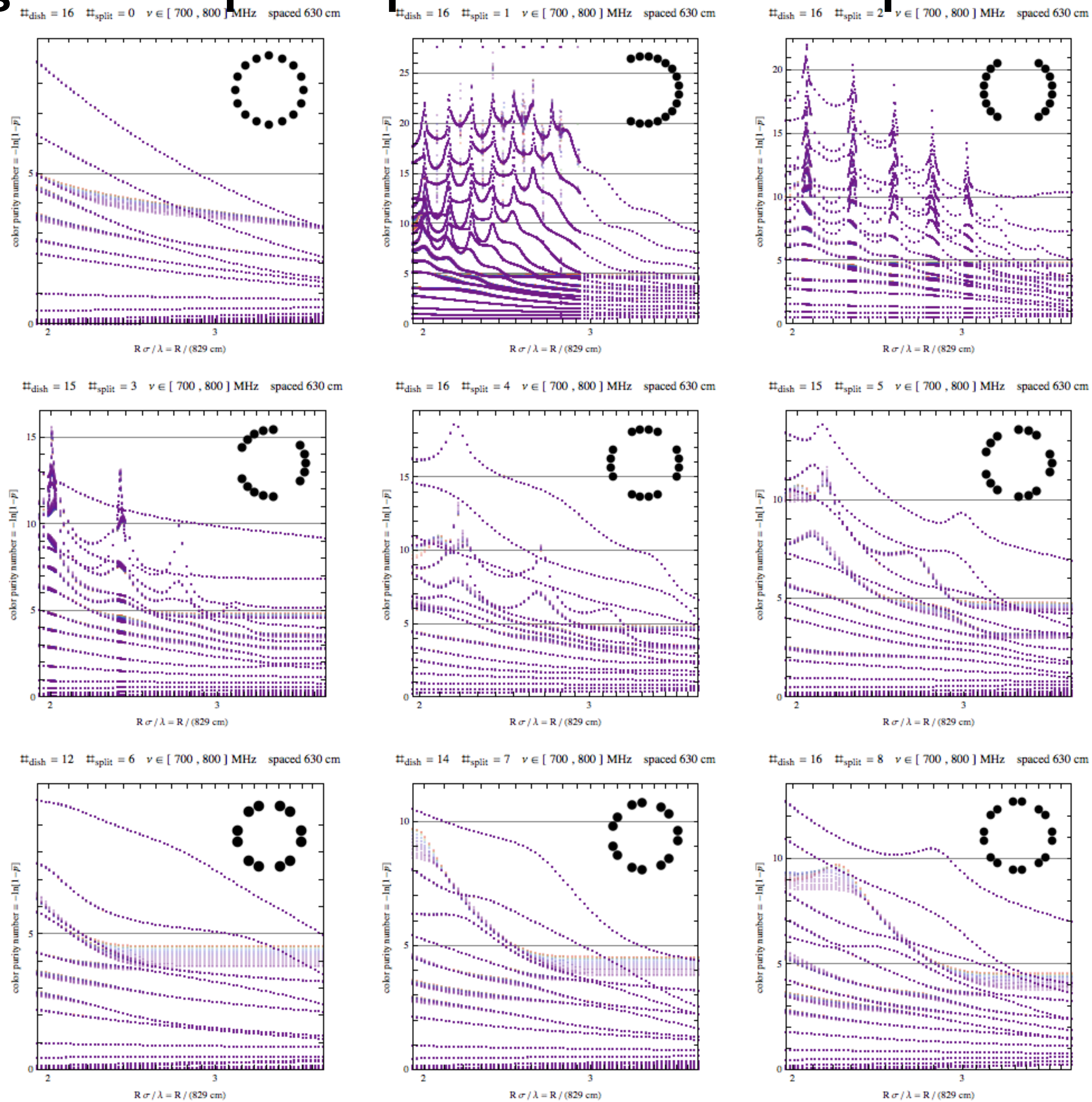


Purity and Telescope Design





configuration space: split circle into *n* compact subarrays



best performance: split into two compact subarrays

there exist purity "resonances" where astounding purity is attained.

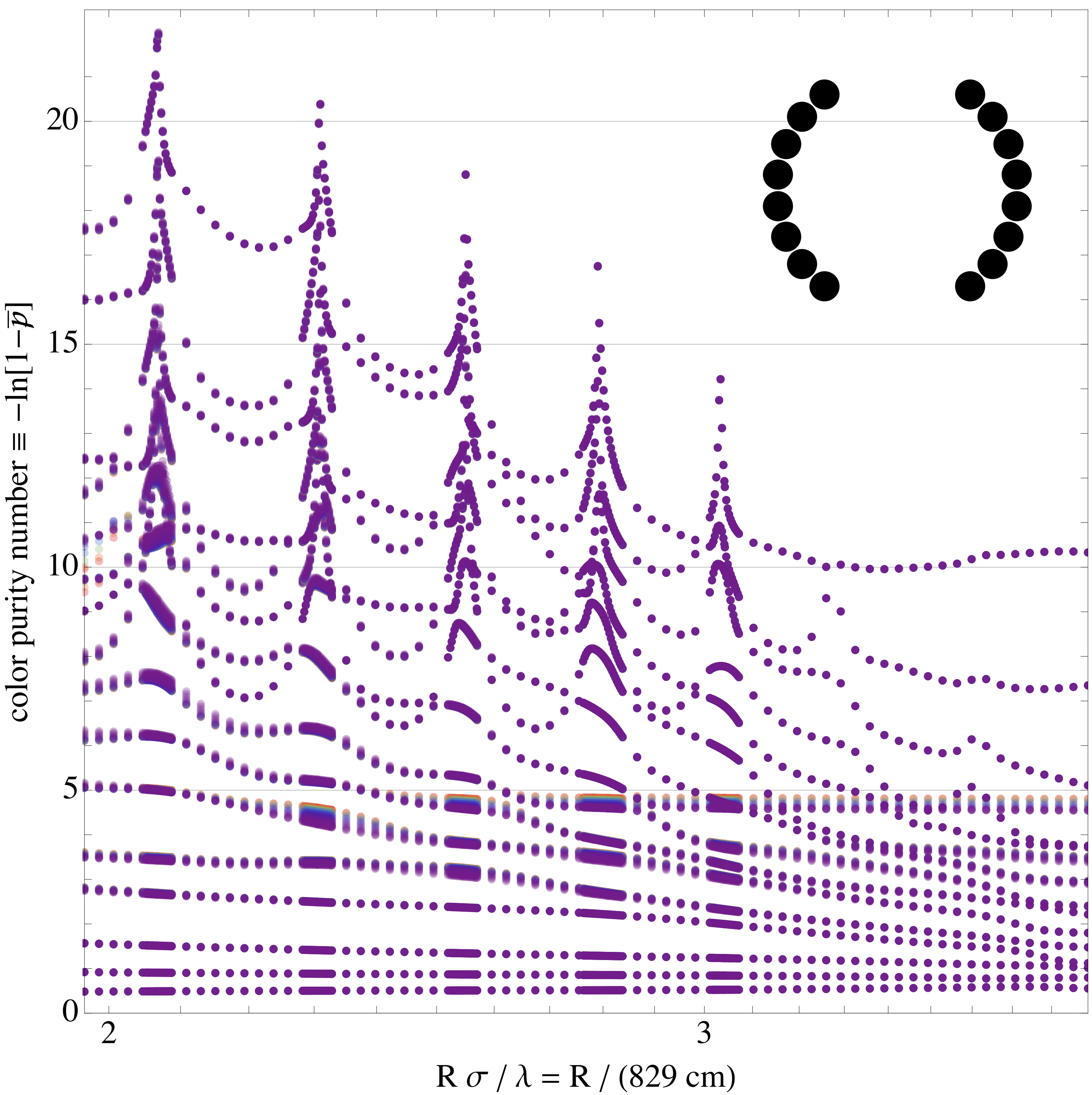
resonances are "narrow" w/ few cm tolerance

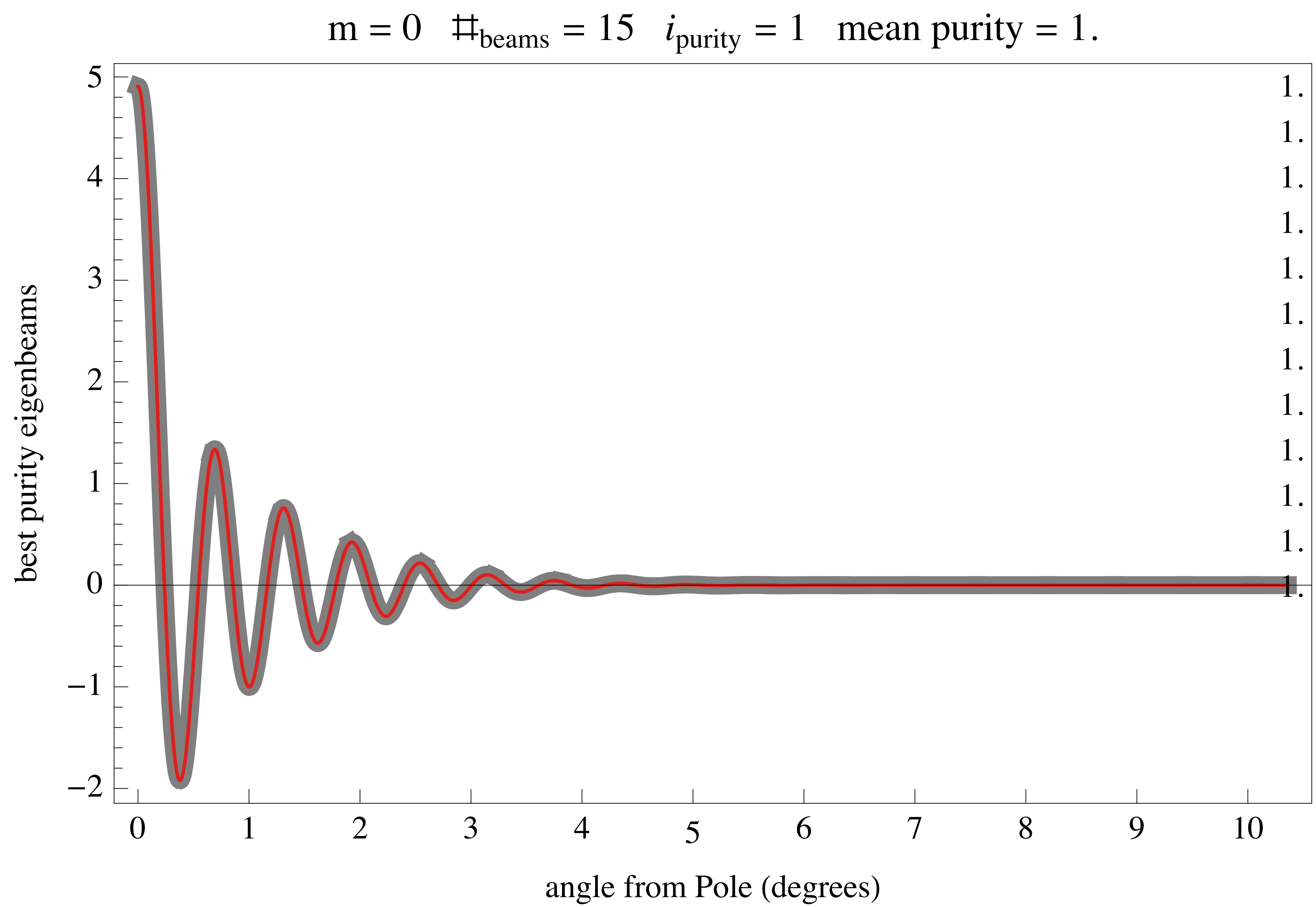
lowest purity attained near "singularities"

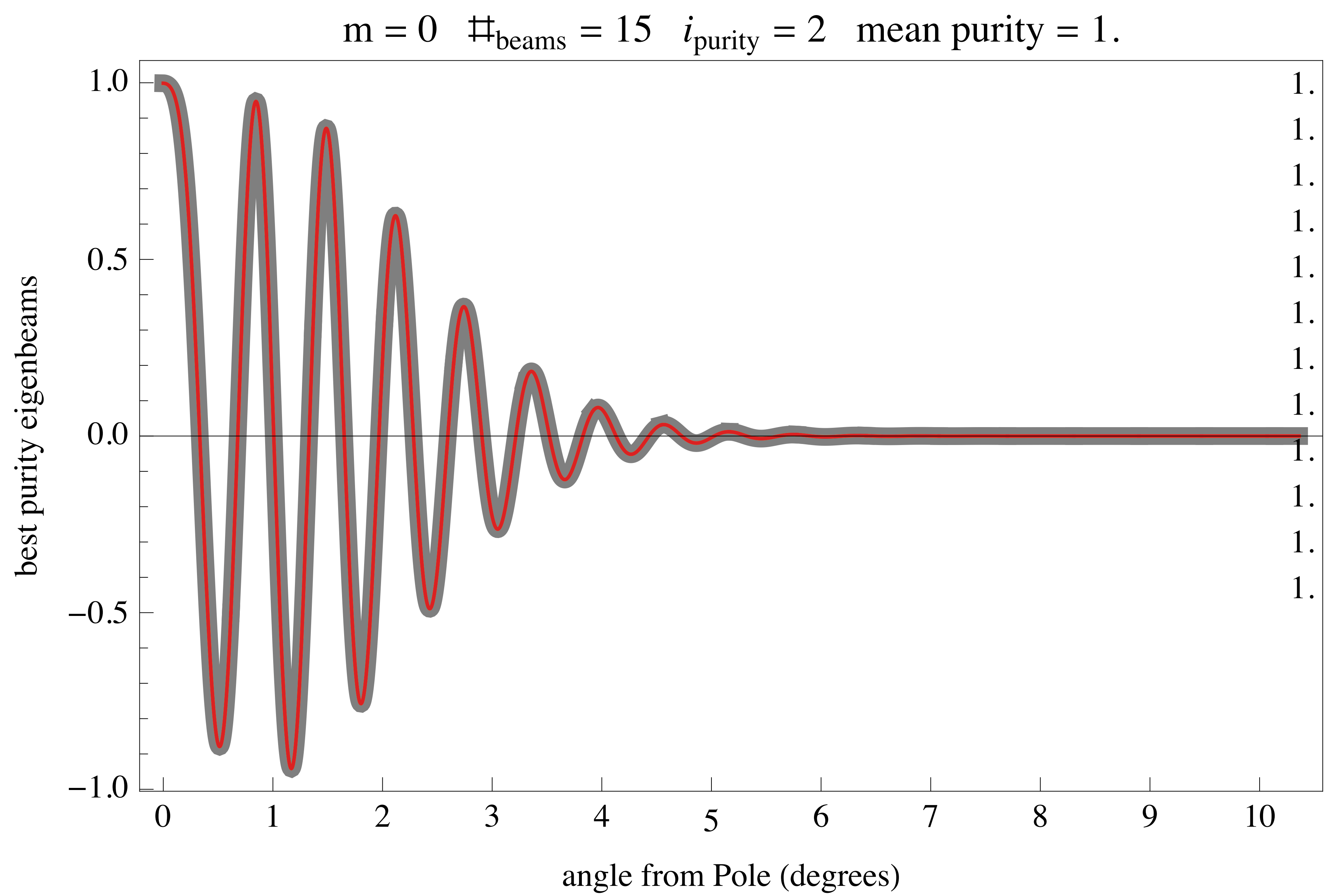
singularities are array configurations where two baselines become equal and the number of independent beams decreases.

resonances are not the most compact configuration

$\#_{dish} = 16 \quad \#_{split} = 2 \quad \nu \in [\ 700\ ,\ 800\ ]$ MHz spaced 630 cm




m = 0 ♯beams = 15 ipurity = 1 mean purity = 1.
best purity eigenbeams
angle from Pole (degrees)
1.
1.
1.
1.
1.
1.
1.
1.
1.
1.
1.
1.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best purity eigenbeams
angle from Pole (degrees)
1.0
0.5
0.0
−0.5
−1.0
0
1
2
3
4
5
6
7
8
9
10
1.
1.
1.
1.
1.
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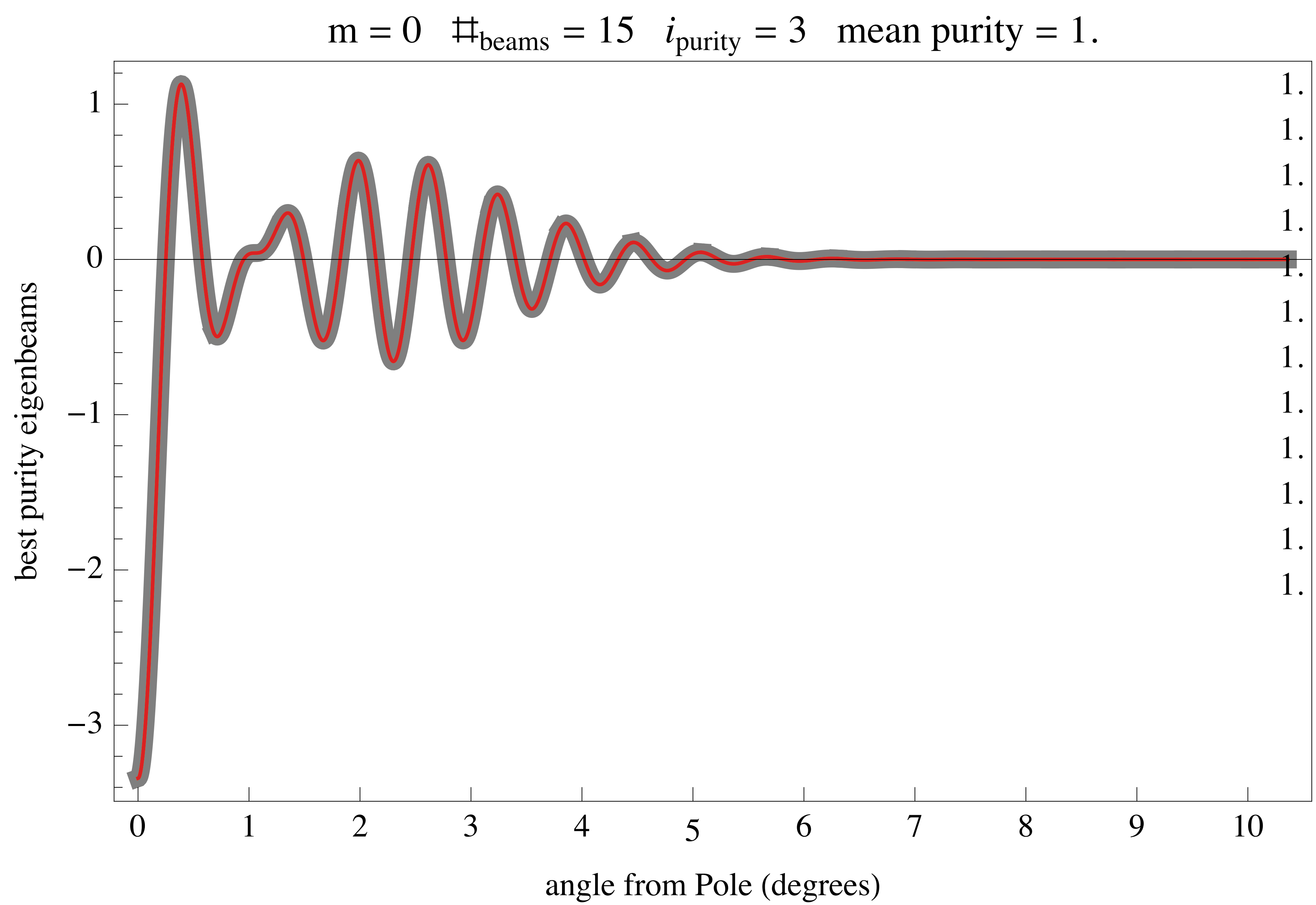
1.


m = 0 ♯beams = 15 ipurity = 3 mean purity = 1.
best purity eigenbeams
angle from Pole (degrees)
1
0
−1
−2
−3
0 1 2 3 4 5 6 7 8 9 10
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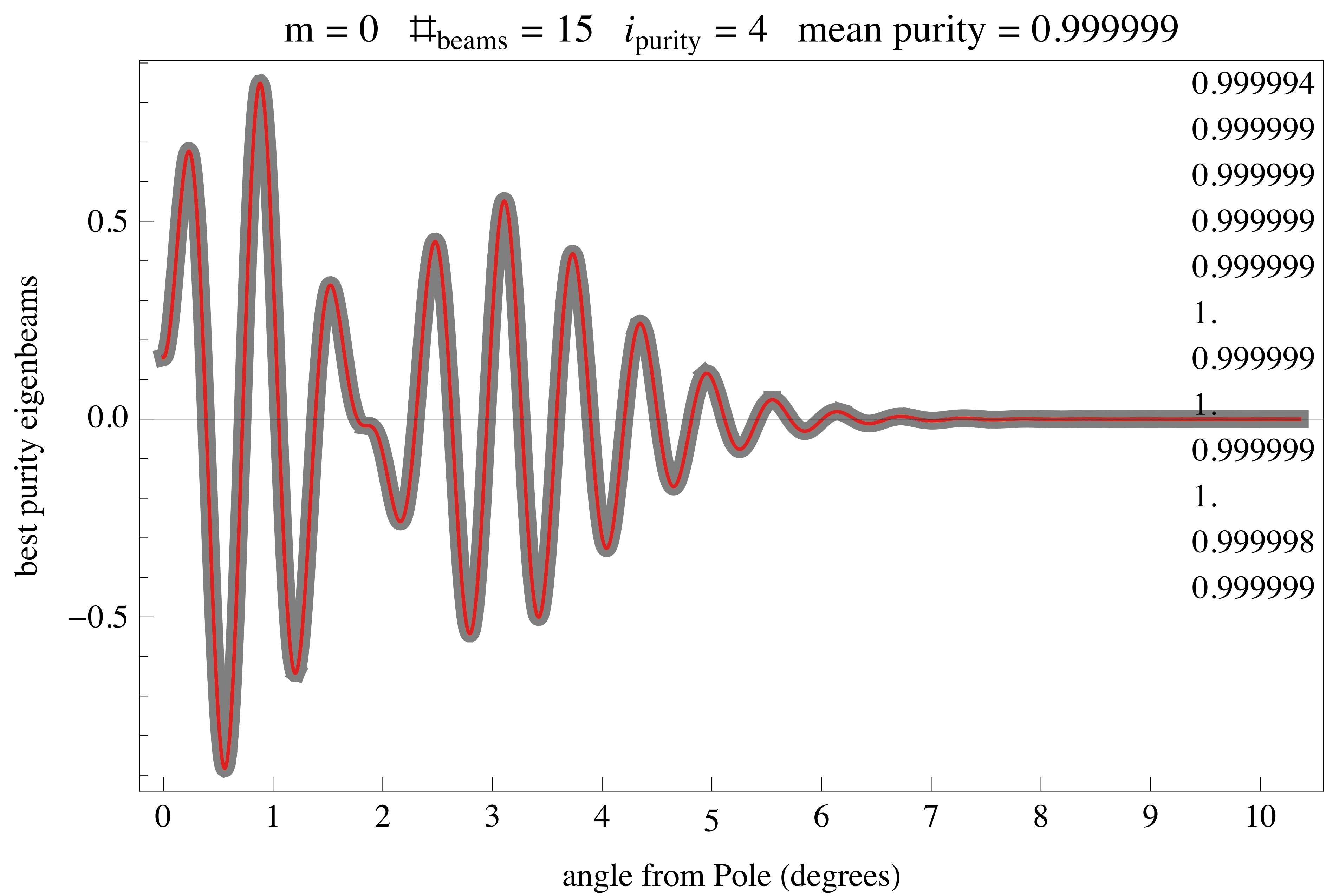
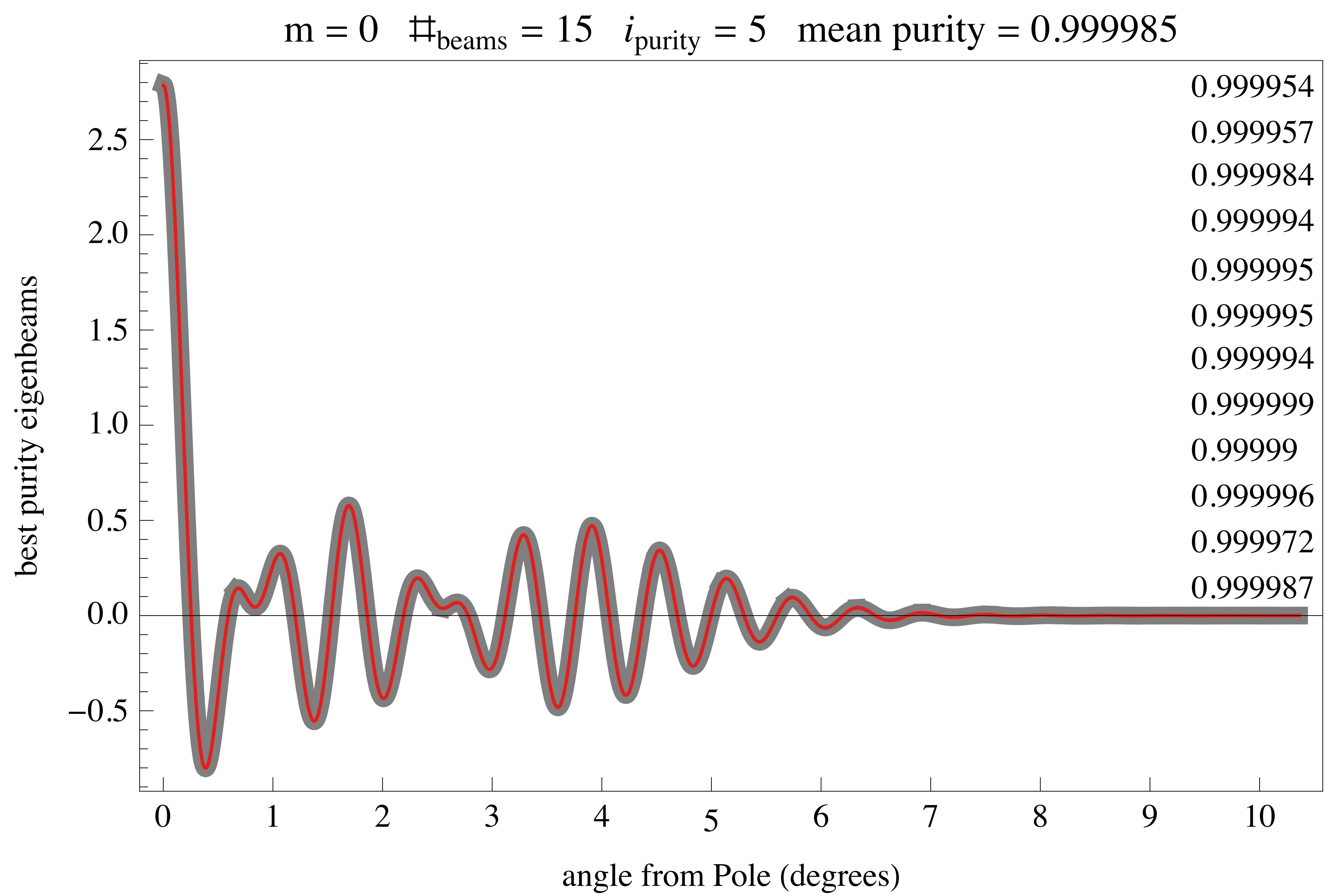
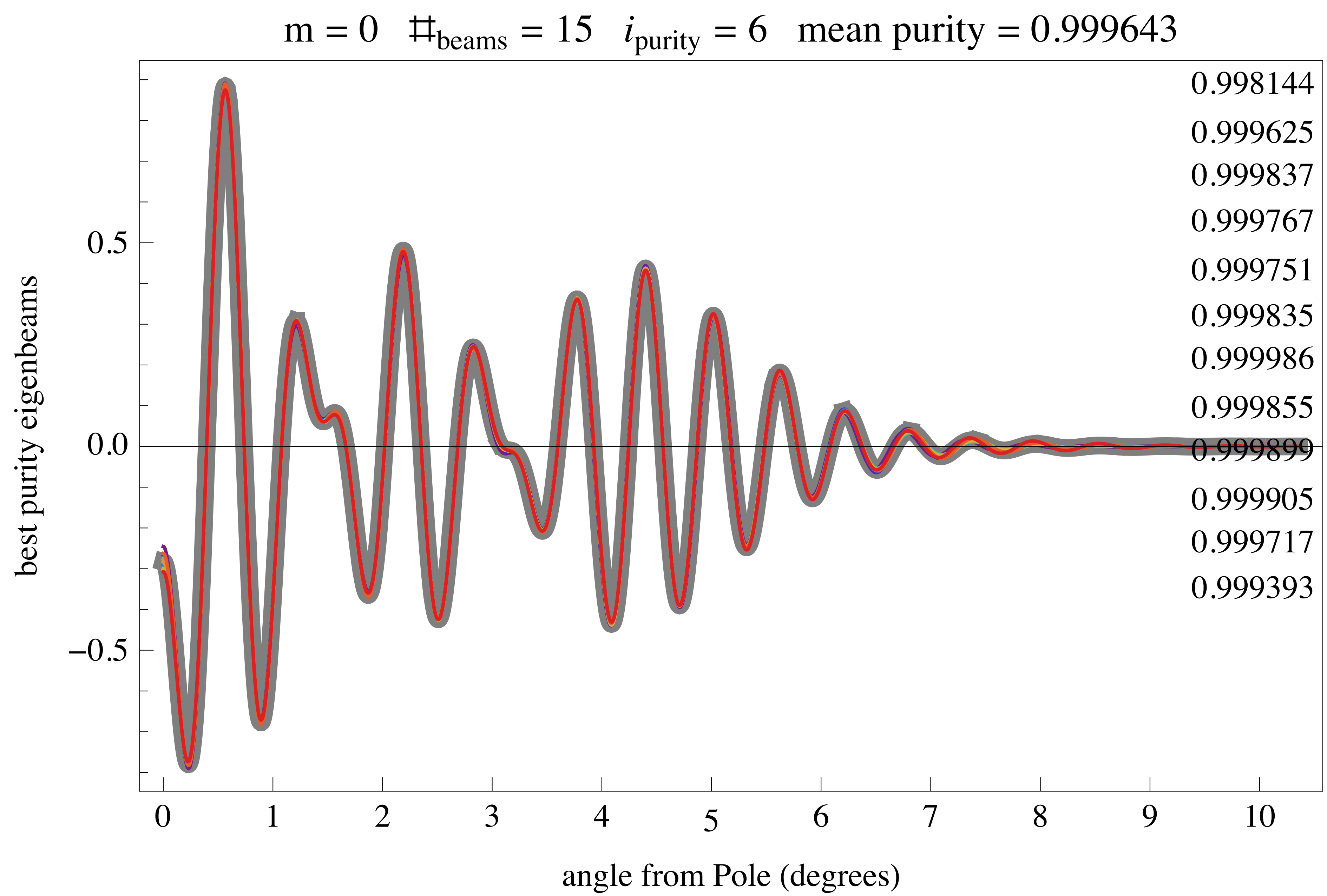
1.
1.
1.
1.



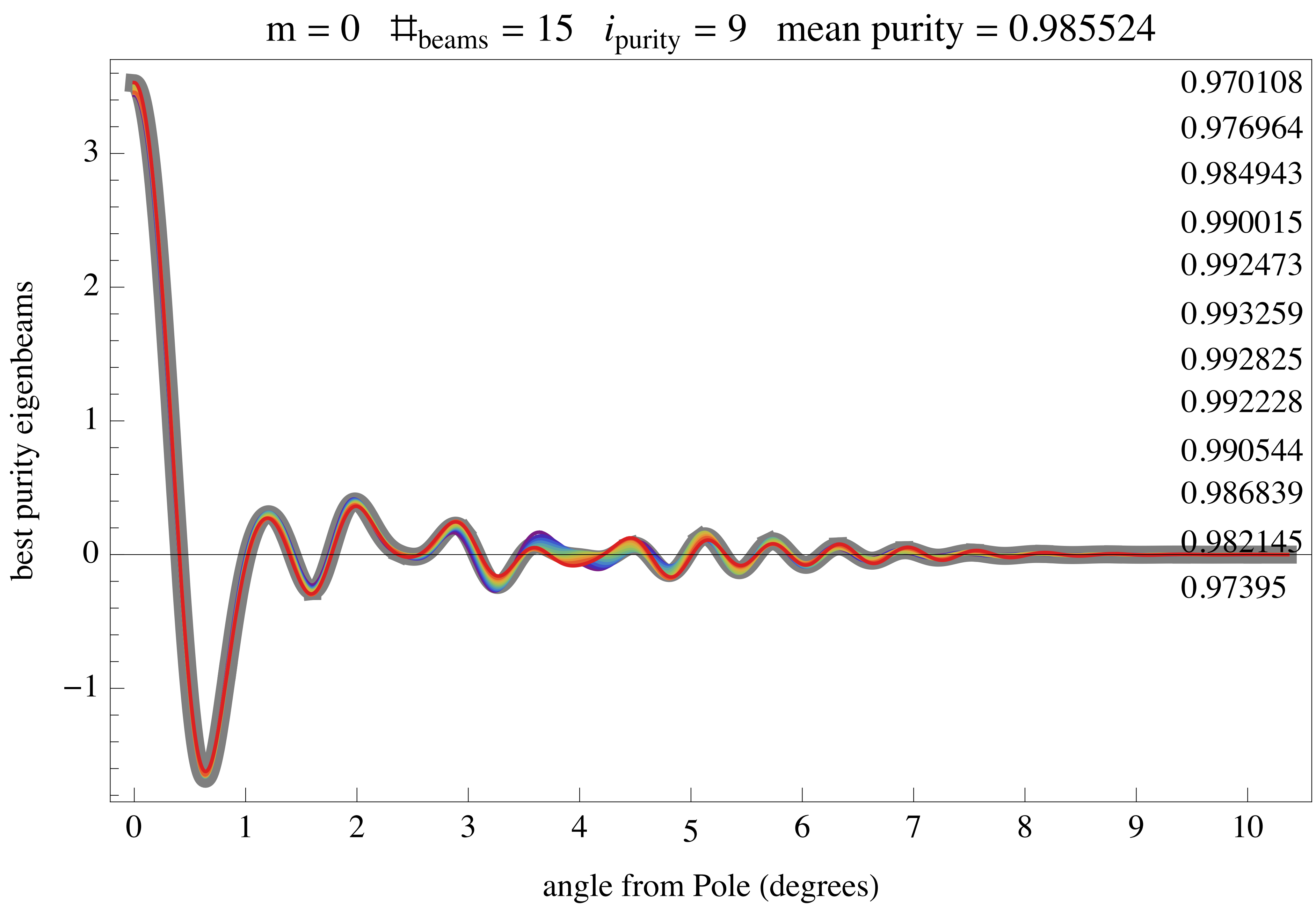
m = 0 ⌗beams = 15 ipurity = 4 mean purity = 0.999999
0.999994
0.999999
0.999999
0.999999
0.999999
1.
0.999999
1.
0.999999
1.
0.999998
0.999999
0.5
0.0
−0.5
best purity eigenbeams
0
1
2
3
4
5
6
7
8
9
10
angle from Pole (degrees)

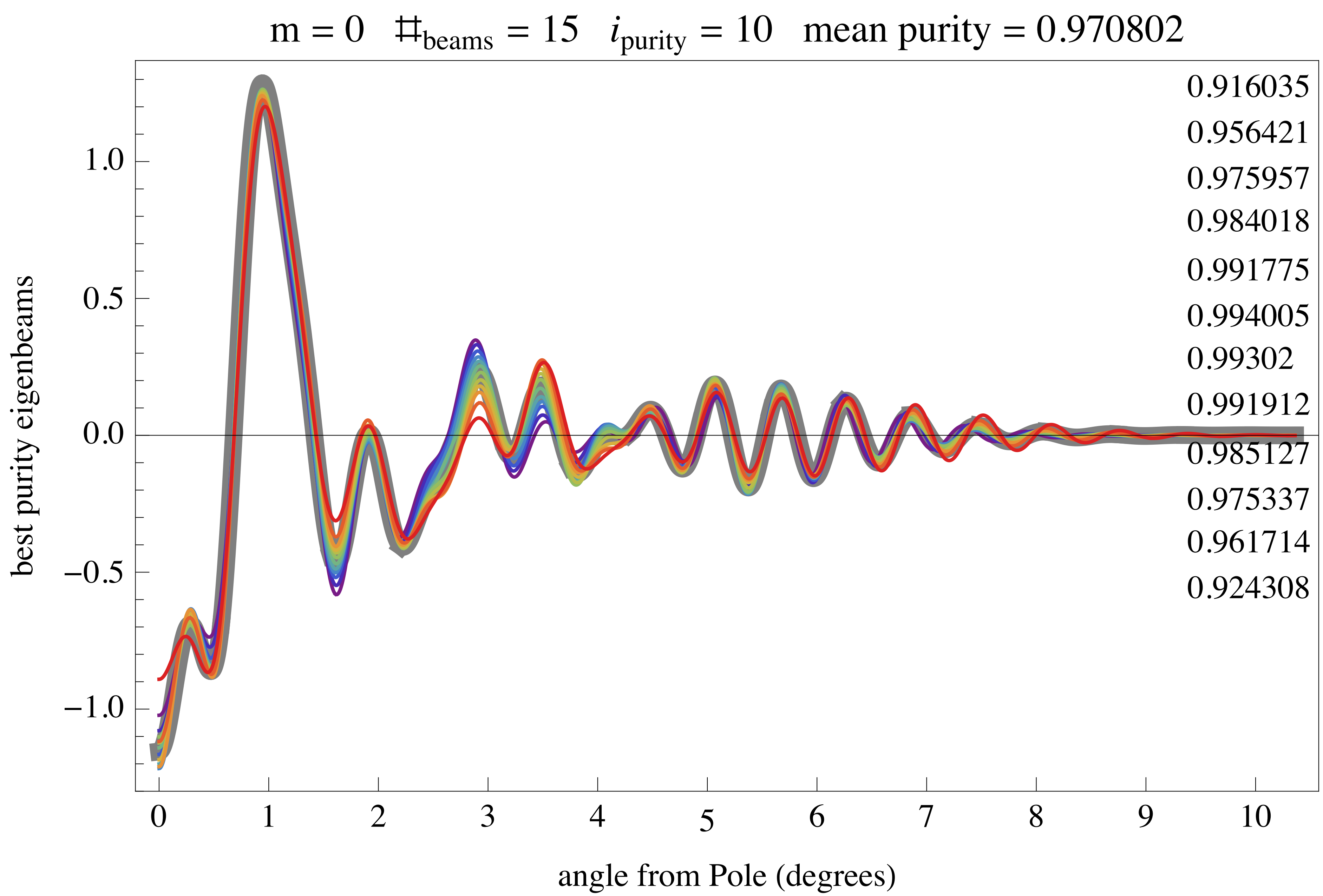

m = 0 ♯beams = 15 ipurity = 5 mean purity = 0.999985
0.999954
0.999957
0.999984
0.999994
0.999995
0.999995
0.999994
0.999999
0.99999
0.999996
0.999972
0.999987
best purity eigenbeams
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
0
1
2
3
4
5
6
7
8
9
10
angle from Pole (degrees)



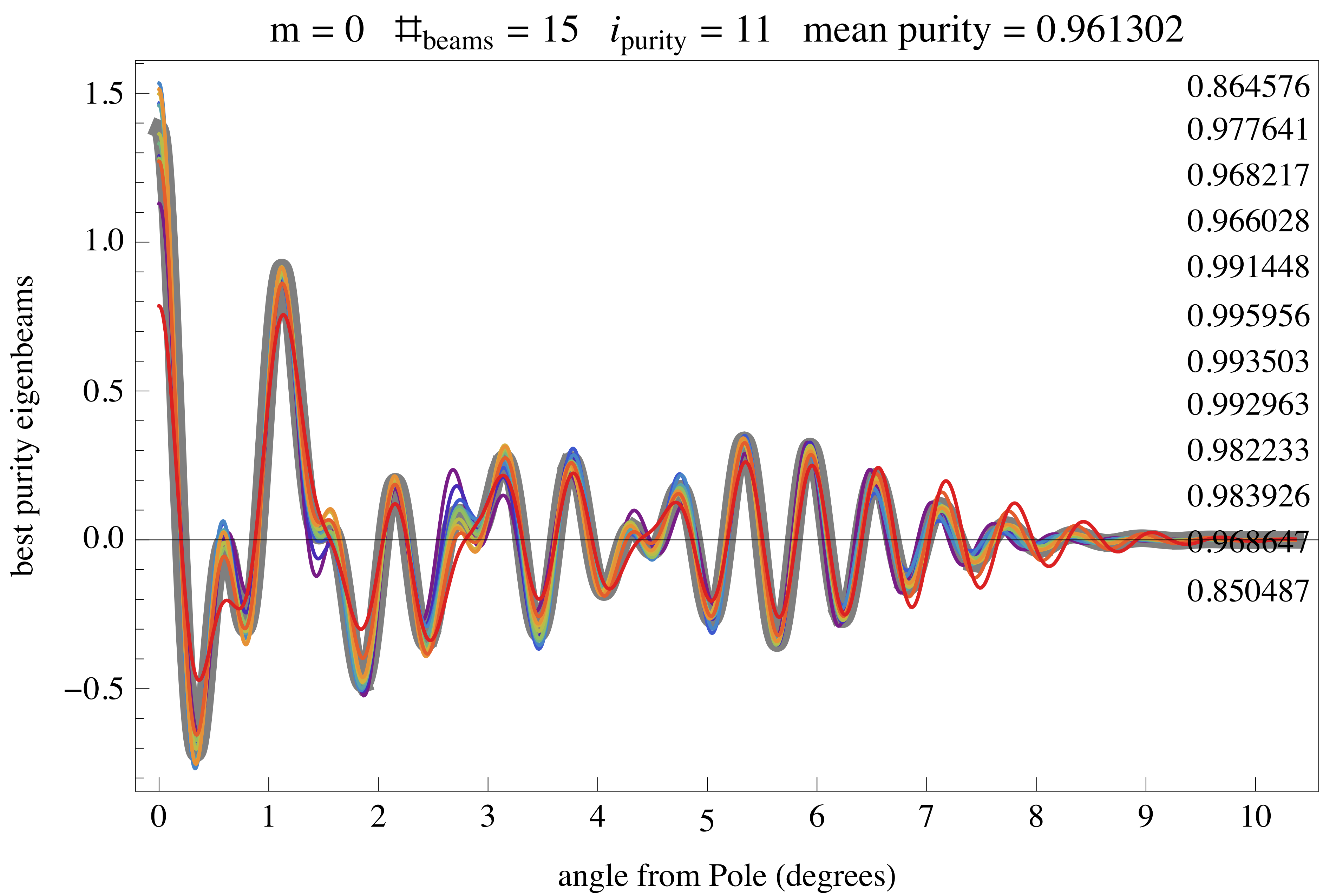
m = 0 ♯beams = 15 ipurity = 6 mean purity = 0.999643
0.998144
0.999625
0.999837
0.999767
0.999751
0.999835
0.999986
0.999855
0.999899
0.999905
0.999717
0.999393
0.5
0.0
−0.5
best purity eigenbeams
0
1
2
3
4
5
6
7
8
9
10
angle from Pole (degrees)

Skip to 9th purity eigenmode





m = 0 ♯beams = 15 ipurity = 10 mean purity = 0.970802
0.916035
0.956421
0.975957
0.984018
0.991775
0.994005
0.99302
0.991912
0.985127
0.975337
0.961714
0.924308
1.0
0.5
0.0
−0.5
−1.0
best purity eigenbeams
0
1
2
3
4
5
6
7
8
9
10
angle from Pole (degrees)


m = 0   ♯beams = 15   ipurity = 11   mean purity = 0.961302
best purity eigenbeams
angle from Pole (degrees)
1.5
1.0
0.5
0.0
−0.5
0
1
2
3
4
5
6
7
8
9
10
0.864576
0.977641
0.968217
0.966028
0.991448
0.995956
0.993503
0.992963
0.982233
0.983926
0.968647
0.850487

Take Aways

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- Small optical design changes can greatly increase achromaticity