THERE AND BACK AGAIN RECOVERING AUTOSPECTRA INFORMATION WITH MULTIPLE CROSSCORRELATION MEASUREMENTS

Lisa Mc Bride LIM 2025

anr agence nationale de la recherche

AU SERVICE DE LA SCIENCE



Trottier Space Institute Trottier at McGill de McGill

Institut spatia



Line Intensity Mapping (LIM)





Line-Intensity Mapping simulation with galaxy distributions



Line Intensity Mapping (LIM)



Growth of Structure



Line-Intensity Mapping simulation with galaxy distributions

z = 0

13.8 Gyr

Image Credit NASA



Line Intensity Mapping (LIM)



Growth of Structure

Still very few auto spectra detections, particularly for 21cm

z = 1z = 08 Gyr 13.8 Gyr

Line-Intensity Mapping simulation with galaxy distributions

Image Credit NASA





Can we extract auto spectra information without a direct detection?



THE LINEAR BIASING MODEL



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On large scales, spectral lines are biased tracers of the underlying matter density field





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autocorrelation (power spectrum)

 $P_{ii} = \beta_i^2 P_m$





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Measuring the EoR Power Spectrum Without Measuring the EoR Power Spectrum

ANGUS BEANE,^{1,2} FRANCISCO VILLAESCUSA-NAVARRO,¹ AND ADAM LIDZ²

¹Center for Computational Astrophysics, Flatiron Institute, 162 5th Ave., New York, NY 10010, USA ²Department of Physics & Astronomy, University of Pennsylvania, 209 South 33rd St., Philadelphia, PA 19104, USA

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crosscorrelation (cross power spectrum) $P_{ij} = \beta_i \beta_j P_m$





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Stayed tuned for Adrian and Hannah's talks!



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a Frequentist estimator that is optimal (minimum variance) and unbiased under certain assumptions





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a Frequentist estimator that is optimal (minimum variance) and unbiased under certain assumptions

$$P_{ij} = \beta_i \beta_j P_m + n_{ij}$$
$$P_{jk} = \beta_j \beta_k P_m + n_{jk}$$
$$P_{ki} = \beta_k \beta_i P_m + n_{ki}$$
$$\beta_0 = \beta_i + n_{b_0}$$





a Frequentist estimator that is optimal (minimum variance) and unbiased under certain assumptions data $\hat{x} = [\mathbf{A}^T \mathbf{N}^{-1} \mathbf{A}]^{-1} \mathbf{A}^T \mathbf{N}^{-1} \mathbf{d}$

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model noise data

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$$\ln P_{ij} = \eta_i + \eta_j + \ln[P_m(k)] + w_{ij}$$

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parameters model noise data



 $\begin{pmatrix} \hat{\beta}_i \\ \hat{\beta}_j \\ \hat{\beta}_k \\ \hat{P}_m \end{pmatrix} = \begin{pmatrix} \beta_0 \\ \beta_0 P_{jk} / P_{ki} \\ \beta_0 P_{jk} / P_{ij} \\ \frac{P_{ij} P_{ki}}{\beta_0^2 P_{jk}} \end{pmatrix}$

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 $P_{ij}P_{ik}$ $\hat{P}_{ii,\mathrm{B19}}$ P_{jk}

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$$\hat{P}_{ii,B19} = \frac{P_{ij}P_{ik}}{P_{jk}} \quad \sigma_{P_{ii}}^2 = \left(\frac{\sigma_{ij}^2}{P_{ij}^2} + \frac{\sigma_{jk}^2}{P_{jk}^2} + \frac{\sigma_{ik}^2}{P_{ik}^2}\right) P_{ii}^2$$

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3 lines (1 prior, 1 autocorrelation, 3 crosscorrelations)



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$$\begin{split} \ln \hat{P}_{ii} &= \left[\frac{\tilde{\sigma}_{ii}^{-2}}{\left(\tilde{\sigma}_{ij}^2 + \tilde{\sigma}_{jk}^2 + \tilde{\sigma}_{ki}^2 \right)^{-1} + \tilde{\sigma}_{ii}^{-2}} \right] \ln P_{ii} \\ &+ \left[\frac{\left(\tilde{\sigma}_{ij}^2 + \tilde{\sigma}_{jk}^2 + \tilde{\sigma}_{ki}^2 \right)^{-1}}{\left(\tilde{\sigma}_{ij}^2 + \tilde{\sigma}_{jk}^2 + \tilde{\sigma}_{ki}^2 \right)^{-1} + \tilde{\sigma}_{ii}^{-2}} \right] \ln \hat{P}_{ii,B19}, \end{split}$$



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An inverse-variance weighted linear combination of the two types of observations!



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An inverse-variance weighted linear combination of the two types of observations! 4 lines (1 prior, 6 crosscorrelations)



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$$\begin{split} \ln \hat{P}_{21\text{cm}} &= \frac{\tilde{\sigma}_{jl}^2 \tilde{\sigma}_{il}^2 + \tilde{\sigma}_{kl}^2 \tilde{\sigma}_{il}^2 + \tilde{\sigma}_{kl}^2 \tilde{\sigma}_{jl}^2}{\Xi^2} \ln \left(\hat{P}_{\text{B19},ijk} \right) \\ &+ \frac{\tilde{\sigma}_{kl}^2 \tilde{\sigma}_{jk}^2 + \tilde{\sigma}_{ik}^2 \tilde{\sigma}_{jk}^2 + \tilde{\sigma}_{ik}^2 \tilde{\sigma}_{kl}^2}{\Xi^2} \ln \left(\hat{P}_{\text{B19},ijl} \right) \\ &+ \frac{\tilde{\sigma}_{ij}^2 \tilde{\sigma}_{jk}^2 + \tilde{\sigma}_{jl}^2 \tilde{\sigma}_{jk}^2 + \tilde{\sigma}_{ij}^2 \tilde{\sigma}_{jl}^2}{\Xi^2} \ln \left(\hat{P}_{\text{B19},ikl} \right) \\ &+ \frac{\tilde{\sigma}_{ik}^2 \tilde{\sigma}_{il}^2}{\Xi^2} \ln \left(\hat{P}_{\text{B19},ijkl} \right) + \frac{\tilde{\sigma}_{ij}^2 \tilde{\sigma}_{il}^2}{\Xi^2} \ln \left(\hat{P}_{\text{B19},ikl} \right) \\ &+ \frac{\tilde{\sigma}_{ij}^2 \tilde{\sigma}_{ik}^2}{\Xi^2} \ln \left(\hat{P}_{\text{B19},iljk} \right), \end{split}$$

 $\Xi^2 \equiv (\tilde{\sigma}_{ij}^2 + \tilde{\sigma}_{kl}^2)(\tilde{\sigma}_{ik}^2 + \tilde{\sigma}_{il}^2 + \tilde{\sigma}_{jk}^2 + \tilde{\sigma}_{jl}^2) + (\tilde{\sigma}_{ik}^2 + \tilde{\sigma}_{jl}^2)(\tilde{\sigma}_{il}^2 + \tilde{\sigma}_{jk}^2)$





Want to investigate the Frequentist estimator framework using simulated signals...







Want to investigate the Frequentist estimator framework using simulated signals... ...and conduct a numerical Bayesian analysis of the full posterior

MATTER DENSITY FIELD







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MATTER DENSITY FIELD DARK MATTER HALO CATALOG































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Matter Density Field







The importance of galaxy formation histories in models of reionization

Jordan Mirocha⁽⁾,¹^{*}[†] Paul La Plante²[‡] and Adrian Liu⁽⁾ ¹McGill University Department of Physics & McGill Space Institute, 3600 Rue University, Montréal, QC, H3A 2T8, Canada ²Department of Astronomy and Radio Astronomy Laboratory, University of California Berkeley, Berkeley, CA 94720, USA









Simulations from Mirocha et al. (2021): arXiv: 2012.09189





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$$= L_{0,i} \left(\frac{M}{M_0}\right)^{\alpha_i}$$



BUBBLE MODEL

Addition of bubble morphology to 21cm model via the zreion package.





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Addition of bubble morphology to 21cm model via the zreion package.

plaplant / zreion Public





Fitting cosmological signal models

- Optimistic scenario: Perfect tracer + 1% fractional noise
- Conservative scenario: Power law + 10% fractional noise
- Pessimistic scenario: Bubble + 15% fractional noise







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Technique fails for the Bubble model because fields are more decorrelated!







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RESULTS 2/2

Fitting current and futuristic surveys

• Current (or upcoming) surveys: HERA, FYST-like, EXCLAIM-like

 Futuristic survey: improved instrumental specifications

see Padmanabhan et al. (2020):

arXiv: 2105.12148





RESULTS 2/2

Fitting current and futuristic surveys

- Current (or upcoming) surveys: HERA, FYST-like, EXCLAIM-like
- Current (or upcoming) surveys + prior: above + additional prior on C[II]
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RESULTS 2/2

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 Futuristic survey: improved instrumental specifications

see Padmanabhan et al. (2020): arXiv: 2105.12148 Technique fails without additional information because ratios are tricky!





SUMMARY

- taken if ...
 - ...the three fields are decorrelated with each other
 - ...the instrumental noise is large, in particular if one line is very noisy
- However theoretical modeling and simulations can improve...
 - ...our understanding of what lines are tracing what underlying fields
 - ...the physical constraints on the bias factors, thus improving the priors on the model parameters

• On large scales, in the linear biasing regime, it may be possible to reconstruct the 21cm power spectrum from three crosscorrelations. But caution must be

arXiv:2308.00749



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ADRIAN'S

On the importance of priors





SNEAKY ERROR PROPAGATION 1/3





SNEAKY ERROR PROPAGATION 2/3





SNEAKY ERROR PROPAGATION 3/3





CAREFUL IN K-SPACE





CAREFUL IN K-SPACE









CAREFUL IN K-SPACE





10⁰



SURVEYS





INTRINSIC BIAS















HERA Telescope at SARAO

0

Radio interferometer based in the Karoo Desert in South Africa. Currently responsible for the world's leading upper limits of the P21 at EOR redshifts, but always looking to improve.

Hoping to meet a complementary survey who shares the same values (and sky coverage). Let's crosscorrelate some data and see how it goes. Looking to avoid drama, and foregrounds.

HERA'S TINDER PROFILE

