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08/07/25

# Exclusive vector-quarkonium photoproduction at NLO in collinear factorisation with GPD evolution and high-energy resummation

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# Scope

- Infamous "scale-stability" problem of exclusive quarkonium production in pQCD

  Problem that has been known for more than 20 years

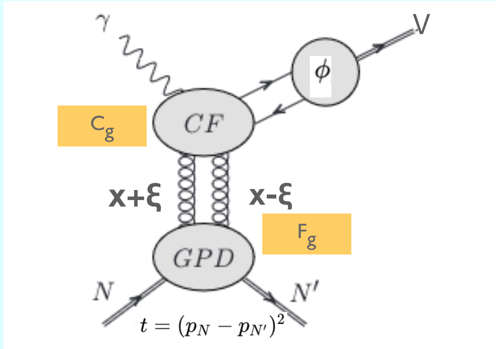
  *Ivanov, Schäfer, Szymanowski, Krasnikov, 04*

  **Our solution:** Consistent integration of Collinear + High-energy Factorisation with full GPD evolution

  ***CAF, Lansberg, Nabeebaccus, Nefedov, Sznajder, Wagner, Phys. Lett. B 859 (2024) 139117***

# Framework

- Fluctuation of incoming photon into pair of heavy quarks
- Pair interacts with proton P (or nucleus N) via two-parton colour-singlet exchange
- Modelling of heavy-quark pair recombination into time-like exclusive vector meson made within NRQCD




$$F_{q/g} \otimes C_{q/g} \otimes \phi^{V}_{Q\bar{Q}}$$

NRQCD: $\phi(z) \sim \delta(z - 1/2)$

**Coefficient functions**

- Photoproduction (on-shell photon)

  Factorisation not proven but holds to fixed-order NLO

  *Ivanov, Schäfer, Szymanowski, Krasnikov, Eur. Phys. J. C 34 (2004) 3, 297-316*

- Electroproduction (off-shell photon)

  *Chen, Qiao, Phys. Lett. B 797 (2019) 134816, Phys. Lett. B (2020) 135759 (erratum)*

  *CAF, Gracey, Jones, Teubner, JHEP 08 (2021) 150*

$$A \propto \int_{-1}^{1} \mathrm{d}x \left[ C_g(x,\xi) F_g(x,\xi) + \sum_{q=u,d,s} C_q(x,\xi) F_q(x,\xi) \right]$$

# GPD modeling

Construct GPDs via Radyushkin's Double Distributions that respect polynomiality, Lorentz invariance, correct forward limits, ...

Ji hep-ph/9801260 hep-ph/9603249 Radyushkin hep-ph/9604317 hep-ph/9605431

$$H_i(x,\xi) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(\beta + \xi\alpha - x)\, d_i(\beta,\alpha)$$

$$d_i(\beta,\alpha) = f_i(\beta) \times h_i(\beta,\alpha)$$

$$h_i(\beta,\alpha) = \frac{\Gamma(2n_i+2)}{2^{2n_i+1}\Gamma^2(n_i+1)} \frac{\left((1-|\beta|)^2-\alpha^2\right)^{n_i}}{(1-|\beta|)^{2n_i+1}}$$

$$\int_{-1+|\beta|}^{1-|\beta|} d\alpha\, h_i(\beta,\alpha) = 1,$$

: $n_i$ parameters control skewness in the input distribution

- Approach **inspired** from **GK model:** $n_g$=$n_{qsea}$=2, $n_{qval}$=1 and parameters in $f_i$ fixed by fitting CTEQ6M PDFs to low-x DVMP (rho,phi) electroproduction data from HERA

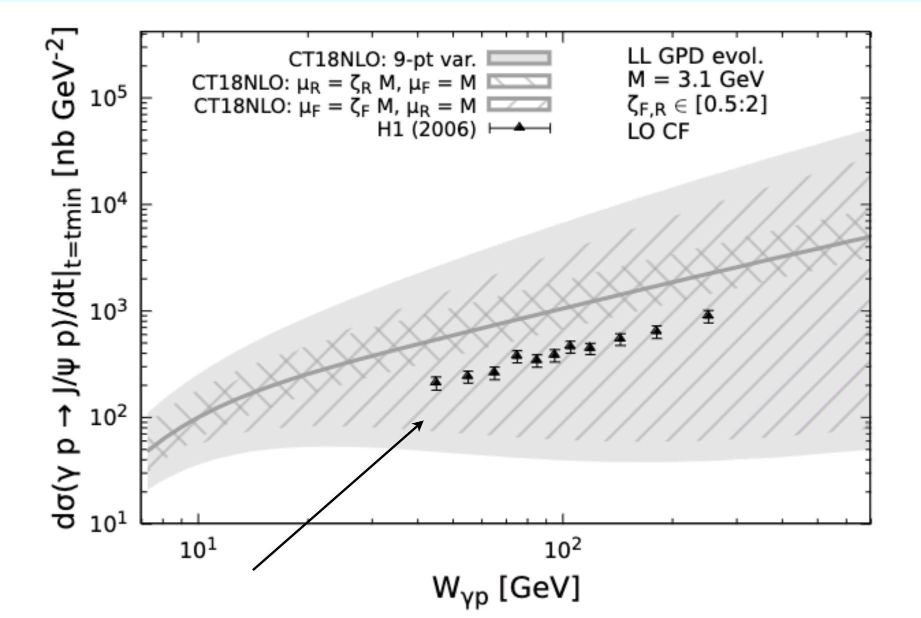
Goloskokov, Kroll, EPJC 50 (2007) 829

- **We** use input CT18NLO partons at scale $\mu_0$ = 2 GeV and perform full leading-logarithmic GPD evolution with APFEL++ as implemented in PARTONS.

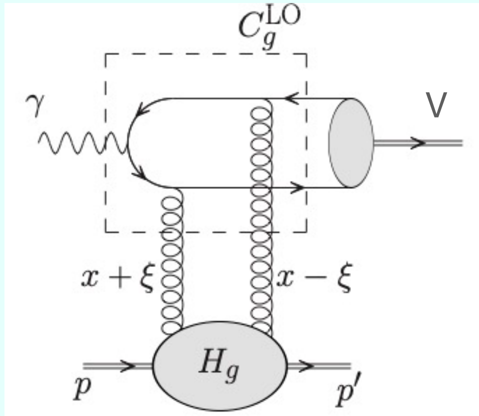
Bertone et al., EPJC 82 (2022) 888 EPJC 78 (2018) 6, 478

# **Illustration** of the scale-stability problem

**LO:**




Lower boundary: for small values of the factorisation scale, the cross section does not increase with energy, exhibits local maximum




**(gluon initiated only)**

$$\mathcal{T}^{\mu\nu}_{\rm LO} = -g_\perp^{\mu\nu} \int_{-1}^{1} \frac{dx}{x} \left[ C_g^{\rm LO}\left(\frac{\xi}{x}\right) \frac{F_g(x,\xi,\mu_F)}{x} \right]$$

$$C_g^{\rm LO}\left(\frac{\xi}{x}\right) = \frac{F_{\rm LO}}{\left[1+\frac{\xi}{x} - i\delta\,\mathrm{sgn}(x)\right]\left[1-\frac{\xi}{x} + i\delta\,\mathrm{sgn}(x)\right]}$$

**small ξ <-> large W**

$$F_{\rm LO} = \frac{4\pi\alpha_s e e_Q R_Q(0)}{m_Q^{3/2}\sqrt{2\pi N_c}}\,, \qquad \xi = \frac{M^2}{2W_{\gamma p}^2 - M^2} \sim \frac{M^2}{2W_{\gamma p}^2}$$

$$\mathrm{Im}\, C_g^{\rm LO}\left(\frac{\xi}{x}\right) = -\pi \frac{F_{LO}}{2}\left[\delta\left(\frac{\xi}{x}-1\right) + \delta\left(\frac{\xi}{x}+1\right)\right]$$

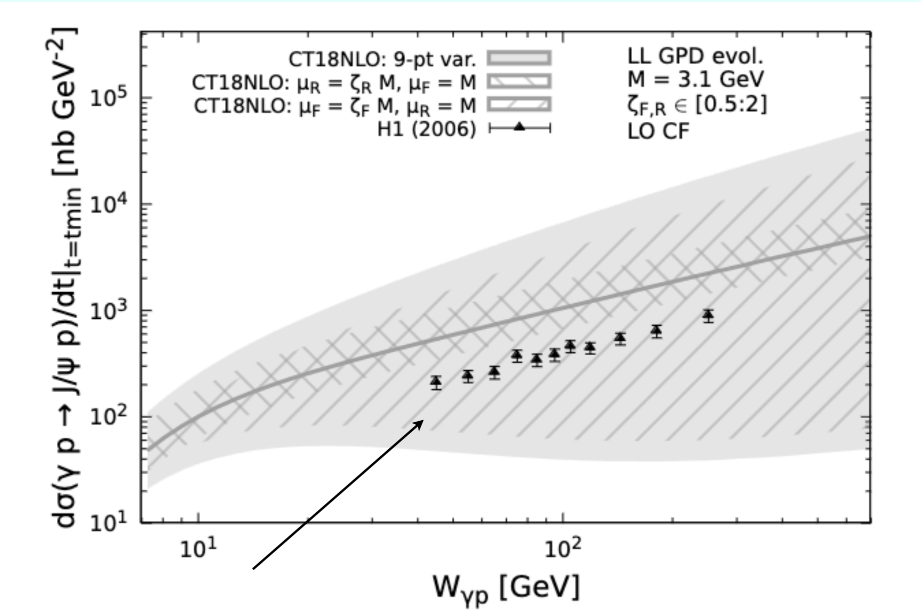
$$\mathrm{Im}\,\mathcal{T}^{\mu\nu}_{\rm LO} = \pi \frac{g_\perp^{\mu\nu} F_{LO}}{\xi} F_g(\xi,\xi)$$

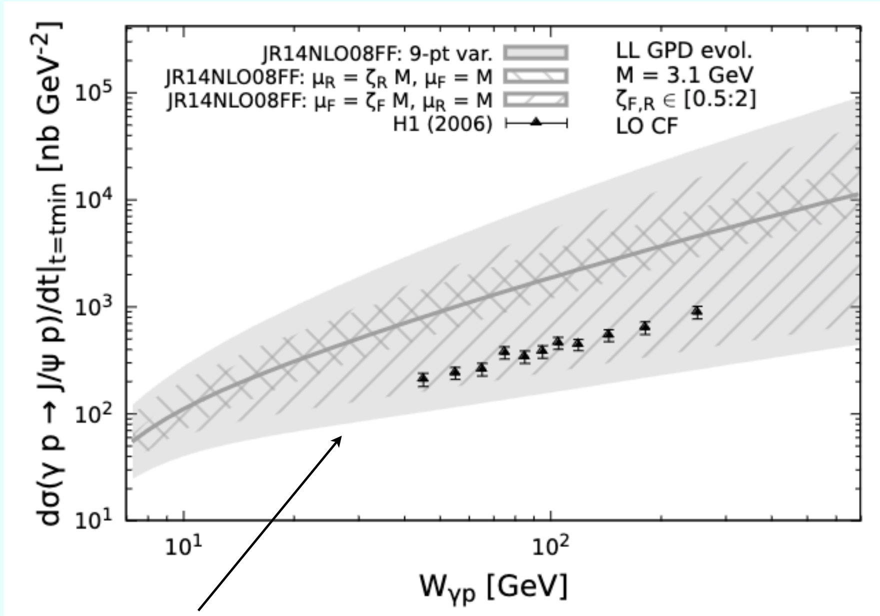
Two-body *t*-differential cross section:

$$\frac{d\sigma}{dt}(\gamma^* p \to Vp)|_{t=t_{\rm min}} = \frac{1}{16\pi\Lambda(W^2, Q^2, m_p^2)} \overline{\sum}_{\lambda,\lambda'=\pm,0} \overline{\sum}_{s,s'} |A^{\lambda\lambda'}|^2,$$

# **Illustration** of the scale-stability problem

**LO:**




Lower boundary: for small values of the factorisation scale, the cross section does not increase with energy, exhibits local maximum

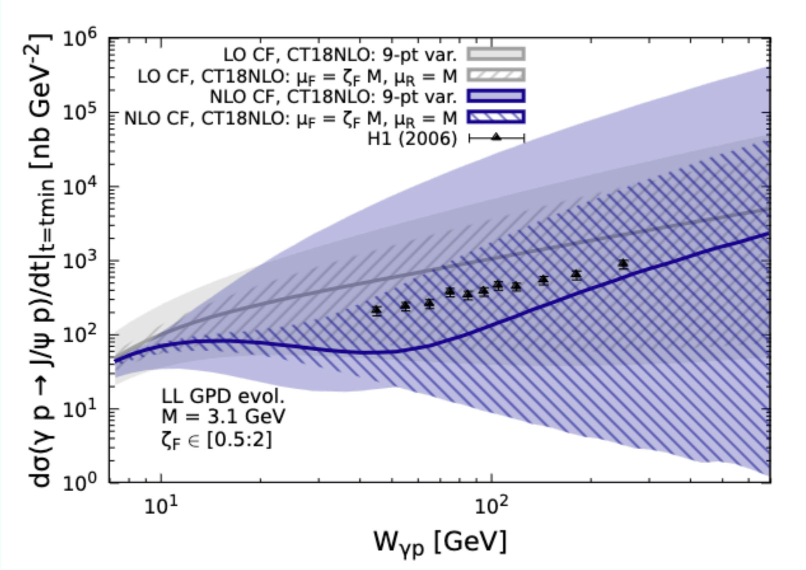
....however absent with PDFs with larger interval of evolution

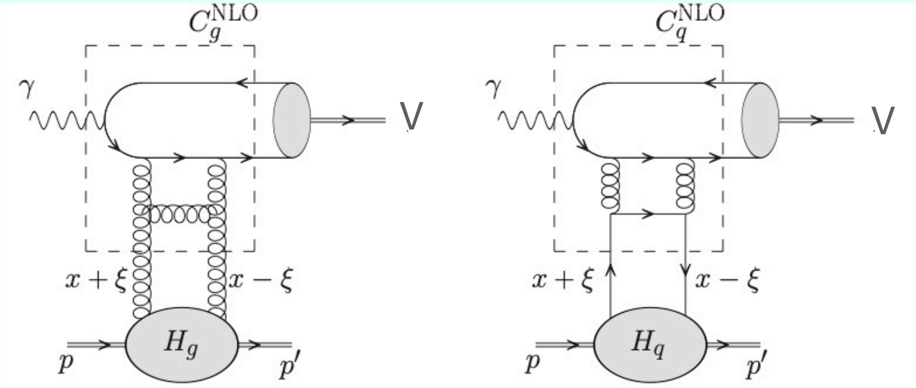
Two-body $t$-differential cross section:

$$\frac{d\sigma}{dt}(\gamma^* p \to Vp)|_{t=t_{\rm min}} = \frac{1}{16\pi\Lambda(W^2, Q^2, m_p^2)} \overline{\sum}_{\lambda,\lambda'=\pm,0} \overline{\sum}_{s,s'} |A^{\lambda\lambda'}|^2,$$

# **Illustration** of the scale-stability problem

**NLO:**



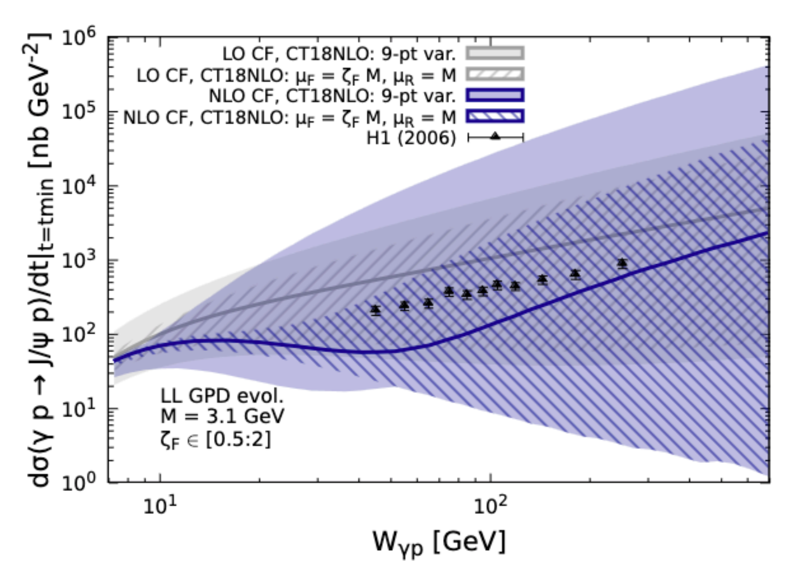

**(quark & gluon initiated)**

Increasing uncertainty with larger energy and oscillatory energy dependence (!)

This problem observed also in Ivanov, Schäfer, Szymanowski, Krasnikov, Eur. Phys. J. C 34 (2004) 3, 297-316 20 years ago

# **Origin** of the scale-stability problem

**NLO:**




Increasing uncertainty with increasing energy, oscillatory energy dependence (!)

**High-energy limit** $W_{\gamma p}^2 \gg M^2 \quad (\xi \to 0)$

$$\mathcal{T}^{\mu\nu}_{\rm NLO} \supset i\pi \frac{g_\perp^{\mu\nu} F_{LO}}{\xi} \left[ H_g(\xi,\xi) + \frac{\alpha_s(\mu_R) C_A}{\pi} \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 \frac{dx}{x} H_g(x,\xi) \right.$$
$$\left. + \frac{\alpha_s(\mu_R) C_A}{\pi} \frac{C_F}{C_A} \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 dx\, (H_q(x,\xi) - H_q(-x,\xi)) \right]$$

both terms scale $\sim \alpha_s \ln(1/\xi) \ln\left(\frac{M^2}{4\mu_F^2}\right)$

This problem observed also in Ivanov, Schäfer, Szymanowski, Krasnikov, Eur. Phys. J. C 34 (2004) 3, 297-316 20 years ago

# **Origin** of the scale-stability problem

- In the DGLAP evolution of **low ξ GPDs,** the probability of emitting a new gluon is strongly enhanced by the **large value of ln(1/ξ)**

**Back-of-envelope estimate:**

Integrate gluon-emission probability $d\xi/\xi \; dk_t^2/k_t^2$ over phase space to give average no. of gluons:

$$\langle n \rangle \simeq \frac{\alpha_s N_C}{\pi} \ln(1/\xi) \, \Delta \ln \mu_F^2$$

- Mean number of gluons in logarithmic interval between conventional scale variation ($\mu_F/2$ to $2\,\mu_F$ ) is

For $\xi \sim 10^{-3} - 10^{-4}$, $\ln(1/\xi) \sim 8$ and $\langle n \rangle \approx 8$

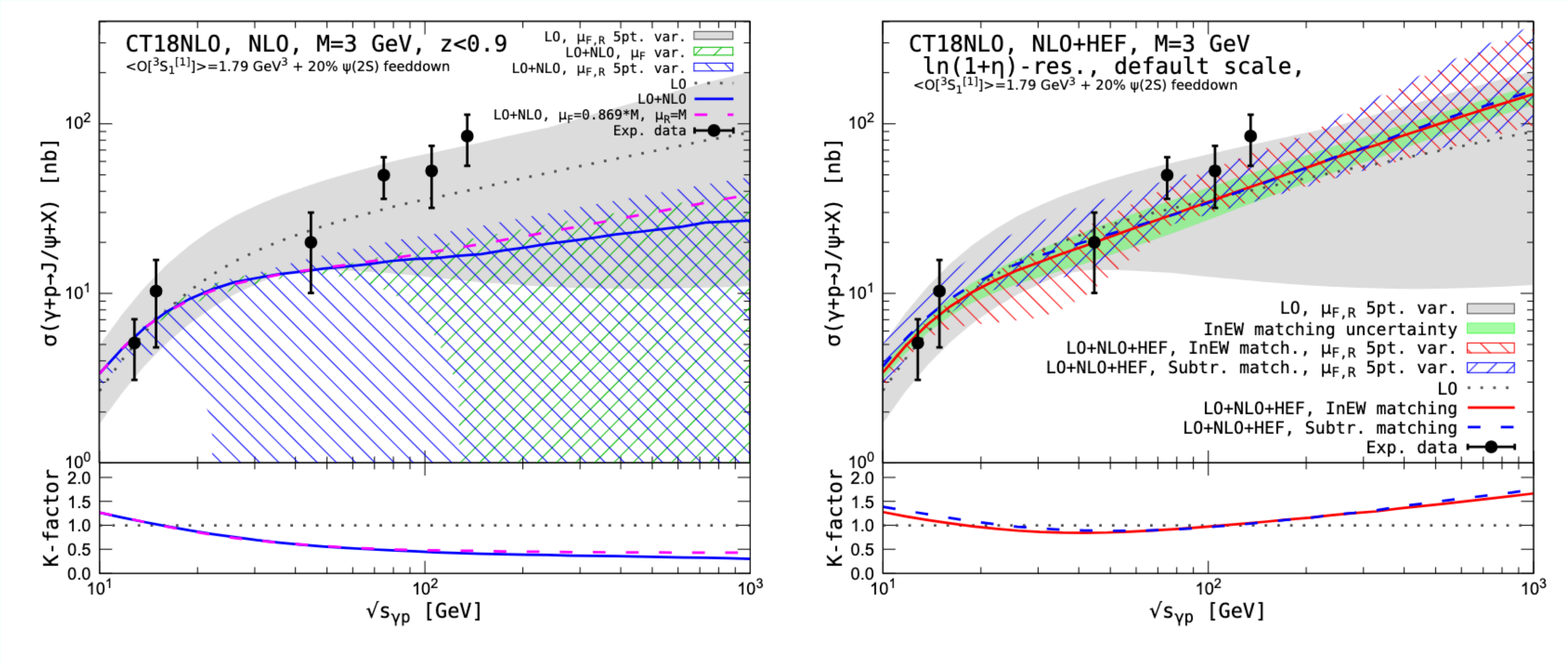
- In contrast, the NLO coefficient function allows for the emission (and reabsorption) of only **one** gluon.

- Therefore we **cannot expect compensation** between the contributions coming from the GPD and the coefficient function as we vary the scale $\mu_F$ .

- This implies the need for **resummation of logarithmic terms ~ ln(1/ξ)**

- At large **ξ ,** the compensation is much more complete and provides reasonable stability of the predictions to variations of the scale $\mu_F$

# Analogies with the inclusive case




**Left**: LO CF (grey) & LO+NLO CF (blue/green)

**Right**: LO CF (grey) & LO+NLO CF **+ HEF** (blue/green)

Instabilities in total inclusive photoproduction cross sections of S-wave quarkonium production largely alleviated via approaches akin to that described here:

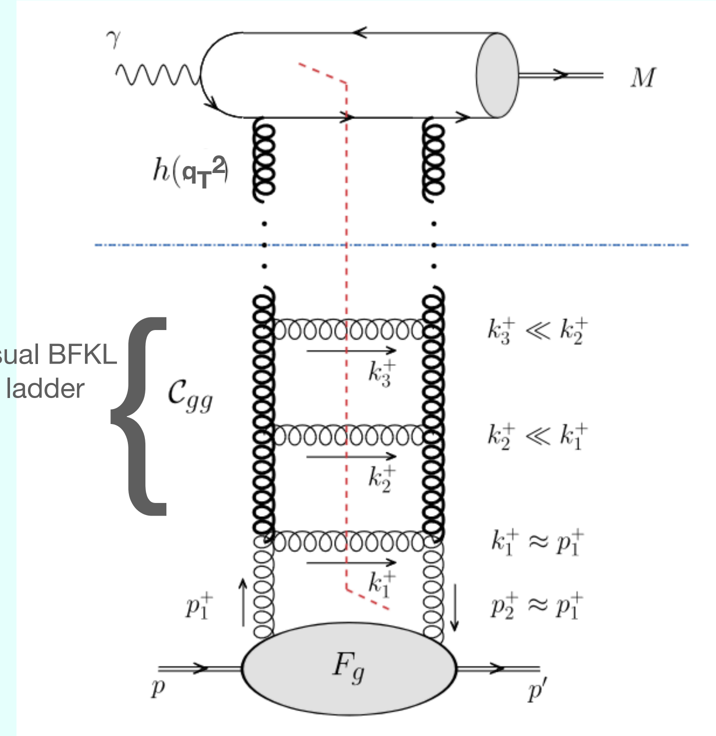
a) **through a scale-fixing approach** Lansberg, Ozcelik, EPJC 81 (2021) 6, 497

b) **resumming high-energy logarithms** Lansberg, Nefedov, Ozcelik, JHEP 05 (2022) 083 & EPJC 84 (2024) 351

# Our solution

- Integration of Collinear Factorisation (CF) with High-energy Factorisation (HEF)

**CAF, Lansberg, Nabeebaccus, Nefedov, Sznajder, Wagner, Phys. Lett. B 859 (2024) 139117**




Resum gluon emissions into hard-scattering to compensate evolution in GPD

**Our strategy:** We implement a resummation of these logarithms in a manner **consistent** with CF

→ truncation to Double-Leading-Logarithmic (DLA) approximation

$C_{gi}$ is known and process independent at LLA & NLLA

$$C_g^{\text{HEF}}\left(\frac{\xi}{x}\right) = \frac{-i\pi}{2}\frac{F_{\text{LO}}}{\left(\frac{\xi}{x}\right)}\int_0^\infty d\mathbf{q}_T^2\, \mathcal{C}_{gi}\left(\frac{\xi}{x}, \mathbf{q}_T^2, \mu_F, \mu_R\right) h(\mathbf{q}_T^2),$$

$$h(\mathbf{q}_T^2) = \frac{M^2}{M^2+4\mathbf{q}_T^2}.$$

← process-dependent factor

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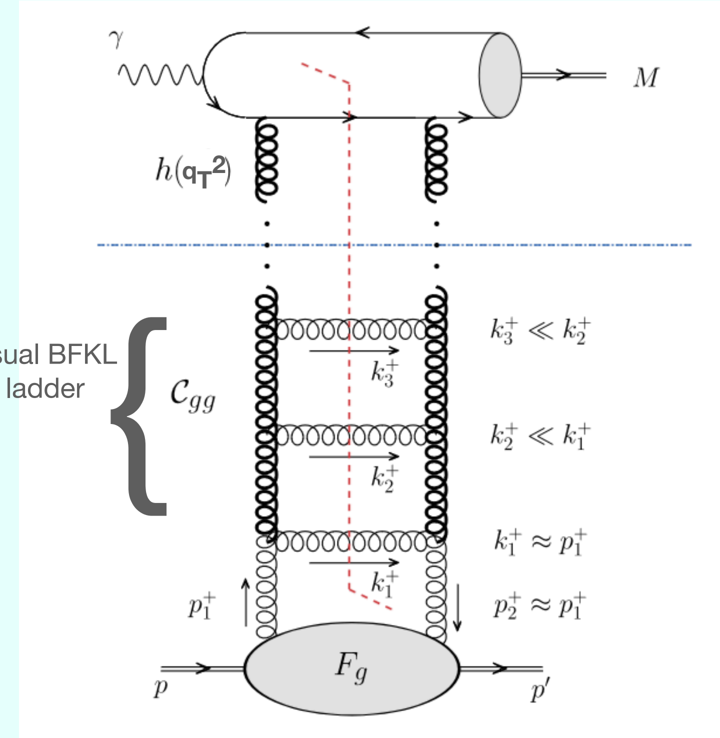
usual BFKL ladder




Convert $C_{gi}$ to Mellin space, insert into $Cg^{HEF}$ to get its Mellin-space rep and then invert again:

$$C_g^{\rm HEF}\left(\frac{\xi}{x}\right) = \frac{-i\pi\hat{\alpha}_s F_{\rm LO}}{2\left|\frac{\xi}{x}\right|}\sqrt{\frac{L_\mu}{L_x}}\left\{I_1\left(2\sqrt{L_x L_\mu}\right) - 2\sum_{k=1}^{\infty}{\rm Li}_{2k}(-1)\left(\frac{L_x}{L_\mu}\right)^k I_{2k-1}\left(2\sqrt{L_x L_\mu}\right)\right\}$$

where $L_\mu = \ln[M^2/(4\mu_F^2)]$ and $L_x = \hat{\alpha}_s \ln\left|\frac{x}{\xi}\right|$. **closed form solution!**

This yields, when expanded in $\alpha_s$,

$$C_g^{\rm HEF}\left(\frac{\xi}{x}\right) = \frac{-i\pi F_{\rm LO}}{2}\left(\underbrace{\delta\left(\left|\frac{\xi}{x}\right| - 1\right) + \frac{\hat{\alpha}_s}{\left|\frac{\xi}{x}\right|}\ln\left(\frac{M^2}{4\mu_F^2}\right)}_{\to C_g^{\rm asy.}} + \frac{\hat{\alpha}_s^2}{\left|\frac{\xi}{x}\right|}\ln\frac{1}{\left|\frac{\xi}{x}\right|}\left[\frac{\pi^2}{6} + \frac{1}{2}\ln^2\left(\frac{M^2}{4\mu_F^2}\right)\right] + \ldots\right)$$

first two terms match the LO and NLO CF results at small ξ

Quark coefficient function:

$$C_q^{\rm HEF}\left(\frac{\xi}{x}\right) = \frac{2C_F}{C_A}C_g^{\rm HEF}\left(\frac{\xi}{x}\right)$$

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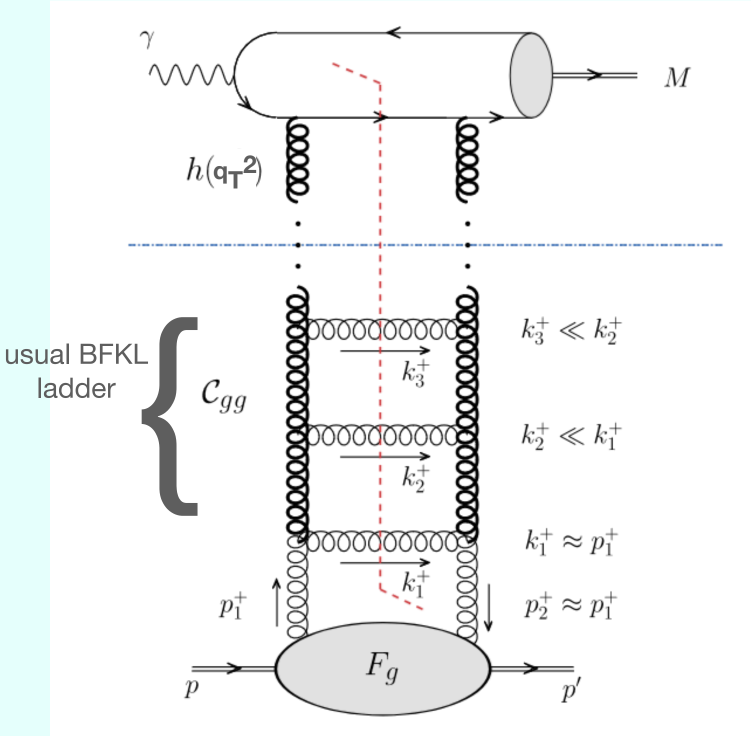


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**Tells us that scale-fixing approach fails at NNLO….**

Quark coefficient function:

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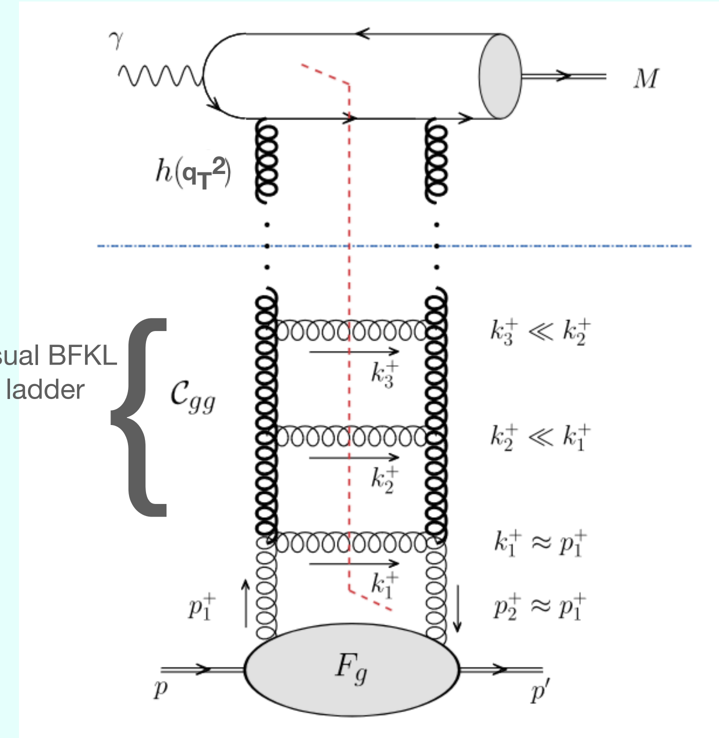



$$C^{\text{match.}}_{g,q}\left(\frac{\xi}{x}\right) = C^{\text{NLO CF}}_{g,q}\left(\frac{\xi}{x}\right) - C^{\text{asy.}}_{g,q}\left(\frac{\xi}{x}\right) + C^{\text{HEF}}_{g,q}\left(\frac{\xi}{x}\right)$$

$$C^{\text{asy.}}_{g}\left(\frac{\xi}{x}\right) = \frac{C_A}{2C_F} C^{\text{asy.}}_{q}\left(\frac{\xi}{x}\right)$$

$$= \frac{-i\pi F_{\text{LO}}}{2}\left[\delta\left(\left|\frac{\xi}{x}\right| - 1\right) + \frac{\hat{\alpha}_s}{\left|\frac{\xi}{x}\right|}\ln\left(\frac{M^2}{4\mu_F^2}\right)\right]$$

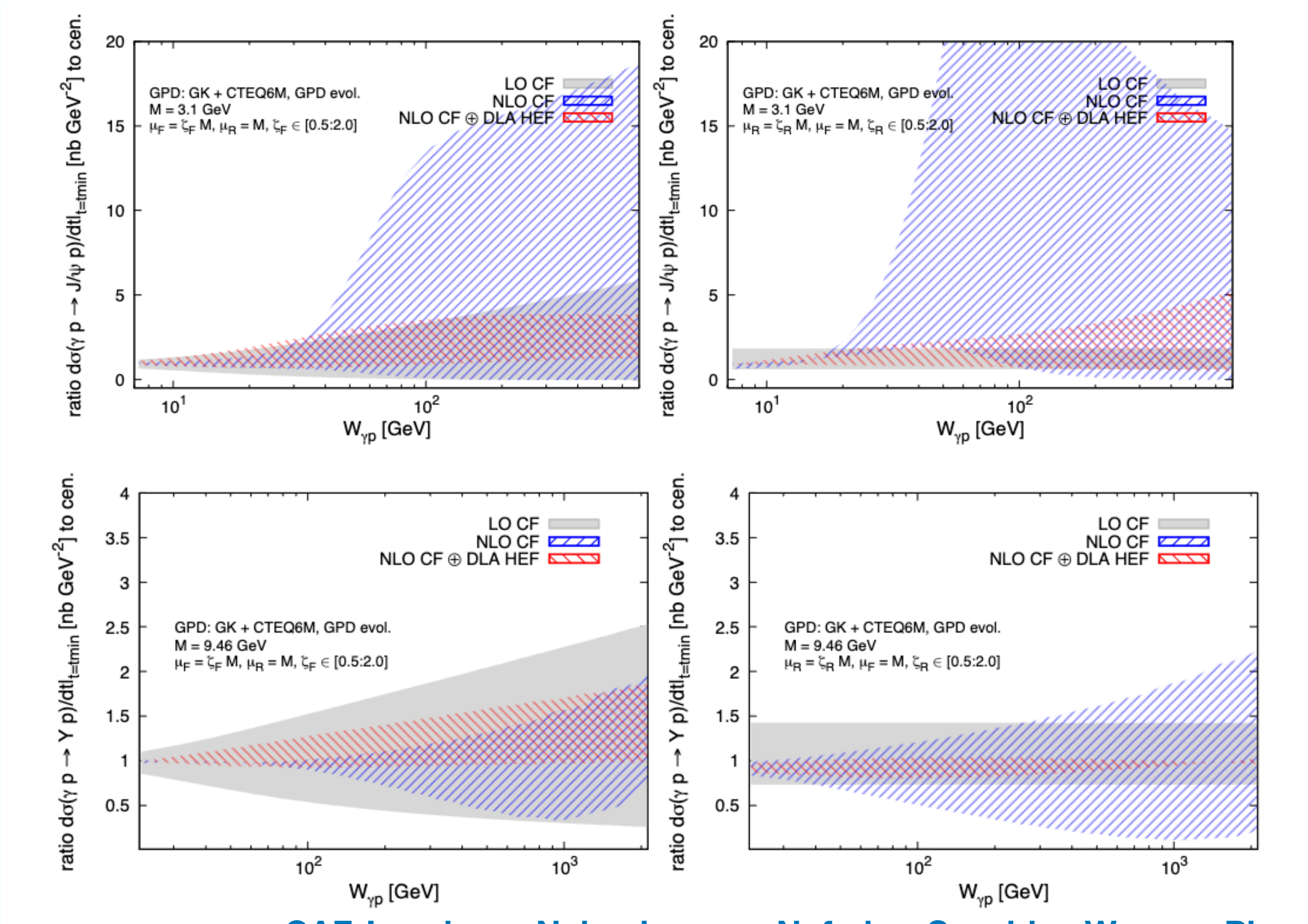
first two terms in $a_S$ expansion of $Cg^{HEF}$

Matching performed before x-integration

# Results

J/ψ

Υ

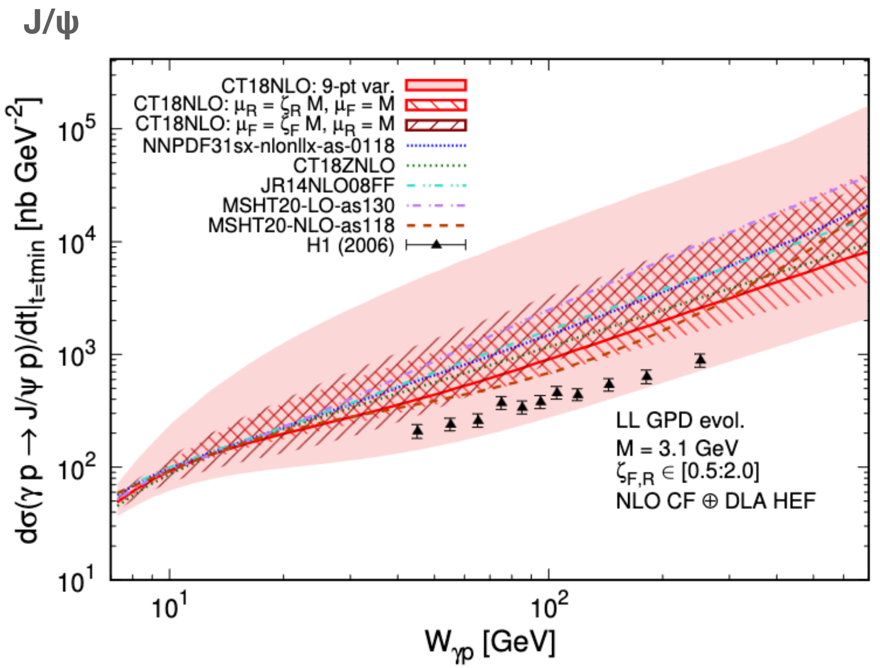


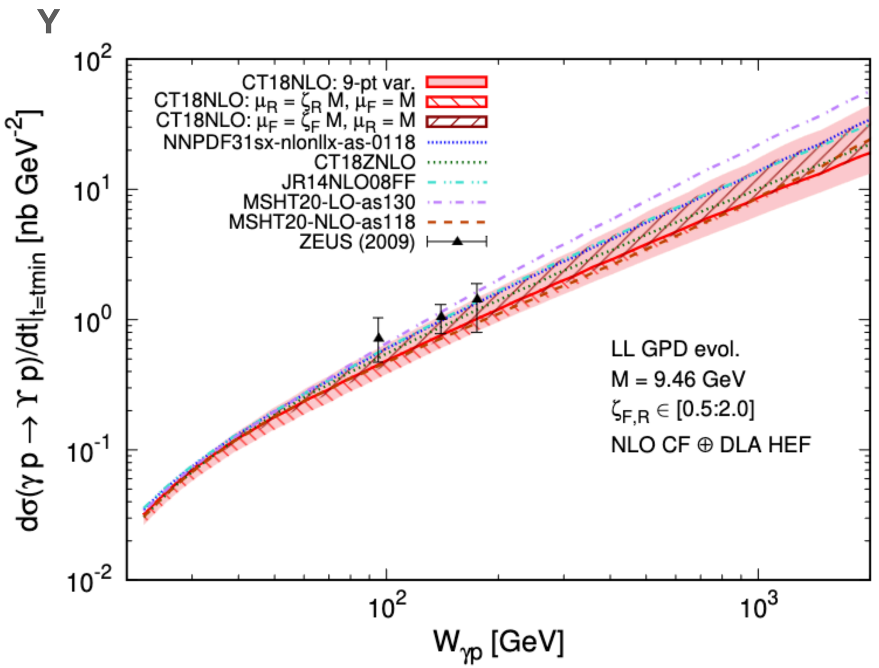

μF uncertainty smaller in both J/ψ & Υ cases for NLO CF + DLA HEF

Υ production exhibits better perturbative convergence but resummation still important!

CAF, Lansberg, Nabeebaccus, Nefedov, Sznajder, Wagner, Phys. Lett. B 859 (2024) 139117

# Results



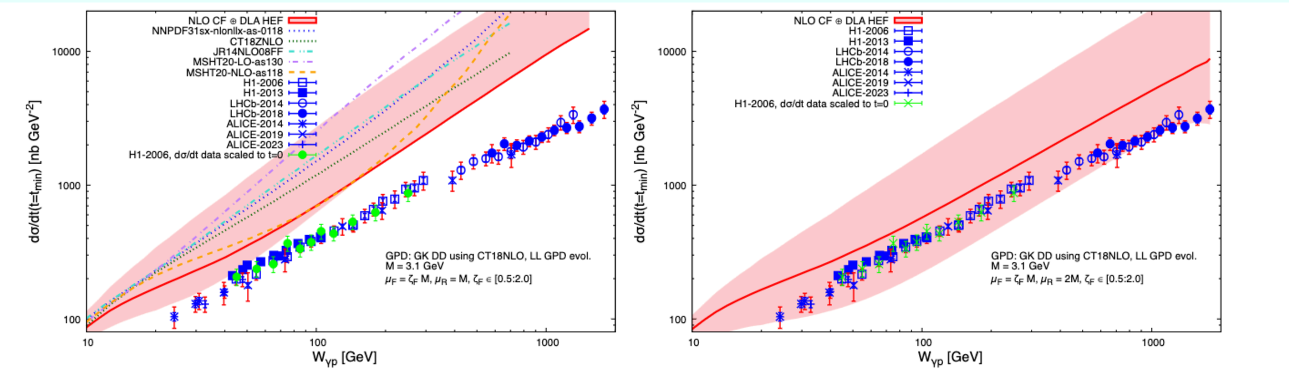

Much milder scale dependence of resummed NLO CF + DLA HEF result

- NLO CF + DLA HEF cures the instabilities of the NLO CF result

**CAF, Lansberg, Nabeebaccus, Nefedov, Sznajder, Wagner, Phys. Lett. B 859 (2024) 139117**

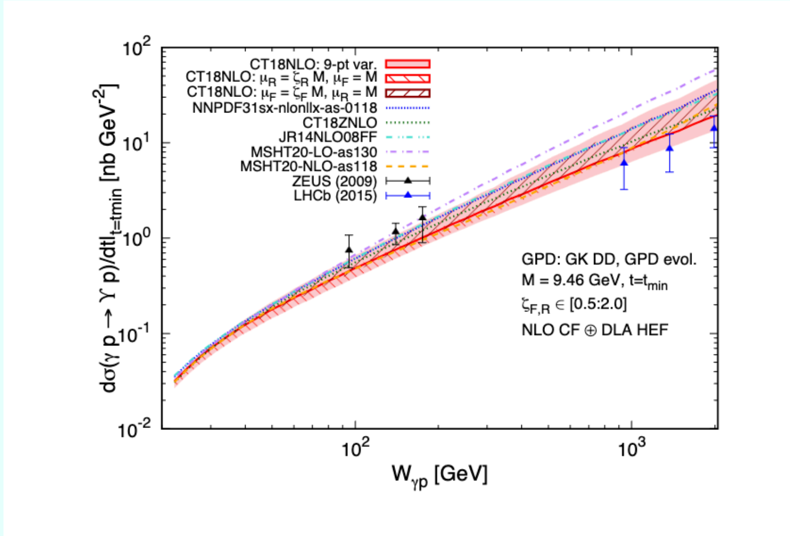
# Results UPC region @ LHC

**J/ψ**




$\mu_R = M$ $\mu_R = 2M$

**Υ**




Care to be taken to unfold photoproduction "data" from high-energy UPC measurements in hadron-hadron collisions!

Have shown unfolded pp UPC data, can also unfold pPb data with reduced uncertainties

see our ESPPU review for motivating pPb data-taking in Run3 of LHC: arXiv:2504.0426862

see e.g. CAF, Jones, Martin, Ryskin, Teubner Phys. Rev. D 105 (2022) 3, 034008 for some arguments for **Υ** production

# Conclusions

- Exclusive J/$\psi$ and Y photoproduction at increasing $W_{\gamma p}$ suffers from perturbative instabilities at NLO, similar to inclusive photoproduction and hadroproduction.

- Scale fixing accounts for effective resummation of large double-logarithmic corrections but
- a) misses terms $\ln(1/\xi)$*const
- b) Insufficient at NNLO and higher orders

- We performed high-energy resummation of large logarithms of $\ln 1/\xi$ matched consistently to NLO CF, employing full LL GPD evolution

- Like in the inclusive case, the NLO CF + DLA HEF results have a milder scale dependence and agree with t-differential data from HERA.

- Future: More flexible GPD modeling away from Radyushkin's double distribution, extend approach to NLLA, impact of relativistic corrections... ultimately want to fit gluon PDFs/GPDs and work can naturally be extended to exclusive quarkonium production in heavy-ion nuclear-nuclear collisions.

**Thank you**

# Generalised Parton Distribution (GPD) spin projections

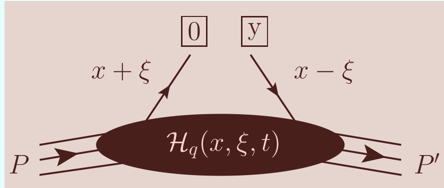
Non-perturbative contributions of the quark and gluon amplitudes can be expressed in terms of GPDs contained in so-called Parton density matrices

Ji hep-ph/9801260 hep-ph/9603249 Radyushkin hep-ph/9604317 hep-ph/9605431

*Quark:* Quark GPD contraction implemented as a spin projection of the *on-shell* quark scattering matrix

$$M^q_{\alpha\beta}(x,\xi) = 2\int \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle P + \frac{\Delta}{2} | \bar{\psi}^q_\beta \left(\frac{\lambda}{2}\right) W\left[-\frac{\lambda}{2}, \frac{\lambda}{2}\right] \psi^q_\alpha \left(-\frac{\lambda}{2}\right) | P - \frac{\Delta}{2} \rangle$$

$$= F^q(x,\xi) \not{p}_{\alpha\beta} + \tilde{F}^q(x,\xi)(\gamma_5 \not{p})_{\alpha\beta} + \ldots$$




$\langle P' | \overline{\psi}_q(y) \mathcal{P}\{\} \psi_q(0) | P \rangle$

*Gluon:* Gluon GPD contraction implemented as a spin projection of the *on-shell* gluon scattering matrix

→ *analogous expression*

Decomposition into leading-twist parton helicity-conserving GPDs (j=q,g)

$$F_{j,ss'} = \frac{1}{2P^+}\left[\bar{u}_{s'}(p')\left(H_j \gamma^+ + E_j \frac{i\sigma^{+\Delta}}{2m_p}\right) u_s(p)\right]$$

# Scale fixing

**Scale fixing** resum $a_s^n \ln^n 1/\xi \ln^n \mu_F$ to **amplitude**

Jones et al., J.Phys.G 43 (2016) 3, 035002

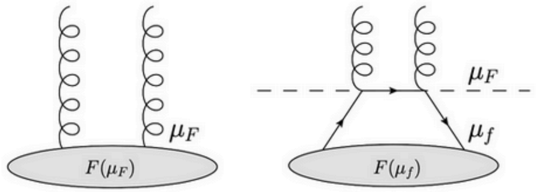
**Idea**: to effectively resum the NLO correction that generates high-energy double logarithms into the PDFs through the scale choice $\mu_F = m = M/2$.

## NLO High-energy limit:

$$\mathcal{M} \approx \frac{-4\,i\,\pi^2\sqrt{4\pi\alpha}\,e_q(e_V^* e_\gamma)}{N_c\,\xi}\left(\frac{\langle O_1\rangle_V}{m^3}\right)^{1/2} \times$$

$$\times\left[\alpha_S(\mu_R)F^g(\xi,\xi,t) + \frac{\alpha_S^2(\mu_R)N_c}{\pi}\ln\left(\frac{m^2}{\mu_F^2}\right)\int_\xi^1 \frac{dx}{x}F^g(x,\xi,t)\right.$$

$$\left. + \frac{\alpha_S^2(\mu_R)C_F}{\pi}\ln\left(\frac{m^2}{\mu_F^2}\right)\int_\xi^1 dx\,\left(F^{q,S}(x,\xi,t) - F^{q,S}(-x,\xi,t)\right)\right]$$



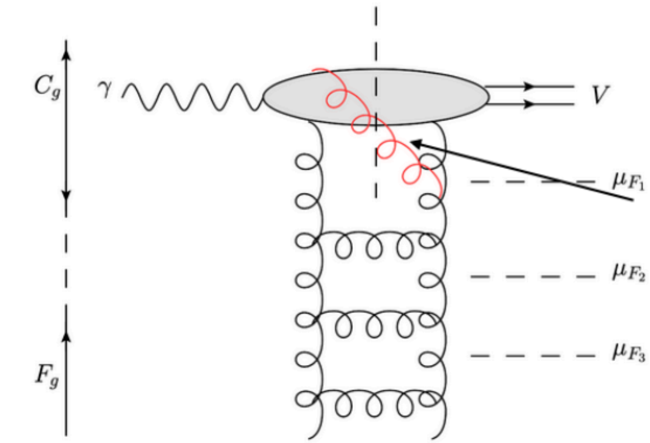

Ideology: Use scale shifting to find optimal scale that removes the largest contribution from the NLO correction *

At fact. scale. $\mu_f$ , quark contribution is part of NLO hard matrix element

At fact. scale $\mu_F$ , absorbed quark contribution into LO result

Effect of scale change driven by (generalised, skewed) DGLAP evolution:

$$A^{(0)}(\mu_f) = \left(C^{(0)} + \frac{\alpha_s}{2\pi}\ln\left(\frac{\mu_f^2}{\mu_F^2}\right)C^{(0)} \otimes V\right) \otimes F(\mu_F)$$




The red gluon cannot be resummed in this scale shifting approach and so will always be treated as part of the higher order correction

# Scale fixing

**Scale fixing** resum $a_S^n \ln^n 1/\xi \ln^n \mu_F$ to **amplitude**

Jones et al., J.Phys.G 43 (2016) 3, 035002

**Idea**: to effectively resum the NLO correction that generates high-energy double logarithms into the PDFs through the scale choice $\mu_F = m = M/2$.

**But**: procedure misses potentially large corrections from $\ln 1/\xi$ not equipped with a $\ln \mu_F$

**NLO High-energy limit:**

$$\mathcal{M} \approx \frac{-4\,i\,\pi^2\sqrt{4\pi\alpha}\,e_q(e_V^* e_\gamma)}{N_c\,\xi}\left(\frac{\langle O_1\rangle_V}{m^3}\right)^{1/2} \times$$

$$\times\left[\alpha_S(\mu_R)F^g(\xi,\xi,t) + \frac{\alpha_S^2(\mu_R)N_c}{\pi}\ln\left(\frac{m^2}{\mu_F^2}\right)\int_\xi^1 \frac{dx}{x}F^g(x,\xi,t)\right.$$

$$\left.+\frac{\alpha_S^2(\mu_R)C_F}{\pi}\ln\left(\frac{m^2}{\mu_F^2}\right)\int_\xi^1 dx\,\left(F^{q,S}(x,\xi,t) - F^{q,S}(-x,\xi,t)\right)\right]$$

# Scale fixing

**Scale fixing** resum $a_S^n \ln^n 1/\xi \ln^n \mu_F$ to **amplitude**

Jones et al., J.Phys.G 43 (2016) 3, 035002

**Idea**: to effectively resum the NLO correction that generates high-energy double logarithms into the PDFs through the scale choice $\mu_F = M/2$.

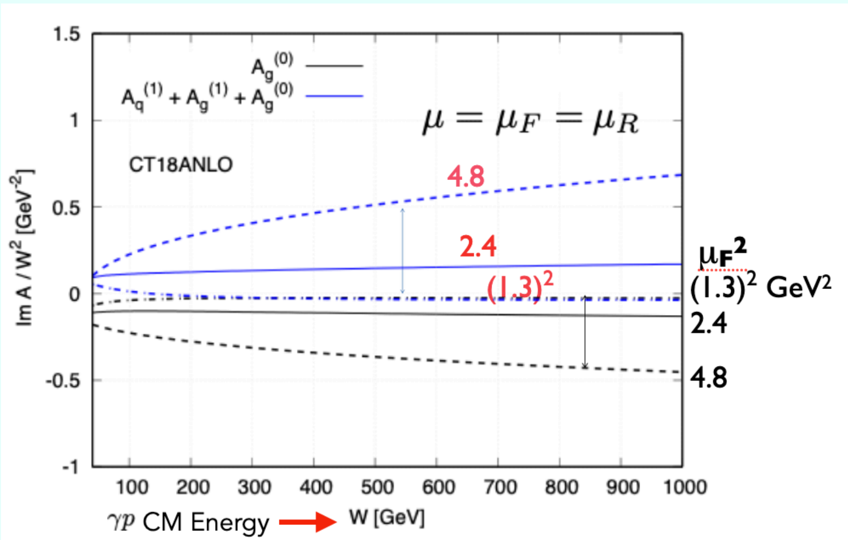
**Upshot**: Alleviates scale dependence but still NLO > LO and opposite sign

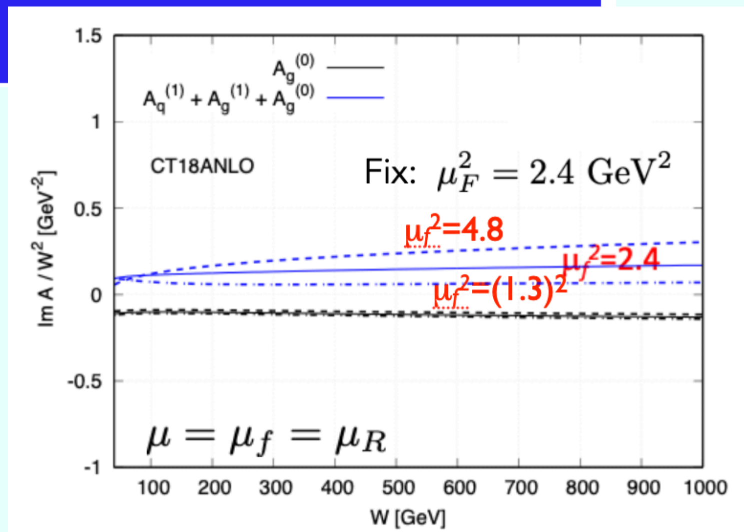
## NLO High-energy limit:

$$\mathcal{M} \approx \frac{-4\,i\,\pi^2\sqrt{4\pi\alpha}\,e_q(e_V^* e_\gamma)}{N_c\,\xi}\left(\frac{\langle O_1\rangle_V}{m^3}\right)^{1/2} \times$$

$$\times\left[\alpha_S(\mu_R)F^g(\xi,\xi,t) + \frac{\alpha_S^2(\mu_R)N_c}{\pi}\ln\left(\frac{m^2}{\mu_F^2}\right)\int_\xi^1 \frac{dx}{x}F^g(x,\xi,t)\right.$$

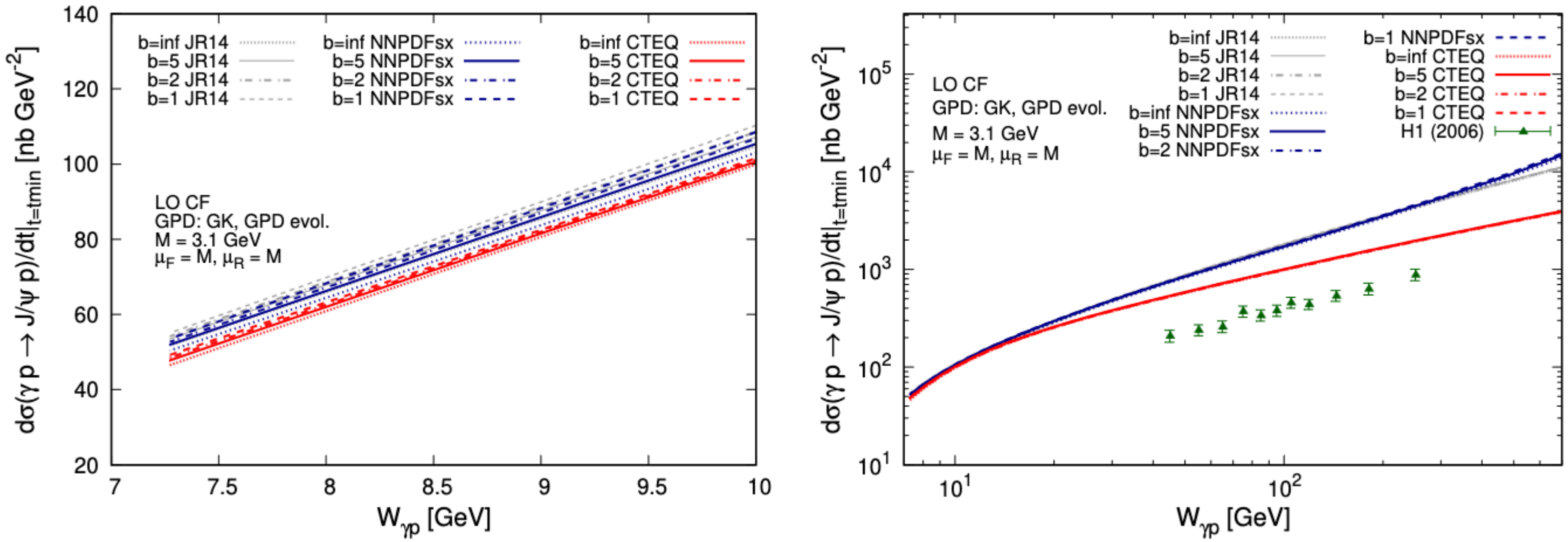
$$\left. + \frac{\alpha_S^2(\mu_R)C_F}{\pi}\ln\left(\frac{m^2}{\mu_F^2}\right)\int_\xi^1 dx\,\left(F^{q,S}(x,\xi,t) - F^{q,S}(-x,\xi,t)\right)\right]$$



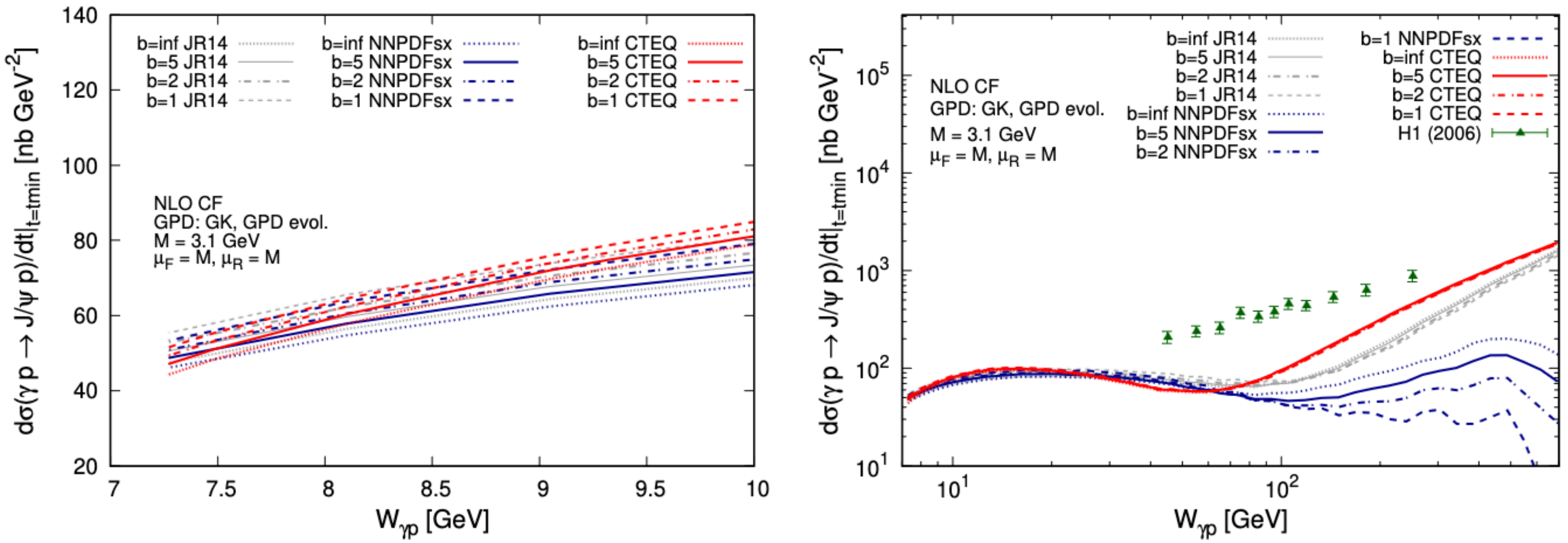

$A(\mu_f) = C^{LO} \times GPD(\mu_F) + C^{NLO}(\mu_F) \times GPD(\mu_f)$

# Results: comparing different GPD inputs for LO CF



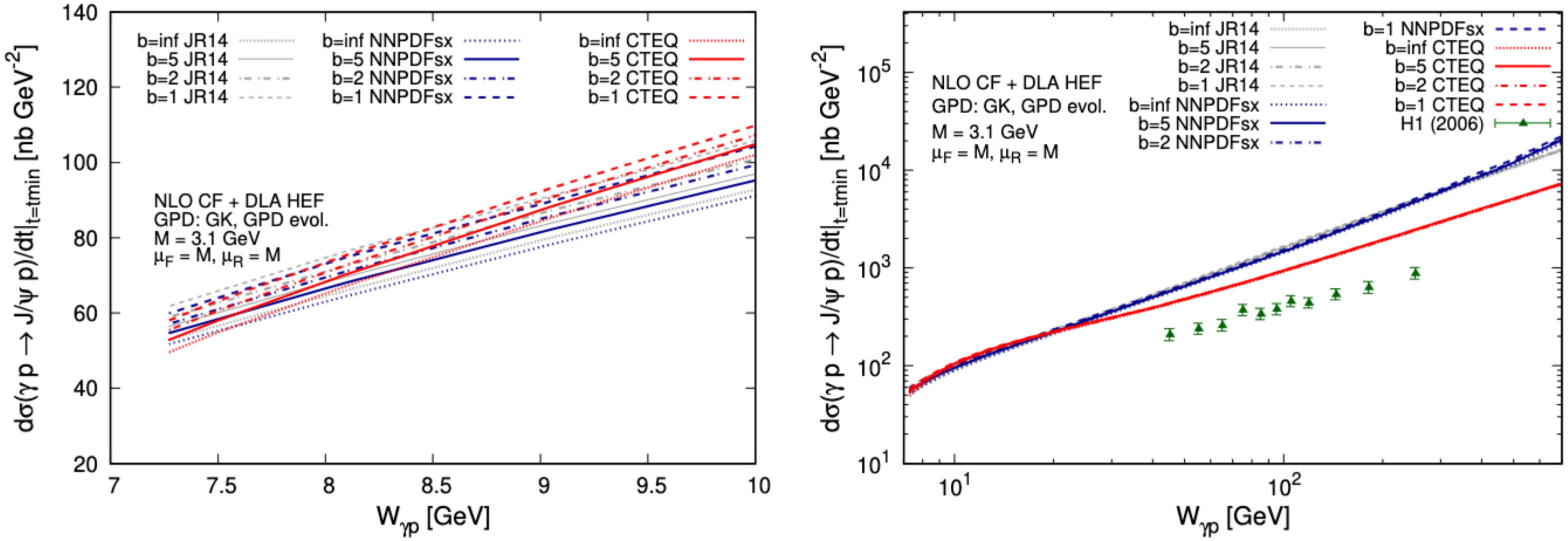

- GPD with PDF input using CTEQ6 (like GK) and 2 more modern PDFs not showing a dip (JR14 and NNPDFsx)
- Strength of $\xi$ dependence of GPD from Double Distribution encoded in *b*
- Low energies (left)
  - Slight effect from *b* variation but smaller than changing PDFs
  - Also much smaller than scale uncertainties (previous slides)
- Higher energies (right) [log plot]
  - Effect from *b* variation negligible

# Different GPD inputs for NLO CF




- GPD with PDF input using CTEQ6 (like GK) and 2 more modern PDFs not showing a dip (JR14 and NNPDFsx)
- Strength of $\xi$ dependence of GPD from Double Distribution encoded in *b*
- Low energies (left)
  - Similar observations than for LO
- Higher energies (right) [log plot]
  - NLO results out of control at higher energies

# Different GPD inputs for NLO CF ⊕ HEF DLA




- GPD with PDF input using CTEQ6 (like GK) and 2 more modern PDFs not showing a dip (JR14 and NNPDFsx)
- Strength of $\xi$ dependence of GPD from Double Distribution encoded in *b*
- Low energies (left)
  - Slight effect from *b* variation but smaller than changing PDFs
  - Also much smaller than scale uncertainties (previous slides)
- Higher energies (right) [log plot]
  - Effect from *b* variation negligible

# Solutions II

Leading-logarithmic approximation (LLA) resummation

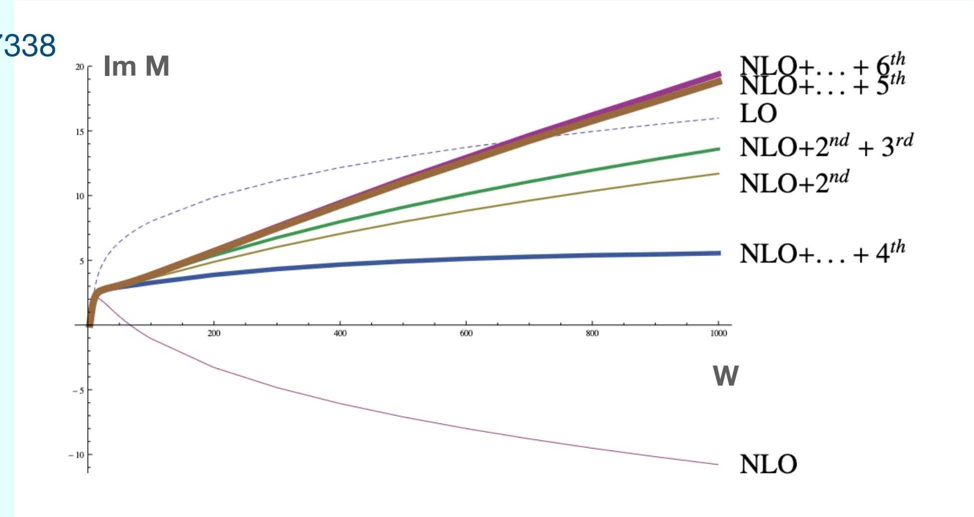
Ivanov 0712.3193
Ivanov et al., 1601.07338

*resums $a_s^n \ln^n 1/\xi$ to **amplitude***

$$\mathcal{I}m\mathcal{M}^g \sim H^g(\xi,\xi) + \int_\xi^1 \frac{dx}{x} H^g(x,\xi) \sum_{n=1} C_n(L) \frac{\bar{\alpha}_s^n}{(n-1)!} \log^{n-1}\frac{x}{\xi}$$

$$\sim 1 + z\ln\left(\frac{m^2}{\mu_F^2}\right) + z^2\left[\frac{\pi^2}{6} + \frac{1}{2}\ln^2\left(\frac{m^2}{\mu_F^2}\right)\right] + \ldots, \quad z^n \sim \alpha_s^n \ln^n(1/\xi)$$

$C_1(L)$ $C_2(L)$ $L = \ln(m^2/\mu_F^2)$




Clear from this formula that scale-fixing approach used at NNLO misses the const*$z^2$ term

Good scale stability observed by including 5 or 6 terms in this LLA resummation

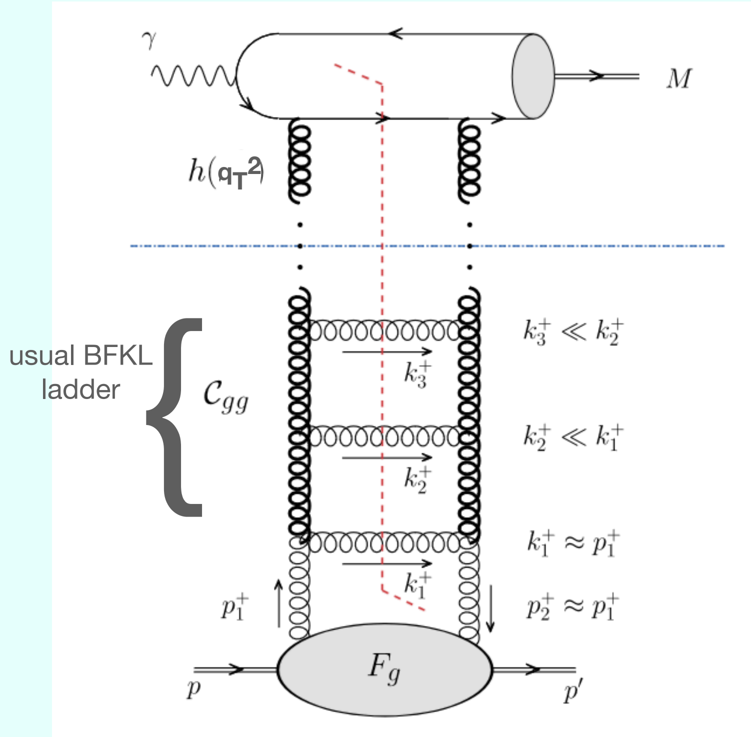
**But** based on above equation, considering **only high-energy terms**

# Solutions III

- Integration of Collinear Factorisation (CF) with High-energy Factorisation (HEF)

in a manner consistent with the NLO DGLAP evolution of GPDs in CF allows for a consistent matching procedure

CAF, Lansberg, Nabeebaccus, Nefedov, Sznajder, Wagner, Phys.Lett.B 859 (2024) 139117




$\Longrightarrow$ resums terms scaling like $\left(\hat{\alpha}_s \ln(x/\xi) \ln(\mu_F^2/\mathbf{q}_T^2)\right)^n$ to all orders in perturbation theory. at ***amplitude*** level

Cgi is known and process independent at LLA & NLLA

$$C_g^{\text{HEF}}\left(\frac{\xi}{x}\right) = \frac{-i\pi}{2}\frac{F_{\text{LO}}}{\left(\frac{\xi}{x}\right)}\int_0^\infty d\mathbf{q}_T^2\, \mathcal{C}_{gi}\left(\frac{\xi}{x}, \mathbf{q}_T^2, \mu_F, \mu_R\right) h(\mathbf{q}_T^2),$$

$$h(\mathbf{q}_T^2) = \frac{M^2}{M^2 + 4\mathbf{q}_T^2}.$$

← process-dependent factor

**LLA in x/ξ**: $a_s^n \ln^{n-1} x/\xi$ *F(Q2), exact in Q.

=> Cgi in **LLA in x/ξ** contains Q dep terms not included in DGLAP

=> to be consistent with NLO DGLAP we **truncate** this resummation to the **Double-Leading-Logarithmic (DLA)** approximation (intersection of LLA in x and Q2).

HEF DLA resummation of terms ~ $a_s^n \ln^{n-1} x/\xi \ln^{n-1} \mu_F$ at ***integrand*** level to the imaginary part of $C_{gi}$

Cgg in DLA is given by

$$\mathcal{C}_{gg}^{(\text{DL})}\left(\frac{\xi}{x}, \mathbf{q}_T^2, \mu_F^2, \mu_R^2\right) = \frac{\hat{\alpha}_s}{\mathbf{q}_T^2}\begin{cases} J_0\left(2\sqrt{\hat{\alpha}_s \ln\left(\frac{x}{\xi}\right)\ln\left(\frac{\mu_F^2}{\mathbf{q}_T^2}\right)}\right) & \text{if } \mathbf{q}_T^2 < \mu_F^2, \\ I_0\left(2\sqrt{\hat{\alpha}_s \ln\left(\frac{x}{\xi}\right)\ln\left(\frac{\mathbf{q}_T^2}{\mu_F^2}\right)}\right) & \text{if } \mathbf{q}_T^2 > \mu_F^2. \end{cases} \quad (*)$$

# General Set up and Framework

***ccbar->J/psi:***

- Effective field theory for production of heavy quarkonium [ Bodwin et al. 1995 ]

$$\sigma_V = \sigma_{q\bar{q}} \cdot \langle O \rangle_V$$

- Relativistic corrections systematically computed by expanding matrix elements in powers of $r$:

$$\mathcal{M}[J/\psi] \propto (\mathcal{A}_\rho + \mathcal{B}_{\rho\sigma} r^\sigma + \mathcal{C}_{\rho\sigma\tau} r^\sigma r^\tau + \ldots)\epsilon^\rho_{J/\psi} \qquad r^\mu = q_1^\mu - q_2^\mu$$

$\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ - matrix elements $\quad \epsilon^\rho_{J/\psi}$ - $J/\psi$ polarization

- We will compute to leading order in relative quark velocity $v$, for $J/\psi$:

$$\mathcal{M}[J/\psi] = \left(\frac{\langle O_1 \rangle_{J/\psi}}{2N_c m_C}\right)^{\frac{1}{2}} \mathcal{A}_\rho \epsilon^\rho_{J/\psi}$$

$$\langle O_1 \rangle_{J/\psi} \equiv \langle O_1(^3S_1) \rangle_{J/\psi}$$

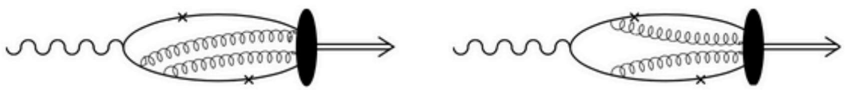
$$\mathcal{O}_1(^3S_1) = \psi^\dagger \boldsymbol{\sigma} \chi \cdot \chi^\dagger \boldsymbol{\sigma} \psi$$

- Compute $\Gamma_{ee} \propto \langle O_1 \rangle_{J/\psi}$
  - Extract $\langle O_1 \rangle_{J/\psi}$ from measurement of $\Gamma_{ee}$

$$\langle O_1 \rangle_V = \frac{N_c}{2\pi} |R_S(0)|^2 + \mathcal{O}(v^2)$$

- Leading zeroth order term in rel. velocity (NRQCD)
- First non-vanishing O(v^2) relativistic correction small AFTER additional ccbar+gg Fock state component considered for gauge invariance

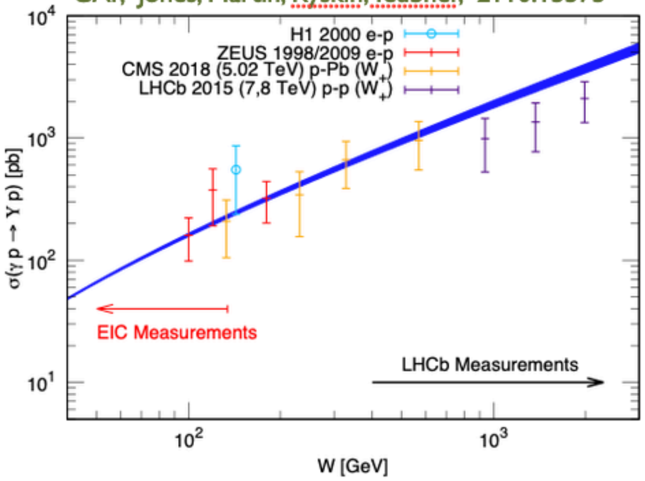
*Hoodbhoy 97*



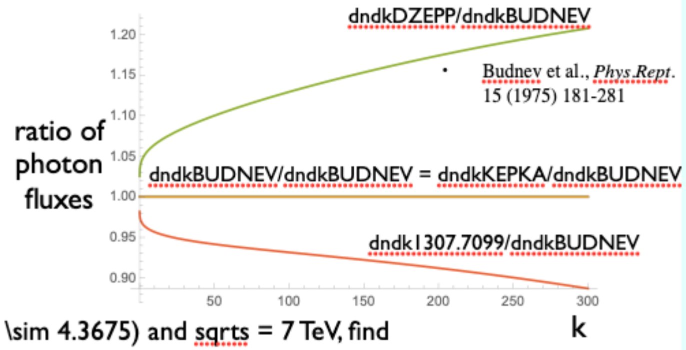
- O(6%) cross section correction factor proportional to derivative of *square* of J/psi w.f. at origin (and affecting normalisation only and not energy dependence)

# Other results in UPC: Photon flux in Upsilon photoprod. in pp

CAF, Jones, Martin, Ryskin, Teubner, 2110.15575




-DGLAP evolve gluon PDF obtained from fit to J/psi data to scale of Upsilon photoproduction and use as input to make cross-section prediction (blue band)




For J/psi rapidity at border of LHCb acceptance (y \sim 4.3675) and sqrts = 7 TeV, find
**(ss1307.7099*flux1307.7099)/(ssBudnev*fluxBudnev)= 0.94901**
**~ 5% effect**

For J/psi rapidity *outside* border of LHCb acceptance (y \sim 5.125) and sqrts = 7 TeV, find
**(ss1307.7099*flux1307.7099)/(ssBudnev*fluxBudnev)= 1.24832**
**~ 25% effect**

Upsilon photoproduction photon energies will be larger so discrepancy between fluxes (and survival factors) will be larger and we enter the region where the approximation of 1307.7099 flux breaks down at much lower rapidities and, importantly, within the acceptance of LHCb

**=> use Budnev flux (without negligence of O(x) terms)**

=> large W unfolded photoproduction LHCb data should be shifted upwards

$$\frac{d\sigma(pp)}{dy} = S^2(W_+)\left(k_+\frac{dn}{dk_+}\right)\sigma_+(\gamma p) + S^2(W_-)\left(k_-\frac{dn}{dk_-}\right)\sigma_-(\gamma p).$$

## Higher twist contributions

- Absorptive corrections, which provide the saturation, are described by higher-twist operators and formally not known within the collinear factorisation approach.
- The relative size of the contribution of the next twist absorptive correction is driven by parameter:

$$c = \alpha_s \frac{xg(x)}{R^2\mu_0^2}$$

- Factor appearing in GLR equation (Phys. Rept. 100 (1983) 1–150) provides non-linear terms through computation of so-called 'fan' diagrams in pQCD that tame (linear) BFKL evolution

- Semi-quantitative estimate based on this scaling gives higher-twist term of O(few percent*). Details in 2006.13857.

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*If one takes into consideration the colour factor calculated assuming that the low x gluon is emitted by the valence quark in the proton, then there is an additional factor of 81/16 which enhances the estimate to ~6.5%. However, the point is that the higher-twist contribution may be relatively small and that, together with the additional factor of alphas from <v2> \sim alphas, all the parametric dependence is included in the GLR factor c.