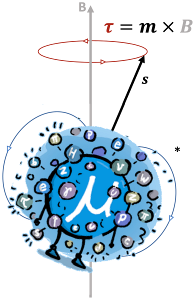
Standard model prediction for the muon $g-2$

Laurent Lellouch

CNRS & Aix-Marseille U.

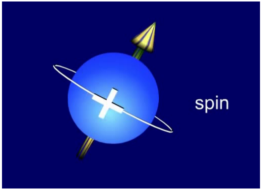
[Budapest-Marseille-Wuppertal collaboration [BMW]]



Charged lepton magnetic moments

Muons are tiny magnets

A massive elementary particle w/ electric charge and spin behaves like a tiny magnet



(← Silver Swan)

Magnetic moment of the muon

$$\vec{\mu}_\mu = \pm g_\mu \frac{e}{2m_\mu}\vec{S}$$

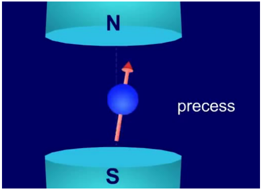
g_μ = Landé factor

In uniform magnetic field $\vec{B}$, $\vec{S}$ precesses w/ angular frequency

$$\omega_S = g_\mu \frac{e}{2m_\mu}|\vec{B}|$$

7×10^6 rotations per second for $|\vec{B}| = 1.45$ T

→ same principle as for MRI

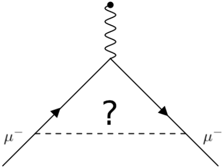


(Silver Swan)

Crucial point:

- g_μ can be **measured** & **calculated** very, very ... precisely
- measurement = SM prediction ?
  → Yes: another victory for the SM
  → No: we have uncovered new fundamental physics

Experimental measurement of $a_\mu \equiv (g_\mu - 2)/2$



Fermilab's Muon $g-2$ Collaboration measurements

Beautiful experiment, very successful six-year data-taking campaign, hugely impressive analyses and results

PHYSICAL REVIEW LETTERS **126,** 141801 (2021)

Editors' Suggestion | Featured in Physics

Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm

B. Abi,[44] T. Albahri,[39] S. Al-Kilani,[36] D. Allspach,[7] L. P. Alonzi,[48] A. Anastasi,[11,a] A. Anisenkov,[4,b] F. Azfar,[44] K. Badgley,[7] S. Baeßler,[47,c] I. Bailey,[19,d] V. A. Baranov,[17] E. Barlas-Yucel,[37] T. Barrett,[6] E. Barzi,[7] A. Basti,[11,32] F. Bedeschi,[11]

(Received 14 March 2021; accepted 25 March 2021; published 7 April 2021)

We present the first results of the Fermilab National Accelerator Laboratory (FNAL) Muon $g-2$ Experiment for the positive muon magnetic anomaly $a_\mu \equiv (g_\mu - 2)/2$. The anomaly is determined from the precision measurements of two angular frequencies. Intensity variation of high-energy positrons from muon decays directly encodes the difference frequency ω_a between the spin-precession and cyclotron frequencies for polarized muons in a magnetic storage ring. The storage ring magnetic field is measured using nuclear magnetic resonance probes calibrated in terms of the equivalent proton spin precession frequency $\tilde{\omega}'_p$ in a spherical water sample at 34.7 °C. The ratio $\omega_a/\tilde{\omega}'_p$, together with known fundamental constants, determines $a_\mu(\mathrm{FNAL}) = 116\,592\,040(54) \times 10^{-11}$ (0.46 ppm). The result is 3.3 standard deviations greater than the standard model prediction and is in excellent agreement with the previous Brookhaven National Laboratory (BNL) E821 measurement. After combination with previous measurements of both μ^+ and μ^-, the new experimental average of $a_\mu(\mathrm{Exp}) = 116\,592\,061(41) \times 10^{-11}$ (0.35 ppm) increases the tension between experiment and theory to 4.2 standard deviations.

PHYSICAL REVIEW LETTERS **131,** 161802 (2023)

Editors' Suggestion

Measurement of the Positive Muon Anomalous Magnetic Moment to 0.20 ppm

D. P. Aguillard,[33] T. Albahri,[30] D. Allspach,[7] A. Anisenkov,[4,a] K. Badgley,[7] S. Baeßler,[35,b] I. Bailey,[17,c] L. Bailey,[27] V. A. Baranov,[15,d] E. Barlas-Yucel,[28] T. Barrett,[6] E. Barzi,[7] F. Bedeschi,[10] M. Berz,[18]

(Received 10 August 2023; accepted 5 September 2023; published 17 October 2023)

We present a new measurement of the positive muon magnetic anomaly, $a_\mu \equiv (g_\mu - 2)/2$, from the Fermilab Muon $g-2$ Experiment using data collected in 2019 and 2020. We have analyzed more than 4 times the number of positrons from muon decay than in our previous result from 2018 data. The systematic error is reduced by more than a factor of 2 due to better running conditions, a more stable beam, and improved knowledge of the magnetic field weighted by the muon distribution, $\tilde{\omega}'_p$, and of the anomalous precession frequency corrected for beam dynamics effects, ω_a. From the ratio $\omega_a/\tilde{\omega}'_p$, together with precisely determined external parameters, we determine $a_\mu = 116\,592\,057(25) \times 10^{-11}$ (0.21 ppm). Combining this result with our previous result from the 2018 data, we obtain $a_\mu(\mathrm{FNAL}) = 116\,592\,055(24) \times 10^{-11}$ (0.20 ppm). The new experimental world average is $a_\mu(\mathrm{exp}) = 116\,592\,059(22) \times 10^{-11}$ (0.19 ppm), which represents a factor of 2 improvement in precision.

arXiv:2506.03069 [hep-ex]

Measurement of the Positive Muon Anomalous Magnetic Moment to 127 ppb

D. P. Aguillard,[33] T. Albahri,[30] D. Allspach,[7] J. Annala,[7] K. Badgley,[7] S. Baeßler,[35] I. Bailey,[16, a] L. Bailey,[27] E. Barlas-Yucel,[28, b] T. Barrett,[6] E. Barzi,[7] F. Bedeschi,[10] M. Berz,[17] M. Bhattacharya,[7] H. P. Binney,[36]

A new measurement of the magnetic anomaly a_μ of the positive muon is presented based on data taken from 2020 to 2023 by the Muon $g-2$ Experiment at Fermi National Accelerator Laboratory (FNAL). This dataset contains over 2.5 times the total statistics of our previous results. From the ratio of the precession frequencies for muons and protons in our storage ring magnetic field, together with precisely known ratios of fundamental constants, we determine $a_\mu = 116\,592\,0710(162) \times 10^{-12}$ (139 ppb) for the new datasets, and $a_\mu = 116\,592\,0705(148) \times 10^{-12}$ (127 ppb) when combined with our previous results. The new experimental world average, dominated by the measurements at FNAL, is $a_\mu(\mathrm{exp}) = 116\,592\,0715(145) \times 10^{-12}$ (124 ppb). The measurements at FNAL have improved the precision on the world average by over a factor of four.

$$a_\mu|_{\mathrm{expt}} = (11659207.15 \pm 1.45) \times 10^{-10} \quad [124\ \mathrm{ppb}]$$

$\longrightarrow$ see talks by Estifa'a Zaid, Elia Bottalico and Alberto Lusiani later in session

Reference standard model calculation of a_μ

[Aliberti et al '25 = WP '25 [Muon $g-2$ Theory Initiative, member of SC]]

All three interactions and all SM particles are needed

$$
\begin{aligned}
a_\mu^{\rm SM} &= a_\mu^{\rm QED} + a_\mu^{\rm had} + a_\mu^{\rm EW} \\
&= O\left(\frac{\alpha}{2\pi}\right) + O\left(\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_\mu}{M_\rho}\right)^2\right) + O\left(\left(\frac{\alpha}{16\pi \sin^2\theta_W}\right)\left(\frac{m_\mu}{M_W}\right)^2\right) \\
&= O\left(10^{-3}\right) + O\left(10^{-7}\right) + O\left(10^{-9}\right)
\end{aligned}
$$

QED contributions to a_ℓ

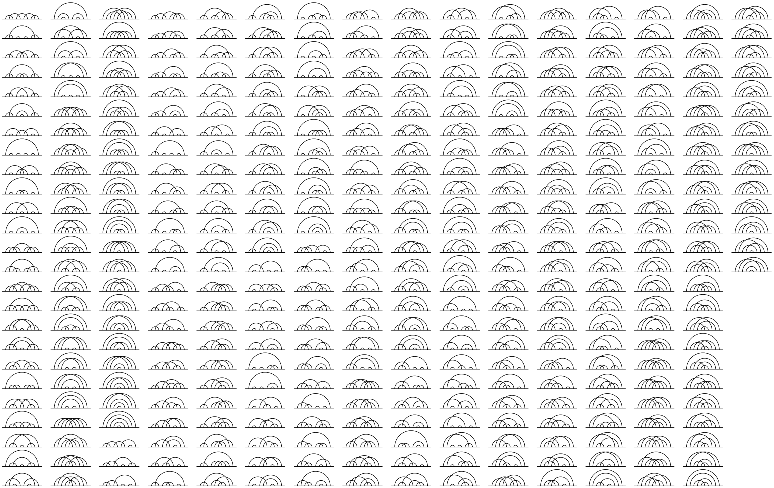
Loops with only photons and leptons: can expand in $\alpha = e^2/(4\pi) \ll 1$

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

- $A_1^{(2)}$, $A_1^{(4)}$, $A_1^{(6)}$, $A_2^{(4)}$, $A_2^{(6)}$, $A_3^{(6)}$ known analytically [Schwinger '48; Sommerfield '57, '58; Petermann '57; ...]
- $O((\alpha/\pi)^3)$: 72 diagrams [Laporta et al '91, '93, '95, '96; Kinoshita '95)
- $O((\alpha/\pi)^4; (\alpha/\pi)^5)$: 891;12,672 diagrams [Laporta '95; Aguilar et al '08; Aoyama et al '96-'19, Volkov '19-'24]
  - Automated generation of diagrams
  - Numerical evaluation of loop integrals
  - Calculations cross-checked

5-loop QED diagrams



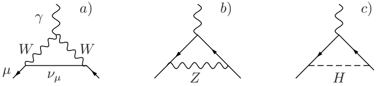
[Aoyama et al '15, '19]

$$a_\mu^{\mathrm{QED}} \times 10^{10} = 11\,658\,471.88(2)\ [1.7\ \mathrm{ppb}]$$

[99.994% of a_μ]

Electroweak contributions to a_μ: Z, W, H, etc. loops

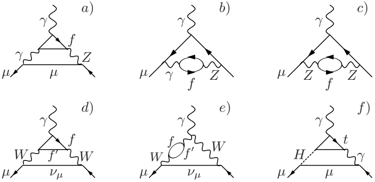
1-loop



$$a_\mu^{\text{EW},(1)} = O\left(\frac{\sqrt{2}G_F m_\mu^2}{16\pi^2}\right) = 19.479(1) \times 10^{-10}$$

(Gnendiger et al ’15, Aoyama et al ’20 and refs therein)

2-loop

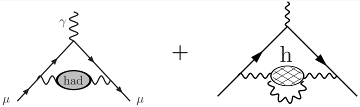


$$a_\mu^{\text{EW},(2)} = O\left(\frac{\sqrt{2}G_F m_\mu^2}{16\pi^2}\frac{\alpha}{\pi}\right) = -4.038(30) \times 10^{-10}$$

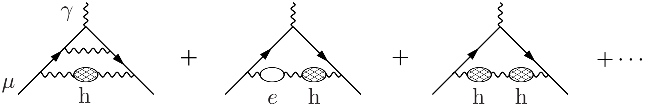
(Gnendiger et al ’15 and refs therein)

$$a_\mu^{\text{EW}} = 15.44(4) \times 10^{-10}$$

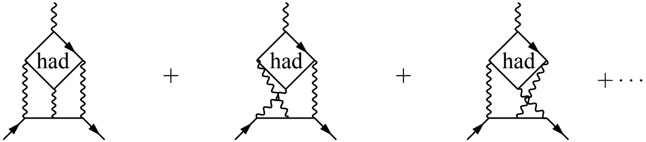
Hadronic contributions to a_μ



$\rightarrow a_\mu^{\text{LO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^2\right)$



$\rightarrow a_\mu^{\text{NLO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$



$\rightarrow a_\mu^{\text{HLbL}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$

Involve q and g in low-energy regime of QCD $\Rightarrow$ nonperturbative tools ($\neq a_\mu^{\text{QED}}, a_\mu^{\text{weak}}$)

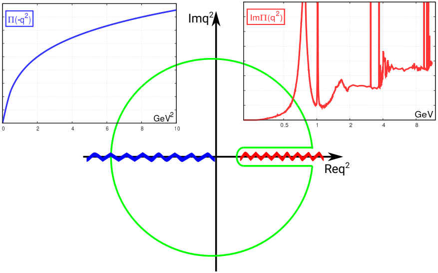
Big challenge: needed to $\lesssim 0.2\%$ to fully leverage Fermilab a_μ measurement !

Data-driven determination of HVP contribution

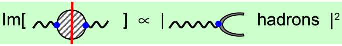
- $\Pi_{\mu\nu}(q) =$  $= (q_\mu q_\nu - g_{\mu\nu} q^2)\, \Pi(q^2)$

- $a_\mu^{\text{LO-HVP}} =$ weighted integral of $\hat{\Pi}(q^2) \equiv \Pi(q^2) - \Pi(0)$ for $q^2 = -Q^2$, $Q^2 = 0 \to \infty$

- $\hat{\Pi}(q^2)$ is real and analytic except for cut along real, positive q^2 axis



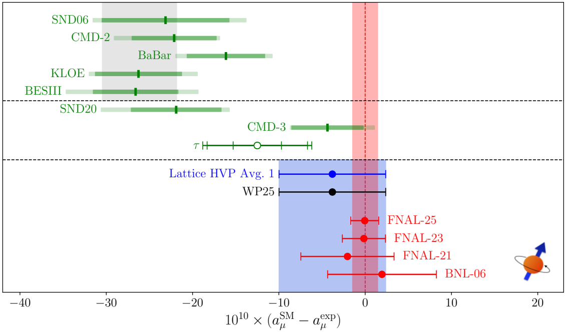
- **Analyticity**: can get $\hat{\Pi}(q^2)$ for $q^2 \leq 0$ from $\text{Im}\Pi(q^2)$ w/ $q^2 > 0$ via contour integral (**QCD asymptotics** $\Rightarrow$ [once subtracted] dispersion relation)

- **Unitarity** [Bouchiat et al '61]: 

$$\text{Im}\Pi(s) = -\frac{R(s)}{12\pi}, \quad R(s) \equiv \frac{\sigma(e^+e^- \to \text{had})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

HVP results

→ for lattice HVP see talks by Alessandro Lupo and Pierre Vanhove



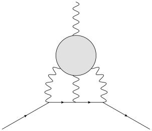
- Situation of data-driven results is currently very confusing
- BMW '20 [Nature 2021] gave first indication that there was a problem w/ data-driven results
- $\sigma(e^+e^- \to \text{hadrons})$ from CMD3 '23 put nail in (temporary) coffin
- WP20: HVP from $\sigma(e^+e^- \to \text{hadrons})$ ⟶ WP25: HVP from lattice [BMW, ETM, Fermilab/HPQCD, Mainz, RBC/UKQCD, . . .]

$$a_\mu^{\text{LO-HVP}}|_{\text{WP25}} = 713.2(6.1) \times 10^{-10} \quad [0.85\%]$$

Data-driven determination of HLbL contribution

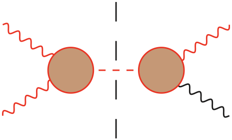
Master formula [G. Colangelo et al, '17]

$$a_\mu^{\rm HLbL} = \frac{2\alpha^3}{48\pi^2}\int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau\sqrt{1-\tau^2}\sum_{i=1}^{12} T_i(Q_1,Q_2,\tau)\bar{\Pi}_i(Q_1,Q_2,\tau)$$



Use: **analyticity**, **unitarity**, **QCD asymptotics** & **experimental data** [and models]

$$\Pi_{\mu\nu\rho\sigma} = \Pi_{\mu\nu\rho\sigma}^{\pi^0\text{-pole}} +$$



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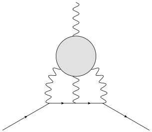
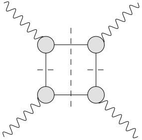


Use: **analyticity**, **unitarity**, **QCD asymptotics** & **experimental data** [and models]

$$\Pi_{\mu\nu\rho\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\rho\sigma} + \Pi^{\pi\text{-box}}_{\mu\nu\rho\sigma} +$$



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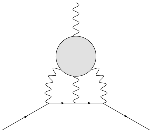
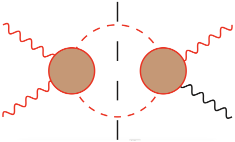


Use: **analyticity**, **unitarity**, **QCD asymptotics** & **experimental data** [and models]

$$\Pi_{\mu\nu\rho\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\rho\sigma} + \Pi^{\pi\text{-box}}_{\mu\nu\rho\sigma} + \Pi^{2\pi}_{\mu\nu\rho\sigma} + \cdots$$



Data-driven determination of HLbL contribution

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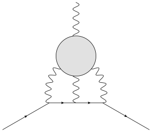
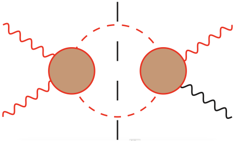


Use: **analyticity**, **unitarity**, **QCD asymptotics** & **experimental data** [and models]

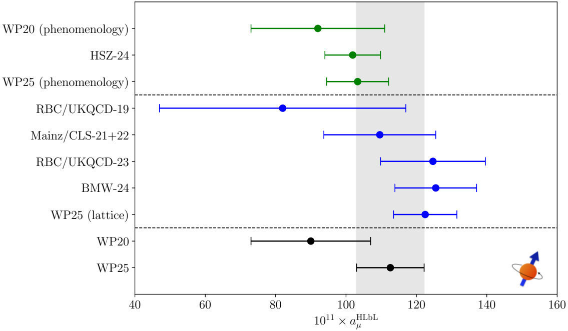
$$\Pi_{\mu\nu\rho\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\rho\sigma} + \Pi^{\pi\text{-box}}_{\mu\nu\rho\sigma} + \Pi^{2\pi}_{\mu\nu\rho\sigma} + \cdots$$



- Much more complex analytic and QCD asymptotic behavior [e.g. J. Bijnens et al '21]
- Much less data and of lesser precision
- Modeling needed for $\cdots$
- However, "only" 10% precision needed to fully leverage Fermilab measurement

HLbL results

→ for lattice HLbL see talk by Antoine Gérardin



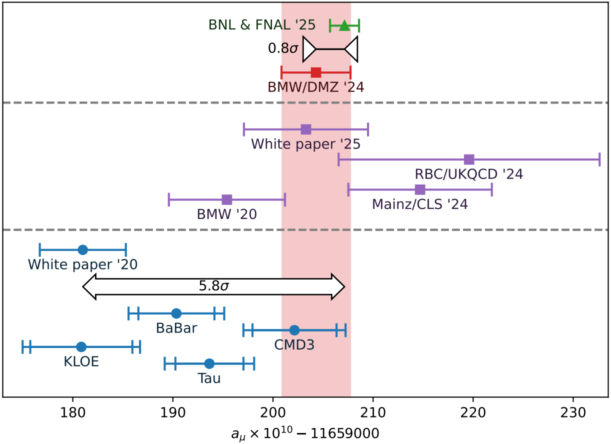
- Lattice and pheno determinations of a_μ^{HLbL} have agreed for years
- Central value has barely changed since model-based 2009 Glasgow consensus
- Now error reduced and fully under control
- WP20 & WP25 use combination of lattice & pheno results

$$a_\mu^{\mathrm{HLbL}}|_{\mathrm{WP25}} = 11.26(96) \times 10^{-10} \quad [8.5\%]$$

SM vs experiment

$$a_\mu|_{\text{WP25}} = (11659203.3 \pm 6.2) \times 10^{-10}$$
$$a_\mu|_{\text{BMW24+WP25}} = (11659204.2 \pm 3.4) \times 10^{-10}$$

[BMW '24 a hybrid, lattice calculation of $a_\mu^{\text{LO-HVP}}$ [2407.10913 [hep-lat], see A. Lupo's talk]]



Conclusions and outlook

Conclusions and outlook

- On June 3rd Fermilab announced measurement of a_μ with incredible 127 ppb precision [WA has 124 ppb]
- . . . BMW '24 + WP '25 indicates that SM confirmed to within 0.3 ppb (0.4 ppb w/ WP '25 alone)
- Decades of perturbative QED & EW calculations combined w/ fully nonperturbative QCD (+QED) ones predict g_μ to a 0.3 ppb precision!
- Stringent, single test of the complete SM (all particles & interactions)
- HVP
  - WP '20: SM prediction given by data-driven methods
  - WP '25: SM prediction given by lattice calculations, w/ consolidated averages from many independent calculations
  - Responsible for 92% of total error-squared (of $a_\mu|_{\text{BMW24+WP25}}$)
  - BMW '24 uncertainty must be reduced by factor ~ 2 to match experiment (~ 4 for WP '25)

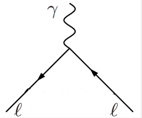
Conclusions and outlook

- HLbL
  - WP '20 & WP '25: SM prediction given by lattice and pheno that agree well
  - Responsible for 8% of total error-squared (of $a_\mu|_{\text{BMW24+WP25}}$)
  - Uncertainty small enough already
- $\tau^\pm \to \pi^\pm\pi^0\nu_\ell$ decays are making a comeback
- Future
  - J-PARC entirely new method for a_μ measurement
  - More lattice results for complete $a_\mu^{\text{LO-HVP}}$ expected soon
  - New BABAR, KLOE, BES III, BELLE-II, SND-2 $e^+e^- \to$ hadrons analyses (and data) for $e^+e^- \to$ hadrons & $\tau^\pm \to \pi^\pm\pi^0\nu_\ell$
  - MUonE @ CERN for spacelike HVP

BACKUP

Early history: the electron

- 1928 : Dirac's new theory predicts the existence of the positron and



$$g_e|_{\text{Dirac}} = 2$$

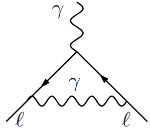
"That was really an unexpected bonus for me" (P.A.M. Dirac)

- 1934 : Kinsler & Houston confirm $g_e = 2$, w/ permil precision by studying spectrum of neon atom

- 1947 : Nafe, Nelson & Rabi, then Kusch & Foley measure hyperfine structure of hydrogen and deuterium, showing that $g_e > 2$ by 0.1%

  $\rightarrow$ there is a problem w/ Dirac!

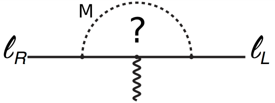
- 1947 : Schwinger understands very quickly that Dirac's theory neglects **quantum fluctuations** and manages to compute them to obtain the **"anomalous"** contribution



$$a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi} = 0.00116\ldots$$

$\rightarrow$ birth of QED and relativistic quantum field theory

Why are a_ℓ interesting?



$$\longrightarrow \qquad \mathcal{L}_{\rm eff} = -\frac{Qe}{2}\frac{a_\ell}{2m_\ell} F^{\mu\nu}[\bar\ell_L \sigma_{\mu\nu} \ell_R] + \text{hc}$$

- $a_{e,\mu}$ are parameter-free predictions of the SM that can be measured very precisely
  $\Rightarrow$ excellent tests of SM

- Loop induced $\Rightarrow$ sensitive to new dofs that may be too heavy or too weakly coupled to be produced directly

- Flavor and CP conserving, chirality flipping $\Rightarrow$ sensitive to muon mass generation mechanism & complementary to: EDMs, s and b decays, LHC direct searches, ...

- Contribution of particle w/ $M \gg m_\ell$ is generically

$$a_\ell^{\rm M} = C \left(\frac{\Delta_{LR}}{m_\ell}\right)\left(\frac{m_\ell}{M}\right)^2$$

- In EW theory $M \simeq M_W$ and only source of chirality flipping is Yukawa

$$\Delta_{LR} = y_\ell \langle H\rangle = m_\ell \qquad \text{and} \qquad C \sim \frac{\alpha}{4\pi \sin^2\theta_W}$$

- In BSM, can have enhancements: e.g. SUSY $M = M_{\rm SUSY}$ and $C \sim \alpha/\left(4\pi\sin^2\theta_W\right)$ & $\Delta_{LR} = \left(\mu/M_{\rm SUSY}\right) \times \tan\beta \times m_\ell$; or radiative m_ℓ model, $\Delta_{LR} \simeq m_\ell$, $C \sim 1$ and $M = M_{\rm N\Phi}$

- $m_\mu/m_e \simeq 207 \Rightarrow a_\mu \sim 43,000\times$ more sensitive to possible new heavy particles than a_e

White Paper ’25 result compilation

Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E989, E821)		Eq. (9.5)	116 592 071.5(14.5)	Refs. [5–8, 10–13]
HVP LO (lattice)	Sec. 3.6.1	Eq. (3.37)	7132(61)	Refs. [14–30]
HVP LO (e^+e^-, τ)	Sec. 2	Table 5	Estimates not provided at this point	
HVP NLO (e^+e^-)	Sec. 2.9	Eq. (2.47)	−99.6(1.3)	Refs. [31, 32]
HVP NNLO (e^+e^-)	Sec. 2.9	Eq. (2.48)	12.4(1)	Ref. [33]
HLbL (phenomenology)	Sec. 5.10	Eq. (5.69)	103.3(8.8)	Refs. [34–57]
HLbL NLO (phenomenology)	Sec. 5.10	Eq. (5.70)	2.6(6)	Ref. [58]
HLbL (lattice)	Sec. 6.2.8	Eq. (6.34)	122.5(9.0)	Refs. [59–63]
HLbL (phenomenology + lattice)	Sec. 9	Eq. (9.2)	112.6(9.6)	Refs. [34–57, 59–63]
QED	Sec. 7.5	Eq. (7.27)	116 584 718.8(2)	Refs. [64–70]
EW	Sec. 8	Eq. (8.12)	154.4(4)	Refs. [51, 71–73]
HVP LO (lattice) + HVP N(N)LO (e^+e^-)	Sec. 9	Eq. (9.1)	7045(61)	Refs. [14–33]
HLbL (phenomenology + lattice + NLO)	Sec. 9	Eq. (9.3)	115.5(9.9)	Refs. [34–63]
Total SM Value	Sec. 9	Eq. (9.4)	116 592 033(62)	Refs. [14–73]
Difference: $\Delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM}$	Sec. 9	Eq. (9.6)	38(63)	