Precision calculation of the hadronic vacuum polarization contribution to the muon anomaly

Based on **arXiv:2407.10913**

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on behalf of BMW and DMZ

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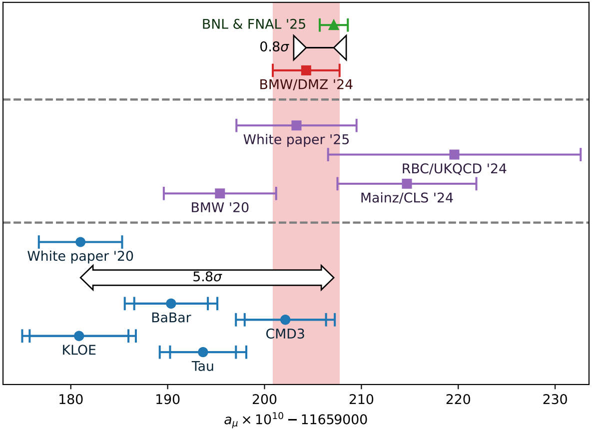

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Lattice QCD

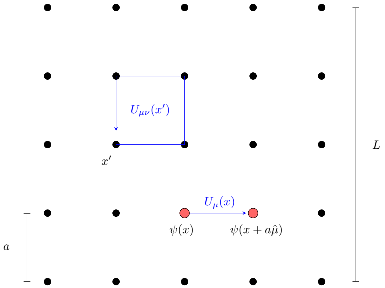
- Regularisation of QCD (IR & UV cutoff) that leads to a well-defined path integral...

- ... in Euclidean spacetime: suitable for Monte Carlo evaluation of observables

$$\langle \mathcal{O} \rangle = \mathcal{Z}^{-1} \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A\ \mathcal{O}[U, A, \psi, \bar{\psi}] e^{-S[U,A,\psi,\bar{\psi}]}$$

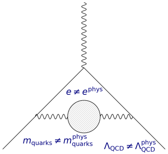
- Framework to precisely quantify systematics: only way to provide ab-initio predictions in QCD (+QED) with subpercent precision

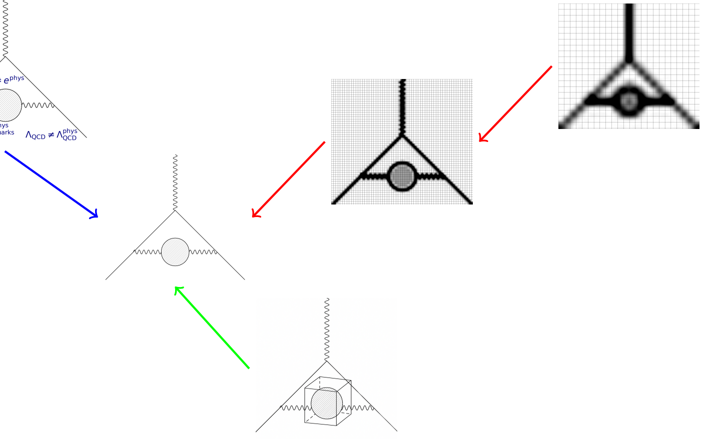
Fermions on site $\psi(x)$ Gluons, photons on links $U_\mu(x)$



Lattice QCD: systematics

- Continuum limit
- Finite volume
- Calibrating parameters to reproduce QCD and QED
- Finite statistics





$$\langle \mathcal{O} \rangle = \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi \, \mathcal{O}[U, \psi, \bar{\psi}] e^{-S_{\text{gluon}} - \int \bar{\psi} D[U] \psi}$$

$$= \int \mathcal{D}U e^{-S_{\text{gluon}}} \det D[U] \; \mathcal{O}[U]_{\text{Wick}}$$

$$= \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N} \mathcal{O}[U_n]$$

- Isospin breaking (QED and $m_u \neq m_d$)

Strategy

- Time + zero-momentum representation [hep-lat/1107.4388 Meyer, Bernecker]

$$a_\mu^{\mathrm{LO-HVP}} = \alpha^2 \int_0^\infty dt\, K(t) C(t)$$

- Two-point function of electromagnetic currents:

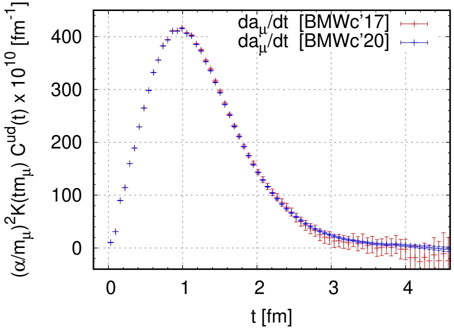
$$C(t) = \frac{1}{3} \sum_{i=1}^{3} \langle J_i(t) J_i(0) \rangle$$

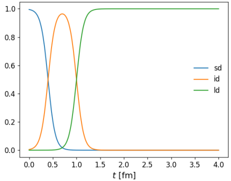
- The kernel K(t), related to the leptonic part, is known

$$K(t) = \int_0^\infty \frac{dQ^2}{m_\mu^2}\, \omega(\frac{Q^2}{m_\mu^2})\, [t^2 - \frac{4}{Q^2} \sin^2(\frac{Qt}{2})]\ ,$$

$$\omega(r) = [r + 2 - \sqrt{r(r+4)}]^2 / \sqrt{r(r+4)}$$

- Construct and compare window observables [RBC/UKQCD '18] to reduce/enhance certain systematics

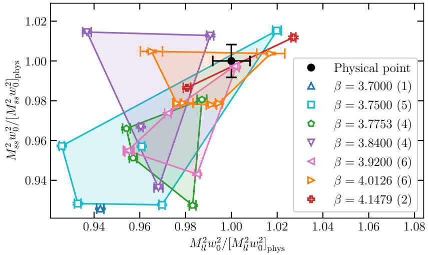


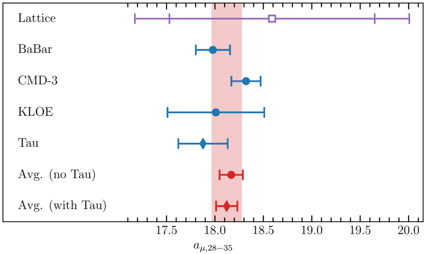


- 28 large-scale simulations, scattered around the physical point
- Blinded analysis
- Including new lattice spacing, closer to the continuum

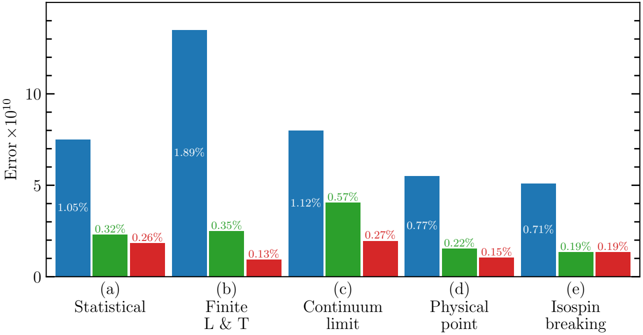
$$a = 0.064\text{fm } [96^3 \times 144] \longrightarrow 0.048\text{fm } [128^3 \times 192]$$

- Continuum extrapolation of $I = 0$ instead of disconnected
- Analysis from optimised set of windows: $[0, 0.4] \cup [0.4, 0.6] \cup [0.6, 1.2] \cup [1.2, 2.8]$fm Lattice QCD+QED : (96.1% of total a_μ)
- Evaluation of $[2.8, \infty]$ fm (3.9%): computed from low-energy region of e^+e^- annihilation data, no tension between datasets (see Fig. $\rightarrow$)





Error budget and improvement



References:

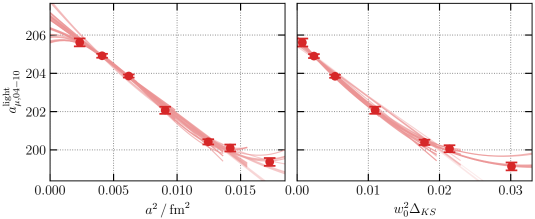
BMW 20 ≡ Borsanyi, S. *et al.* Leading hadronic contribution to the muon magnetic moment from lattice QCD. *Nature* **593,** 51–55. arXiv: 2002.12347 [hep-lat] (2021).

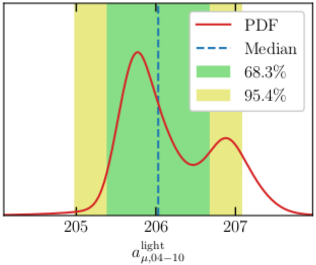
BMW 17 ≡ Borsanyi, S. *et al.* Hadronic vacuum polarization contribution to the anomalous magnetic moments of leptons from first principles. *Phys. Rev. Lett.* **121,** 022002. arXiv: 1711.04980 [hep-lat] (2018).

- Extrapolation of a given quantity Y to the continuum, iso-symmetric point
- X_l, X_s deviations from physical values of light / strange quarks
- a^2 and Δ_{KS} parametrise deviations from the continuum results

$$Y(a, X_l, X_s) = A(a^2) + A'(\Delta_{KS}) + X_l B(a^2) + X_s C(a^2)$$

- Combinations of different models, cuts on data, ... weighted with AIC





- Typical lattice volumes $L_{\rm ref} \sim 6$ fm

- The remaining 2% needs to be estimated:

  · ChPT [next talk by Pierre Vanhove]

  · e^+e^- data into finite-volume physics

  [Lüscher '91, Lellouch-Lüscher '01, H.B.Meyer '11], Hansen-Patella ['19, '20]

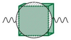
- BMW20: $L_{\rm big} \sim 11$fm box

- BMW24: e^+e^- tail [2.8 fm, inf] reduces the part of the calculation that is in a finite volume

$$18.5(2.5) \to 9.3(9) \quad \textbf{[0.13\%]}$$

- BMW24: for our estimate we use lattice in the $L_{\rm big} = 11$fm box, and we check consistency with other methods (Fig. →)

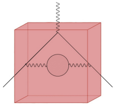
97% of a_μ



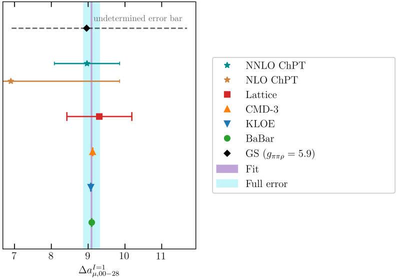
$L_{\rm ref}$

(missing $\sim$ 20 out of 715 units 10^{-10})

99.9% of a_μ

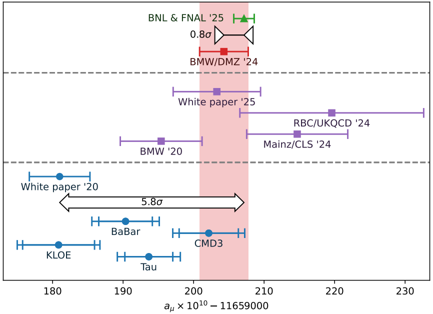


$L_{\rm big}$



Conclusions

light and disconnected $00-28$	$618.6(1.9)(2.3)[3.0]$	this work, Equation (34)
strange $00-28$	$53.19(13)(16)[21]$	this work, Equation (37)
charm $00-28$	$14.64(24)(28)[37]$	this work, Equation (40)
light qed	$-1.57(42)(35)$	[5], Table 15 corrected in Equation (45)
light sib	$6.60(63)(53)$	[5], Table 15
disconnected qed	$-0.58(14)(10)$	[5], Table 15
disconnected sib	$-4.67(54)(69)$	[5], Table 15
disconnected charm	$0.0(1)$	[31], Section 4 in Supp. Mat.
strange qed	$-0.0136(86)(76)$	[5], Table 15
charm qed	$0.0182(36)$	[43]
bottom	$0.271(37)$	[44]
tail from data-driven $28-\infty$	$27.59(17)(9)[26]$	this work, Equation (50)
total	$714.1(2.2)(2.5)[3.3]$	



References:

5. BMW20
31. BMW17
43. Giusti et al. [hep-lat/1901.10462]
44. Colquhoun et al. [hep-lat/1408.5768]

- Agreement between lattice collaborations and experiments: no indication of new physics
- Tension with e^+e^- based determination remains