LEVERHULME TRUST

UNIVERSITY OF LIVERPOOL

Beam dynamics corrections at the Muon g-2 experiment at Fermilab

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FERMILAB-SLIDES-25-0152-V

Beam Dynamics Correction

- The formula shows the R'_μ calculation to extract a_μ.
- The corrections applied to the measured anomalous precession frequency ω_a^m are necessary to get an unbiased a_μ result.

Estifa'a Parallel Talk - 723

Beam dynamics corrections

Saskia Plenary Talk - 136

$$R'_\mu \approx \frac{f_{clock}\,\omega_a^m \left(1 + C_e + C_p + C_{ml} + C_{pa} + C_{dd}\right)}{f_{calib} < \omega'_p(x, y, \phi) \times M(x, y, \phi) > \left(1 + B_k + B_q\right)}$$

- The total correction applied to ω_a^m in **Run-456** is: **515 ppb** with a systematic of **42 ppb**

Beam Dynamics Correction

- We must distinguish these corrections into two different categories:

Spin Dynamics

Time varying phase

Beam Dynamics Correction

- We must distinguish these corrections into two different categories:

Spin Dynamics

Time varying phase

Spin Dynamics

- Considering the extended version of the anomalous precession frequency we find two contributions that affect the measured $\vec{\omega}_a$:

$$\vec{\omega}_a = \frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)(\vec{\beta}\times\vec{E}) - a_\mu \left(\frac{\gamma}{\gamma + 1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta}\right]$$

E-field correction

- The formula represents the anomalous precession frequency:

$$\vec{\omega}_a = \frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)(\vec{\beta}\times\vec{E}) - a_\mu\left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta}\right]$$

This term is reduced by the choice of magic momentum, but not completely cancelled (due to muon momentum distribution)

- The residual effect of the electric field is computed as follow:

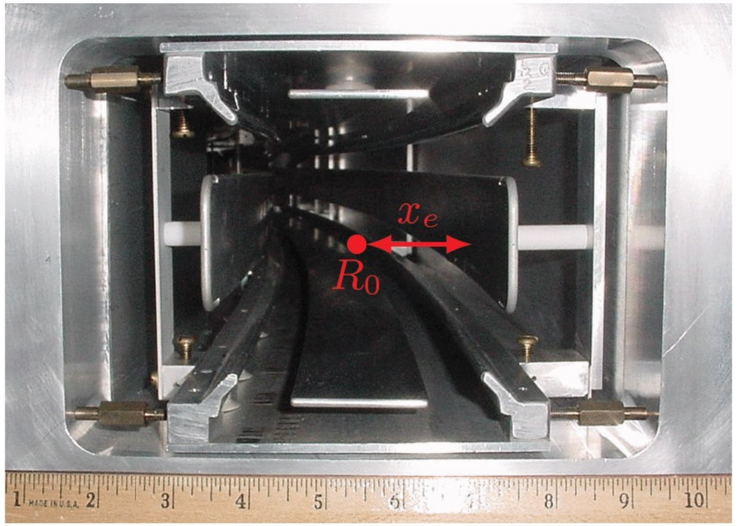
$$C_E \approx 2\frac{\beta_0}{B}\left\langle E_r \frac{\Delta p}{p_0}\right\rangle$$

Radial Electric field (w/o quads C_E=0)

Momentum spread (w/ mono-energetic muon C_E=0)

- Both E_r and $\Delta p/p_0$ relate to the **radial offset** x_e:

$$E_r \approx \left(\frac{\beta_0 B}{R_0} n\right) x_e \qquad \frac{\Delta p}{p_0} \approx \left(\frac{1-n}{R_0}\right) x_e$$



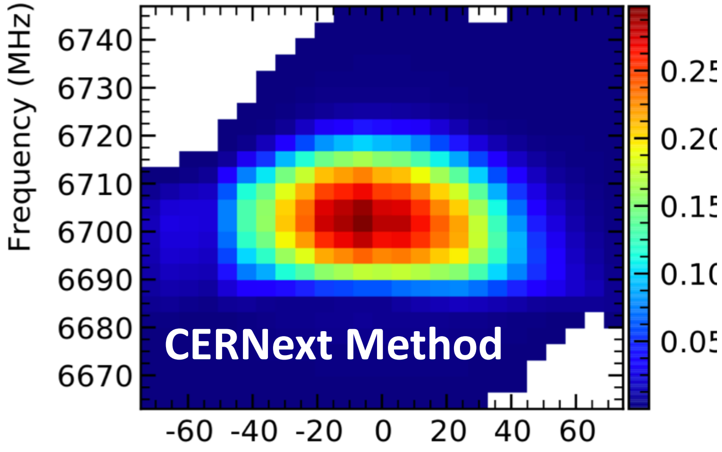
- The $\boldsymbol{C_e}$ can be written as function of equilibrium radii or momentum:

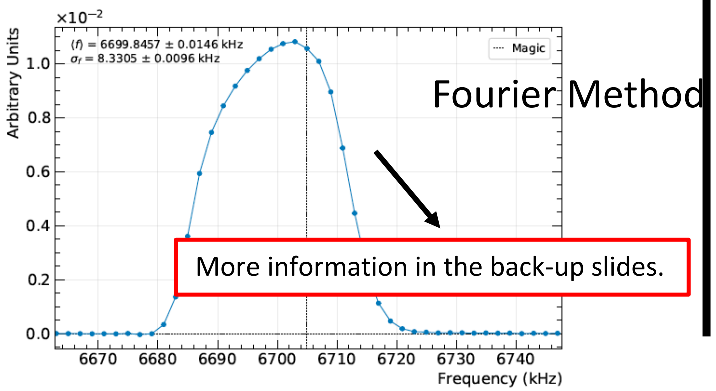
$$\boldsymbol{C_E \approx 2n(1-n)\frac{\beta_0^2}{R_0}\langle x_e^2\rangle = \frac{2n\beta_0^2}{1-n}\langle \delta^2\rangle}$$

C_e correction – Calculation methods

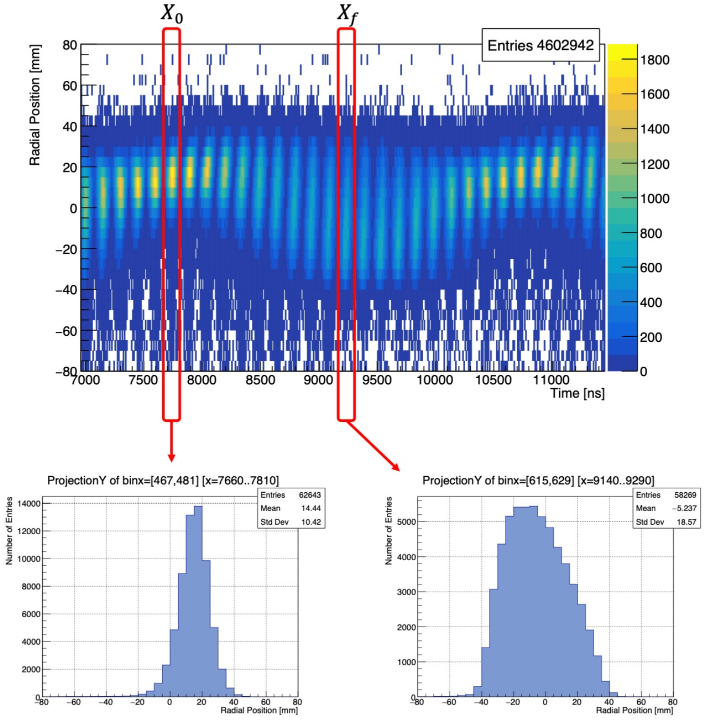
- Three different methods have been developed to compute the C_e correction:

Calo method

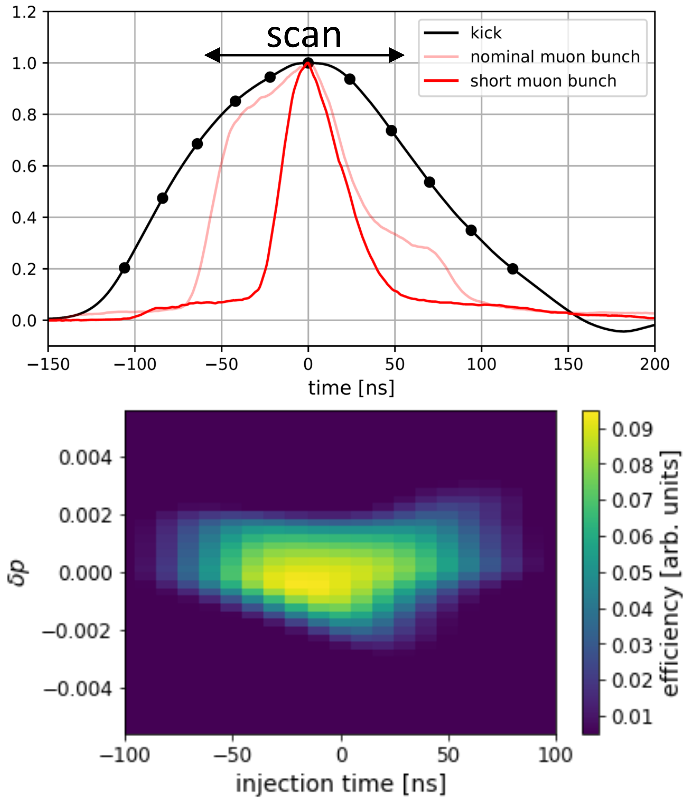




Tracker method



MiniSciFi method



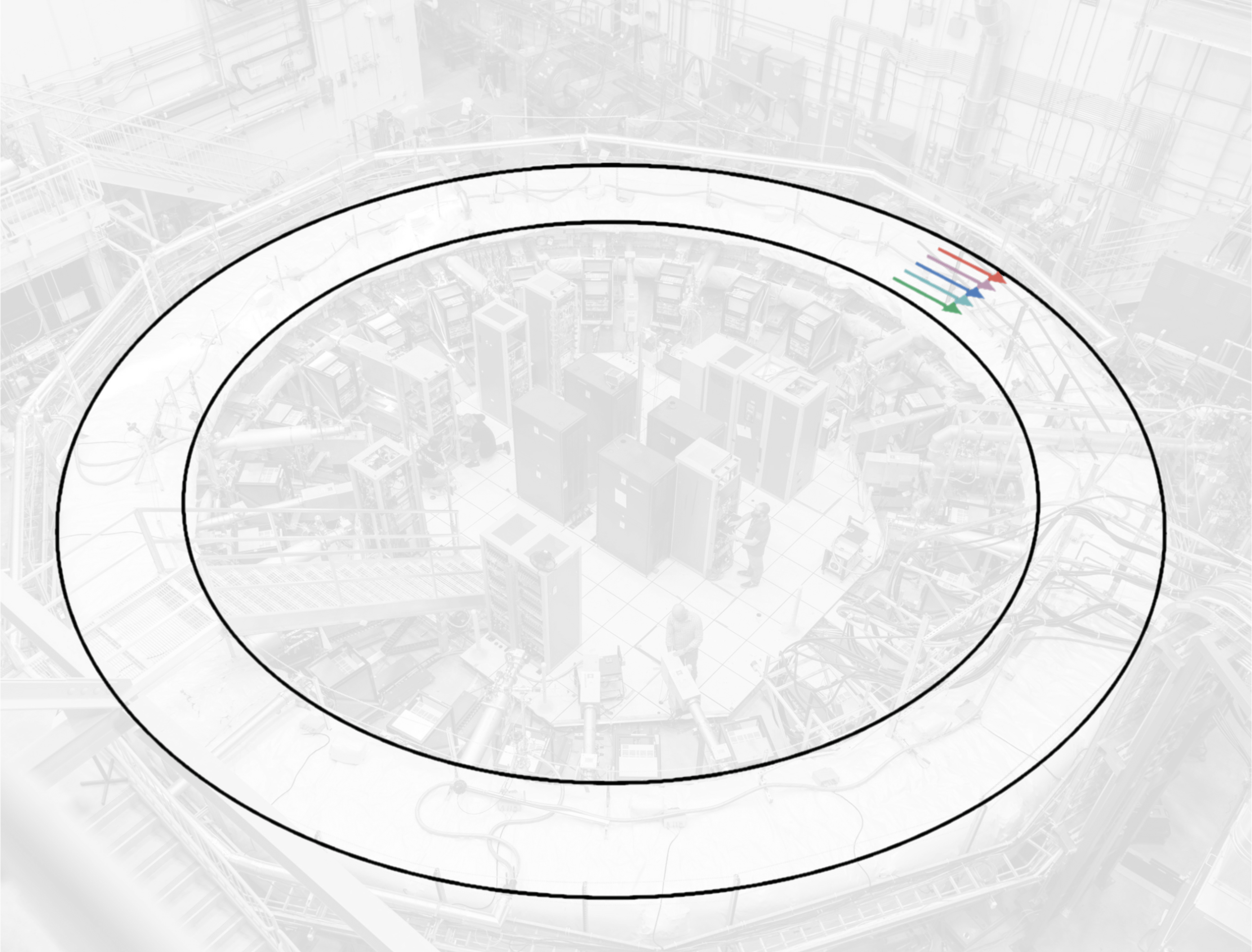
Higher momentum muons:

- stored at **larger radii**
- take **longer** to go around

Lower momentum muons:

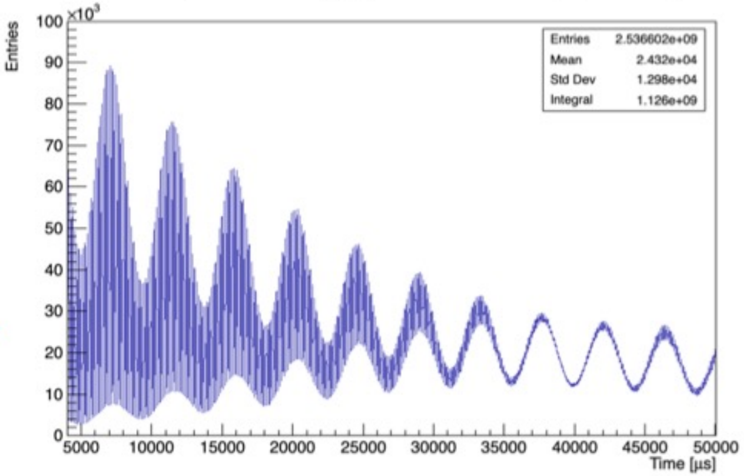
- stored at **smaller radii**
- take **shorter** to go around

Dephasing and **radial position** used to extract the stored momentum spectrum



Calo Method – Fast Rotation Signal

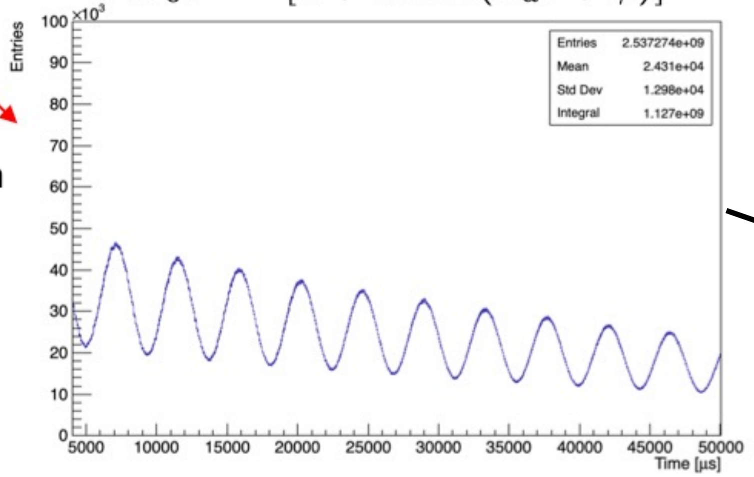
$$N_0 e^{-t/\tau}[1 + A\cos(\omega_a t + \phi)][1 + A_{FR}(t)\cos(\omega_C t + \phi_C)]^*$$



Split data in two

Randomize using uniform distribution with width 149 ns

$$N_0 e^{-t/\tau}[1 + A\cos(\omega_a t + \phi)]^*$$

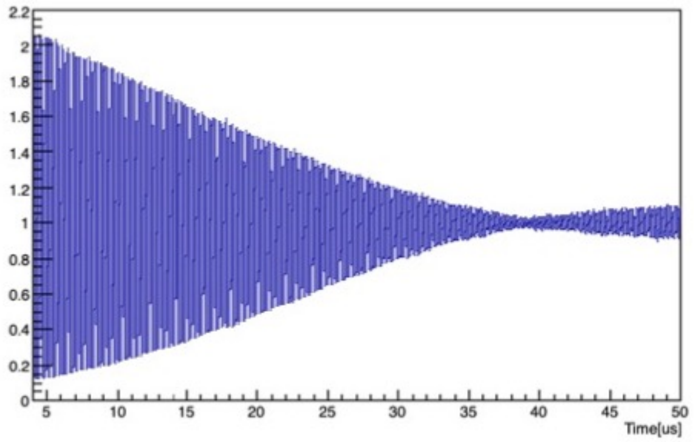


Take ratio

=

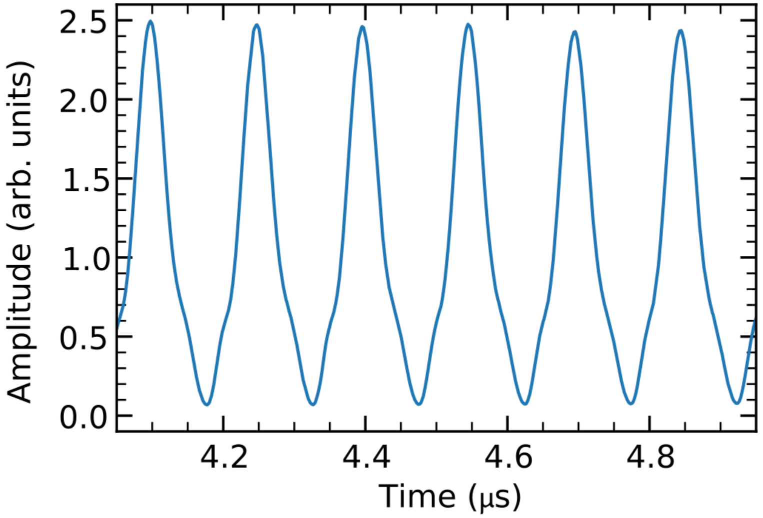
FR signal isn't really sinusoidal but is a sum of lots of them. See Tyler's notes.

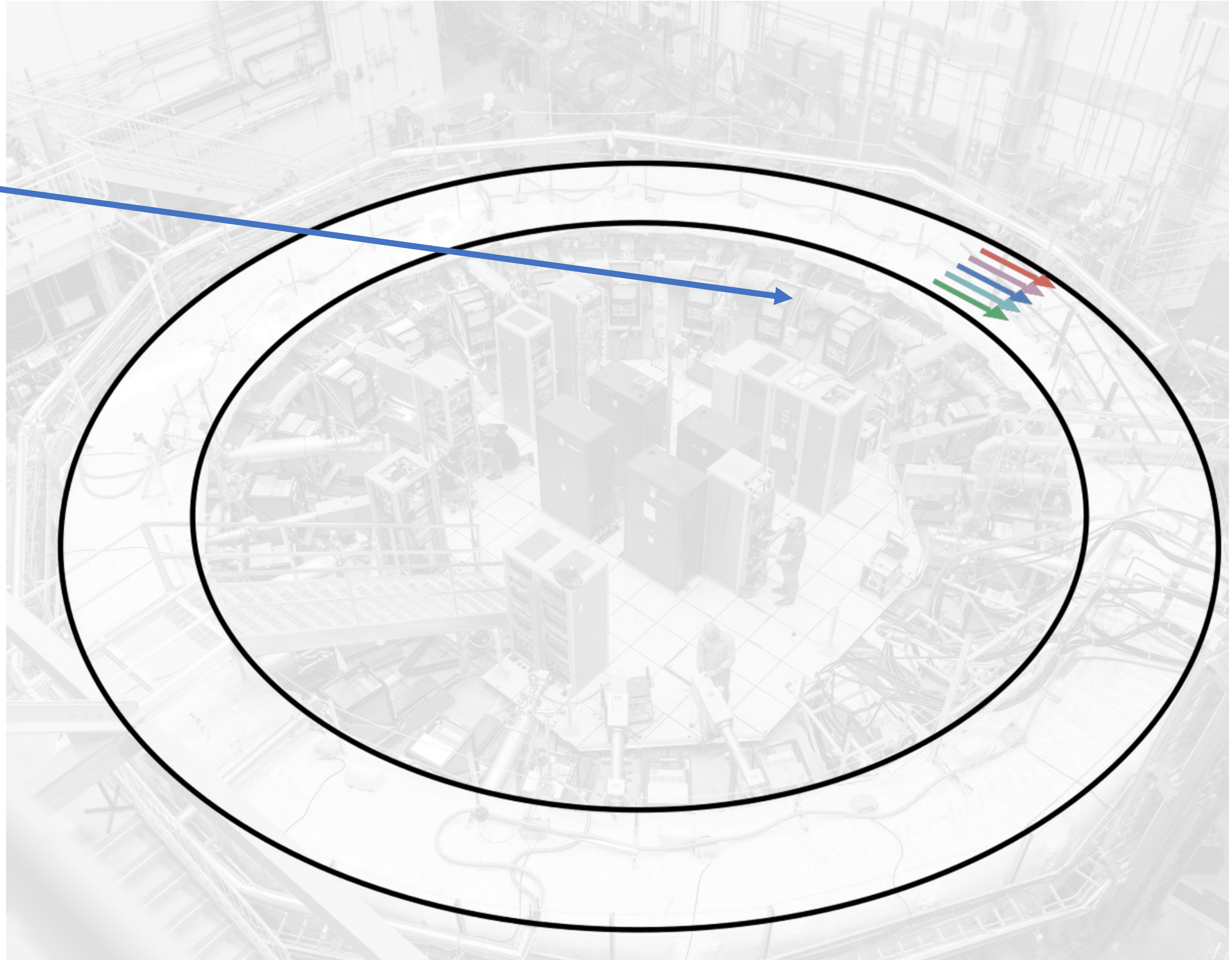
$$1 + A_{FR}(t)\cos(\omega_C t + \phi_C)^*$$



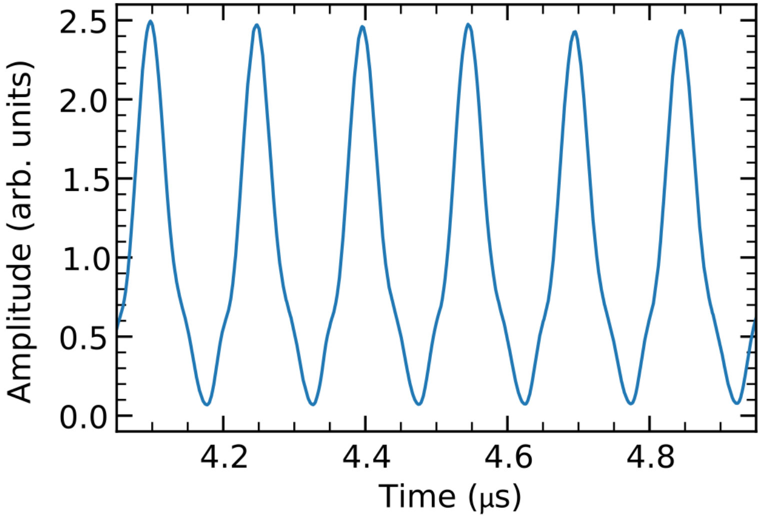
Randomization with a triangular kernel

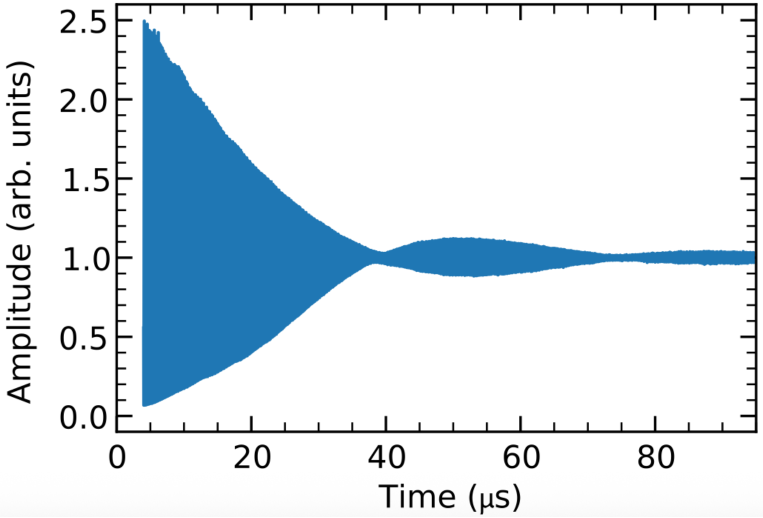
Fast rotation signal

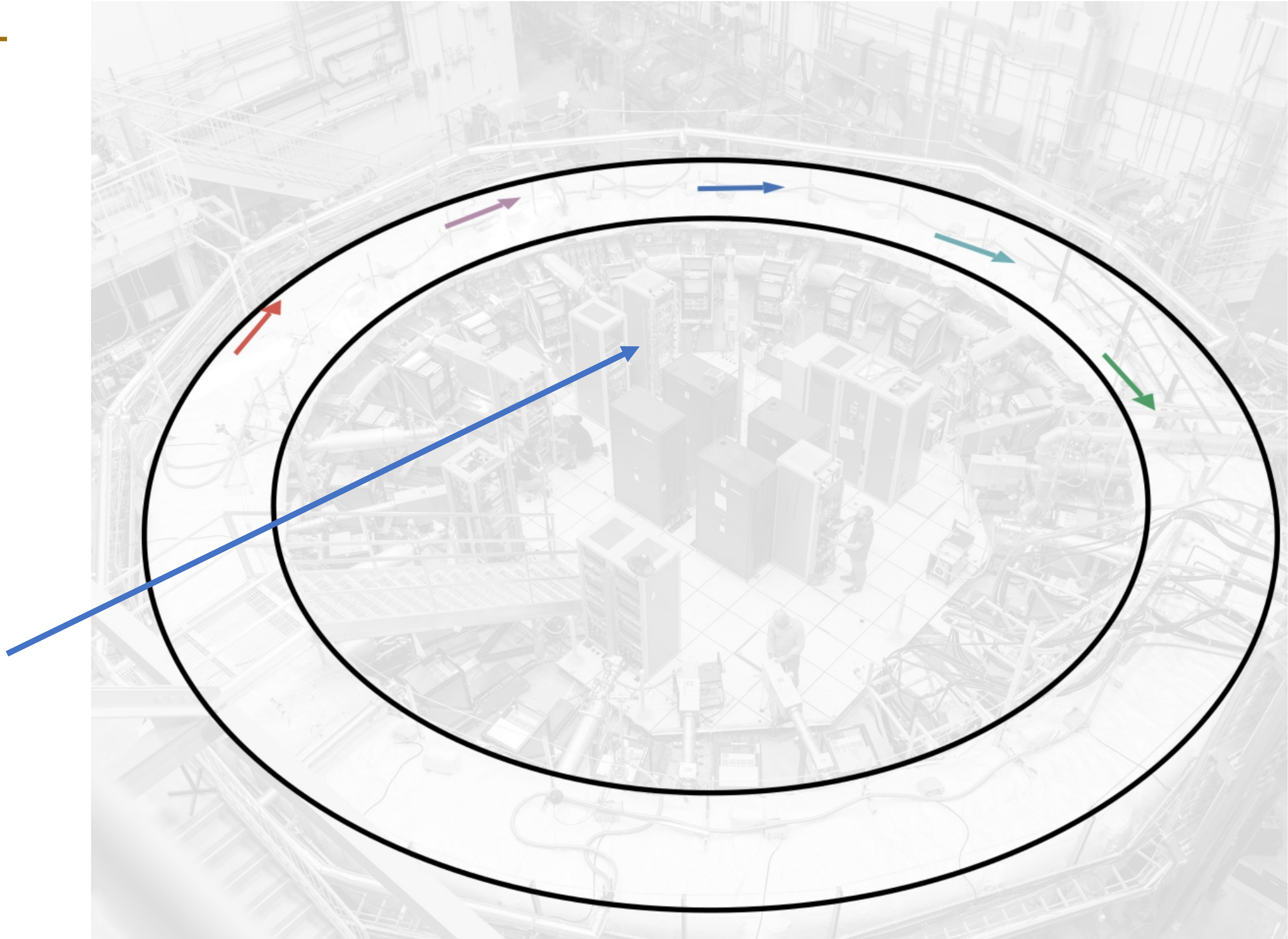




Fast rotation signal







C_e correction – Cernext Method

- In E821 and Run-1 we used the CERN method to extract the electric field correction, building the χ^2 from the Fast rotation signal:

Propagator function

Injection time

$$\chi^2 = \sum_k \frac{(S_k - \beta_{ijk} R_i T_j)}{\sigma_k}$$

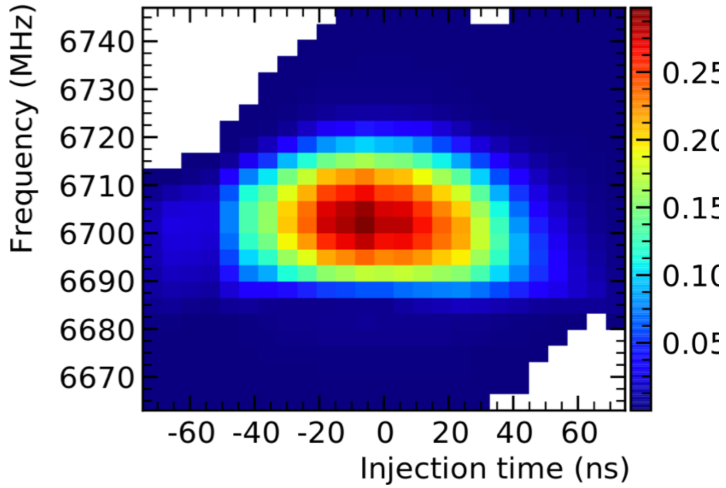
Fast rotation signal

Equilibrium radius

- This method doesn't account for correlation between R_i and T_j, so an extra term needs to be added at the χ^2:

$$\chi^2 = \sum_k \frac{(S_k - \beta_{ijk} \Sigma_m R_i T_j \epsilon_{ijm})}{\sigma_k}$$

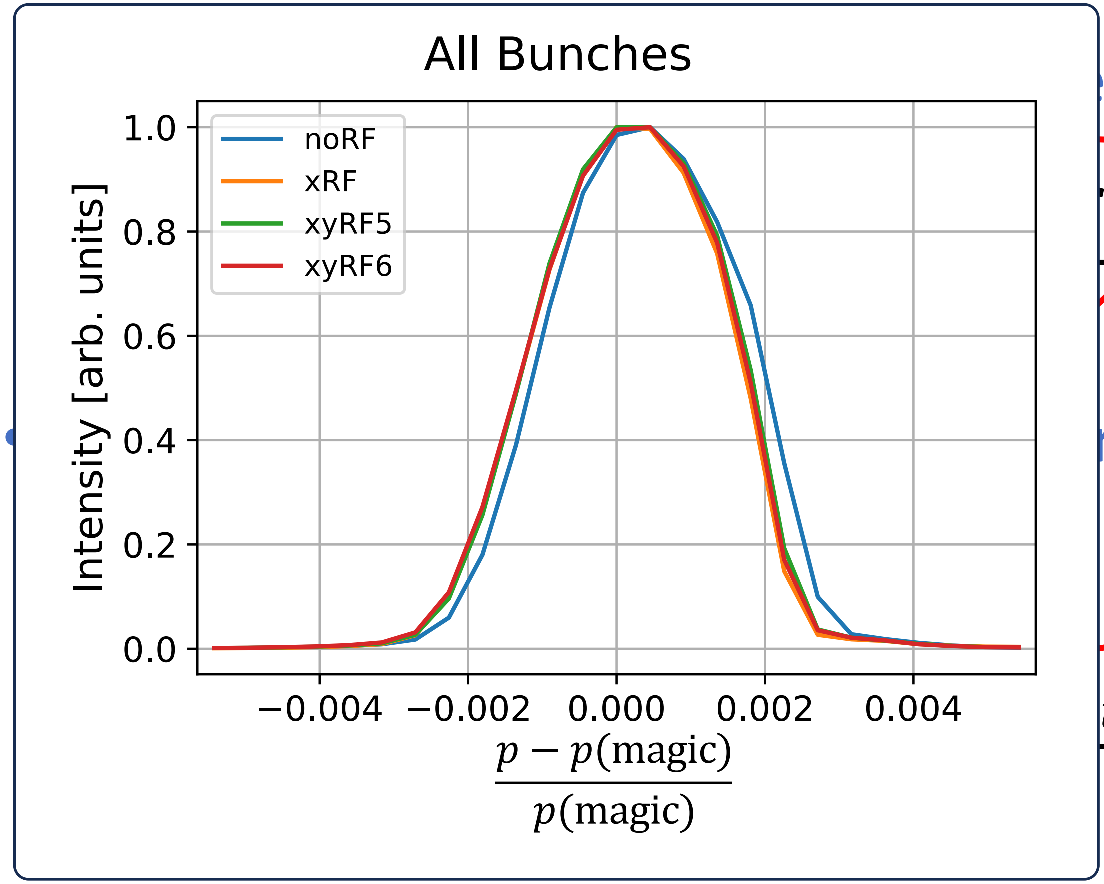
Time Momentum Correlation



- 12 parameters are needed to describe the **time momentum correlation**

C_e correction – Cernext Method

- In E821 and Run-1 we used the CERN method to extract the electric field



Fast rotation signal:

$$\ldots_k - \beta_{ijk} R_i T_j) \over \sigma_k$$

Injection time

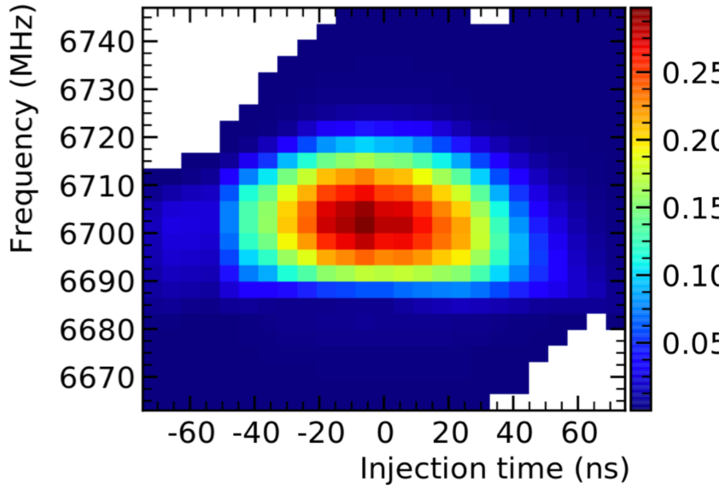
Equilibrium radius

- ...rrelation between R_i and T_j, so an extra term

$$\ldots_{ijm})$$

Vertical projection

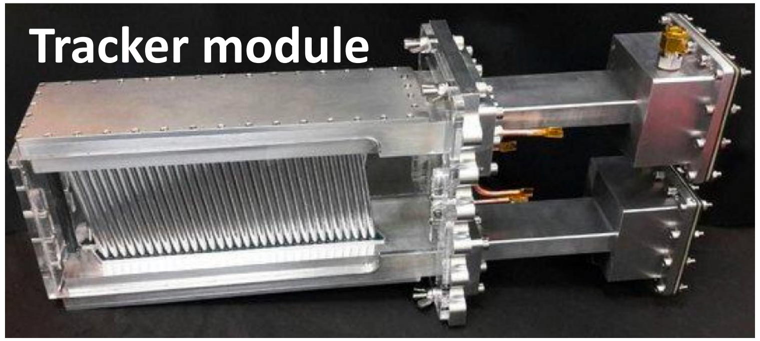
Time Momentum Correlation



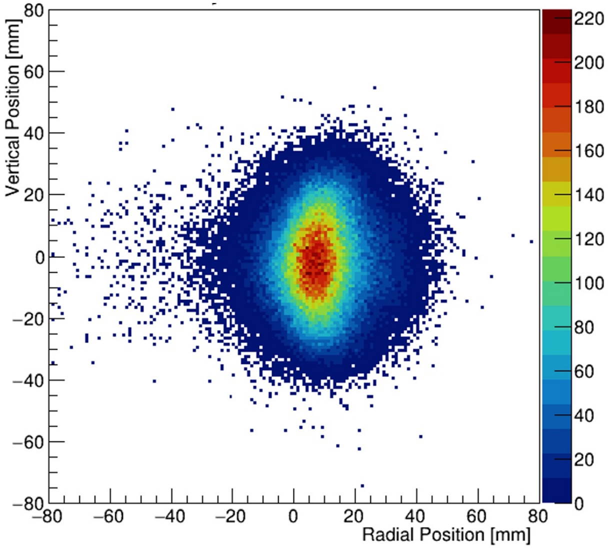
- 12 parameters are needed to describe the **time momentum correlation**

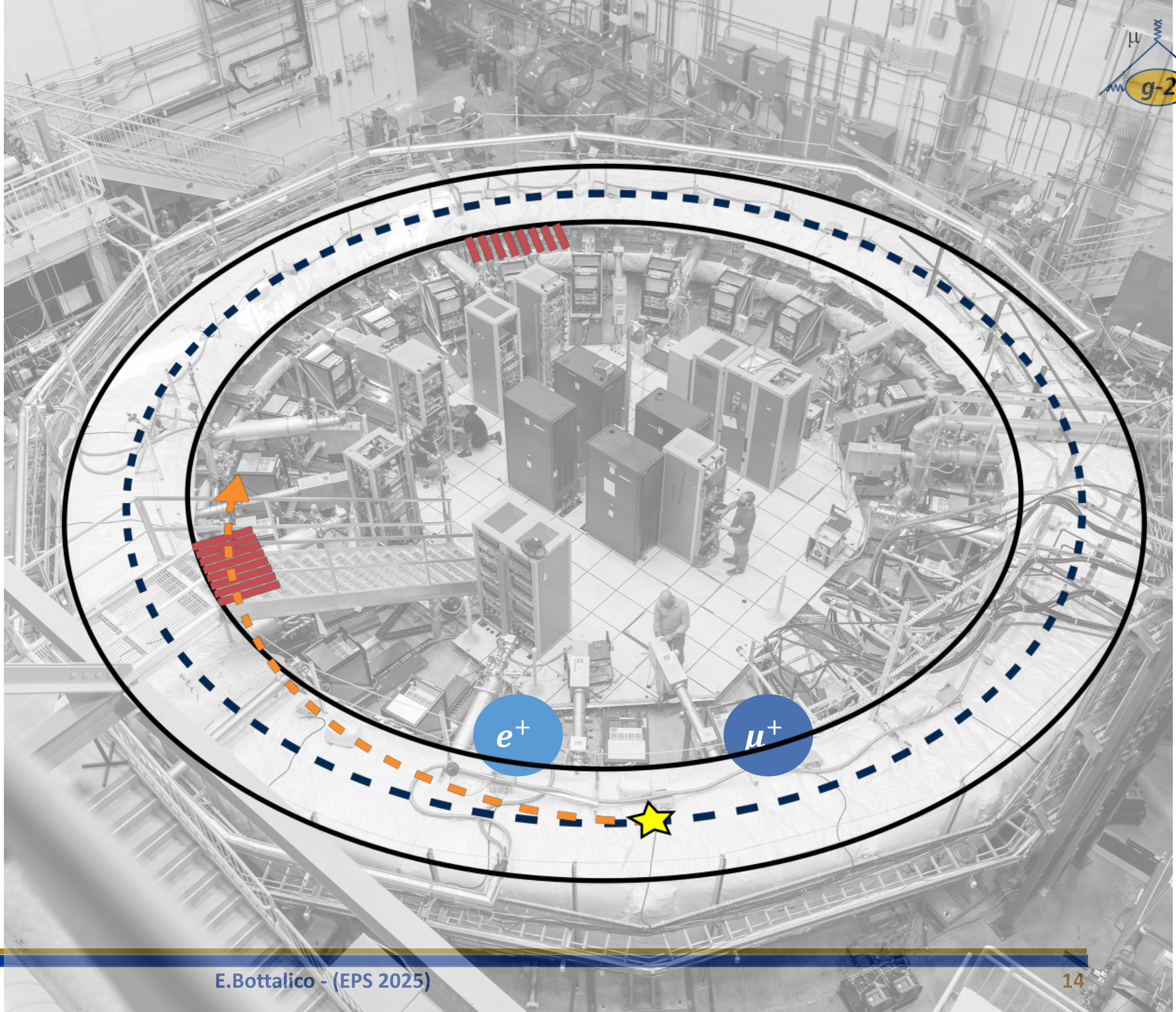
2 straw-tracker stations

each 8 modules, 4 layers of 32 straws, 50:50 Ar:Ethane



Reconstruct Muon Distribution



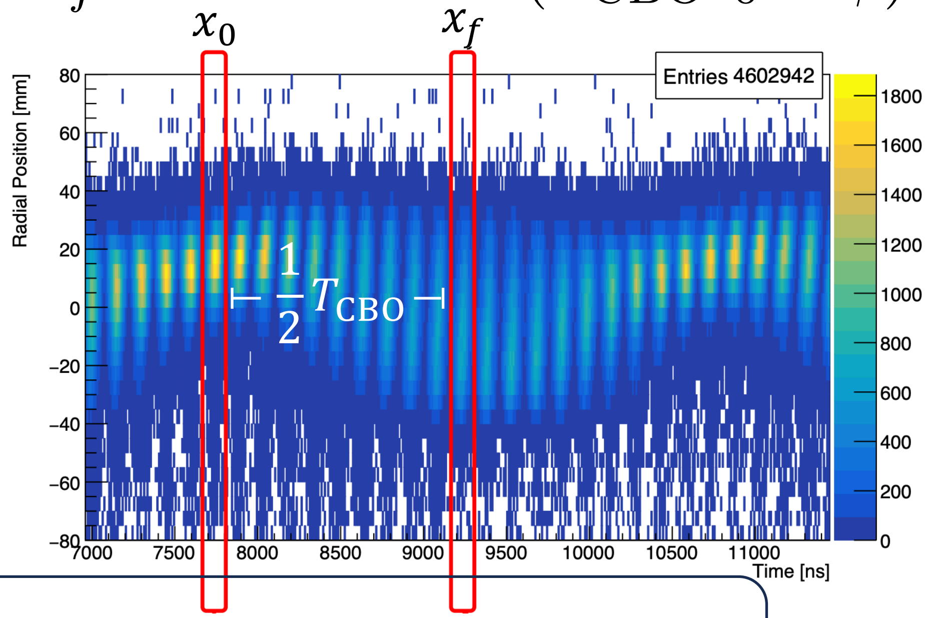


Tracker method

- The ring is used a **momentum spectrometer**
- Take the radial coordinates of muon under harmonic motion, separated in time by ½ a CBO period:

$$x_0 = D\delta + A\cos(\omega_{\mathrm{CBO}} t_0 + \phi) \quad , \quad x_f = D\delta - A\cos(\omega_{\mathrm{CBO}} t_0 + \phi)$$

- Square x_0 and x_f,
- average over the muon beam detected by a tracker station within one cyclotron period, and
- solve for $\langle \delta^2 \rangle$:



$$\langle \delta^2 \rangle = \frac{1}{2D_x^2}\left[\langle x_0^2 \rangle + \langle x_f^2 \rangle - \langle A^2 \cos(2\omega_{\mathrm{CBO}} t_0 + 2\phi) \rangle - \langle A^2 \rangle\right]$$

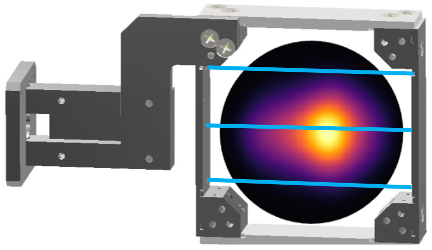
from NNLS minimizer

$$C_e = \frac{\sum_i C_{e,i} \exp(-t_i/\gamma_0 \tau_\mu)}{\sum_i \exp(-t_i/\gamma_0 \tau_\mu)} \qquad C_{e,i} = \frac{2 n_i \beta_0^2}{1 - n_i} \langle \delta^2 \rangle_i$$

MiniSciFi method

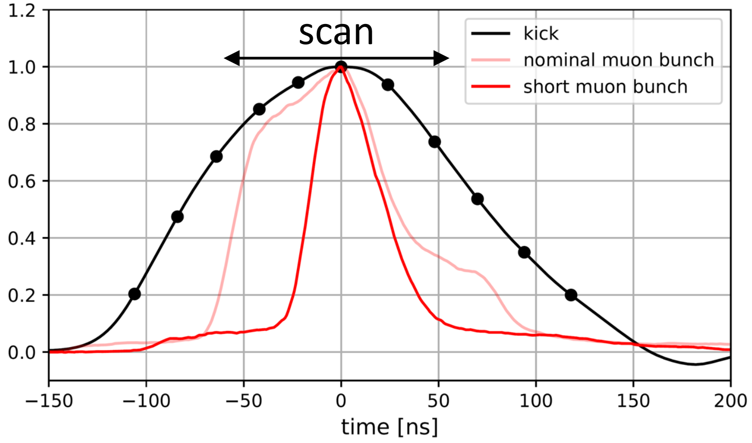
- During Run-6 data taking a Minimal Intrusive Scintillator Detector (MiniSciFi) has been installed to measure the time-momentum correlation

Horizontal MiniSciFi



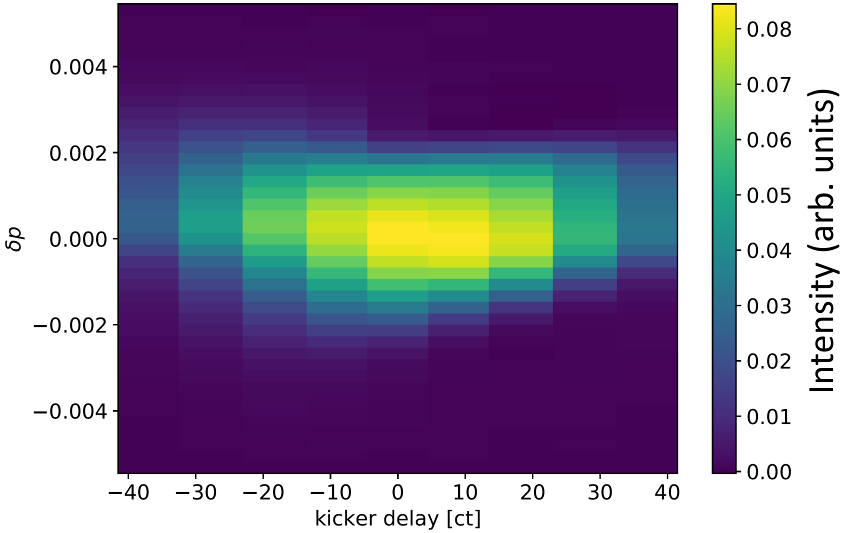
Fibers measure circulating beam fast rotation intensity

Scanned over kicker delays to map momentum vs. injection time



Each kicker delay measures stored momentum at an injection time slice

RF on (summed fibers)



kicker delay time = - injection time
1 c.t. = 2 ns delay

Extract the fundamental t-p distribution

Nominal stored distribution

$$\rho(p,\tau) = \rho_0(\tau)\epsilon(p,\tau)$$

T0 bunch τ-p efficiency

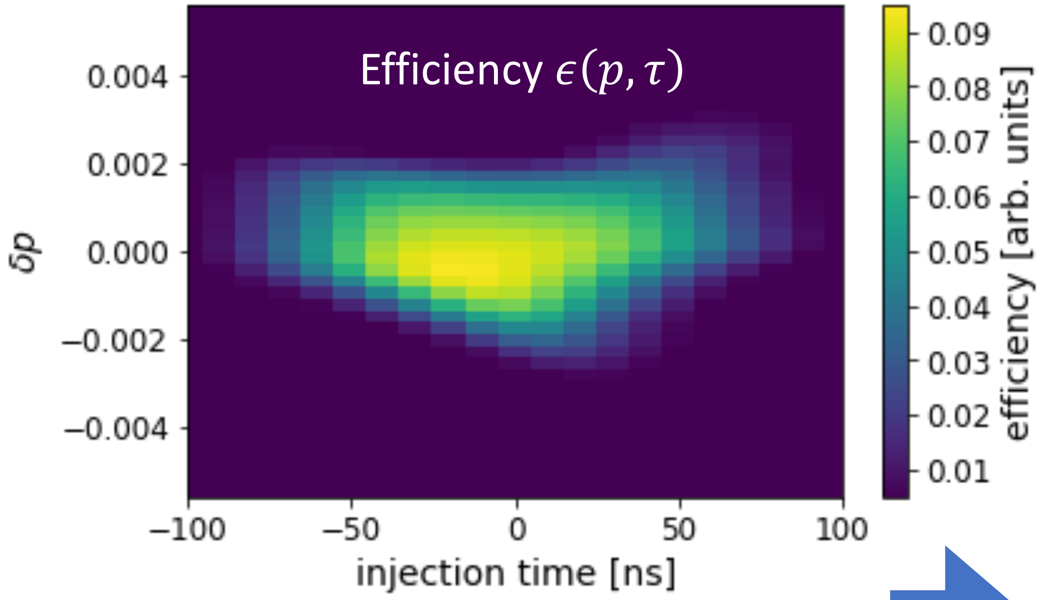
Distribution measured in scan

$$\rho(p,t_k) = \int \rho_{0,short}(\tau + t_k)\epsilon(p,\tau)d\tau$$

kicker delay

Convolution of short bunch with τ-p efficiency

MiniSciFi method



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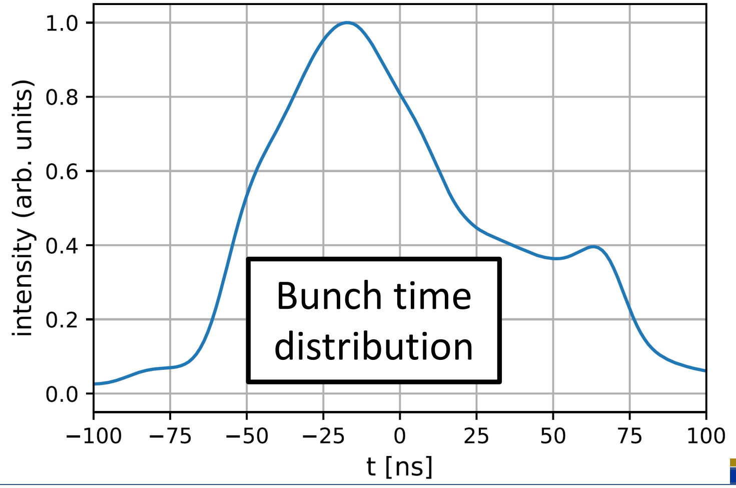
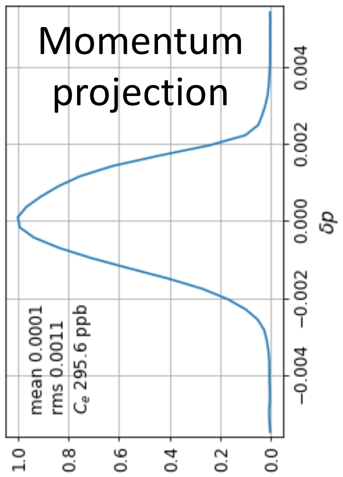
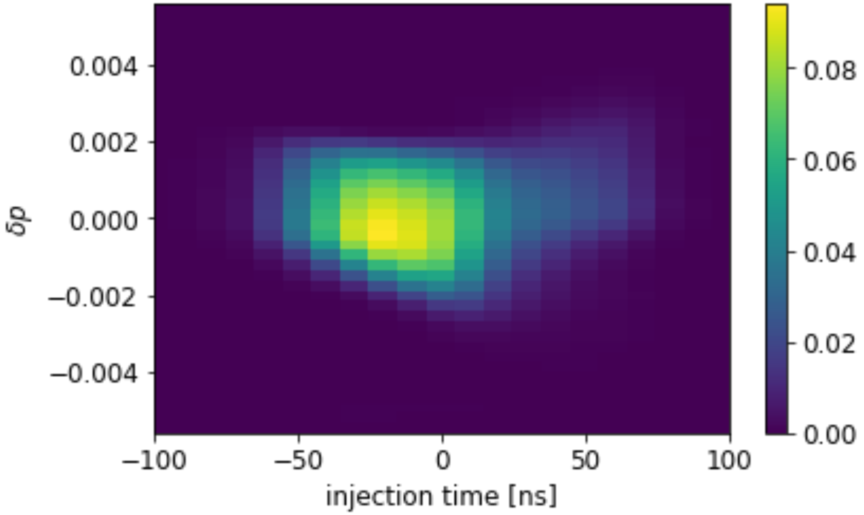


Reconstruct stored t-p distribution

$$\rho(p,\tau) = \rho_0(\tau)\epsilon(p,\tau)$$



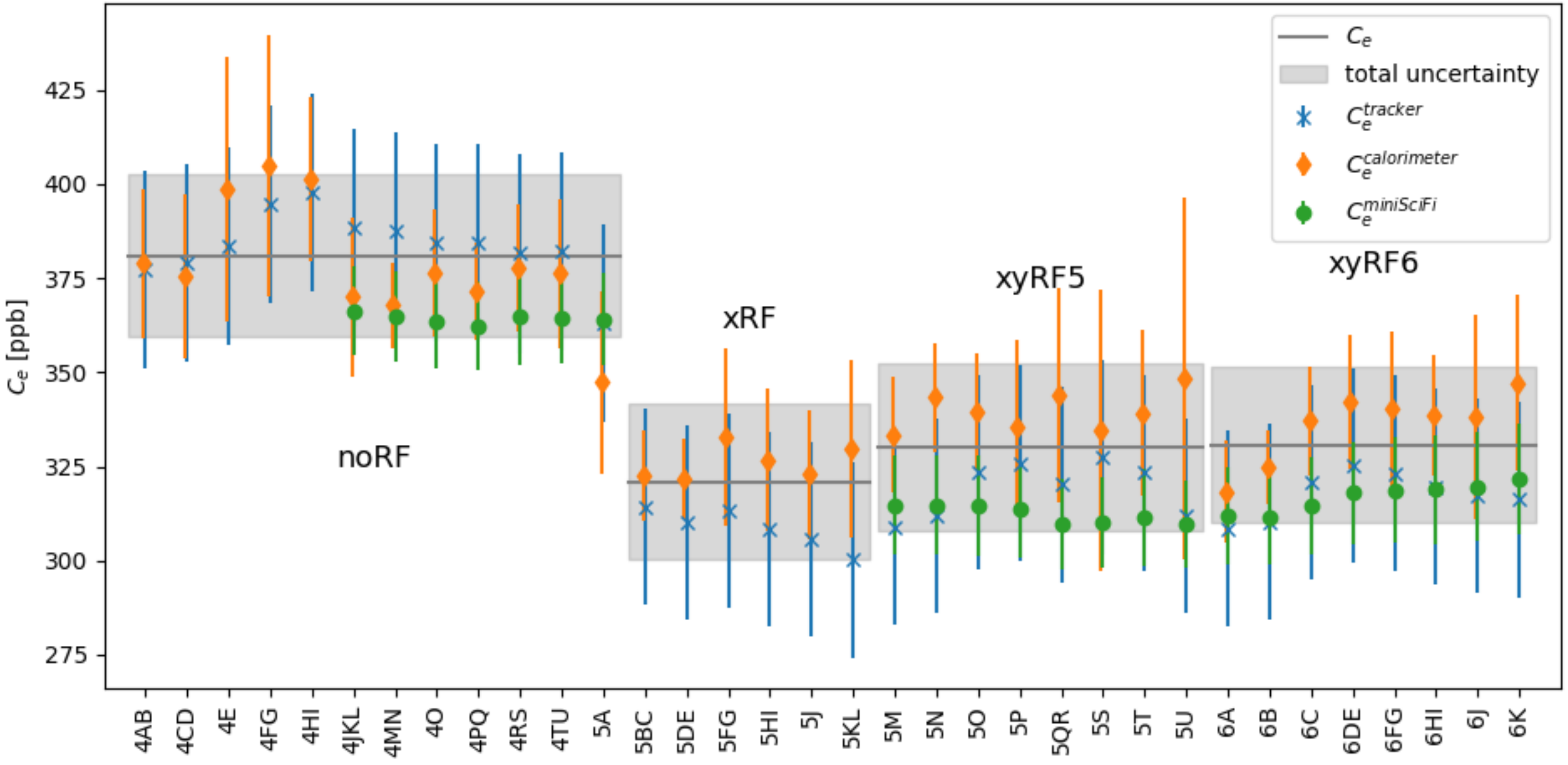


Calculate C_E as usual from momentum distribution

$$C_E = \frac{2n\beta_0^2}{(1-n)}\left(\langle\delta p\rangle^2 + \sigma_{\delta p}^2\right)$$

C_e correction results

- In Run-456 the main results have been provided by CERNext and tracker method, while MiniSciFi methos provided a solid cross check.



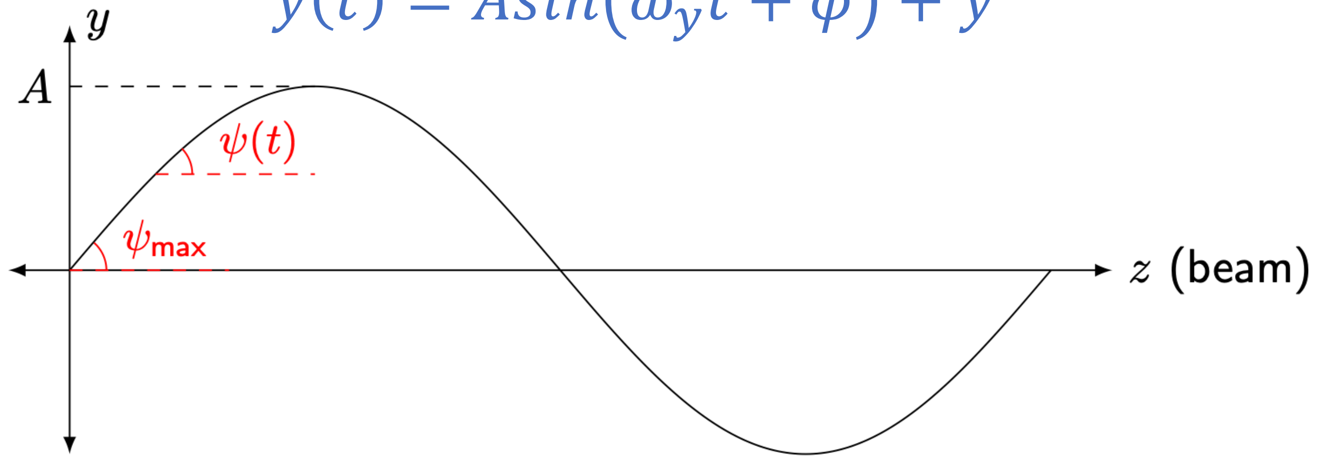
$$C_e = 347(27)\ ppb$$

Pitch correction

$$\overrightarrow{\omega_a} = \frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)(\vec{\beta}\times\vec{E}) - a_\mu\left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta}\right]$$

Each muon undergoes simple harmonic motion in the vertical direction:

$$y(t) = A sin(\omega_y t + \phi) + \bar{y}$$



The pitch of each muon would be:

$$C_p = \frac{1}{2}\langle\psi^2\rangle$$

Pitch correction

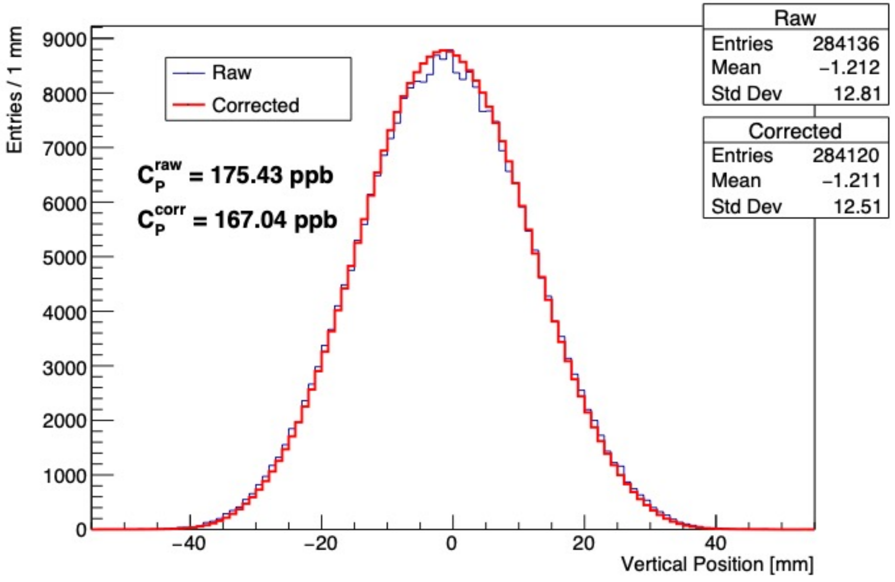
We don't measure ψ directly, but we approximate -> $\psi \sim \tan\psi = \frac{dy}{dz}$

Calculating $\langle\psi^2\rangle$, the correction can be revised as:

$$\langle\psi^2\rangle = \frac{n}{2R_0^2}A^2 \rightarrow \langle C_p\rangle = \frac{n}{4R_0^2}\langle A^2\rangle$$

Trackers can only measure the decay positions:

$$\langle(y-\bar{y})^2\rangle = \frac{1}{2}A^2 \rightarrow \langle C_p\rangle = \frac{n}{2R_0^2}\sigma_y^2$$



$$C_p = 175(9)\ ppb$$

Beam Dynamics Correction

- We must distinguish these corrections into two different categories:

Spin Dynamics

Time varying phase

Time varying phase

Many systematics come from effects that **change** the **phase** of the detected positrons **over time** and introduce a bias on ω_a:

$$cos(\omega_a t + \phi(t)) = cos(\omega_a t + \phi_0 + \phi' t + ...)$$
$$= cos((\omega_a + \phi')t + \phi_0 + ...)$$

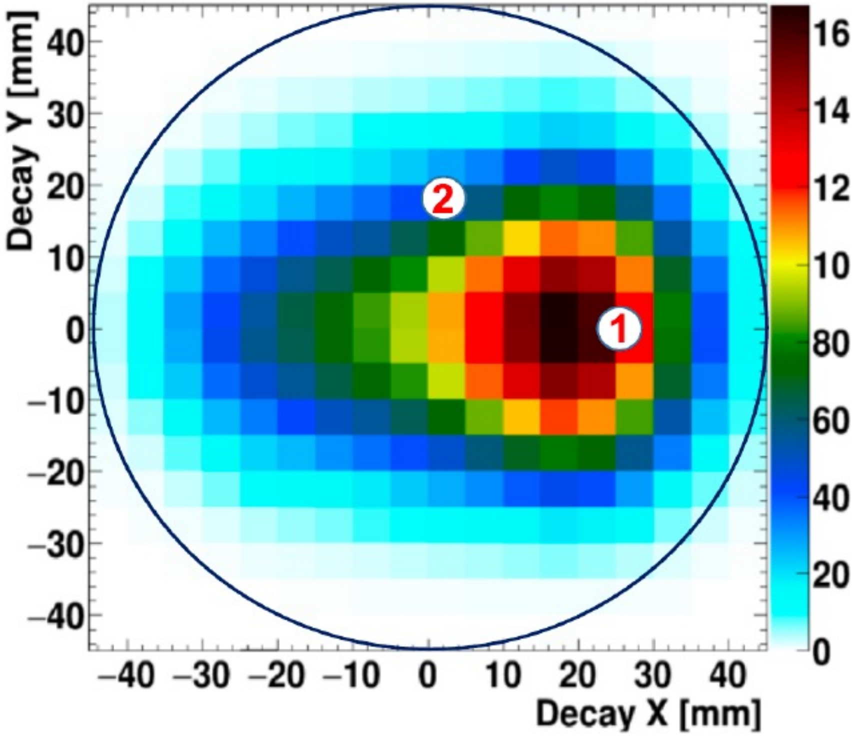
In general, anything that changes from **early-to-late** within each muon fill can be a cause of systematic error, as:

- Beam distortion
- Muon losses
- Varying lifetime
- Rate dependent reconstruction

More information in the back-up slides.

Phase acceptance

- The measured *g-2* phase of the muon is decay vertex position dependent.
- It is obtained as weighted average of the phases measured by each (x,y) pair position.



$$N_2(t) = N_{02} e^{-t/\tau}\left[1 + A_2 \cos(\omega_a t + \phi_2)\right]$$

$$N_1(t) = N_{01} e^{-t/\tau}\left[1 + A_1 \cos(\omega_a t + \phi_1)\right]$$

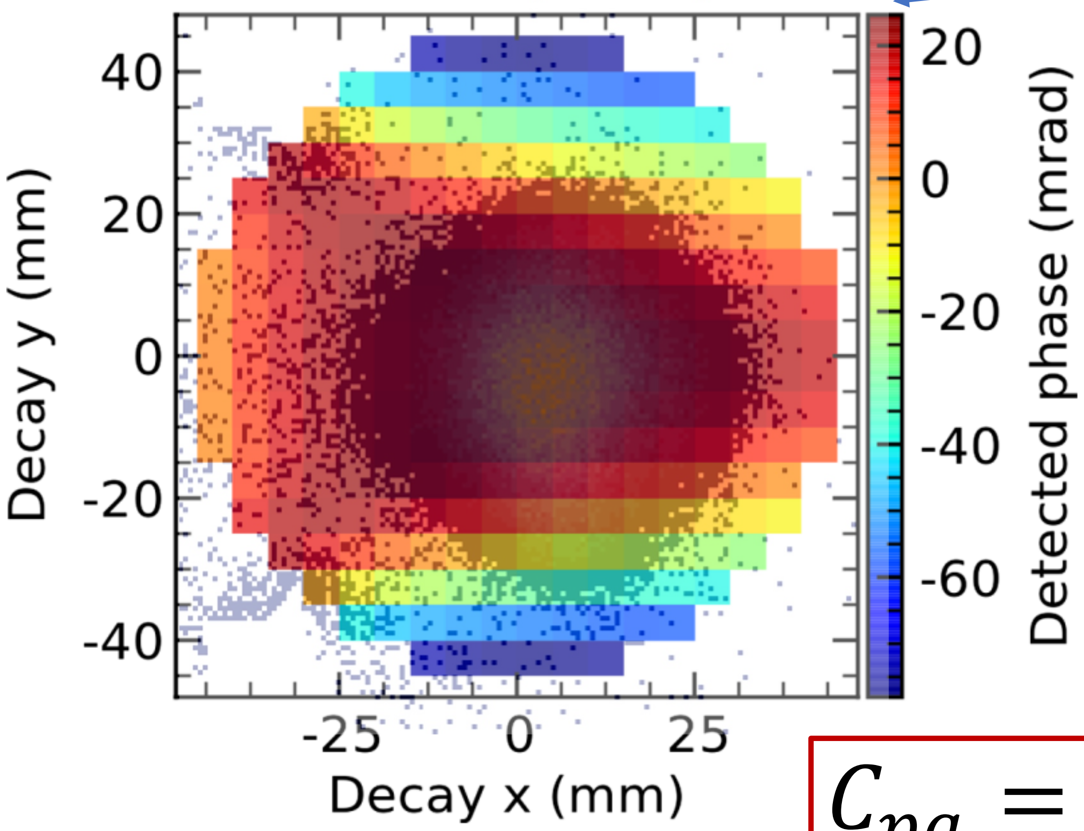
$$N(t) = N_1(t) + N_2(t) = N_\Sigma e^{-t/\tau}\left[1 + A_\Sigma \cos(\omega_a t + \phi_\Sigma)\right]$$

$$\phi_\Sigma = \arctan \frac{N_{01} A_1 \sin(\phi_1) + N_{02} A_2 \sin(\phi_2)}{N_{01} A_1 \cos(\phi_1) + N_{02} A_2 \cos(\phi_2)}$$

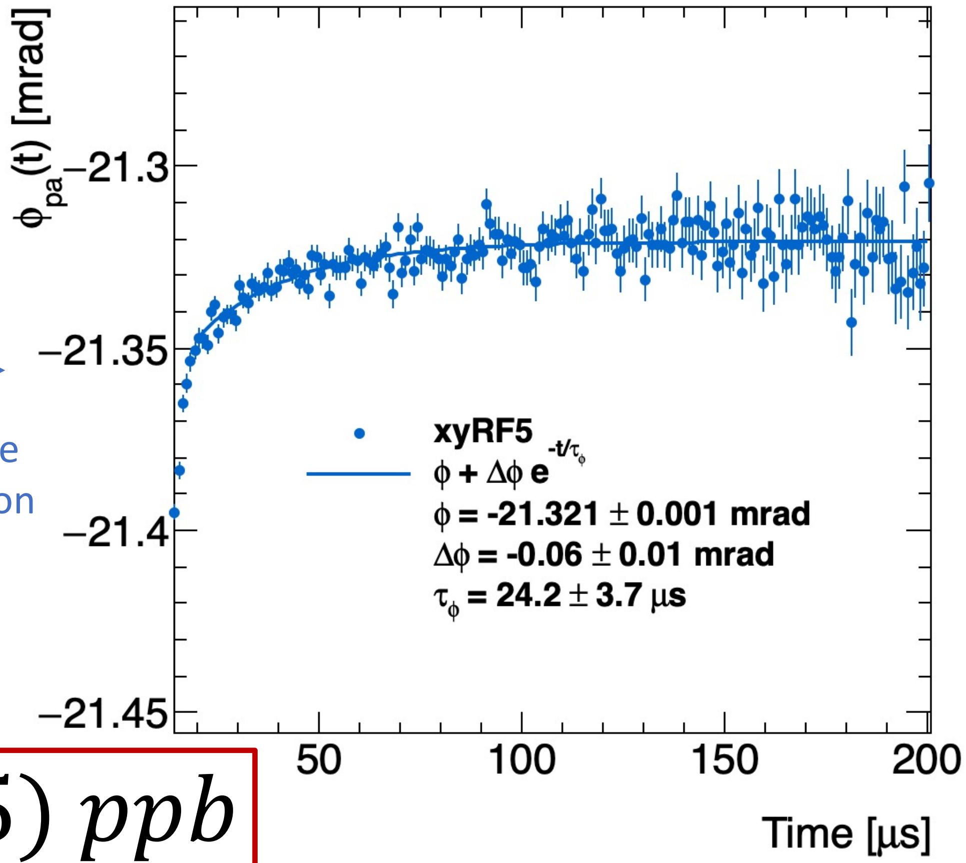
Phase acceptance

Simulation

$$\varphi_0^{c_k}(t) = \arctan \frac{\sum_{ij} M_{T,k}(x_i, y_j, t) \cdot \varepsilon_{c,k}(x_i, y_j) \cdot A_k(x_i, y_j) \cdot \sin(\varphi_{0,k}(x_i, y_j))}{\sum_{ij} M_{T,k}(x_i, y_j, t) \cdot \varepsilon_{c,k}(x_i, y_j) \cdot A_k(x_i, y_j) \cdot \cos(\varphi_{0,k}(x_i, y_j))}$$



This is the average phase as a function of the time.



$C_{pa} = -33(15)\ ppb$

Differential Decay

$$C_{dd} = -\frac{\Delta\omega_a}{\omega_a} = \left(\frac{1}{\omega_a}\right)\left(\frac{d\phi_0}{d\delta}\right)\left(\frac{d\delta}{dt}\right)_{dd}$$

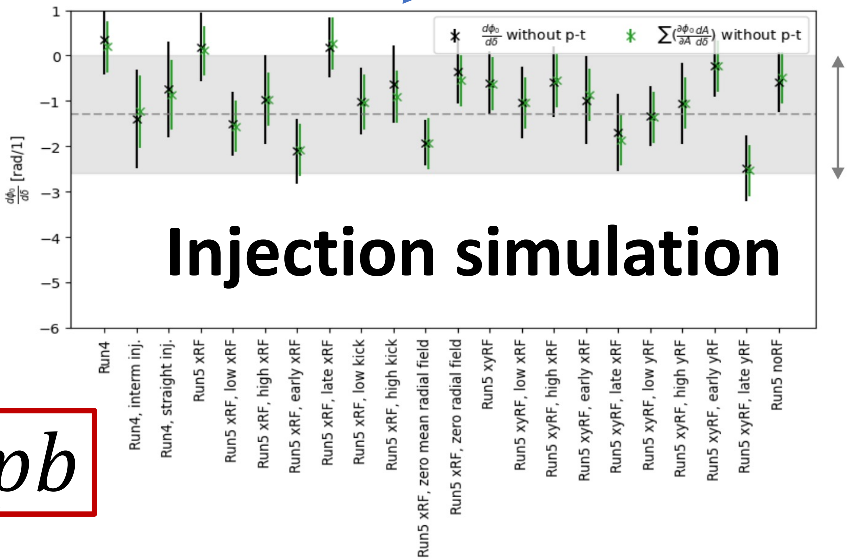
$$\left(\frac{dp}{dt}\right)_{dd} \approx \frac{p_0}{\gamma_0\tau_\mu}\sigma_\delta^2$$

$\sigma_\delta \approx 0.001$

from CernExt/Fourier

$$\frac{d\phi_0}{d\delta} = \underbrace{\left.\frac{\partial\phi_0}{\partial\delta}\right|_s}_{\text{direct}} + \underbrace{\left.\frac{\partial\phi_0}{\partial x}\right|_s \frac{dx(s)}{d\delta} + \left.\frac{\partial\phi_0}{\partial x'}\right|_s \frac{dx'(s)}{d\delta}}_{\text{radial}} + \underbrace{\left.\frac{\partial\phi_0}{\partial y}\right|_s \frac{dy(s)}{d\delta} + \left.\frac{\partial\phi_0}{\partial y'}\right|_s \frac{dy'(s)}{d\delta}}_{\text{vertical}} + \underbrace{\frac{\partial\phi_0}{\partial t_0}\frac{dt_0}{d\delta}}_{\text{longitudinal}}$$

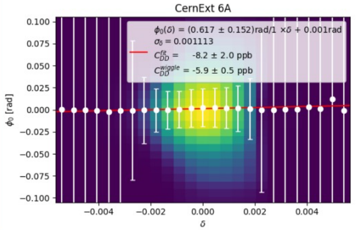
$\sim 20ppb$

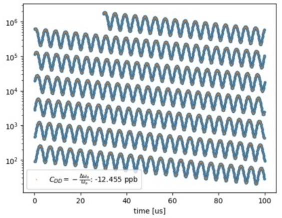


Injection simulation

"$p - t_0$" **Data-driven**

from CernExt dist. + wiggle





additional uncertainty

$\sim 4/8ppb$

$$C_{dd} = 26(27)\ ppb$$

Conclusion

- The beam dynamics study in g-2 is a crucial component in comprehending the behavior of the beam and determining an unbiased value of ω_a.
- The ω_a needs to be corrected for two main effects:
  - 1) **spin dynamics** and 2) **phase-varying effects**
- In Run-456 analysis, new methodologies have been implemented to improve the beam dynamics calculation, resulting in a more robust understanding of these effects.

	Run-2/3		Run-4/5/6	
Quantity	Cor. (ppb)	Unc. (ppb)	Cor. (ppb)	Unc. (ppb)
C_e	451	32	347	27
C_p	170	10	175	9
C_{pa}	-27	13	-33	15
C_{dd}	17	22	26	27
C_{ml}	0	3	0	2

“The closer you look the more there is to see”

F. Jegerlehner

Thank you!!!

- For any question or just to have a chat – elia.bottalico@liverpool.ac.uk

BACK-UP

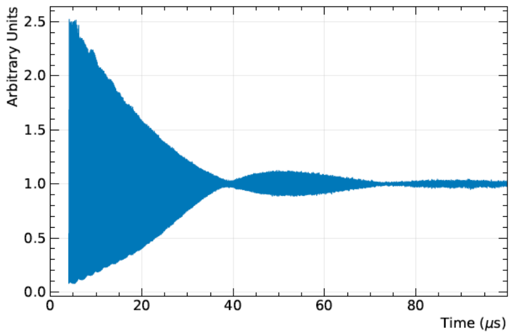
Calo Method – Fourier Method

1) produce fast rotation signal → 2) calculate cosine transform → 3) fit and subtract background → 4) convert frequency to radius → 5) calculate electric field correction

(3) → (2): scan over t_0 for best fit)

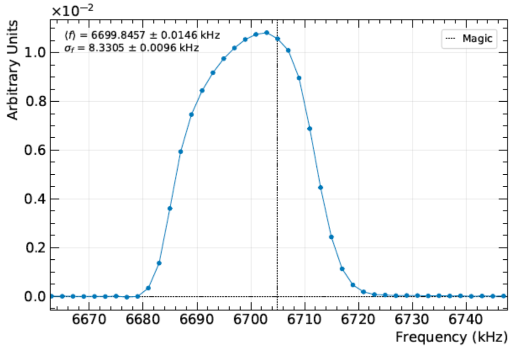
We get the x_e distribution from the cyclotron frequencies in the fast rotation signal:

1)

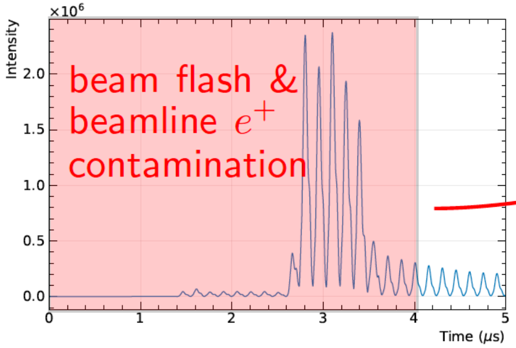


2)

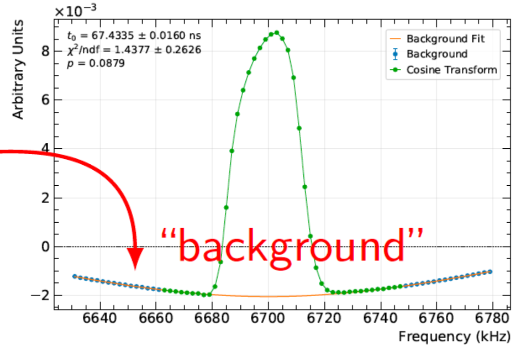
$$\Longrightarrow \underbrace{\int_{t_0}^{\infty} S(t+t_0)\cos(\omega t)\,dt}_{\text{cosine transform with signal phase } t_0} \Longrightarrow$$



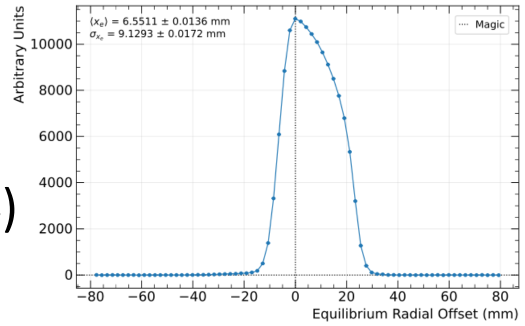
Starting the integral at a later time introduces a background, which we must fit and subtract:



3)



4)



Muon loss

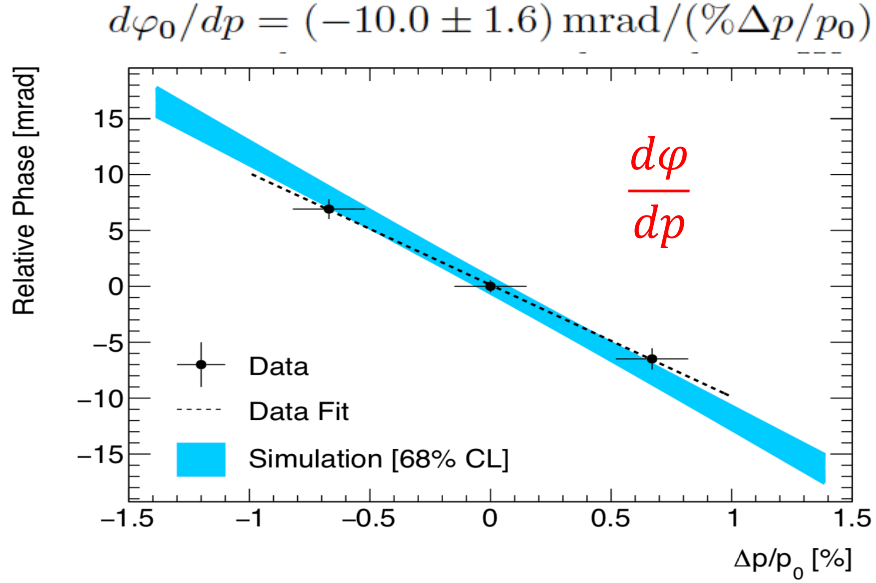
C_{lm}: describes the induced effect on the phase of ω_a due to muon losses during the fill. This is caused by:

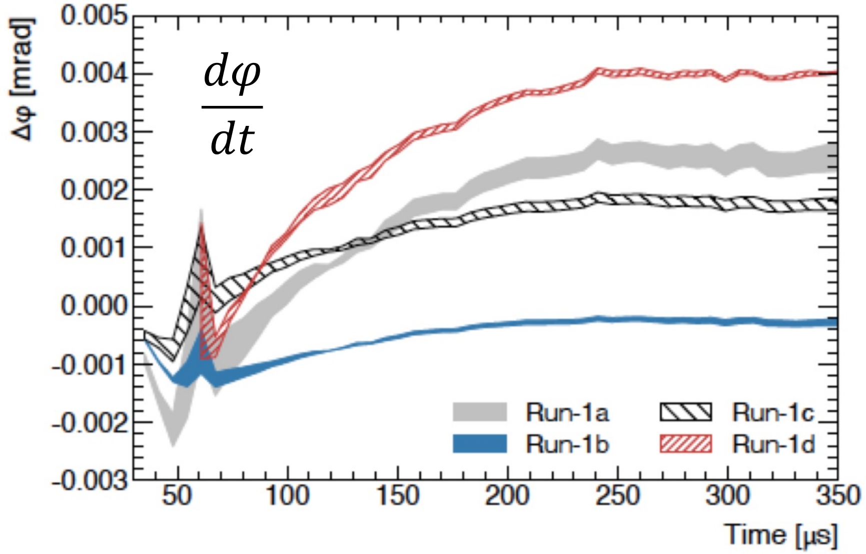
1. Muons with different momenta having different phases;
2. The number of lost muons varying as a function of momentum.

$$\Delta\omega_a = \frac{d\varphi}{dt} = \overset{1)}{\frac{d\varphi}{dp}} \cdot \overset{2)}{\frac{dp}{dt}}$$

$$C_{ml} = 0(2)\ ppb$$

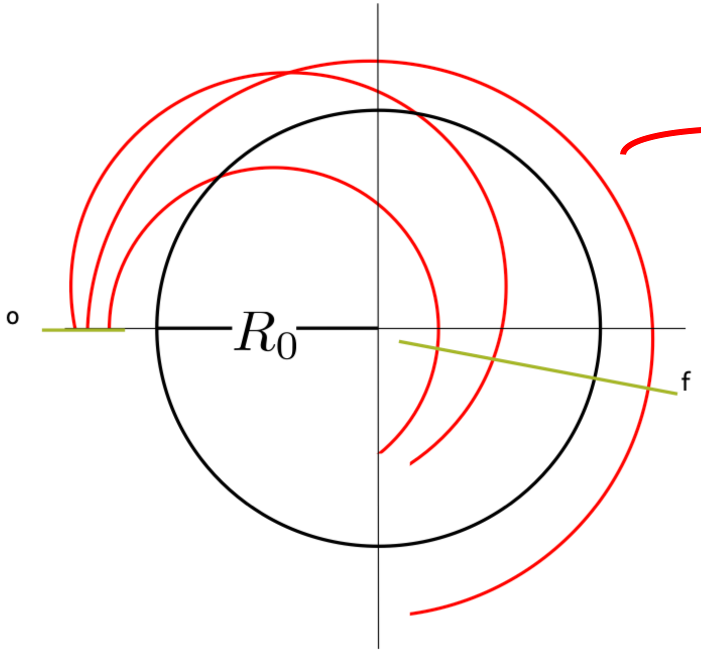
$d\varphi_0/dp = (-10.0 \pm 1.6)\ \mathrm{mrad}/(\%\Delta p/p_0)$





C_e correction – Tracking Method

- The tracker method is based on the muon propagation within the ring.
- The ring is used as a **momentum spectrometer**.
- To describe the motion between 2 points the following transformation is used:



Transfer matrix

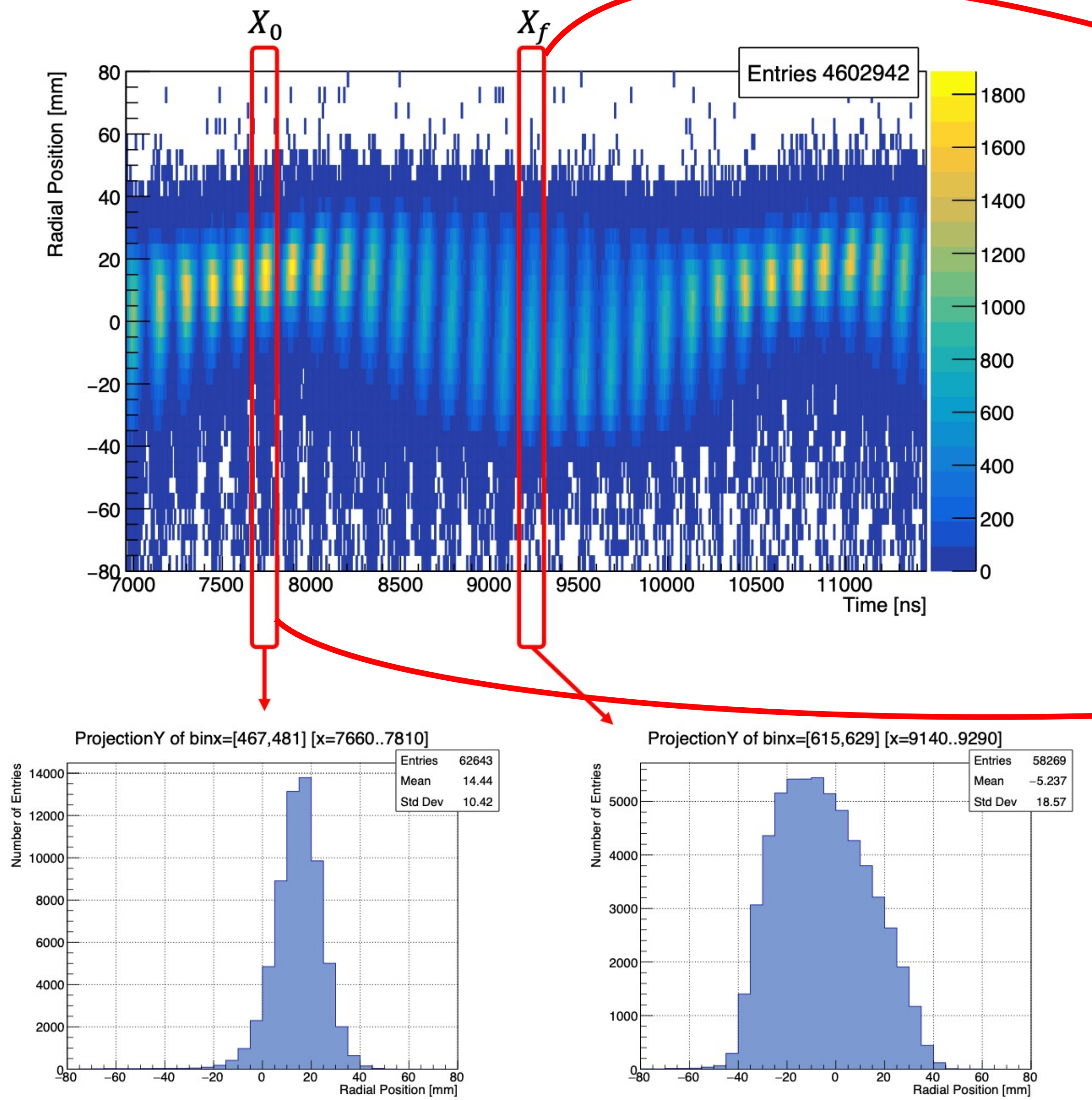
$$\begin{pmatrix} x \\ x' \\ \delta \end{pmatrix}_f = \begin{pmatrix} \cos(\sqrt{1-n}\phi) & \frac{R_0}{\sqrt{1-n}}\sin(\sqrt{1-n}\phi) & \frac{R_0}{1-n}[1-\cos(\sqrt{1-n}\phi)] \\ -\frac{\sqrt{1-n}}{R_0}\sin(\sqrt{1-n}\phi) & \cos(\sqrt{1-n}\phi) & \frac{1}{\sqrt{1-n}}\sin(\sqrt{1-n}\phi) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \\ \delta \end{pmatrix}_0$$

For a **phase advance** of $\phi = \pi/\sqrt{1-n}$

$$\begin{pmatrix} x \\ x' \\ \delta \end{pmatrix}_f = \begin{pmatrix} -1 & 0 & 2\frac{R_0}{1-n} \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \\ \delta \end{pmatrix}_0$$

Computing the product we obtain: $\delta = \frac{1-n}{2R_0}(x_0 + x_f)$

C_e correction – Tracking Method



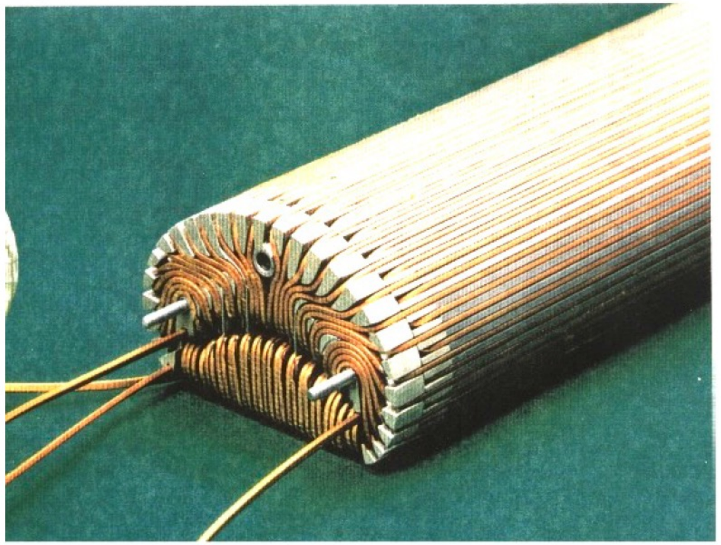
- … propagation within the ring.
- … points the following transformation is

$$\delta = \frac{1-n}{2R_0}\left(x_0 + x_f\right)$$

For a **phase advance** of $\phi = \pi/\sqrt{1-n}$

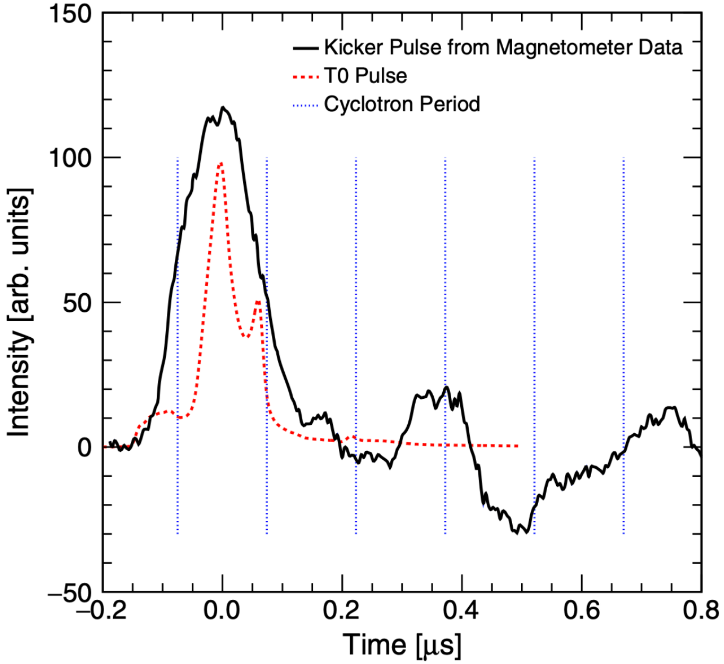
$: \delta = \frac{1-n}{2R_0}\left(x_0 + x_f\right)$

Kickers and Inflector



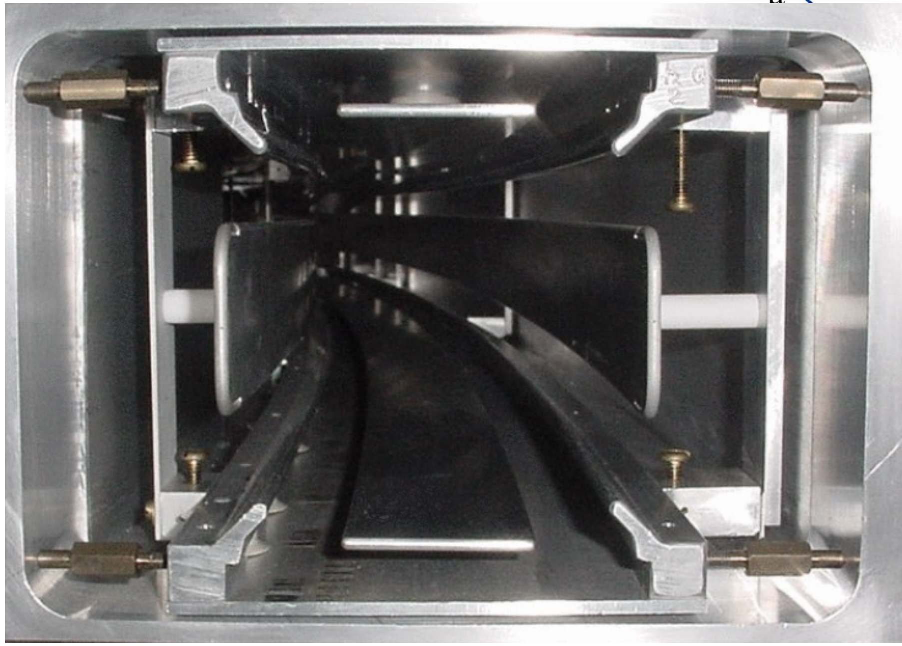
- The **inflector** cancels the storage ring field such that the muons are not deflected by the main **1.45 T** field.
- Superconducting, operational current ∼2.6 kA.

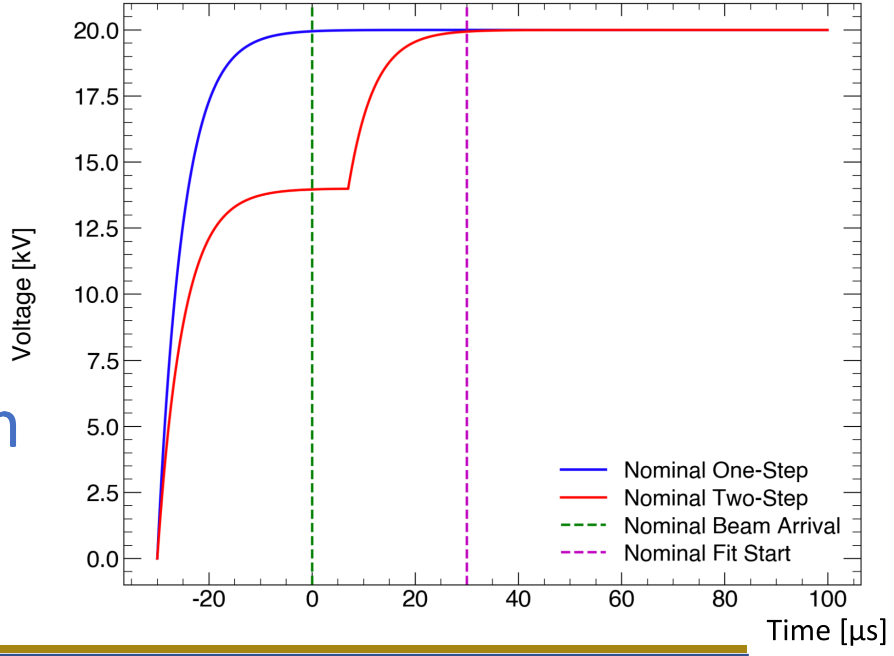
- **3 Kickers** are necessary to inject magic momentum muons along the magic radius (7.11 m) with a required kick at order of 10 mrad.
- 4 kA current in 200 ns pulse.
- Design kick strength has been reached in Run-3 (∼160 kV).



Quadrupoles

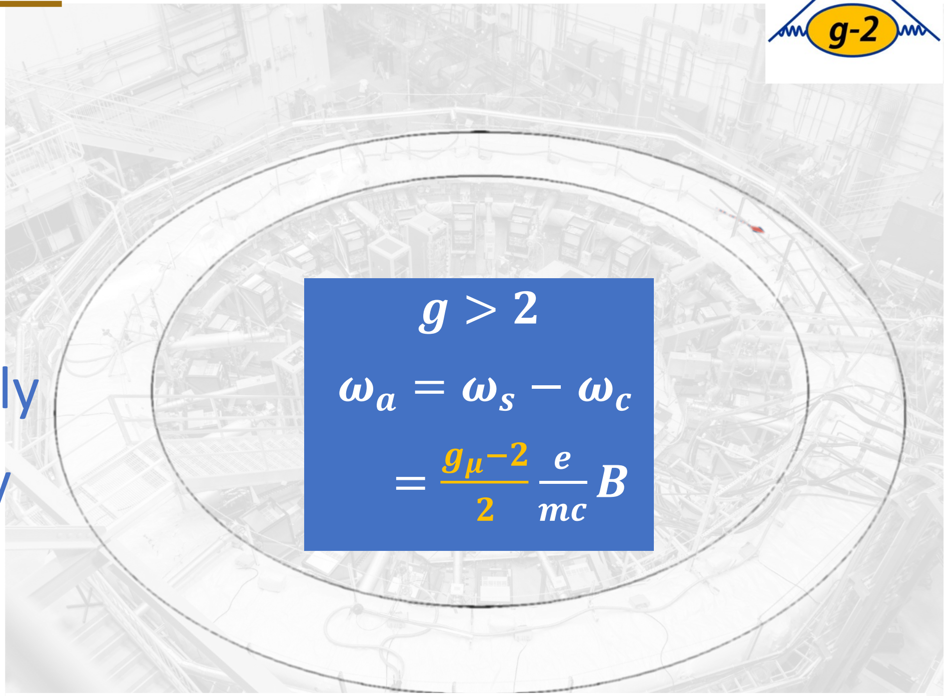
- The Electrostatic Quadrupoles (ESQ) system allows to strongly focus the beam vertically, four ESQ stations are symmetrically placed around the ring.
- The plates are raised from ground to operating voltage prior to each *fill* with RC charging time constants of **$\sim 5\ \mu s$**.
- This procedure, known as **scraping**, initially displaces the beam vertically and horizontally with respect to the central closed orbit.



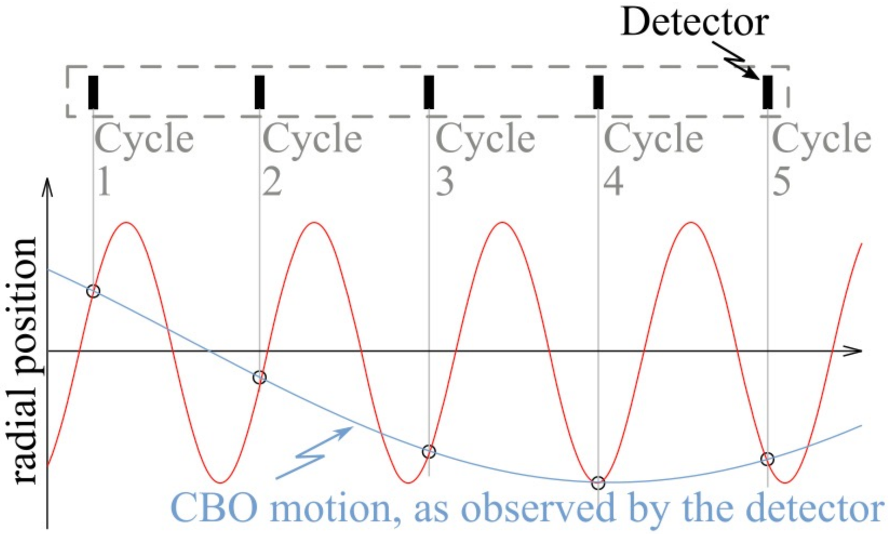


Betatron motion

- An ideal muon at the magic momentum, 3.09 MeV/c, in a 1.45T magnetic field is stored with a period of $T_c = 149.2$ ns
- We need quadruples to store the muon.
- **The Electrostatic Quadrupoles (ESQ)** system allows to strongly focus the beam vertically, four ESQ stations are symmetrically placed around the ring, introducing a field index -> $\boldsymbol{n}$ ~**0.108**
- Given the restoring force by radial magnetic field, the beam oscillates radially (vertically too) as the betatron frequency: $\omega_{BO} = \omega_c\sqrt{1-n}$, where $\boldsymbol{n}$ is the field-index.
- $\omega_{BO} < \omega_C$, so calorimeters see a different phase at each turn, measuring an alias-oscillation called **Coherent Betatron Oscillation** (**CBO**), given by $\omega_{CBO} = \omega_C - \omega_{BO}$



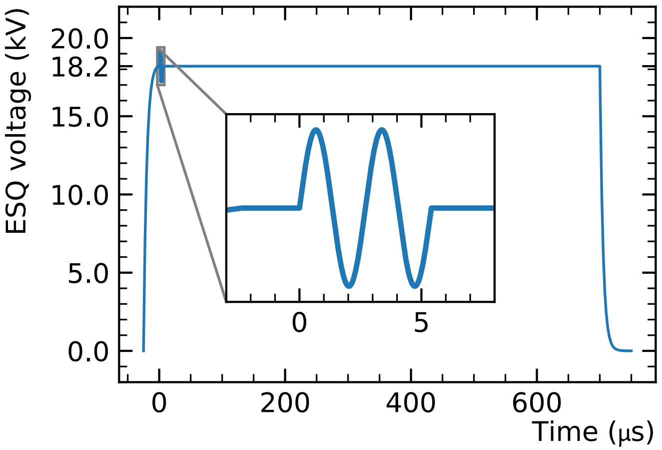
$$g > 2$$
$$\omega_a = \omega_s - \omega_c = \frac{g_\mu - 2}{2}\frac{e}{mc}B$$



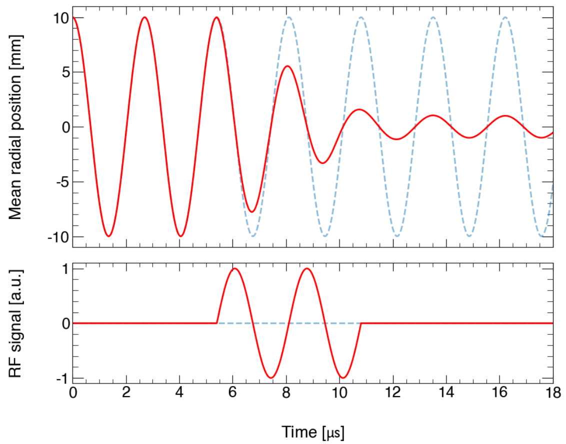
Real world beam dynamics

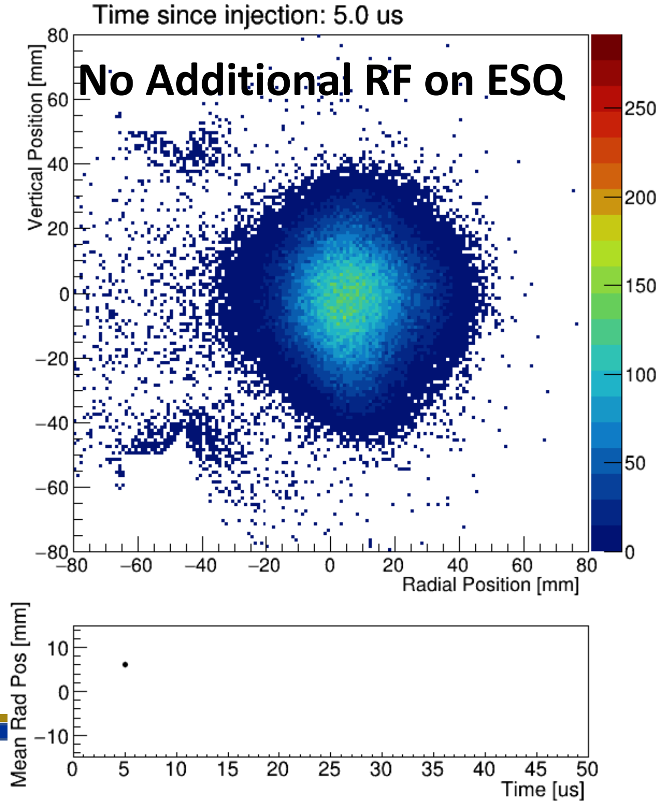
- In Run456 a **RF system** have been implemented to dump down the CBO oscillation, reducing the CBO amplitude by a factor ~10.

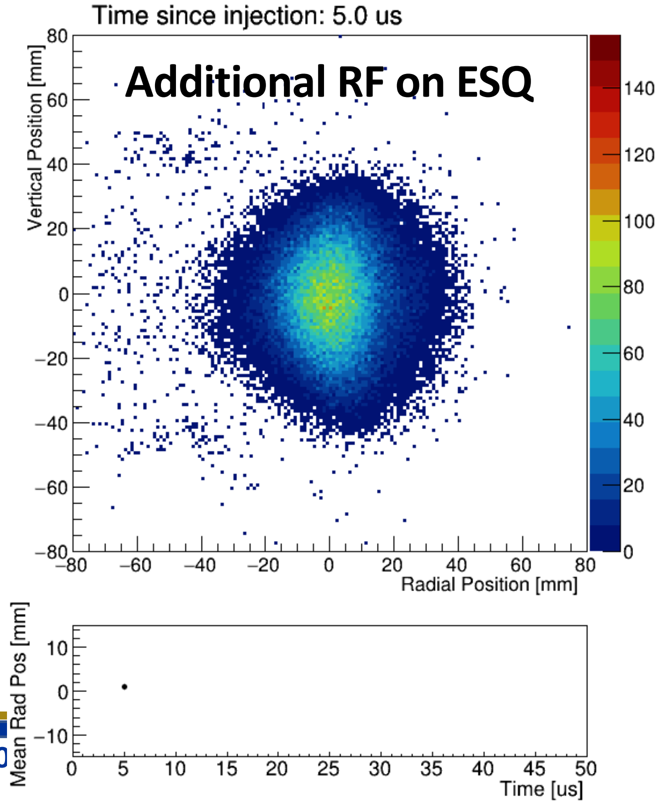
RF Signal



RF effect

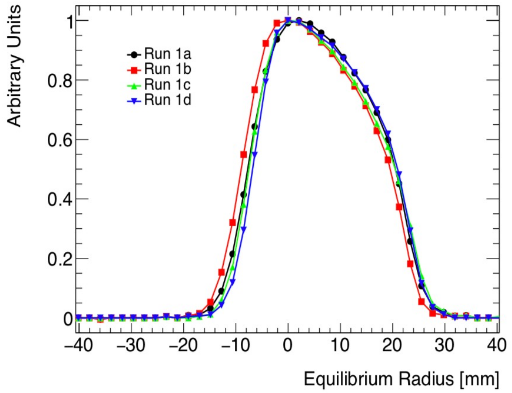




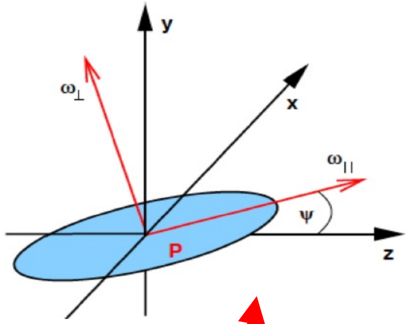


Beam Dynamics Correction

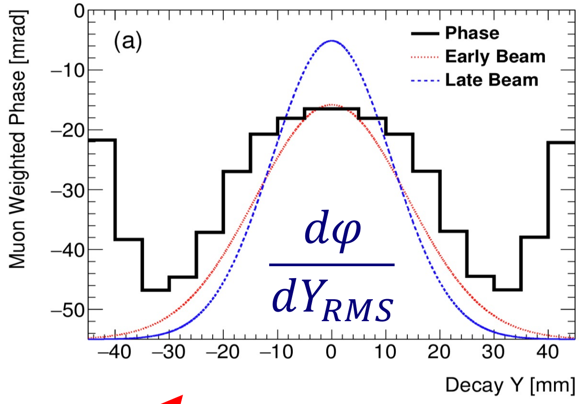
E-Field: 347(27) ppb



Pitch: 175(9) ppb

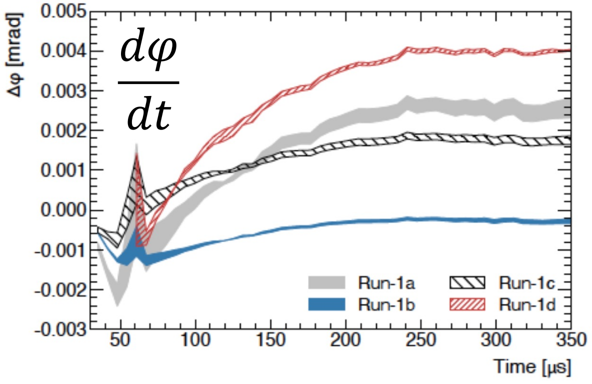


PA: -33(15) ppb



$$R'_\mu \approx \frac{f_{clock}\,\omega_a^m(1+C_e+C_p+C_{ml}+C_{pa}+C_{dd})}{f_{calib}\langle \omega'_p(x,y,\phi)\times M(x,y,\phi)\rangle(1+B_k+B_q)}$$

Muon Loss: 0(2) ppb



Differential Decay: 26(27) ppb