General Linblad equation for in-QGP quarkonia evolution

EPS-HEP 2025 ; Marseille (France)

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Goal: Describe the entire time evolution of in-QGP quarkonia...

...through a general Lindblad equation, encompassing both the high and low T regimes.

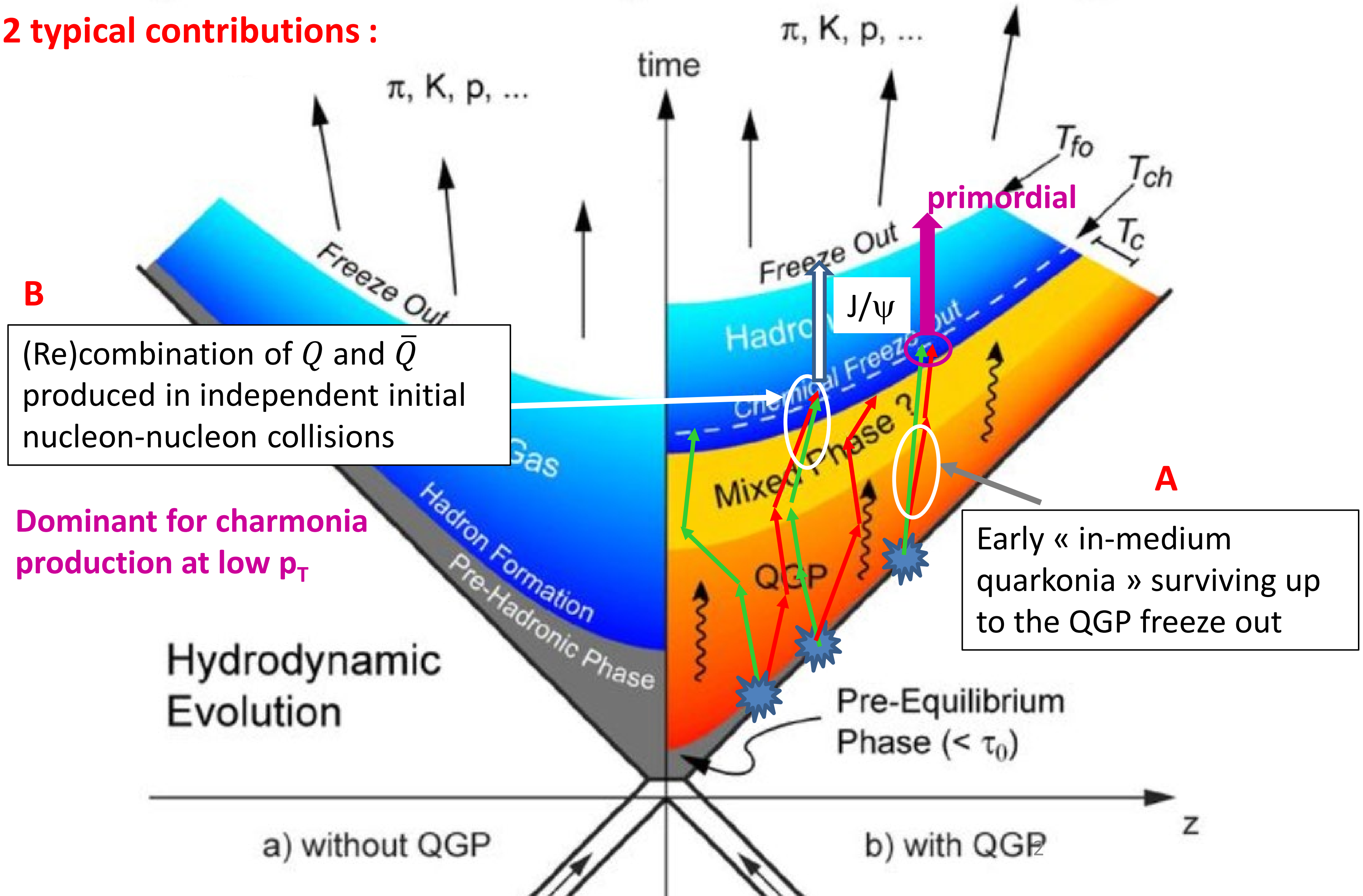
1. Pretty longuish introduction and motivation
2. Indecipherable derivation of a Universal Lindblad equation
3. Some illustration

With: J.P. Blaizot, Th. Gousset

and Pays de la Loire

Probing URHIC with quarkonia production

2 typical contributions :



B

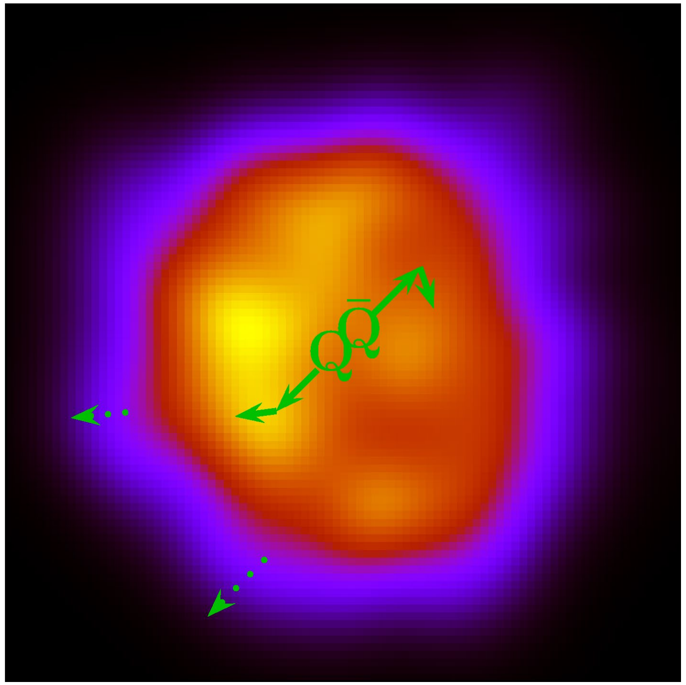
(Re)combination of Q and $\bar{Q}$ produced in independent initial nucleon-nucleon collisions

Dominant for charmonia production at low p_T

A

Early « in-medium quarkonia » surviving up to the QGP freeze out

What is a quarkonia… in a hot QGP medium ?



Answer may vary depending on how hot is the QGP, and how long you observe

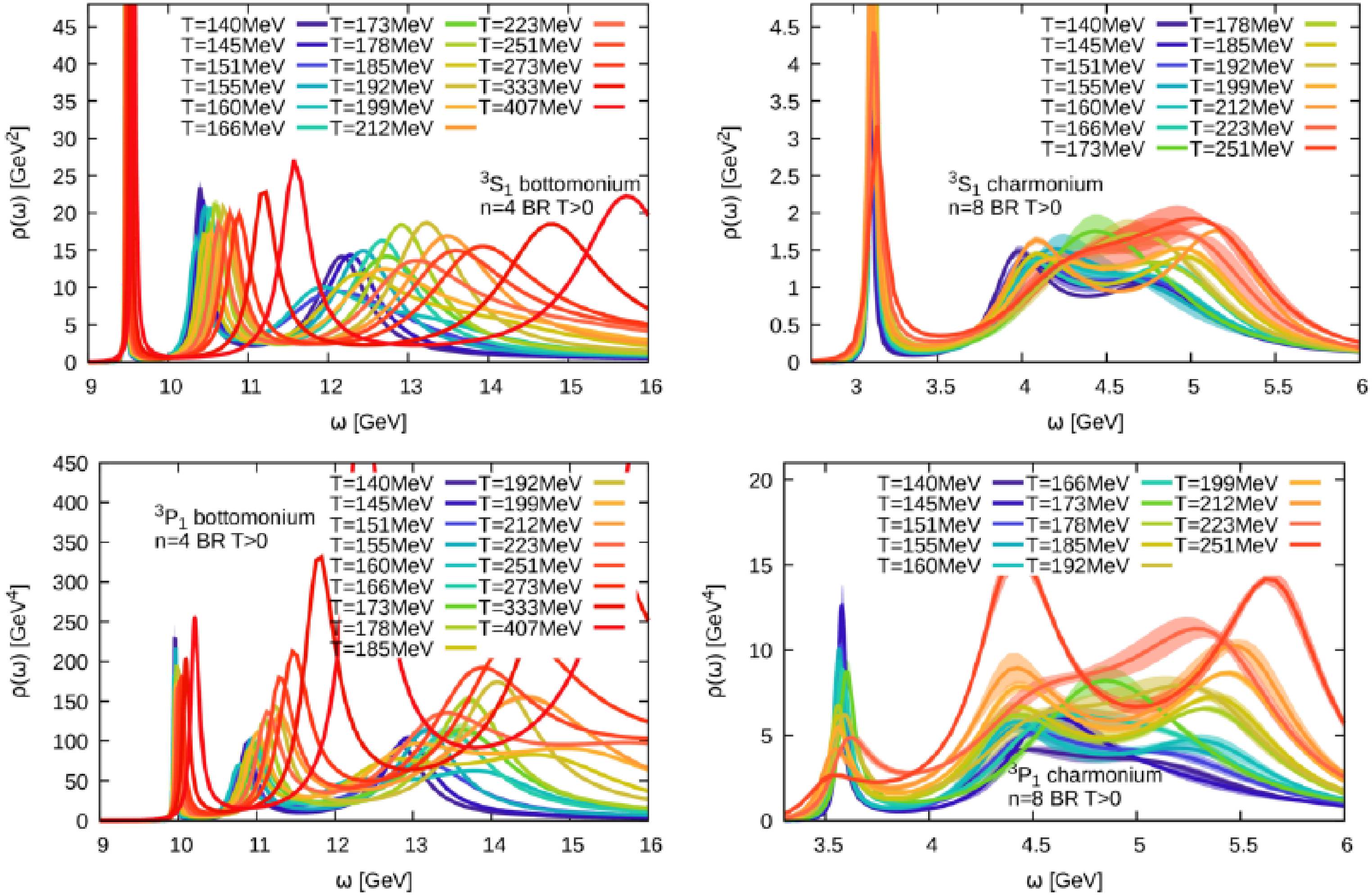
Not too high T, not too long : Same as in vacuum + some external perturbation

If not : probably better to speak a $Q\bar{Q}$ pair

When is it legitimate to speak of a bound state ?... And deal with it as such in the transport theory. Answer may vary depending on the fundamental ingredients

lQCD perspective : spectral function

Kim et al, JHEP11(2018)088



Many such kind of results in the literature

Rich structure : broadening and mass shift. What are the underlying “ingredients’” ?

The 3 pillars of quarkonia production in AA

The present challenges for Quarkonium modelling in URHIC

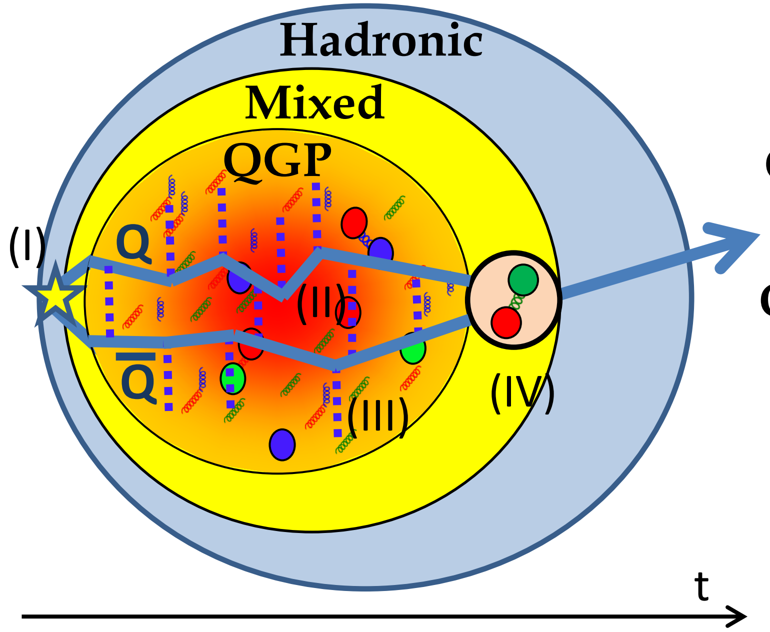
Meet the higher and higher precision of experimental data (already beyond the present model uncertainties)

Unravel the Q-Qbar interactions under the influence of the surrounding QGP and with the QGP

Develop a scheme able to deal with the evolution of one (or many) $Q\bar{Q}$ pair(s) in a QGP, fulfilling all fundamental principles (quantum features, gauge invariance, equilibration,…)

Need for lQCD constraints / inputs

The full transport scheme



Quarkonium

?

Open mesons

Strictly speaking, only resolved at the end of the evolution

Beware of quantum coherences during the whole evolution !

Especially at early time…

In practice, what counts is the so-called decoherence time, not the "Heisenberg time"

Complicated QFT problem (also due to the evolving nature of the QGP that mixes several scales)… only started to be addressed at face value recently

How to proceed ?

1) Initial state
2) (Screened) interaction between both HQ
3) Interactions with surrounding QGP partons
4) Projection on the final quarkonia

First incomplete QM treatments dating back to Blaizot & Ollitrault, Thews, Cugnon and Gossiaux; early 90's

Reminder : the density operator

- One has to deal with statistical averaging of a Q-Qbar subsystem in a QGP…
- Operationnally, this **cannot** be treated dealing with usual quantum states $|\psi\rangle = |\psi_{Q\bar{Q}}| \otimes |\psi_{\rm QGP}\rangle$
- One needs to resort to the density operator / matrix
  - ✓ Simple form (pure state) : $\hat{\rho}_{Q\bar{Q}} = |\psi_{Q\bar{Q}}\rangle\langle\psi_{Q\bar{Q}}|$
  - ✓ More involved form : $\hat{\rho}_{Q\bar{Q}} = \sum_{\alpha,\beta} d_{\alpha,\beta} |\alpha_{Q\bar{Q}}\rangle\langle\beta_{Q\bar{Q}}|$

QME deal with the (coupled) evolution of probabilities ($d_{\alpha,\alpha}$) and “quantum coherences” ($d_{\alpha,\beta\neq\alpha}$)

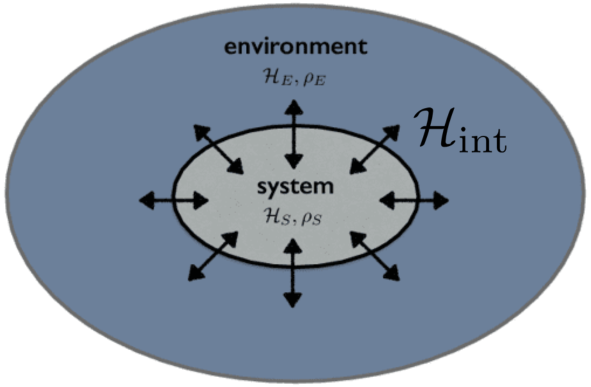
- Probability to observe a quarkonium Φ in a given Q-Qbar state

$$\text{prob}(\Phi) = \langle\psi_\Phi|\hat{\rho}_{Q\bar{Q}}|\psi_\Phi\rangle = \text{tr}(\hat{\rho}_{Q\bar{Q}}\hat{\rho}_\Phi)$$

Should be always positive !

First incomplete QM treatments dating back to Blaizot & Ollitrault, Thews, Cugnon and Gossiaux; early 90’s

Open Quantum Systems & Quantum Master Equations



Quite generally, system (Q-Qbar pair) builds correlation with the environment thanks to the Hamiltonian $\hat{H} = \hat{H}^{(0)}_{Q\bar{Q}} + \hat{H}_E + \hat{H}_{\rm int}$ with $\hat{H}_E = \hat{H}_{\rm QGP}$

Von Neumann equation for the total density operator ρ

$\frac{\mathrm{d}}{\mathrm{d}t}\rho = -i[H, \rho]$

System + environment (QGP)

$\rho(t=0) = \rho_{Q\bar{Q}} \otimes \rho_{QGP}$

Evolution of the total system

$\rho(t) = U(t,0)\left[\rho_{Q\bar{Q}} \otimes \rho_{QGP}\right] U(t,0)^\dagger$

Trace out QGP degrees of freedom =>
Reduced density operator $\rho_{Q\bar{Q}}$

System ($Q\bar{Q}$ pair)

$\rho_{Q\bar{Q}}(t=0)$

Evolution of the system

$\rho_{Q\bar{Q}}(t) = Tr_{QGP}\left[U(t,0)\rho(t=0)U(t,0)^\dagger\right]$

Can be formulated differentially ./. time :

$$\frac{\mathrm{d}\rho_{Q\bar{Q}}}{\mathrm{d}t} = \mathcal{L}[\rho_{Q\bar{Q}}]$$

Definition of $\mathcal{L}[\cdot]$

However, $\mathcal{L}[\cdot]$ is generically a non local super-operator in time (linear map)

A special Quantum Master Equation: The Lindblad Equation

There are many different QME… a special one :

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)\right] + \sum_i \boxed{\gamma_i}\left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \frac{1}{2}\left\{L_i L_i^\dagger, \rho_{Q\bar{Q}}(t)\right\}\right]$$

γ_i Characterize the coupling of the system (Q-Qbar) with the environment

$H_{Q\bar{Q}}$: $\{Q, \bar{Q}\}$ $\underbrace{\text{kinetics + Vacuum potential V}}_{\hat{H}^{(0)}_{Q\bar{Q}}}$ + Lamb shift / screening (every unitary term that is generated by tracing out the environment ⬄ Von Neumann)

L_i : Collapse (or Lindblad) operators, depend on the properties of the medium

3 important conservation properties :

$\rho^\dagger_{Q\bar{Q}} = \rho_{Q\bar{Q}}$	$\mathrm{Tr}[\rho_{Q\bar{Q}}] = 1$	$\langle\varphi\|\rho_{Q\bar{Q}}\|\varphi\rangle > 0, \forall\|\varphi\rangle$
(Hermiticity)	(Norm)	(Positivity)

… but in general, non unitary !!! (relaxation)

Nice feature : Can be brought to the form of a stochastic Schroedinger equation (quantum jump method : QTRAJ) => more effective numerically

A special QME: The Lindblad Equation

Non unitary / dissipative evolution ≡ decoherence

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}},\rho_{Q\bar{Q}}(t)\right] + \boxed{\sum_i \gamma_i\left[L_i\rho_{Q\bar{Q}}(t)L_i^\dagger - \frac{1}{2}\left\{L_iL_i^\dagger,\rho_{Q\bar{Q}}(t)\right\}\right]}$$

Genuine transitions :

- ✓ Singlet <-> octet
- ✓ Octet <-> octet

Can be reshuffled into non Hermitic effective hamiltonian

$$\hat{H}_{Q\bar{Q},\mathrm{eff}} = \hat{H}_{Q\bar{Q}} - i\boxed{\sum_j \gamma_j \frac{L_jL_j^\dagger}{2}}$$ ≡ Dissociation width

Indeed, starting from a singlet density matrix : $\rho^{\mathrm{s}}_{Q\bar{Q}} = |s\rangle\langle s|$, one generates an octet composant :

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle o|\rho_{Q\bar{Q}}|o\rangle = \sum_i \gamma_i\langle o|L_i|s\rangle\langle s|L_i^\dagger|o\rangle = \sum_i \gamma_i|\langle o|L_i|s\rangle|^2$$

Usual transition rate

Starting from generic $\hat{H} = \hat{H}^{(0)}_{Q\bar{Q}} + \hat{H}_E + \hat{H}_{\mathrm{int}}$ with $\hat{H}_{\mathrm{int}} = \sum A_{QGP} \times S$ how to obtain a Linblad equation ?

QGP operator (field) Q-Qbar operator (charge)

Pictorial summary

Inspired from Yao Int. J. of Mod. Phys. A, Vol. 36, No. 20, 2130010 (2021)

τ_E: environment autocorrelation time **τ_R: system relaxation time**

Subsystem + environment: von Neumann equation

Iterate Von Neumann eq. + **Trace out environment**

Subsystem: non-unitary, time-irreversible evolution

Weak syst-environment coupling **+ Markovian limit** $\tau_E \ll \tau_R$ Ok QCD : $\tau_R \propto m_Q^2$

Redfield equation

A-A correlator

$$\frac{\partial}{\partial t}\rho_{\rm I}(t) = -\frac{1}{\hbar^2}\sum_{m,n}\int_0^\infty d\tau\left(C_{mn}(\tau)\Big[S_{m,{\rm I}}(t), S_{n,{\rm I}}(t-\tau)\rho_{\rm I}(t)\Big] - C^*_{mn}(\tau)\Big[S_{m,{\rm I}}(t), \rho_{\rm I}(t)S_{n,{\rm I}}(t-\tau)\Big]\right)$$

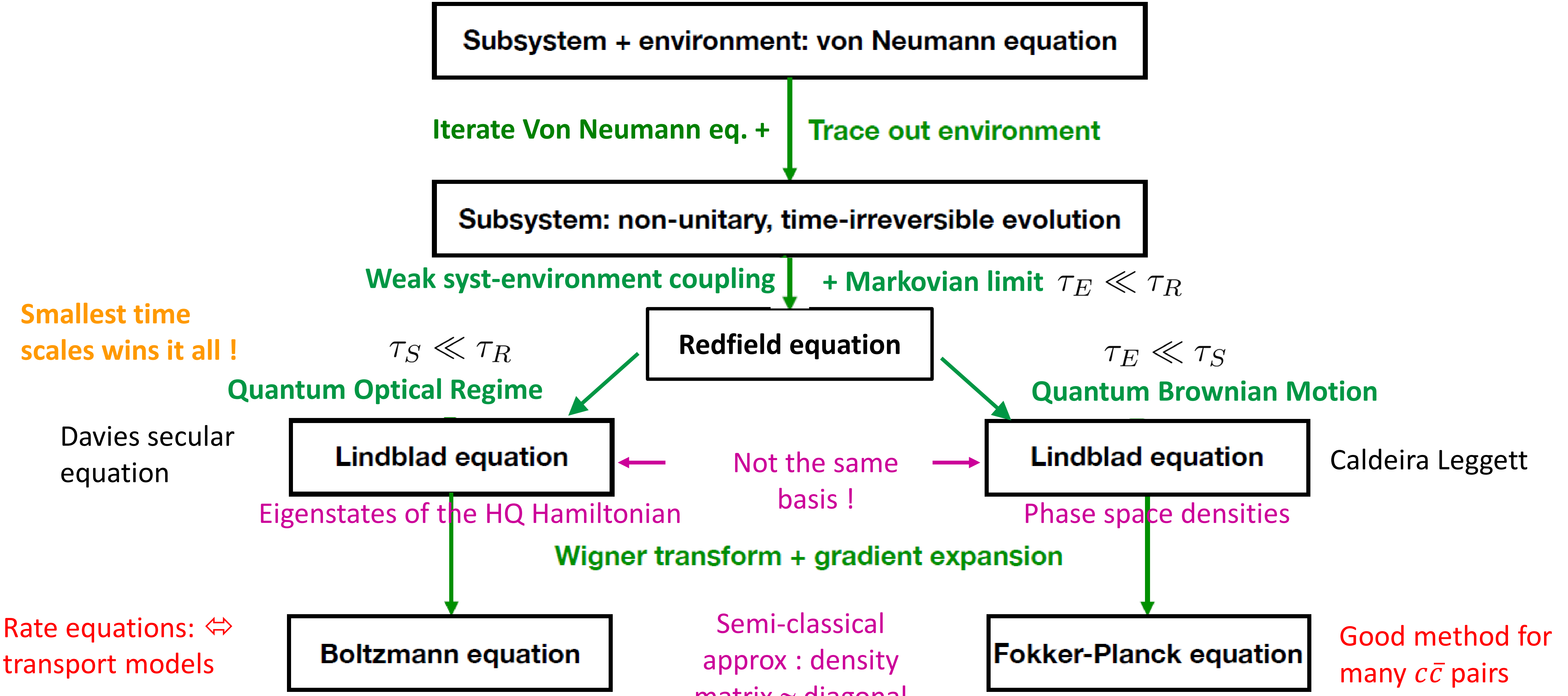
Some similitude with the Linblad equation but with time delay effects => **Not Lindbladian**

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)\right] + \sum_i \gamma_i\left[L_i\rho_{Q\bar{Q}}(t)L_i^\dagger - \tfrac{1}{2}\left\{L_iL_i^\dagger, \rho_{Q\bar{Q}}(t)\right\}\right]$$

Pictorial summary

Inspired from Yao Int. J. of Mod. Phys. A, Vol. 36, No. 20, 2130010 (2021)

τ_E: environment autocorrelation time τ_R: system relaxation time τ_S: system intrinsic time scale



Subsystem + environment: von Neumann equation

Iterate Von Neumann eq. + Trace out environment

Subsystem: non-unitary, time-irreversible evolution

Weak syst-environment coupling + Markovian limit $\tau_E \ll \tau_R$

Redfield equation

Smallest time scales wins it all !

$\tau_S \ll \tau_R$

Quantum Optical Regime

$\tau_E \ll \tau_S$

Quantum Brownian Motion

Davies secular equation

Lindblad equation

Not the same basis !

Lindblad equation

Caldeira Leggett

Eigenstates of the HQ Hamiltonian

Phase space densities

Wigner transform + gradient expansion

Rate equations: ⇔ transport models

Boltzmann equation

Semi-classical approx : density matrix ≈ diagonal

Fokker-Planck equation

Good method for many $c\bar{c}$ pairs

Two types of dynamical modelling

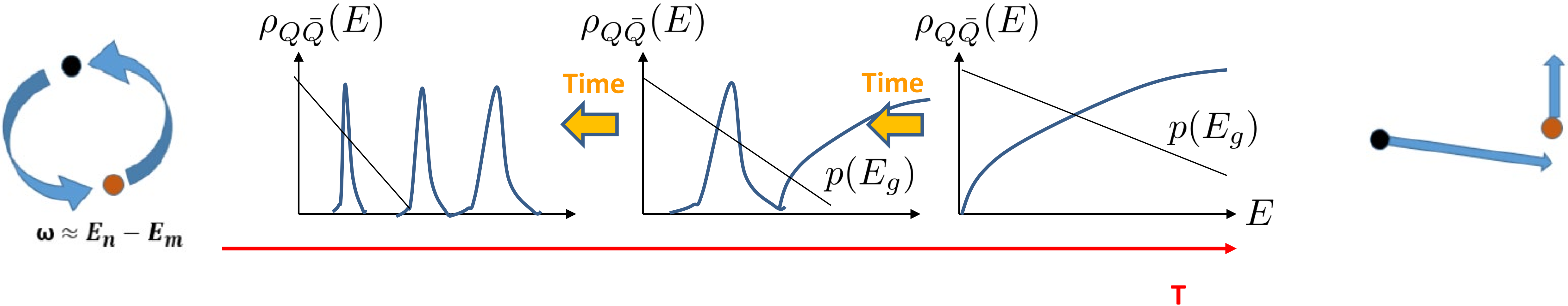
$m_D \ll E_{\rm bind}$

Quantum Optical Regime

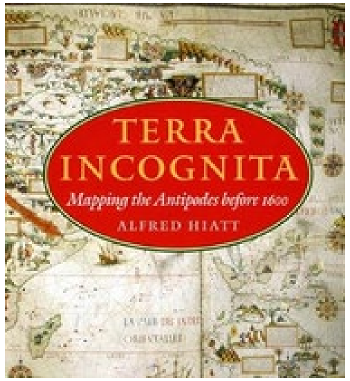
$m_D \sim E_{\rm bind}$

$m_D \gg E_{\rm bind}$

Quantum Brownian Motion



Dissociation of well identified levels by scarce "high-energy" modes (dilute medium => cross section ok) **.**
Best described using the eigenstates of the $H_{Q\bar{Q}}$



Multiple scattering on quasi free states.
Best described in position-momentum space

QCD time scales

τ_E: environment autocorrelation time

$$\tau_E \approx \frac{1}{m_D} \approx \frac{1}{CT} \approx \frac{1}{T}$$ (C taken as close to unity)

τ_S: system intrinsic time scale

$$\tau_S \approx \frac{1}{\underbrace{\Delta E}} \approx \frac{1}{m_Q v^2} \quad \text{with} \quad v \approx \alpha_S$$ … at the beginning of the evolution

Difference btwn energy levels

τ_R: system relaxation time

$$\Gamma = \tau_R^{-1} \sim 2\langle\psi|W\psi\rangle \approx \alpha_s T \times \Phi(m_D r) \approx \alpha_s T \times \Phi(\frac{CT}{m_Q \alpha_s})$$

At "small" T $\left(T \lesssim \frac{m_Q \alpha_S}{C}\right)$: dipole approximation : $\Gamma = \tau_R^{-1} \approx \frac{C^2 T^3}{\alpha_s m_Q^2}$

$$\boxed{\frac{\tau_R}{\tau_E} = \frac{\alpha_s m_Q^2}{CT^2} \gg 1}$$

And $\frac{\tau_R}{\tau_S} = \frac{\alpha_s^3 m_Q^3}{C^2 T^3} \gg 1$ for $T \lesssim m_Q \frac{\alpha_S}{C^{2/3}}$

Fine with the Markovian assumption

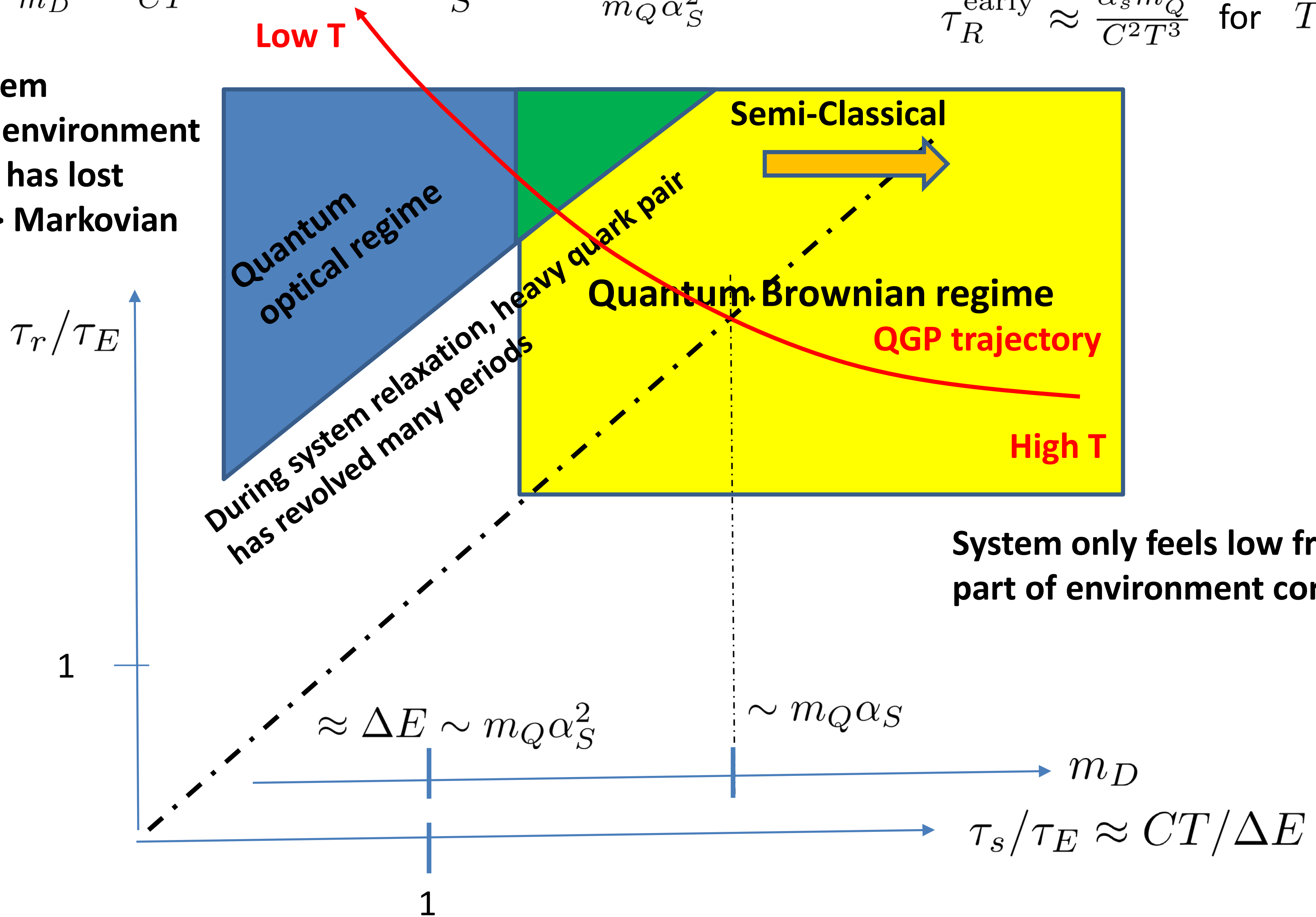
QCD time scales

$\tau_E \approx \frac{1}{m_D} = \frac{1}{CT}$

$\tau_S^{\text{early}} \approx \frac{1}{m_Q \alpha_S^2}$

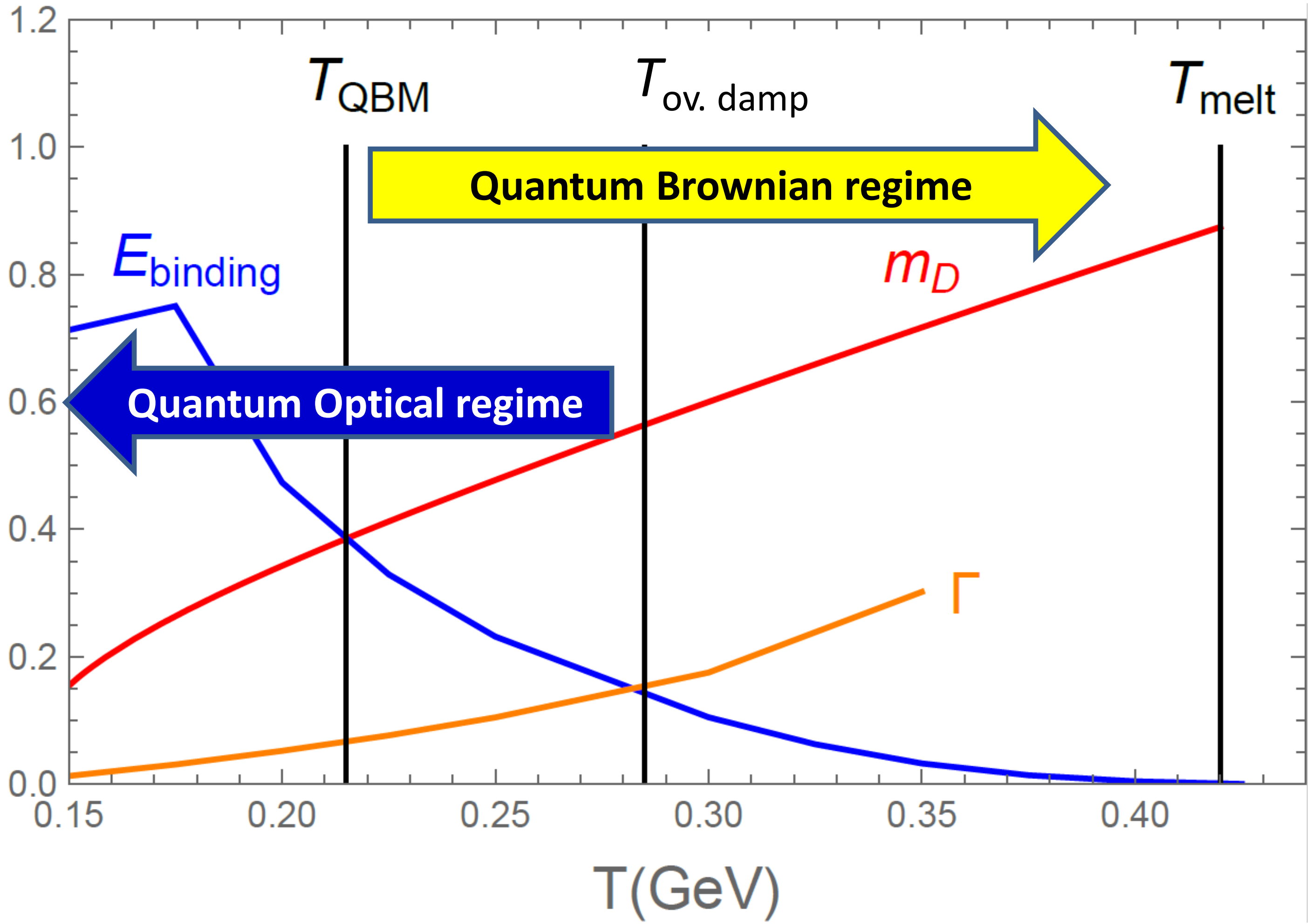
$\tau_R^{\text{early}} \approx \frac{\alpha_s m_Q^2}{C^2 T^3}$ for $T \lesssim \frac{m_Q \alpha_S}{C}$

During system relaxation, environment correlation has lost memory => Markovian process



System only feels low frequency part of environment correlation

Two types of dynamical modelling



$c\bar{c}$ pair

Numbers extracted from a specific potential model : Katz et al, Phys. Rev. D 101, 056010 (2020)

Recent OQS implementations (single $Q\bar{Q}$ pair)

regime	SU3 ?	Dissipation ?	3D / 1D	Num method	year	remark	ref
NRQCD ⇔ QBM	No	No	1D	Stoch potential	2018		Kajimotoet al. , Phys. Rev. D 97, 014003 (2018), 1705.03365
	Yes	No	3D	Stoch potential	2020	Small dipole	R. Sharma et al Phys. Rev. D 101, 074004 (2020), 1912.07036
	Yes	No	3D	Stoch potential	2021		Y. Akamatsu, M. Asakawa, S. Kajimoto (2021), 2108.06921
	No	Yes	1D	Quantum state diffusion	2020		T. Miura, Y. Akamatsu et al, Phys. Rev. D 101, 034011 (2020), 1908.06293
	Yes ✓	Yes ✓	1D	Quantum state diffusion	**2021**		Akamatsu & Miura, EPJ Web Conf. 258 (2022) 01006, 2111.15402
	No	Yes	1D	Direct resolution	**2021**		O. Ålund, Y. Akamatsu et al, Comput. Phys. 425, 109917 (2021), 2004.04406
	Yes ✓	Yes ✓	1D	Direct resolution	**2022**		S Delorme et al, https://inspirehep.net /literature/ 2026925
pNRQCD (i)	Yes	No	1D+	Direct resolution	2017	S and P waves	N. Brambilla et al, Phys. Rev. D96, 034021 (2017), 1612.07248
(i) Et (ii)	Yes	No	1D+	Direct resolution	2017	S and P waves	N. Brambilla et al, Phys. Rev. D 97, 074009 (2018), 1711.04515
(i)	Yes	No	Yes	Quantum jump	**2021**	See SQM 2021	N. Brambilla et al. , JHEP 05, 136 (2021), 2012.01240 & *Phys.Rev.D* 104 (2021) 9, 094049, 2107.06222
(i)	Yes ✓	Yes ✓	Yes ✓	Quantum jump	**2022**		N. Brambilla et al. 2205.10289
(iii)	Yes ✓	Yes ✓	Yes ✓	Boltzmann (?)	2019		Yao & Mehen, Phys.Rev.D 99 (2019) 9, 096028, 1811.07027
NRQCD & « pNRQCD »	Yes	Yes	1D	Quantum state diffusion	**2022**		Miura et al. http://arxiv.org/abs/2205.15551v1
Other	No	Yes	1D	Stochastic Langevin Eq.	2016	Quadratic W	Katz and Gossiaux

(Year > 2015)

Not exhaustive

See as well table in 2111.15402v1

And recent review : A. Andronic et al, *Eur.Phys.J.A* 60 (2024) 4, 88

Either the one or the other … nothing in between ?

…

The core of the ULE derivation

Lindblad structure $\frac{d\rho(t)}{dt} = -i\left[H_Q + H_{LS}, \rho(t)\right] + \sum_n \gamma_n \left(L_n\rho(t)L_n^\dagger - \frac{1}{2}\left\{L_n^\dagger L_n, \rho(t)\right\}\right)$

Starting from the NRQCD Hamiltonian $H_{\rm tot} = \left(\frac{\boldsymbol{p}_Q^2}{2M} + \frac{\boldsymbol{p}_{\bar{Q}}^2}{2M}\right) \otimes I_{\rm QGP} + I_{Q\bar{Q}} \otimes H_{\rm QGP} + \int_{\boldsymbol{x}} n_{\boldsymbol{x}}^a \otimes gA_0^a\left(\boldsymbol{x}\right)$

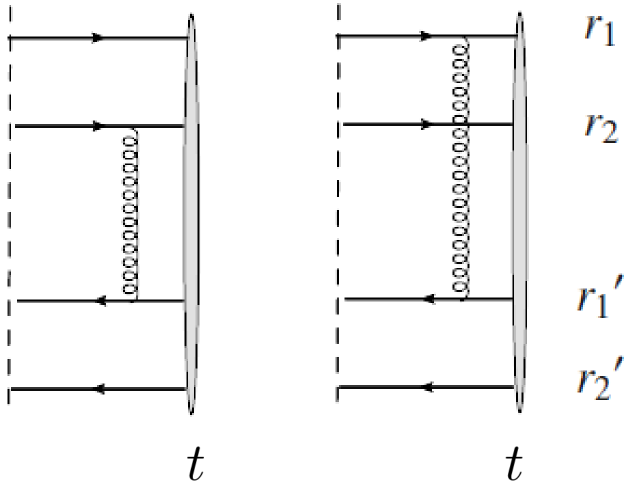
Iterating the Von Neumann equation, tracing out the QGP d.o.f. and assuming a slow evolution of ρ^I in the interaction representation (Born - Markov), one arrives at the Redfield equation

$$\frac{d\rho^I(t)}{dt} = -\int_{t_0}^{t} dt' \int_{xx'} \left[n^a(t,x), n^a(t',x')\rho^I(t)\right] \Delta^>(t-t', x-x') + h.c.$$

Where the gluon propagator is

$$\delta^{ab} \underbrace{\Delta^>\left(t_1 - t_2, \boldsymbol{x} - \boldsymbol{x}'\right)} = g^2 \left\langle T_C \left[A_0^a\left(t_1, \boldsymbol{x}\right) A_0^b\left(t_2, \boldsymbol{x}'\right)\right]\right\rangle_0$$

IMPORTANT : localized on $t_1 - t_2 \lesssim \tau_E$



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Lindblad structure $\frac{d\rho(t)}{dt} = -i\left[H_Q + H_{LS}, \rho(t)\right] + \sum_n \gamma_n \boxed{\left(L_n\rho(t)L_n^\dagger\right.} - \frac{1}{2}\left\{L_n^\dagger L_n, \rho(t)\right\}\Big)$

Jumps -> other states

Redfield equation : $$\frac{d\rho^I(t)}{dt} = -\int_{t_0}^{t} dt' \int_{xx'} \left[n^a(t,x), n^a(t',x')\rho^I(t)\right] \Delta^>(t-t', x-x') + h.c.$$

$$\delta^{ab}\Delta^>(t_1-t_2, \boldsymbol{x}-\boldsymbol{x}') = g^2\left\langle T_C\left[A_0^a(t_1,\boldsymbol{x})\,A_0^b(t_2,\boldsymbol{x}')\right]\right\rangle_0$$

Jump contribution : $$\frac{d\rho^I(t)}{dt} = \int_{t_0}^{t} dt' \int_{xx'} \Delta^>(t-t', x-x')n^a(t',x')\rho^I(t)n^a(t,x) + h.c. + \cdots$$

No symetrized form ! => not Linblad structure

For t < τ_R: $$\rho^I(t) - \rho^I(t_0) = \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \int_{xx'} \Delta^>(t'-t'', x-x')n^a(t'',x')\rho^I(t)n^a(t',x) + h.c. + \cdots$$

$$= \int_{t_0}^{t} dt' \int_{t_0}^{+\infty} dt''\theta(t'-t'') \int_{xx'} \Delta^>(t'-t'', x-x')n^a(t'',x')\rho^I(t)n^a(t',x) + h.c. + \cdots$$

$$\theta(t'-t'') = \boxed{\frac{1}{2}} + \frac{1}{2}\mathrm{sign}(t'-t'')$$

« Lamb shift »

The core of the ULE derivation

Lindblad structure $\frac{d\rho(t)}{dt} = -i\left[H_Q + H_{LS}, \rho(t)\right] + \sum_n \gamma_n \left(L_n \rho(t) L_n^\dagger - \frac{1}{2}\left\{L_n^\dagger L_n, \rho(t)\right\}\right)$

$$\rho^I(t) - \rho^I(t_0) = \frac{1}{2}\int_{t_0}^{t} dt' \int_{t_0}^{+\infty} dt'' \int_{x'x''} \underset{\uparrow}{\Delta^>}(t'-t'', x'-x'')\, n^a(t'', x'')\, \rho^I(t)\, n^a(t', x') + h.c. + \cdots$$

t’ and t’’ still intricated

Trick : $$\Delta^>(t'-t'', x'-x'') = \int_{-\infty}^{+\infty} dv \int_y g(v-t'', y-x'')\, g(t'-v, x'-y)$$ g : jump correlators

with $g(t'-v, x'-y) = g^\star(v-t', y-x')$

$$\rho^I(t) = \underbrace{\frac{1}{2}\int_y \int dv}_{\sim\sum_n \gamma_n} \int_{t_0}^{t} dt' \int_{t_0}^{+\infty} dt'' \int_{x''} g(v-t'', y-x'')\, \hat{n}^a(t'', x'')\, \rho^I(t) \int_{x'} g^\star(v-t', y-x')\, \hat{n}^a(t', x') + \cdots$$

$$= \underbrace{\frac{1}{2}\int_y \int dv}_{\sim\sum_n \gamma_n} \underbrace{\int_{t_0}^{+\infty} dt'' \int_{x''} g(v-t'', y-x'')\, \hat{n}^a(t'', x'')}_{\equiv L_n???}\, \rho^I(t)\, \underbrace{\int_{t_0}^{\mathbf{t}} dt' \int_{x'} g^\star(v-t', y-x')\, \hat{n}^a(t', x')}_{\equiv L_n^\dagger???} + h.c. + \cdots$$

Not quite ! As time derivation will break this structure !!! **Redfield : NOT LINDBLAD**

The core of the ULE derivation

Lindblad structure $$\frac{d\rho(t)}{dt} = -i\left[H_Q + H_{LS}, \rho(t)\right] + \sum_n \gamma_n \left(L_n\rho(t)L_n^\dagger - \frac{1}{2}\left\{L_n^\dagger L_n, \rho(t)\right\}\right)$$

$$\rho^I(t) = \frac{1}{2}\int_y \int_{-\infty}^{+\infty} dv \int_{t_0}^{t} dt' \int_{t_0}^{+\infty} dt'' \int_{x''} g(v-t'', y-x'')\hat{n}^a(t'',x'')\rho^I(t)\int_{x'} g^\star(v-t', y-x')\hat{n}^a(t',x') + \cdots$$

How to proceed without introducing further approximations (than $\tau_E << \tau_R \approx t-t_0$) ? **In fact, there is an {} of equivalent QME**

Nathan and Rudner (2020): In significant contributions to the integrals stem from regions where the 3 times v, t' and t'' are separated by τ_E at most => one can **permutate the boundaries on v and t'** (at the price of a small correction in τ_E / τ_R)

$$\rho^I(t) = \frac{1}{2}\int_y \int_{t_0}^{t} dv \int_{-\infty}^{+\infty} dt' \int_{t_0}^{+\infty} dt'' \int_{x''} g(v-t'', y-x'')\hat{n}^a(t'',x'')\rho^I(t)\int_{x'} g^\star(v-t', y-x')\hat{n}^a(t',x') + \cdots$$

Then proceeding with time derivative:

$$\frac{d\rho^I(t)}{dt} = \frac{1}{2}\int_y \int_{-\infty}^{+\infty} dt' \int_{-\infty}^{+\infty} dt'' \int_{x''} g(t-t'', y-x'')\hat{n}^a(t'',x'')\rho^I(t)\int_{x'} g^\star(t-t', y-x')\hat{n}^a(t',x') + h.c. + \cdots$$

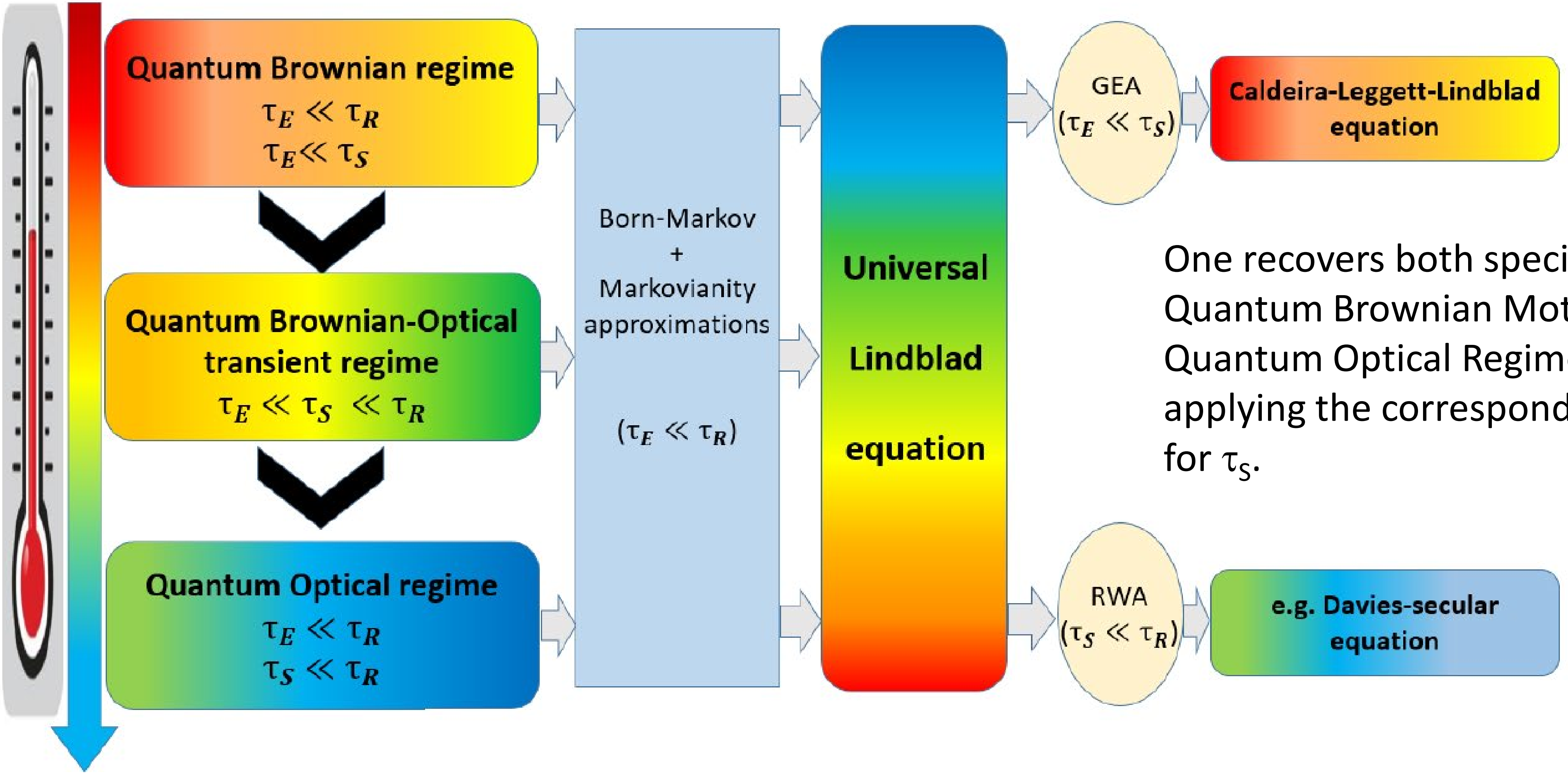
Markovianity

Linblad structure provided one defines $$\hat{L}_y = \int_{-\infty}^{+\infty} \int_{x''} dt' g(t-t'', y-x'')\hat{n}^a(t'',x'')$$

Contains all spectral information of the QGP heat bath !

Same structure in the Schroedinger representation

Schematic applicability of the ULE :



Quantum Brownian regime
$\tau_E \ll \tau_R$
$\tau_E \ll \tau_S$

Quantum Brownian-Optical transient regime
$\tau_E \ll \tau_S \ll \tau_R$

Quantum Optical regime
$\tau_E \ll \tau_R$
$\tau_S \ll \tau_R$

Born-Markov
+
Markovianity approximations

$(\tau_E \ll \tau_R)$

Universal Lindblad equation

GEA
$(\tau_E \ll \tau_S)$

Caldeira-Leggett-Lindblad equation

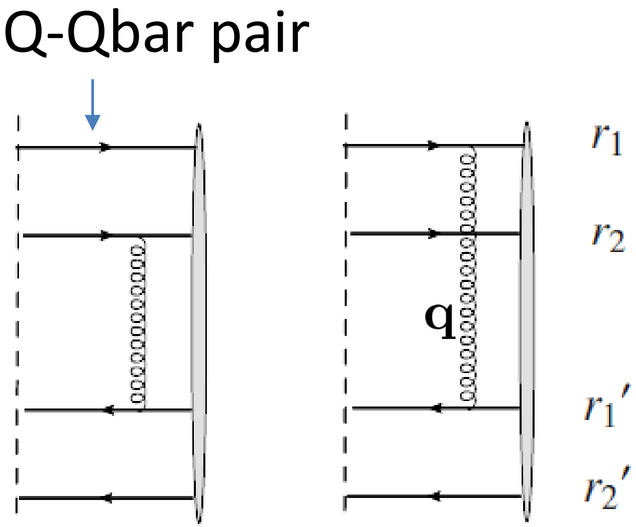
RWA
$(\tau_S \ll \tau_R)$

e.g. Davies-secular equation

QGP Temperature

One recovers both specific QME in the Quantum Brownian Motion and in the Quantum Optical Regimes when applying the corresponding hierarchies for τ_S.

A coupled singlet-octet universal Lindblad equations



- Scattering from gluons change the color representation : o <-> s

$$\mathcal{D}_Q = \begin{pmatrix} \mathcal{D}_s \\ \mathcal{D}_o \end{pmatrix}$$

- One assumes some binding Poschl-Teller potential in the singlet representation => in-QGP Bound states (|n>, |m>,…)
- For the octet representation, no binding potential => diffusion states |**k**> => « large » energy gap.
- Illustration for the singlet -> octet (center of mass integrated)

$$\frac{d\rho}{dt} = \cdots + \int_y \hat{L}_1(y)\rho\hat{L}_1^\dagger(y) + \cdots$$

with $$\hat{L}_1(y) = \int d\mathbf{k}\sum_n \tilde{L}_1(\mathbf{k},n,y)|\mathbf{k}\rangle\langle n|$$

Transition s -> o

$$\tilde{L}_1(\mathbf{k},n,y) = -i\sqrt{\frac{1}{2N_c}}\int d\mathbf{q}\, e^{i\mathbf{q}\cdot\mathbf{y}}\tilde{g}\left(q^0 = E_n - \frac{k^2}{M},\mathbf{q}\right)\langle\mathbf{k}|\sin\frac{\mathbf{q}\cdot\mathbf{s}}{2}|n\rangle$$

Fourier transform of g, sqrt of the QGP spectral density

Dipolar transition matrix element

Illustrations of transition rates

- Starting form a singlet state : $\hat{\rho}(0) = |n\rangle\langle n|$

- The transition rate towards octet state |k> (⬄ differential dissociation rate) is $d\Gamma/d\mathbf{k} = \dfrac{d}{dt}\langle \mathbf{k}|\hat{\rho}(t)|\mathbf{k}\rangle$

Probability to find the Q-Qbar in an octet state

- With previous expressions :

$$d\Gamma/d\mathbf{k} = \frac{N_c^2-1}{2N_c}\int d\mathbf{q}\, \underbrace{\tilde{g}^2\left(q^0 = E_n - \frac{k^2}{M}, \mathbf{q}\right)}_{\text{QGP spectral density}} |\langle \mathbf{k}|\sin\frac{\mathbf{q}\cdot\mathbf{s}}{2}|n\rangle|^2$$

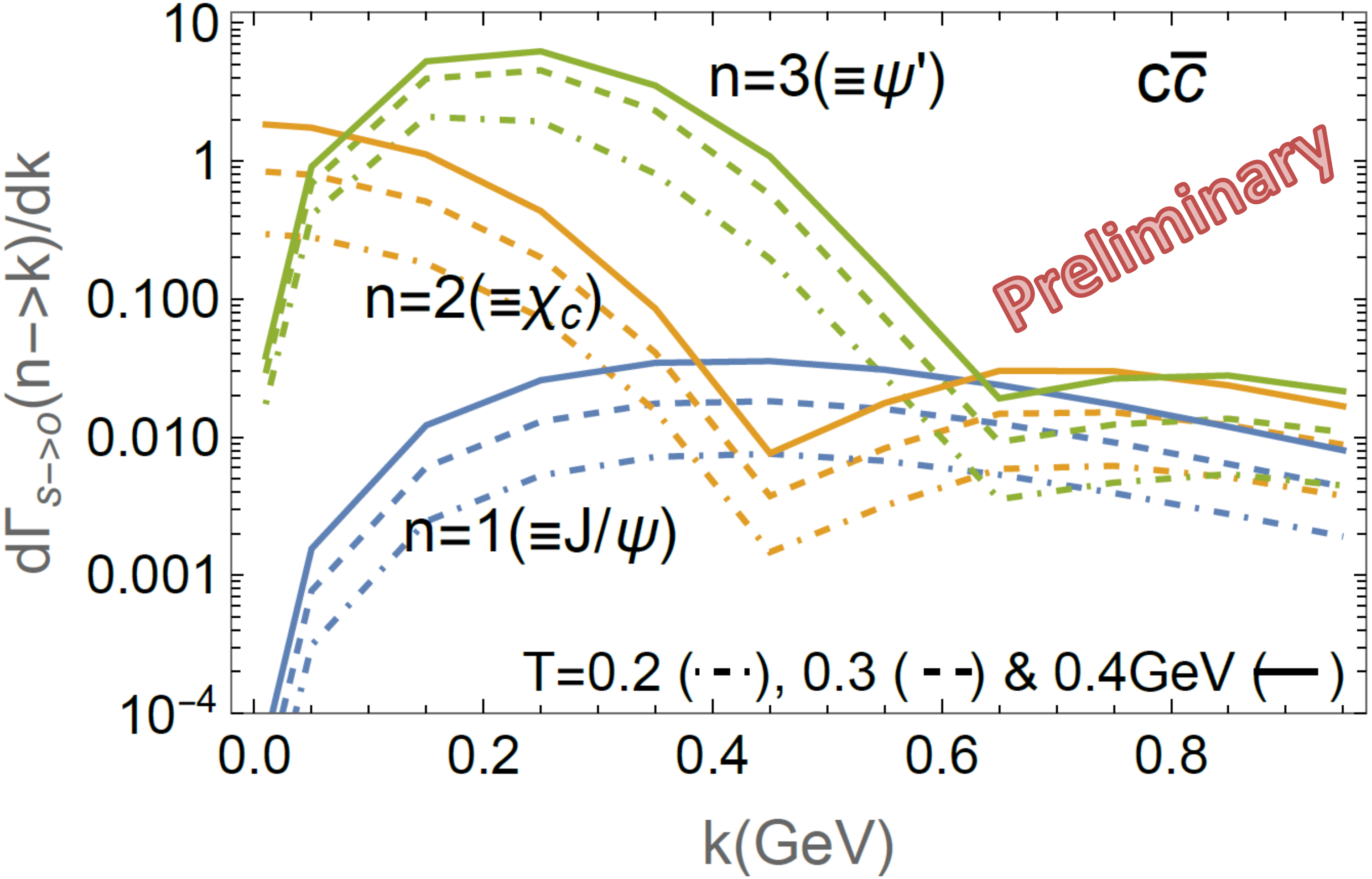
Dipolar transition matrix element

Result quite similar to the Fermi Golden rule

- But also genuine quantum coherences: $\dfrac{d}{dt}\langle \mathbf{k}'|\hat{\rho}(t)|\mathbf{k}\rangle \neq 0$

Illustrations of transition rates

$$d\Gamma_{\mathrm{s}\to\mathrm{o}}(n)/d\mathbf{k} = \frac{N_c^2-1}{2N_c}\int d\mathbf{q}\,\tilde{g}^2\left(q^0 = E_n - \frac{k^2}{M},\mathbf{q}\right)|\langle\mathbf{k}|\sin\frac{\mathbf{q}\cdot\mathbf{s}}{2}|n\rangle|^2$$

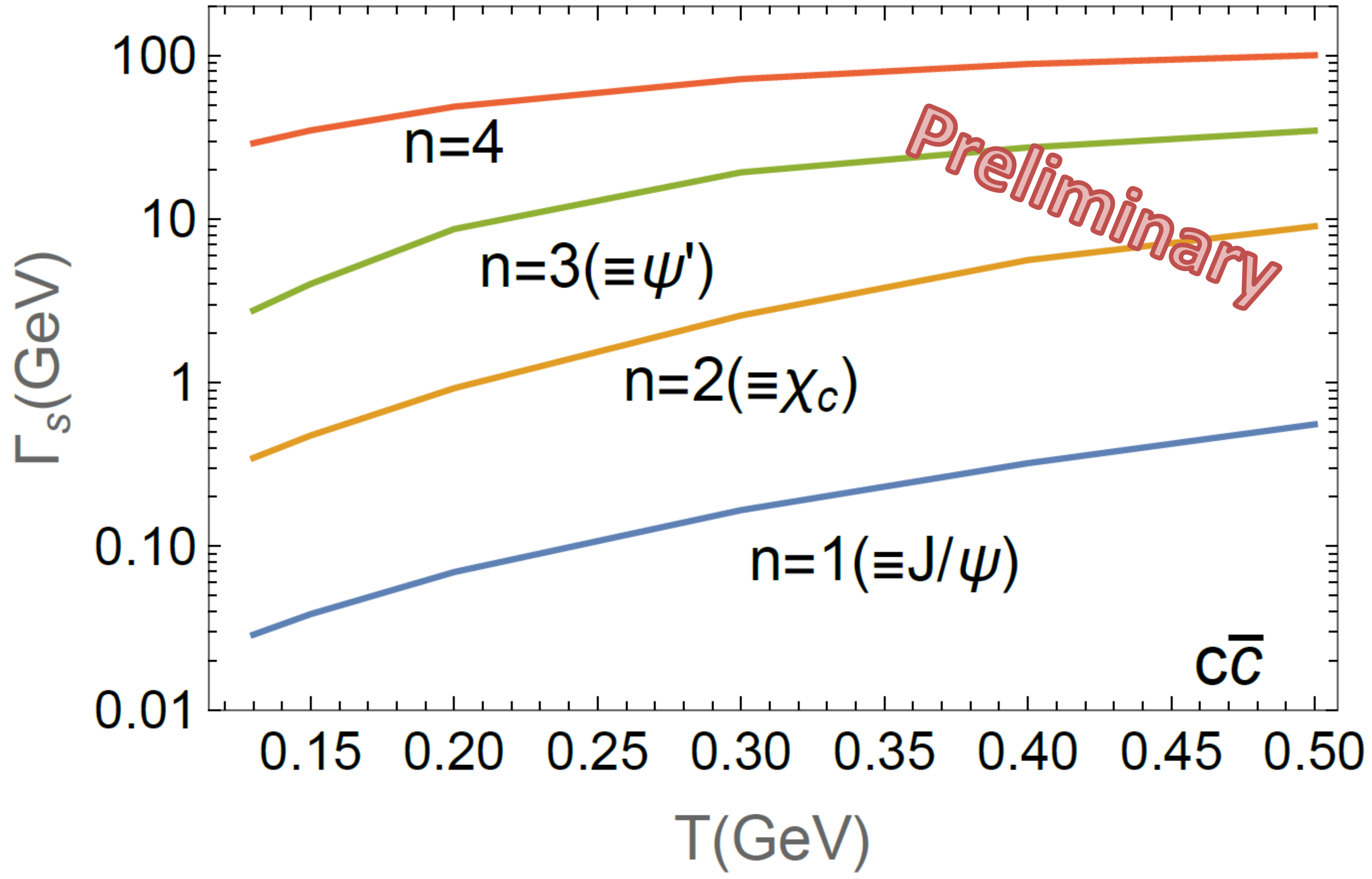


- ➢ Hierarchy from ground state -> excited state
- ➢ The transition rate towards octet state |k> (⇔ differential dissociation rate) is
- ➢ For k close to 0 : selection rule for even bound states

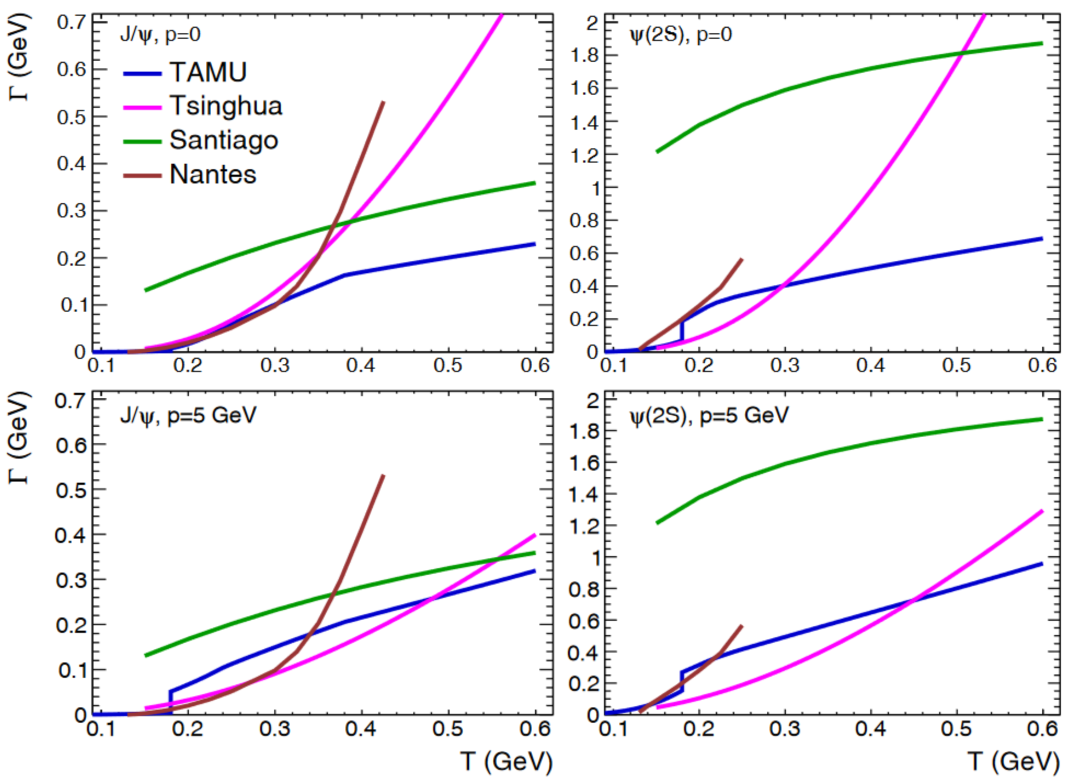
$$\langle\mathbf{k}=0|\sin\frac{\mathbf{q}\cdot\mathbf{s}}{2}|n_{\mathrm{even}}\rangle = 0$$

Illustrations of transition rates

Total transition rate : $\Gamma_{\mathrm{s}\to\mathrm{o}}(n) = \int d\mathbf{k} d\Gamma_{\mathrm{s}\to\mathrm{o}}(n)/d\mathbf{k}$



A. Andronic et al, *Eur.Phys.J.A* 60 (2024) 4, 88



Compatible with recent compilation of dissociation rates

Illustrations of spectral densities for charmonia

Lindblad equation : $\frac{d\rho(t)}{dt} = \boxed{-i\left[H_Q + H_{LS}, \rho(t)\right]} + \sum_n \gamma_n \left(L_n \rho(t) L_n^\dagger \boxed{-\frac{1}{2}\left\{L_n^\dagger L_n, \rho(t)\right\}} \right)$

▭ : Von Neumann equation with

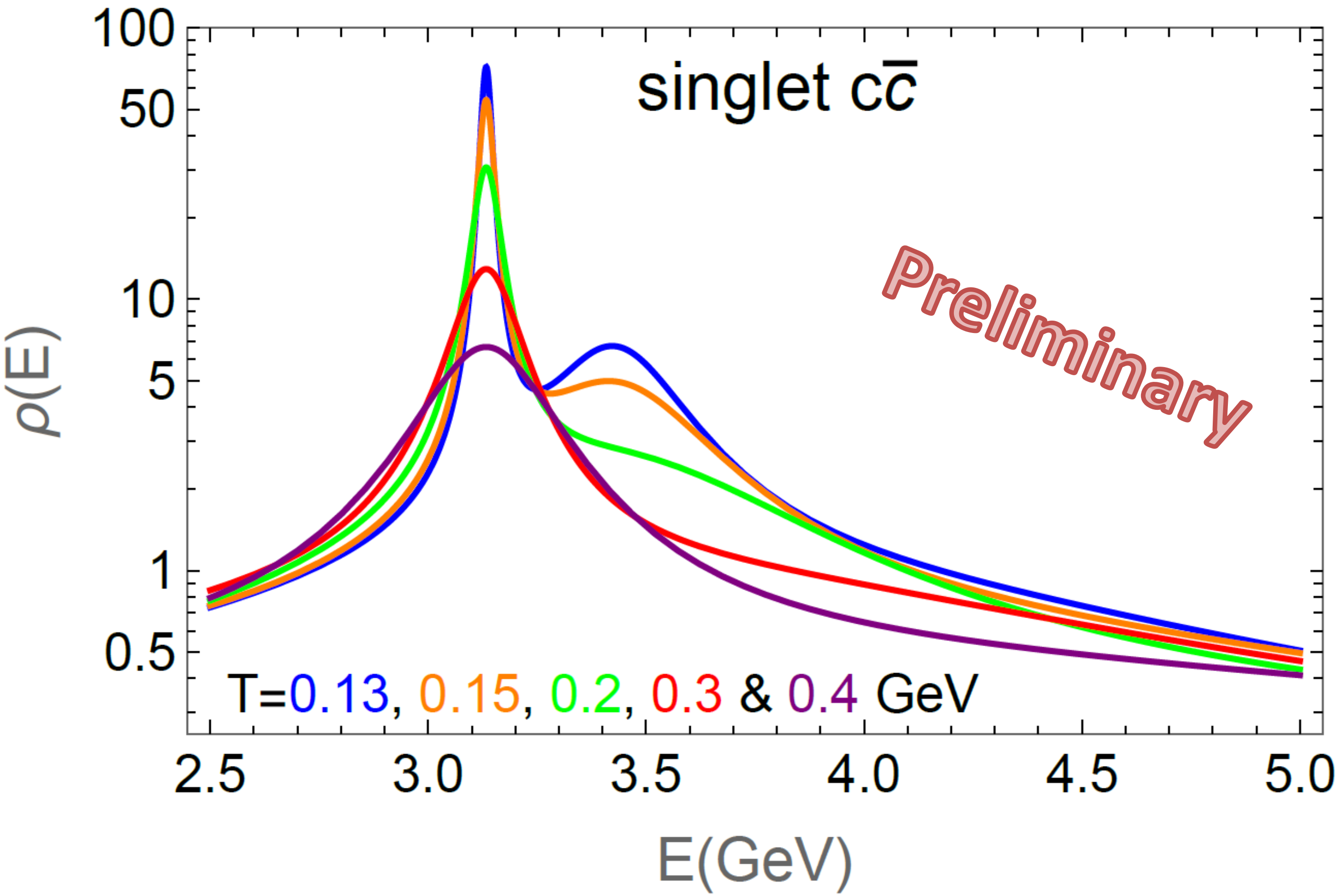
$$H_{\text{eff}} = H_Q + H_{\text{LS}} - i \sum_n \frac{\gamma_n}{2} L_n^\dagger L_n$$

Non Hermitic

Definition of the spectral density :

$$\rho(E) = \frac{1}{\pi} \Im \, \text{tr} \frac{1}{E - H_{\text{eff}} - i\epsilon}$$

At T=0, a sum of Dirac peaks



Main result: fast melting of the χ_c state, progressive melting of the J/ψ state beyond Tc…

… correct interpolation between the Quantum Optical regime at low T and the Quantum Brownian Motion regime at high T

Conclusions

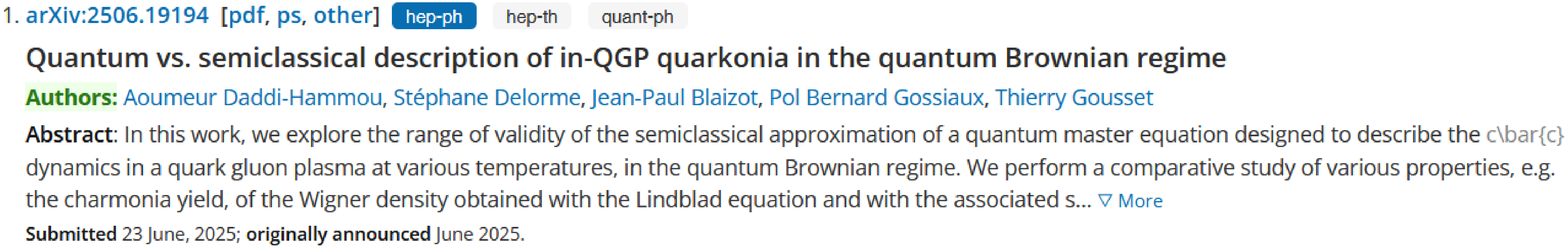
Goal: Describe the entire time evolution of in-QGP quarkonia...

...through a general Lindblad equation, encompassing both the high and low T regimes.

- The proper QME was identified
- First numerical applications show the « interpolating » features of such Universal Lindblad equation

Manuscript coming soon

- Further numerical studies of dynamical evolution on the way...
- Future work : establishing a semiclassical equation and checking its validity... see as well :



1. arXiv:2506.19194 [pdf, ps, other] hep-ph hep-th quant-ph

Quantum vs. semiclassical description of in-QGP quarkonia in the quantum Brownian regime

Authors: Aoumeur Daddi-Hammou, Stéphane Delorme, Jean-Paul Blaizot, Pol Bernard Gossiaux, Thierry Gousset

Abstract: In this work, we explore the range of validity of the semiclassical approximation of a quantum master equation designed to describe the c\bar{c} dynamics in a quark gluon plasma at various temperatures, in the quantum Brownian regime. We perform a comparative study of various properties, e.g. the charmonia yield, of the Wigner density obtained with the Lindblad equation and with the associated s... ▽ More

Submitted 23 June, 2025; **originally announced** June 2025.

Back up

The master equations of the two regimes

Having $H_{\text{int}} = \sum_i A^{(i)} \otimes B^{(i)}$, the quantum Brownian regime is described by the Caldeira-Leggett-Lindblad equation

$$\frac{\mathrm{d}\hat{\rho}_{Q\bar{Q}}(t)}{\mathrm{dt}} = -i\left[\hat{H}_{Q\bar{Q}} + \hat{H}_{\mathrm{LS}}, \hat{\rho}_{Q\bar{Q}}(t)\right] + \sum_{i,j} \gamma_{ij}(0)\left(\bar{A}^{(j)}\hat{\rho}_{Q\bar{Q}}(t)\bar{A}^{(i)\dagger} - \frac{1}{2}\left\{\bar{A}^{(i)\dagger}\bar{A}^{(j)}, \hat{\rho}_{Q\bar{Q}}(t)\right\}\right), \tag{7}$$

with

$$\bar{A}^{(i)} \equiv \hat{A}^{(i)} + \frac{i}{4T}\dot{\hat{A}}^{(i)}, \ \dot{\hat{A}}^{(i)} \equiv i\left[\tilde{H}_{Q\bar{Q}}, \hat{A}^{(i)}\right]. \tag{8}$$

The quantum optical regime is described e.g. by The Davies secular equation

$$\frac{\mathrm{d}\hat{\rho}_{Q\bar{Q}}(t)}{\mathrm{dt}} = -i\left[\hat{H}_{Q\bar{Q}} + \hat{H}_{\mathrm{LS}}, \hat{\rho}_{Q\bar{Q}}(t)\right] + \sum_{\omega}\sum_{i,j} \gamma_{ij}(\omega)\left(\hat{A}^{(j)}(\omega)\hat{\rho}_{Q\bar{Q}}(t)\hat{A}^{(i)\dagger}(\omega) - \frac{1}{2}\left\{\hat{A}^{(i)\dagger}(\omega)\hat{A}^{(j)}(\omega), \hat{\rho}_{Q\bar{Q}}(t)\right\}\right), \tag{9}$$

Two types of dynamical modelling

$m_D \ll E_{\text{bind}}$

Quantum Optical Regime

$m_D \sim E_{\text{bind}}$

$m_D \gg E_{\text{bind}}$

Quantum Brownian Motion

- **Well identified resonances**
- Time long enough wrt quantum decoherence time (once we reach this regime)

Good description with transport models (TAMU, Tsinghua, Duke)

Central quantities :
2->2 and 2->3 Cross sections,
decay rates

Equilibrium : exp(-E_n/T) (theorem)

SC Approx: rate equations

?

- Correlations growing with cooling QGP
- **Best described in position-momentum space**
- Time short wrt quantum decoherence time ?

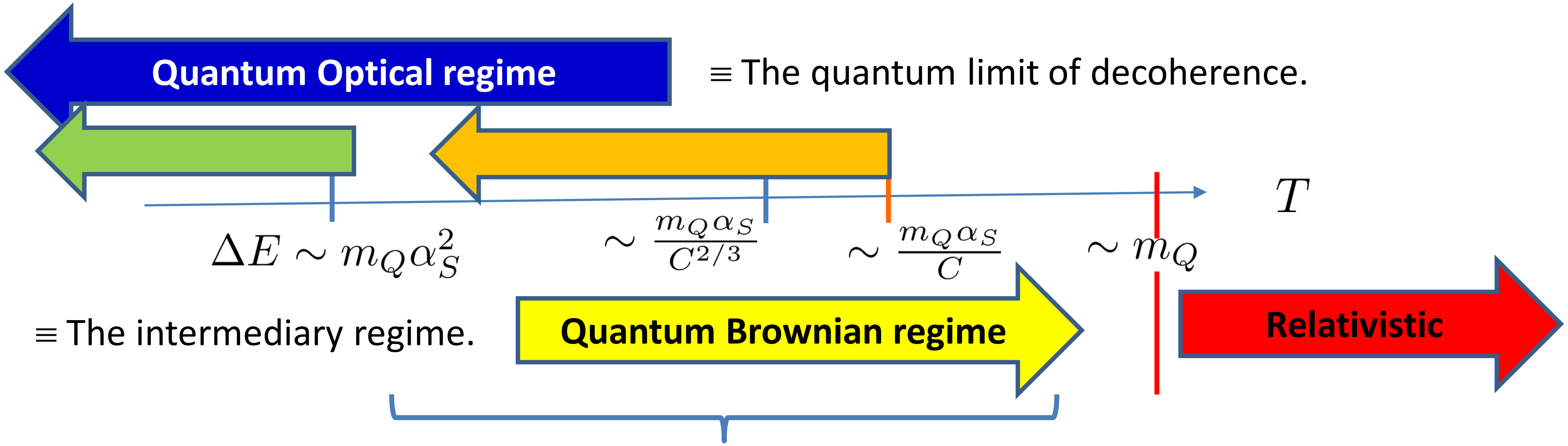
Quantum Master Equations for **microscopic dof (QS and Qbars)**

Equilibrium / asympt* : some limiting cases

SC Approx: Fokker-Planck equations
in position-momentum space

* Since one is facing both dissociation and recombination, obtaining a correct equilibrium limit of these models is an important prerequisite !!!

QCD Temperature scales



Quantum Optical regime ≡ The quantum limit of decoherence.

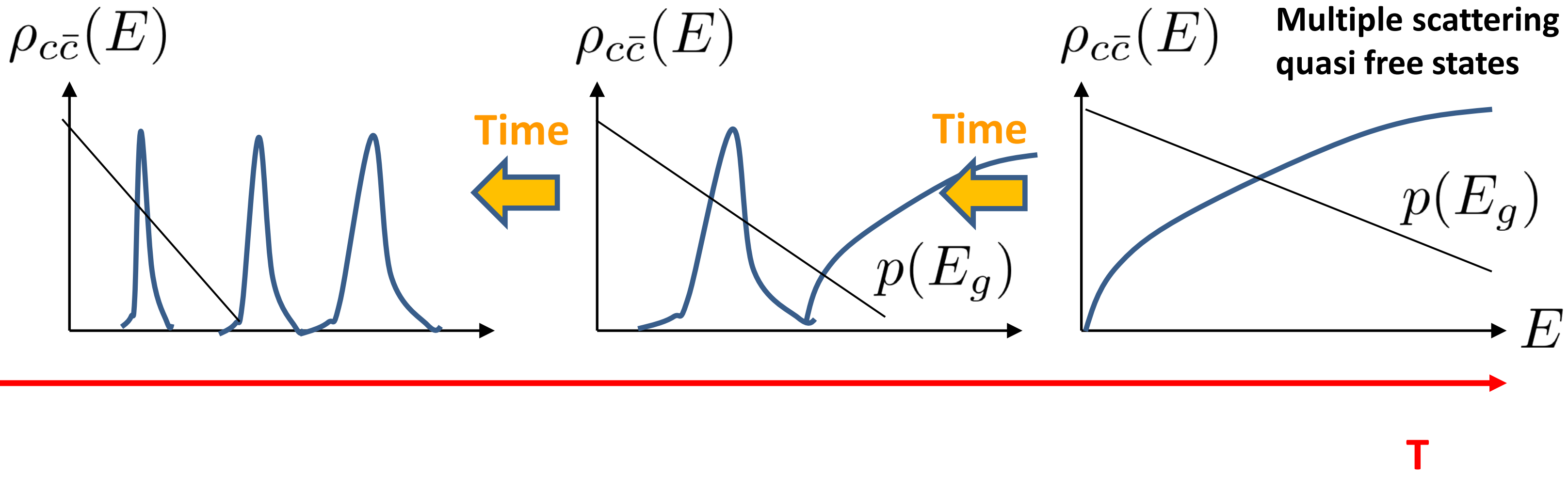
$\Delta E \sim m_Q \alpha_S^2$ $\sim \frac{m_Q \alpha_S}{C^{2/3}}$ $\sim \frac{m_Q \alpha_S}{C}$ $\sim m_Q$ T

≡ The intermediary regime. **Quantum Brownian regime**

Relativistic

For these « large » temperatures, the Q-Qbar gain enough energy to overwhelm the real binding potential => larger distance => larger decoherence ….

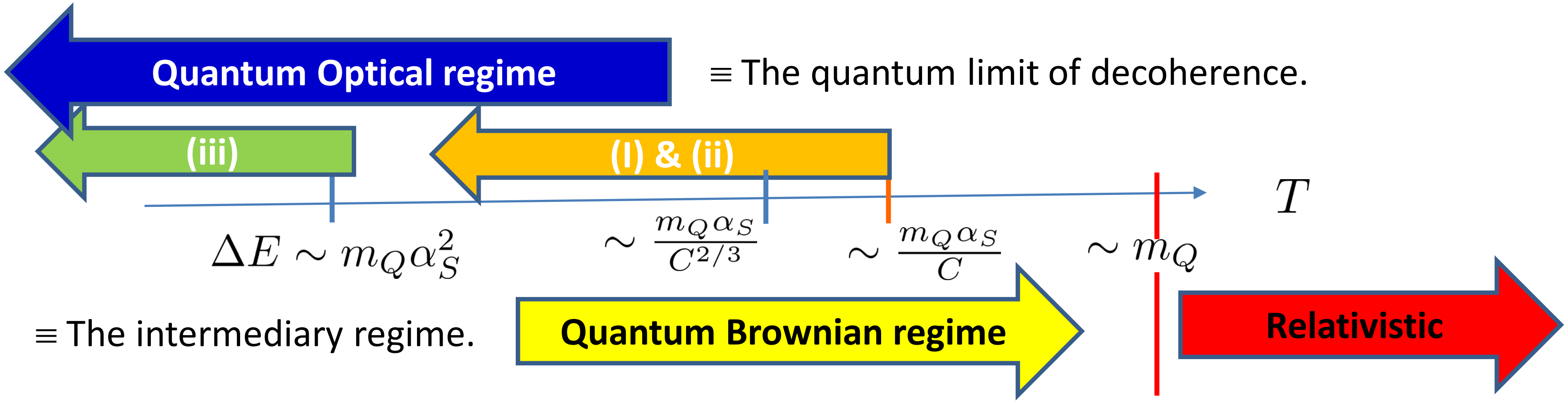
dissociation of well identified levels by scarce "high-energy" modes (dilute medium => cross section ok)



Multiple scattering on quasi free states

In // : continuous evolution of the $Q\bar{Q}$ spectral function

QCD Temperature scales



Quantum Optical regime ≡ The quantum limit of decoherence.

(iii)

(I) & (ii)

T

$\Delta E \sim m_Q \alpha_S^2$ $\sim \frac{m_Q \alpha_S}{C^{2/3}}$ $\sim \frac{m_Q \alpha_S}{C}$ $\sim m_Q$

≡ The intermediary regime. **Quantum Brownian regime**

Relativistic

Refined subregimes when playing with the scales of NRQCD / pNRQCD (series of recent papers by N. Brambilla, M.A. Escobdo, A. Vairo, M Strickland et al, Yao, Müller and Mehen,…)

NRQCD: $Mv, \Lambda_{\rm QCD}, T \ll \mu_{\rm NR} \ll M$: most general scheme for markovian OQS !

pNRQCD:

(Singlet and octet quarkonium fields)

- (i) $1/r \gg T \sim m_D \gg E$: « strongly coupled » QME same as small dipole limit of NRQCD (applies for small time evolution) : **See talk by Tom Magorsch on Monday**
- (ii) $1/r \gg T \gg E \gg m_D$: « weakly coupled » : g T << T : essential contribution is gluo – dissociation from hard mode T : does not apply in QCD
- (iii) $1/r \gg T \sim E \gg m_D$: Quantum optical regime

Quantum Brownian Motion : The Blaizot-Escobedo QME

Series expansion in τ_E/τ_S

Compact form: $\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} = \mathcal{L}\,\mathcal{D}_Q$ with $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \cdots$

$$\mathcal{L}_0\,\mathcal{D}_Q \equiv -i[H_Q, \mathcal{D}_Q],$$

$$\mathcal{L}_1\,\mathcal{D}_Q \equiv -\frac{i}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} V(\boldsymbol{x}-\boldsymbol{x}')[n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q],$$

Mean field hamiltonian

$$\mathcal{L}_2\,\mathcal{D}_Q \equiv \frac{1}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left(\{n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q\} - 2 n^a_{\boldsymbol{x}} \mathcal{D}_Q n^a_{\boldsymbol{x}'}\right),$$

Fluctuations, Linblad form

$$\mathcal{L}_3\,\mathcal{D}_Q \equiv \frac{i}{4T}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left([n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] + [n^a_{\boldsymbol{x}}, \mathcal{D}_Q \dot{n}^a_{\boldsymbol{x}'}]\right)$$

Dissipation

External "ingredients" : complex potential V + I W

N.B. : Friction is NOT of the Linbladian form => the evolution breaks positivity.

Positivity and Linblad form can be restored at the price **of extra subleading terms** :

$$\left\{\left(n^a_{\mathbf{x}} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\left(n^a_{\mathbf{x}'} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right), \mathcal{D}_{Q\bar{Q}}\right\} - 2\left(n^a_{\mathbf{x}} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\mathcal{D}_{Q\bar{Q}}\left(n^a_{\mathbf{x}'} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right)$$

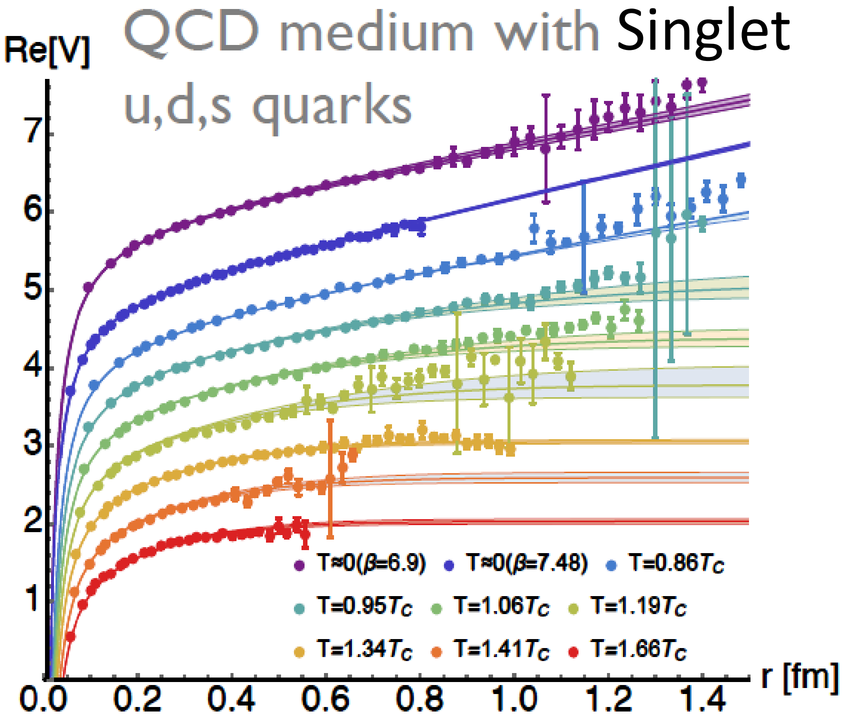
$\mathcal{L}_4$

EPS-HEP 2025

Application to QED-like and QCD for both cases of 1 body and 2 body densities…

Screening of the real potential

Potential (recent lQCD calculations)



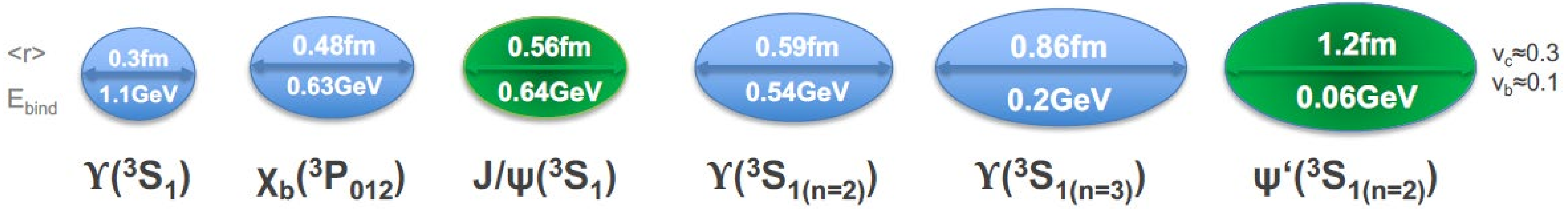
At T=0, well described by the Cornell shape:

$$V(r) = -\frac{\alpha}{r} + Kr$$

Quarkonia scales

- m_Q
- In vacuum: Binding energy / separation energy btwn levels: ΔE α m_Q g^4 (Coulomb part) => v α g^2
- Radius : (m_Q g^2) $^{-1}$
- For a linear potental $\hbar\omega_0 = \left(\frac{\hbar^2 K_l^2}{m_b/2}\right)^{\frac{1}{3}} \approx 0.504\ \text{GeV}$

$$v \propto \left(\frac{K_l}{m_b^2}\right)^{\frac{1}{3}}$$

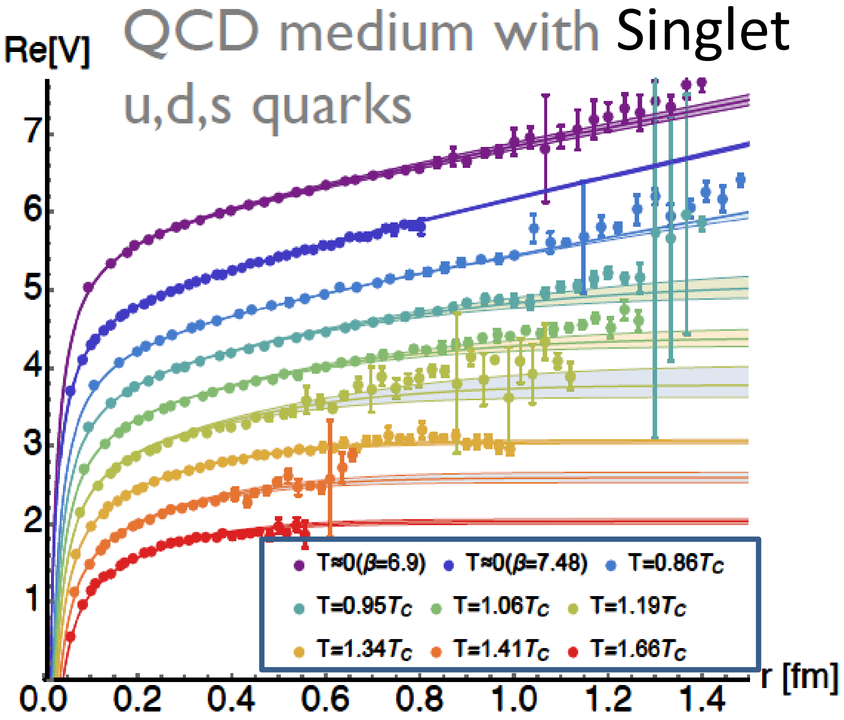


<r>	0.3fm	0.48fm	0.56fm	0.59fm	0.86fm	1.2fm
E_{bind}	1.1GeV	0.63GeV	0.64GeV	0.54GeV	0.2GeV	0.06GeV
	$\Upsilon(^3S_1)$	$\chi_b(^3P_{012})$	$J/\psi(^3S_1)$	$\Upsilon(^3S_{1(n=2)})$	$\Upsilon(^3S_{1(n=3)})$	$\psi'(^3S_{1(n=2)})$

$v_c \approx 0.3$
$v_b \approx 0.1$

Screening of the real potential

Potential (recent lQCD calculations)



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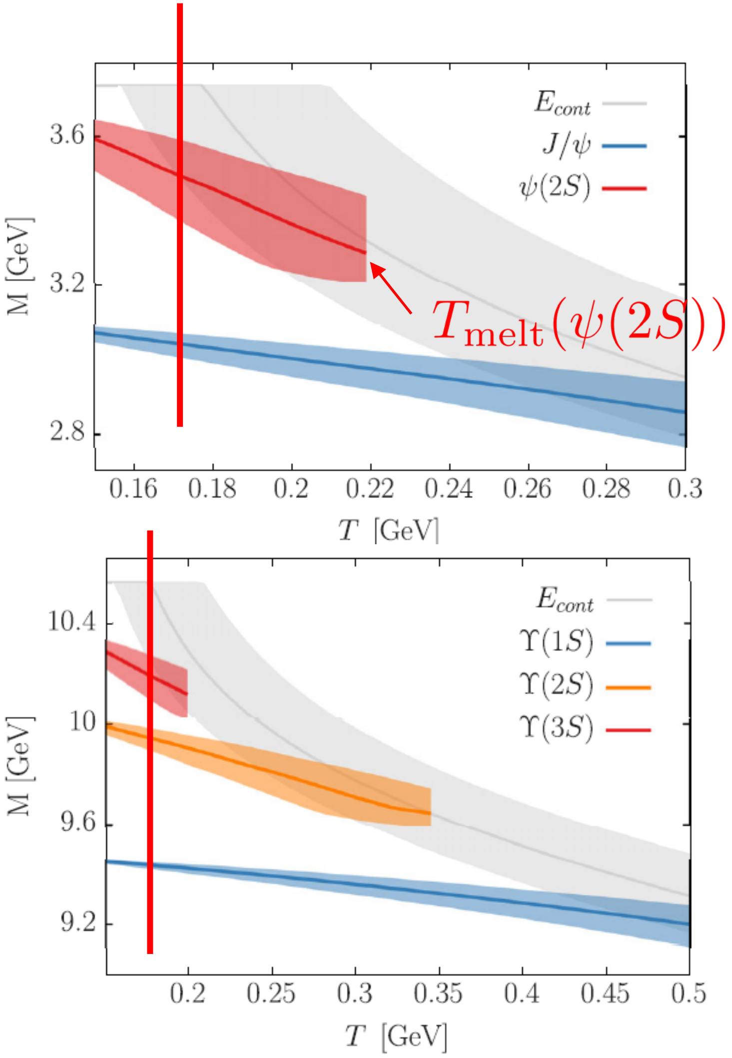
$$v \propto \left(\frac{K_l}{m_b^2}\right)^{\frac{1}{3}}$$

Compact and tightly bound states (at least for the lowest ones) => could survive QGP at low/mid T as well as to interactions with hadronic matter.

Screening of the real potential

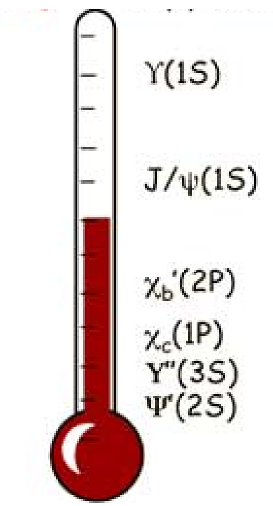
Recent In-medium spectrum (Lafferty and Rothkopf 2020)



« all or nothing scenario»:

- If $T_{\text{early QGP}} > T_{\text{melt}}$ => the state is not produced
- If $T_{\text{early QGP}} < T_{\text{melt}}$ => the state is produced like in pp

=> *SEQUENTIAL SUPPRESSION; Quarkonia as early QGP thermometer*



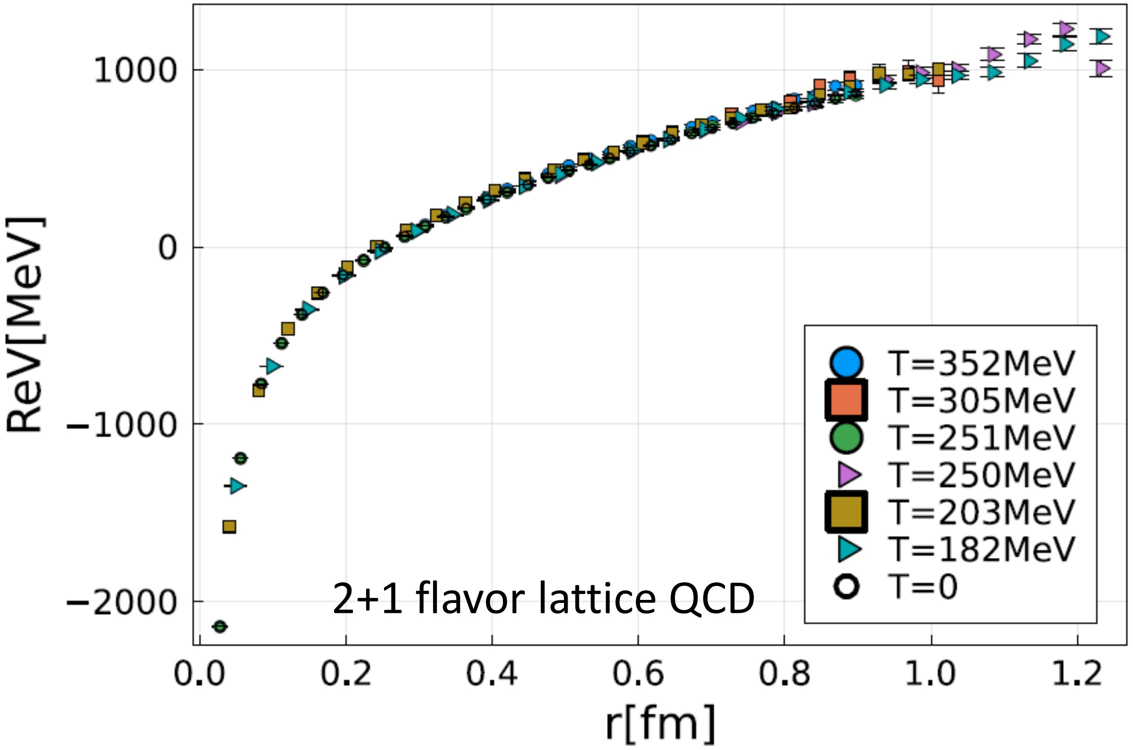
Most prominently : probing new state of matter in AA collision: Original idea by Matsui and Satz (86)…

… and advertized as a motivation in hundreds of talks (and papers) since then

David Lafferty, Alexander Rothkopf , Phys. Rev. D 101, 056010 (2020)

Screening of the real potential

Recent news : the real potential is not screened at temperatures reached in AA collisions !!!



Bazazov et al 2023 (Hot QCD collaboration)

How to define properly a “potential” on the lattice ?

Historically : thermodynamical potential like the free energy (in presence of a static dipole) or the total internal energy.

Modern approach : evaluate the Wilson loop and connect it to the r-dependent spectral density

$$W(\tau, r, T) = \int_{-\infty}^{+\infty} d\omega e^{-\omega\tau} \rho_r(\omega, T)$$

A “peak” contribution in the spectral density modelled as

$$\rho_r^{\rm peak}(\omega, T) = \frac{1}{\pi} {\rm Im} \frac{A_r(T)}{\omega - {\rm Re}V(r,T) - i\Gamma(\omega, r, T)}$$

=> Lattice data then unfolded with this Ansatz.

Does not seems quite intuitive, may not be the end of the story

Screening of the real potential

Recent news : the real pote ened at temperatures reached in AA collisions !!!

Prog. Part. Nucl. Phys., Vol. 30, pp. 405–406, 1993.
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0146–6410/93 $24.00
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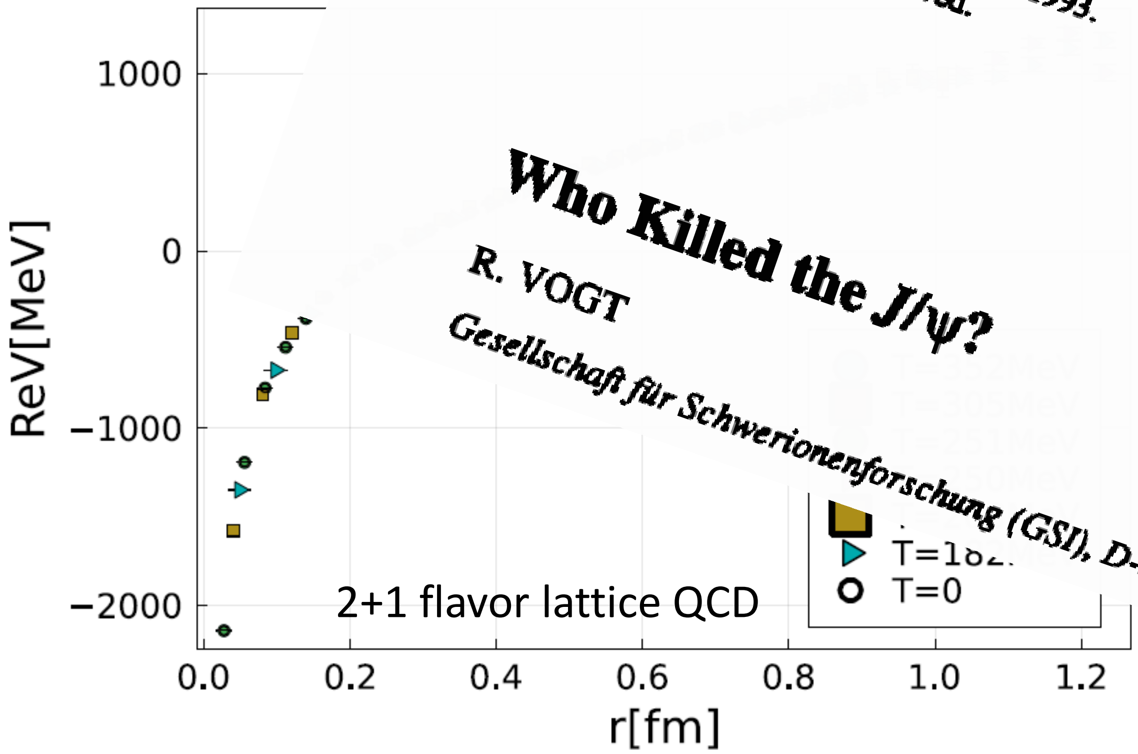
Who Killed the J/ψ?

R. VOGT

Gesellschaft für Schwerionenforschung (GSI), D-6100 Darmstadt 11, Germany



2+1 flavor lattice QCD

Bazazov et al 2023 (Hot QCD collaboration)

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A “peak” contribution in the spectral density modelled as

$$\rho_r^{\rm peak}(\omega, T) = \frac{1}{\pi}{\rm Im}\frac{A_r(T)}{\omega - {\rm Re}V(r,T) - i\Gamma(\omega, r, T)}$$

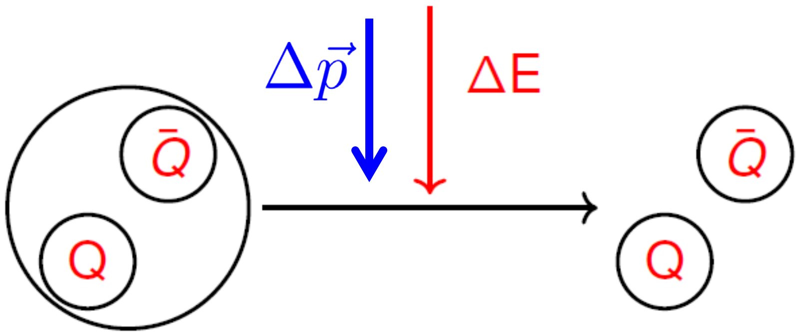
=> Lattice data then deconvoluted with this Ansatz.

Does not seems quite intuitive, may not be the end of the story

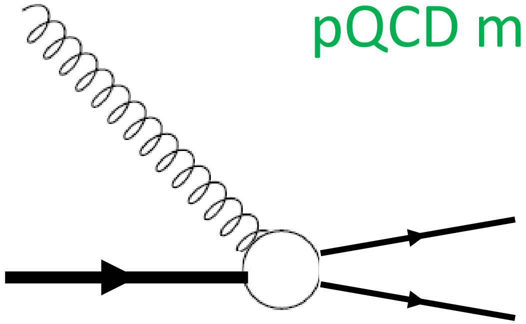
Collisions with the QGP

- Besides arguments based on the Debye mass / screening, it was pointed out already in the 90's that interactions with partons in the QGP could lead to dissociation of bound states (whose spectral function thus acquire some width Γ corresponding to the dissociation rate)

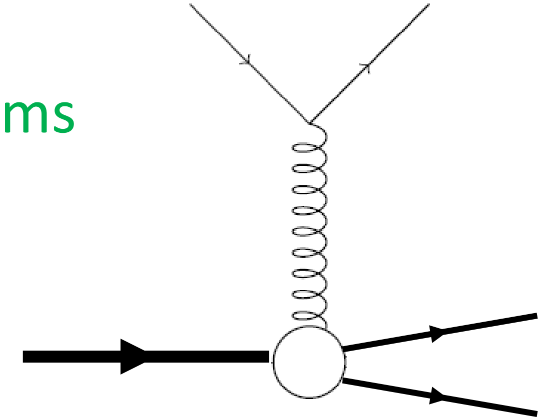
- Energy-momentum exchange with the QGP (gluo-dissociation, q – quarkonia quasi elastic scattering)



pQCD mechanisms



Gluo-dissociation



Inelastic scattering

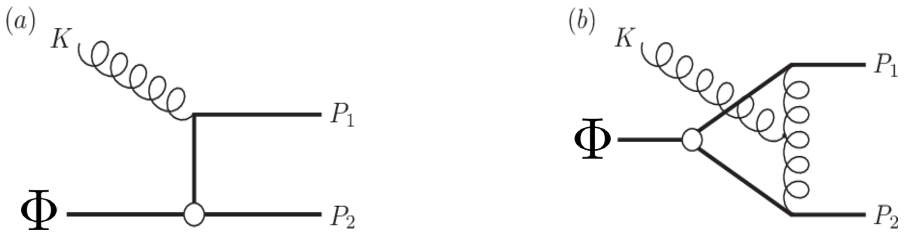
- => pair dissociation => **Suppression**

- ⬄ loss of probability of the quarkonia … Often described by some imaginary potential W in modern approaches

A central quantity: the decay rate Γ

Many approaches

pQCD view (Bhanot & Peskin), later on consolidated by NRQCD (Brambilla & Vairo)



$$\Phi + g \to Q + \bar{Q}$$

Dissociation cross section σ

$$\Gamma_\Phi(T) = \langle \sigma n_g \rangle_T$$

Other mechanisms : $x + \Phi \to x + Q + \bar{Q}$

QFT/Lattice QCD

Time correlator

$$\mathcal{C}_>(t,\vec{r}) \approx \langle \psi(t,\tfrac{\vec{r}}{2})\bar{\psi}(t,-\tfrac{\vec{r}}{2})\psi(0,0)\bar{\psi}(0,0)\rangle$$

Satisfies Schroedinger equation with complex potential V+iW . Breakthrough by Laine et al. (2006)

$$\Gamma_\Phi(T) = -2\langle \Phi | W | \Phi \rangle$$

Concept better suited at it genuinely encodes the “in medium” propagation

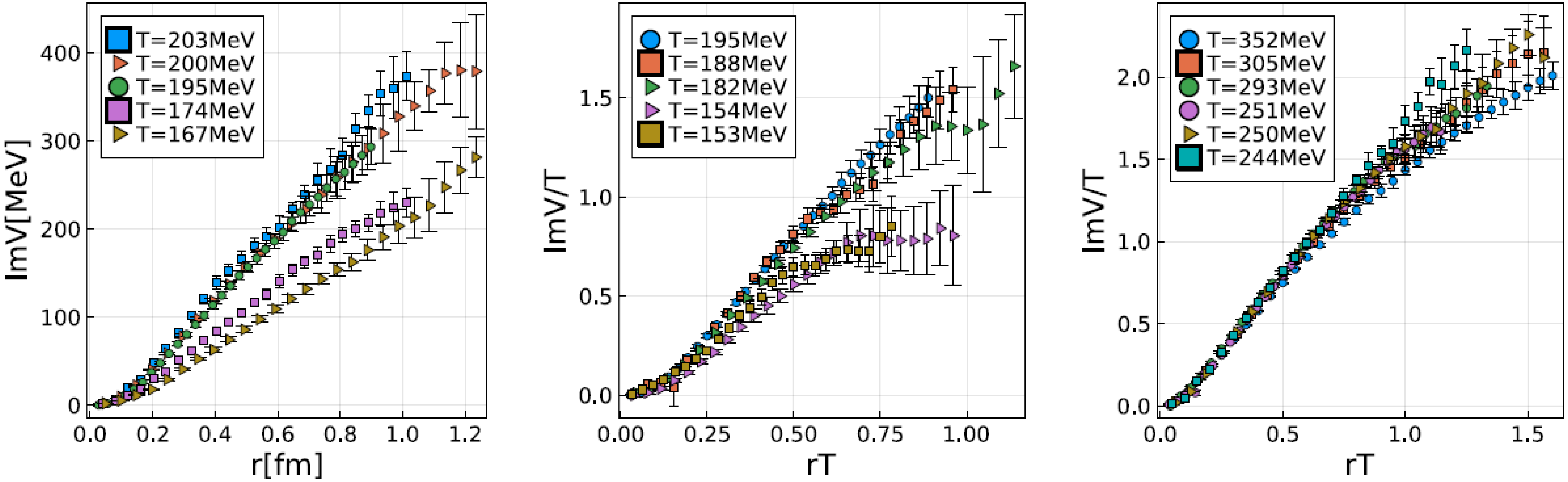
=> Simple decay law : Prob survival $= \exp\left(-\int_{t_0}^{t_{\rm fin}} \Gamma(T(t))dt\right)$

A central quantity: the decay rate Γ

Recent lQCD calculations of W(r) = Im(V(r)) (at ω=0)

$$\rho_r^{\mathrm{peak}}(\omega,T)=\frac{1}{\pi}\mathrm{Im}\frac{A_r(T)}{\omega-\mathrm{Re}V(r,T)-i\Gamma(\omega,r,T)}$$

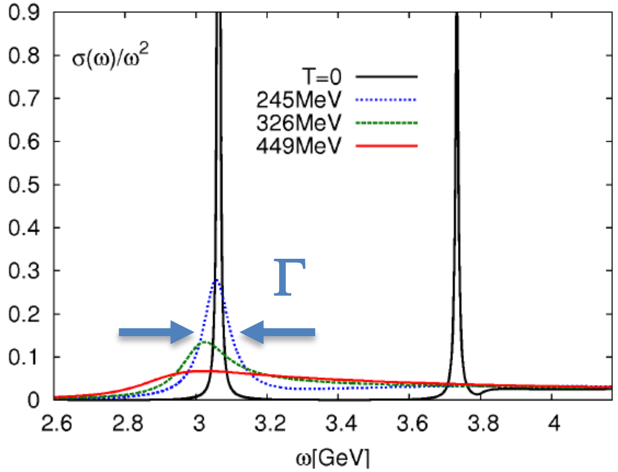
Bazazov et al 2023 (Hot QCD collaboration)



- Nice r T scaling
- Dipole structure at small r, no saturation seen at “large” r

Quarkonia at finite T

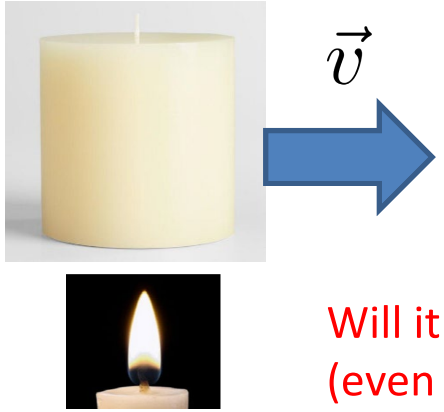
- Pheno: Yet, these pictures might still be compatible with the notion of sequential « suppression »…
- However, this notion has to be made more precise : (LQCD) spectral function lQCD



$$\rho(\omega,p,T)=\frac{1}{2\pi}\mathrm{Im}\int_{-\infty}^{\infty}dt e^{i\omega t}\int d^3x e^{ipx}\langle[J(x,t),J(0,0)]\rangle_T$$

At T=245 MeV, ψ' has disappeared but J/ψ still surviving for $\approx 1/\Gamma \approx$ a couple of fm/c … which needs to be compared with the local QGP cooling time τ_{cool} : Γ x τ_{cool} > 1 ⇔ suppressed

- N.B.: The opposite phenomenom might also be relevant: some state above the « melting » temperature can survive (for a short while < $1/\Gamma$) before getting lost definitively.



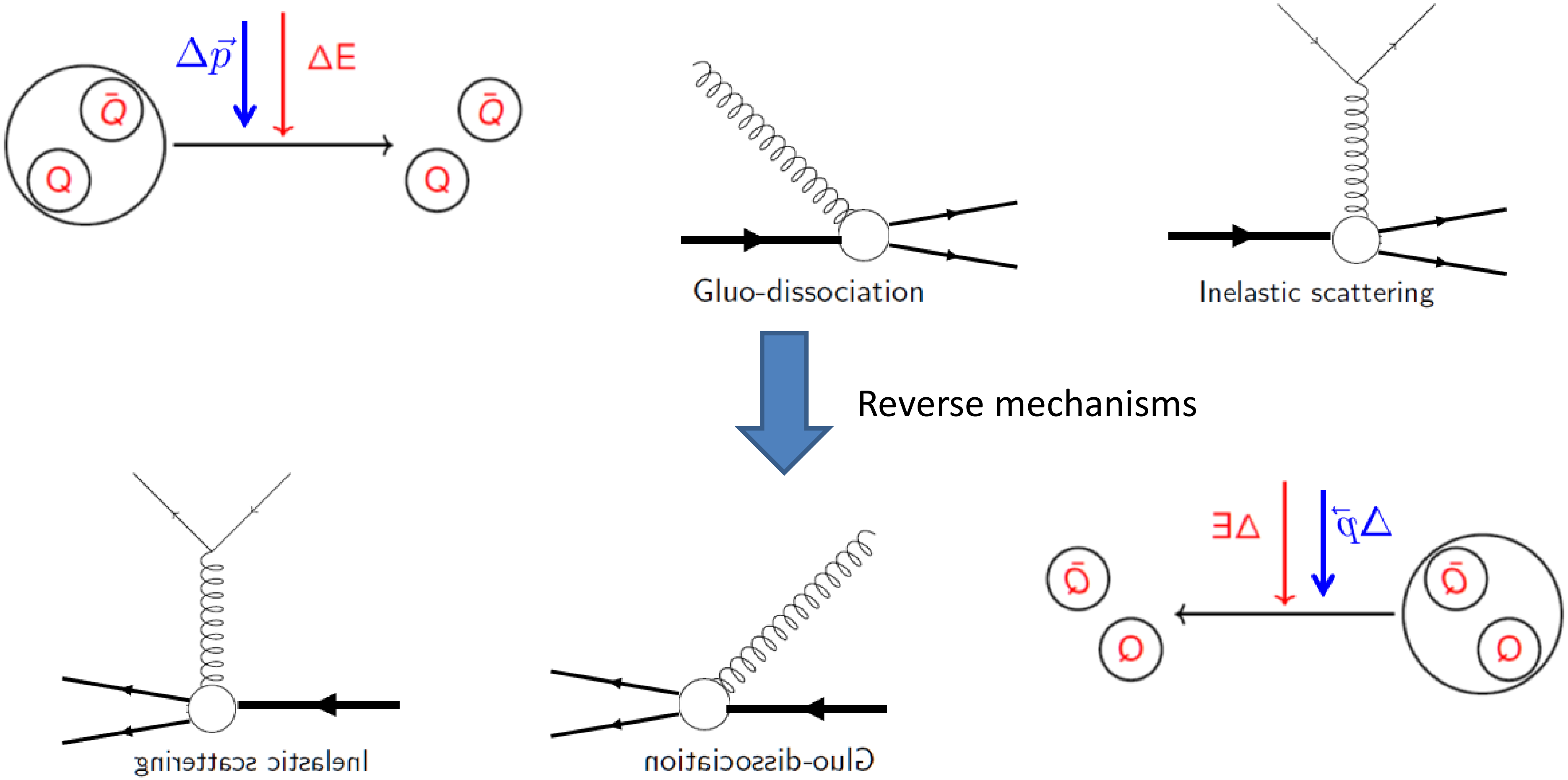
Will it melt
(even party) ?

- Key question : do the quarkonia states (chemically) equilibrate with the QGP ?

Modern era

Regeneration

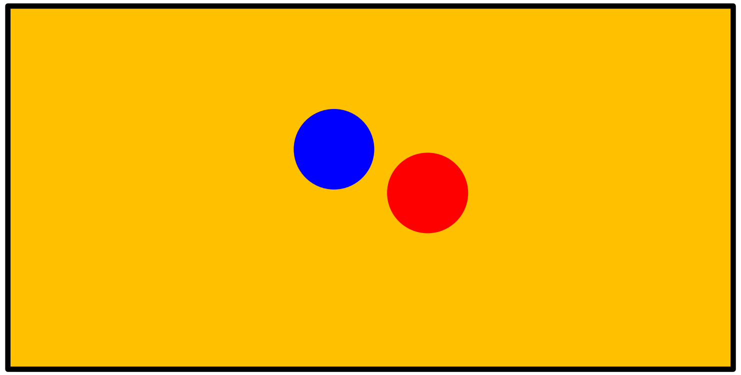
Detailed balance :



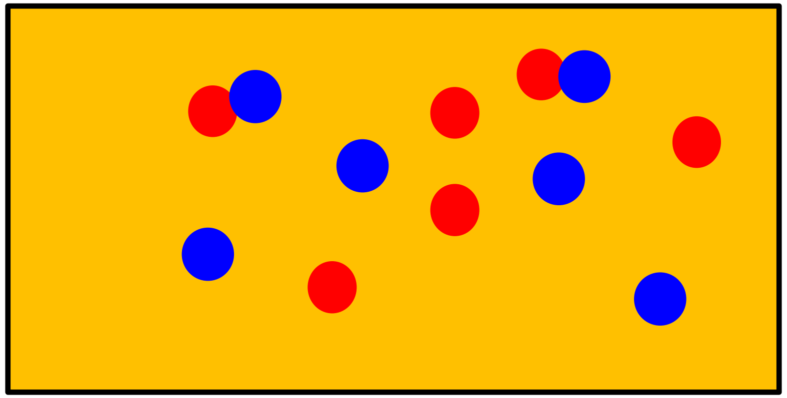
Reverse mechanisms

Regeneration: Dilute vs Dense

Bottomia

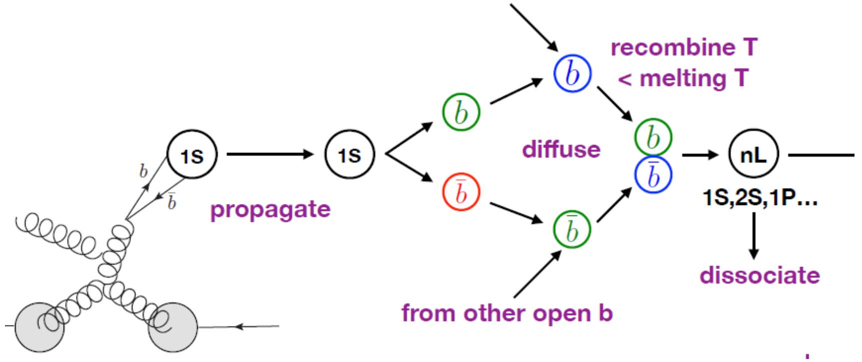


Charmonia



No exogenous recombination : only the b-bbar pairs which are initially close together will emerge as bottomia states

In some SC formalisms : intermediate regeneration



Yao, Mehen, Müller

Exogenous recombination : c & cbar initially far from each other may recombine and emerge as charmonia states

No full quantum treatment possible => need semi-classical approximation(s)

Key question : when does the recombination (dominantly) happen ? Crucial role of the binding force.

One extreme viewpoint : regeneration happens at the end of the QGP (Statistical Hadronization Model)