Factorization and virtuality evolution of jets in QCD medium

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Motivation:

Vacuum vs medium-induced evolution

The evolution of jets in heavy-ion collisions

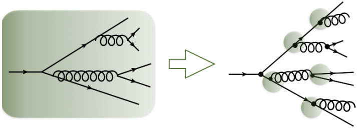
Disputes on how to incorporate vacuum and medium-induced radiation:

- Two different approaches:

Change in the jet evolution:

Medium-induced modifications can take place throughout the parton evolution

Medium-modifications at all momentum scales

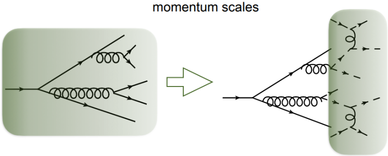


E.g: JETSCAPE, JEWEL, MATTER, Q-PYTHIA,...

Modifications on a developed shower

Vacuum (hard and collinear) parton structure unmodified

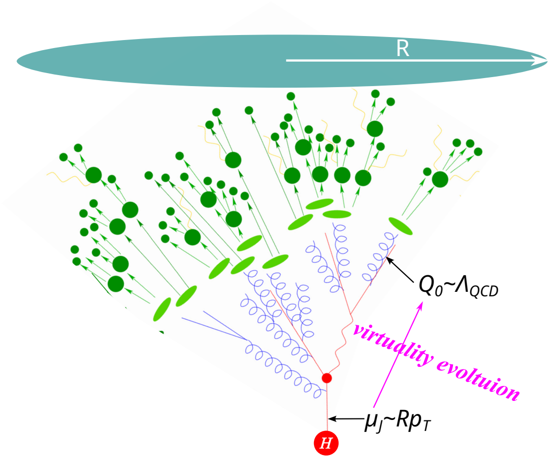
Medium-modifications dominate low momentum scales



E.g:(Co-)LBT, Hybrid, MARTINI, PyQUEN..

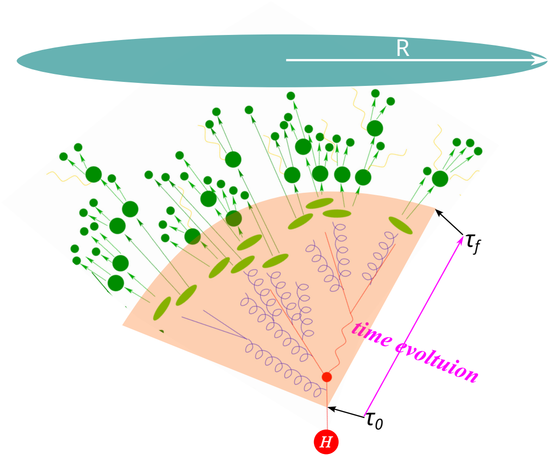
For a nice review: Liliana Apolinário's talk at HP2023.

Virtuality evolution of jets in vacuum



For a review: S. Marzani, G. Soyez and M. Spannowsky, vol. 958, Springer (2019), [1901.10342].

Time evolution of jets in medium



For an introduction: J. Casalderrey-Solana and C. A. Salgado, Acta Phys. Polon. B **38**, 3731-3794 (2007) [arXiv:0712.3443 [hep-ph]]..

Let us combine virtuality and time evolution of jets using QCD factorization!

Jet evolution in heavy-ion collisions

Our factorization approach:

1. Quantity to be factorized: the impact-parameter dependent cross section

   BW, JHEP 07 (2021) 002 [arXiv:2102.12916].

2. Factorization of initial states: cold nuclear effects

   N. Armesto, F. Cougoulic and BW, JHEP **11**, 081 (2024) [arXiv:2407.19243 [hep-ph]].

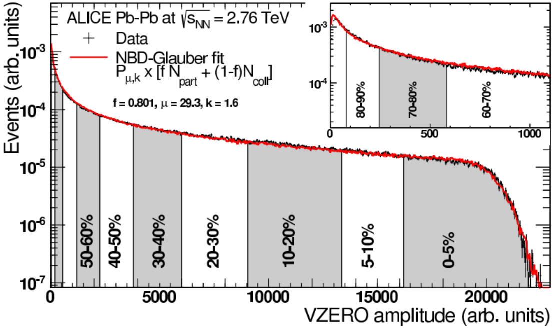
3. Factorization of final states: jet evolution in medium

   C. L. Rodriguez, C. Salgado and BW, work in progress (see PoS **ICHEP2024**, 626 (2025)).

Impact-parameter dependent cross sections

Centrality determination and collision geometry

In heavy-ion collisions, centrality c $\Rightarrow$impact parameter b:

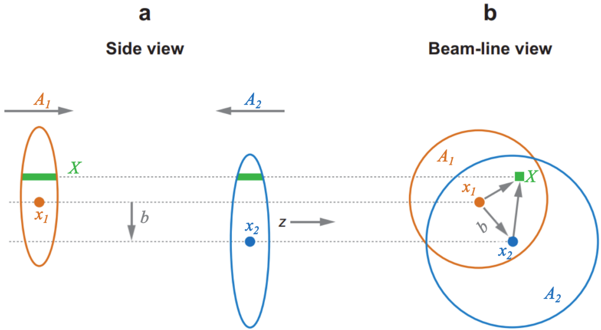


$$c \approx \underbrace{\frac{1}{\sigma}\int_{N_{ch}^{THR}}^{\infty}\frac{d\sigma}{dNch'}\,dNch' \approx \frac{1}{\sigma}\int_{0}^{E_{ZDC}^{THR}}\frac{\mathrm{d}\sigma}{\mathrm{d}E'_{ZDC}}\,\mathrm{d}E'_{ZDC}}_{\text{measurement}} \approx \underbrace{\frac{1}{\sigma}\int_{0}^{b}\frac{\mathrm{d}\sigma}{\mathrm{d}b'}\,\mathrm{d}b'}_{\text{theory}}$$

For example, ALICE, Phys. Rev. C **88**, no.4, 044909 (2013) [arXiv:1301.4361 [nucl-ex]].

The classical picture

The CLASSICAL picture of collision geometry in the Glauber model:

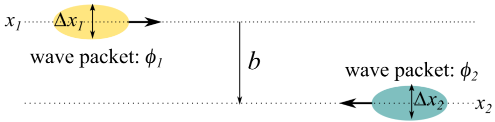


Miller, Reygers, Sanders and Steinberg, *Ann. Rev. Nucl. Part. Sci.* **57** (2007) 205 [arXiv:nucl-ex/0701025].

Topic 1: Impact-parameter dependent cross section in QFT

The quantum picture

In quantum theory, the probability for producing any observable O:



$$\frac{dP}{dO} \equiv \int \prod_f [d\Gamma_{p_f}]\,\delta(O - O(\{p_f\}))\langle\phi_1\phi_2|\hat{S}^\dagger|\{p_f\}\rangle\langle\{p_f\}|\hat{S}|\phi_1\phi_2\rangle$$

$$= \int \prod_f [d\Gamma_{p_f}]\,\delta(O - O(\{p_f\}))\mathrm{Tr}[\hat{S}^\dagger|\{p_f\}\rangle\langle\{p_f\}|\hat{S}|\phi_1\phi_2\rangle\langle\phi_1\phi_2|] \rightarrow \frac{d\sigma}{d^2\mathbf{b}dO}$$

where the wave packages of the colliding particles with $\mathbf{x}_i$ the transverse location of nucleus i:

$$|\phi_i\rangle = \int \frac{d^3\vec{p}}{(2\pi)^3}\frac{e^{-i\mathbf{p}\cdot\mathbf{x}_i}}{\sqrt{2E_p}}\underbrace{\phi_i(\vec{p})}_{\text{unknown}}|\vec{p}\rangle \qquad \int d\Gamma_p \equiv \int \frac{d^4p}{(2\pi)^4}(2\pi)\delta(p^2 - m^2)\theta(p^0).$$

Conditions for defining $\frac{d\sigma}{d^2\mathbf{b}dO}$

$\frac{dP}{dO} \to \frac{d\sigma}{d^2\mathbf{b}dO}$ defines cross section only if ϕ_i's can be integrated out!

ϕ_i's depend on how the beams are prepared and the cross section should be independent of it.

M.E. Peskin and D.V. Schroeder, Addison-Wesley, Reading, USA (1995).

For this, one needs to impose one more condition:

i) high energy: $|P_{iz}| \gg |\mathbf{P}_i|, \Delta p_T, \Delta p_z$;
ii) localization: $|\mathbf{b}| \gg \Delta x_T$.

It would be questionable if one extrapolates the CLASSICAL picture into low-energy or small collision systems.

BW, JHEP 07 (2021) 002 [arXiv:2102.12916].

Impact-parameter dependent cross sections

After integrating the two wave packets:

$$\frac{d\sigma}{d^2\mathbf{b}dO} = \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} \int \prod_f [d\Gamma_{p_f}]\,\delta(O - O(\{p_f\}))$$
$$\times \frac{1}{2s} M(p_1, p_2 \to \{p_f\}) M^*(\bar{p}_1, \bar{p}_1 \to \{p_f\}) (2\pi)^4 \delta^{(4)}(p_1 + p_2 - \sum p_f)$$

where

$$p_1^\mu = p_1^+ \frac{n_1^\mu}{2} + \frac{q_T^\mu}{2} - \frac{q_T^2}{4p_1^+}\frac{\bar{n}_1^\mu}{2}, \qquad p_2^\mu = p_2^- \frac{n_2^\mu}{2} - \frac{q_T^\mu}{2} - \frac{q_T^2}{4p_2^-}\frac{\bar{n}_2^\mu}{2},$$
$$\bar{p}_2^\mu = p_1^+ \frac{n_1^\mu}{2} - \frac{q_T^\mu}{2} - \frac{q_T^2}{4p_1^+}\frac{\bar{n}_1^\mu}{2}, \qquad \bar{p}_2^\mu = p_2^- \frac{n_2^\mu}{2} + \frac{q_T^\mu}{2} - \frac{q_T^2}{4p_2^-}\frac{\bar{n}_2^\mu}{2}.$$

with $n_{1,2} = (1, 0, 0, \pm 1)$ and $\bar{n}_{1,2} = (1, 0, 0, \mp 1)$.

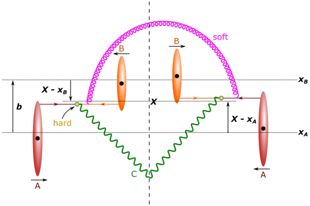
BW, JHEP 07 (2021) 002 [arXiv:2102.12916].

Factorization of initial states

1. At leading order in $1/Q$ (leading twist)

The factorization formula using SCET

For the Drell-Yan process: BW, JHEP 07 (2021) 002 [arXiv:2102.12916].



$$\frac{d\sigma}{d^2\mathbf{b}\,dy_C\,d^2\mathbf{p}_C} = \frac{1}{4\pi s}\sum_{j,k}\int d^2\mathbf{X}\int d^2\mathbf{x}\,e^{i\mathbf{p}_C\cdot\mathbf{x}}\int_0^1 \frac{dz_A}{z_A}\frac{dz_B}{z_B}$$

$$\times \int \prod_f [d\Gamma_{p_f}] \prod_{i=A,B} \delta\Big(z_i\bar{n}_i\cdot P_i - \bar{n}_i\cdot p_C - \sum \bar{n}_i\cdot p_f\Big)$$

$$\times \underbrace{\mathcal{T}_{j/A}(\mathbf{X}, z_A, \mathbf{x})\,\mathcal{T}_{k/B}(\mathbf{X}-\mathbf{b}, z_B, \mathbf{x})}_{\text{Thickness beam functions}}$$

$$\times \underbrace{H^{\bar{a}_A\bar{a}_B}_{a_Aa_B}(z_AP_A, z_BP_B \to p_C, \{p_f\})}_{\text{hard function}}\underbrace{\mathcal{S}^{\bar{a}_A\bar{a}_B}_{a_Aa_B}(\mathbf{x})}_{\text{soft function}}.$$

Thickness beam functions

Definition:

$$\mathcal{T}_{q/i}(\mathbf{r}, z, \mathbf{x}) = \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{r}} \int \frac{dt}{2\pi} e^{-izt\bar{n}\cdot P} \times \left\langle \bar{n}\cdot P, -\frac{\mathbf{q}}{2} \right| \bar{\chi}_n\left(\frac{t\bar{n}}{2} + \frac{x_T}{2}\right) \frac{\bar{n}}{2}\chi_n\left(-\frac{t\bar{n}}{2} - \frac{x_T}{2}\right) \left| \bar{n}\cdot P, \frac{\mathbf{q}}{2} \right\rangle$$

$$\mathcal{T}_{g/i}(\mathbf{r}, z, \mathbf{x}) = z\bar{n}\cdot P(-g_{T\,\alpha'\alpha}) \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{r}} \int \frac{dt}{2\pi} e^{-izt\bar{n}\cdot P} \times \left\langle \bar{n}\cdot P, -\frac{\mathbf{q}}{2} \right| \mathcal{B}^{a\alpha'}_{nT}\left(\frac{t\bar{n}}{2} + \frac{x_T}{2}\right) \mathcal{B}^{a\alpha}_{nT}\left(-\frac{t\bar{n}}{2} - \frac{x_T}{2}\right) \left| \bar{n}\cdot P, \frac{\mathbf{q}}{2} \right\rangle .$$

where

$$\text{For } q/\bar{q} : \chi_{n_i}(x) \equiv W^\dagger_{n_i}(x)\frac{n_i\bar{n}_i}{4}\psi_{n_i}(x), \qquad \bar{\chi}_{n_i}(x),$$

$$\text{For } g : \mathcal{B}^\mu_{n_i T} = \frac{1}{g_s} W^\dagger_{n_i}(x) iD^\mu_{n_i T} W_{n_i}(x)$$

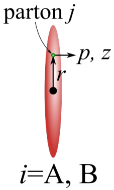
with $D^\mu_{n_i T} \equiv \partial^\mu_T - ig_s A^\mu_{n_i T}$ and W_{n_i} the n_i-collinear Wilson line.

Thickness beam functions

Physical interpretation:

Fourier transform of transverse phase space (TPS) PDF with respect to $\mathbf{x}$

$$f_{j/i}(\mathbf{r}, z, \mathbf{p}) =$$



$$= \int d^2\mathbf{x} e^{i\mathbf{p}\cdot\mathbf{x}} \mathcal{T}_{j/i}(\mathbf{r}, z, \mathbf{x}).$$

TMD PDF at r! (up to gauge link prescription etc)

In the limit $\mathbf{x} \to 0$: $\mathcal{T}_{j/i}(\mathbf{r}, z, \mathbf{0})$ admits the interpretation as the PDF at $\mathbf{r}$.

Given experimental uncertainties, we shall use the Glauber model below.

The Glauber modelling of heavy nuclei

Neglecting correlations between nucleons, for the right-moving A_1:

$$|\phi_1\rangle\langle\phi_1| = \prod_{i=1}^{A_1} \int \frac{dP_i^+ d^2\mathbf{P}_i}{(2\pi)^3 2P_i^+} \frac{1}{2} \int db_i^- d^2\mathbf{b}_i W_{A_1}(P_i, b_i)$$
$$\times \int \frac{dq_i^+ d^2\mathbf{q}_i}{(2\pi)^3} e^{\frac{i}{2} q_i^+ b_i^- - i\mathbf{q}_i\cdot\mathbf{b}_i} |P_i + q_i/2\rangle\langle P_i - q_i/2|,$$

where the Wigner distribution function for one nucleon is defined as

$$W_{A_1}(P, b) \equiv \int \frac{dq^+ d^2\mathbf{q}}{(2\pi)^3 2P^+} e^{-\frac{i}{2} q^+ b^- + i\mathbf{q}\cdot\mathbf{b}} \langle P + q/2|\phi_1\rangle\langle\phi_1|P - q/2\rangle.$$

Replacing W_{A_1} with the following form:

$$W_{A_1}(p, b) = \hat{\rho}_{A_1}(b^-, \mathbf{b}) 2(2\pi)^3 \delta(p^+ - P_1^+)\delta^{(2)}(\mathbf{p}),$$

where P_1^+ denotes the "+" momentum of the nucleon, and $\hat{\rho}_{A_1} = \rho_{A_1}/A_1$ with $\rho_{A_1}(b^-, \mathbf{b})$ the distribution of nucleons (say a Woods-Saxon functional form).

Kovchegov and Sievert, Phys. Rev. D **89**, no.5, 054035 (2014) [arXiv:1310.5028 [hep-ph]];

BW and Kovchegov, JHEP **03**, 158 (2018)[arXiv:1709.02866 [hep-ph]].

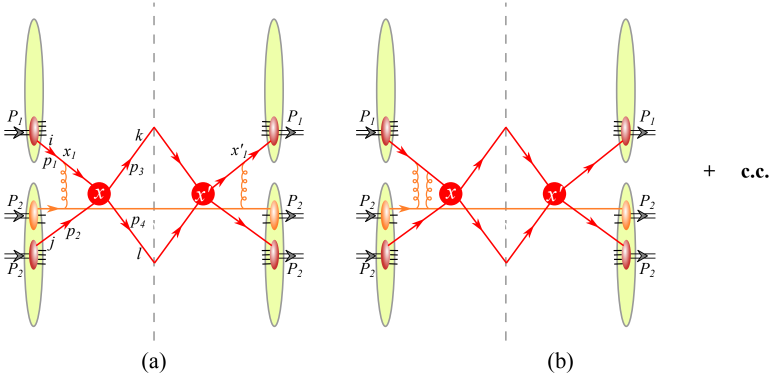
We deal with soft (semi-hard) and hard prcesses at the same footing!

Factorization of initial states

2. Beyond leading twist

Initial-state single scattering

All the single-scattering diagrams



plus those with nuclei 1 and 2 swapped. Diagrams with the Glauber gluon connecting two partons collinear to the same direction vanish in eikonal limit.

$$\frac{d\sigma_{A_1A_2\to l^+l^-}}{d^2\mathbf{b}d\eta_3 d\eta_4 dp_T d^2\mathbf{Q}_T} = 2\int d^4X \rho_{A_1}(X^-,\mathbf{X})\rho_{A_2}(X^+,\mathbf{X}-\mathbf{b})\sum_{ij}\frac{d\sigma^{(0)}_{ij\to l^+l^-}}{d\eta_3 d\eta_4 dp_T}$$
$$\times\int\frac{d^2\mathbf{x}}{(2\pi)^2}e^{-i\mathbf{x}\cdot\mathbf{Q}_T}\left[1-\frac{|\mathbf{x}|^2}{4}\int_{-\infty}^{X^+}dX_1^+\hat{q}_{i/A_2}(X_1^+,\mathbf{X}-\mathbf{b},|\mathbf{x}|)\right.$$
$$\left.-\frac{|\mathbf{x}|^2}{4}\int_{-\infty}^{X^-}dX_1^-\hat{q}_{j/A_1}(X_1^-,\mathbf{X},|\mathbf{x}|)\right].$$

More about $\hat{q}$: the jet quenching parameter

$$\hat{q}_{j/A_i}(X^\pm,\mathbf{X},|\mathbf{x}|) = \frac{4\pi^2\alpha_s C_j}{N_c^2-1}\rho_{A_i}(X^\pm,\mathbf{X})xG(x,1/|\mathbf{x}|^2).$$

The same as that in parton saturation physics: *Baier, Dokshitzer, Mueller, Peigne and Schiff, Nucl. Phys. B* ***484****, 265 (1997).*

Here, xG is the TMD PDF at this order in the limit $x\to 0$

$$xG(x,1/|\mathbf{x}|^2) = \frac{\alpha_s C_F}{\pi}\ln\left(\frac{1}{\mu^2|\mathbf{x}|^2}\right)\int d\xi f_q(\xi,\mu).$$

Resumming multiple scattering

Factorized multiple scattering due to the fact that diagrams with the Glauber gluon connecting two partons collinear to the same direction vanish.

In heavy nuclei, the summation of multiple scattering results can be substituted by the exponentiation of single scattering results, namely:

$$\frac{d\sigma_{A_1A_2\to l^+l^-}}{d^2\mathbf{b}d\eta_3 d\eta_4 dp_T d^2\mathbf{Q}_T} = 2\int d^4X \rho_{A_1}(X^-,\mathbf{X})\rho_{A_2}(X^+,\mathbf{X}-\mathbf{b})\sum_{ij}\frac{d\sigma^{(0)}_{ij\to l^+l^-}}{d\eta_3 d\eta_4 dp_T}$$
$$\times\int\frac{d^2\mathbf{x}}{(2\pi)^2}e^{-i\mathbf{x}\cdot\mathbf{Q}_T-\frac{|\mathbf{x}|^2}{4}\int_{-\infty}^{X^+}dX_1^+\hat{q}_{i/A_2}(X_1^+,\mathbf{X}-\mathbf{b},|\mathbf{x}|)}$$
$$\times\, e^{-\frac{|\mathbf{x}|^2}{4}\int_{-\infty}^{X^-}dX_1^-\hat{q}_{j/A_1}(X_1^-,\mathbf{X},|\mathbf{x}|)}.$$

where $\mathbf{Q}_T = \mathbf{p}_{3T} + \mathbf{p}_{4T}$.

Could be measured via azimuthal decorrelation of the l^+l^- pair.

N. Armesto, F. Cougoulic and BW, JHEP **11**, 081 (2024) [arXiv:2407.19243 [hep-ph]].

Recall parton saturation in DIS

$$\frac{dN}{d^2bd^2Q_T} = \int\frac{d^2x_\perp}{(2\pi)^2}e^{-iQ_T\cdot x_\perp}\rho_0 x q_N(x)\int_0^L dz\, e^{-\frac{1}{4}\hat{q}x_\perp^2 z}$$

Kovchegov and Mueller, Nucl. Phys. B **529**, 451 (1998).

Factorization of jet cross sections

(Take $AA \to \gamma$+jet as an example)

The factorization formula

For a jet initially emerges along the direction:

$$n^\mu = (1, \vec{n}) = (1, \cos\phi_I / \cosh\eta_I, \sin\phi_I / \cosh\eta_I, \tanh\eta_I),$$

where η_I and ϕ_I denote the initial pseudorapidity and azimuthal angle.

Up to $O(\alpha_s)$, we obtain:

$$\frac{d\sigma}{d^2\mathbf{b}dy_J d^2\mathbf{p}_J dm_J^2 d\eta_\gamma d^2\mathbf{p}_\gamma} = \int d^2\mathbf{X}\ T_{A_1}(\mathbf{X}) T_{A_2}(\mathbf{X}-\mathbf{b})$$
$$\times \int d\eta_I d^2\mathbf{p}_I dm_I^2 \sum_{ijk} \frac{d}{dm_I^2} \mathcal{J}_k(p_I; p_J)$$
$$\times \int d\xi d\xi' f_i(\xi) f_j(\xi') \frac{d\hat{\sigma}_{ij\to\gamma k}}{d\eta_I d^2\mathbf{p}_I d\eta_\gamma d^2\mathbf{p}_\gamma}(P_1, P_2 \to p_\gamma, \hat{p}_I),$$

where $\hat{\sigma}_{ij\to\gamma k}$ denotes the cross section for the partonic process: $ij \to \gamma k$ and $\mathcal{J}_k$ represents the jet function with final-state jet momentum p_J. Here, the interactions between the jet and QCD medium are counted as $O(1)$.

C. L. Rodriguez, C. Salgado and BW, work in progress (see PoS **ICHEP2024**, 626 (2025)).

The jet function

The jet function: in lightcone gauge $\bar{n} \cdot A = 0$ with $\bar{n}^\mu = (1, -\vec{n})$, we have

$$\frac{d}{dm_l^2} \mathcal{J}_k(p_l; p_J) P^{\alpha\beta}(p_l) = \int \frac{d^4x}{2(2\pi)^4} e^{ip_l \cdot x} \mathcal{J}_k^{\alpha\beta}(X_T + x/2, X_T - x/2; p_J).$$

where in terms of the parton field Φ_α^c with c and α color and spinor/lorentz indices, respectively,

$$\begin{aligned} &\mathcal{J}_k^{\alpha\beta}(X_T + x/2, X_T - x/2; p_J) \\ &\equiv \frac{1}{2d_C} \prod_i \int d\Gamma_{p_i} \langle\langle\langle 0|\bar{T}\Phi_\alpha^{c\dagger}(x_0')|\{p_i\}\rangle\langle\{p_i\}|T\Phi_\beta^c(x_0)|0\rangle\rangle\rangle \delta^{(4)}(p_J - \sum_i p_i) \end{aligned}$$

and

$$P^{\alpha\beta}(p_l) = p_l^+ \frac{n_{\alpha\beta}}{2} \ \text{ for } q/\bar{q} \qquad \text{and} \qquad P^{\alpha\beta}(p_l) = -g_\perp^{\mu\nu} \ \text{ for } g.$$

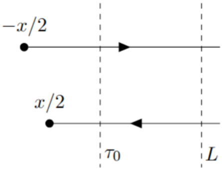
The medium average: $\langle\langle\langle 0| \cdots 0\rangle\rangle\rangle$ denotes $\langle\langle A_1 - 1, A_2 - 1| \otimes \langle 0| \cdots 0 \otimes |A_1 - 1, A_2 - 1\rangle\rangle$. That is, all the other nucleons create background QCD matter coupling to the jet.

The jet function at LO in BDMPS-Z formalism

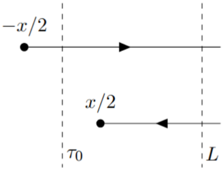
For simplicity, we carry out a fixed-order calculatoin using the BDMPS-Z formalism for the medium average.

Baier, Dokshitzer, Mueller, Peigne and Schiff, Nucl. Phys. B **483**, 291-320 (1997) doi:10.1016/S0550-3213(96)00553-6 [arXiv:hep-ph/9607355 [hep-ph]]; Nucl. Phys. B **484**, 265-282 (1997) doi:10.1016/S0550-3213(96)00581-0 [arXiv:hep-ph/9608322 [hep-ph]]; Zakharov, JETP Lett. **63**, 952-957 (1996) doi:10.1134/1.567126 [arXiv:hep-ph/9607440 [hep-ph]].

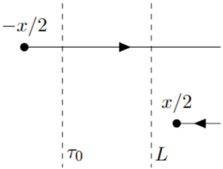
At LO, one has for QCD matter existing between τ_0 and L:



(a) $2\tau_0 > x^+ > -2\tau_0$



(b) $2L > |x^+| > 2\tau_0$



(c) $|x^+| > 2L$

Virtuality evolution in medium at LO

Let us calculate:

$$\frac{d\mathcal{S}_k}{dm_l^2}(\underline{x}_f) \equiv \int d^2\underline{p}_J e^{-i\underline{x}_f\cdot\underline{p}_J} \frac{d\mathcal{J}_k}{dm_l^2 d^2\underline{p}_l}.$$

where $\underline{x}_f$ represents the final transverse size of the dipole with $p^- = m_l^2/p^+$.
Note that in harmonic oscillator approximation, one has

$$S_k(t_2, t_1, \underline{r}_1) = e^{-\frac{1}{4}\hat{q}_k|\underline{x}_1|^2(t_2-t_1)}$$

with $\hat{q}_k$ the jet quenching parameter for color representation of k.

In vacuum: one has

$$\frac{d\mathcal{J}_k}{dm_l^2 d^2\underline{p}_l} = \delta(m_l^2)\delta^{(2)}(\underline{p}_f) \Leftrightarrow \frac{d\mathcal{S}_k}{dm_l^2}(\underline{x}_f) = \delta(m_l^2)$$

with the choice of the coordinates $-\underline{p}_l = 0$.

In medium: we have

$$\frac{d\mathcal{J}_k}{dm_l^2 d^2\underline{p}_l} = (a) + (b) + (c).$$

Virtuality evolution in medium at LO

In details, we have

$$(a) = \frac{1}{\pi m_I^2} \sin\left(\frac{\tau_0 m_I^2}{p_J^+}\right) S_2(L, \tau_0; \underline{x}_f) = \frac{1}{\pi m_I^2} \sin(\frac{\tau_0 m_I^2}{p_J^+}) e^{-\frac{1}{4}\hat{q}_R |\underline{x}_f|^2 (L-\tau_0)},$$

$$(b) = 2\text{Re} \int_{2\tau_0}^{2L} \frac{dx^+}{2p_J^+(2\pi)} e^{\frac{im_I^2}{2p_J^+} x^+} S_2(L, x^+/2, \underline{x}_f) e^{-\frac{1}{2\lambda_R}(\frac{x^+}{2}-\tau_0)},$$

$$(c) = e^{-\frac{1}{2}\frac{1}{\lambda_R}(L-\tau_0)} 2\text{Re} \int_{L^+}^{+\infty} \frac{dx^+}{2p_J^+(2\pi)} e^{\frac{im_I^2}{2p_J^+} x^+}$$

$$= e^{-\frac{1}{2}\frac{1}{\lambda_R}(L-\tau_0)} \left[\delta(m_I^2) - \frac{1}{\pi m_I^2} \sin(\frac{L m_I^2}{p_J^+})\right].$$

That is, interactions between the jet and medium broaden the vacuum distribution $\delta(m_I^2)$! Note it could be negative at given m_I^2!

Summary and Perspective

1. We define, and investigate the factorization of, the impact-parameter dependent cross section for heavy-ion collisions.
2. Using Glauber modelling of heavy nuclei, we present a formalism to deal with both hard and soft (semi-hard) processes.
3. In initial-state factorization, we discuss the cold nuclear effects, which could be observable in azimuthal decoorelation in the Drell-Yan prcess ($AA \to l^+ l^-$).
4. In final-state factorization, we define a jet function includes both virtuality and time evoltuion.
5. Efforts under way to elucidate the inerplay between virtuality and time evolution in parton cascade.

Backup slides

Why do we need the two conditions?

$$\frac{dP}{dO} = (\phi_A, \phi_B \to \{p_f\}) = \int d^2\mathbf{X}_A d^2\mathcal{P}_A d^2\mathbf{X}_B d^2\mathcal{P}_B \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot(\mathbf{b}+\mathbf{X}_B-\mathbf{X}_A)}$$
$$\times W_A(\mathbf{X}_A, \mathcal{P}_A) W_B(\mathbf{X}_B, \mathcal{P}_B) \sigma(p_A, p_B \to \{p_f\} \leftarrow \bar{p}_A, \bar{p}_B)$$

where the transverse Wigner functions are given by

$$W_i(\mathbf{X}, \mathcal{P}) \equiv \int \frac{d^2\chi}{(2\pi)^2} \int dz e^{i\mathcal{P}\cdot\chi} \tilde{\phi}_i \left(\mathbf{X} - \frac{\chi}{2}, z\right) \tilde{\phi}_i^* \left(\mathbf{X} + \frac{\chi}{2}, z\right),$$

$$\sigma(p_A, p_B \to \{p_f\} \leftarrow \bar{p}_A, \bar{p}_B) \equiv \frac{(2\pi)^4 \delta^{(4)}(p_A + p_B - \sum p_f)}{\sqrt{2E_{p_A} 2E_{\bar{p}_A} 2E_{p_B} 2E_{\bar{p}_B}} |\bar{v}_{Az} - \bar{v}_{Bz}|}$$
$$\times M(p_A, p_B \to \{p_f\}) M^*(\bar{p}_A, \bar{p}_B \to \{p_f\}).$$

The two conditions means $|\mathbf{X}_i| \ll b$ and $|\mathcal{P}_i|$ can be neglected in $\sigma(p_A, p_B \to \{p_f\} \leftarrow \bar{p}_A, \bar{p}_B)$ and, accordingly, the Wigner functions (wave packets) can be integrated out.

Soft and hard functions

The soft function:

$$\mathcal{S}^{\bar{a}_A\bar{a}_B}_{a_A a_B}(\mathbf{x}) = \langle 0|\,\bar{T}[S^{\dagger a'_B\bar{a}_B}_{n_B}(\mathbf{x})S^{\dagger a'_A\bar{a}_A}_{n_A}(\mathbf{x})]\,T[S^{a_A a'_A}_{n_A}(0)S^{a_B a'_B}_{n_B}(0)]|0\rangle$$

The hard function:

$$H^{\bar{a}_A\bar{a}_B}_{a_A a_B} \equiv \frac{P^{\bar{\alpha}_A\alpha_A}}{d_{c_A}}\frac{P^{\bar{\alpha}_B\alpha_B}}{d_{c_B}}\tilde{\mathcal{C}}^{*\bar{a}_A\bar{a}_B}_{\bar{\alpha}_A\bar{\alpha}_B}\tilde{\mathcal{C}}^{a_A a_B}_{\alpha_A\alpha_B}$$

with $\tilde{\mathcal{C}}$ given by

$$\tilde{\mathcal{C}}(\epsilon, z_A\bar{n}_A\cdot P_A, z_B\bar{n}_B\cdot P_B) = \int dt_A dt_B e^{i(t_A z_A\bar{n}_A\cdot P_A + t_B z_B\bar{n}_B\cdot P_B)}\mathcal{C}(\epsilon, t_A, t_B).$$

The same as for pp collisions!

The Glauber model

Nuclei are modelled as uncorrelated nucleons:

the nucleons are distributed according to the charge distribution

$$\rho_A(r)/\rho_0 = \frac{1}{1+e^{\frac{r-R_{ws}}{a}}} \qquad \text{with } R_{ws} = 6.62 \text{ fm and } a = 0.55 \text{ fm for } ^{208}\text{Pb}.$$

where ρ_0 is the density at the center.

C. W. De Jager, H. De Vries and C. De Vries, Atom. Data Nucl. Data Tabl. **14** (1974), 479-508.

The thickness function

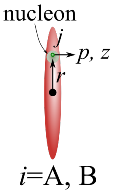
$$T_i(\mathbf{r}_i) \equiv \int dz \rho_i(\mathbf{r}_i, z).$$

Miller, Reygers, Sanders and Steinberg, *Ann. Rev. Nucl. Part. Sci.* **57** (2007) 205 [arXiv:nucl-ex/0701025].

Thickness beam functions in the Glauber model

In terms of the thickness functions

$\mathcal{T}_{j/i}(\mathbf{r}_i, z, \mathbf{x}) \to$



$$= T_i(\mathbf{r}_i)\left[\frac{Z_i}{Z_i + N_i} B_{j/p}(z, \mathbf{x}) + \frac{N_i}{Z_i + N_i} B_{j/n}(z, \mathbf{x})\right]$$

$=$ the thickness functions $\times$ the beam functions

where $B_{j/p}$ and $B_{j/n}$ are the beam functions for protons and neutrons, respectively, and the nucleus is assumed to contain Z_i protons and N_i neutrons.

The factorization formula gives

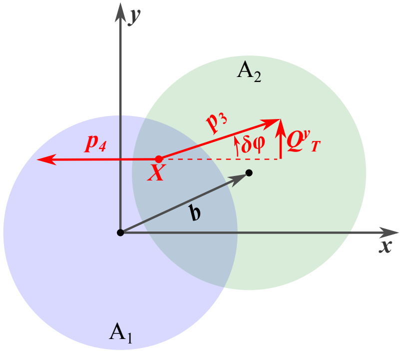
$$R_{AA} \equiv \frac{\frac{d\sigma_{AB}}{d^2\mathbf{b}\, dy_C\, d^2\mathbf{p}_C}}{T_{AB}(\mathbf{b})\frac{d\sigma_{nn}}{dy_C\, d^2\mathbf{p}_C}} = 1,$$

with $T_{AB}(\mathbf{b}) \equiv \int d^2\mathbf{X}\, T_A(\mathbf{X})\, T_B(\mathbf{X} - \mathbf{b})$.

Azimuthal decorrelation in AA

The focus of this talk: the first step toward verifying factorization in AA when both radiation and multiple scattering are include.

The observable: $Q \gg |Q_T^y| \gg \Lambda_{QCD}$



$$\frac{1}{p_T}\frac{d\sigma_{A_1A_2}}{d^2\mathbf{b}d\eta_3 d\eta_4 dp_T d\delta\varphi} \approx \frac{d\sigma_{A_1A_2}}{d^2\mathbf{b}d\eta_3 d\eta_4 dp_T dQ_T^y}$$

with $\delta\varphi = \arcsin(Q_T^y/|\mathbf{p}_3|)$, $\mathbf{Q}_T \equiv \mathbf{p}_3 + \mathbf{p}_4$, $Q = p_T \equiv |\mathbf{p}_4| \approx |\mathbf{p}_3|$.

The formula for perturbative calculations

After first factoring the hadronic cross section into the convolution of nucleon PDFs and the partonic cross section, we have

$$\frac{d\sigma}{d^2\mathbf{b}dO} = \int \prod_f [d\Gamma_{p_f}]\, \delta(O - O(\{p_f\}))$$

$$\times \sum_{\{a_i, b_j\}} \left(\prod_{i=1}^{A_1} \frac{1}{P_1^+} \int \frac{d\xi_i}{\xi_i} f_{a_i}(\xi_i) \right) \left(\prod_{j=1}^{A_2} \frac{1}{P_2^-} \int \frac{d\xi_j'}{\xi_j'} f_{b_j}(\xi_j') \right)$$

$$\times \langle \{\xi_i P_1\}, \{\xi_j' P_2\} | \hat{S}^\dagger | \{p_f\} \rangle \langle \{p_f\} | \hat{S} | \{\xi_i P_1\}, \{\xi_j' P_2\} \rangle$$

$$\otimes \left(\prod_{i=1}^{A_1} \hat{\rho}_{A_1}(X_i^-, \mathbf{X}_i) \right) \left(\prod_{j=1}^{A_2} \hat{\rho}_{A_2}(Y_j^+, \mathbf{Y}_j - \mathbf{b}) \right),$$

where a_i and b_j iterate over all the parton species, and the operator $\otimes$ indicates that the incoming partons i and j enter the diagrams in the amplitude (the conjugate amplitude) at x_i (x_i') and y_j (y_j') respectively with $X_i = (x_i + x_i')/2$ and $Y_j = (y_j + y_j')/2$. Here, the initial- and final-state spins and colors are respectively averaged and summed over in the square of the partonic S-matrix element.