Beautiful Majorana Higgses at Colliders

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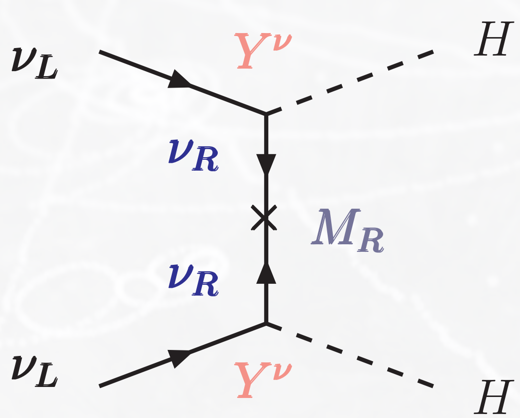
Based on 2403.07756 and 2503.21354 with B. Fuks, F. Nesti and M. Nemevšek

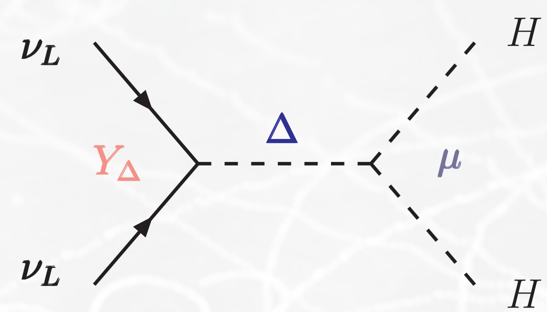
Making Majorana neutrinos

Effective mass term $\mathscr{L}_{\text{eff}} \sim \frac{m_{LL}}{2}\,\bar{\nu}_L\,\nu_L^C$ from Weinberg operator: $\mathscr{L}^{d=5} \sim \frac{h_{ij}}{2\Lambda}(H\,L_i\,H\,L_j)$



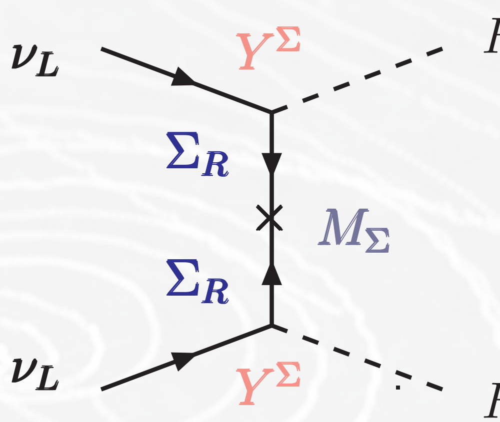
Type I (fermion singlet)

(Minkowski ’77)



Type II (scalar triplet)

(e.g. Schechter & Valle ’80)



Type III (fermion triplet)

(e.g. Foot et al. ’89)

Mass terms: $m_\nu^I \sim -\,v^2 Y_\nu^T \frac{1}{M_R} Y_\nu$, $\quad m_\nu^{II} \sim -\,v^2 Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} \sim -\,Y_\Delta v_\Delta$, $\quad m_\nu^{III} \sim -\,Y_\Sigma^T \frac{v^2}{2M_\Sigma} Y_\Sigma$

Countless more possibilities with higher odd-dimensional operators or loop-level realisations… ♪

(Actually they are countable, see e.g. [John Gargalionis and Ray Volkas: 2009.13537]

Making neutrino masses

Effective … $I\,L_j)$

Different … $)(L_j^T \sigma_a H)$

T …)

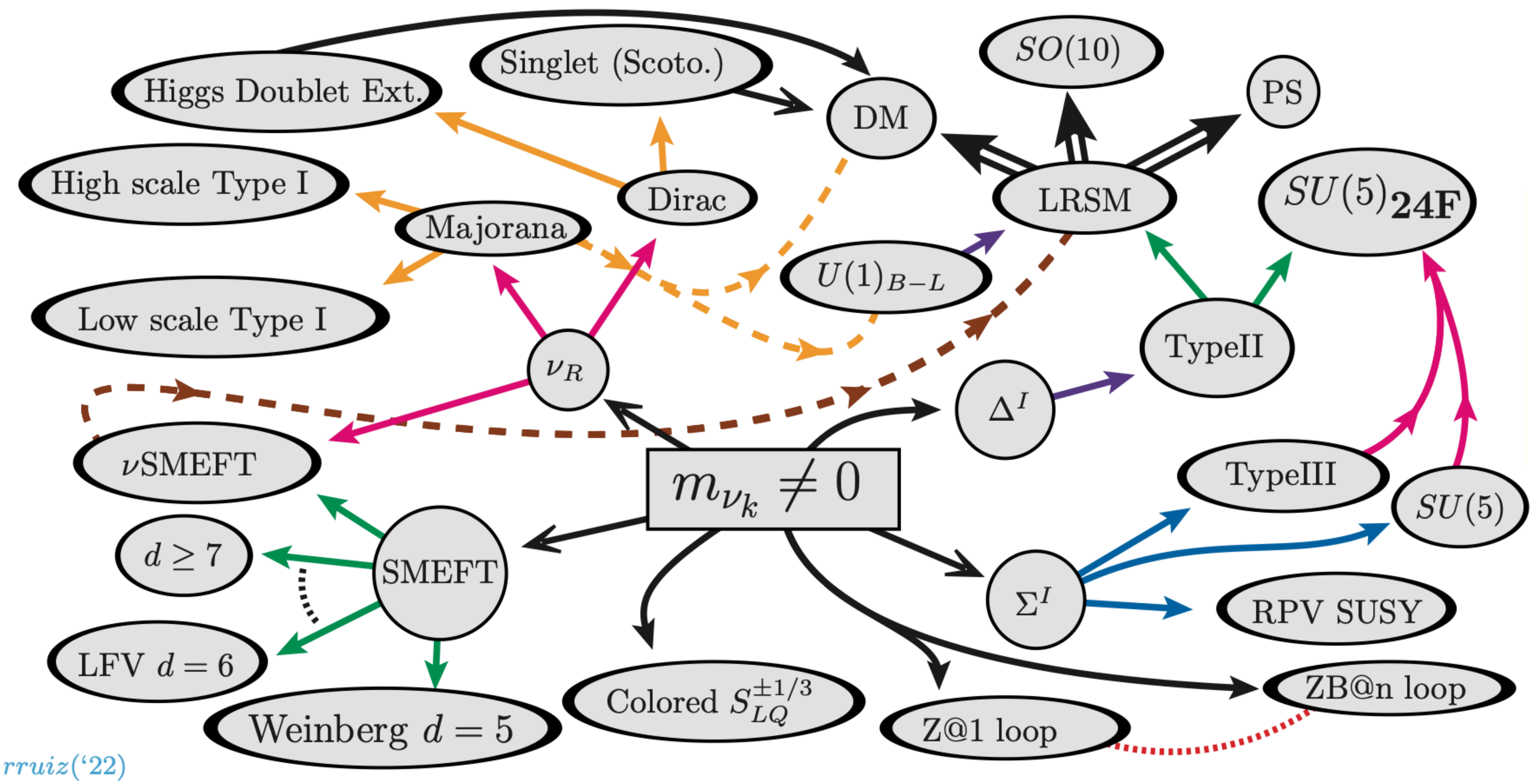
Mass terr…

Countless … tions…

09.13537]

These core ideas can be realized in ***many*** ways!

Minkowski ('77); Yanagida ('79); Glashow & Levy ('80); Gell-Mann et al., ('80); Mohapatra & Senjanović ('82); + *many* others

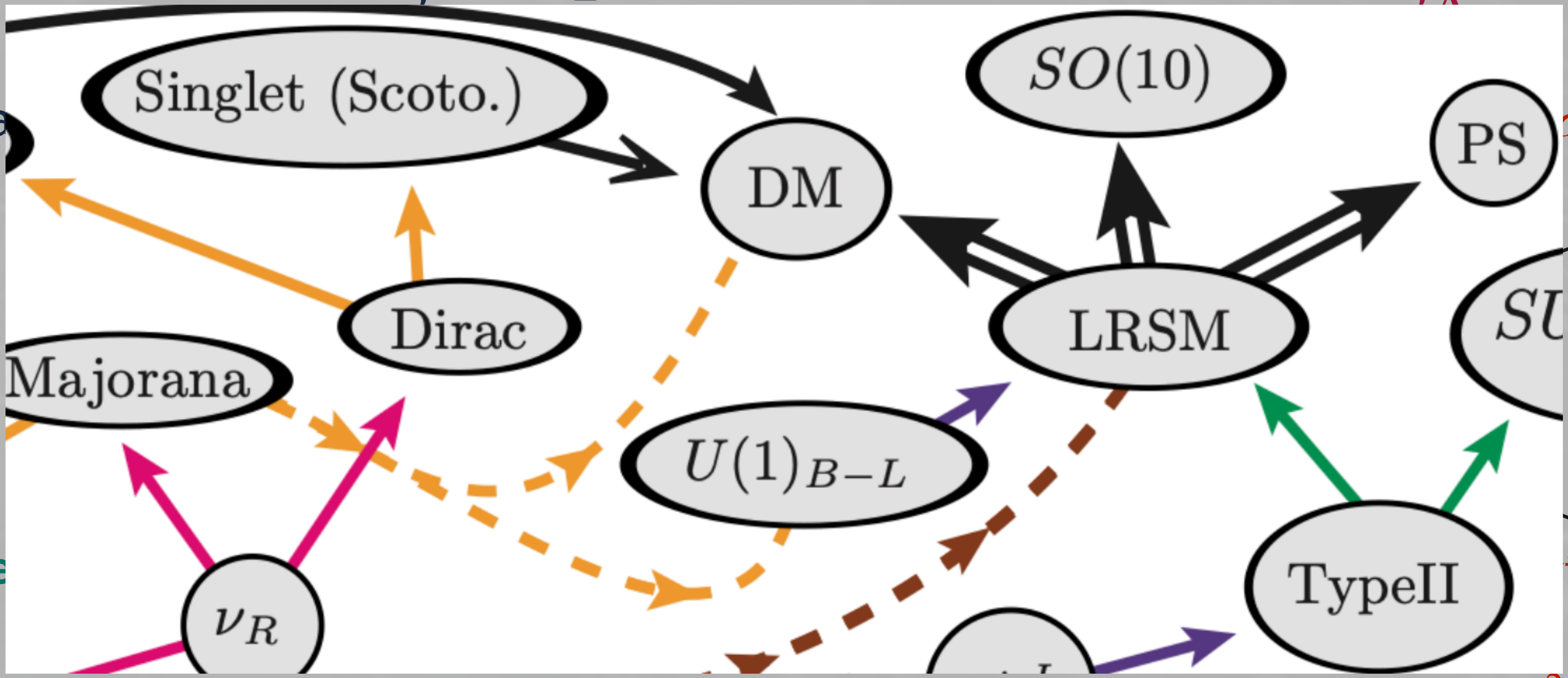


rruiz('22)

Making neutrino masses

Effective mass term $\mathscr{L}_{\text{eff}} \sim \frac{m_{LL}}{2} \bar{\nu}_L \nu_L^C$ from Weinberg operator: $\mathscr{L}^{d=5} \sim \frac{h_{ij}}{2\Lambda}(H\, L_i\, H\, L_j)$

Different rea... $\sigma_a H)(L_j^T \sigma_a H)$



ν_L H

ν_L H

Type... ...iplet)

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Countless more possibilities with higher odd-dimensional operators or loop-level realisations… ♪

(Actually they are countable, see e.g. [John Gargalionis and Ray Volkas: 2009.13537]

Introducing Left-Right: Motivation

Features:

Mohapatra, Senjanović '75

- Combination of **type I** & **type II** seesaw mechanism, new states $\sim \mathcal{O}(\mathrm{TeV})$
- Can address the **strong CP problem** (see e.g. [2107.10852])
- Lightest **right-handed neutrino** can be a **Dark Matter candidate** [2312.00129]
- Low(ish)-scale **leptogenesis** can be implemented [C. Hati et al. '18]
- Left-right symmetry $\mathcal{G}_{\mathrm{LR}} = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

  appears in the breaking of **GUTs**, e.g.:

$$SO(10) \to SU(4)_c \otimes SU(2)_L \otimes SU(2)_R \to \mathcal{G}_{LR} \to \mathcal{G}_{\mathrm{SM}}$$

Introducing Left-Right: Model overview

SM Gauge group is extended: $\mathscr{G}_{\rm LR} = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

Right-handed SM fermion singlets are promoted to $SU(2)_R$**-doublets**

$\Rightarrow$ **Add RH neutrinos,** $U(1)_{B-L}$-anomalies automatically cancelled

(E6 models)

Scalar sector: different (minimal) possibilities, bi-doublet + 2 doublets
or (like here) bi-doublet + 2 triplets

Physical spectrum: **SM +** $N_R\,, W_R^\pm\,, Z_R\,, \Delta_{R,L}^{\pm\pm}\,, \Delta_L^+\,, \Delta_L^0\,, \chi_L^0\,, \Delta_R^0\,, A^0\,, H^0\,, H^\pm$

Field content: $\mathcal{G}_{\mathrm{LR}} = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

Fermions: $Q_{L,R} = \begin{pmatrix} u \\ d \end{pmatrix}_{L,R}$, $L_{L,R} \begin{pmatrix} \nu \\ \ell \end{pmatrix}$ **(3 generations)**

Gauge Fields: $SU(2)_{L,R}$-gauge fields, $A_{L,R} = A^a_{L,R} \frac{\sigma^a}{2}$, $A^{\pm}_{L,R} = \frac{A^1_{L,R} \mp i A^2_{L,R}}{\sqrt{2}}$

$U(1)_{B-L}$-gauge field B + **QCD** $SU(3)_c$

Scalar Fields: $SU(2)_{L,R}$ triplets, $\Delta_{L,R} = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$, $(1,3,1,2)_L$ & $(1,1,3,2)_R$

$SU(2)_{L,R}$ bi-doublet, $\phi = \begin{pmatrix} \phi_1^{0*} & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}$, **(1, 2, 2, 0)**

Electrical charge: $Q = T^3_L + T^3_R + \frac{B-L}{2}$

Making Neutrino Masses

Majorana Mass from $SU(2)_R$ breaking

Discrete $\mathscr{C}$-symmetry:

$\mathscr{C} : \phi \leftrightarrow \phi^T \,, \Delta_L \leftrightarrow \Delta_R^*$

$$\mathscr{L}_Y^\ell \supseteq \bar{L}_L'(Y_\ell \phi + \tilde{Y}_\ell \tilde{\phi}) L_R' + \bar{L}_L'^c i\sigma_2 \Delta_L Y_L^M L_L' + L_R'^c i\sigma_2 \Delta_R Y_R^M L_R'$$

$$\Rightarrow Y_\ell = Y_\ell^T \,,\; \tilde{Y}_\ell = \tilde{Y}_\ell^T \,,\; Y_L^M = Y_R^M \,,\; M_D = M_D^T \,,\; M_L = \frac{v_L}{v_R} M_R$$

Majorana Mass terms

From the light and heavy masses

$$M_\nu \simeq M_L - M_D M_R^{-1} M_D^T \,, \qquad M_N \simeq M_R$$

All Yukawas **fully determined** by measurable inputs $(m_{\nu_i}, m_{N_i}, \mathscr{U}_L, \mathscr{U}_R)$

$$M_D = M_N \sqrt{\frac{v_L}{v_R} \mathbb{1} - M_N^{-1} M_\nu}$$

Nemevšek, Senjanović, Tello PRL'13

JK, Nemevšek, Nesti EPJC'24

Analytical matrix square-root

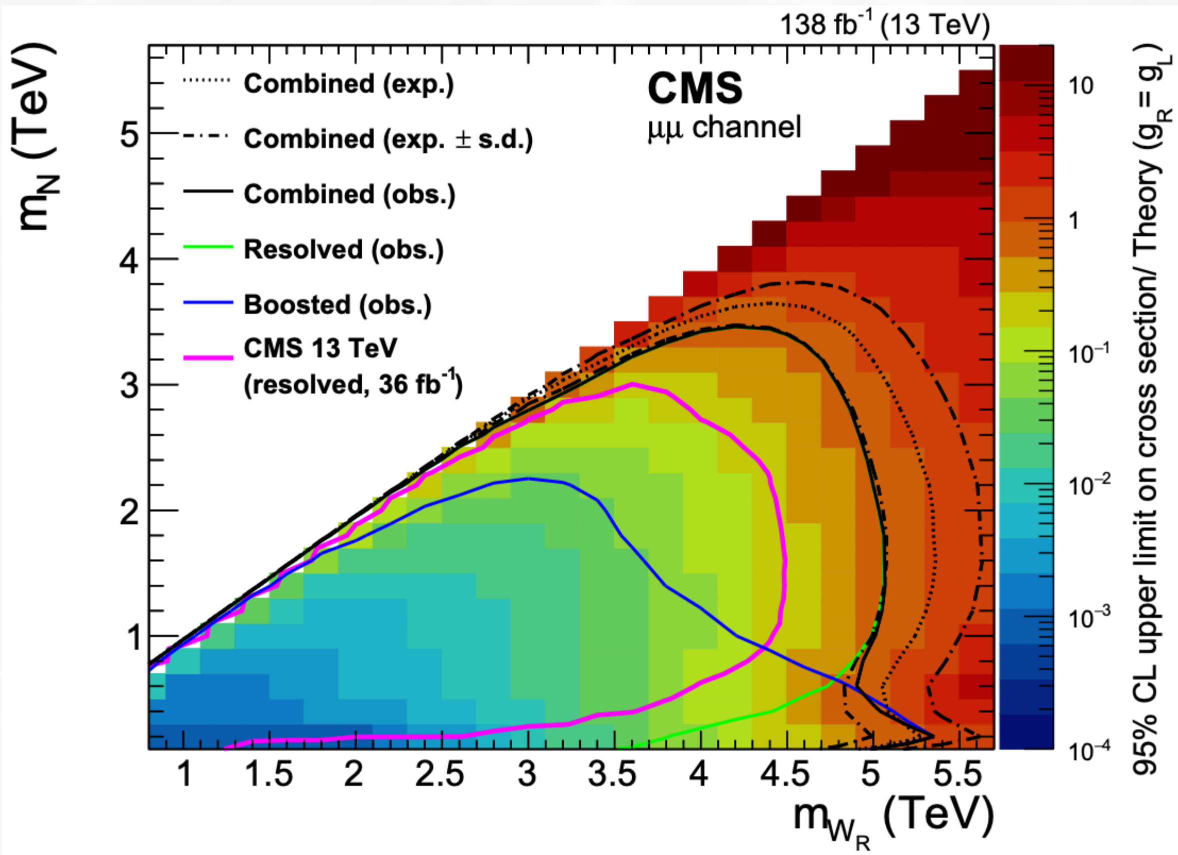
$$\sqrt{A} = c_0 \, \mathbb{1} + c_1 \, A + c_2 \, A.A$$

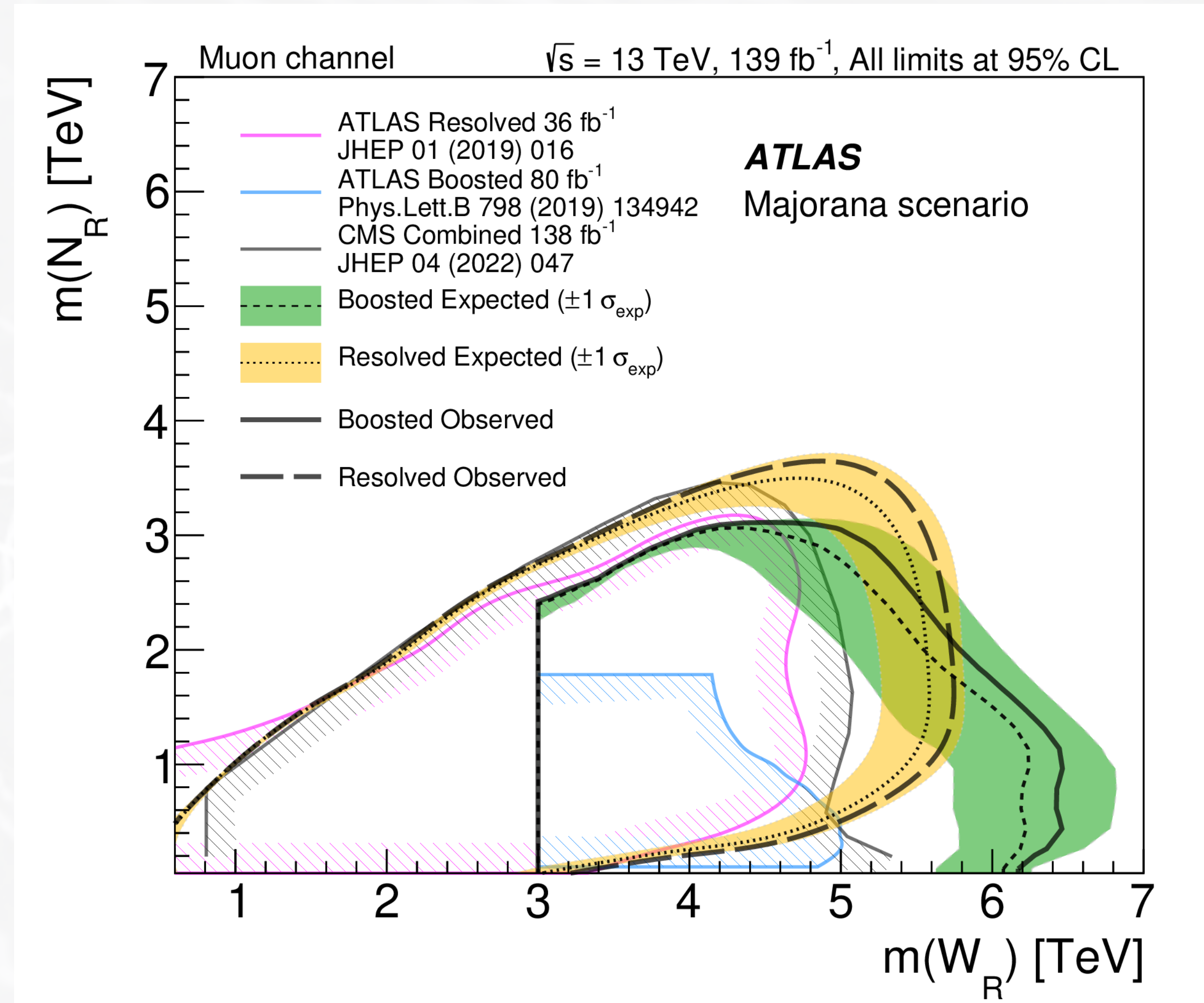
c_i are functions of invariants of A

LNV at LHC in Left-Right: Keung Senjanović process

[ATLAS: 2304.09553]

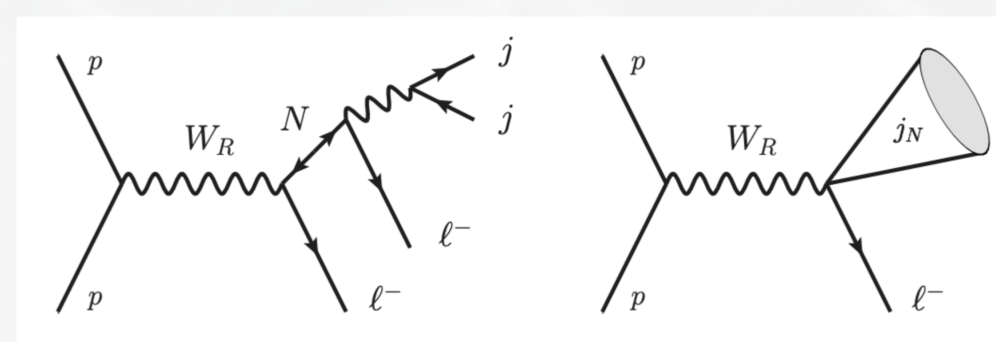
[CMS: 2112.03949]





- Exclusion $m_{W_R} \gtrsim 6 - 7$ TeV

- Di-jet searches $m_{W_R} \gtrsim 4.5$ TeV



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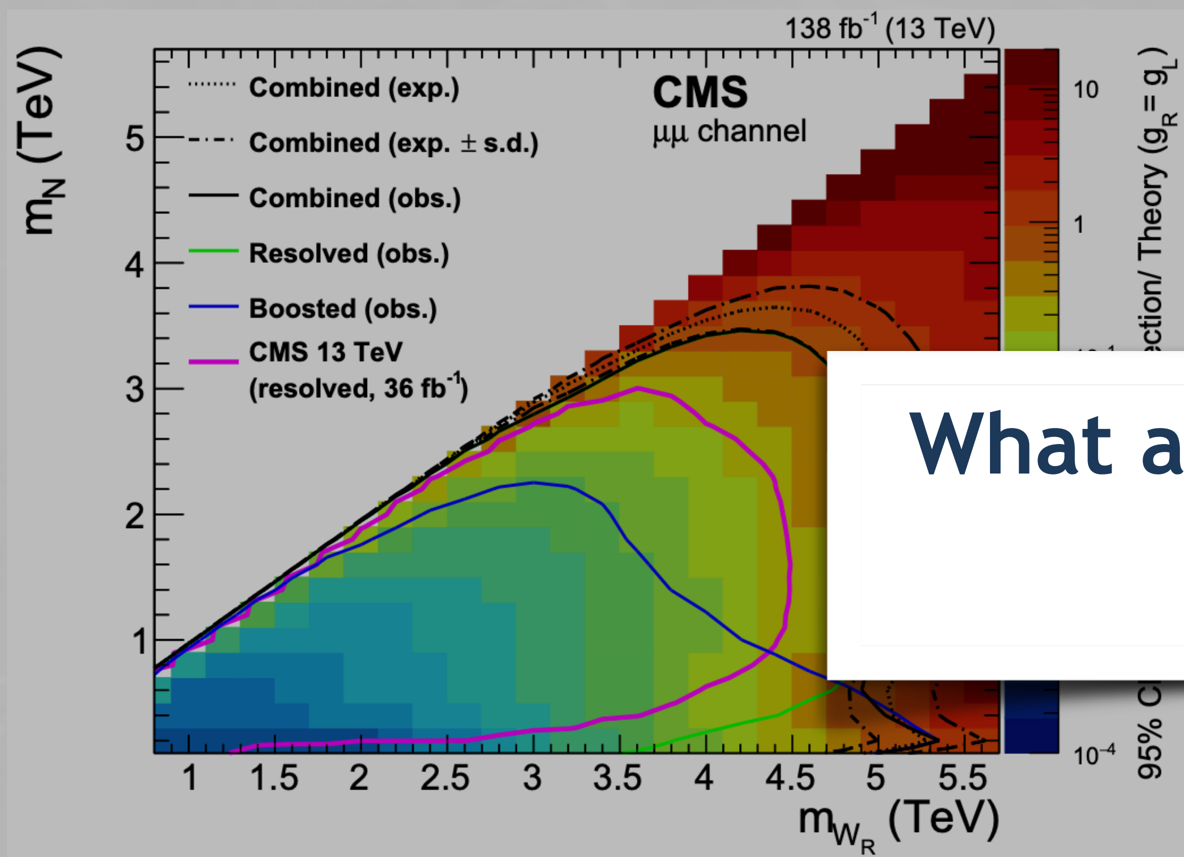

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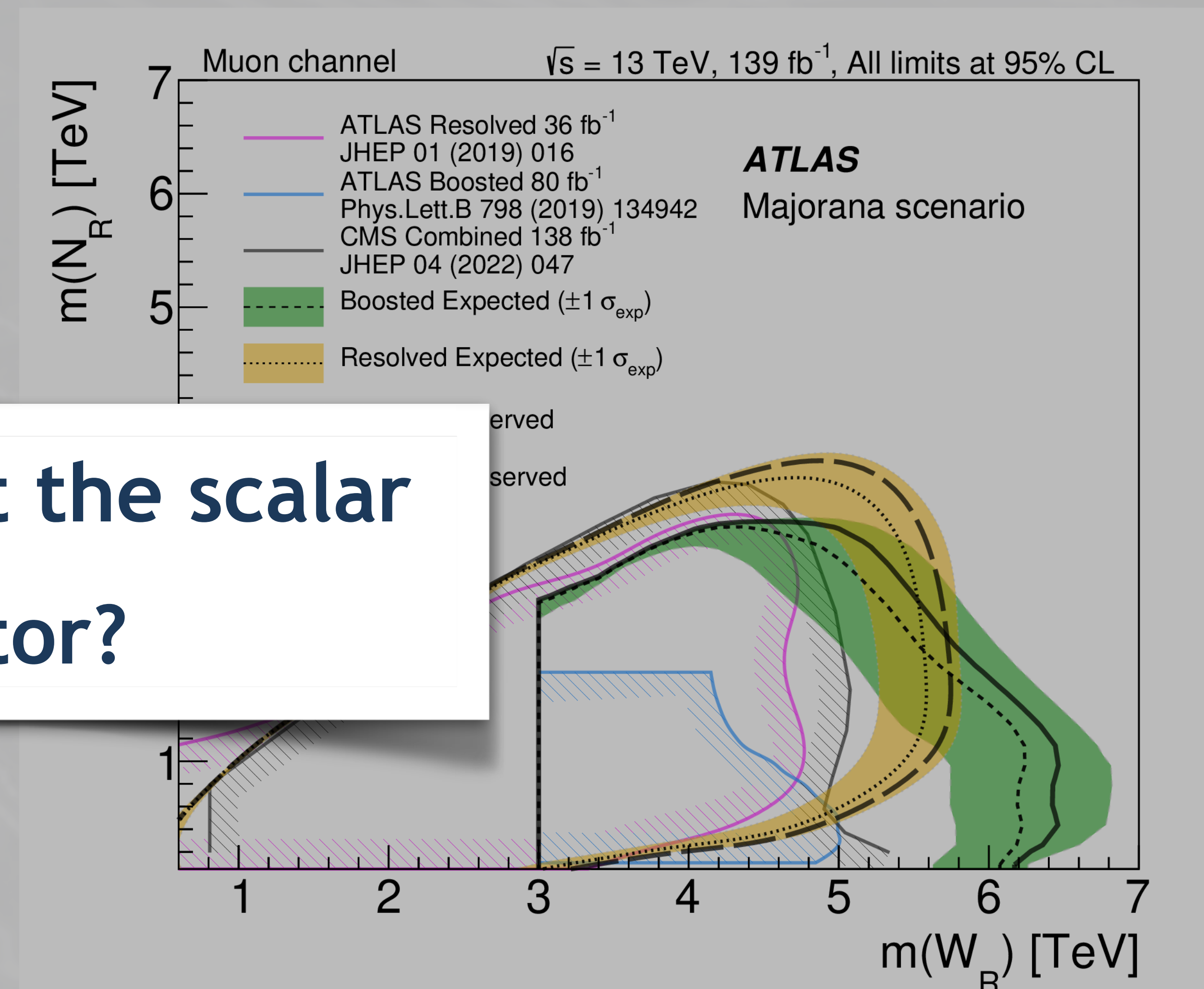
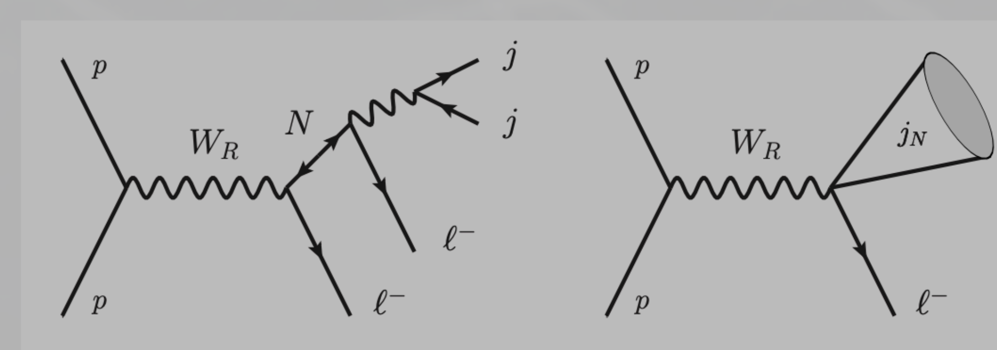


What about the scalar sector?

- Exclusion $m_{W_R} \gtrsim 6 - 7$ TeV
- Di-jet searches $m_{W_R} \gtrsim 4.5$ TeV



Diagonalising the Lagrangian: Scalar sector

The most general $\mathscr{P}$- (and $\mathscr{C}$-) symmetric potential is given by:

$$
\begin{aligned}
\mathcal{V} = & -\mu_1^2\left[\phi^\dagger\phi\right] - \mu_2^2\left(\left[\tilde{\phi}\phi^\dagger\right] + \left[\tilde{\phi}^\dagger\phi\right]\right) - \mu_3^2\left(\left[\Delta_L\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R^\dagger\right]\right) \\
& + \lambda_1\left[\phi^\dagger\phi\right]^2 + \lambda_2\left(\left[\tilde{\phi}\phi^\dagger\right]^2 + \left[\tilde{\phi}^\dagger\phi\right]^2\right) + \lambda_3\left[\tilde{\phi}\phi^\dagger\right]\left[\tilde{\phi}^\dagger\phi\right] + \lambda_4\left[\phi^\dagger\phi\right]\left(\left[\tilde{\phi}\phi^\dagger\right] + \left[\tilde{\phi}^\dagger\phi\right]\right) \\
& + \rho_1\left(\left[\Delta_L\Delta_L^\dagger\right]^2 + \left[\Delta_R\Delta_R^\dagger\right]^2\right) + \rho_2\left(\left[\Delta_L\Delta_L\right]\left[\Delta_L^\dagger\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R\right]\left[\Delta_R^\dagger\Delta_R^\dagger\right]\right) + \rho_3\left[\Delta_L\Delta_L^\dagger\right]\left[\Delta_R\Delta_R^\dagger\right] \\
& + \rho_4\left(\left[\Delta_L\Delta_L\right]\left[\Delta_R^\dagger\Delta_R^\dagger\right] + \left[\Delta_L^\dagger\Delta_L^\dagger\right]\left[\Delta_R\Delta_R\right]\right) + \alpha_1\left[\phi^\dagger\phi\right]\left(\left[\Delta_L\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R^\dagger\right]\right) \\
& + \left(\alpha_2\left(\left[\tilde{\phi}\phi^\dagger\right]\left[\Delta_L\Delta_L^\dagger\right] + \left[\tilde{\phi}^\dagger\phi\right]\left[\Delta_R\Delta_R^\dagger\right]\right) + \text{h.c.}\right) + \alpha_3\left(\left[\phi\phi^\dagger\Delta_L\Delta_L^\dagger\right] + \left[\phi^\dagger\phi\Delta_R\Delta_R^\dagger\right]\right) \\
& + \beta_1\left(\left[\phi\Delta_R\phi^\dagger\Delta_L^\dagger\right] + \left[\phi^\dagger\Delta_L\phi\Delta_R^\dagger\right]\right) + \beta_2\left(\left[\tilde{\phi}\Delta_R\phi^\dagger\Delta_L^\dagger\right] + \left[\tilde{\phi}^\dagger\Delta_L\phi\Delta_R^\dagger\right]\right) + \beta_3\left(\left[\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger\right] + \left[\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger\right]\right)
\end{aligned}
$$

Minimisation conditions $\dfrac{\partial\mathscr{V}}{\partial S_i} = 0$ and $\dfrac{\partial^2\mathscr{V}}{\partial S_i\partial S_j} > 0$:

$$
\begin{aligned}
\mu_1^2 &= 2\left(\lambda_1 + s_{2\beta}c_\alpha\lambda_4\right)v^2 + \left(\alpha_1 - \alpha_3\frac{s_\beta^2}{c_{2\beta}}\right)v_R^2\,, \\
\mu_2^2 &= \left(s_{2\beta}\left(2c_{2\alpha}\lambda_2 + \lambda_3\right) + \lambda_4\right)v^2 \\
&\quad + \frac{1}{2c_\alpha}\left(2c_{\alpha+\delta_2}\alpha_2 + \alpha_3\frac{t_{2\beta}}{2c_\alpha}\right)v_R^2\,, \\
\mu_3^2 &= \left(\alpha_1 + \left(2c_{\alpha+\delta_2}\alpha_2 s_{2\beta} + \alpha_3 s_\beta^2\right)\right)v^2 + 2\rho_1 v_R^2 \\
\alpha_2 s_{\delta_2} &= \frac{s_\alpha}{4}\left(\alpha_3 t_{2\beta} + 4\left(\lambda_3 - 2\lambda_2\right)s_{2\beta}\epsilon^2\right).
\end{aligned}
$$

$$
\begin{aligned}
v_L = & \frac{\epsilon^2 v_R}{\left(1+t_\beta^2\right)\left(2\rho_1 - \rho_3\right)}\Big(-\beta_1 t_\beta\cos(\alpha-\theta_L) \\
& + \beta_2\cos(\theta_L) + \beta_3 t_\beta^2\cos(2\alpha-\theta_L)\Big).
\end{aligned}
$$

EWPO/**Flavour physics**: $v_L \ll v \ll v_R \simeq \mathscr{O}(\text{TeV})$

Assume $\beta_i = v_L = 0$ for exact solvability

Scalar Sector: the bottom line

All potential parameters are cast as physical masses and mixings

(see **JK**, Nemevšek, Nesti 2403.07756 for the gruesome details)

Scalar Sector: the bottom line

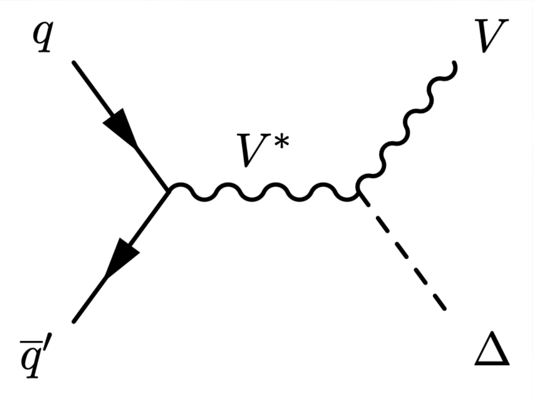
All potential parameters are cast as physical masses and mixings

(see **JK**, Nemevšek, Nesti 2403.07756 for the gruesome details)

New model file (FeynRules/UFO):

- **All mixings** are calculated
- New parameter inversion: cast all parameters in **physical (measurable) parameters**
- Includes full **QCD NLO corrections** for the first time
- Also a parity violating version of the model file where $g_L \neq g_R$

LNV at LHC in Left-Right: "Majorana Higgs"



Here: production and decay of $\Delta_R^0 \to NN$

$$\mathscr{L}_Y^\ell \supseteq \bar{L}_L^{'}(Y_\ell \phi + \tilde{Y}_\ell \tilde{\phi})L_R^{'} + \bar{L}_L^{'c} i\sigma_2 \, \Delta_L \, Y_L^M L_L^{'} + L_R^{'c} i\sigma_2 \, \Delta_R \, Y_R^M L_R^{'}$$

$$\Gamma(\Delta_R^0 \to NN) \propto m_{\Delta_R^0} \frac{m_N^2}{m_{W_R}^2}$$

$$\Delta_R = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta_R^{++} \\ \Delta_R^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$$

Δ_R^0 mixes with **SM-like Higgs** $\propto \sin\theta$

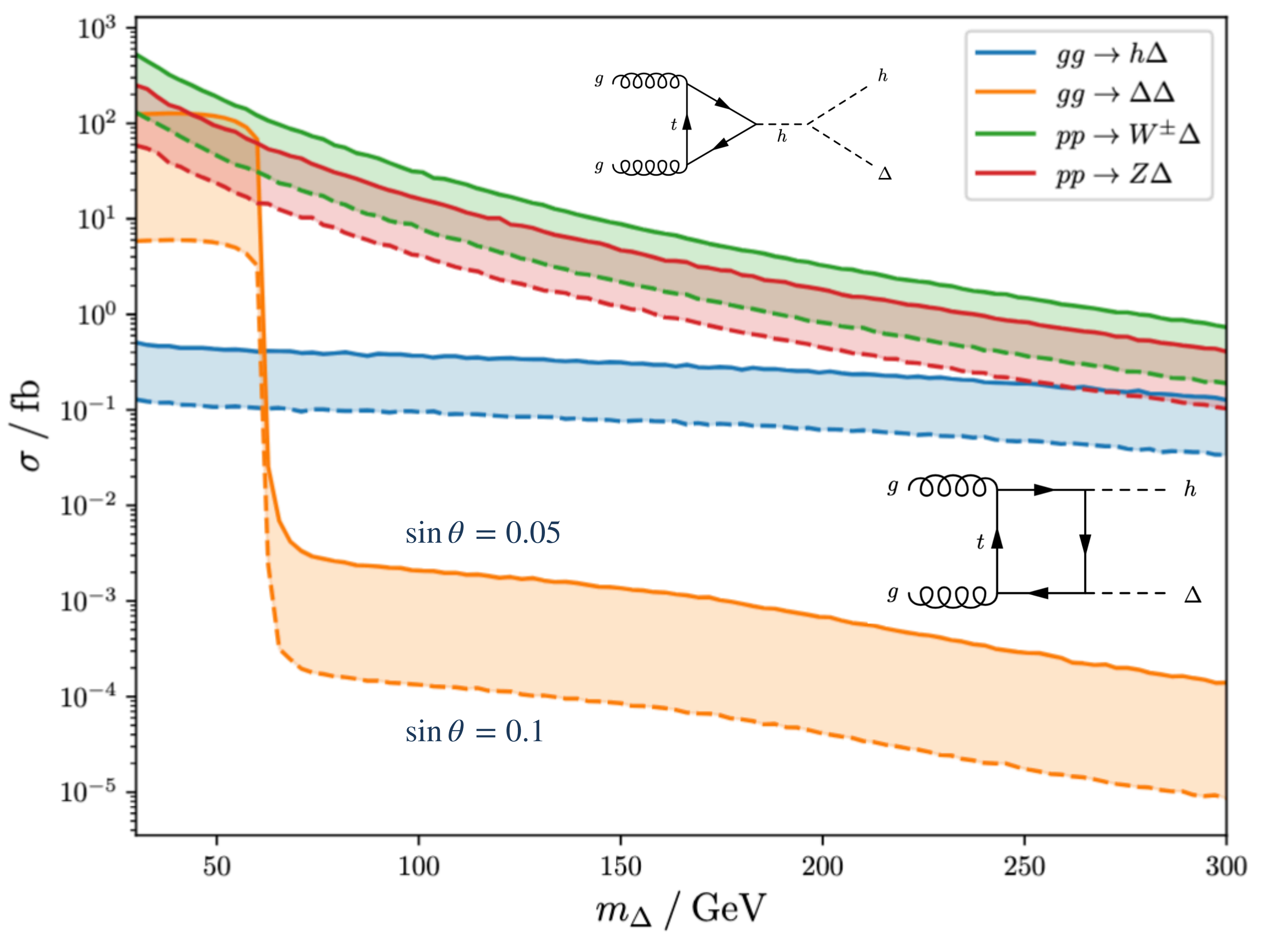
$\Rightarrow \Delta_R^0$ decays into **SM states**

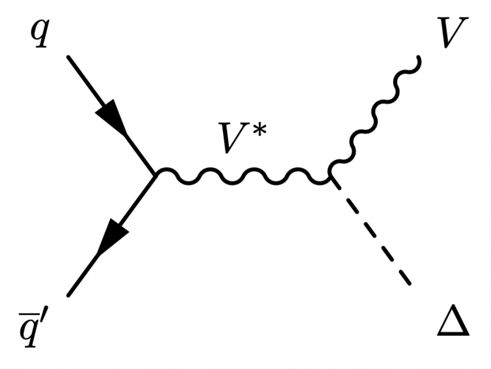
$$\Gamma(\Delta_R^0 \to VV^{(*)}) \simeq \sin^2\theta \, \Gamma(h \to VV^{(*)})$$

$$\Gamma(\Delta_R^0 \to f\bar{f}) \simeq \sin^2\theta \, \Gamma(h \to f\bar{f})$$

LNV at LHC in Left-Right: Production of Δ

Fuks, **JK,** Nemevšek, Nesti **arXiv:2503.21354**





Sizeable production in "**Δ-strahlung**" and **gluon fusion**

Resonant production via Higgs decay for very light Δ

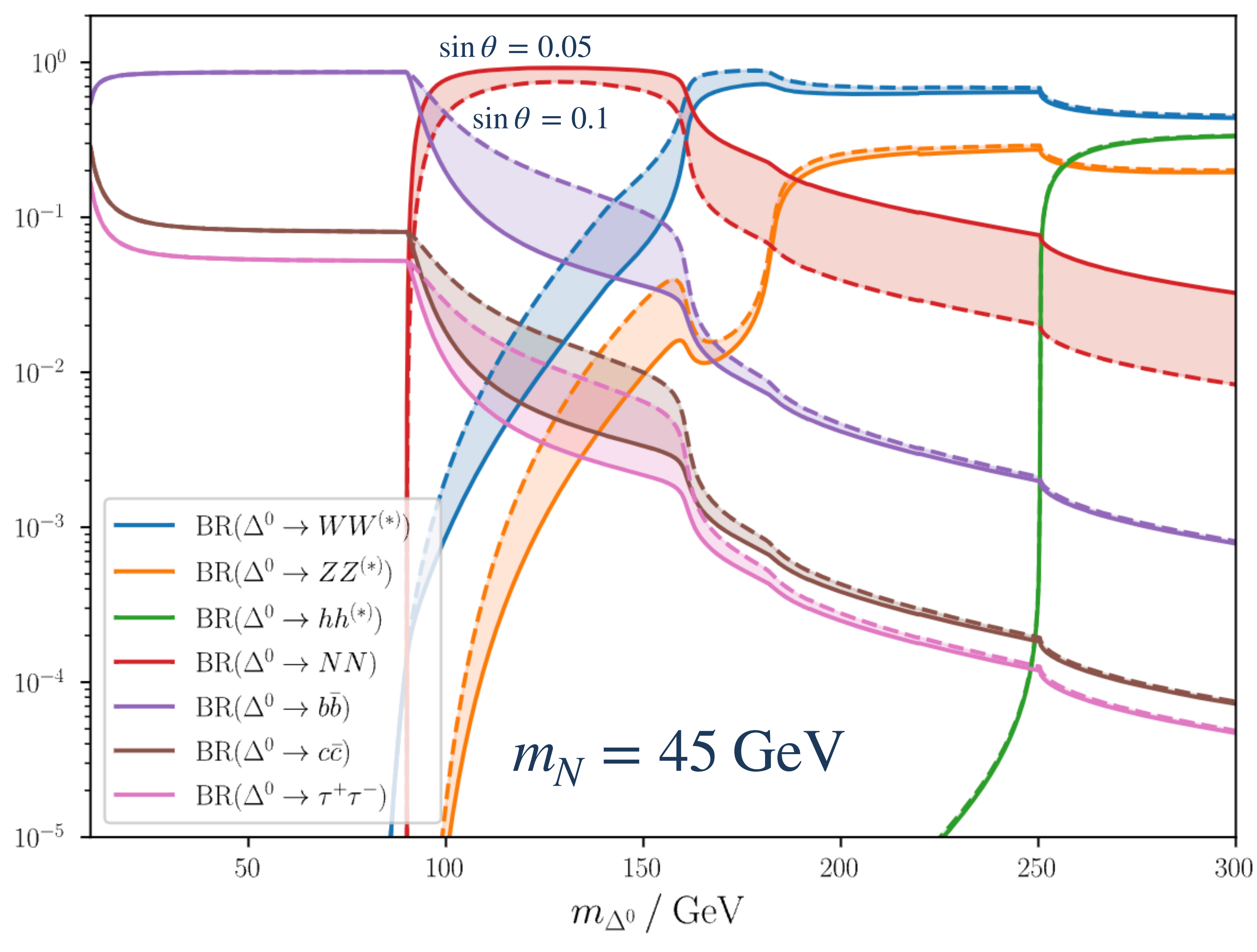
NLO model-file **JK,** Nemevšek, Nesti EPJC'24

See also: https://sites.google.com/site/leftrighthep

LNV at LHC in Left-Right: Decays of the Δ

$$\Gamma(\Delta_R^0 \to NN) \propto m_{\Delta_R^0} \frac{m_N^2}{m_{W_R}^2}$$

Fuks, **JK,** Nemevšek, Nesti **arXiv:2503.21354**



Ample room for $\Delta_R^0 \to NN$!

$$\Gamma(\Delta_R^0 \to VV^{(*)}) \simeq \sin^2\theta\, \Gamma(h \to VV^{(*)})$$

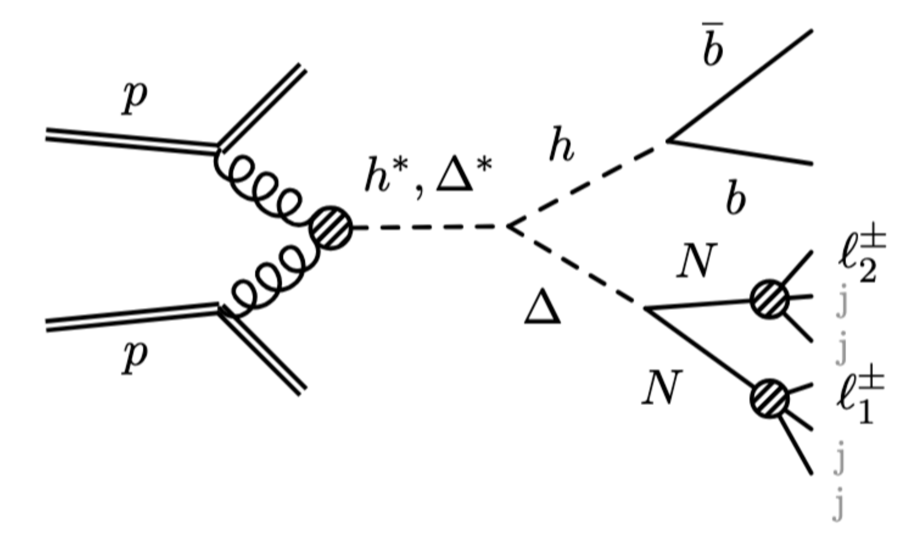
$$\Gamma(\Delta_R^0 \to f\bar{f}) \simeq \sin^2\theta\, \Gamma(h \to f\bar{f})$$

(Transverse) displacement of N

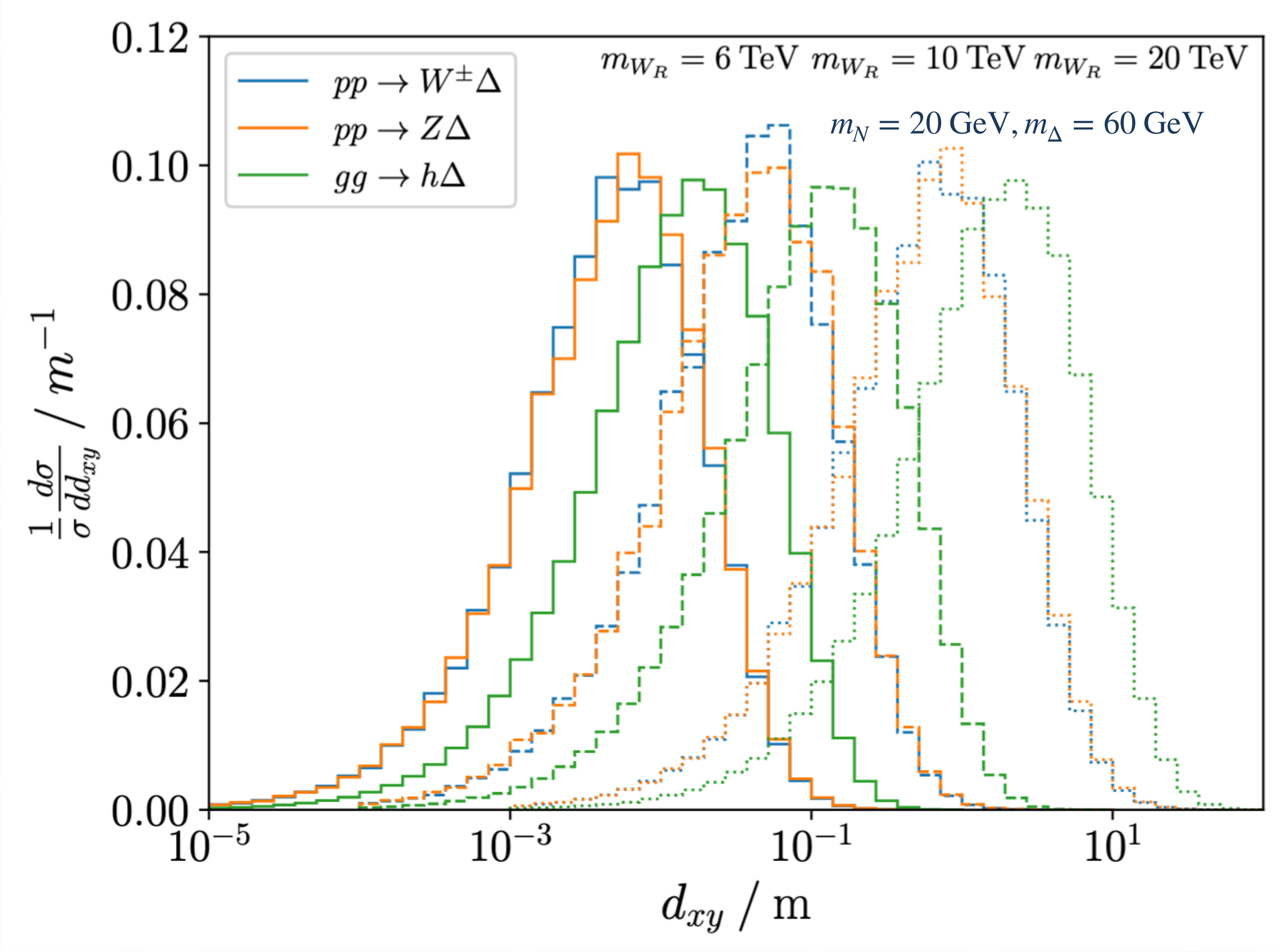
Fuks, **JK,** Nemevšek, Nesti arXiv:2503.21354

Sizeable displacement from N decay

⇒ Focus on displaced leptons/jets



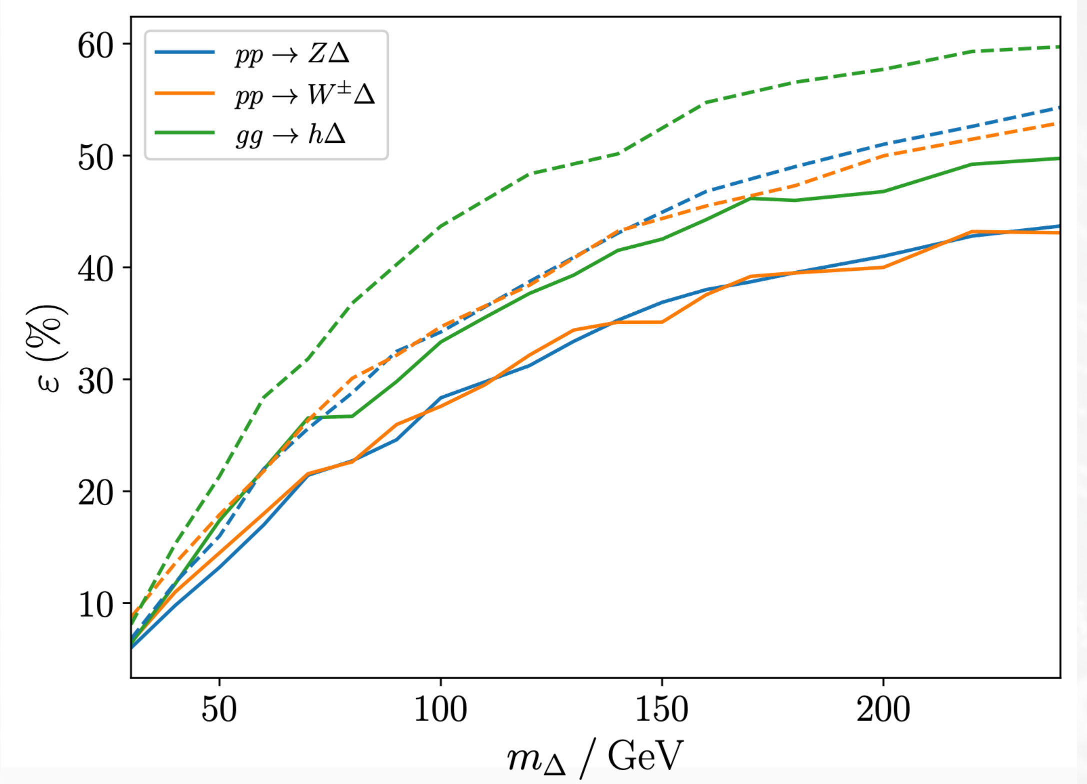
⇒ Decay of associated boson triggers event



$$N \text{ lifetime } \approx 2.5\text{mm}\,\frac{(m_{W_R}/3\text{ TeV})^4}{(m_N/10\text{ GeV})^5}$$

Analysis outline

Fuks, **JK,** Nemevšek, Nesti **arXiv:2503.21354**



Select events with 2 **same-sign** leptons with

$\Delta R(\ell, j_c) > 0.25$ (lose most events due to soft-lepton isolation)

$|\eta(\ell)| < 2.4$ kinematic/isolation efficiencies: $20 - 40\,\%$

$p_T(\ell) > 10$ GeV

0.1 mm $< d_{xy} < 30$ cm (Decay in **inner tracker**)

Simulation with MadGraph5, Pythia, Delphes, MadAnalysis tool-chain

NLO model-file **JK,** Nemevšek, Nesti EPJC'24

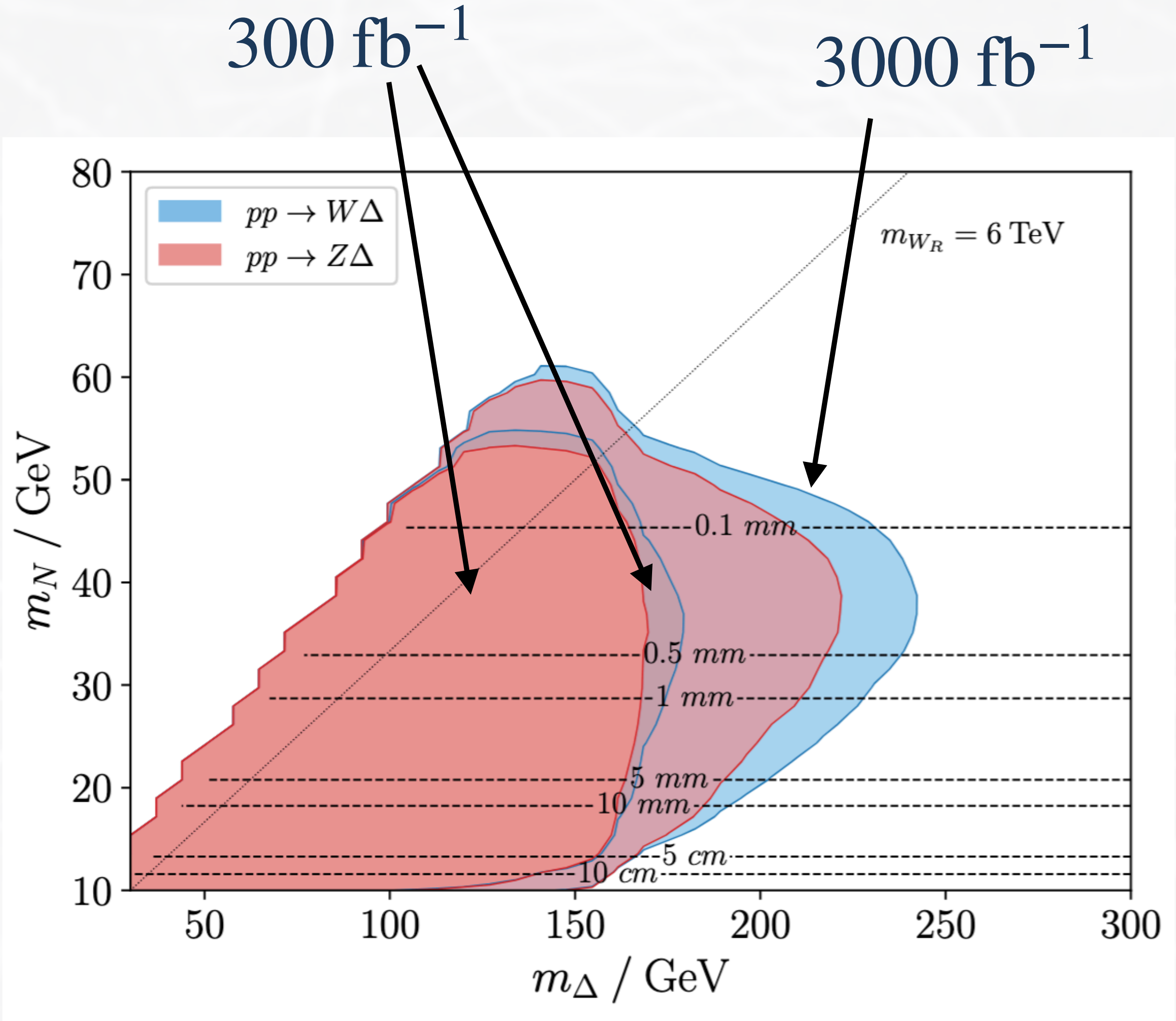
See also: https://sites.google.com/site/leftrighthep

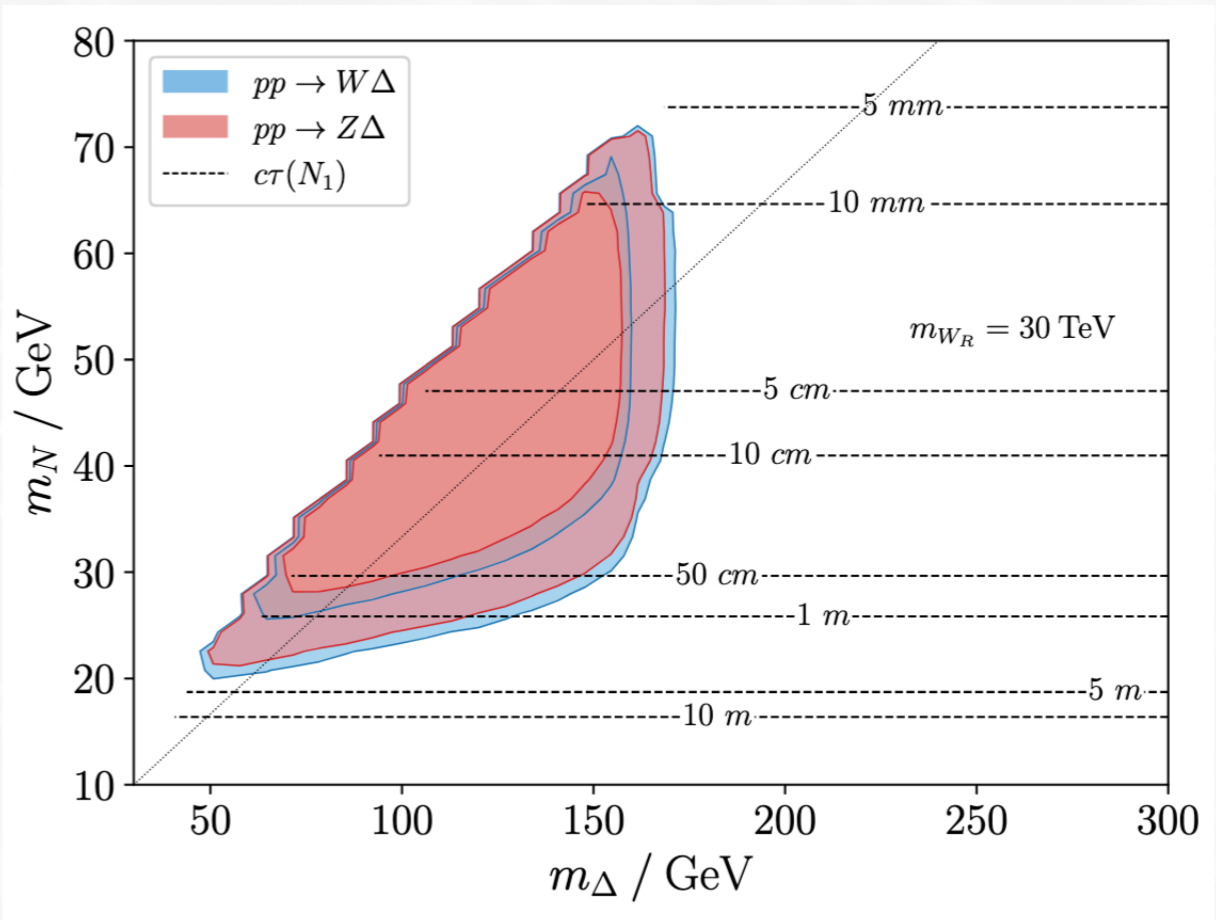
Large displacements, same-sign leptons, $m_V(\ell jj) \gtrsim 10$ GeV : no prompt backgrounds

Sensitivities at (HL)-LHC

Detection/Reconstruction of at least 3 Events

Fuks, **JK,** Nemevšek, Nesti arXiv:2503.21354





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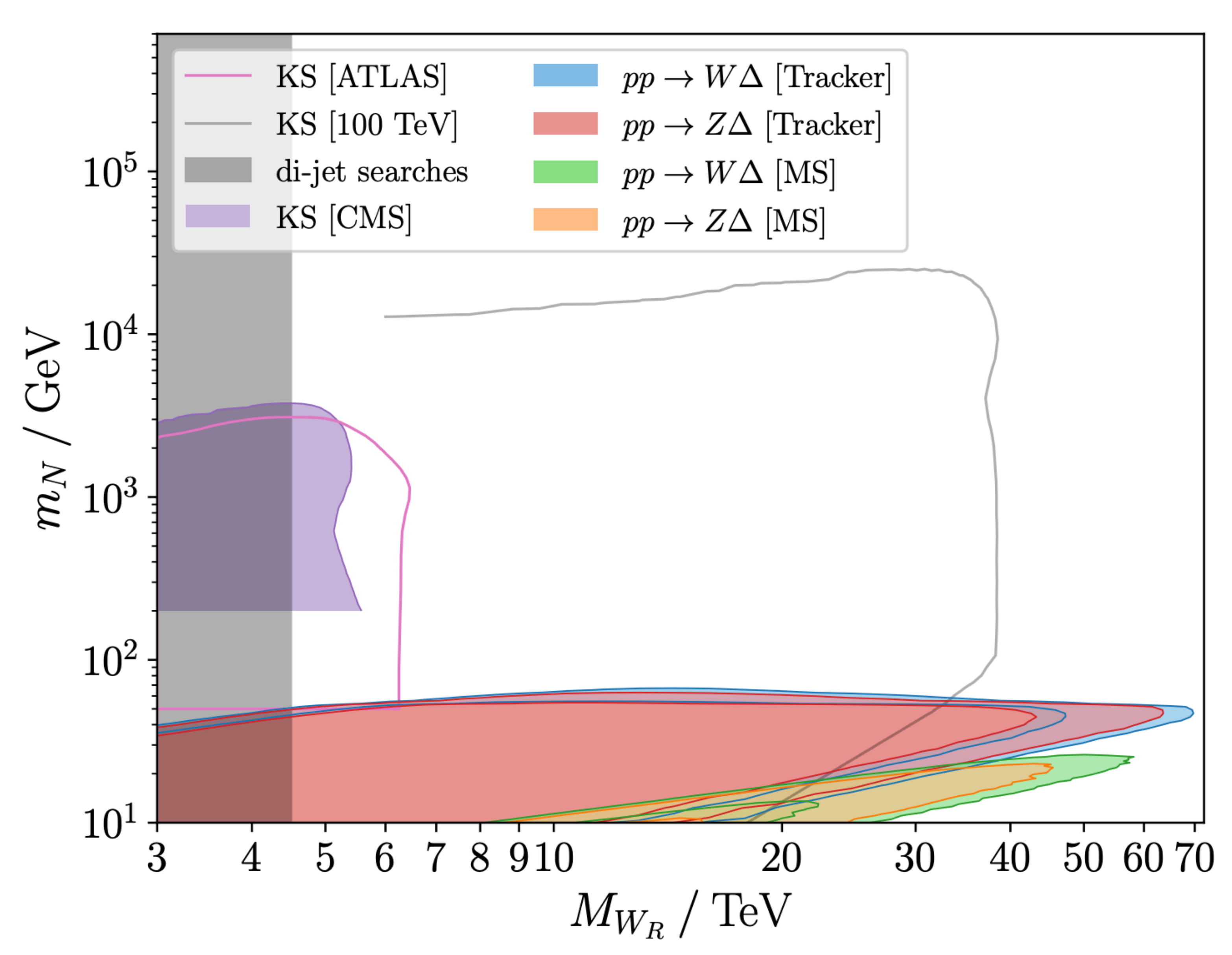
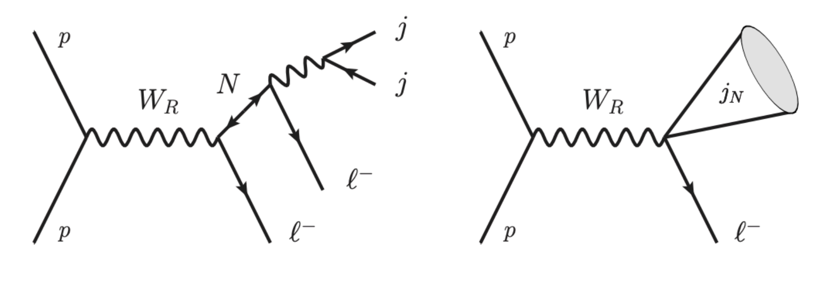




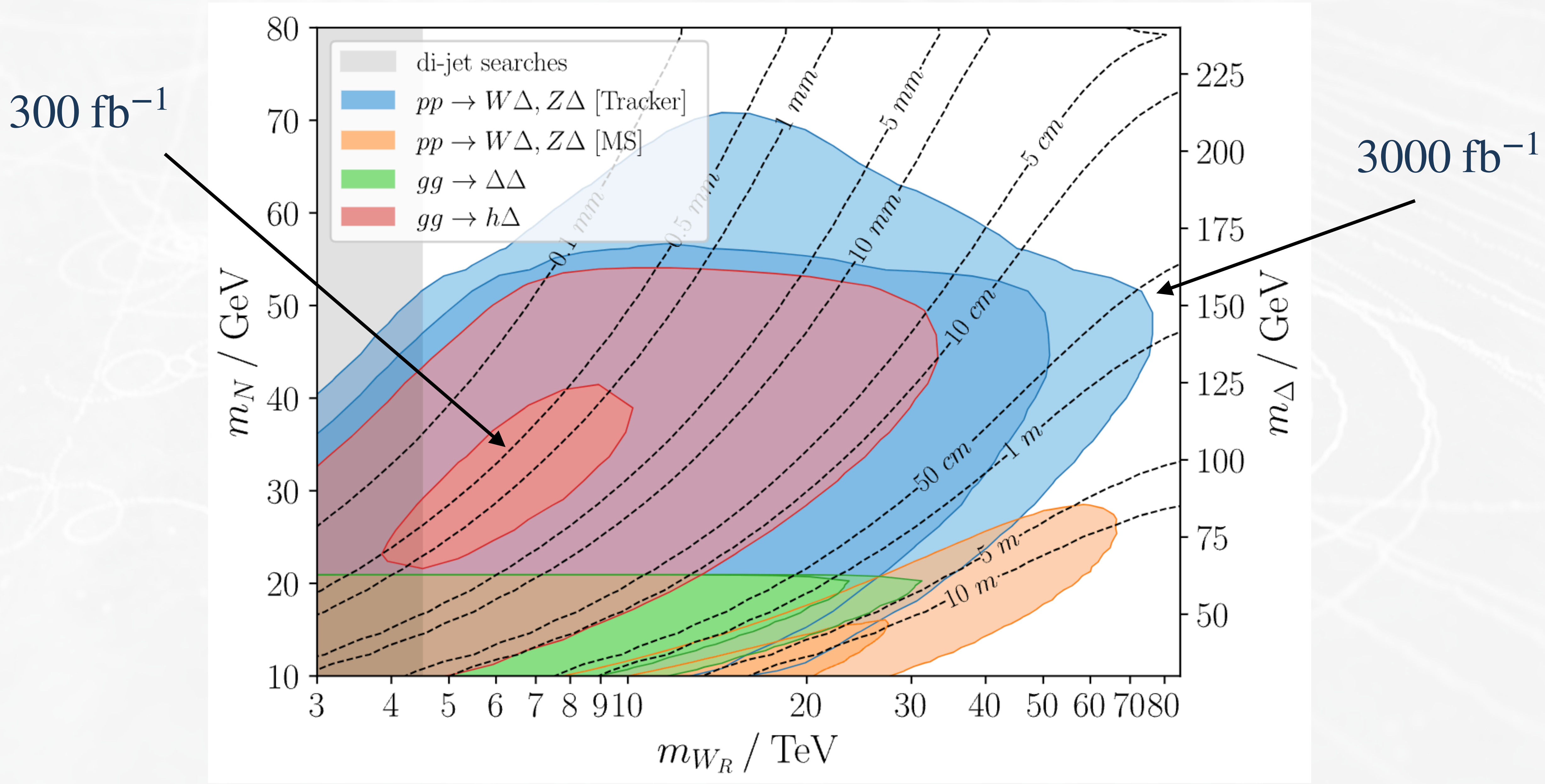
ATLAS & CMS: Keung-Senjanović process (direct search for $W_R \to \ell N$)

$\Rightarrow$ complementary parameter space, exclusion up to $m_{W_R} \gtrsim 70 - 80$ TeV!

Sensitivities at (HL)-LHC

Detection/Reconstruction of at least 3 Events

Fuks, **JK,** Nemevšek, Nesti arXiv:2503.21354



⇒ complementary parameter space, exclusion up to $m_{W_R} \gtrsim 70 - 80$ TeV!

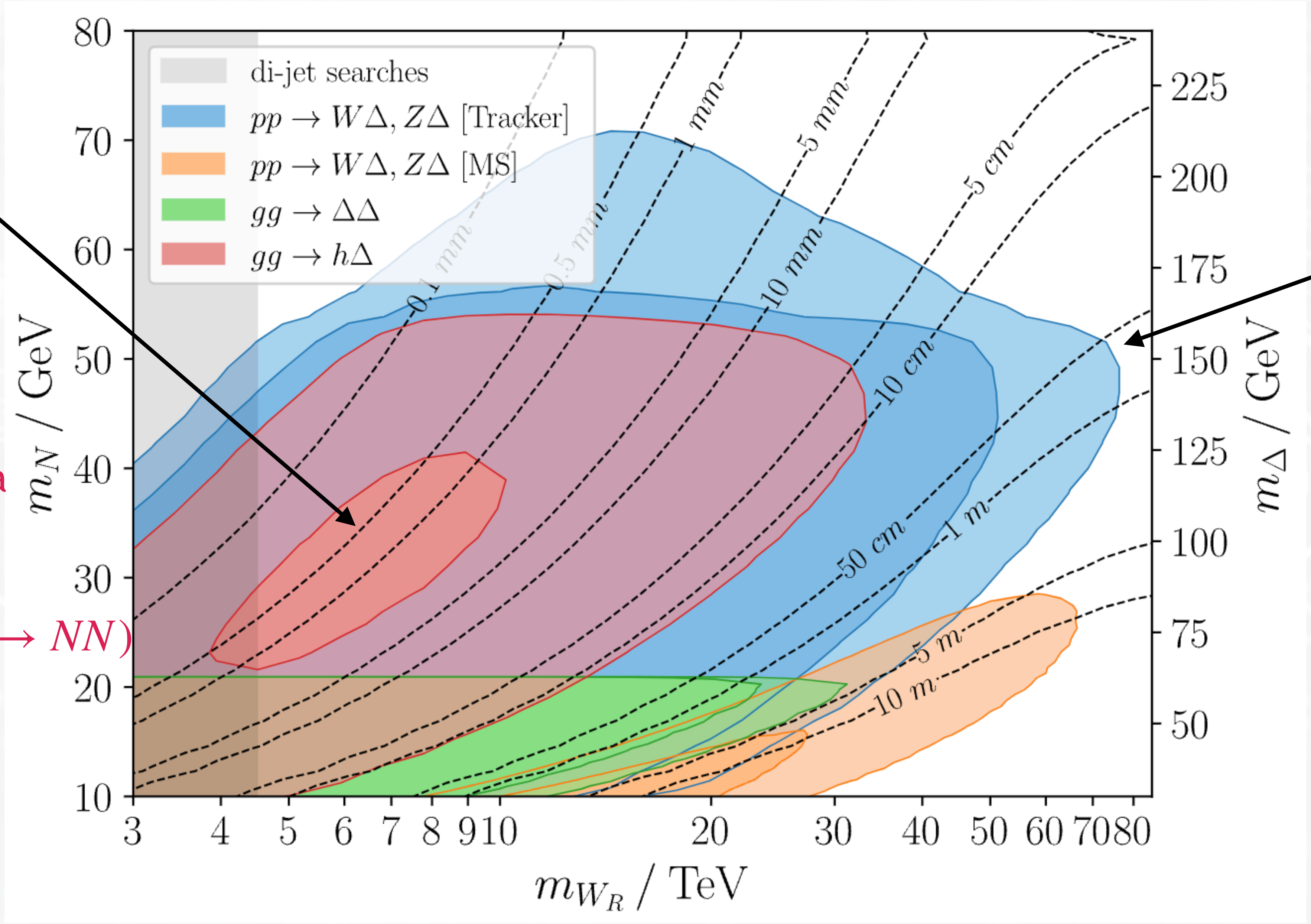
⇒ Large displacements up to Muon Spectrometer [MS] possible! $8 \text{ m} < d_{xy} < 13 \text{ m}$

Sensitivities at (HL)-LHC

Detection/Reconstruction of at least 3 Events

Fuks, **JK,** Nemevšek, Nesti **arXiv:2503.21354**

$300\ \text{fb}^{-1}$



$3000\ \text{fb}^{-1}$

b-eautiful Majorana Higgses:

$gg \to h(\to b\bar{b})\Delta(\to NN)$

⇒ complementary parameter space, exclusion up to $m_{W_R} \gtrsim 70-80$ TeV!

⇒ Large displacements up to Muon Spectrometer [MS] possible! $8\ \text{m} < d_{xy} < 13\ \text{m}$

Conclusions & Outlook

- Suggest new search for (light) Δ_R^0 in **Left-Right symmetric model**
- **Same-sign leptons** from $\Delta_R^0 \to NN \to \ell^\pm \ell^\pm + \text{jets decay} \Rightarrow$ **LNV**
- b**-eautiful** signature: from $gg \to h(\to b\bar{b})\Delta(\to NN)$ decay

  $\Rightarrow$ simultaneously measure spontaneous mass origin of **Dirac** and **Majorana** states
- Dedicated **displaced vertex analysis**: $\Rightarrow m_{W_R} \gtrsim 70 - 80\ \text{TeV}$

Conclusions & Outlook

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Thanks for your attention!

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Bonus slides

ay $\Rightarrow$ **LNV**

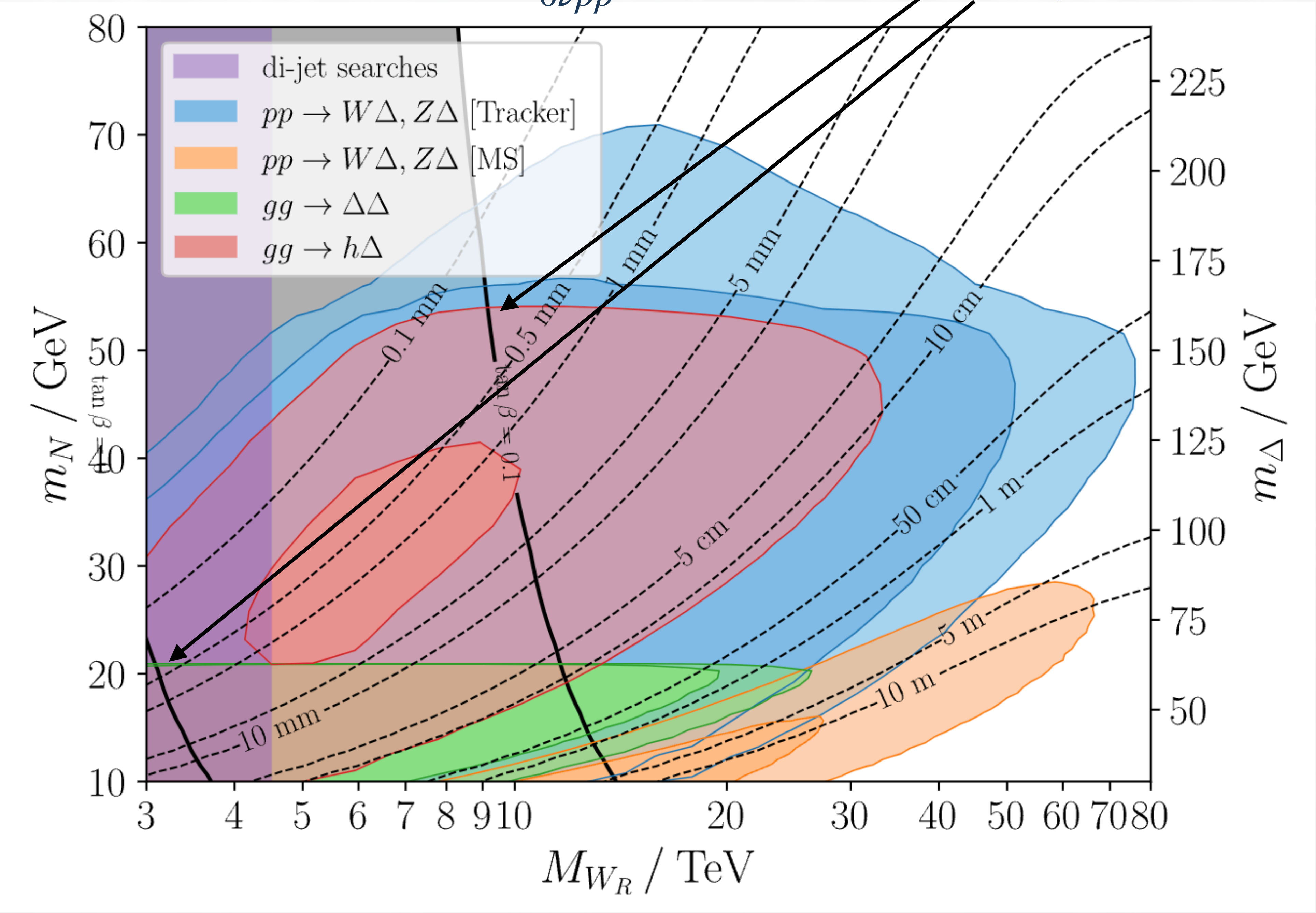
- b**-eautiful** signature: from $gg \to h(\to b\bar{b})\Delta(\to NN)$ decay

  $\Rightarrow$ simultaneously measure spontaneous mass origin of **Dirac** and **Majorana** states

- Dedicated **displaced vertex analysis**: $\Rightarrow m_{W_R} \gtrsim 70 - 80\,\text{TeV}$

Interplay with $0\nu\beta\beta$

KamLAND-Zen: $T^{1/2}_{0\nu\beta\beta}(^{136}\mathrm{Xe}) \gtrsim 2.3 \times 10^{26}\mathrm{yrs}$



Diagonalising the Lagrangian: Scalar sector

Scalars are complex

$$\Rightarrow \Delta_{L,R} = \begin{pmatrix} \frac{\Delta^{+}_{L,R}}{\sqrt{2}} & \Delta^{++}_{L,R} \\ v_{L,R} + \mathrm{Re}\Delta^{0}_{L,R} + i\mathrm{Im}\Delta^{0}_{L,R} & -\frac{\Delta^{+}_{L,R}}{\sqrt{2}} \end{pmatrix}, \qquad \phi = \begin{pmatrix} v_1 + \mathrm{Re}\phi_1^0 - i\mathrm{Im}\phi_1^0 & \phi_2^+ \\ \phi_1^- & v_2 + \mathrm{Re}\phi_2^0 + i\mathrm{Im}\phi_2^0 \end{pmatrix}$$

$\Rightarrow$ some of the **pseudo-scalar** excitations are eaten by the (massive) gauge bosons

The most general $\mathscr{P}$- (and $\mathscr{C}$-) symmetric potential is given by:

$$\mathscr{P} : \phi \to \phi^{\dagger},\ \Delta_L \leftrightarrow \Delta_R \qquad\qquad \mathscr{C} : \phi \to \phi^{T},\ \Delta_L \leftrightarrow \Delta_R^{*}$$

$$\begin{aligned}
\mathcal{V} = & -\mu_1^2 \left[\phi^\dagger\phi\right] - \mu_2^2 \left(\left[\tilde{\phi}\phi^\dagger\right] + \left[\tilde{\phi}^\dagger\phi\right]\right) - \mu_3^2 \left(\left[\Delta_L\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R^\dagger\right]\right) \\
& + \lambda_1 \left[\phi^\dagger\phi\right]^2 + \lambda_2 \left(\left[\tilde{\phi}\phi^\dagger\right]^2 + \left[\tilde{\phi}^\dagger\phi\right]^2\right) + \lambda_3 \left[\tilde{\phi}\phi^\dagger\right]\left[\tilde{\phi}^\dagger\phi\right] + \lambda_4 \left[\phi^\dagger\phi\right]\left(\left[\tilde{\phi}\phi^\dagger\right] + \left[\tilde{\phi}^\dagger\phi\right]\right) \\
& + \rho_1 \left(\left[\Delta_L\Delta_L^\dagger\right]^2 + \left[\Delta_R\Delta_R^\dagger\right]^2\right) + \rho_2 \left(\left[\Delta_L\Delta_L\right]\left[\Delta_L^\dagger\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R\right]\left[\Delta_R^\dagger\Delta_R^\dagger\right]\right) + \rho_3 \left[\Delta_L\Delta_L^\dagger\right]\left[\Delta_R\Delta_R^\dagger\right] \\
& + \rho_4 \left(\left[\Delta_L\Delta_L\right]\left[\Delta_R^\dagger\Delta_R^\dagger\right] + \left[\Delta_L^\dagger\Delta_L^\dagger\right]\left[\Delta_R\Delta_R\right]\right) + \alpha_1 \left[\phi^\dagger\phi\right]\left(\left[\Delta_L\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R^\dagger\right]\right) \\
& + \left(\alpha_2 \left(\left[\tilde{\phi}\phi^\dagger\right]\left[\Delta_L\Delta_L^\dagger\right] + \left[\tilde{\phi}^\dagger\phi\right]\left[\Delta_R\Delta_R^\dagger\right]\right) + \text{h.c.}\right) + \alpha_3 \left(\left[\phi\phi^\dagger\Delta_L\Delta_L^\dagger\right] + \left[\phi^\dagger\phi\Delta_R\Delta_R^\dagger\right]\right) \\
& + \beta_1 \left(\left[\phi\Delta_R\phi^\dagger\Delta_L^\dagger\right] + \left[\phi^\dagger\Delta_L\phi\Delta_R^\dagger\right]\right) + \beta_2 \left(\left[\tilde{\phi}\Delta_R\phi^\dagger\Delta_L^\dagger\right] + \left[\tilde{\phi}^\dagger\Delta_L\phi\Delta_R^\dagger\right]\right) + \beta_3 \left(\left[\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger\right] + \left[\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger\right]\right)
\end{aligned}$$

In the case of $\mathscr{C}$, additional phases appear:

$\Rightarrow$ the parameters μ_2, λ_2, λ_4, ρ_4 and β_i can now be complex, in $\mathscr{P}$ only α_2 carries the phase δ_2

Diagonalising the Lagrangian: Scalar sector

The most general $\mathscr{P}$- (and $\mathscr{C}$-) symmetric potential is given by:

$$\begin{aligned}\mathcal{V} = &-\mu_1^2\left[\phi^\dagger\phi\right] - \mu_2^2\left(\left[\tilde{\phi}\phi^\dagger\right] + \left[\tilde{\phi}^\dagger\phi\right]\right) - \mu_3^2\left(\left[\Delta_L\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R^\dagger\right]\right)\\
&+ \lambda_1\left[\phi^\dagger\phi\right]^2 + \lambda_2\left(\left[\tilde{\phi}\phi^\dagger\right]^2 + \left[\tilde{\phi}^\dagger\phi\right]^2\right) + \lambda_3\left[\tilde{\phi}\phi^\dagger\right]\left[\tilde{\phi}^\dagger\phi\right] + \lambda_4\left[\phi^\dagger\phi\right]\left(\left[\tilde{\phi}\phi^\dagger\right] + \left[\tilde{\phi}^\dagger\phi\right]\right)\\
&+ \rho_1\left(\left[\Delta_L\Delta_L^\dagger\right]^2 + \left[\Delta_R\Delta_R^\dagger\right]^2\right) + \rho_2\left(\left[\Delta_L\Delta_L\right]\left[\Delta_L^\dagger\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R\right]\left[\Delta_R^\dagger\Delta_R^\dagger\right]\right) + \rho_3\left[\Delta_L\Delta_L^\dagger\right]\left[\Delta_R\Delta_R^\dagger\right]\\
&+ \rho_4\left(\left[\Delta_L\Delta_L\right]\left[\Delta_R^\dagger\Delta_R^\dagger\right] + \left[\Delta_L^\dagger\Delta_L^\dagger\right]\left[\Delta_R\Delta_R\right]\right) + \alpha_1\left[\phi^\dagger\phi\right]\left(\left[\Delta_L\Delta_L^\dagger\right] + \left[\Delta_R\Delta_R^\dagger\right]\right)\\
&+ \left(\alpha_2\left(\left[\tilde{\phi}\phi^\dagger\right]\left[\Delta_L\Delta_L^\dagger\right] + \left[\tilde{\phi}^\dagger\phi\right]\left[\Delta_R\Delta_R^\dagger\right]\right) + \text{h.c.}\right) + \alpha_3\left(\left[\phi\phi^\dagger\Delta_L\Delta_L^\dagger\right] + \left[\phi^\dagger\phi\Delta_R\Delta_R^\dagger\right]\right)\\
&+ \beta_1\left(\left[\phi\Delta_R\phi^\dagger\Delta_L^\dagger\right] + \left[\phi^\dagger\Delta_L\phi\Delta_R^\dagger\right]\right) + \beta_2\left(\left[\tilde{\phi}\Delta_R\phi^\dagger\Delta_L^\dagger\right] + \left[\tilde{\phi}^\dagger\Delta_L\phi\Delta_R^\dagger\right]\right) + \beta_3\left(\left[\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger\right] + \left[\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger\right]\right)\end{aligned}$$

The minimisation conditions $\dfrac{\partial\mathscr{V}}{\partial S_i} = 0$ and $\dfrac{\partial^2\mathscr{V}}{\partial S_i\partial S_j} > 0$ lead us to:

$$\begin{aligned}\mu_1^2 &= 2\left(\lambda_1 + s_{2\beta}c_\alpha\lambda_4\right)v^2 + \left(\alpha_1 - \alpha_3\frac{s_\beta^2}{c_{2\beta}}\right)v_R^2\,,\\
\mu_2^2 &= \left(s_{2\beta}\left(2c_{2\alpha}\lambda_2 + \lambda_3\right) + \lambda_4\right)v^2\\
&\quad + \frac{1}{2c_\alpha}\left(2c_{\alpha+\delta_2}\alpha_2 + \alpha_3\frac{t_{2\beta}}{2c_\alpha}\right)v_R^2\,,\\
\mu_3^2 &= \left(\alpha_1 + \left(2c_{\alpha+\delta_2}\alpha_2 s_{2\beta} + \alpha_3 s_\beta^2\right)\right)v^2 + 2\rho_1 v_R^2\\
\alpha_2 s_{\delta_2} &= \frac{s_\alpha}{4}\left(\alpha_3 t_{2\beta} + 4\left(\lambda_3 - 2\lambda_2\right)s_{2\beta}\epsilon^2\right).\end{aligned}$$

$$\begin{aligned}v_L = &\frac{\epsilon^2 v_R}{\left(1+t_\beta^2\right)\left(2\rho_1 - \rho_3\right)}\Big(-\beta_1 t_\beta\cos(\alpha-\theta_L)\\
&+ \beta_2\cos(\theta_L) + \beta_3 t_\beta^2\cos(2\alpha-\theta_L)\Big).\end{aligned}$$

For exact solvability we assume $\beta_i = v_L = 0$ and keep only the phase δ_2 (no impact on collider pheno)

In any case: $v_L \ll v \ll v_R \simeq \mathscr{O}(\text{TeV})$

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

Let's start with the "easy" ones that don't mix (in units of v_R) :

$$m^2_{\Delta_R^{++}} = 4\rho_2 + \frac{c_{2\beta}}{c_\beta^4}\alpha_3\epsilon^2 ,$$

$v_L = 0 \Rightarrow$ no mixing of Δ_L , Δ_R^{++}

$$m^2_{\Delta_L^{++}} = (\rho_3 - 2\rho_1) - \frac{t_\beta^4 - 2c_{2\alpha}t_\beta^2 + 1}{t_\beta^4 - 1}\alpha_3\epsilon^2 ,$$

$$m^2_{\Delta_L^{+}} = (\rho_3 - 2\rho_1) - \frac{\left(t_\beta^2 + 1\right)^2 - 4t_\beta^2 c_{2\alpha}}{2\left(t_\beta^4 - 1\right)}\alpha_3\epsilon^2 ,$$

$$m^2_{\Delta_L^{0}} = m^2_{\chi_L^{0}} = (\rho_3 - 2\rho_1) + s_{2\beta}t_{2\beta}s_\alpha^2\alpha_3\epsilon^2 ,$$

Take as input parameters: $m_{\Delta_R^{++}}$, $m_{\Delta_L^0}$, (and $\tan\beta$ and α), solve for $\rho_{2,3}$

ρ_1 and α_3 are fixed by other masses

$\Rightarrow$ Mass spectrum of Δ_L follows a sum rule:

$$m^2_{\Delta_L^{++}} - m^2_{\Delta_L^{+}} = m^2_{\Delta_L^{+}} - m^2_{\Delta_L^{0}} = v^2\alpha_3\frac{c_{2\beta}}{2}$$

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

Now the singly charged scalars:

$$(\phi_1^-, \phi_2^-, \Delta_R^-)\, M_+^2 \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \\ \Delta_R^+ \end{pmatrix} \quad \text{with} \quad M_+^2 = \alpha_3 v_R^2 \begin{pmatrix} \frac{s_\beta^2}{c_{2\beta}} & -e^{-i\alpha}\frac{t_{2\beta}}{2} & -\epsilon e^{-i\alpha}\frac{s_\beta}{\sqrt{2}} \\ -e^{i\alpha}\frac{t_{2\beta}}{2} & \frac{c_\beta^2}{c_{2\beta}} & \epsilon\frac{c_\beta}{\sqrt{2}} \\ -\epsilon e^{i\alpha}\frac{s_\beta}{\sqrt{2}} & \epsilon\frac{c_\beta}{\sqrt{2}} & \epsilon^2\frac{c_{2\beta}}{2} \end{pmatrix}$$

M_+ is diagonalised with a unitary rotation (up to $\mathscr{O}(\epsilon^2)$:

$$\begin{pmatrix} \phi_1^+ \\ \phi_2^+ \\ \Delta_R^+ \end{pmatrix} = U_+ \begin{pmatrix} \varphi_L^+ \\ H^+ \\ \varphi_R^+ \end{pmatrix}$$

$$\begin{aligned} U_+ &= \begin{pmatrix} c_\beta & -e^{-i\alpha}s_\beta & 0 \\ e^{i\alpha}s_\beta & c_\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1+\frac{1}{4}\epsilon^2 c_{2\beta}^2 & \frac{\epsilon c_{2\beta}}{\sqrt{2}} \\ 0 & -\frac{\epsilon c_{2\beta}}{\sqrt{2}} & -1+\frac{1}{4}\epsilon^2 c_{2\beta}^2 \end{pmatrix} \begin{pmatrix} 1-\frac{1}{4}\epsilon^2 s_{2\beta}^2 & 0 & \frac{\epsilon s_{2\beta}}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{\epsilon s_{2\beta}}{\sqrt{2}} & 0 & 1-\frac{1}{4}\epsilon^2 s_{2\beta}^2 \end{pmatrix} \\ &\simeq \begin{pmatrix} c_\beta & e^{-i\alpha}s_\beta & 0 \\ e^{i\alpha}s_\beta & -c_\beta & 0 \\ 0 & 0 & -1 \end{pmatrix} + \frac{\epsilon}{\sqrt{2}} \begin{pmatrix} 0 & 0 & e^{-i\alpha}s_\beta \\ 0 & 0 & c_\beta \\ e^{i\alpha}s_{2\beta} & -c_{2\beta} & 0 \end{pmatrix} + \frac{\epsilon^2}{2} \begin{pmatrix} -4c_\beta s_\beta^4 & -e^{-i\alpha}s_\beta c_{2\beta}^2 & 0 \\ -4e^{i\alpha}s_\beta c_\beta^4 & c_\beta c_{2\beta}^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

$\Rightarrow$ $\varphi_{L,R}^\pm$ are the goldstones of $W_{L,R}^\pm$ and remain massless

The remaining (fixed) mass is

$$m_{H^+} \simeq \sqrt{\frac{\alpha_3}{c_{2\beta}}} v_R \left(1 + \epsilon^2 \frac{c_{2\beta}^{3/2}}{4}\right)$$

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

Goldstones are decoupled in an independent rotation

We are left with 4 neutral states in the basis $(\mathrm{Re}\varphi_{10}, \mathrm{Re}\Delta_R^0, \mathrm{Re}\varphi_{20}, \mathrm{Im}\varphi_{20})$

$$m_0^2 = \begin{pmatrix} 4\epsilon^2\left(\lambda_1 + \frac{4tc_\alpha\left(\lambda_4\left(t^2+1\right)+4\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2}\right) & 2\epsilon\left(\alpha_1 - \frac{t^2X\left(t^2-s_{2\alpha+\delta_2}/s_{\delta_2}\right)}{\left(t^2+1\right)^2}\right) & \frac{4\epsilon^2\left(t^2c_{2\alpha}-1\right)\left(\lambda_4\left(t^2+1\right)+8\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2} & \frac{4t^2\epsilon^2 s_{2\alpha}\left(\lambda_4\left(t^2+1\right)+8\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2} \\ 2\epsilon\left(\alpha_1 - \frac{t^2X\left(t^2-s_{2\alpha+\delta_2}/s_{\delta_2}\right)}{\left(t^2+1\right)^2}\right) & Y & \frac{2tX\epsilon\left(t^2c_{2\alpha}-1\right)s_{\alpha+\delta_2}/s_{\delta_2}}{\left(t^2+1\right)^2} & \frac{2t^3X\epsilon s_{2\alpha}s_{\alpha+\delta_2}/s_{\delta_2}}{\left(t^2+1\right)^2} \\ \frac{4\epsilon^2\left(t^2c_{2\alpha}-1\right)\left(\lambda_4\left(t^2+1\right)+8\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2} & \frac{2tX\epsilon\left(t^2c_{2\alpha}-1\right)s_{\alpha+\delta_2}/s_{\delta_2}}{\left(t^2+1\right)^2} & X + \frac{16\lambda_2\epsilon^2\left(t^2c_{2\alpha}-1\right)^2}{\left(t^2+1\right)^2} & \frac{16\lambda_2 t^2\epsilon^2 s_{2\alpha}\left(t^2c_{2\alpha}-1\right)}{\left(t^2+1\right)^2} \\ \frac{4t^2\epsilon^2 s_{2\alpha}\left(\lambda_4\left(t^2+1\right)+8\lambda_2 tc_\alpha\right)}{\left(t^2+1\right)^2} & \frac{2t^3X\epsilon s_\alpha s_{\alpha+\delta_2}/s_{\delta_2}}{\left(t^2+1\right)^2} & \frac{16\lambda_2 t^2\epsilon^2 s_{2\alpha}\left(t^2c_{2\alpha}-1\right)}{\left(t^2+1\right)^2} & X + \frac{16\lambda_2 t^4\epsilon^2 s_{2\alpha}^2}{\left(t^2+1\right)^2} \end{pmatrix}$$

First we decouple the **SM**-like Higgs h from the rest via a 2-1 rotation around θ:

$$m_h^2 = v^2\left(4\lambda_1 + \frac{64\lambda_2 t^2 c_\alpha^2}{(t^2+1)^2} + \frac{16\lambda_4 tc_\alpha}{t^2+1} - Y\tilde{\theta}^2\right)$$

$$\tilde{\theta} = \frac{\theta}{\epsilon} = \left(\frac{2\alpha_1}{Y} + \frac{X}{Y}\frac{t^2(1-t^2)}{1+t^2}\frac{\sin(2\alpha+\delta_2)}{\sin(\delta_2)}\right)$$

m_h and θ will be taken as input to solve for λ_1 and α_1

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

We are left with 4 neutral states in the basis $(\mathrm{Re}\varphi_{10}, \mathrm{Re}\Delta_R^0, \mathrm{Re}\varphi_{20}, \mathrm{Im}\varphi_{20})$

Remarkably, setting $\lambda_3 = 2\lambda_2$ allows to determine the remaining rotations *exactly*:

We rotate $(\mathrm{Re}\varphi_{10}, \mathrm{Re}\Delta_R^0, \mathrm{Re}\varphi_{20}, \mathrm{Im}\varphi_{20})^T = O_N(h, \Delta_R, H, A)^T$

$$\theta \equiv \epsilon\,\tilde{\theta} \equiv \theta_{21} = \epsilon\left[\frac{2\alpha_1}{Y} - \frac{2X\left(t^4 - t^2\, s_{2\alpha+\delta_2}/s_{\delta_2}\right)}{Y\left(t^2+1\right)^2}\right],$$

$$\phi \equiv \epsilon^2\tilde{\phi} \equiv \theta_{31} = \epsilon^2\frac{\left(t^2 c_{2\alpha}-1\right)}{(1+t^2)^2}\left[\frac{32tc_\alpha\lambda_2 + 4\lambda_4(1+t^2)}{X} - 2\,t\,\tilde{\theta}\, s_{\alpha+\delta_2}/s_{\delta_2}\right],$$

$$\theta_{41} = \epsilon^2\frac{t^2 s_{2\alpha}}{\left(1+t^2\right)^2}\left[\frac{32tc_\alpha\lambda_2 + 4\lambda_4(t^2+1)}{X} - 2\,t\,\tilde{\theta}\, s_{\alpha+\delta_2}/s_{\delta_2}\right],$$

$$\theta_{34} = \cot^{-1}\left[\cot(2\alpha) - \frac{\csc(2\alpha)}{t^2}\right],$$

$$\eta \equiv \theta_{23} = -\frac{1}{2}\tan^{-1}\left[\frac{4tX\epsilon\sqrt{t^4 - 2c_{2\alpha}t^2+1}\, s_{\alpha+\delta_2}/s_{\delta_2}}{\left(t^2+1\right)^2\left(Y\tilde{\theta}^2\epsilon^2 - \frac{16(t^4-2c_{2\alpha}t^2+1)\lambda_2\epsilon^2}{(t^2+1)^2} - X + Y\right)}\right]$$

$$h \text{ part of } \Re\Delta_R: \quad \theta \equiv \theta_{21} \simeq -(O_N)_{2,1},$$

$$H \text{ part of } \Re\Delta_R: \quad \eta \equiv \theta_{23} = \arcsin[(O_N)_{2,3}/c_\theta],$$

$$h \text{ part of } \Re\phi_{20}: \quad \phi \equiv \theta_{31} \simeq -(O_N)_{3,1},$$

θ, ϕ, η can be taken as **input** parameters!

Mixing angles also control scalar couplings to SM-gauge bosons & quarks!

Diagonalising the Lagrangian: Scalar sector

Inserting the minimisation conditions into $\mathscr{V}$ gives us the mass terms…

We rotate $(\mathrm{Re}\varphi_{10}, \mathrm{Re}\Delta_R^0, \mathrm{Re}\varphi_{20}, \mathrm{Im}\varphi_{20})^T = O_N(h, \Delta_R, H, A)^T$ and get the mass eigenvalues:

$$m_h^2 = \epsilon^2\left(4\lambda_1 + \frac{64\lambda_2 t^2 c_\alpha^2}{(t^2+1)^2} + \frac{16\lambda_4 t c_\alpha}{t^2+1} - Y\tilde{\theta}^2\right),$$
$$m_\Delta^2 = Y + \sec(2\eta)\left[(Y-X)s_\eta^2 + \epsilon^2\left(Y\tilde{\theta}^2 c_\eta^2 - \frac{16\lambda_2\left(t^4 - 2c_{2\alpha}t^2+1\right)}{(t^2+1)^2}s_\eta^2\right)\right]$$
$$m_H^2 = X - \sec(2\eta)\left[(Y-X)s_\eta^2 + \epsilon^2\left(Y\tilde{\theta}^2 s_\eta^2 - \frac{16\lambda_2\left(t^4 - 2c_{2\alpha}t^2+1\right)}{(t^2+1)^2}c_\eta^2\right)\right]$$
$$m_A^2 = X\,,$$

The masses m_h , m_Δ , m_H, m_A are taken as input parameters to determine the potential

And we get another sum rule:

$$m_{H^+}^2 - m_A^2 = v^2\alpha_3\frac{c_{2\beta}}{2} \sim O(150\,\mathrm{GeV})^2$$

Mass splitting $|m_H^2 - m_A^2|$ must be small to ensure perturbativity of λ_2 : $|m_H^2 - m_A^2| \lesssim 16v^2$

Diagonalising the Lagrangian: Fermions

The various vevs in the model induce tree-level mass terms for all fermions:

Only the bi-doublet couples to quarks, giving masses to up- and down-type quarks

$$\mathcal{L}_Y^q = \bar{Q}'_L \left(Y_q \, \phi + \tilde{Y}_q \, \tilde{\phi} \right) Q'_R + \text{H.c.} \,,$$
$$\mathcal{L}_Y^\ell = \bar{L}'_L \left(Y_\ell \, \phi + \tilde{Y}_\ell \, \tilde{\phi} \right) L'_R + $$
$$+ \bar{L}'^c_L i\sigma_2 \Delta_L Y_L^M L'_L + \bar{L}'^c_R i\sigma_2 \Delta_R Y_R^M L'_R + \text{H.c.} \,.$$

$$M_u = Y_q \, v_1 + \tilde{Y}_q \, e^{-i\alpha} v_2$$
$$M_d = -Y_q \, e^{i\alpha} v_2 - \tilde{Y}_q \, v_1$$

Which are diagonalised as:

$$M_u = U_{uL} \, m_u \, U_{uR}^\dagger \,, \qquad M_d = U_{dL} \, m_d \, U_{dR}^\dagger$$

From these mixings we can define the **CKM** and its **right-handed** (measurable) analogue:

$V_L^{\text{CKM}} \equiv U_{uL}^\dagger U_{dL}$, $V_R^{\text{CKM}} \equiv U_{uR}^\dagger U_{dR}$ (V_R can has additional phases in the case of $\mathscr{C}$)

The quark Yukawas are then fully determined from measurable inputs: quark masses and mixings

$$Y_q = \frac{1}{v_1^2 - v_2^2} \left(M_u \, v_1 + e^{-i\alpha} M_d \, v_2 \right)$$
$$\tilde{Y}_q = -\frac{1}{v_1^2 - v_2^2} \left(M_d \, v_1 + e^{i\alpha} M_u \, v_2 \right)$$

Diagonalising the Lagrangian: Fermions

Both triplets and the bi-doublet induce mass-terms for the leptons:

$$M_\ell = -Y_\ell v_2 e^{i\alpha} + \tilde{Y}_\ell v_1\,, \quad M_D = Y_\ell v_1 - \tilde{Y}_\ell v_2 e^{-i\alpha}\,, \quad M_L = v_L Y_L^M\,, \quad M_R = v_R Y_R^M$$

In which M_D is a mass-term between **LH** and RH neutrinos, M_L and M_R are Majorana

The charged lepton mass M_ℓ is easily diagonalised: $M_\ell = U_{\ell L}\, m_\ell\, U_{\ell R}^\dagger$

And the Yukawas of the bi-doublet are given by:

$$Y_\ell = \frac{1}{v_1^2 - v_2^2}\left(M_D\, v_1 + M_\ell\, e^{-i\alpha} v_2\right)$$

$$\tilde{Y}_\ell = -\frac{1}{v_1^2 - v_2^2}\left(M_\ell\, v_1 + M_D\, e^{i\alpha} v_2\right)$$

Due to M_D, **LH** and **RH** neutrinos mix with each other, we need to diagonalise the mass matrix:

$$\bar{n}_L' M_n n_L^{'c} = \left(\bar{\nu}_L'\ \bar{\nu}_R^{\prime c}\right)\begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}\begin{pmatrix} \nu_L^{\prime c} \\ \nu_R' \end{pmatrix}$$

Diagonalising the Lagrangian: Fermions

Due to M_D, **LH** and **RH** neutrinos mix with each other, we need to diagonalise the mass matrix:

$$\bar{n}'_L M_n n'^c_L = (\bar{\nu}'_L \; \bar{\nu}'^c_R) \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu'^c_L \\ \nu'_R \end{pmatrix}$$

Majorana mass matrix is complex symmetric: (block-) diagonalised via Autonne-Takagi factorisation:

$$\tilde{W}^T M_n \tilde{W} = \begin{pmatrix} m_{\text{light}} & \mathbb{0} \\ \mathbb{0} & m_{\text{heavy}} \end{pmatrix}$$

Perturbative diagonalisation (expand in M_R^{-1}) gives us:

$$M_\nu \simeq M_L - M_D M_R^{-1} M_D^T \,, \qquad M_N \simeq M_R$$

In which the blocks are diagonalised via the unitary matrices V_ν and V_N:

$$V_\nu^T M_\nu V_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \,, \quad V_N^\dagger M_N V_N^* = \text{diag}(m_{N_1}, m_{N_2}, m_{N_3})$$

Diagonalising the Lagrangian: Fermions

The full rotation matrix is approximately given by (up to M_R^{-1}):

$$W = \begin{pmatrix} \sqrt{\mathbb{1} - BB^\dagger} V_\nu & BV_N^* \\ -B^\dagger V_\nu & \sqrt{\mathbb{1} - B^\dagger B} V_N^* \end{pmatrix} \simeq \begin{pmatrix} V_\nu & B_1 V_N^* \\ -B_1^\dagger V_\nu & V_N^* \end{pmatrix} .$$

With $B_1 = M_D^\dagger M_R^{-1\dagger}$

Charged lepton currents can be cast as:

$$\mathcal{L}_{cc}^{\ell} = \frac{g_L}{\sqrt{2}} \bar{\ell}_L \gamma^\mu \mathcal{U}_L n_L W_L^\mu + \frac{g_R}{\sqrt{2}} \bar{\ell}_R \gamma^\mu \mathcal{U}_R n_R W_R^\mu$$

With the 3×6 mixing matrices given by:

$$(\mathcal{U}_L)_{\alpha i} = \sum_{k=1}^{3} (V_{\ell L}^\dagger)_{\alpha k} W_{ki} ,$$
$$(\mathcal{U}_R)_{\alpha i} = \sum_{k=1}^{3} (V_{\ell L}^\dagger)_{\alpha k} W_{(k+3)i} .$$

The first 3×3 block of $\mathcal{U}_L$ can be identified as the **LH** would-be PMNS, the second 3×3 block of $\mathcal{U}_R$ as its **RH** analogue

$\mathcal{U}_R$ could be measured in $W_R^\pm \to \ell N$ decays