Local basis truncation for Quantum Field Theories

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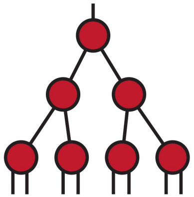
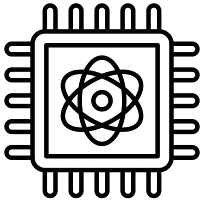
Motivation

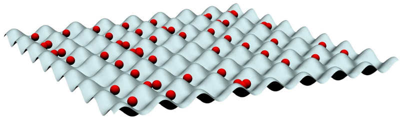
- Tensor Network (TN) methods: Extension to perturbation theory and Monte Carlo

Pros:
State of the art in 1d
non-perturbative and sign-problem free
non-equilibrium dynamics is accessible

Cons:
Efficient for low entanglement states
Hard in d>1


Tensor Network


Quantum Computing

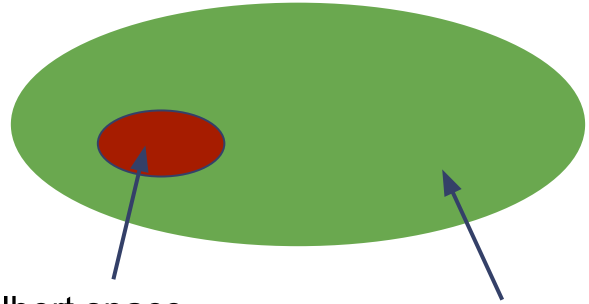

Quantum Simulations

Toolkit:

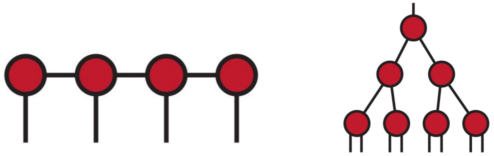
Quantum many body wavefunction:

$$|\psi\rangle = \sum_{\{i_1,\dots,i_n\}} C_{i_1,\dots,i_n} |i_1, \dots, i_n\rangle$$

$$\sim d^N$$



Ansätze:



Computational cost, example:

$$\mathcal{O}(d \cdot L \cdot BD^3)$$

Why does it work?

Many physically relevant states obey entanglement area laws.

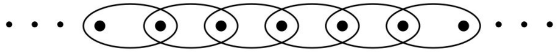
Collaborators: Giovanni Cataldi, Giuseppe Magnifico, Pietro Silvi, Simone Montangero

Optimal local basis truncation of lattice quantum many-body systems

Method:

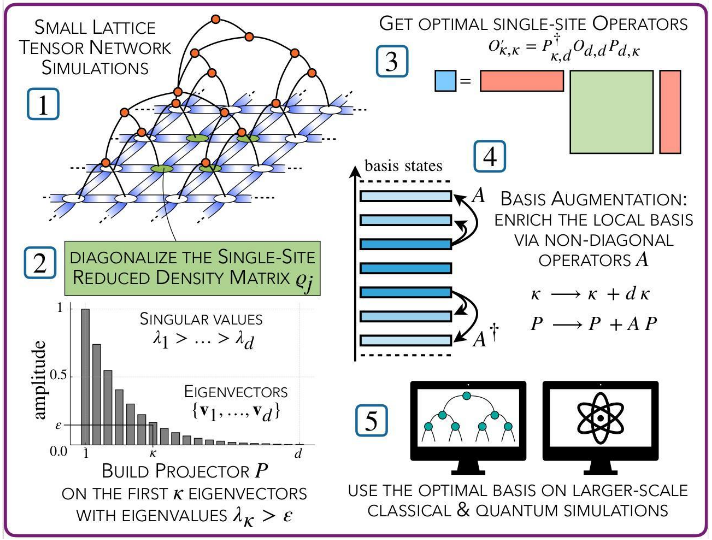
- Estimate reduced density ρ_i

$\longrightarrow$ MF-theory, TN, Semi-definit programming



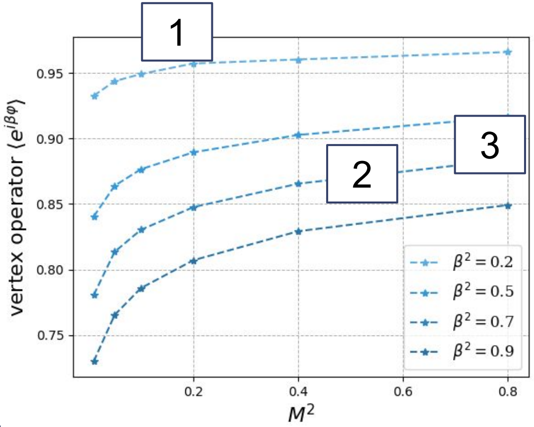
- Define projection P_i

- Effective Hamiltonian $O_i^{\text{eff}} = P^\dagger O_i P$

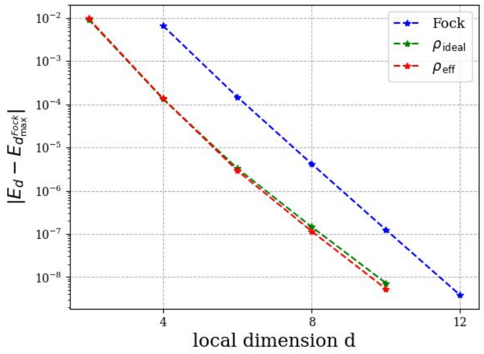


sG-model:

$$aH = \sum_{j\in\Lambda} \pi_j^2 + \frac{1}{2}\left[(\varphi_{i+1} - \varphi_j)^2 - \frac{M^2}{\beta^2}\cos(\beta\varphi_j)\right]$$



1



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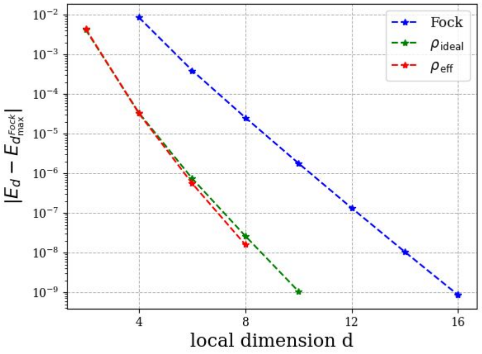

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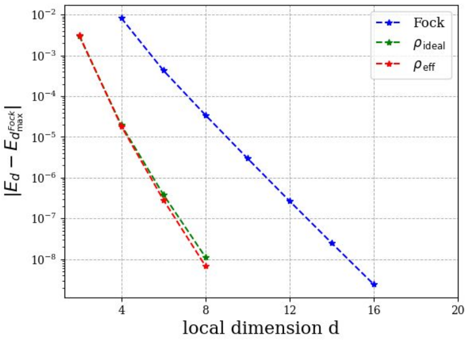
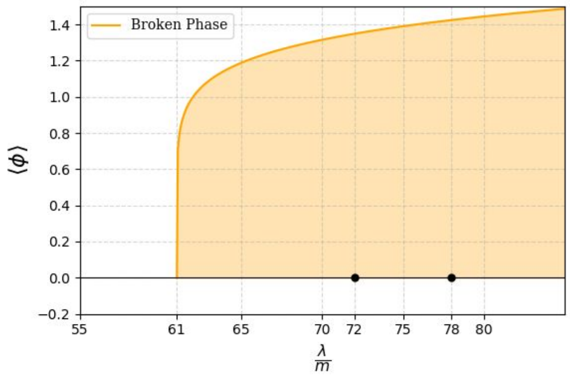
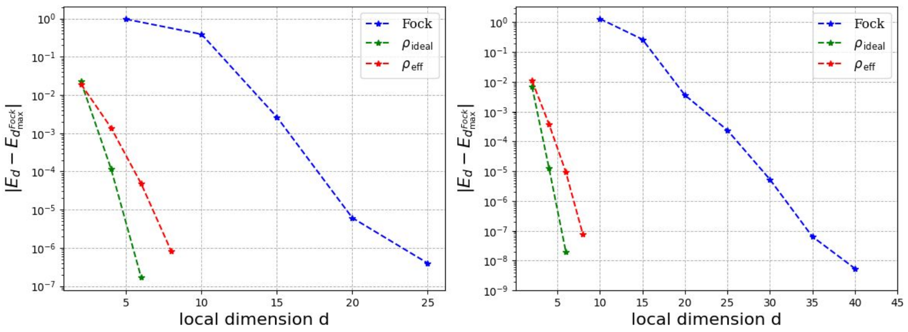


φ^4 -model:

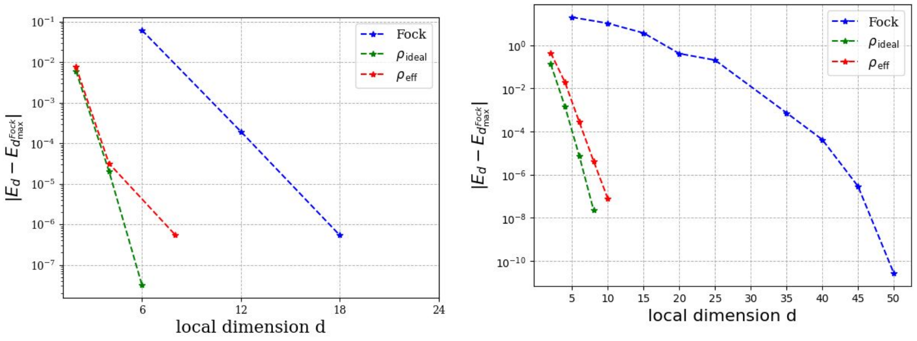
$$aH = \sum_{j\in\Lambda}[\frac{1}{2}\pi_j^2 + \frac{(\varphi_{j+1}-\varphi_j)^2}{2} + \frac{a^2m^2}{2}\varphi_j^2 + \frac{a^2\lambda}{4!}\varphi_j^4]$$



1D:



2D:

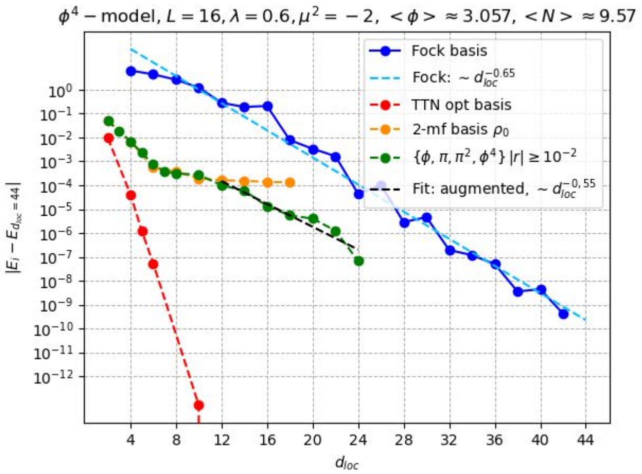


Basis augmentation:

- Estimate reduced density ρ_i
- A priori, we don't know how good ρ_i is
- Construct again the projector P_i

→ extent basis by acting with local operators

$$\left| v_i' - \sum_l \langle v_i' | v_l \rangle \, v_{l_2} \right| > r$$



Collaborators: Marco Rigobello, Mattia Morgavi

Lattice gauge theory dressed sites

Credits: Marco Rigobello

Hamiltonian LGT:

$$\mathrm{H} = \underbrace{\frac{i}{2}\sum_{x,k}\psi_x \overset{\text{parallel transporter}}{\boxed{U_{x,k}}}\psi^{\dagger}_{x+k} + \mathrm{h.c}}_{\text{fermion hopping}} + \underbrace{m\sum_x(-1)^x\psi^{\dagger}_x\psi_x}_{\text{fermion mass}} + \underbrace{\frac{g^2}{2}\sum_{x,k}E^2_{x,k} + \frac{1}{g^2}\sum_{\Box} P}_{\text{electric+magnetic energy}}$$

Gauss law :

$$G^a_x\,|\psi_{\text{phys}}\rangle = \left[\sum_{k=1}^{D}\boxed{\left(L^a_{x,k} - R^a_{x,k}\right)} - \boxed{Q^a_x}\right]|\psi_{\text{phys}}\rangle = 0 \quad \forall x, a$$

Truncation of gauge-dof:

- The change of basis from group element basis $\{|g\rangle\}$ to the representation basis $\{|jmn\rangle\}$ is given by

$$\langle g|jmn\rangle = \sqrt{\frac{\dim(j)}{\mathrm{Vol}(G)}}[\pi_j(g)]_{mn}$$

- Naturally the parallel transporter can then be decomposed as:

$$\langle j'm'n'|U^J_{M,N}|jmn\rangle = \sqrt{\frac{\dim(j)}{\dim(j')}}\sum_\alpha \quad \overline{C^{j'n',\alpha}_{jmJM}}C^{j'm',\alpha}_{jnJN}$$

Encoding for TN*:

- Discard un-physical states: $\mathcal{H} = \otimes_x \mathcal{H}_x$

$$\boxed{[G_x, G_y] = 0}$$

- To preserve the tensor product structure, we “double” symmetry content :

$$|j, m_L, m_R\rangle \longrightarrow |j, m_L\rangle \otimes |j, m_R\rangle$$

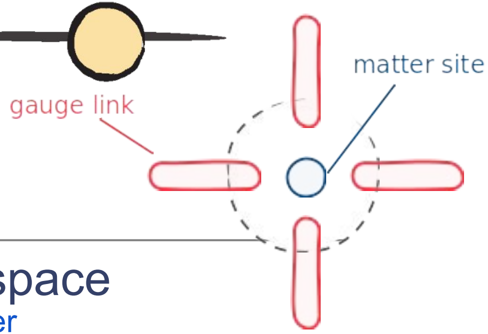
(in truncated irrep-basis)

split in semi-links

- Fuse site and surrounding semi-links and find singlets

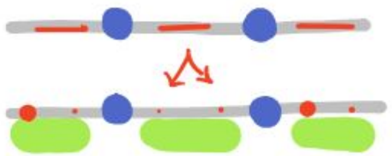
- Impose abelian constraint on semi-links

*Silvi et al. PRD 100 074512 (2019)

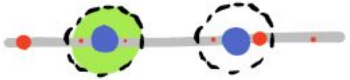


Algorithm

❶ split links



❷ combine site + semi-links



❸ select colour singlets (**Gauss law**)

local Hilbert space

fermionic matter

single component
$\{\psi_x, \psi_y^\dagger\} = \delta_{xy}$

bosonic gauge link

$|g\rangle \ \forall\, g \in \mathbf{SU}(3)$
infinite dimensional

Peter-Weyl $\mathscr{H} = \bigoplus_j (j \otimes j^\star)$
$|j, m, \bar{m}\rangle \ \forall$ irrep $j,\ m \in j,\ \bar{m} \in j^\star$

algebra $[E^a, U^\dagger] = iT^a U^\dagger$
$U\,|j, m, \bar{m}\rangle =$ Clebsch-Gordan
$E^2\,|j, m, \bar{m}\rangle = C_2(j)$ **Casimir**

Input:
(1+d)D
background charge
matter content
gauge group
truncation scheme

What gauge groups?

$$Z_{\mathrm{N}}, U(1), SU(N)$$

Non-simple groups:

$$U(1) \otimes SU(2) \otimes SU(3)$$

```
links: 2
gauge:
  G:
    group: SU 3
    num-C2-levels: 1
matter:
  u:
    irreps:
      G: 1 0 0
  d:
    irreps:
      G: 1 0 0
```

Output:
Operators

(gauge invariant)
(abelian constraint)
(local interaction)

It's modular! You can add your favourite gauge group or truncation.

ℓ	d					
	(2 + 1)-dimensions			(3 + 1)-dimensions		
	U(1)	SU(2)	SU(3)	U(1)	SU(2)	SU(3)
1	35	30	164	267	178	3096
2	165	168	752	3437	3670	52476
3	455	600	3738	18487	35280	813438
4	969	1650	19878	64953	214958	17490134
5	1771	3822	43698	177155	967466	69232482
6	2925	7840	82128	408421	3509062	228461186
7	4495	14688	212496	835311	10828494	1245755754
8	6545	25650	333538	1561841	29473038	2782999996

Magnifico, Cataldi, Rigobello, PM et al.
arXiv: 2407.03058

Summary:

Estimating reduced density matrix ρ_i

efficient representation of local degrees of freedom

TN-simulation   Quantum simulation   Quantum computing

Dressuag package: Efficient representaion of local degrees of freedom of gauge theories for TN-simulations

Collaborators:

Giovanni Cataldi Marco Rigobello

Mattia Morgavi

Pietro Silvi Simone Montangero

Thank you!

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