Scalable quantum algorithm for Meson Scattering

Yahui Chai,

In collaboration with Yibin Guo, Stefan Kuehn

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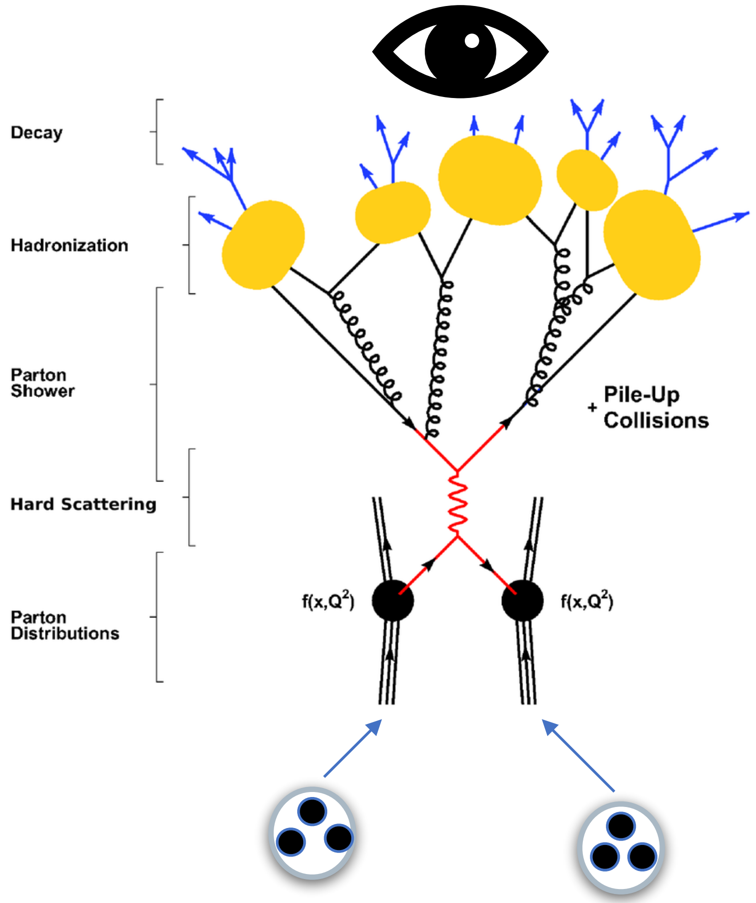
Motivation

- Experiments on colliders: verify the theory, probe the particle structure.



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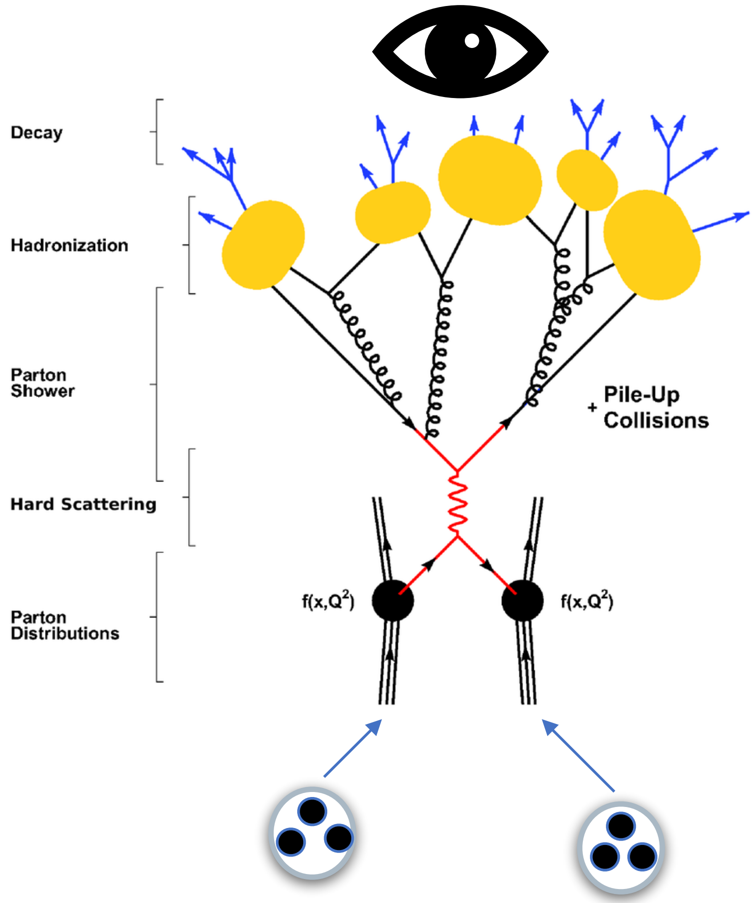
- Real time dynamics in classical simulation
  - Conventional Monte Carlo: sign problem
  - Tensor Network : increasing entanglement with time-- increasing computation resource



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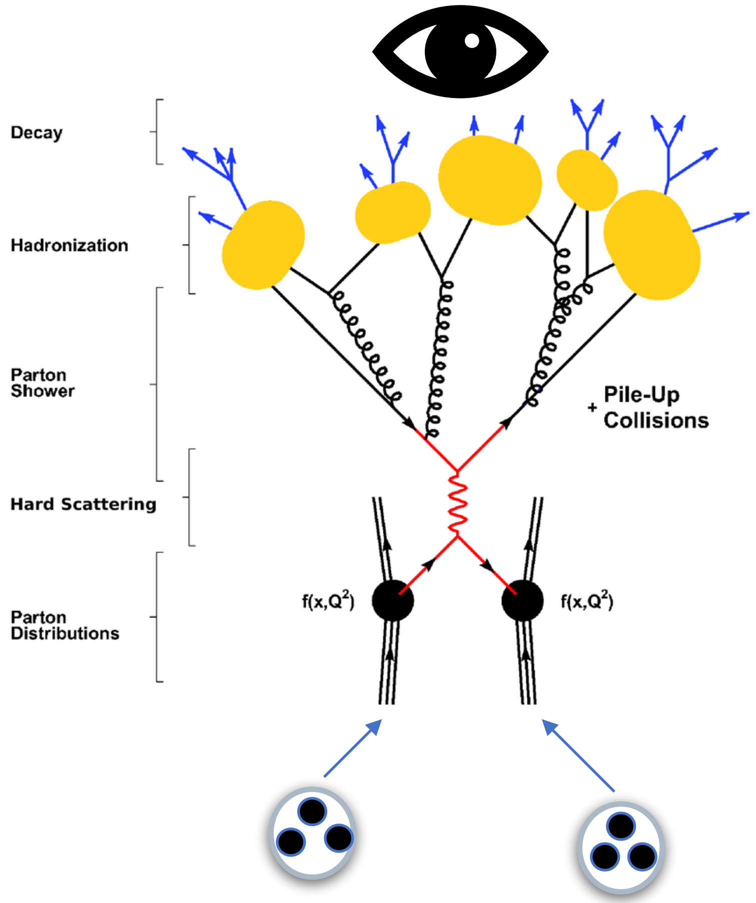
- Quantum computers promise to efficiently simulate real-time dynamics



Scattering process on QC

1. Vacuum state $|\Omega\rangle$: ground state of Hamiltonian

QC: VQE 😎

2. Initial state $|\psi(t=0)\rangle = B_1^\dagger B_2^\dagger |\Omega\rangle$: wave packets of particles

QC: only unitary operator ☹️

3. Time evolution: $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$

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Lattice $\mathbb{Z}_2$ gauge theory in 1+1 D

- Staggered fermion coupled with $\mathbb{Z}_2$ gauge field, periodic condition

$$H = \frac{1}{2a}\sum_{n=1}^{L}\left(\xi_n^\dagger Z_{g,n}\xi_{n+1} + \text{h.c.}\right) + m\sum_{n=1}^{L}(-1)^n \xi_n^\dagger \xi_n + \varepsilon \sum_{n=1}^{L} X_{g,n}$$

Kinetic term

Mass term

Electric field term

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Kinetic term

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- Quantum number: charge conjugation and momentum

$$C\xi_n C^{-1} = (-1)^n \xi_{n+1}^\dagger \qquad CZ_{g,n}C^{-1} = Z_{g,n+1}$$

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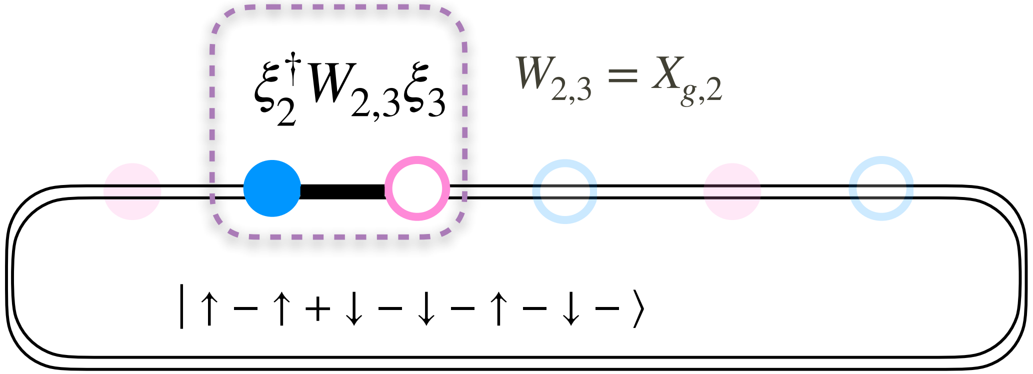
- Strong coupling limit $\varepsilon \to \infty$

$W_{2,3} = X_{g,2}$

Lightest meson:



How to create a meson state?

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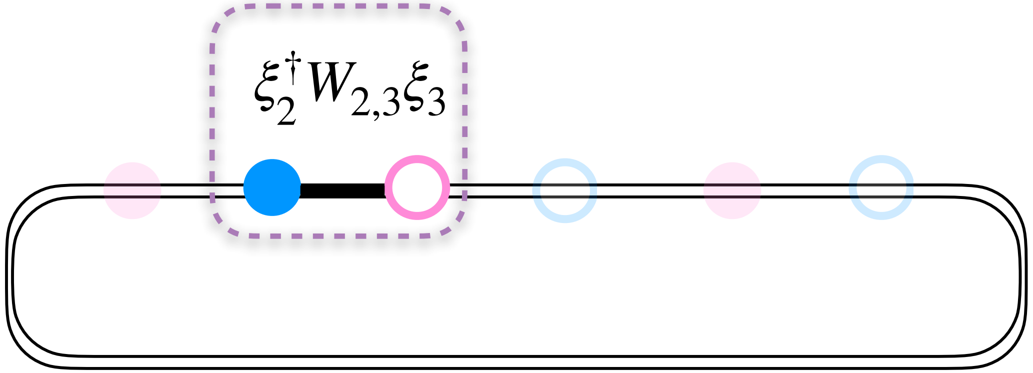


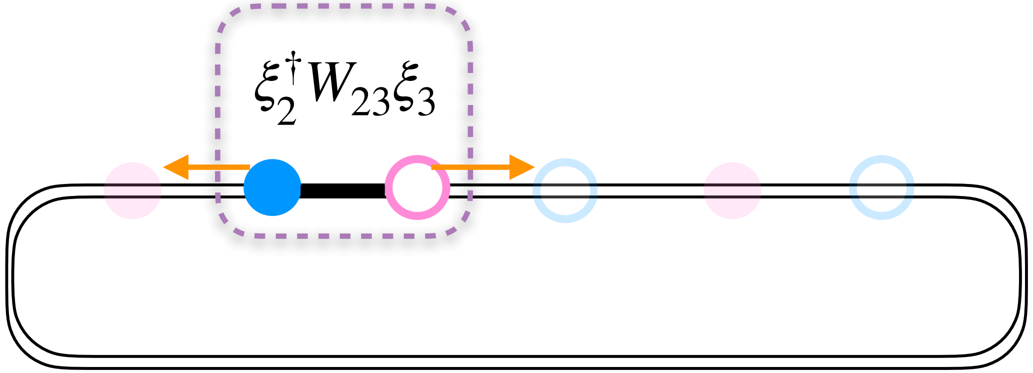
- Meson eigenstate

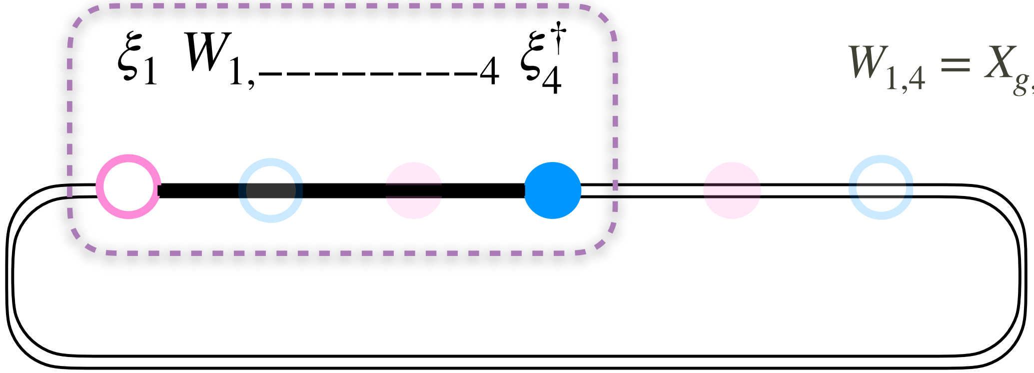
$$|k = 0,\ c = -1\rangle_b = \sum_n \left(\xi_n^\dagger Z_{g,n} \xi_{n+1} - \text{h.c.} \right) |\Omega\rangle_\infty$$

Momentum — Charge conjugation

How to create a meson state?

- General coupling ε





$W_{1,4} = X_{g,1} \; X_{g,2} \; X_{g,3}$

Meson Creation Operator

$$b^{\dagger}_{k,c} = \sum_I a_I^{(k,c)} M_I$$

Operators, $M_I \equiv M_{(n,l)} \in \{\xi_n^{\dagger} W_{n,l} \xi_l \,|\, n, l = 1,2,\cdots L\}$

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$$b^\dagger_{k,c} = \sum_I a_I^{(k,c)} \; M_I$$

$$|\, k, c\rangle_b = b^\dagger_{k,c}\,|\,\Omega\rangle = \sum_I a_I^{(k,c)} \; M_I\,|\,\Omega\rangle$$

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$$H|k,c\rangle_b = E|k,c\rangle_b$$

$$C|k,c\rangle_b = ce^{ik}|k,c\rangle_b$$

$c = -1$, vector meson;
$c = 1$, scalar meson

Quantum subspace expansion

$$b_{k,c}^{\dagger} = \sum_I a_I^{(k,c)} M_I$$

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- QSE equation

$$\left(\mathscr{H} + \mathscr{C}\right) \vec{a}^{(k,c)} = \lambda \; \mathscr{S} \vec{a}^{(k,c)},$$

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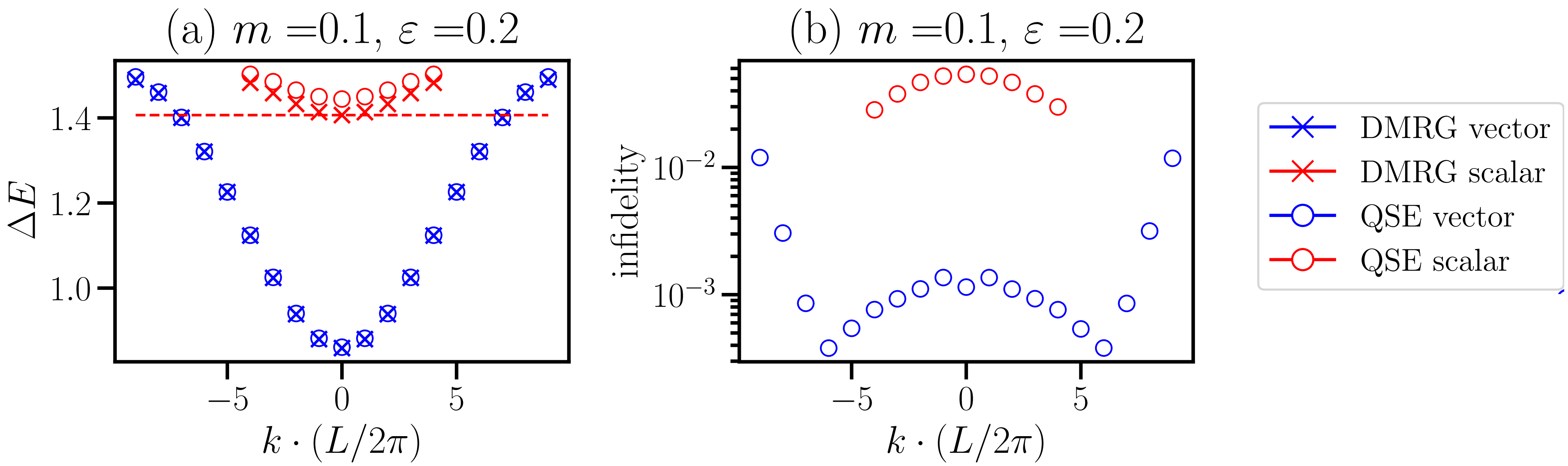
$$\lambda = E + c e^{-ika}$$

Results from QSE for $L = 30$

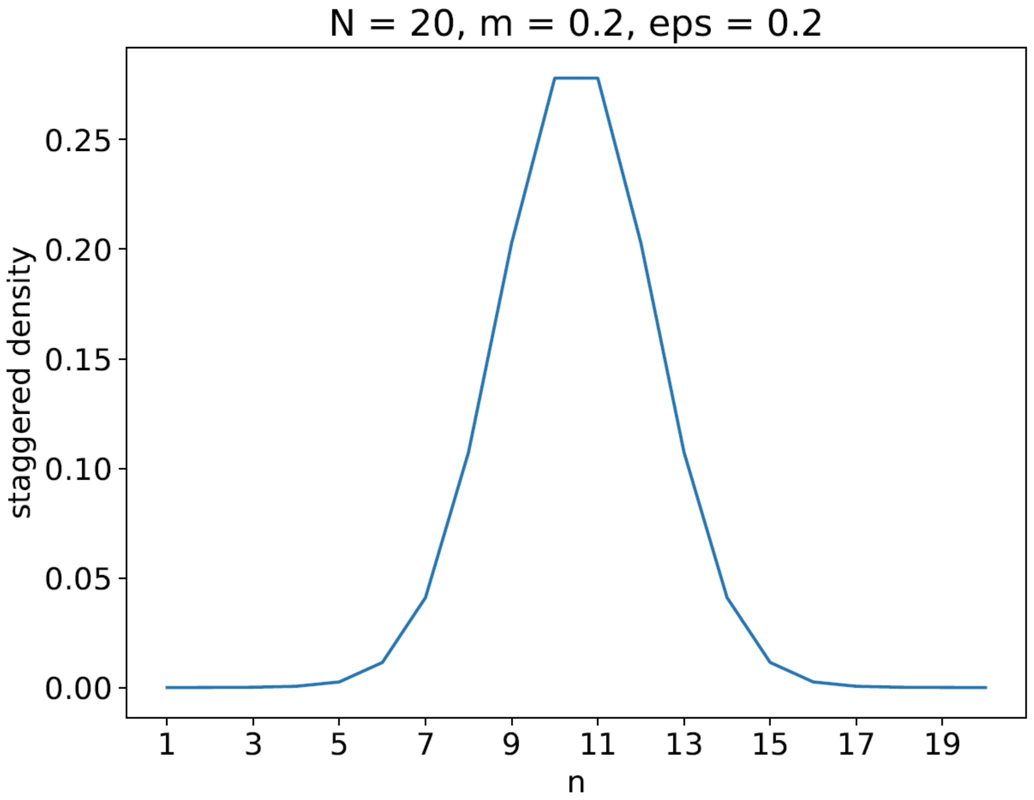
- With coefficients from QSE: $|k, c\rangle_b = \sum_I a_I^{k,c} M_I |\Omega\rangle$



Wave packet of meson

$$B^\dagger_{\bar{k},\bar{x}} = \sum_{k\in\Lambda^*} \phi(k)_{\bar{k},\bar{x}}\ b^\dagger_{k,-1} \qquad \phi(k)_{\bar{k},\bar{x}} = \frac{1}{\sqrt{\mathcal{N}_\phi}} \exp(-ik\bar{x})\ \exp\left(-(k-\bar{k})^2/(4\sigma_k^2)\right)$$

$B^\dagger_{\bar{k},\bar{x}}|\Omega\rangle$:



Staggered fermion density $\Delta\chi_n$

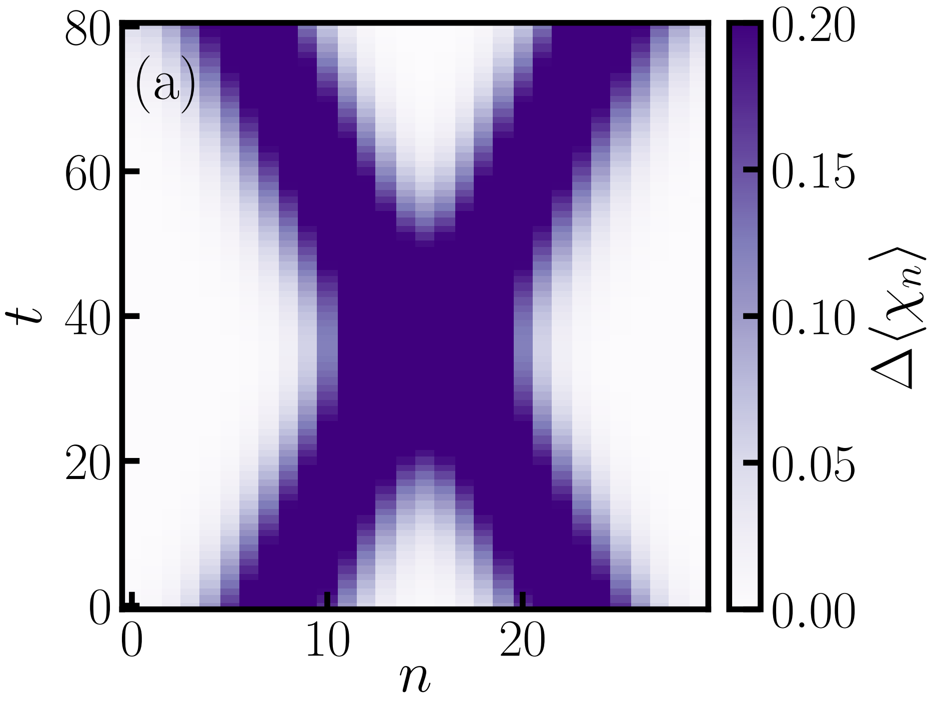
$$\chi_n = \begin{cases} 1 - \langle\psi(t)|\xi_n^\dagger\xi_n|\psi(t)\rangle, & n \in \text{odd}, \\ \langle\psi(t)|\xi_n^\dagger\xi_n|\psi(t)\rangle, & n \in \text{even}. \end{cases}$$

Scattering

$$B^\dagger_{\bar{k},\bar{x}} = \sum_{k\in\Lambda^*} \phi(k)_{\bar{k},\bar{x}}\; b^\dagger_{k,-1} \qquad \phi(k)_{\bar{k},\bar{x}} = \frac{1}{\sqrt{\mathcal{N}_\phi}} \exp(-ik\bar{x})\;\; \exp\left(-(k-\bar{k})^2/(4\sigma_k^2)\right)$$

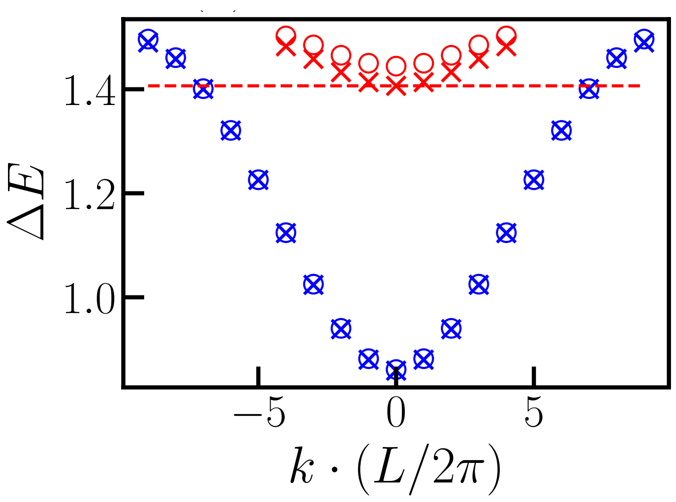
- Scattering:
  1. Initial state: $|\psi(t=0)\rangle = B^\dagger_{\bar{k},\bar{x}_1} B^\dagger_{-\bar{k},\bar{x}_2} |\Omega\rangle$
  2. Time evolution: $|\psi(t)\rangle = e^{-iHt} |\psi(t=0)\rangle$

Trotterization



Elastic Scattering

Inelastic Scattering



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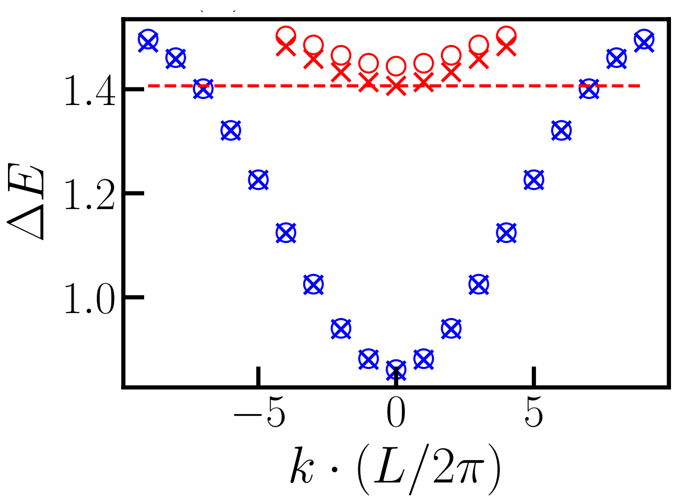

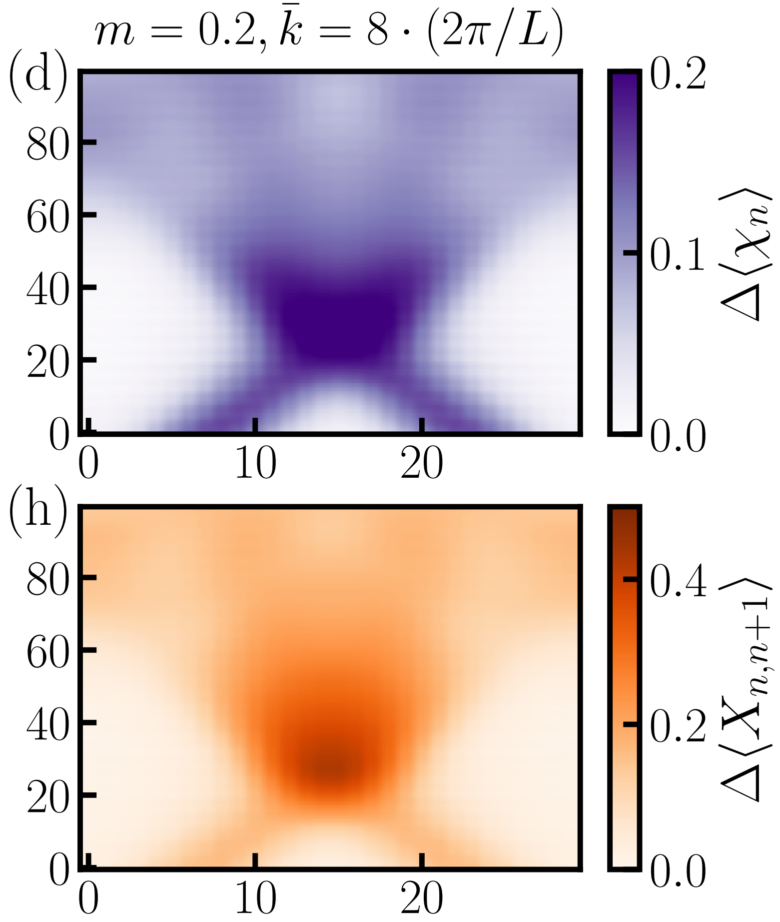

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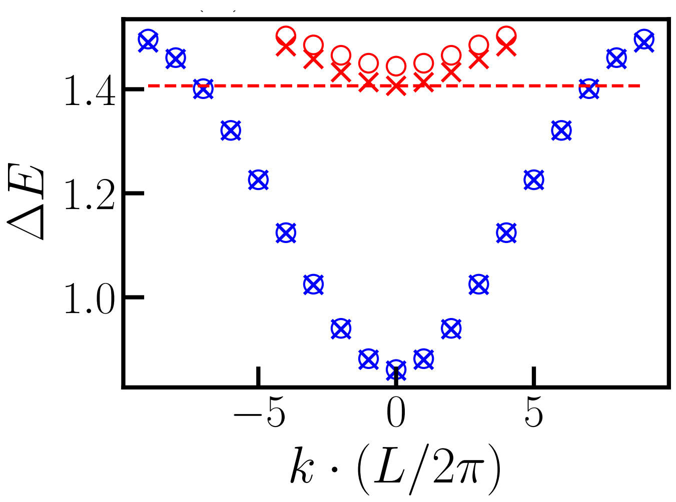
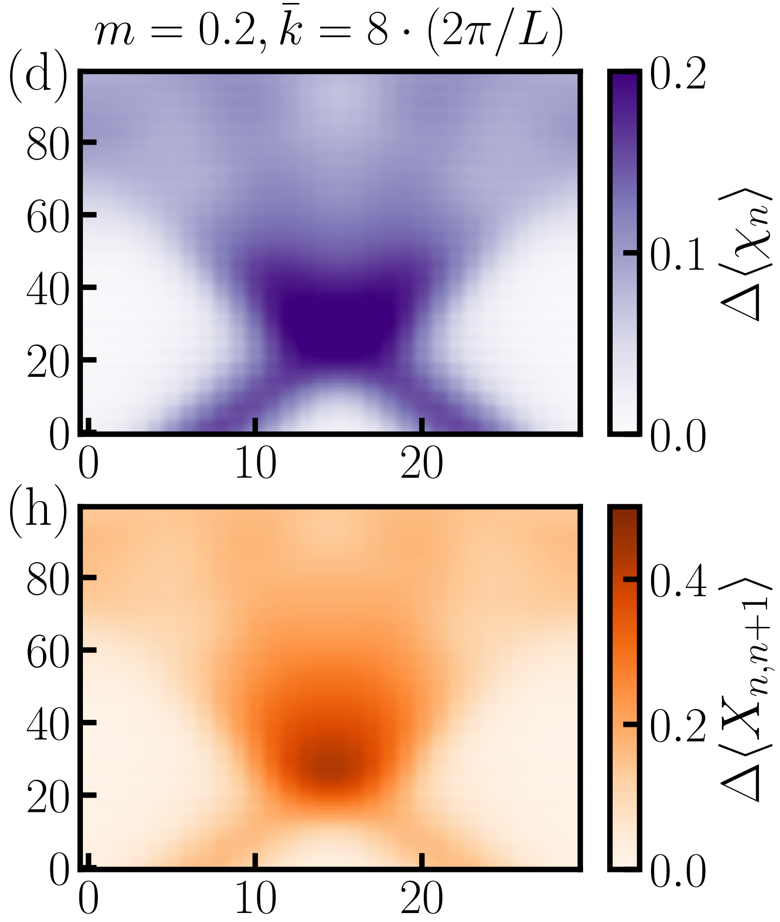
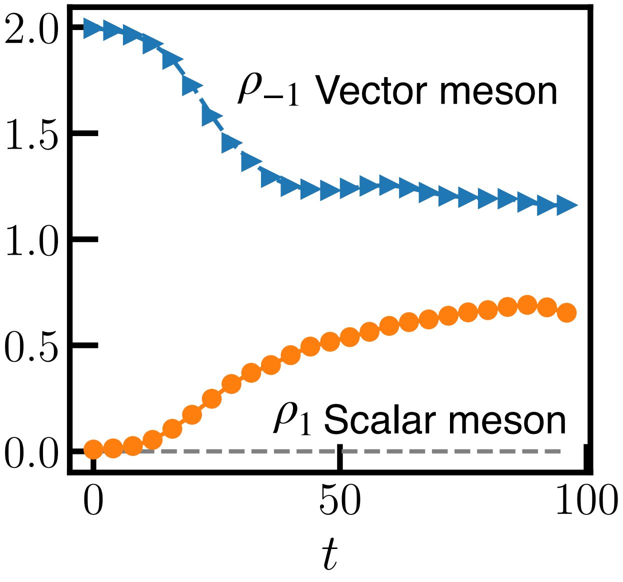




meson number

$$\rho_c = \sum_{k\in\Lambda^*} \langle\psi(t)\,|\,b^\dagger_{k,c} b_{k,c}\,|\,\psi(t)\rangle$$



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Quantum circuit for wave packet

- Non-unitary $$B^\dagger_{\bar{k},\bar{x}} = \sum_{k\in\Lambda^*} \phi(k)_{\bar{k},\bar{x}}\, b^\dagger_{k,-1},$$
$$= \sum_{k\in\Lambda^*} \sum_{nl} \phi(k)_{\bar{k},\bar{x}} a^{k,-1}_{nl} \; \xi^\dagger_n W_{n,l} \xi_l,$$

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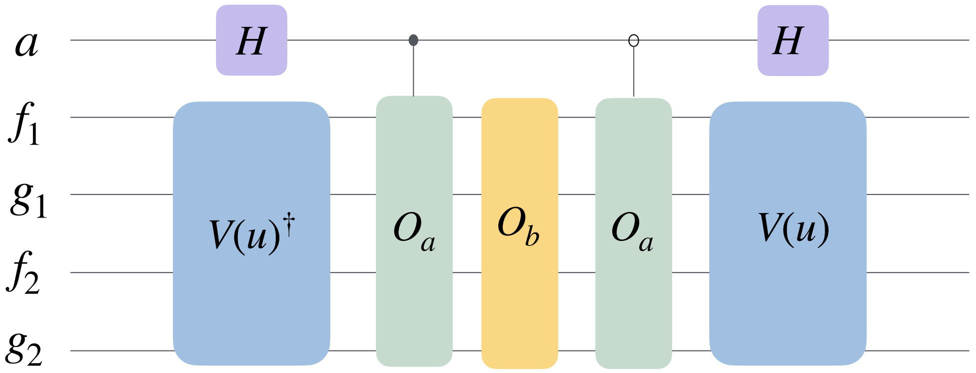

	CNOT number	CNOT depth
$V(u,\tilde{\xi})$ or $V(u,\tilde{\xi})^\dagger$	$2L(L-1)$	$4(2L-3)$
O_a	$4(L-1)$	$4(L-1)$
O_b	$12(L-1)$	$12(L-1)$
Total	$4L^2+16L-20$	$36L-44$

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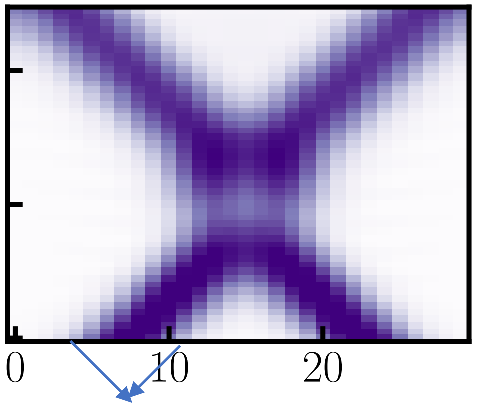
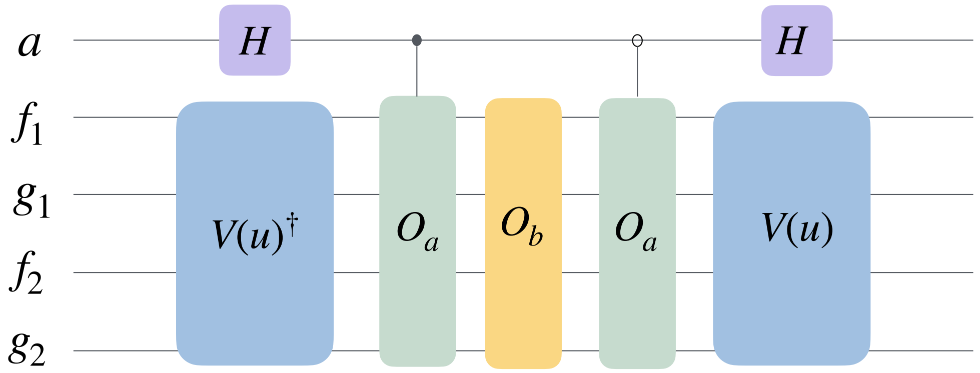
$m = 0.1, \bar{k} = 4 \cdot (2\pi/L)$



$L \to \sigma_x \sim 1/\sigma_k$



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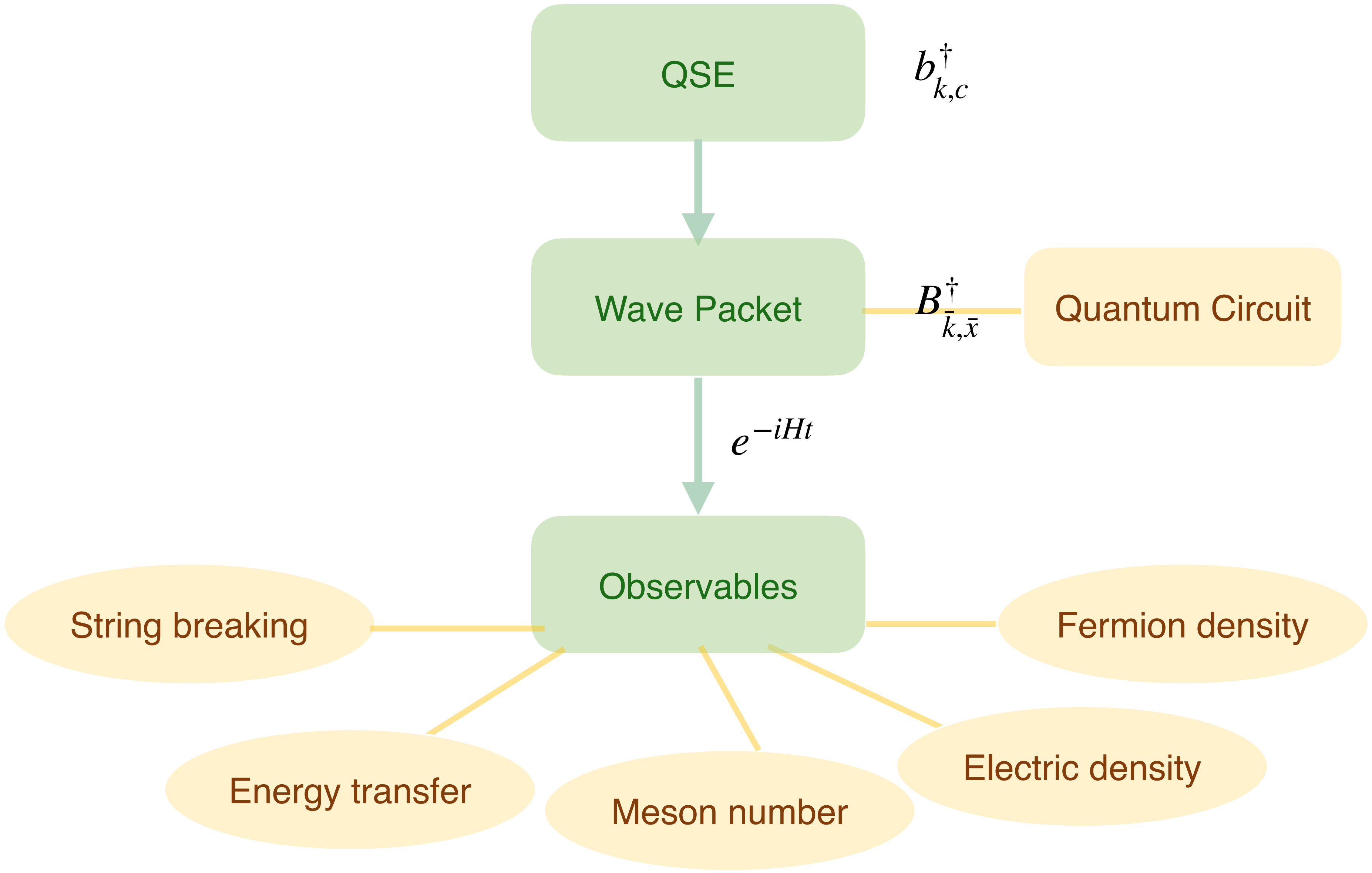
Summary: framework of meson scattering in DQC



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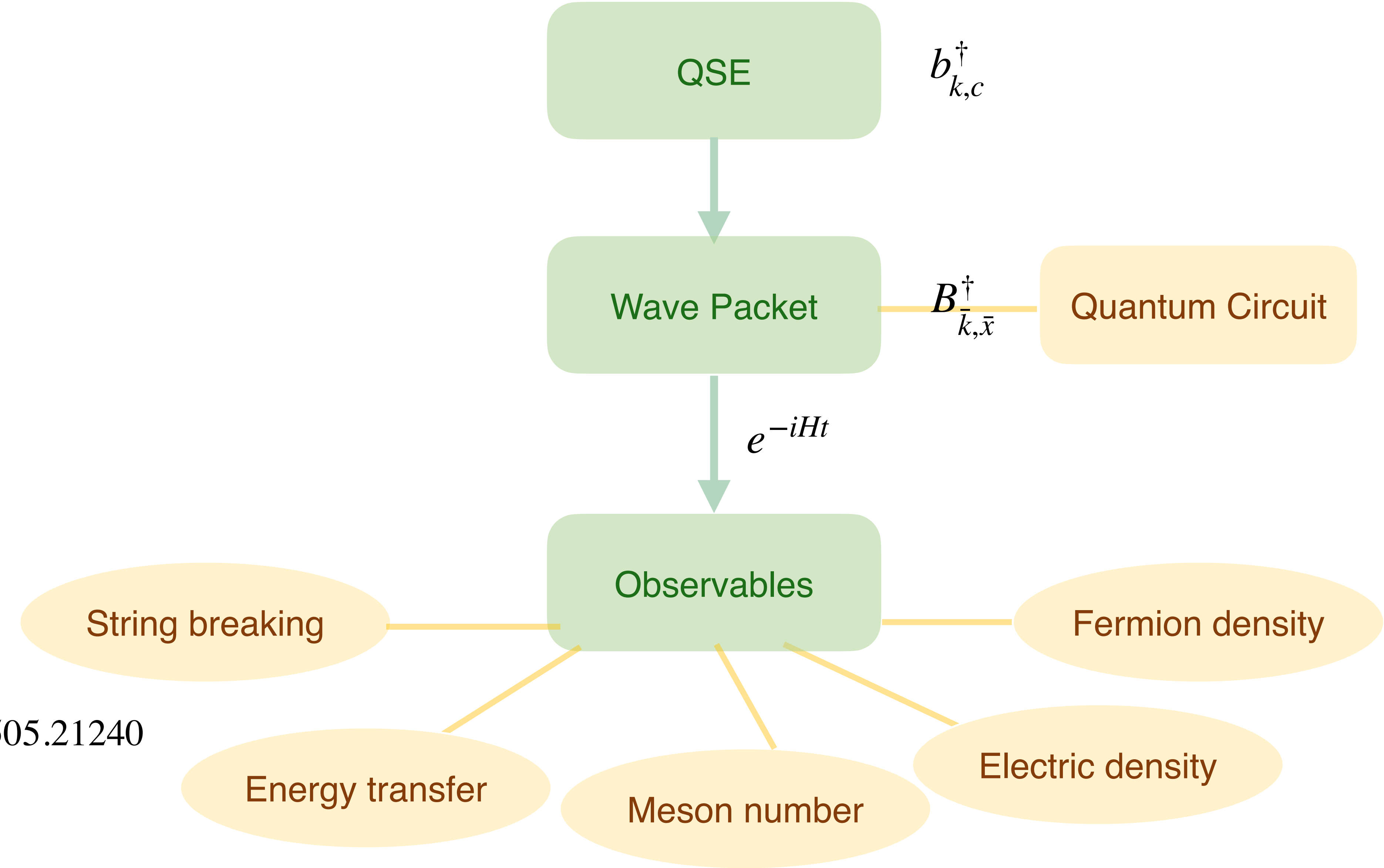

arXiv:2505.21240

Outlook

- Other Gauge theory, e.g., QED
- Higher dimension
- Improvement QSE
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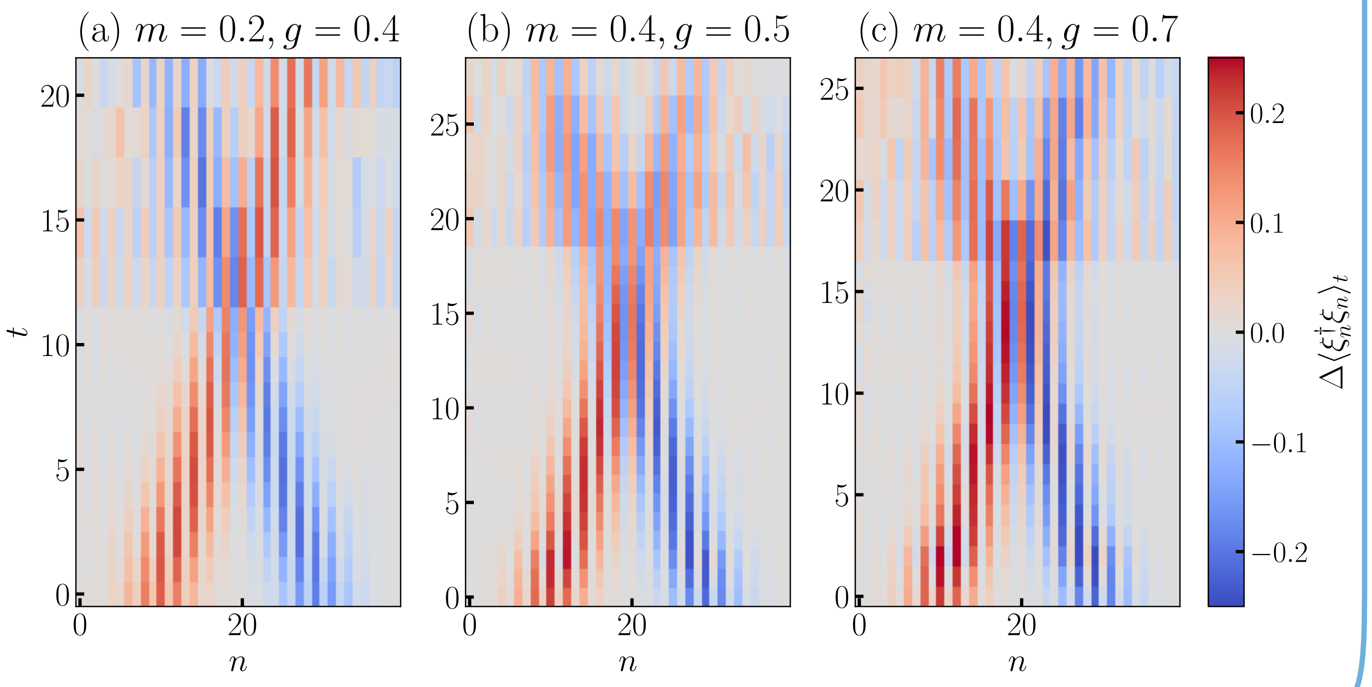
Hardware run for fermion scattering
40 qubits, ibm_fez



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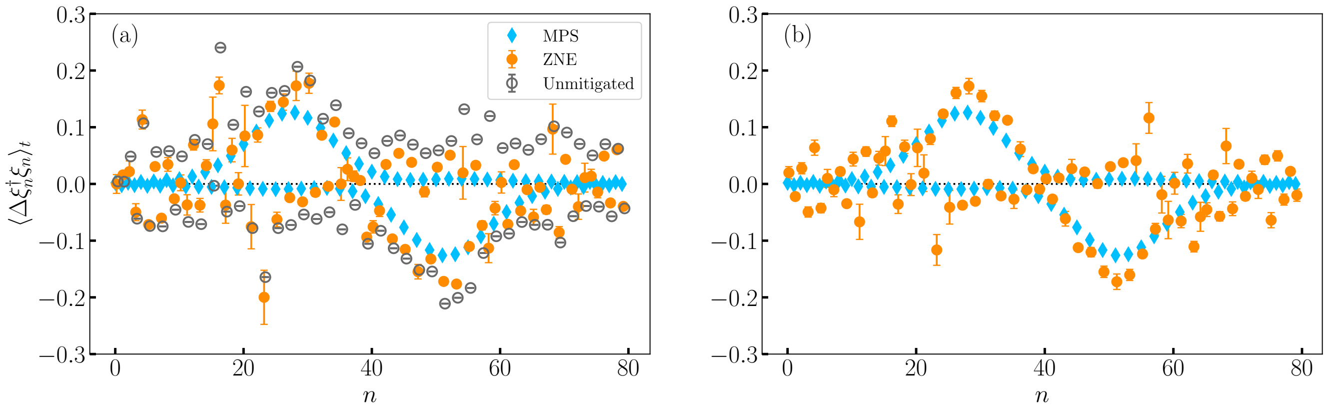
- Local observables

Hardware run for state preparation
80 qubits, ibm_fez



Thank you!

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Francesco Tacchino (IBM Zürich)

Ivano Tavernelli (IBM Zürich)

Yibin Guo (CQTA)

Joe Gibbs
University of Surrey

Zoe Holmes
EPFL

First fermion scattering paper

Y. Chai, A. Crippa, K. Jansen, S. Kühn, V. R. Pascuzzi, F. Tacchino, and I. Tavernelli, Quantum **9**, 1638 (2025).

Meson scattering paper

arXiv:2505.21240

Fermion scattering hardware run for 40 and 80 qubits: soon?

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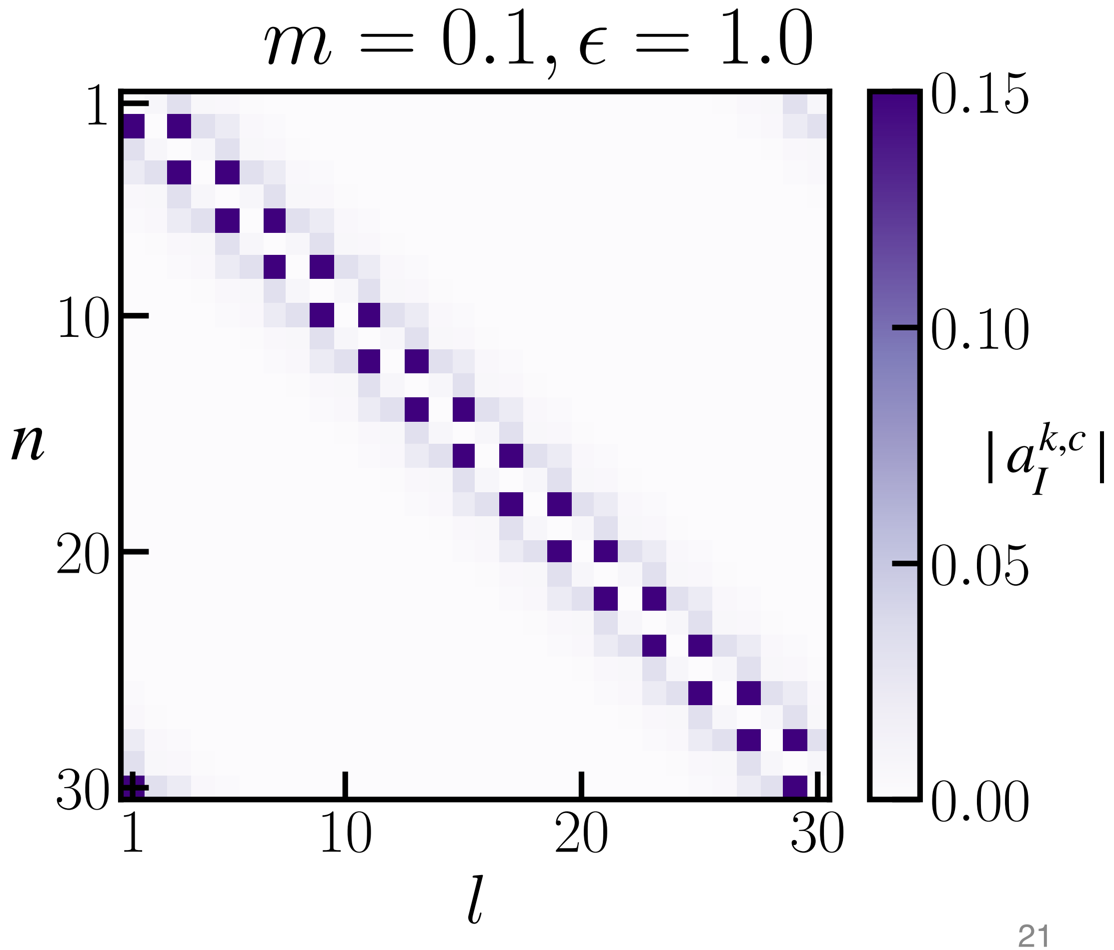
Results from QSE

- With coefficients from QSE: $|k,c\rangle_b = \sum_I a_I^{k,c} M_I \, |\Omega\rangle$



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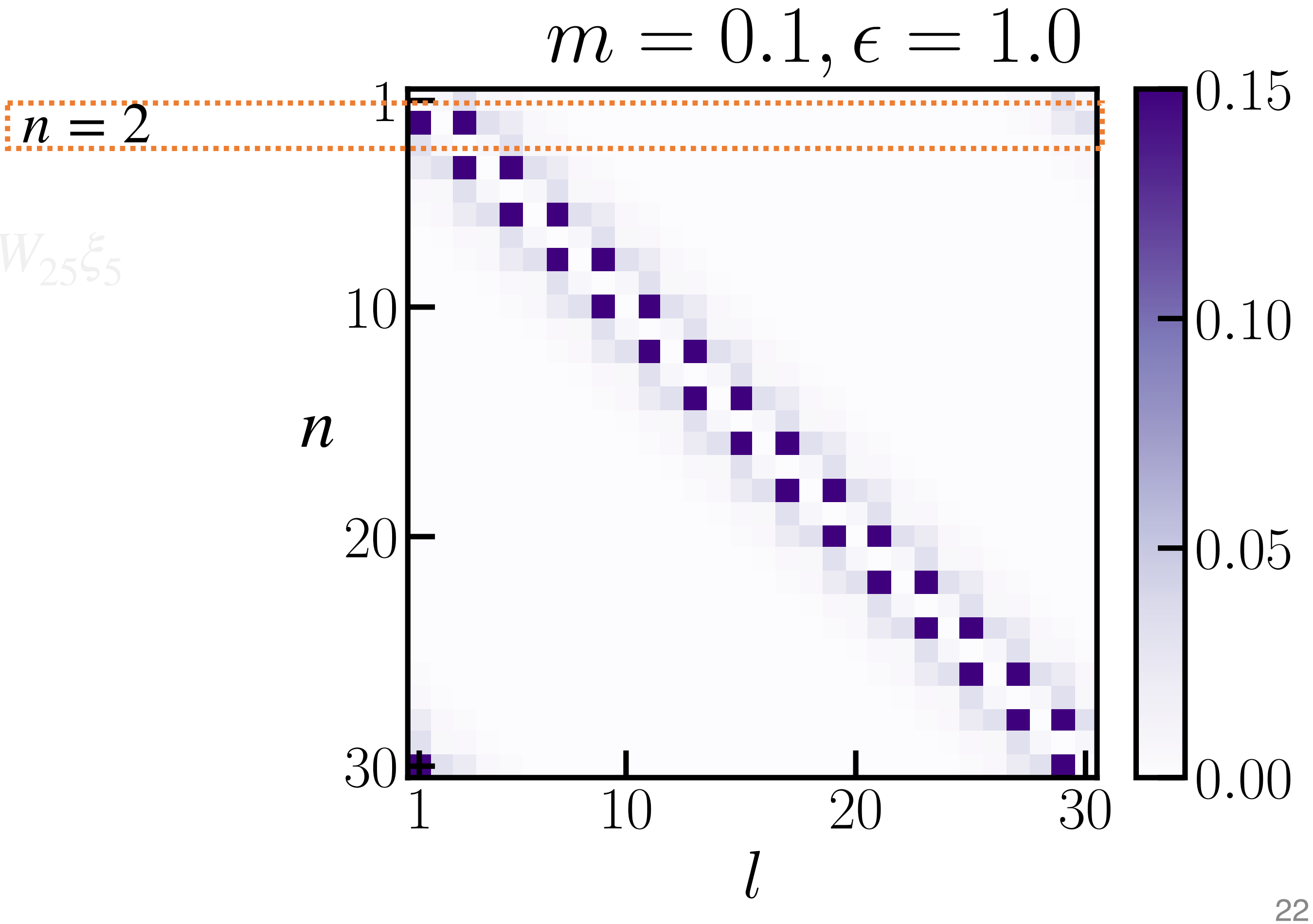
$\xi_2^\dagger W_{21}\xi_1$ $\quad \xi_2^\dagger \xi_2$ $\quad \xi_2^\dagger W_{23}\xi_3$ $\quad \xi_2^\dagger W_{24}\xi_4$ $\quad \xi_2^\dagger W_{25}\xi_5$



Results from QSE

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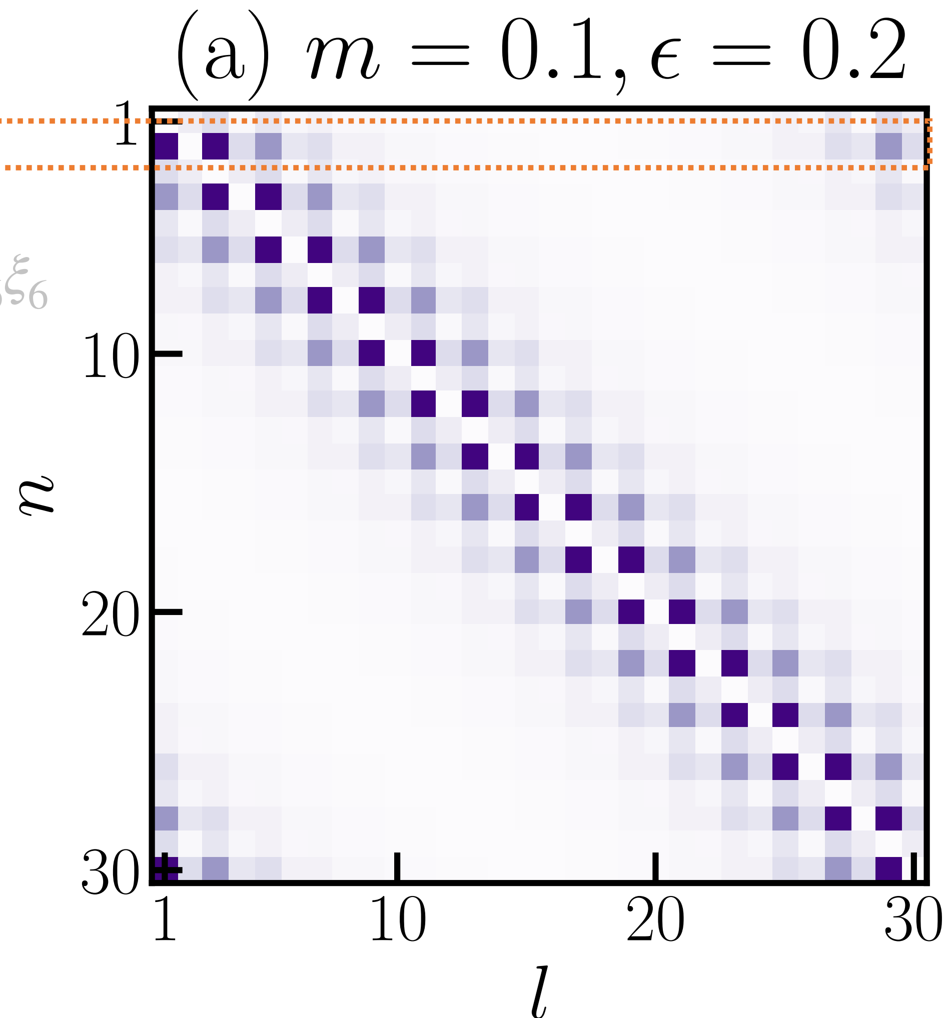
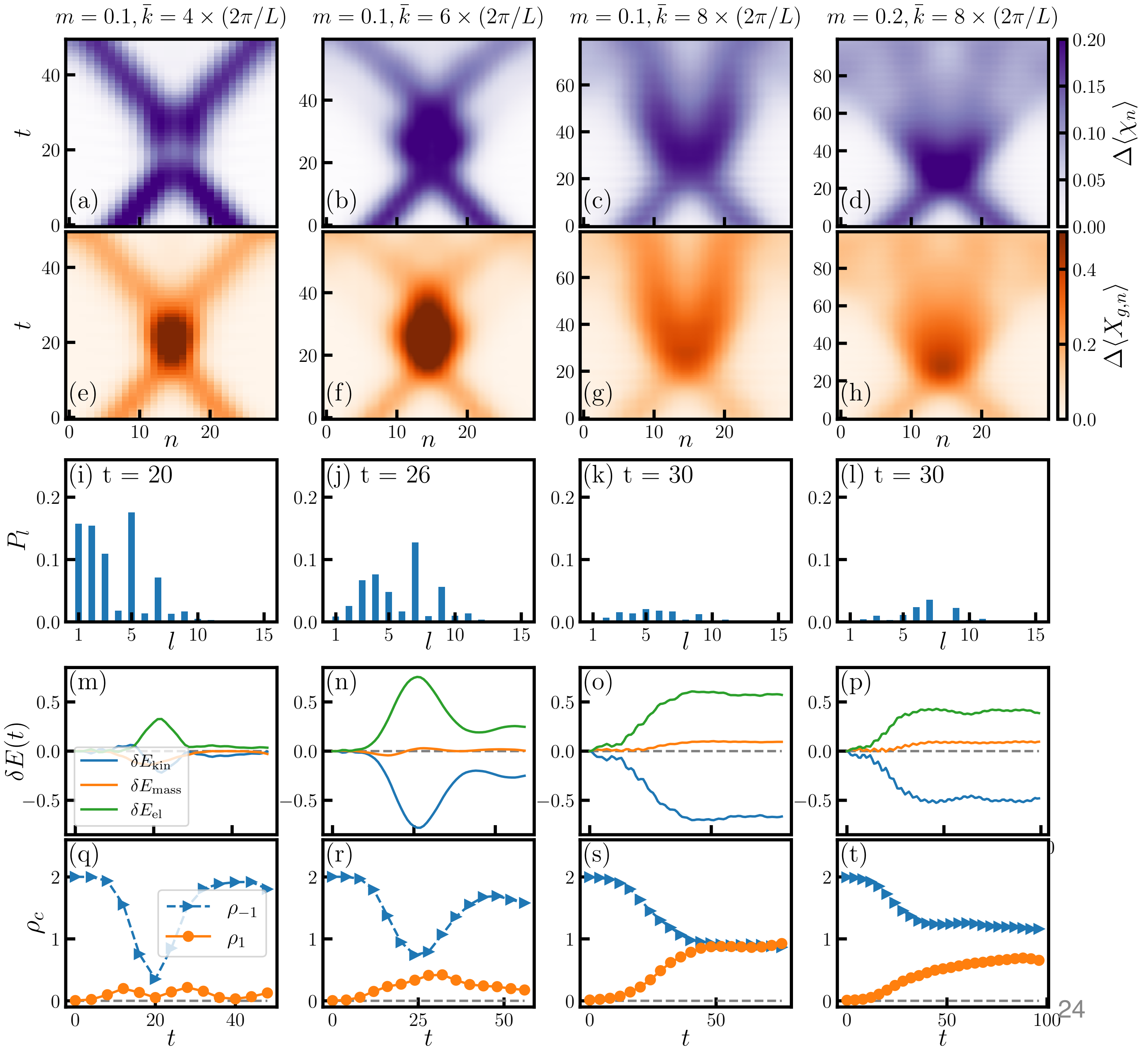
$n = 2$

$\xi_2^\dagger W_{21}\xi_1$ $\xi_2^\dagger \xi_2$ $\xi_2^\dagger W_{23}\xi_3$ $\xi_2^\dagger W_{24}\xi_4$ $\xi_2^\dagger W_{25}\xi_5$ $\xi_2^\dagger W_{26}\xi_6$



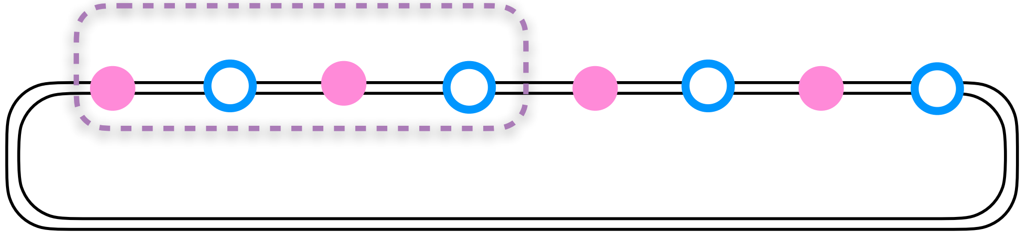
Scattering

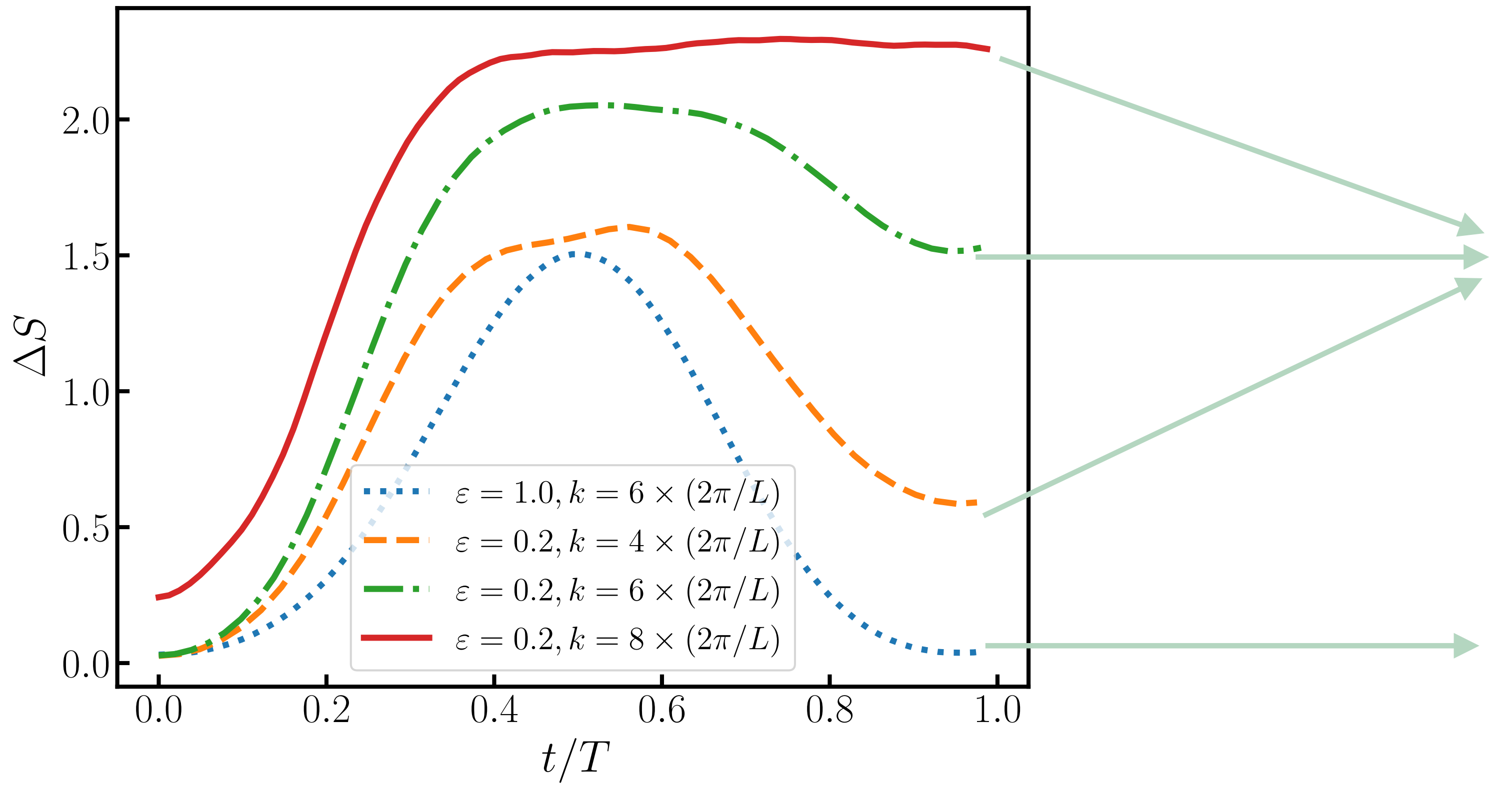
$$P_l = \sum_{n=1}^{L-l} |\langle \psi(t) | \xi_n^\dagger W_{n,n+l} \xi_{n+l} | \Omega \rangle|^2 + \sum_{n=l+1}^{L} |\langle \psi(t) | \xi_n^\dagger W_{n,n-l} \xi_{n-l} | \Omega \rangle|^2$$



Entropy

$$S(t) = -Tr[\rho(t)\log_2 \rho(t)]$$





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