A Quantum Adaptive Importance Sampling Algorithm for Multidimensional Integration

Paper

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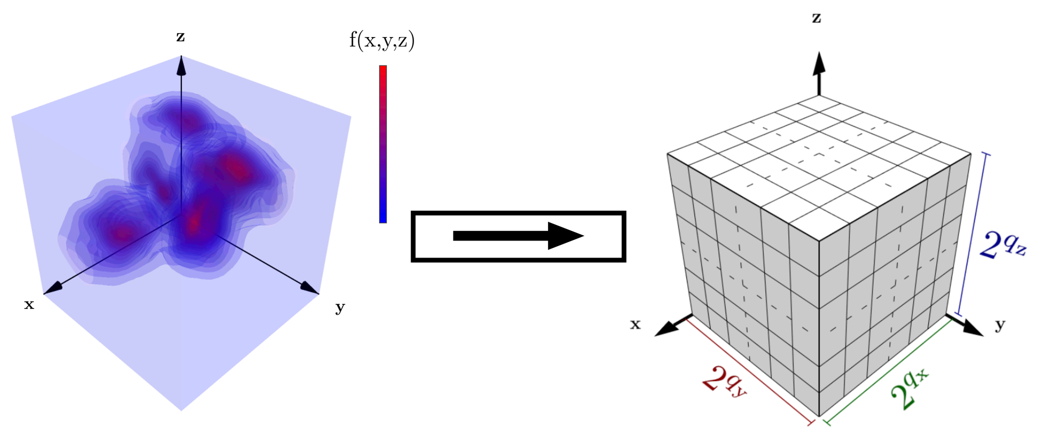
1. Introduction and Objectives

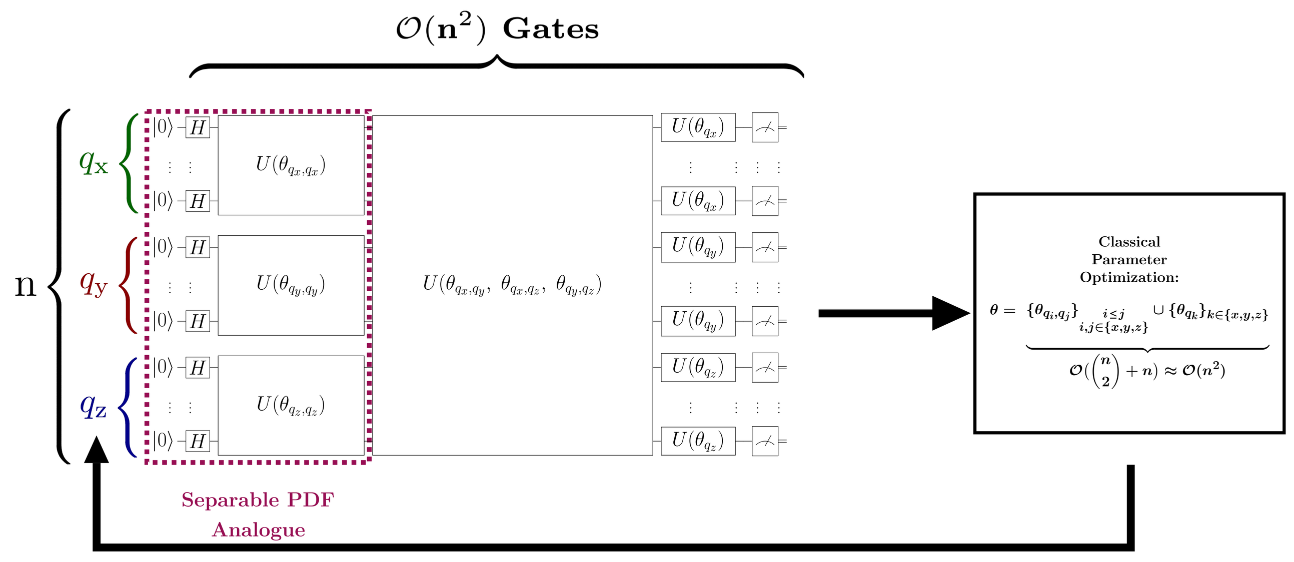
- **Precision in Perturbative QFT:**
Numerical integration of computationally demanding multi-loop Feynman diagrams and phase-space integrals.
- **Classical Approach: Adaptive Importance Sampling (VEGAS):**
Effective for sharp peaks. Allocates samples via adapting a grid. Mitigates the curse of dimensionality with a separable probability density function (PDF).
- **Quantum Adaptive Importance Sampling (QAIS):**
Exploit the exponentially large Hilbert space of a parameterised quantum circuit (PQC) to load and sample from a non-separable proposal PDF.
- **Why Importance Sampling (IS) with Quantum Computing?**
IS targets small, high-impact regions. Direct sampling with a quantum circuit is efficient when the answer lies in a small corner of the full Hilbert space.

2. The Quantum Algorithm

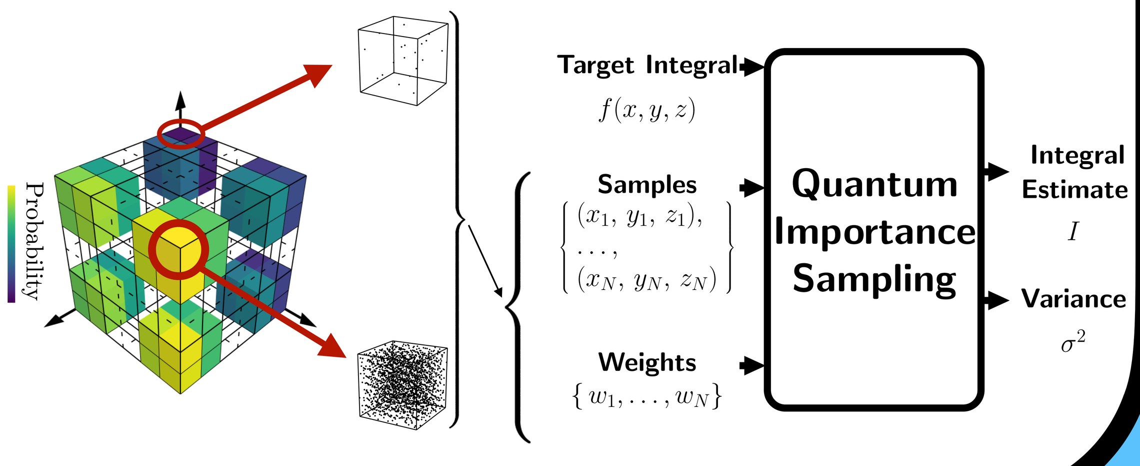
I) Discretise the multidimensional integration domain



II) Map the grid to a PQC. Quantum Generative Modelling, in particular a Quantum Circuit Born Machine (QCBM), adapts the PDF over the grid

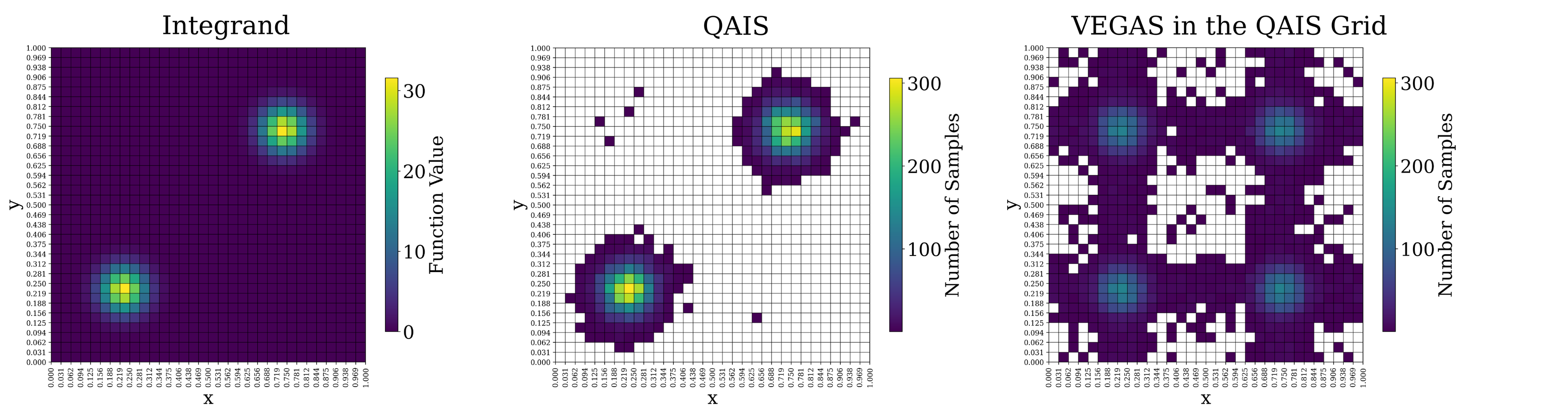


III) Sample from the optimised PQC. Shot counts per state set the quasi-random points allocation. Samples and weights produce the estimation.



3. Two-Dimensional Gaussian Case

$$\int_{[0,0]}^{[1,1]} f(\mathbf{x})d\mathbf{x} = \int_{[0,0]}^{[1,1]} \left(\sum_{i=0}^{1} e^{\left(-200|\mathbf{x}-\mathbf{r}_i|^2\right)}\right) d\mathbf{x}\,, \quad \mathbf{r}_0 = (0.23, 0.23), \quad \mathbf{r}_1 = (0.74, 0.74)$$



Naive QAIS integral estimator: $\hat{I}_N^{(\mathrm{QAIS})} = \frac{1}{N}\sum_{j=1}^{N} w^{(i)} f(\mathbf{x}_j^{(i)}),\quad w^{(i)} = \frac{|\Omega^{(i)}|\,N}{N_i}.$

$$\left(\hat{\sigma}_N^{(\mathrm{QAIS})}\right)^2 = \frac{1}{N-1}\left(\frac{1}{N}\sum_{j=1}^{N}\left[w^{(i)} f(\mathbf{x}_j^{(i)})\right]^2 - \left(\hat{I}_N^{(\mathrm{QAIS})}\right)^2\right)$$

$\Omega^{(i)}$: i-th grid cell ≡ i-th quantum state
$\mathbf{x}_j^{(i)}$: j-th random point at grid cell i
N: Number of shots
N_i: Number of times state i was measured

4. Debiasing Through Tiling Algorithm

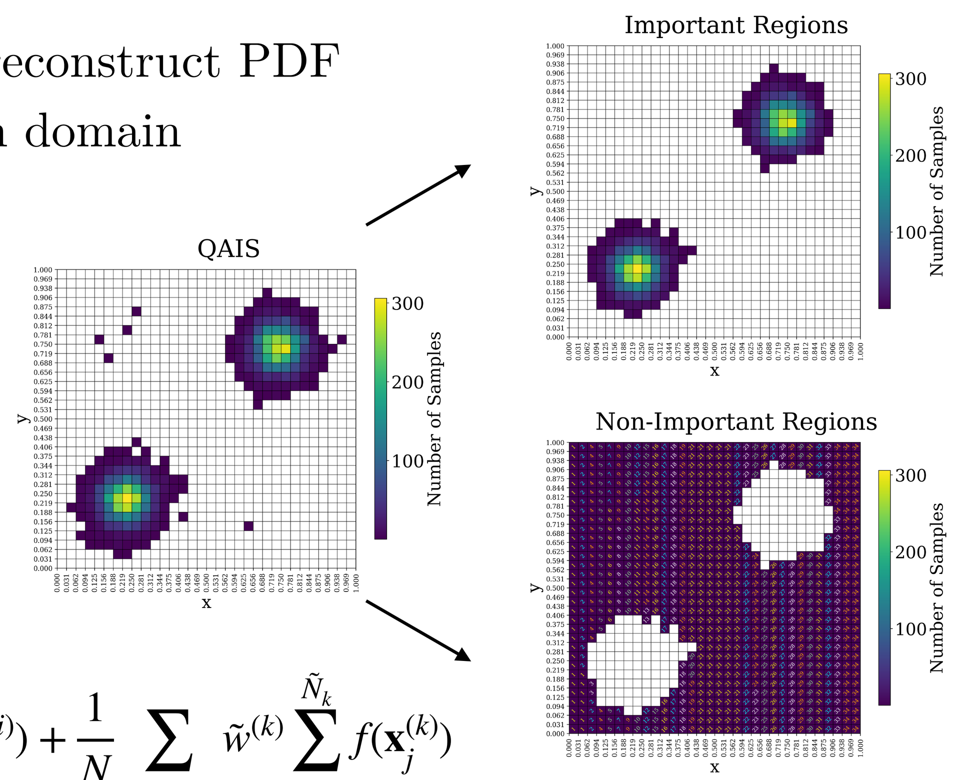
1. Challenge: Unmeasured states → Integral underestimation (Bias)

2. Conflicting objectives:
- **Efficient Quantum Measurement:** Target at few states
- **Unbiasedness:** Evaluate every grid cell

3. Solution: **Tiling Algorithm** to reconstruct PDF over the full integration domain

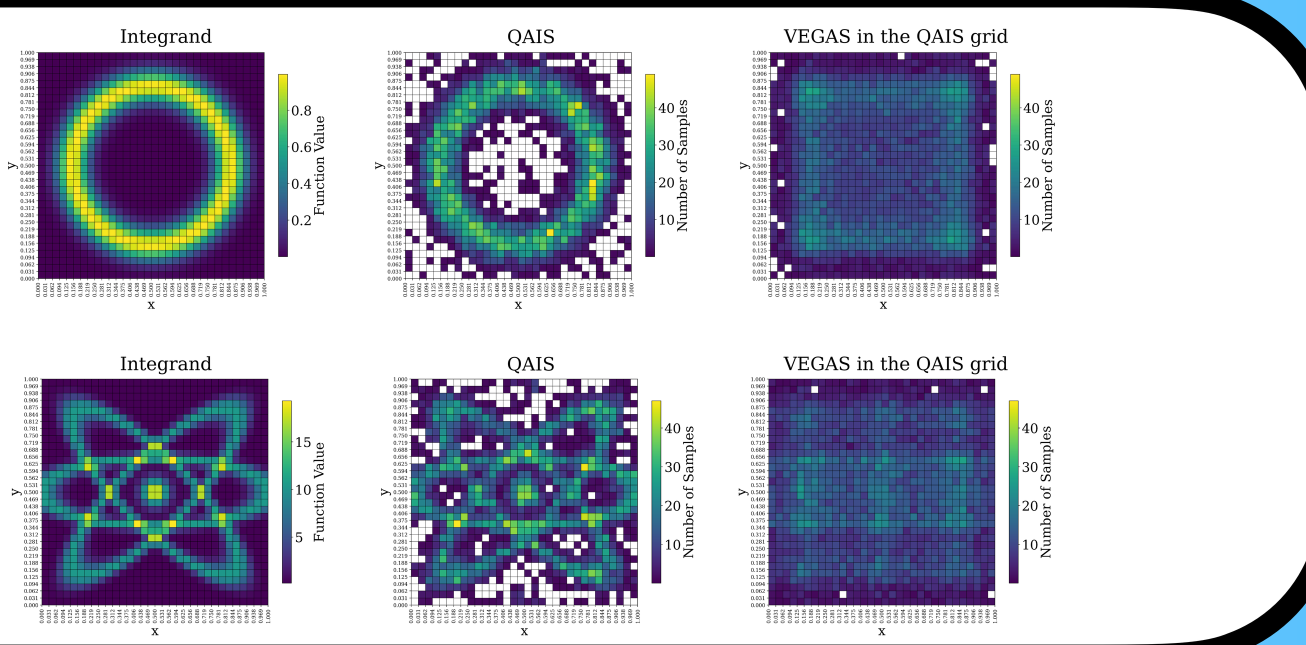
I) Detailed sampling in Important Regions

II) Sparse uniform sampling in the Non-Important Region



QAIS estimator: $\hat{I}_N^{(\mathrm{QAIS})} = \frac{1}{N}\sum_{i\in\Omega_I} w^{(i)}\sum_{j=1}^{N_i} f(\mathbf{x}_j^{(i)}) + \frac{1}{N}\sum_{k\in\Omega_{N-I}} \tilde{w}^{(k)}\sum_{j=1}^{\tilde{N}_k} f(\mathbf{x}_j^{(k)})$

5. Non-Trivial Two-Dimensional Cases

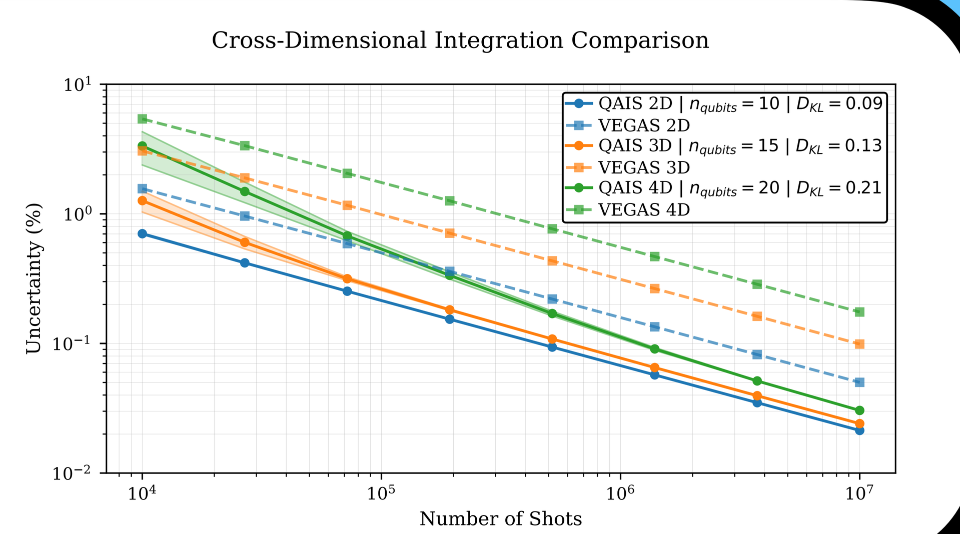


6. Cross-Dimensional Results

Dimensions: 2-4

$$\int_{[0,1]^d} f(\mathbf{x})\,d\mathbf{x} = \int_{[0,1]^d}\left(\sum_{i=0}^{2} e^{-50|\mathbf{x}-\mathbf{r}_i|}\right)d\mathbf{x}$$

$\mathbf{r}_0 = (0.23,\ldots,0.23)$,
$\mathbf{r}_1 = (0.39,\ldots,0.39)$,
$\mathbf{r}_2 = (0.74,\ldots,0.74)$



7. Summary

- **Entangled PQC:** Encodes non-separable proposal PDFs that can capture the intricate structures of integrands.
- **High-precision:** Integral estimates stay robust even as the dimensionality increases.
- **Current bottleneck:** Optimisation through QCBM is significantly more expensive than VEGAS.
- **Future work:** Loading the proposal PDFs more efficiently.

References

[1] G. P. Lepage, A New Algorithm for Adaptive Multidimensional Integration, J. Comput. Phys. 27 (1978) 192.

[2] K. Pyretzidis, J. J. Martínez de Lejarza and G. Rodrigo, Unlocking Multi-Dimensional Integration with Quantum Adaptive Importance Sampling, arXiv:2506.19965 [quant-ph] (2025).

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