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Gravity induced CP violation in $K^0/\overline{K}^0$ and $B^0/\overline{B}^0$ mixing, decays and interferences experiments

Violation de CP induite par la gravitation dans les experiences de mélange, de desintegration et d'interferences de $K^0/\overline{K}^0$ et $B^0/\overline{B}^0$

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Abstract. The impact of earth's gravity on neutral mesons dynamics is analyzed. The main effect of a Newtonian potential is to couple the strangeness and bottomness flavor oscillations with the quarks zitterbewegung oscillations. This coupling is responsible of the observed CP violations in the three types of experiments analyzed here: (*i*) indirect violation in the mixing, (*ii*) direct violation in the decay to one final state and (*iii*) violation in interference between decays with and without mixing. The three violation parameters associated with these experiments are predicted in agreement with the experimental data. The amplitude of the violation is linear with respect to the strength of gravity so that this new mechanism allows to consider matter dominated cosmological evolutions providing the observed baryon asymmetry of the universe.

Résumé. L'impact de la gravité terrestre sur la dynamique des mésons neutres est analysé. L'effet principal d'un potentiel newtonien est de coupler les oscillations de saveurs avec les oscillations zitterbewegung des quarks. Ce couplage est responsable des violations de CP observées dans les trois types d'expériences analysées ici : (*i*) violation indirecte dans le mélange, (*ii*) violation directe dans la désintégration vers un état final et (*iii*) violation dans l'interférence entre les désintégrations avec et sans mélange. Les trois paramètres de violation associés à ces experiences sont prédits en accord avec les données expérimentales. L'amplitude de la violation est linéaire par rapport à la force de gravité, aussi ce nouveau mécanisme permet d'envisager des évolutions cosmologiques asymétriques, dominées par la matière, expliquant ainsi l'asymétrie baryons-antibaryons dans l'univers.

Keywords. Gravitation, CP violation, Neutral mesons.

Mots-clés. Gravitation, Violation de CP, Mésons neutres.

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1. Introduction

Since the first observation of long-lived kaons decays into pairs of charged pions, reported sixty years ago by Christenson, Cronin, Fitch and Turlay [1], many complementary observables associated with flavored neutral mesons CP violation (CPV) have been identified, measured and interpreted. The canonical framework of interpretation is the standard model (SM) through the adjustment between the Kobayashi-Maskawa (KM) [2] complex phase and the experimental values of the violation parameters. In this study, we focus on the most documented and clearest experimental evidences of CPV and we demonstrate that *gravity induced CPV* provides a pertinent framework to interpret these experiments and to predict the violation parameters, as a function of earth's gravity, in agreement with the experimental data. As a consequence, far from any massive object, i.e. in a flat Lorentzian space-time, the Cabibbo-Kobayashi-Maskawa (CKM) [2, 3] matrix must be considered free from any CPV phase as CPV effects are just gravity induced near massive objects like earth.

Among the measured CPV observables, three types (*i*, *ii* and *iii*) of effects will be considered here: (*i*) indirect CPV in the mixing which has been observed with neutral kaons $K^0/\overline{K}^0$, this CPV is described by the parameter $\mathrm{Re}\,\varepsilon$ [4]; (*ii*) direct CPV in decays into one final state which has also been observed in neutral kaons decays and is characterized by the parameter $\mathrm{Re}\,\varepsilon'/\varepsilon$ [5]; (*iii*) CPV in interference between decays with and without mixing, which has been observed in $B^0/\overline{B}^0$ decays and is described by the angle β [5].

Beside these (*i*, *ii* and *iii*) types of CPV, a forth additional experimental evidence of CPV must be considered: (*iv*) the observed dominance of baryons over antibaryons in our universe as CPV is one of the necessary condition to build cosmological evolution models compatible with this baryon asymmetry [6].

Despite its success to provide a framework to interpret earth based experiments such as (*i*, *ii* and *iii*), the KM mechanism, incorporated into cosmological evolution models, fails, by several orders of magnitude, to account for this (*iv*) major CPV evidence. To explain how our matter-dominated universe emerged during its early evolution we need to identify a CPV mechanism different from the KM one. Beside its potential to predict the measured parameters associated with types (*i*, *ii* and *iii*) CPV experiments on earth, the new gravity induced CPV mechanism opens very interesting perspectives to set up cosmological models with asymmetric baryogenesis compatible with the present state of our universe. During the early stages of evolution of the universe, gravity/curvature was far more larger than on earth today and gravity induced CPV identified and described here, which is a linear function of the gravitational field, opens an avenue to resolve the present contradiction between the very small value of the KM mechanism and the very large CPV needed to build a pertinent model of our matter-dominated universe.

To summarize, gravity induced CPV, not only explain (*i*, *ii* and *iii*) CPV effects and predict observables such as ε, ε' and β, but it also renews, in depth, the baryons asymmetry (*iv*) cosmological issue.

In this study, we demonstrate that a small secular coupling, induced by earth's gravity, between fast *quarks zitterbewegung oscillations* at the velocity of light inside the mesons and strangeness oscillations $\Delta S = 2$, or bottomness oscillations $\Delta B' = 2$, provides both a qualitative explanation of CPV and a quantitative prediction of the CPV parameters ε, ε' and β in agreement with the experimental measurements.

The new interpretation of CPV experiments presented below is based on a careful analysis of the impact of earth gravity on the dynamics of strangeness and bottomness oscillations. To do so we use the effective Hamiltonian of Lee, Oehme and Yang (LOY) [7, 8], completed here with Newtonian gravity.

Neutral mesons oscillations such as $K^0 \rightleftharpoons \overline{K}^0$ and $B^0 \rightleftharpoons \overline{B}^0$ are very low energy oscillations

($10^{-6} - 10^{-4}$ eV). The typical earth's gravity coupling parameter $\hbar g/c \sim 10^{-23}$ eV (g is the acceleration due to gravity on earth) is very small with respect to the various energy scales involved in neutral mesons oscillations. Given the smallness of these parameters, there are no needs to rely on quantum field theory and the usual two states LOY model offers the pertinent framework to describe the interplay between two low energy quantum oscillations: quarks zitterbewegung vertical oscillations at the velocity of light inside the meson on the one hand and the strangeness oscillations ($\Delta S = 2$), or bottomness oscillations $\left(\Delta B' = 2\right)$, on the other hand.

The three types of CPV experimental evidences are usually analyzed under the assumption of CPT conservation. The CPT theorem is demonstrated within the framework of three hypothesis: Lorentz group invariance, spin-statistics relations and local field theory. In the rest frame of a meson interacting with a massive spherical object like earth the first hypothesis is not satisfied. Thus, when earth influence is considered, we must not be surprised that CPT theorem, apparently, no longer holds. Within the framework of a gravity induced CPV mechanism earth's gravity is described as an external field and the evolution of a meson state $|M\rangle$ alone, as a linear superposition of two flavor eigenstates $\left|M^0\right\rangle$ and $\left|\overline{M}^0\right\rangle$, does not provide the complete picture of the dynamical system and so can not be considered as a good candidate displaying CPT invariance. But CPT might be restored for the global three bodies $\left(M^0/\overline{M}^0/\oplus\right)$ evolution of the state $|M\oplus\rangle$ describing both the meson-antimeson pair and earth. In this study we consider only the evolution of $|M\rangle$ and earth's effect is described as an external static field so that CPT will appear to be violated because of this restricted two bodies $\left(M^0/\overline{M}^0\right)$ model of the system.

The study presented below complements a previous study based on two coupled Klein-Gordon equations describing $K^0/\overline{K}^0$ evolution on a Schwarzschild metric [9], rather than a Newtonian framework with two coupled Schrödinger equations used here. The results on $K^0/\overline{K}^0$ dynamics given by the Newtonian model, presented below, are similar to those of this previous Einsteinian model [9], these results are thus model independent. Moreover, with gravity induced CPV there is no T violation at the microscopic level and, for example in the $K^0/\overline{K}^0$ case, the observed T violation stems from the irreversible decay of the short-lived kaons K_S continuously regenerated from the long-lived one K_L by the gravity induced coupling.

This paper is organized as follows, in the next section we briefly review the LOY model without CPV. In section 3 we review the usual modifications to $K^0/\overline{K}^0$ and $B^0/\overline{B}^0$ mass eigenstates needed to accommodate CPV experimental results. The impact of earth's gravity is considered in section 4 where, to describe neutral mesons oscillations $M^0 \rightleftharpoons \overline{M}^0$ on earth, the CP conserving LOY model, presented in section 2, is completed with a Newtonian gravity term. We carefully analyze the nature and the impact of this additional term and discover that it contains the zitterbewegung motion of the quarks inside the meson. The study of type (*i*), (*ii*) and (*iii*) gravity induced CPV are developed in sections 5, 6 and 7. We consider specifically type (*i*) and (*ii*) CPV for $K^0/\overline{K}^0 \sim \left(d\overline{s}\right)/\left(\overline{d}s\right)$ and type (*iii*) CPV for $B^0/\overline{B}^0 \sim \left(d\overline{b}\right)/\left(\overline{d}b\right)$. Section 8 provides a brief comment on others, $D^0/\overline{D}^0$ and $B_s^0/\overline{B_s}^0$, neutral mesons and gives our conclusions. In sections 2 and 4, $M^0/\overline{M}^0$ will stand for $K^0/\overline{K}^0$ or $B^0/\overline{B}^0$. In sections 5, 6 and 7 the experimental numerical values used to evaluate the expressions are taken from the *PDG 2024* Ref. [5]

2. Mass and CP eigenstates without CPV

Consider a generic neutral meson pair $M^0/\overline{M}^0$, either $K^0/\overline{K}^0$ or $B^0/\overline{B}^0$.

The meson state $|M(\tau)\rangle$ is a linear superposition of the flavor eigenstates $\left|M^0\right\rangle$ and $\left|\overline{M}^0\right\rangle$ ($\left\langle M^0 \middle| M^0\right\rangle = \left\langle \overline{M}^0 \middle| \overline{M}^0\right\rangle = 1$ and $\left\langle \overline{M}^0 \middle| M^0\right\rangle = 0$) and the amplitudes (a, b) of this superposition are functions of the meson proper time τ.

This state is also coupled to a set of final states $\left|f,\mathbf{p},Q,...\right\rangle$, with quantum number Q and momentum $\mathbf{p}$ in the $M^0/\overline{M}^0$ meson rest frame, described by the amplitudes w_f,

$$\left|M(\tau)\right\rangle = a(\tau)\left|M^0\right\rangle + b(\tau)\left|\overline{M}^0\right\rangle + \sum_f w_f(\tau)\left|f\right\rangle. \quad (1)$$

The Weisskopf-Wigner (WW) approximation [10] is used to describe the coupling to the final states $\left|f\right\rangle$ as an irreversible decay. Within the framework of this usual approximation we introduce a non-Hermitian decay operator $j\widehat{\gamma}$ capturing the effects of the w_f amplitudes and describing $M \to f$ transitions as irreversible decay processes. It is to be noted that, as the possibilities of $f \to M$ transitions are neglected by this approximation, the use of $j\widehat{\gamma}$ is thus the source of a T violation which must not be attributed to fundamental interactions but to the WW model.

The time evolution of $\left|M(\tau)\right\rangle$ can thus be restricted to a two states Hilbert sub-space: $\left|M^0\right\rangle, \left|\overline{M}^0\right\rangle$, at the cost of the loss of unitarity $d\left\langle M|M\right\rangle/d\tau < 0$ induced by the decay operator $j\widehat{\gamma}$. This restriction of the Hilbert space to $\left|M^0\right\rangle, \left|\overline{M}^0\right\rangle$, allowed by the WW approximation, leads to the effective LOY Hamiltonian.

The LOY Hamiltonian without CPV is the sum of the mass energy (mc^2), plus a strangeness/bottomness ($S=\pm1$ / $B'=\pm1$) coupling operator ($\widehat{\delta mc^2}$), plus the WW irreversible decay ($j\hbar\widehat{\gamma}$), according to the Schrődinger equation

$$j\hbar\frac{d\left|M(\tau)\right\rangle}{d\tau} = mc^2\left|M(\tau)\right\rangle - \left[\frac{\widehat{\delta m}}{2}c^2 + j\hbar\frac{\widehat{\gamma}}{2}\right]\cdot\left|M(\tau)\right\rangle. \quad (2)$$

The coupling operator $\widehat{\delta m}$ and the decay operator $\widehat{\gamma}$ are given by

$$\widehat{\delta m} = \delta m\left[\left|M^0\right\rangle\left\langle\overline{M}^0\right| + \left|\overline{M}^0\right\rangle\left\langle M^0\right|\right], \quad (3)$$

$$\widehat{\gamma} = \Gamma\left[\left|M^0\right\rangle\left\langle M^0\right| + \left|\overline{M}^0\right\rangle\left\langle\overline{M}^0\right|\right] - \delta\Gamma\left[\left|M^0\right\rangle\left\langle\overline{M}^0\right| + \left|\overline{M}^0\right\rangle\left\langle M^0\right|\right], \quad (4)$$

where $\delta m > 0$ is the mass splitting between the heavy and light mass eigenstates and $\Gamma > 0$, $\delta\Gamma < 0$ are respectively the average and the splitting between the decay widths of the these eigenstates [4]. These mass eigenstates are: the long-lived L and short-lived S states ($K_{S/L}$) for $K^0/\overline{K}^0$, and the heavy H and light L states ($B_{L/H}$) for $B^0/\overline{B}^0$. We take the convention $\widehat{CP}\left|M^0\right\rangle = \left|\overline{M}^0\right\rangle$. The CP eigenstates $\left|M_1\right\rangle$ and $\left|M_2\right\rangle$ are related to the flavor eigenstates by

$$\left|M_1\right\rangle = \frac{\left|M^0\right\rangle}{\sqrt{2}} + \frac{\left|\overline{M}^0\right\rangle}{\sqrt{2}} = \widehat{CP}\left|M_1\right\rangle, \quad (5)$$

$$\left|M_2\right\rangle = \frac{\left|M^0\right\rangle}{\sqrt{2}} - \frac{\left|\overline{M}^0\right\rangle}{\sqrt{2}} = -\widehat{CP}\left|M_2\right\rangle. \quad (6)$$

These CP eigenstates, M_1 and M_2, are also energy/mass eigenstates of Eq. (2), thus the time evolution of the CP and mass eigenstates without CPV is given by

$$\left|M_1(\tau)\right\rangle = \left|M_1\right\rangle \exp - j\frac{c^2}{\hbar}\left[m - \frac{\delta m}{2} - j\hbar\frac{\Gamma - \delta\Gamma}{2c^2}\right]\tau \quad (7)$$

$$\left|M_2(\tau)\right\rangle = \left|M_2\right\rangle \exp - j\frac{c^2}{\hbar}\left[m + \frac{\delta m}{2} - j\hbar\frac{\Gamma + \delta\Gamma}{2c^2}\right]\tau \quad (8)$$

The above symmetric picture where CP commute with the Hamiltonian is no longer valid when the experimental results of CPV are to be taken into account.

3. Mass eigenstates with types (*i*) and (*iii*) CPV

When CPV comes into play, the Hamiltonian (2) is modified and the mass eigenstates $K_{S/L}$ or $B_{L/H}$ are no longer the CP eigenstates $K_{1/2}$ or $B_{1/2}$ (5, 6). The mass eigenvalues (7, 8) are not significantly changed by CPV.

For types (*i*) and (*iii*) CPV, experimental evidences require to modify the Hamiltonian and the resulting mass eigenstates. Type (*ii*) direct CPV in the decay to one final state is also due to earth's gravity, as will be demonstrated in section 6, but the associated ε' parameter is not involved in the LOY Hamiltonian describing mixing. The parameters ε and β are introduced in order to describe $K^0/\overline{K}^0$ type (*i*) and $B^0/\overline{B}^0$ type (*iii*) effects.

For $K^0/\overline{K}^0$ type (*i*) CPV, the indirect CPV effects are described by the small parameter ε and the mass eigenstates $|K_{S/L}\rangle$ are related to the CP eigenstates $|K_{1/2}\rangle$ (5, 6) by

$$|K_S\rangle = |K_1\rangle + \varepsilon|K_2\rangle, \tag{9}$$

$$|K_L\rangle = |K_2\rangle + \varepsilon|K_1\rangle. \tag{10}$$

As $|\varepsilon| = 2.2\times10^{-3}$ we have neglect $O\left[10^{-6}\right]$ corrections associated with the normalization $\langle K_{S/L}|K_{S/L}\rangle = 1$. The quantity $\langle K_{S/L}|K_{L/S}\rangle = 2\,\mathrm{Re}\,\varepsilon$ is an observable.

For $B^0/\overline{B}^0$ type (*iii*) CPV, it is convenient to introduce an angle β and to consider mass eigenstates $|B_{L/H}\rangle$ related to CP eigenstates $|B_{1/2}\rangle$ (5,6) by

$$|B_L\rangle = \cos\beta|B_1\rangle + j\sin\beta|B_2\rangle, \tag{11}$$

$$|B_H\rangle = \cos\beta|B_2\rangle + j\sin\beta|B_1\rangle. \tag{12}$$

The overlap of mass eigenstates $\langle B_L|B_S\rangle = 0$, thus there is no type (*i*) CPV with this parametrization and normalization is ensured as $\langle B_{L/H}|B_{L/H}\rangle = 1$.

The CP symmetry is restored when $\varepsilon = 0$, $\varepsilon' = 0$ and $\beta = 0$. In the usual KM interpretation these parameters are related to combinations of CKM matrix elements where the KM phase is adjusted to the measured CPV amplitude. Rather than adjusting a complex phase, an other interpretation of the experiments is proposed below: we simply take into account the impact of earth's gravity on the experiments without the need to introduce a new parameters in a CP conserving CKM matrix which is thus free of CPV far from any massive object.

The final quantitative results predicted with this new mechanism leads to the conclusion that CPV observed in the three canonical types of flavored neutral mesons experiments (*i, ii* and *iii*) is (earth) gravity induced, and not fundamental at the level of the CKM matrix elements.

4. Strangeness and bottomness oscillations on earth

The Schrődinger equation Eq. (2) is pertinent far from any massive object, but, on earth, we have to consider the very small Newtonian potential energy $mG_NM_\oplus/R_\oplus \sim 10^{-9}mc^2$. We can restrict the description of this new coupling to the first term of the Taylor expansion of $mG_N M_\oplus/(R_\oplus+X+x)$ with respect to a vertical position $X+x$ where $x \ll X \ll R_\oplus$. The position X is the vertical average position of the meson with respect to the level $R_\oplus$. This is an external degree of freedom: it can not enter in the τ dynamics (2) as τ is the meson proper time. The vertical position $x(\tau)$ describes the internal vertical fluctuations around this average X. This is an internal degree of freedom: it must enter the proper time Hamiltonian (2). Thus, we consider an additional energy term $mg\,\widehat{x}(\tau)$ in (2) with $g = G_NM_\oplus/R_\oplus^2 = 9.8\ \mathrm{m/s^2}$,

$$j\hbar\frac{d|M(\tau)\rangle}{d\tau} = mc^2|M\rangle - \frac{\widehat{\delta m}}{2}c^2\cdot|M\rangle - j\hbar\frac{\widehat{\gamma}}{2}\cdot|M\rangle + mg\ \widehat{x}(\tau)\cdot|M\rangle. \tag{13}$$

We have just applied here the *correspondence principle* between classical mechanics and quantum mechanics: classical variables becomes operators. It is very important to note that, as τ is the meson proper time, the position operator $\widehat{x}(\tau)$ in (13) must not be interpreted as the vertical position of the meson with respect to a reference vertical level in the laboratory. As τ is the rest frame proper time of the meson, the motion described by the operator $\widehat{x}(\tau)$ is associated with the (unknown) internal quark vertical motions, inside the mesons, as a function of the meson proper time τ: the zitterbewegung motion inherent to all, free or bound, spin 1/2 fermions [11].

The operator $\widehat{x}(\tau)$ in (13) describes the fast fluctuating vertical motion of the quarks inside the meson with respect to the meson average position defining the rest frame of the meson. This rest frame has a proper time τ and its free fall does not affect (13) on the time scale of the experiment.

The separation between a slow average and fast fluctuations is based on a three time scales ordering of the dynamics: a very slow time scale of the meson free fall, which does not enter in the meson proper time dynamics Eq. (13), a slow time scale of the order of $\hbar/\delta mc^2$ associated with flavor oscillations, and the fast fluctuating motions associated with quarks zitterbewegung internal oscillations, with a zitterbewegung time scale of the order of the Compton wavelength divided by the velocity of light $\lambda_C/c \sim \hbar/mc^2$. This separation between fast zitterbewegung fluctuations and mixing oscillations displays a strong ordering. The mixing and zitterbewegung time scales entering in (13) are ordered according to $\hbar/mc^2 \sim (10^{-15} - 10^{-13})\hbar/\delta mc^2$.

The mesons, $\left|M^0\right\rangle$ and $\left|\overline{M}^0\right\rangle$, are stationary diquarks bound states ultimately described by Dirac spinors associated with one light quark q' and one heavier quark q: $\left|M^0\right\rangle \sim \left|q'\overline{q}\right\rangle$ and $\left|\overline{M}^0\right\rangle \sim \left|\overline{q}'q\right\rangle$. The Dirac spinors are combined into *singlet* spin zero states. In the restricted Hilbert space, $\left|M^0\right\rangle, \left|\overline{M}^0\right\rangle$, the internal vertical position operator $\widehat{x}(\tau)$ is thus represented by the following four matrix elements $\langle|\widehat{x}|\rangle$ of the stationary Dirac spinors states $\left|q'\overline{q}\right\rangle$

$$\begin{aligned}\widehat{x}(\tau) = &\left\langle q'\overline{q}\right|\widehat{x}(\tau)\left|q'\overline{q}\right\rangle\left|M^0\right\rangle\left\langle M^0\right| + \left\langle\overline{q}'q\right|\widehat{x}(\tau)\left|\overline{q}'q\right\rangle\left|\overline{M}^0\right\rangle\left\langle\overline{M}^0\right| \\ &+ \left\langle\overline{q}'q\right|\widehat{x}(\tau)\left|q'\overline{q}\right\rangle\left|\overline{M}^0\right\rangle\left\langle M^0\right| + \left\langle q'\overline{q}\right|\widehat{x}(\tau)\left|\overline{q}'q\right\rangle\left|M^0\right\rangle\left\langle\overline{M}^0\right|. \end{aligned} \tag{14}$$

The internal vertical position operator $\widehat{\mathbf{x}}(\tau)$ fulfils Heisenberg's equation $j\hbar d\widehat{\mathbf{x}}/d\tau = \widehat{\mathbf{x}}\cdot\widehat{H} - \widehat{H}\cdot\widehat{\mathbf{x}}$ where $\widehat{H}$ is the Dirac Hamiltonian describing quarks confinement inside the meson. The values of the internal vertical position matrix elements $\langle|\widehat{x}(\tau)|\rangle$ depend of the model of their confinement inside the meson. The typical size of the meson $\left\langle x^2\right\rangle$ is the Compton wavelength λ_C^2.

The instantaneous velocity operator $d\widehat{\mathbf{x}}/d\tau$ of spin 1/2 particles/antiparticles pairs, either free or bound, is well known to display the so called zitterbewegung (nonintuitive) behavior: a quiver (zitter) motion (bewegung), on a length scale given by the Compton wavelength, at an instantaneous velocity equal to the velocity light [11].

It is important to note that the values of the instantaneous velocity matrix elements $\langle|d\widehat{\mathbf{x}}/d\tau|\rangle$ are independent of the charge and mass of the fermions as well as of the shape and strength of the effective confinement potential involved in the Dirac Hamiltonian $\widehat{H}$ describing confinement. This zitterbewegung universality is a consequence of Heisenberg's equation

$$j\hbar\frac{d\widehat{\mathbf{x}}}{d\tau} = \left[\widehat{\mathbf{x}}, \widehat{H}\left(\widehat{\mathbf{x}}, \widehat{\mathbf{p}}\right)\right] = j\hbar c\boldsymbol{\alpha}. \tag{15}$$

We have introduced the usual 4×4 alpha matrices: $\boldsymbol{\alpha} = \left(\alpha_x, \alpha_y, \alpha_z\right)$ [11] which can be expressed in terms of the 2×2 Pauli matrices $\boldsymbol{\sigma} = \left(\sigma_x, \sigma_y, \sigma_z\right)$. Equation (15) imply that the values of the internal fluctuating velocity matrix elements are equal to c or 0. As a consequence of Eq. (15) we have to identify the eigenvalues and the eigenstates of $\boldsymbol{\alpha}$. Without loss of generality we consider α_x and the four Dirac spinors complete orthogonal basis

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\0\end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\-1\\0\end{bmatrix}. \tag{16}$$

The usual physical interpretation of these four spinors (16) is as follows [11].

Starting from the left, the first spinor and the second one describe a symmetric superpositions of one fermion and one antifermion $\left(\left|q\right\rangle+\left|\overline{q}\right\rangle\right)/\sqrt{2}$. These two symmetric superpositions (16) are eigenstates of α_x with the eigenvalue 1. The last two spinors describe an antisymmetric superpositions of one fermion and one antifermion $\left(\left|q\right\rangle-\left|\overline{q}\right\rangle\right)/\sqrt{2}$. These two antisymmetric superpositions (16) are eigenstates of α_x with the eigenvalue -1.

The spinor representation of M_1, the symmetric CP eigenstate (5), is constructed with q' and q quarks spinors (16) of the first two types and M_2, the antisymmetric CP eigenstate (6), is constructed with quarks spinors of the last two types. These M_1 and M_2 diquarks states are spin zero singlet combinations of two Dirac spinors q' and q: d and s for $K^0/\overline{K}^0$ and d and b for $B^0/\overline{B}^0$. We note $\left\langle q'\overline{q}\right|$ the spin zero singlet spinors state of one quark q' and one antiquark $\overline{q}$.

The symmetric and antisymmetric superpositions (16) are eigenstates of α_x with eigenvalues ± 1, so that the matrix elements of the $\alpha_x\otimes\alpha'_x$ operator are given by

$$\begin{aligned}\left[\left\langle q'\overline{q}\right|\pm\left\langle\overline{q'}q\right|\right]\alpha_x\otimes\alpha'_x\left[\left|q'\overline{q}\right\rangle\pm\left|\overline{q'}q\right\rangle\right] &= \pm 1,\\ \left[\left\langle q'\overline{q}\right|\mp\left\langle\overline{q'}q\right|\right]\alpha_x\otimes\alpha'_x\left[\left|q'\overline{q}\right\rangle\pm\left|\overline{q'}q\right\rangle\right] &= 0.\end{aligned} \tag{17}$$

Thus, on the (M_1, M_2) CP basis (5,6), the representation of the zitterbzwzgung velocity operator $d\widehat{x}/d\tau$ is given by

$$\frac{d\widehat{x}}{d\tau}=c\left|M_1\right\rangle\left\langle M_1\right|-c\left|M_2\right\rangle\left\langle M_2\right|. \tag{18}$$

On the flavor basis $(M^0,\overline{M}^0)$ this leads to the relations $\left\langle\overline{M}^0\right|d\widehat{x}/d\tau\left|M^0\right\rangle = c$ and $\left\langle M^0\right|d\widehat{x}/d\tau\left|\overline{M}^0\right\rangle=c$ and the two others matrix elements are equal to zero. In the LOY model (2) the mass m of the antiparticle is positive as the mass of the particle, although, in the Dirac representation, the antiparticle are negative mass solutions. This last point is resolved through the Feynman interpretation of an antiparticle as a particle propagating backward in time. To construct the LOY representation of the fluctuating velocity the Feynman picture leads to the following representation of the internal velocity operator expressed on the flavor basis

$$\frac{d\widehat{x}}{d\tau}=c\left|M^0\right\rangle\left\langle\overline{M}^0\right|-c\left|\overline{M}^0\right\rangle\left\langle M^0\right|. \tag{19}$$

In two previous studies, Ref. [9] and [12], we have given two detailled demonstrations of this result (19) with two different methods. This operator describes the instantaneous velocity (fast time scale) of the quarks. It is to be noted that the operator $\widehat{x}(\tau)$ (14) displays the very high frequency content of the zitterbewegung motion, but $d\widehat{x}/d\tau$ (19) displays no high frequency content.

Beside the time ordering between an average and an instantaneous dynamics, the inclusion of the gravity term in (2), to give (13), introduces an energy ordering. The Compton wavelength of the meson λ_C provides an approximate maximum size of the matrix elements $|\langle|\widehat{x}|\rangle|$ in (14) as quarks are bound states inside the volume of a meson. The very small numerical value of the energy $mg\lambda_C=\hbar g/c\sim 10^{-23}$ eV in front of $\delta mc^2\sim 10^{-6}-10^{-4}$ eV leads to the occurrence of a very strong ordering fulfilled by the four matrix elements in (14), $mg|\langle|\widehat{x}|\rangle|\sim mg\lambda_C\ll\delta mc^2$, in front of the other LOY matrix elements. Note that, beside this energy ordering, the frequency ordering between mixing and zitterbewegung is reversed: $\delta mc^2/\hbar\ll mc^2/\hbar$.

The very strong energy ordering identified here allows to set up a perturbative expansion of (13) with respect to the small expansion parameter $\hbar g/\delta mc^3\sim 10^{-19}-10^{-17}$. We define $|N(\tau)\rangle$ and $|n(\tau)\rangle$ such that the meson dynamics is described by $|N(\tau)\rangle+|n(\tau)\rangle$

$$|M(\tau)\rangle = |N(\tau)\rangle\exp-j\frac{mc^2\tau}{\hbar}+|n(\tau)\rangle\exp-j\frac{mc^2\tau}{\hbar}. \tag{20}$$

The states $|N(\tau)\rangle$ and $|n(\tau)\rangle$ are ordered according to: $|N\rangle \sim O\left(\hbar g/\delta mc^3\right)^0$, $|n\rangle \sim O\left(\hbar g/\delta mc^3\right)^1$ and the first neglected term is $O\left(\hbar g/\delta mc^3\right)^2 \sim 10^{-38} - 10^{-34}$. With this expansion scheme (20), Schrödinger's equation (13) becomes

$$j\hbar\frac{d|N\rangle}{d\tau} = -\frac{1}{2}\left(\widehat{\delta m}c^2 + j\hbar\widehat{\gamma}\right)\cdot|N\rangle, \tag{21}$$

$$j\hbar\frac{d|n\rangle}{d\tau} = -\frac{1}{2}\left(\widehat{\delta m}c^2 + j\hbar\widehat{\gamma}\right)\cdot|n\rangle + mg\,\widehat{x}\cdot|N\rangle. \tag{22}$$

To identify the dominant secular contribution of earth's gravity we introduce the inverse of the operator $\widehat{\delta m}c^2 + j\hbar\widehat{\gamma}$ and then use this operator and (21) to rewrite (22)

$$\begin{aligned} j\hbar\frac{d|n\rangle}{d\tau} = & -\frac{1}{2}\left(\widehat{\delta m}c^2 + j\hbar\widehat{\gamma}\right)\cdot|n\rangle + 2jmg\hbar\frac{d\widehat{x}}{d\tau}\cdot\left(\widehat{\delta m}c^2 + j\hbar\widehat{\gamma}\right)^{-1}\cdot|N\rangle \\ & -2jmg\hbar\frac{d}{d\tau}\left[\widehat{x}\cdot\left(\widehat{\delta m}c^2 + j\hbar\widehat{\gamma}\right)^{-1}\cdot|N\rangle\right]. \end{aligned} \tag{23}$$

The strong time ordering between strangeness (or bottomness) oscillations $\left(\hbar/\delta mc^2\right)$ and zitterbewegung oscillations $\left(\sim\hbar/mc^2\right)$ can be used to simplify (23). We are interested by the strangeness or bottomness dynamics taking place on the *slow* time scale $\hbar/\delta mc^2$, thus we introduce 2θ the period of the (unknown) *fast* periodic functions $\langle|\widehat{x}(\tau)|\rangle$ associated with the zitterbewegung oscillations. This time 2θ is such that $\hbar/mc^2 \sim \theta \ll \hbar/\delta mc^2$. We apply the averaging operator $\widehat{A}_\theta \equiv \int_{\tau-\theta}^{\tau+\theta} dt/2\theta$ on both side of (23) to average out the high frequency $\left(mc^2/\hbar\right)$ components. For any low frequency $\left(\delta mc^2/\hbar\right)$ function $f(t)$: $\widehat{A}_\theta\cdot f(t) = f(\tau)$ and $\widehat{A}_\theta\cdot df/dt = df/d\tau$ and for any high frequency function $g(t)$: $\widehat{A}_\theta\cdot dg/dt = 0$. This usual averaging methods is just *Bogolioubov-Krilov-Mitropolski* method when applied on the dynamical equations, or *Witham* method if we average directly the Lagrangian associated with the evolution [13].

The equations describing strangeness or bottomness oscillations of a neutral meson $|N(\tau)\rangle + |n(\tau)\rangle$ on earth are given by

$$j\hbar\frac{d|N\rangle}{d\tau} = -\frac{1}{2}\left(\widehat{\delta m}c^2 + j\hbar\widehat{\gamma}\right)\cdot|N\rangle, \tag{24}$$

$$j\hbar\frac{d|n\rangle}{d\tau} = -\frac{1}{2}\left(\widehat{\delta m}c^2 + j\hbar\widehat{\gamma}\right)\cdot|n\rangle + j\widehat{G}\cdot|N\rangle. \tag{25}$$

The *gravity-zitterbewegung* operator $\widehat{G}$, capturing the secular interplay between zitterbewegung oscillations and bottomness or strangeness oscillations, is defined as

$$\widehat{G} = 2mg\hbar\left(\frac{d\widehat{x}}{d\tau}\right)\cdot\left(\widehat{\delta m}c^2 + j\hbar\widehat{\gamma}\right)^{-1}. \tag{26}$$

Flavored neutral mesons pairs $K^0/\overline{K}^0$ and $B^0/\overline{B}^0$ display different m, δm, Γ and $\delta\Gamma$ and the impact of earth gravity on their behavior is to be analyzed specifically. In the following we keep the notation of Eqs. (24, 25) and (26) with an additional index K or B for these specific studies.

5. Gravity induced type (*i*) CPV in the mixing of $K^0/\overline{K}^0$

The ordering associated with the specific case of a $K^0/\overline{K}^0$ pair is given by: $\delta m_K/m_K \sim 10^{-15}$ and the lifetime of the K_S is 577 times shorter than the lifetime of K_L. The first step to interpret $K^0/\overline{K}^0$ experiments is to consider a unitary evolution and to neglect the finite lifetime of both particles ($j\hbar\widehat{\gamma} = \widehat{0}$ in Eqs. (24, 25) and (26)). Then, as the lifetime of K_L is 577 times longer than the lifetime of K_S, we set up a steady state balance between the fast decay of the small K_1 component of a K_L, produced initially without K_1, and its gravity induced regeneration from this K_L.

Considering first a unitary evolution we have to solve

$$j\hbar\frac{d\left|N_K\right\rangle}{d\tau} = -\frac{1}{2}\widehat{\delta m_K}c^2\cdot\left|N_K\right\rangle, \tag{27}$$

$$j\hbar\frac{d\left|n_K\right\rangle}{d\tau} = -\frac{1}{2}\widehat{\delta m_K}c^2\cdot\left|n_K\right\rangle + j\widehat{G}_K\cdot\left|N_K\right\rangle. \tag{28}$$

The operator $\widehat{\delta m_K}c^2$ is given by (3), the operator $d\widehat{x}/d\tau$ by (19) and $\widehat{G}_K$ by (26). The action of $\widehat{G}_K$ on the CP eigenstates $\left|K_1\right\rangle$ and $\left|K_2\right\rangle$, defined in Eqs. (5, 6), is

$$\widehat{G}_K\left|K_2\right\rangle = \kappa\delta m_K c^2\left|K_1\right\rangle, \tag{29}$$

$$\widehat{G}_K\left|K_1\right\rangle = \kappa\delta m_K c^2\left|K_2\right\rangle, \tag{30}$$

where we have defined the small parameter κ

$$\kappa = \frac{2m_K g\hbar}{\delta m_K^2 c^3} = 1.7\times 10^{-3}. \tag{31}$$

This small parameter has been identified and discussed by Fishbach, forty five years ago, as the undimensional combination matching approximately the experimental value of $\mathrm{Re}\,\varepsilon$ [14, 15].

If we consider the following CP eigenstate

$$\left|N_{K_2}(\tau)\right\rangle = \left|K_2\right\rangle\exp - j\delta m_K c^2\tau/2\hbar, \tag{32}$$

which is the $(m_K+\delta m_K)$ mass eigenstate without CPV (6,8), it fulfils Eq. (27) and the associated solution of Eq. (28) is

$$\left|n_{K_2}(\tau)\right\rangle = j\kappa\left|K_1\right\rangle\exp - j\delta m_K c^2\tau/2\hbar. \tag{33}$$

Thus, on earth, the mass eigenstates $\left|K_2^{\oplus}\right\rangle$ is not the CP eigenstates $\left|K_2\right\rangle$ (6), but the sum of the previous solutions (32, 33)

$$\left|K_2^{\oplus}\right\rangle = \left|K_2\right\rangle + j\kappa\left|K_1\right\rangle. \tag{34}$$

We neglect the small correction $O[10^{-6}]$ needed for normalization and consider $\left\langle K_2^{\oplus}\right|\left.K_2^{\oplus}\right\rangle = 1$.

A similar result is obtained for the other $(m_K-\delta m_K)$ mass eigenstate without CPV (5,7) by taking

$$\left|N_{K_1}(\tau)\right\rangle = \left|K_1\right\rangle\exp j\delta m_K c^2\tau/2\hbar \tag{35}$$

as a source term on the right hand side of Eq. (28). This leads to a gravity induced correction

$$\left|n_{K_1}(\tau)\right\rangle = -j\kappa\left|K_2\right\rangle\exp j\delta m_K c^2\tau/2\hbar. \tag{36}$$

The other mass eigenstates on earth is not the CP eigenstates $\left|K_1\right\rangle$ (5), but the sum of the previous solutions (35, 36)

$$\left|K_1^{\oplus}\right\rangle = \left|K_1\right\rangle - j\kappa\left|K_2\right\rangle. \tag{37}$$

At the fundamental level of a unitary evolution, without decays, the impact of earth's gravity appears as a CPT violation, with T conservation, because the indirect violation parameter $\left\langle K_1^{\oplus}\right|\left.K_2^{\oplus}\right\rangle = 2j\kappa$ is imaginary [4], rather than a CP and T violation with CPT conservation requiring a non zero real value [4].

We must now take into account the K_1 fast decay. This decay will change the picture, qualitatively: an apparent CP and T violation, with CPT conservation, is measured experimentally rather than a CPT one because of the finite lifetime of K_1, and quantitatively: with the right prediction of $\mathrm{Re}\,\varepsilon$ which is slightly smaller than κ.

The previous results, Eqs. (34, 37), allow to calculate the *gravity induced transition amplitude* $\Omega_{2\to 1}$ describing the transition amplitude per unit time from the state $\left|K_2^{\oplus}\right\rangle\exp - j\delta m_K c^2\tau/2\hbar$ to the state $\left|K_1^{\oplus}\right\rangle\exp j\delta m_K c^2\tau/2\hbar$,

$$\Omega_{2\to 1} = \left\langle\frac{dK_2^{\oplus}}{d\tau}\right|\left.K_1^{\oplus}(\tau)\right\rangle = \kappa\frac{\delta m_K c^2}{\hbar}\exp j\frac{\delta m_K c^2}{\hbar}\tau. \tag{38}$$

This can be viewed as a gravity induced *oscillating regeneration* competing with the short-lived kaon irreversible decay to the set of final states $\{|f\rangle\}$. This decay takes place at a rate $\Gamma_{1\to f}/2 = (\Gamma_K - \delta\Gamma_K)/2 \sim \Sigma_f \left|\langle f|\mathcal{T}|K_1\rangle\right|^2$. Note that $|\Omega_{2\to 1}| \sim O\left[10^{-3}\Gamma_{1\to f}\right]$ so, starting from a pure $O[1]$ K_2 population, an $O\left[10^{-3}\right]$ K_1 steady state satellite will be observed.

We consider now a typical experiment dedicated to indirect CPV. Experimentally K_1 and K_2 are first produced together in equal amounts. Then, after few $1/\Gamma_{1\to f}$ decay times, the initial content of $|K_1\rangle$ disappears and a pure $|K_2\rangle$ state is expected. In fact, the state $\left|K_{L\exp}(\tau)\right\rangle$ observed in such an experiment is not a pure $|K_2\rangle$ state. This observed $\left|K_{L\exp}(\tau)\right\rangle$ state is a linear superposition of $|K_2\rangle$, plus a small amount of $|K_1\rangle$,

$$\left|K_{L\exp}(\tau)\right\rangle = a_2(\tau)|K_2\rangle + a_1(\tau)|K_1\rangle, \tag{39}$$

resulting from the balance between gravity induced regeneration (38) and irreversible decay of the K_1 component. We assume that the K_2 component is stable and that the depletion of its amplitude associated with the *gravitational regeneration* of K_1 is negligible so that $|a_2(\tau)| = 1$

$$a_2(\tau) = \exp - j\delta m_K c^2\tau/2\hbar. \tag{40}$$

The amplitude a_1 of K_1 in (39) is given by the steady-state balance between a decay at the (amplitude) rate $\Gamma_{1\to f}/2$ on the one hand, and a (gravity induced) transition/regeneration $\Omega_{2\to 1}$ (38) from K_2 on the other hand

$$a_2(\tau)\Omega_{2\to 1} = a_1(\tau)\frac{\Gamma_{1\to f}}{2}. \tag{41}$$

The solution is this equation is

$$a_1(\tau) = \frac{\delta m_K c^2}{\hbar\Gamma_1/2}\kappa \exp j\delta m_K c^2\tau/2\hbar, \tag{42}$$

where we have dropped $\to f$ in Γ_1 to simplify the notations. The short-lived $|K_1\rangle$ component is observed through its two pions decay [1]. Thus the observed long-lived mass eigenstate $\left|K_{L\exp}\right\rangle$, obtained after few $1/\Gamma_1$ decay times away from a neutral kaons source, must be represented by

$$\left|K_{L\exp}\right\rangle = |K_2\rangle + \frac{\delta m_K c^2}{\hbar\Gamma_1/2}\kappa|K_1\rangle. \tag{43}$$

This is the usual CPV parametrization of the kaon state Eq. (10). The observed value of the indirect, gravity induced, CPV parameter,

$$\mathrm{Re}\,\varepsilon_{\exp} = \frac{\delta m_K c^2}{\hbar\Gamma_1/2}\frac{2m_K g\hbar}{\delta m_K^2 c^3} = 1.66\times 10^{-3}, \tag{44}$$

is in agreement with the experimental value, reported by Gershon and Nir, page 290 of Ref. [5]:

$$\mathrm{Re}\,\varepsilon_{\,PDG2024} = (1.66\pm 0.02)\times 10^{-3}. \tag{45}$$

We have taken into account here the finite lifetime of the short-lived kaon, to complete this analysis we can also take into account the decay of the other mass eigenstate, and this will reveal a phenomenological dissipative phase of ε. Considering $\Gamma_1 = \Gamma_S = \Gamma_K - \delta\Gamma_K$ for K_1, and $\Gamma_L = \Gamma_K + \delta\Gamma_K$ for K_2 ($\delta\Gamma_K < 0$), beside the usual definition of decay rates $\Gamma_{S/L} = \Sigma_f \left|\langle f|\mathcal{T}|K_{S/L}\rangle\right|^2$ in terms of transition amplitudes, Bell and Steinberger [16] have demonstrated a general relation based on global unitarity starting from the evaluation of $d\langle M|M\rangle/d\tau$ at $\tau = 0$. Using the fact that, for K_S, the sum Σ_f over the final states is dominated (99.9%) by $K_S \to 2\pi$ decays, more precisely by the $K_S \to I_0$ decays (95%) to the isospin-zero combination of $\left|\pi^+\pi^-\right\rangle$ and $\left|\pi^0\pi^0\right\rangle$, the Bell-Steinberger's unitarity relations [16] can be written:

$$j\frac{\delta m_K c^2}{\hbar} + \frac{\Gamma_S}{2} = \frac{\langle I_0|\mathcal{T}|K_L\rangle\langle I_0|\mathcal{T}|K_S\rangle^*}{\langle K_S|K_L\rangle}. \tag{46}$$

The restriction of the sum $\sum_f |f\rangle$ to $|I_0\rangle$ reduces the K_S width to $\Gamma_S = \langle I_0|\mathcal{T}|K_S\rangle \langle I_0|\mathcal{T}|K_S\rangle^*$ so that

$$\frac{\langle I_0|\mathcal{T}|K_L\rangle}{\langle I_0|\mathcal{T}|K_S\rangle} = \frac{\langle I_0|\mathcal{T}|K_L\rangle \langle I_0|\mathcal{T}|K_S\rangle^*}{\Gamma_S}. \tag{47}$$

This expression is then substituted in Bell-Steinberger's relation (46) to obtain the final expression

$$\frac{\langle I_0|\mathcal{T}|K_L\rangle}{\langle I_0|\mathcal{T}|K_S\rangle} = \frac{\langle K_S|K_L\rangle}{2}\left(1 + j\frac{2\delta m_K c^2}{\hbar\Gamma_S}\right). \tag{48}$$

The left hand side of Eq. (48) is just the definition of the complex parameter ε and $\langle K_S|K_L\rangle/2 = \mathrm{Re}\,\varepsilon$, thus the argument of the CPV complex parameter ε is given by

$$\arg\varepsilon = \arctan\left(2\delta m_K c^2/\hbar\Gamma_S\right) = 43.4°, \tag{49}$$

in agreement with the experimental result 43.5° [5]. This last relation (49) complements (44) and confirms that gravity induced CPV provides a global pertinent framework to interpret $K^0/\overline{K}^0$ indirect CPV experiments.

It is very important to note that the fundamental parameter describing indirect CPV is associated with the unitary evolution overlap of the mass eigenstates induced by earth's gravity

$$\frac{\left\langle K_1^{\oplus}|K_2^{\oplus}\right\rangle}{2} = j\frac{2m_K g\hbar}{\delta m_K^2 c^3}, \tag{50}$$

and, as explained above, the measurments of the complex CPV parameter given by

$$\frac{2m_K g\hbar}{\delta m_K^2 c^3}\left[\frac{2\delta m_K c^2}{\hbar\Gamma_S}\left(1 + j\frac{2\delta m_K c^2}{\hbar\Gamma_S}\right)\right], \tag{51}$$

is due to a *dissipative dressing* of this overlap (50) resulting from the finite lifetime of the mesons. This dissipative dressing is not *stricto sensu* a CPV effects but is inherent to the experiments, this point is important to interpret type (*ii*) CPV and to understand the nature of gravity induced CPV.

6. Gravity induced type (*ii*) CPV in the decay of $K^0/\overline{K}^0$

The analysis of type (*ii*) and (*iii*) CPV rely on the measurement of the ratio η_f associated with the the decay amplitudes to one final state $\langle f|$,

$$\eta_f = \frac{\langle f|\mathcal{T}|K_L\rangle}{\langle f|\mathcal{T}|K_S\rangle}, \tag{52}$$

or on the measurement of the phase-convention-independent ratio of amplitudes λ_f,

$$\lambda_f = \frac{\left\langle \overline{K}^0|K_S\right\rangle}{\langle K^0|K_S\rangle}\frac{\left\langle f|\mathcal{T}|\overline{K}^0\right\rangle}{\langle f|\mathcal{T}|K^0\rangle} = -\frac{\left\langle \overline{K}^0|K_L\right\rangle}{\langle K^0|K_L\rangle}\frac{\left\langle f|\mathcal{T}|\overline{K}^0\right\rangle}{\langle f|\mathcal{T}|K^0\rangle}. \tag{53}$$

These parameters η_f and λ_f capture the informations on the CP asymmetry associated with the decays to one final state $\langle f|$. These two CPV parameters are not independent,

$$\eta_f = \frac{1-\lambda_f}{1+\lambda_f}, \lambda_f = \frac{1-\eta_f}{1+\eta_f}. \tag{54}$$

The relations (54) are valid only if CPT invariance is assumed when the mass eigenstates are

$$|K_S\rangle = \frac{1+\varepsilon}{\sqrt{2}}\left|K^0\right\rangle + \frac{1-\varepsilon}{\sqrt{2}}\left|\overline{K}^0\right\rangle, \tag{55}$$

$$|K_L\rangle = \frac{1+\varepsilon}{\sqrt{2}}\left|K^0\right\rangle - \frac{1-\varepsilon}{\sqrt{2}}\left|\overline{K}^0\right\rangle. \tag{56}$$

so that amplitude ratio fulfils

$$\frac{\left\langle \overline{K}^0 \left| K_S \right.\right\rangle}{\left\langle K^0 \left| K_S \right.\right\rangle} = -\frac{\left\langle \overline{K}^0 \left| K_L \right.\right\rangle}{\left\langle K^0 \left| K_L \right.\right\rangle}. \tag{57}$$

Considering the results obtained in the previous section on the unitary evolution of a kaons system on earth, the fundamental mass eigenstates without dissipation (34, 37) are given by

$$\left| K_1^{\oplus} \right\rangle = \frac{1 - j\kappa}{\sqrt{2}} \left| K^0 \right\rangle + \frac{1 + j\kappa}{\sqrt{2}} \left| \overline{K}^0 \right\rangle, \tag{58}$$

$$\left| K_2^{\oplus} \right\rangle = \frac{1 + j\kappa}{\sqrt{2}} \left| K^0 \right\rangle - \frac{1 - j\kappa}{\sqrt{2}} \left| \overline{K}^0 \right\rangle, \tag{59}$$

so that

$$\frac{\left\langle \overline{K}^0 \left| K_1^{\oplus} \right.\right\rangle}{\left\langle K^0 \left| K_1^{\oplus} \right.\right\rangle} \neq -\frac{\left\langle \overline{K}^0 \left| K_2^{\oplus} \right.\right\rangle}{\left\langle K^0 \left| K_2^{\oplus} \right.\right\rangle}. \tag{60}$$

The data analysis protocols used to calculate η_f and λ_f on the basis of the experimental data usually assume CPT invariance and (57), thus it is not straightforward to accommodate the definitions (53) with the relation (60).

The parameter η_f (52) is not invariant under rephasing but the parameter λ_f is constructed to be a phase-convention-independent quantity. The various bra and ket in a quantum model are defined up to an unobservable phase. The arbitrary conventional phases inherent to quantum theoretical models are to be eliminated to define phase-convention-independent observables. However, despite its phase-convention-independent property, λ_f is not adapted to gravity induced CPV because of the relation (60). So we consider an η_f parameter with its rephasing factor to provide a phase-convention-independent quantity.

To interpret the measurements of the direct violation parameter ε' we consider $\left\langle f \right| = \left\langle \pi^0\pi^0 \right|$ and the $2\pi^0$ decays of K_L and K_S. The definition of the direct CPV parameter ε', as a function of the amplitude ratio η_{00}, is given by

$$\eta_{00} = \frac{\left\langle \pi^0\pi^0 \right| \mathcal{T} \left| K_L \right\rangle}{\left\langle \pi^0\pi^0 \right| \mathcal{T} \left| K_S \right\rangle} \equiv \varepsilon - 2\varepsilon', \tag{61}$$

where the direct violation in the decay to one final state ε' is a correction to the indirect violation in the mixing $\mathrm{Re}\,\varepsilon \gg \mathrm{Re}\,\varepsilon'$.

The definition of η_{00} is invariant under rephasing of the pions state $\left\langle \pi^0\pi^0 \right|$, but not with respect to the rephasing of the kaons mass eigenstates $\left| K_{L/S} \right\rangle$. We can define a decay amplitude ratio which is a phase-convention-independent quantity through the multiplication of η_{00} with the factor φ_K

$$\varphi_K = \frac{\left\langle K^0 \left| K_S \right.\right\rangle}{\left\langle K^0 \left| K_L \right.\right\rangle}. \tag{62}$$

If we consider the mass eigenstates (55, 56) used for the usual description of CPV and those obtained at the fundamental level of an unitary evolution (58,59) with gravity induced CPV, we get two different expressions of the rephasing factor φ_K.

For the usual CPV parametrization (55, 56) with CPT conservation

$$\varphi_K = \frac{\left\langle K^0 \left| K_S \right.\right\rangle}{\left\langle K^0 \left| K_L \right.\right\rangle} = 1. \tag{63}$$

For gravity induced CPV (58, 59) we obtain

$$\varphi_K^{\oplus} = \frac{\left\langle K^0 \left| K_1^{\oplus} \right.\right\rangle}{\left\langle K^0 \left| K_2^{\oplus} \right.\right\rangle} = 1 - \left\langle K_1^{\oplus} \left| K_2^{\oplus} \right.\right\rangle, \tag{64}$$

where $O\left[10^{-6}\right]$ and higher orders terms are neglected.

The interaction between a $\left(\pi^0\pi^0\right)$ state and a neutral kaon state, K^0 or $\overline{K}^0$, can not differentiate the K^0 from the $\overline{K}^0$ (a final state phase can be absorbed by a proper phase convention between K^0 and $\overline{K}^0$), thus the amplitude of $K^0 \to \pi^0\pi^0$ can be taken to be equal to the amplitude of $\overline{K}^0 \to \pi^0\pi^0$. Using Eqs. (58, 59), the ratio of amplitudes $\eta_{00}^{\oplus}$ associated with the unitary mass eigenstates resulting from gravity induced CPV is

$$\eta_{00}^{\oplus} = \frac{\left\langle \pi^0\pi^0 \right| \mathcal{T} \left| K_2^{\oplus} \right\rangle}{\left\langle \pi^0\pi^0 \right| \mathcal{T} \left| K_1^{\oplus} \right\rangle} = \frac{\left\langle K_1^{\oplus} \left| K_2^{\oplus} \right. \right\rangle}{2}. \tag{65}$$

We conclude that the physical observable $\eta_{00}\varphi_K$ on earth, without dissipation, within the framework of an unitary evolution, is given by

$$\eta_{00}^{\oplus}\varphi_K^{\oplus} = \frac{\left\langle K_1^{\oplus} \left| K_2^{\oplus} \right. \right\rangle}{2}\left[1 - 2\frac{\left\langle K_1^{\oplus} \left| K_2^{\oplus} \right. \right\rangle}{2}\right]. \tag{66}$$

We have demonstrated in the previous section that the gravity induced mixing between $|K_1\rangle$ and $|K_2\rangle$ leads to an apparent CPT violation with $\left\langle K_1^{\oplus} \left| K_2^{\oplus} \right. \right\rangle = 2j\kappa$ when we neglect the finite lifetime of K_1. When decays are taken into account the finite lifetime of K_1 was shown to induce a rotation from the imaginary value $j\kappa$ to the real observed value $\kappa\left(2\delta m_K c^2/\hbar\Gamma_1\right)$, (34) becomes (43). Taking into account this rotation, the observed amplitude ratio $\eta_{00\,\mathrm{exp}}^{\oplus}\varphi_{K\,\mathrm{exp}}^{\oplus}$ measured in $K_{L/S} \to \pi^0\pi^0$ experiments on earth, [17–19], is thus given by the use of (55, 56), where $\varepsilon = \kappa\left(2\delta m_K c^2/\hbar\Gamma_1\right)$, rather than (58, 59), in the relation (66). This dissipative rotation from $\left\langle K_1^{\oplus} \left| K_2^{\oplus} \right. \right\rangle = 2j\kappa$ to $\langle K_S | K_L\rangle = 2\kappa\left(2\delta m_K c^2/\hbar\Gamma_1\right)$ results in the measured amplitude ratio

$$\eta_{00\,\mathrm{exp}}^{\oplus}\varphi_{K\,\mathrm{exp}}^{\oplus} = \frac{\langle K_S | K_L\rangle}{2}\left[1 - 2\frac{\langle K_S | K_L\rangle}{2}\right]. \tag{67}$$

The phase-convention-independent definition of ε' given by (61) is

$$\eta_{00}\varphi_K = \varepsilon\left[1 - 2\frac{\varepsilon'}{\varepsilon}\right]. \tag{68}$$

This lead to the conclusion $\mathrm{Re}\left(\varepsilon'/\varepsilon\right) = \mathrm{Re}\left(\varepsilon\right)$ if the experiments are interpreted within the gravity induced CPV framework. The gravity induced direct CPV parameter

$$\mathrm{Re}\left(\varepsilon'/\varepsilon\right) = \frac{\delta m_K c^2}{\hbar\Gamma_1/2}\kappa = 1.66 \times 10^{-3}, \tag{69}$$

is in agreement with the experimental value, reported by Gershon and Nir, page 285 of Ref. [5]

$$\mathrm{Re}\left(\varepsilon'/\varepsilon\right) = (1.66 \pm 0.23) \times 10^{-3}. \tag{70}$$

The fact that $\mathrm{Re}\left(\varepsilon'/\varepsilon\right) \sim \mathrm{Re}\left(\varepsilon\right)$ was considered, up to now, as a numerical coincidence and it finds here a simple explanation within the framework of gravity induced CPV.

The precise definition of phase-convention-independent quantities, in order to clearly identify what is measured in an experiment, is also one of the key to interpret the experimental observation of interferences between mixing and decay in CPV dedicated $B^0/\overline{B}^0$ experiments.

7. Gravity induced type (*iii*) CPV in the interference between mixing and decay of $B^0/\overline{B}^0$

Up to 2001, the evidences of CPV where restricted to K mesons experiments and the baryons asymmetry of the universe. In 2001 the first clear identification of CPV with B mesons experiments in B-factories was reported [20, 21]. The mass and width ordering associated with the $B^0/\overline{B}^0$ system is given by : $\delta m_B/m_B \sim 10^{-19}$ and $\delta m_B/\Gamma_B \sim 0.7$. The lifetime of the CP eigenstate B_1 is considered to be equal to the lifetime of the other CP eigenstate B_2 so that $\delta\Gamma_B = 0$. The most pronounced CPV effects in the $B^0/\overline{B}^0$ system is displayed through interference experiments dedicated to the study of the phase difference between the decay path $B_0 \to f$ and the decay path $B_0 \to \overline{B}^0 \to f$ [20–22].

To set up an interpretation of these experiments we keep a finite lifetime Γ_B^{-1} for both particles and consider the decay operator

$$\widehat{\gamma}_B = \Gamma_B \left[\left| B^0 \right\rangle \left\langle B^0 \right| + \left| \overline{B}^0 \right\rangle \left\langle \overline{B}^0 \right| \right], \tag{71}$$

to describe the dissipative part of the bottomness dynamics. Thus, we have to solve Eqs. (24, 25)

$$j\hbar \frac{d\left|N_B\right\rangle}{d\tau} = -\frac{1}{2}\left(\widehat{\delta m_B} c^2 + j\hbar\widehat{\gamma}_B\right)\cdot\left|N_B\right\rangle, \tag{72}$$

$$j\hbar \frac{d\left|n_B\right\rangle}{d\tau} = -\frac{1}{2}\left(\widehat{\delta m_B} c^2 + j\hbar\widehat{\gamma}_B\right)\cdot\left|n_B\right\rangle + j\widehat{G}_B\cdot\left|N_B\right\rangle. \tag{73}$$

The operator $\widehat{\delta m_B} c^2$ is given by (3), $\widehat{G}_B$ by (26), the operator $d\widehat{x}/d\tau$ by (19) and $\widehat{\gamma}_B$ by (71). The action of $\widehat{G}_B$ on the CP eigenstates $\left|B_1\right\rangle$ and $\left|B_2\right\rangle$ (5, 6) is

$$j\widehat{G}_B\left|B_2\right\rangle = -\delta m_B c^2 \varsigma\left(1 - j\chi\right)\left|B_1\right\rangle, \tag{74}$$

$$j\widehat{G}_B\left|B_1\right\rangle = \delta m_B c^2 \varsigma\left(1 + j\chi\right)\left|B_2\right\rangle. \tag{75}$$

Where we define the real parameters χ and ς associated with this gravity induced mixing of the $\left[\left|B_1\right\rangle, \left|B_2\right\rangle\right]$ CP basis

$$\chi = \delta m_B c^2/\hbar\Gamma_B = 0.77, \tag{76}$$

$$\varsigma = 2 m_B g\hbar/\delta m_B^2 c^3 \left(\chi + \chi^{-1}\right) \sim O\left[10^{-6}\right]. \tag{77}$$

In order to solve Eq. (73) and to express the mass eigenstates on earth, we consider the CP eigenstates

$$\left|N_{B_2}(\tau)\right\rangle = \left|B_2\right\rangle \exp - j\frac{\delta m_B c^2 - j\hbar\Gamma_B}{2\hbar}\tau, \tag{78}$$

which is also the $(m_B + \delta m_B)$ mass eigenstate without CPV, it fulfils (72) and the associated solution of (73) is

$$\left|n_{B_2}(\tau)\right\rangle = -\varsigma\left(1 - j\chi\right)\left|B_1\right\rangle \exp - j\frac{\delta m_B c^2 - j\hbar\Gamma_B}{2\hbar}\tau. \tag{79}$$

Then we consider the other $(m_B - \delta m_B)$ CP eigenstate as a drive on the right hand side of Eq. (73)

$$\left|N_{B_1}(\tau)\right\rangle = \left|B_1\right\rangle \exp j\frac{\left(\delta m_B c^2 + j\hbar\Gamma_B\right)}{2\hbar}\tau. \tag{80}$$

It fulfils Eq. (72) and the driven solution of Eq. (73) is

$$\left|n_{B_1}(\tau)\right\rangle = -\varsigma\left(1 + j\chi\right)\left|B_2\right\rangle \exp j\frac{\delta m_B c^2 + j\hbar\Gamma_B}{2\hbar}\tau. \tag{81}$$

Thus, on earth, the CP eigenstates $\left|B_1\right\rangle$ and $\left|B_2\right\rangle$ (5, 6) are no longer the mass eigenstates $B^{\oplus}_{L/H}$ which are given by the sum $\left|N_{B_{1/2}}\right\rangle + \left|n_{B_{1/2}}\right\rangle$ of (78, 80) plus (79, 81)

$$\left|B_L^{\oplus}\right\rangle = \left|B_1\right\rangle - \varsigma\left(1 + j\chi\right)\left|B_2\right\rangle, \tag{82}$$

$$\left|B_H^{\oplus}\right\rangle = \left|B_2\right\rangle - \varsigma\left(1 - j\chi\right)\left|B_1\right\rangle. \tag{83}$$

Using the flavor basis $\left[\left|B^0\right\rangle, \left|\overline{B}^0\right\rangle\right]$, rather than the CP basis $\left[\left|B_1\right\rangle, \left|B_2\right\rangle\right]$, these mass eigenstates (82, 83) become

$$\left|B_L^{\oplus}\right\rangle = \frac{1 - \varsigma\left(1 + j\chi\right)}{\sqrt{2}}\left|B^0\right\rangle + \frac{1 + \varsigma\left(1 + j\chi\right)}{\sqrt{2}}\left|\overline{B}^0\right\rangle, \tag{84}$$

$$\left|B_H^{\oplus}\right\rangle = \frac{1 - \varsigma\left(1 - j\chi\right)}{\sqrt{2}}\left|B^0\right\rangle - \frac{1 + \varsigma\left(1 - j\chi\right)}{\sqrt{2}}\left|\overline{B}^0\right\rangle. \tag{85}$$

The difference between these gravity induced mass eigenstates (82, 83, 84, 85) and the usual type (*iii*) $B^0/\overline{B}^0$ parametrization (11, 12), is that gravity induced CPV requires two real number ς and χ to express the eigenstates $\left|B^{\oplus}_{L/H}\right\rangle$ although type (*iii*) standard CPV parametrization (11, 12)

$$\left|B_L\right\rangle = \frac{\exp+j\beta}{\sqrt{2}}\left|B^0\right\rangle + \frac{\exp-j\beta}{\sqrt{2}}\left|\overline{B}^0\right\rangle, \quad (86)$$

$$\left|B_H\right\rangle = \frac{\exp+j\beta}{\sqrt{2}}\left|B^0\right\rangle - \frac{\exp-j\beta}{\sqrt{2}}\left|\overline{B}^0\right\rangle, \quad (87)$$

is based on a single real parameter β to interpret the experimental results.

This difference is due to the CPT invariance hypothesis associated with the parametrization (11, 12) and (86, 87).

When the decay into one final CP eigenstate $\left|f\right\rangle$ is considered in experiments, the observable λ_f (53) is given by

$$\lambda_f = \frac{\left\langle\overline{B}^0\left|B_L\right\rangle\right.}{\left\langle B^0\left|B_L\right\rangle\right.}\frac{\left\langle f\right|\mathcal{T}\left|\overline{B}^0\right\rangle}{\left\langle f\right|\mathcal{T}\left|B^0\right\rangle} = \exp-2j\beta\frac{\left\langle f\right|\mathcal{T}\left|\overline{B}^0\right\rangle}{\left\langle f\right|\mathcal{T}\left|B^0\right\rangle}, \quad (88)$$

which is obviously phase-convention-independent. This parameter is observable through the measurement of S_f and C_f

$$S_f = 2\,\mathrm{Im}\,\lambda_f/(1+\left|\lambda_f\right|^2),\ C_f = (1-\left|\lambda_f\right|^2)/(1+\left|\lambda_f\right|^2), \quad (89)$$

which can be extracted from the data obtained from interferences between the direct path $B_0 \to f$ and the mixed path $B_0 \to \overline{B}^0 \to f$.

This parameter λ_f is meaningful to characterize type (*iii*) CPV with the CPT invariant parametrization (86, 87) because it captures all the component of the expansion of the mass eigenstates on the bottomness basis as

$$\frac{\left\langle\overline{B}^0\left|B_L\right\rangle\right.}{\left\langle B^0\left|B_L\right\rangle\right.} = -\frac{\left\langle\overline{B}^0\left|B_H\right\rangle\right.}{\left\langle B^0\left|B_H\right\rangle\right.} = \frac{\left\langle\overline{B}^0\left|B_L\right\rangle\right.}{\left\langle B^0\left|B_H\right\rangle\right.} = -\frac{\left\langle\overline{B}^0\left|B_H\right\rangle\right.}{\left\langle B^0\left|B_L\right\rangle\right.}. \quad (90)$$

However, these four amplitudes ratios are different if we consider the gravity induced mass eigenstates (84, 85)

$$\frac{\left\langle\overline{B}^0\left|B^{\oplus}_L\right\rangle\right.}{\left\langle B^0\left|B^{\oplus}_L\right\rangle\right.} \neq -\frac{\left\langle\overline{B}^0\left|B^{\oplus}_H\right\rangle\right.}{\left\langle B^0\left|B^{\oplus}_H\right\rangle\right.} \neq \frac{\left\langle\overline{B}^0\left|B^{\oplus}_L\right\rangle\right.}{\left\langle B^0\left|B^{\oplus}_H\right\rangle\right.} \neq -\frac{\left\langle\overline{B}^0\left|B^{\oplus}_H\right\rangle\right.}{\left\langle B^0\left|B^{\oplus}_L\right\rangle\right.}. \quad (91)$$

Despite this difference between (90) and (91), the experimental results analyzed within a CPT invariant framework (90), can be understood and explained within the framework of gravity induced CPV (91). This situation is similar to the one encountered in section 6 devoted to the study of ε': if CPT is assumed the rephasing factors $\varphi = 1$, and the interpretation of the experimental measurements is based on the hypothesis of direct violation and imply a CPV at the fundamental level of the CKM matrix. However, if earth's gravity effects are taken into account $\varphi \neq 1$ and the very same phase-convention-independent measured quantities agree with the experiments without any additional assumptions. In section 6, earth's gravity was identified as the sole source of ε'.

The analysis below will use two different approaches to interpret the measurement of β, each providing the same final result. The two issues addressed below are: first, the invariance under rephasing of the mass eigenstates, when needed, to define an observable and second, the invariance under rephasing of the flavor eigenstates, when needed, to define an observable.

In order to accommodate the relation (88) with (90, 91), we consider a λ_f parameter constructed with the amplitude ratio $\left\langle\overline{B}^0\left|B_L\right\rangle\right./\left\langle B^0\left|B_H\right\rangle\right.$ which is better suited to characterize the dynamics of oscillating $B_{L/S}$ as it takes into account all the eigenstates: the two flavor eigenstates and the two mass eigenstates involved in experiments. However, this $\widetilde{\lambda}_f$ parameter reflecting the $B_{L/S}$ content of the oscillating and propagating $B^0/\overline{B}^0$,

$$\widetilde{\lambda}_f = \frac{\left\langle \overline{B}^0 \left| B_L \right\rangle \right.}{\left\langle B^0 \left| B_H \right\rangle \right.} \frac{\left\langle f \middle| \mathcal{T} \middle| \overline{B}^0 \right\rangle}{\left\langle f \middle| \mathcal{T} \middle| B^0 \right\rangle} = \exp -2j\beta \frac{\left\langle f \middle| \mathcal{T} \middle| \overline{B}^0 \right\rangle}{\left\langle f \middle| \mathcal{T} \middle| B^0 \right\rangle}, \tag{92}$$

is not phase-convention-independent with respect to the mass eigenstates.

To set up a fully phase-convention-independent parameter we introduce the symmetric rephasing factor

$$\varphi_B = \sqrt{\frac{\langle B_1 \left| B_H \right\rangle}{\langle B_1 \left| B_L \right\rangle} \frac{\langle B_2 \left| B_H \right\rangle}{\langle B_2 \left| B_L \right\rangle}} = 1. \tag{93}$$

We have used $B_{1/2}$ states because they are CP eigenstates like f. The amplitude ratio observed in the experimental measurement are given by phase-convention-independent product $\widetilde{\lambda}_f \varphi_B$

$$\widetilde{\lambda}_f \varphi_B = \frac{\left\langle \overline{B}^0 \left| B_L \right\rangle \right.}{\left\langle B^0 \left| B_H \right\rangle \right.} \frac{\left\langle f \middle| \mathcal{T} \middle| \overline{B}^0 \right\rangle}{\left\langle f \middle| \mathcal{T} \middle| B^0 \right\rangle} \varphi_B = \exp -2j\beta \frac{\left\langle f \middle| \mathcal{T} \middle| \overline{B}^0 \right\rangle}{\left\langle f \middle| \mathcal{T} \middle| B^0 \right\rangle} \tag{94}$$

which is equal to λ_f (88).

When the same rephasing factor $\varphi_B^{\oplus}$ is calculated within the framework of gravity induced CPV with (82, 83) rather than (11, 12), this gives

$$\varphi_B^{\oplus} = \sqrt{\frac{\langle B_1 \left| B_H^{\oplus} \right\rangle}{\langle B_1 \left| B_L^{\oplus} \right\rangle} \frac{\langle B_2 \left| B_H^{\oplus} \right\rangle}{\langle B_2 \left| B_L^{\oplus} \right\rangle}} = \sqrt{\frac{1 - j\chi}{1 + j\chi}}. \tag{95}$$

The phase-convention-independent product,

$$\widetilde{\lambda}_f^{\oplus} \varphi_B^{\oplus} = \frac{\left\langle \overline{B}^0 \left| B_L^{\oplus} \right\rangle \right.}{\left\langle B^0 \left| B_H^{\oplus} \right\rangle \right.} \frac{\left\langle f \middle| \mathcal{T} \middle| \overline{B}^0 \right\rangle}{\left\langle f \middle| \mathcal{T} \middle| B^0 \right\rangle} \varphi_B^{\oplus}, \tag{96}$$

calculated with (84, 85, 95), becomes

$$\widetilde{\lambda}_f^{\oplus} \varphi_B^{\oplus} = \exp\left(-j \arctan \chi\right) \frac{\left\langle f \middle| \mathcal{T} \middle| \overline{B}^0 \right\rangle}{\left\langle f \middle| \mathcal{T} \middle| B^0 \right\rangle} \left(1 + O\left[10^{-6}\right]\right). \tag{97}$$

To compare the interpretations based on the usual CPT eigenstates $|B_{L/H}\rangle$ (86, 87) with the gravity induced mass eigenstates $\left|B_{L/H}^{\oplus}\right\rangle$ (84, 85), we must define β such that $2\beta = \arctan(0.77)$. If $\left\langle f \middle| \mathcal{T} \middle| \overline{B}^0 \right\rangle / \left\langle f \middle| \mathcal{T} \middle| B^0 \right\rangle$ is assumed real and equal to one the experiments dedicated to $\overline{B}^0/B_0 \to f$ interferences between a direct and a mixed path should give a measurement of $\sin 2\beta$ equal to

$$S_f = \sin 2\beta = \sin\left[\arctan(0.77)\right] = 0.61,\ C_f = 0. \tag{98}$$

The modes $\overline{b} \to \overline{s}s\overline{s}$ and $\overline{b} \to \overline{c}c\overline{s}$ have been studied in depth, both from the SM theoretical point of view and from the experimental point of view, through $B_0 \to \phi K_S^0$ and $B_0 \to \psi K^0$ interference measurments. According to the data reported in [5] the present status of the values is

$$\sin 2\beta_{\phi K_S^0} = 0.58 \pm 0.12,\ \ \sin 2\beta_{\psi K^0} = 0.701 \pm 0.01. \tag{99}$$

Other neutral final states, such as $J/\psi K^{*0}$ and $K^0\pi^0$, giving $S_{J/\psi K^{*0}} = 0.60 \pm 0.24 \pm 0.08$, $C_{J/\psi K^{*0}} = 0.025 \pm 0.083 \pm 0.054$ and $S_{K^0\pi^0} = 0.64 \pm 0.13$, $C_{K^0\pi^0} = 0.00 \pm 0.08$, are in good agreement with the gravity induced effect Eq. (98) if $\left\langle f \middle| \mathcal{T} \middle| \overline{B}^0 \right\rangle = \left\langle f \middle| \mathcal{T} \middle| B^0 \right\rangle$. But, for the full set of final states f studied up to now, S_f are centered around (98) but deviate from this value. The difficulty to evaluate $\arg\left\langle f \middle| \mathcal{T} \middle| \overline{B}^0 \right\rangle / \left\langle f \middle| \mathcal{T} \middle| B^0 \right\rangle$ is one source of the dispersion of S_f, note also that the sign of $\left\langle f \middle| \widehat{CP} \middle| f \right\rangle$ is to be considered to analyze the sign of S_f and the fact that CPT invariance is assumed is probably also a source of dispersion. A clear understanding of the $\sin 2\beta$ distribution around $0.6 - 0.7$ requires to drop the CPT assumption and to adopt the mass eigenstates (84, 85), rather than (86, 87), to write down the data analysis protocols used to extract the physical information from the raw experimental data. A precise evaluation of $\left\langle f\right|\mathscr{T}\left|\overline{B}^{0}\right\rangle / \left\langle f\right|\mathscr{T}\left|B^{0}\right\rangle$ is also needed.

Let us adopt a second point of view. We will not consider the interpretation of interferences experiments and, rather than addressing the issue of λ_f, we address directly the issue of β.

We consider the different mass eigenstates expansions on either CP or flavor eigenstates: (11, 12, 86, 87) for the CPT one, and (82, 83, 84, 85) for the gravity induced one. In order to compare the usual eigenstates parametrization (86, 87), based on a single angle β, with the gravity induced mass eigenstates (84, 85), involving two parameters ς and χ, we must define β through a *gedanken* experiment providing $\exp j\beta$ as a phase-convention-independent expression. We consider the symmetric and complete combination

$$\rho_B = \frac{\left\langle B^0 \left| B_L \right. \right\rangle}{\left\langle \overline{B}^0 \left| B_L \right. \right\rangle} \frac{\left\langle B^0 \left| B_H \right. \right\rangle}{\left\langle \overline{B}^0 \left| B_H \right. \right\rangle}, \tag{100}$$

which takes into account the four components at work in the description. This definition of β through ρ_B takes into account all flavor and mass eigenstates but suffers from a lack of (unphysical) phase compensation with respect to the flavor eigenstates. All measured observables, independently of the interpretation of the measurement, are combinations of phase-convention-independent quantities. We introduce the coefficient φ'_B needed to provide a phase-convention-independent observable associated with ρ_B

$$\varphi'_B = \frac{\left\langle \overline{B}^0 \left| B_2 \right. \right\rangle \left\langle B_2 \left| B_H \right. \right\rangle}{\left\langle B^0 \left| B_1 \right. \right\rangle \left\langle B_1 \left| B_L \right. \right\rangle} \frac{\left\langle \overline{B}^0 \left| B_2 \right. \right\rangle \left\langle B_2 \left| B_L \right. \right\rangle}{\left\langle B^0 \left| B_1 \right. \right\rangle \left\langle B_1 \left| B_H \right. \right\rangle}, \tag{101}$$

where we have chosen the two projection operators $\left| B_1 \right\rangle \left\langle B_1 \right|$ and $\left| B_2 \right\rangle \left\langle B_2 \right|$ because they commute with CP.

It can be checked that the product $\rho_B \varphi'_B$ is phase-convention-independent and thus can be measured in a gedanken experiment.

If the usual CPT invariant parametrization of CPV effects is used (11, 12, 86, 87), this rephasing factor φ'_B changes nothing because it is equal to one

$$\rho_B = -\exp j4\beta, \tag{102}$$

$$\varphi'_B = 1, \tag{103}$$

and the product $\rho_B \varphi'_B$ can be measured and interpreted as -exp $j4\beta$.

If CPV is gravity induced, we replace $\left| B_H \right\rangle$ and $\left| B_L \right\rangle$ with $\left| B_H^{\oplus} \right\rangle$ and $\left| B_L^{\oplus} \right\rangle$ given by (82, 83, 84, 85), and the very same observable is the product of the following factors

$$\rho_B^{\oplus} = -1 + O\left[10^{-6}\right], \tag{104}$$

$$\varphi_B'^{\oplus} = \frac{1 + j\chi}{1 - j\chi} = \exp\left(2j \arctan\chi\right). \tag{105}$$

We conclude that, if gravity induced CPV is taken into account, the measurement of the phase-convention-independent observable $\rho_B \varphi'_B$ on earth gives

$$\rho_B^{\oplus} \varphi_B'^{\oplus} = -\exp\left(2j \arctan\chi\right), \tag{106}$$

although if the measurement of the very same phase-convention-independent observable $\rho_B \varphi'_B$ is interpreted within the usual CPT invariant framework it defines β as

$$\rho_B \varphi'_B = -\exp j4\beta. \tag{107}$$

The conclusion of this ρ_B *gedanken* measurement with two frameworks of interpretation is that $\arctan\chi = 2\beta$ and

$$\sin 2\beta = \sin\left[\arctan(0.77)\right] = 0.61. \tag{108}$$

A twelve years old Belle [23] and BaBar [24] average gives $\sin 2\beta = 0.67 \pm 0.02$ [25], onely few percents above (108).

The measurement of this angle β is still one of the major subjects at the forefront of the studies related to the physics of the SM.

8. Gravity induced CPV in $D^0/\overline{D}^0$ and $B_s^0/\overline{B_s}^0$ experiments and conclusions

On the basis of the exact predictions of ε and ε', and of the prediction of $\sin 2\beta$ with an accurracy of few percent with respect to a global average [23] [24], we can state that gravity induced CPV offers a pertinent framework to interpret $K^0/\overline{K}^0$ and $B^0/\overline{B}^0$ experiments dedicated to CPV and that the CKM matrix must be considered free from any CPV phase far from any massive object.

The previous calculations on the impact of earth gravity on neutral mesons oscillations can be extended to $D^0/\overline{D}^0 \sim (c\overline{u})/(\overline{c}u)$ and $B_s^0/\overline{B_s}^0 \sim \left(s\overline{b}\right)/(\overline{s}b)$. The framework of analysis of the experimental data on $D^0/\overline{D}^0$ and $B_s^0/\overline{B_s}^0$ is similar to the methods presented in section 5, 6 and 7. The parameters $m_D g\hbar/\delta m_D^2 c^3$ and $m_{B_s} g\hbar/\delta m_{B_s}^2 c^3$ for both mesons systems are very small so a type (i) indirect violations will be extremely difficult to observe. However type (ii) and type (iii) CPV can be analyzed on the basis of gravity induced CPV, presented in section 6 and 7, and will be considered in a forthcoming analysis.

In any environment where a flavored neutral mesons $|M\rangle$, with mass m, mass spliting δm and Compton wavelength λ_C, experiences a gravity $\mathbf{g}$, i.e. in any curved space-time environment, the amplitude of CP violation will be given by

$$(m/\delta m)^2 \left|\mathbf{g}\right| \lambda_C / c^2. \tag{109}$$

The first factor $m/\delta m$ is associated with electroweak and strong interactions, the second one is the product of a (wave)length, an acceleration and c, quantities related to geometry and space-time rather than to electroweak or strong interactions. The proportionality to $\left|\mathbf{g}\right|$ indicate that this new CPV mechanism allows to set up cosmological evolution models predicting the strong asymmetry between the abundance of matter and the abundance anti-matter in our present universe [6].

Beside the problem of early baryogenesis, neutrinos oscillations near a spherical massive object might be revisited to explore the impact of the interplay between gravity and mixing.

The type (i) CPV observed with $K^0/\overline{K}^0$ stems from a gravity induced interplay between vertical quarks zitterbewegung oscillations at the velocity of light on the one hand and the strangeness oscillations ($\Delta S = 2$) on the other hand.

The type (ii) small CPV observed with $K^0/\overline{K}^0$ is associated with the CPT invariant modelisation of a gravity induced CPT violation and is elucidated through a careful analysis of the rephasing invariance of the observable η_{00}.

The large type (iii) CPV observed with $B^0/\overline{B}^0$ is associated with the CPT invariant modelisation of a gravity induced CPT violation displaying a very small modulus and a significant phase β.

When the mesons are considered stables, the evolution is unitary and there is no T violation, T violation stems from the modelisation of the transition amplitudes w_f in Eq. (1) as irreversible decays in Eq. (2) within the framework of the WW approximation [10].

The very large type (iv) CPV observed in our universe, namely its baryon-antibaryon asymmetry, remains an open issue within the KM framework of interpretation, although gravity induced CPV displays the potential to set up cosmological evolution models in agreement with the present state of our universe.

We have demonstrated that gravity induced CPV allows to predict three experimental CPV parameters ($\varepsilon, \varepsilon', \beta$) and appears to provide the potential to explain the baryon asymmetry of the universe as its amplitude is linear with respect to the strength of gravity.

This set of new results was obtained within the canonical framework of quantum mechanics, on earth, without any speculative assumption on new coupling, or new field, or new physics. From this clear convergence of results, we can conclude that a CKM matrix free of CPV phase is to be considered as the core of the SM in a flat Lorentzian environment and earth's gravity is the sole source of ε, ε' and β CPV effects in $K^0/\overline{K}^0$ and $B^0/\overline{B}^0$ experiments.

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