UT*fit* combined analysis of $D - \overline{D}$ mixing data and extraction of the CKM angle γ

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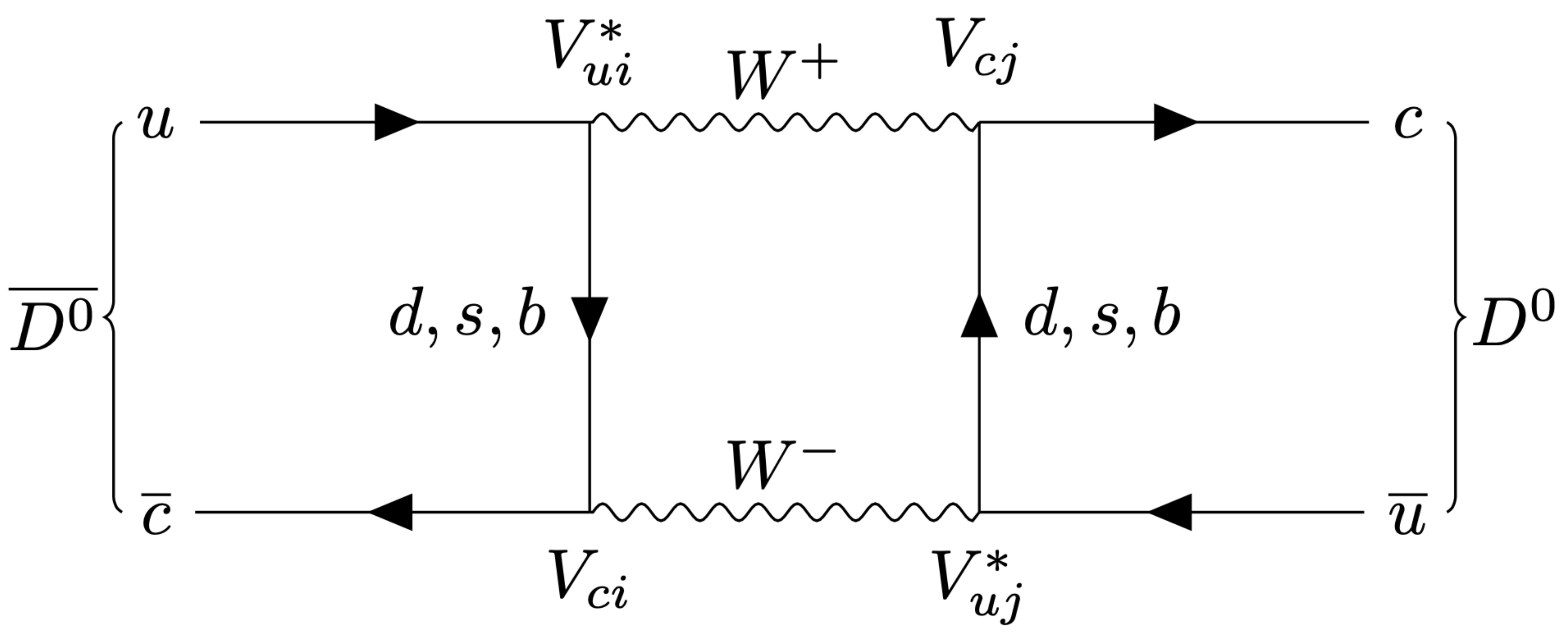
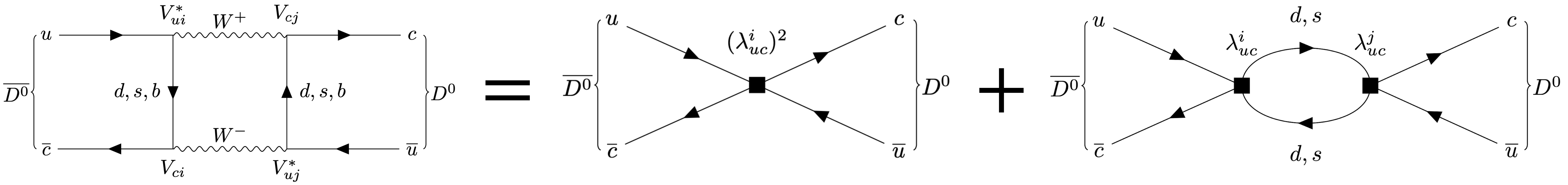
Why charm mixing?



- NO FCNC at the tree-level in the SM
- GIM + CKM suppression. CPV starts at $O(\theta_c^4)$
- Many experimental results. No first-principles predictions yet

Mixing parameters



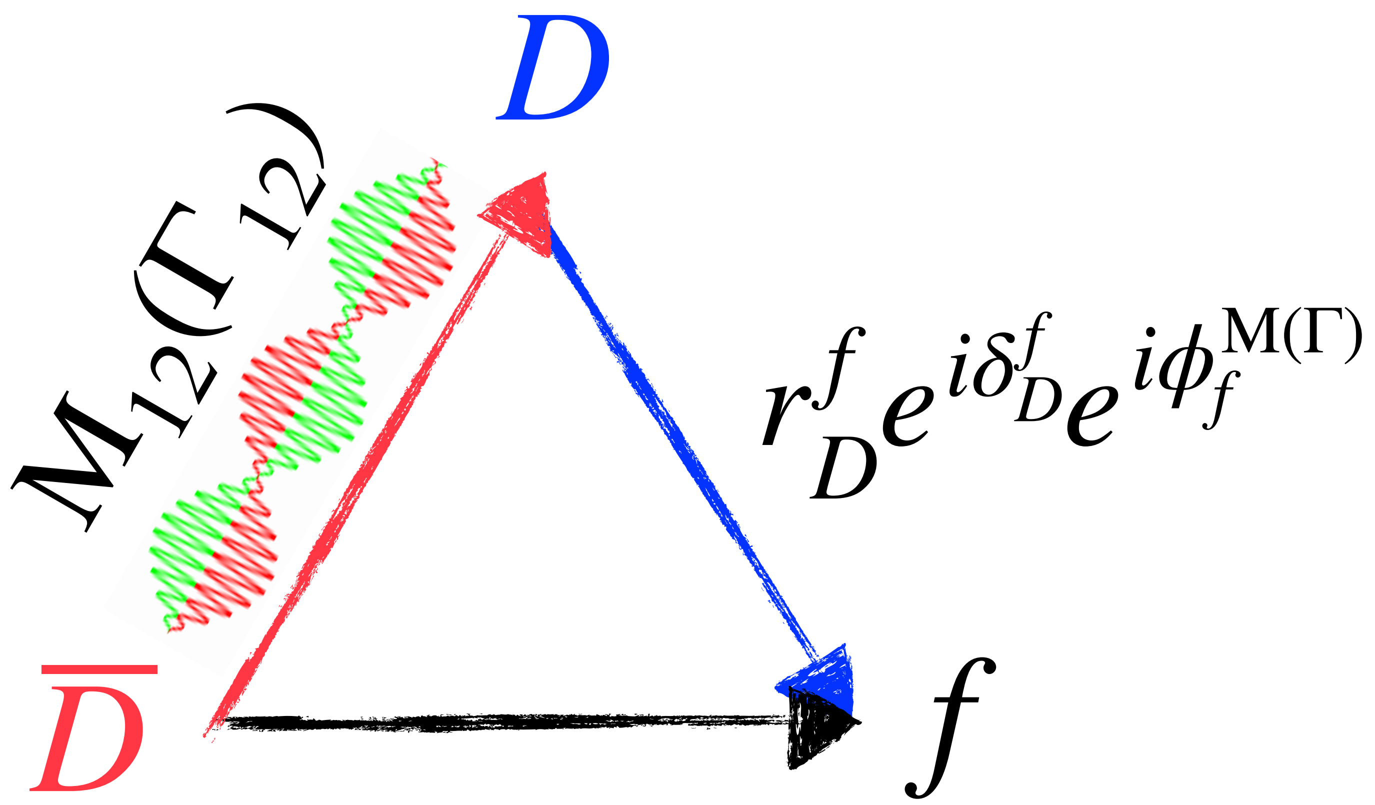
Short-distance (SD) Long-distance (LD)

- In the SM, it is expected LD $\approx 100 \times$ SD

$$\langle D | S - 1 | \overline{D} \rangle = \mathrm{M}_{12} - i/2\Gamma_{12}$$

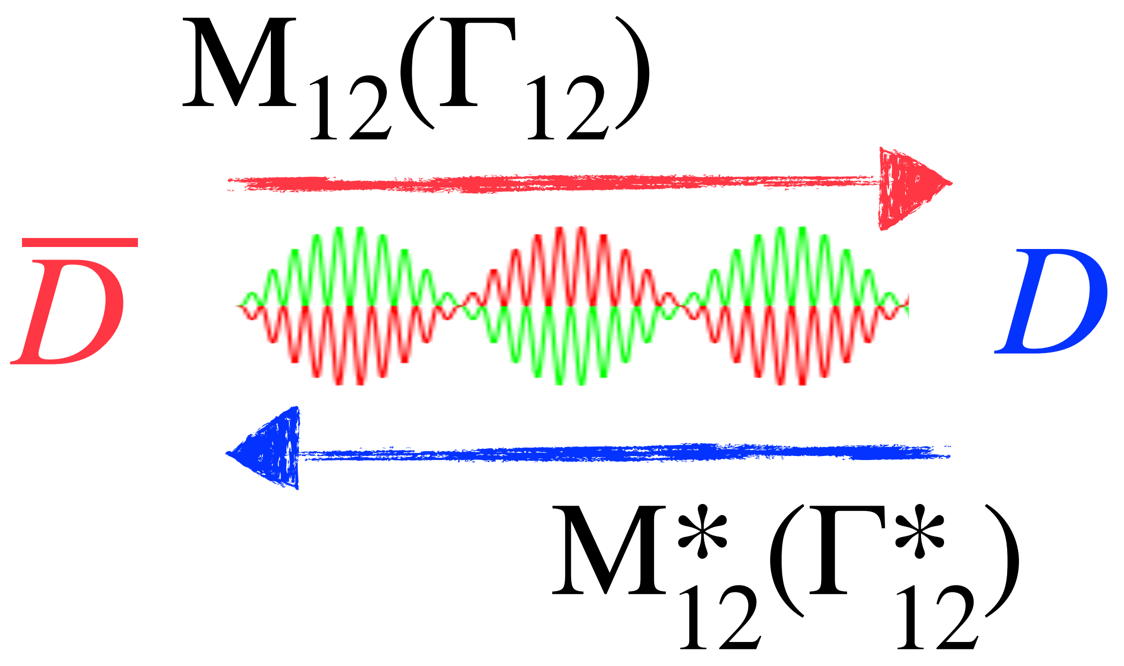
Indirect CPV

- CPV between decay with and without Dispersive (Absorptive) mixing



$$\phi_f^{M(\Gamma)} = \phi^{M(\Gamma)} + \psi_f$$

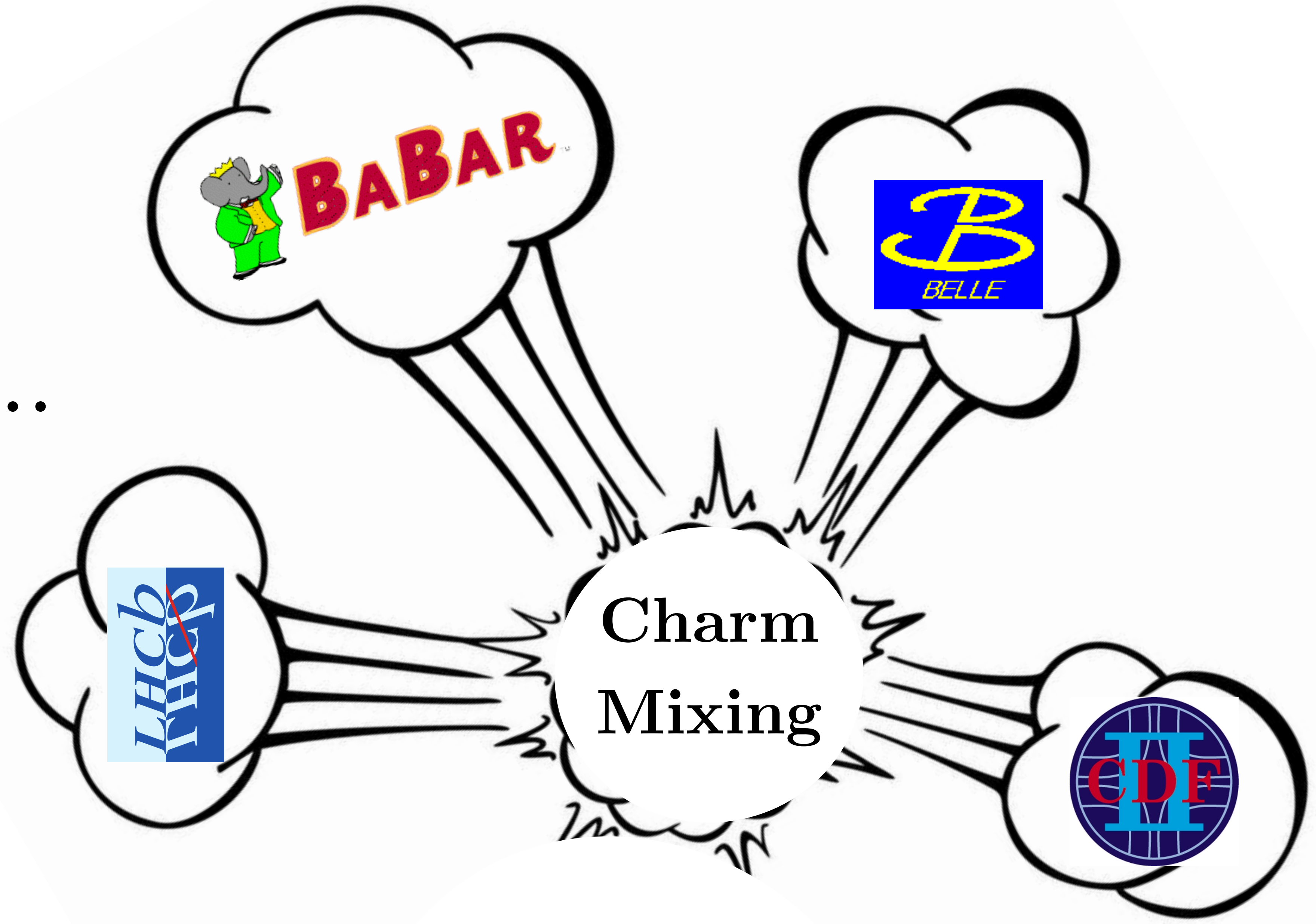
- CPV in pure mixing



$$\phi_{12} = \phi_f^M - \phi_f^\Gamma = \phi^M - \phi^\Gamma$$

Measurements of CPV

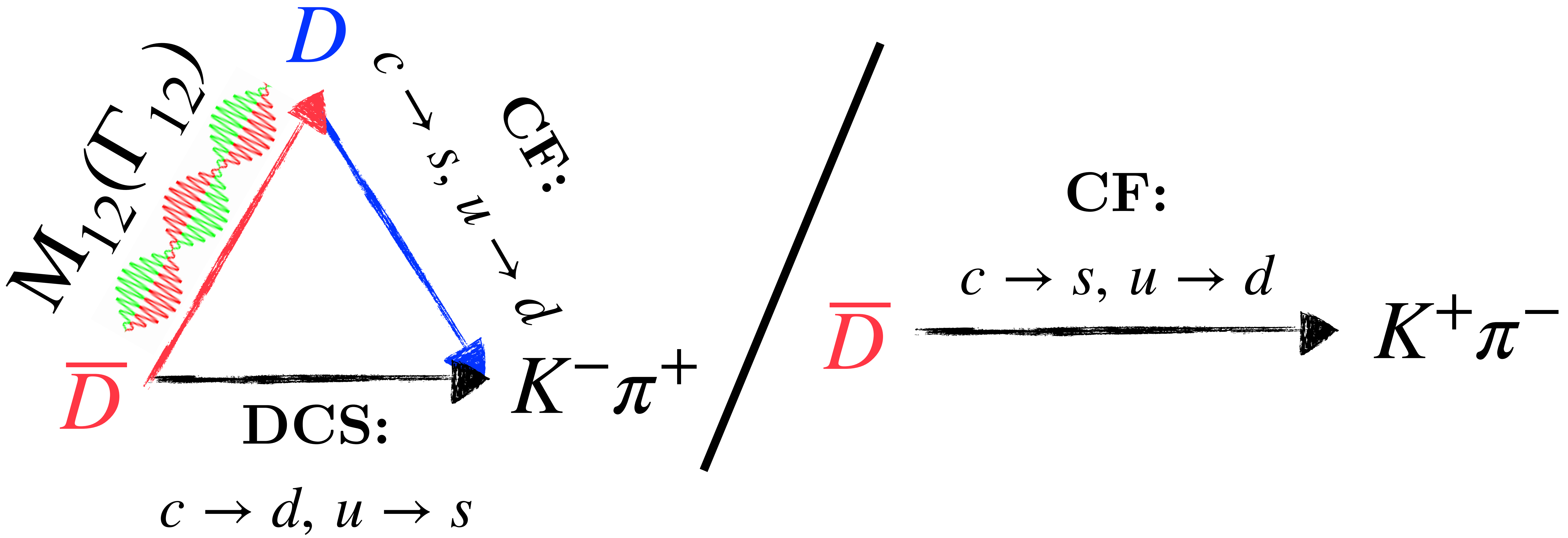
More than 15 years of experiments...



- CF/DCS decays to $K\pi$
- Phase-space analysis of $K_S^0\pi\pi$
- SCS decays to $\pi\pi$, KK

CF/DCS decays to $K\pi$

- Experiments measure the ratio between DCS/CF decay rates



- The coefficients of the expansion in time depend on the charm parameters

(e.g. Belle(2006), Babar(2007), LHCb(2024, 2025))

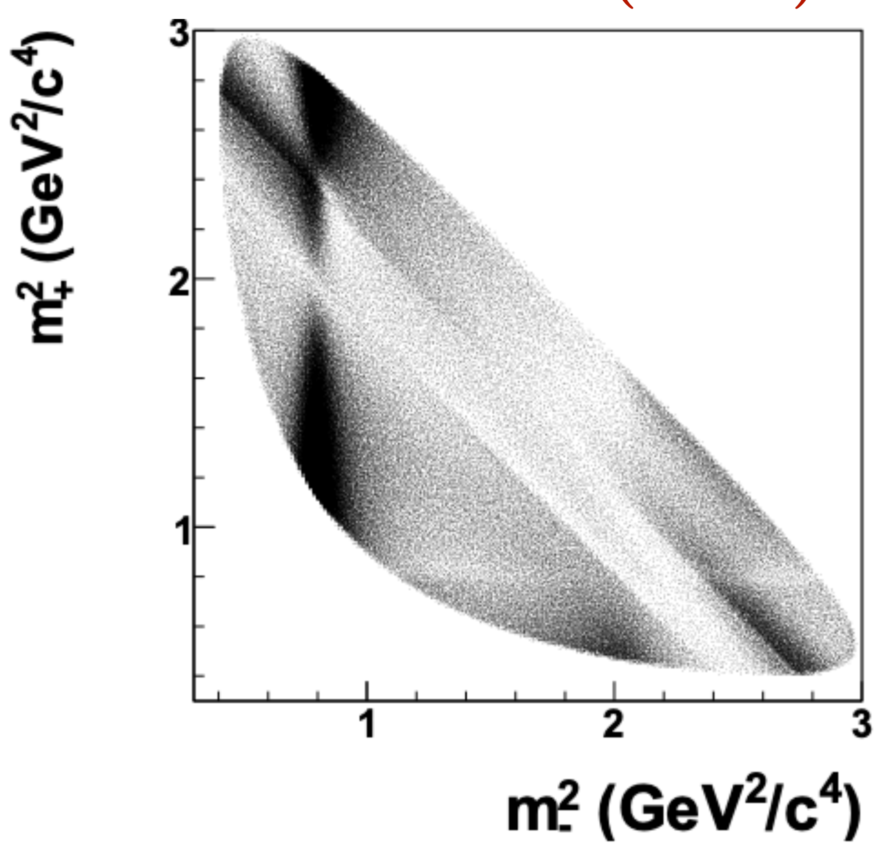
$$\frac{d\Gamma(\overline{D} \to K^+\pi^-)}{d\Gamma(\overline{D} \to K^-\pi^+)}(t) = (r_D^{K\pi})^2 + (t/\tau)L^-(|M_{12}|, |\Gamma_{12}|, \phi^M_{K\pi}, \phi^\Gamma_{K\pi}) + (t/\tau)^2 Q^-(|M_{12}|, |\Gamma_{12}|, \phi^M_{K\pi}, \phi^\Gamma_{K\pi})$$

Three-body states

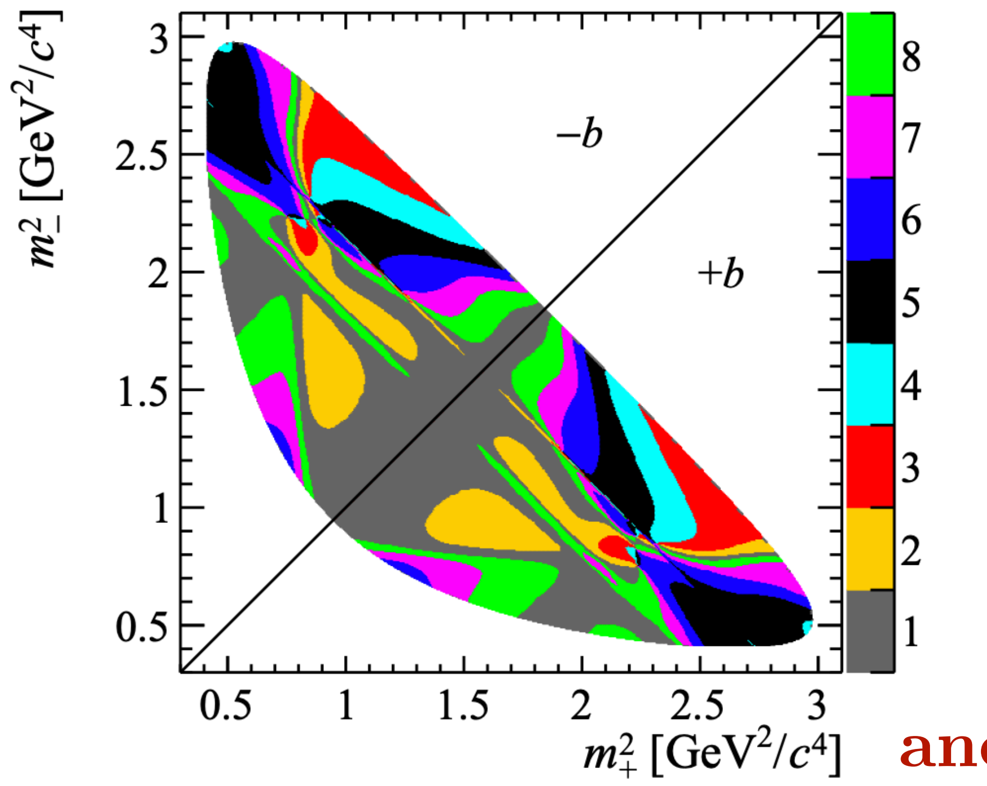
- They measure the distributions of the events

$$\frac{d\Gamma(D^0 \to K_S^0\pi^+\pi^-)(m_+^2, m_-^2)}{d\Gamma(D^0 \to K_S^0\pi^+\pi^-)(m_-^2, m_+^2)}(t)$$

From Belle(2014)

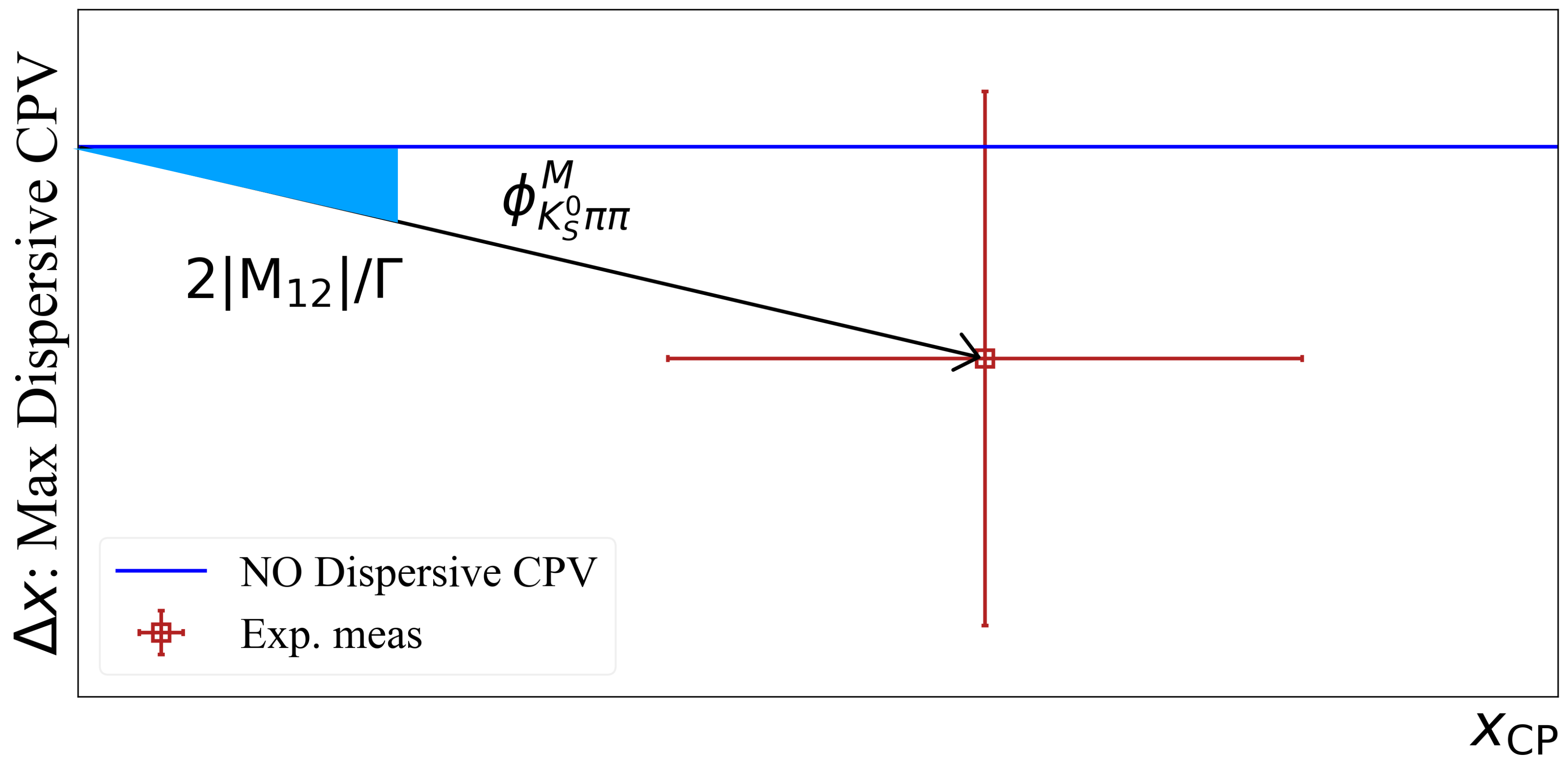


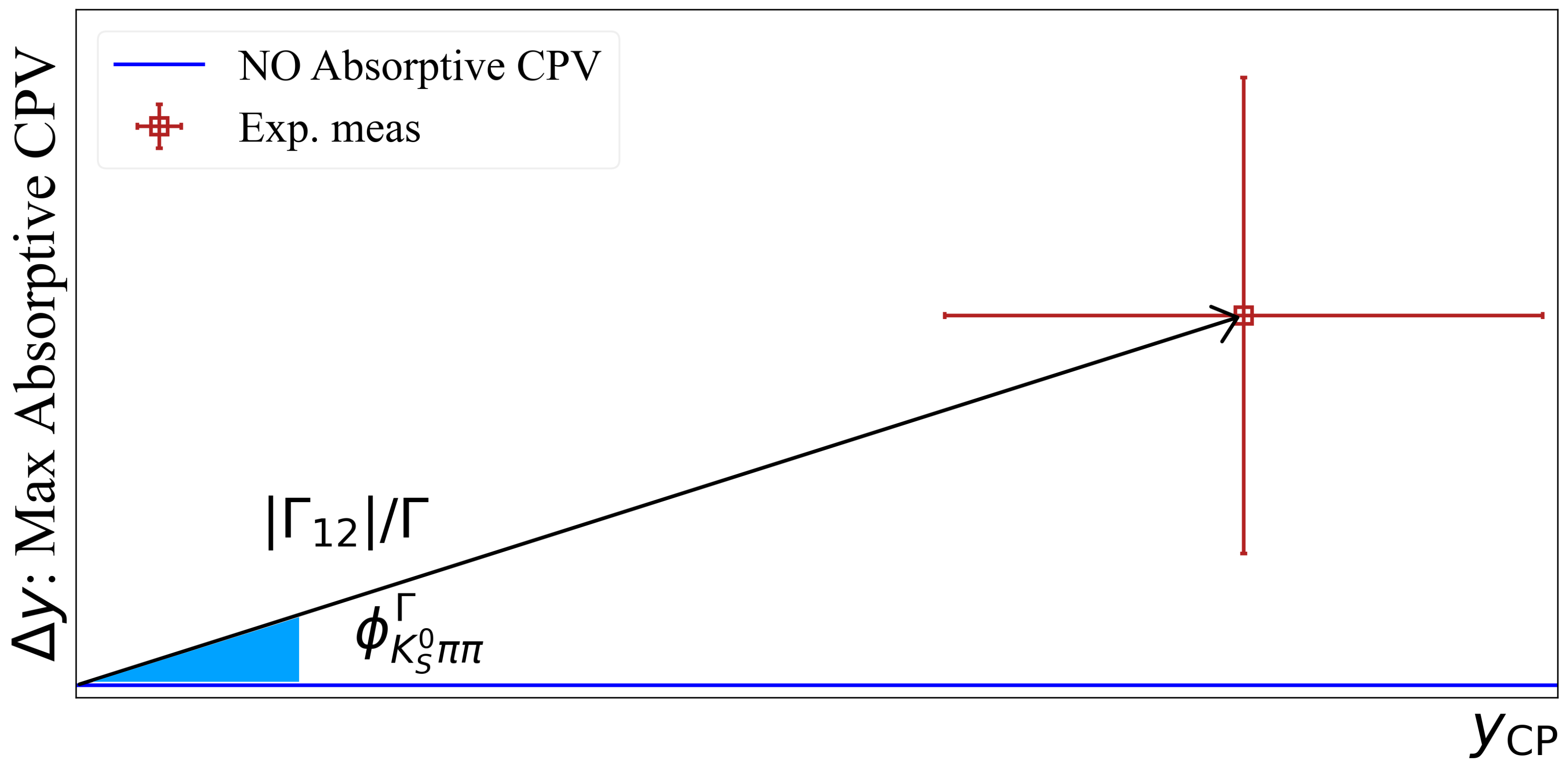
From LHCb(2023)



and Babar(2010)...

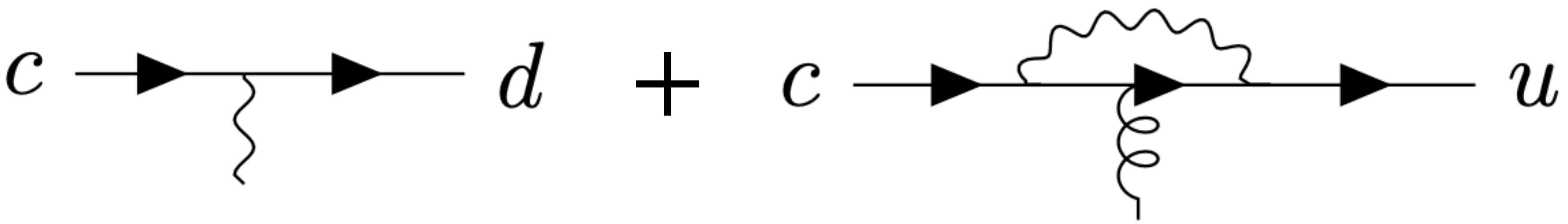
- We fit "polar observables" to extract the charm parameters





SCS decays to CP eigenstates

- SCS decays to $\pi^+\pi^-(K^+K^-)$ in the SM has tree-level + penguin



$$\rightarrow 1 - \frac{A(\overline{D} \rightarrow f)}{A(D \rightarrow f)} = a_D^f \neq 0$$

- Direct and indirect CPV enter the asymmetries

$$A_f(t) = \frac{d\Gamma(D^0 \rightarrow f) - d\Gamma(\overline{D}^0 \rightarrow f)}{d\Gamma(D^0 \rightarrow f) + d\Gamma(\overline{D}^0 \rightarrow f)}(t) = a_D^f + \Delta Y_f t/\tau$$

(e.g. CDF(2014), LHCb(2021))

Both time-dependent

and time-integrated analysis

(e.g. CDF(2012), Babar(2008), Belle(2008), LHCb(2023))

$$\Delta Y_f = (-2|\mathrm{M}_{12}|/\Gamma \sin\phi_f^M + a_D^f|\Gamma_{12}|/\Gamma)$$

Theoretical framework

- A pair of CPV phases for each of the final states

Approximate universality

$$\phi_2^{\mathrm{M}},\ \phi_2^{\Gamma}$$

A. Kagan, L. Silvestrini(2020)

U-spin decomposition

- In the SM, we can write

$$\lambda^j_{uc} = V^*_{uj} V_{cj} \qquad \Gamma^{\rm SM}_{12} = \sum_{i,j=d,s} \lambda^i_{uc}\lambda^j_{uc}\Gamma_{ij} \qquad {\rm M}^{\rm SM}_{12} = \sum_{i,j=d,s,b} \lambda^i_{uc}\lambda^j_{uc}{\rm M}_{ij}$$

- Employing CKM unitarity, we get

$$\Gamma^{\rm SM}_{12} = \frac{(\lambda^s_{uc} - \lambda^d_{uc})^2}{4}\Gamma_2 \times \left[1 + \frac{2\lambda^b_{uc}}{(\lambda^s_{uc} - \lambda^d_{uc})}\frac{\Gamma_1}{\Gamma_2} + \frac{(\lambda^b_{uc})^2}{(\lambda^s_{uc} - \lambda^d_{uc})^2}\frac{\Gamma_0}{\Gamma_2}\right]$$

- The amplitudes have flavour structures

$$\Gamma_0 = (\bar{s}s + \bar{d}d)^2 = O(1) \qquad \Gamma_1 = (\bar{s}s - \bar{d}d)(\bar{s}s + \bar{d}d) = O(\varepsilon) \qquad \Gamma_2 = (\bar{s}s - \bar{d}d)^2 = O(\varepsilon^2)$$

Approximate Universality

- Employing the U-spin decomposition, the Γ_2 term is dominant

$$\Gamma_{12}^{\rm SM} = \frac{(\lambda_{uc}^s - \lambda_{uc}^d)^2}{4}\Gamma_2 \times \left[1 + \left(\frac{0.3}{\varepsilon}\right) \times 10^{-3} + \left(\frac{0.3}{\varepsilon}\right)^2 \times 10^{-7}\right]$$

- Two universal CPV phases can be defined w.r.t. the dominant term

$$\phi_2^{\rm M} = \arg\left[\frac{{\rm M}_{12}}{{\rm M}_2(\lambda_{uc}^s - \lambda_{uc}^d)^2/4}\right], \quad \phi_2^\Gamma = \arg\left[\frac{\Gamma_{12}}{\Gamma_2(\lambda_{uc}^s - \lambda_{uc}^d)^2/4}\right]$$

- They provide good approximations for their final-state dep. counterparts

$$\phi_f^{\rm M(\Gamma)} \to \phi_2^{\rm M(\Gamma)}, \qquad \delta\phi_f = \phi_f^{\rm M} - \phi_2^{\rm M} = \phi_f^\Gamma - \phi_2^\Gamma$$

Is it a good approximation?

- For CF/DCS decays to $K\pi$, the misalignment is a CKM phase

$$\delta\phi_{K\pi} = O\left(\frac{\lambda_{uc}^b}{\lambda_{uc}^d}\right)^2 \simeq 10^{-6}$$

- For $K_S^0\pi\pi$, after correcting for $K-\overline{K}$ mixing

$$\delta\phi_{K_S^0\pi\pi} - 2\epsilon_I - |\lambda_{uc}^b/\lambda_{uc}^s|\sin\gamma = -2\mathrm{Im}[r_0] \simeq 10^{-4}$$

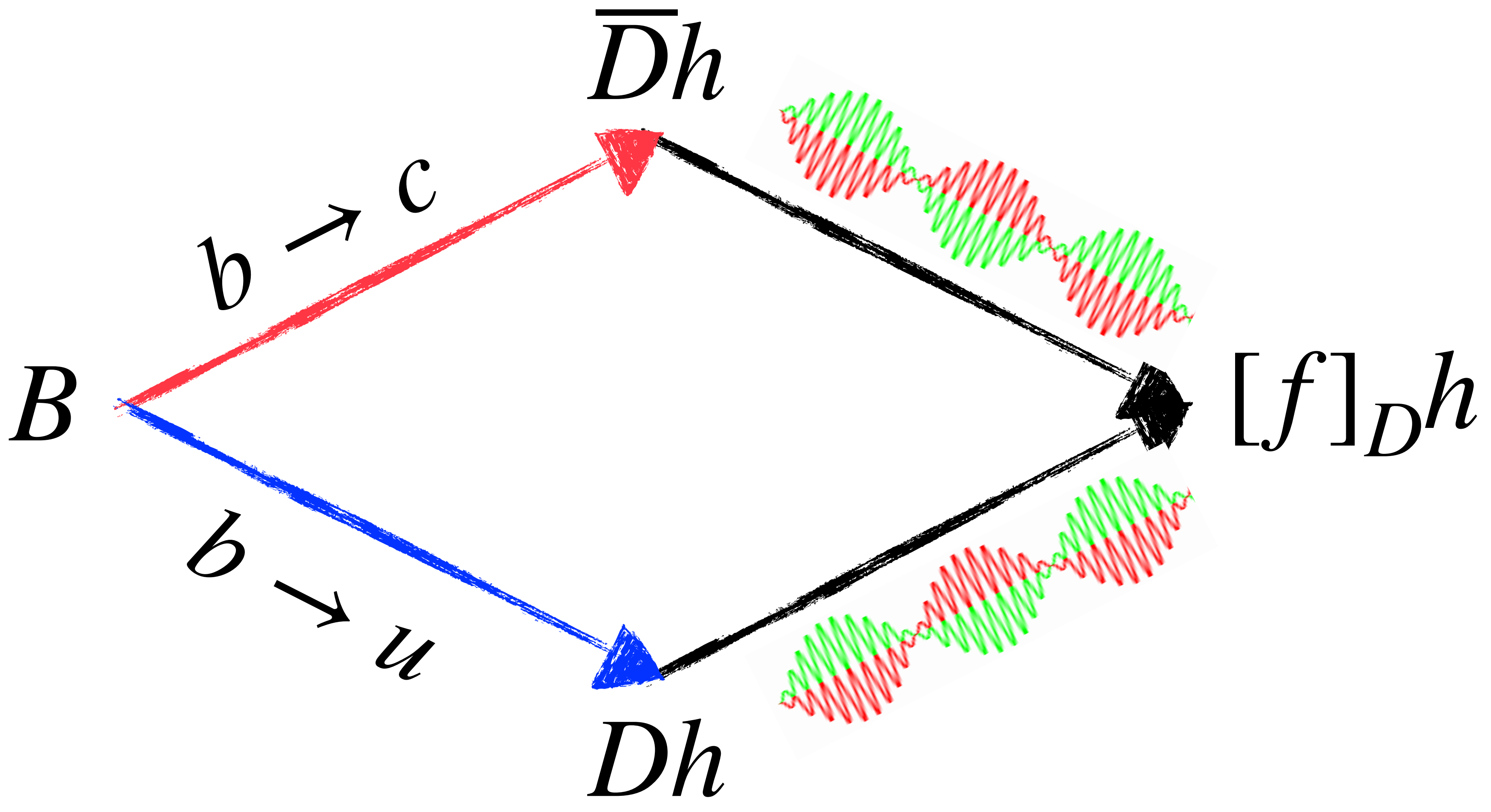
- For SCS decays to $\pi\pi(KK)$, the misalignment is unknown and non-perturbative, but we can get an additional $O(\varepsilon)$ suppression

$$\delta\phi_{KK} = -\delta\phi_{\pi\pi}, \qquad \frac{\phi_{KK}^{\mathrm{M}(\Gamma)} + \phi_{\pi\pi}^{\mathrm{M}(\Gamma)}}{2} = \phi_2^{\mathrm{M}(\Gamma)}\left(1 + O(\varepsilon^2)\right)$$

Beauty observables?

- The observables depend also on $r_D^{K\pi}$, $\delta_D^{K\pi}$, …
- Additional information are provided by beauty observables LHCb(2021)

$$\gamma = \arg[-V_{ud}V_{ub}^*V_{cb}V_{cd}^*]$$



$h = K,\ \pi,\ K\pi\pi,\ K^*, \ldots$

$f = K\pi,\ K_S^0\pi\pi,\ \pi\pi,\ KK, \ldots$

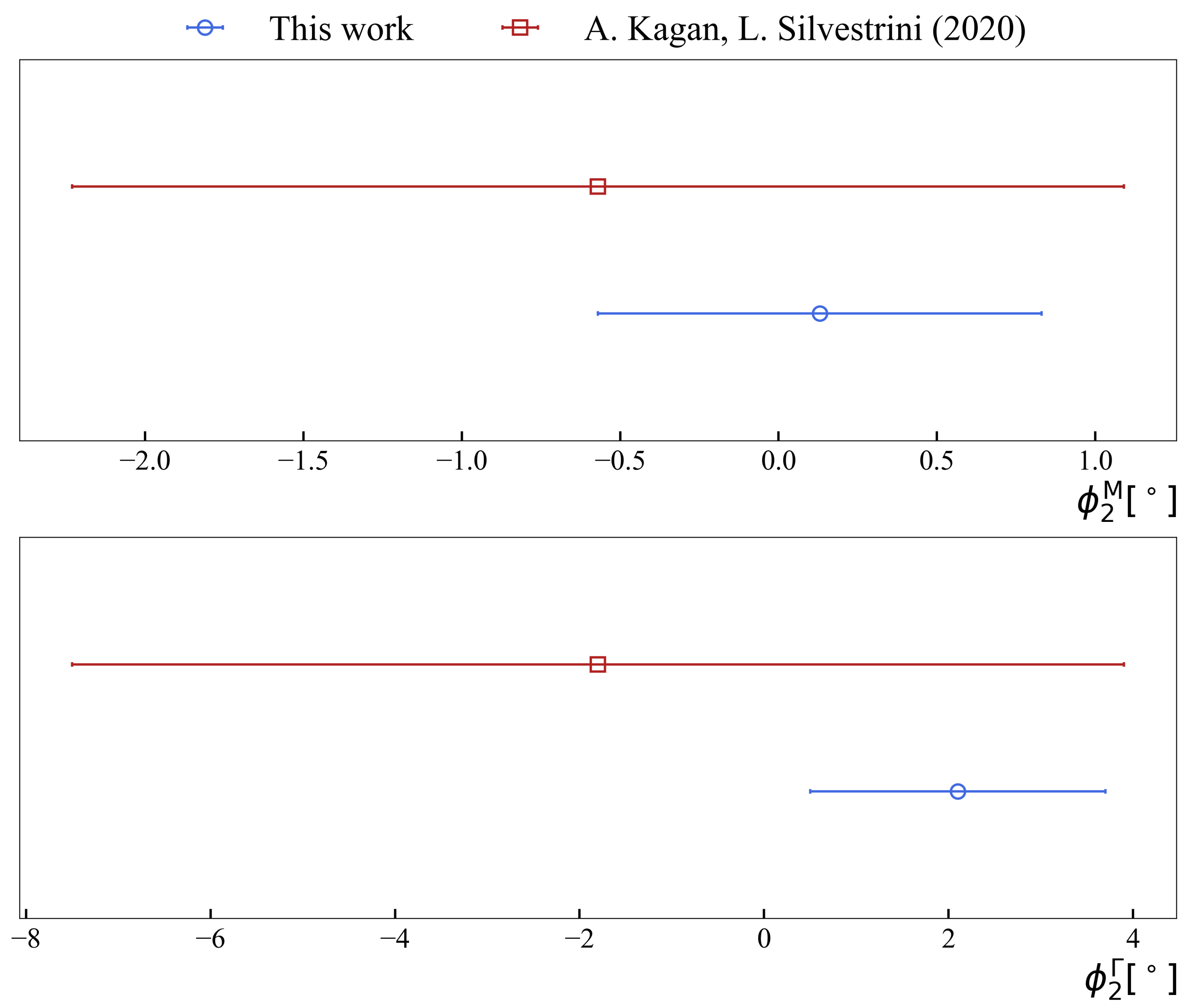
- Bayesian combination of the observables

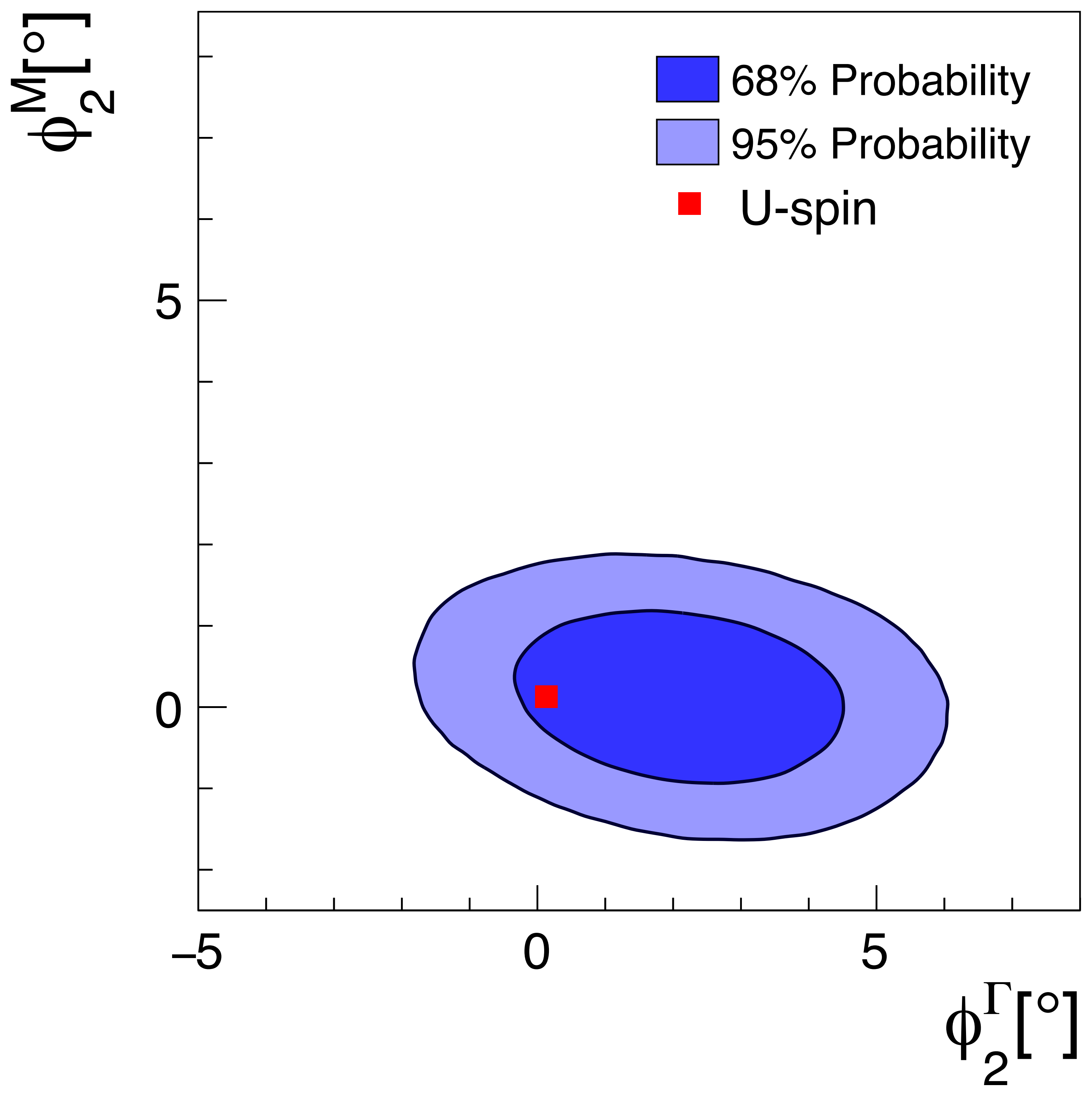
$$P(\vec{\theta}\,|\,\mathbf{O}) \propto P(\mathbf{O}\,|\,\vec{\theta})\ P_0(\vec{\theta})$$

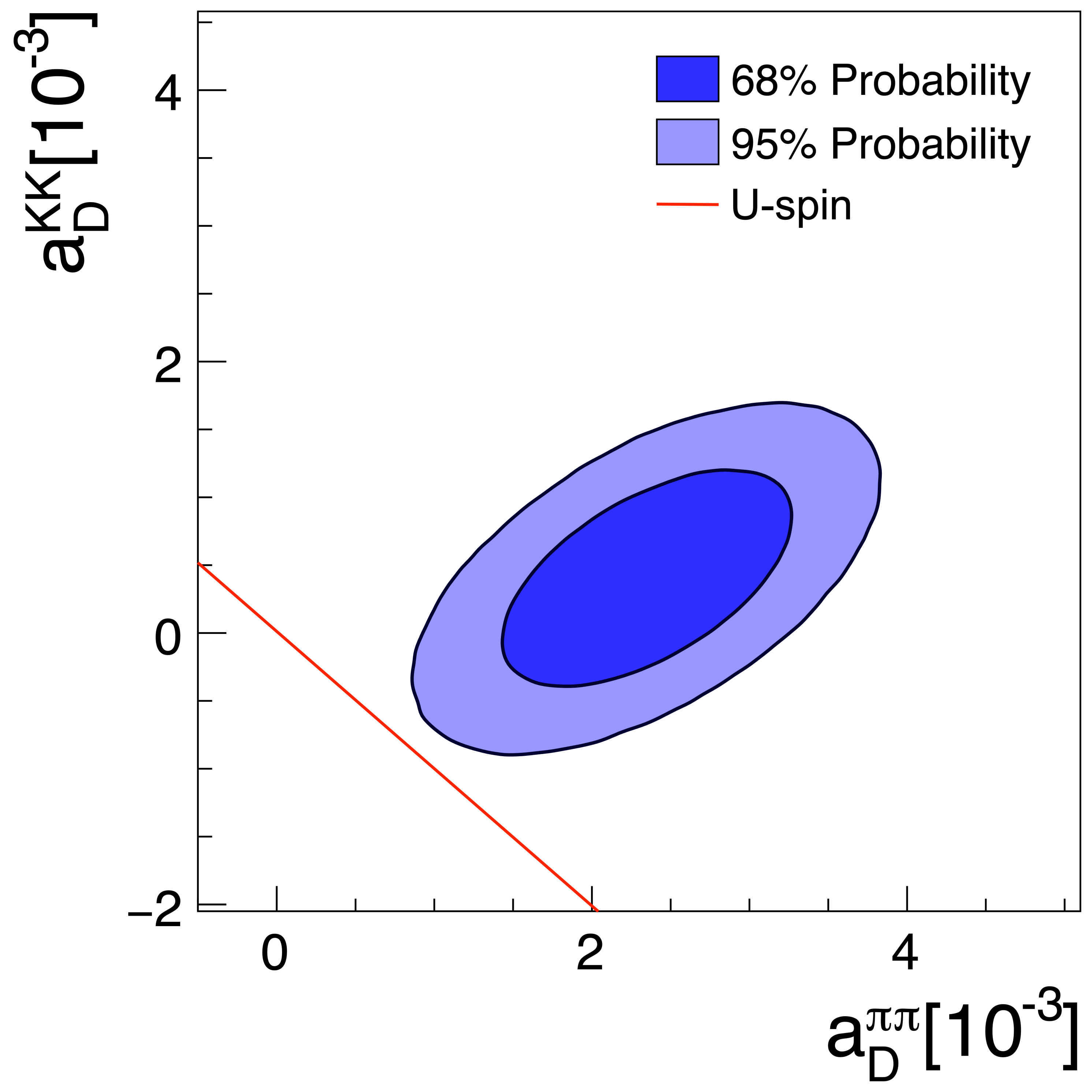
CPV parameters

- In the Approximate Universality framework, the CPV parameters are estimated to be

$$|\phi_2^\Gamma| < 0.3^\circ, \qquad \phi_2^M \sim \phi_2^\Gamma \sim \phi^{\text{U-spin}} = 0.13^\circ, \qquad a_D^{KK} = -a_D^{\pi\pi}$$



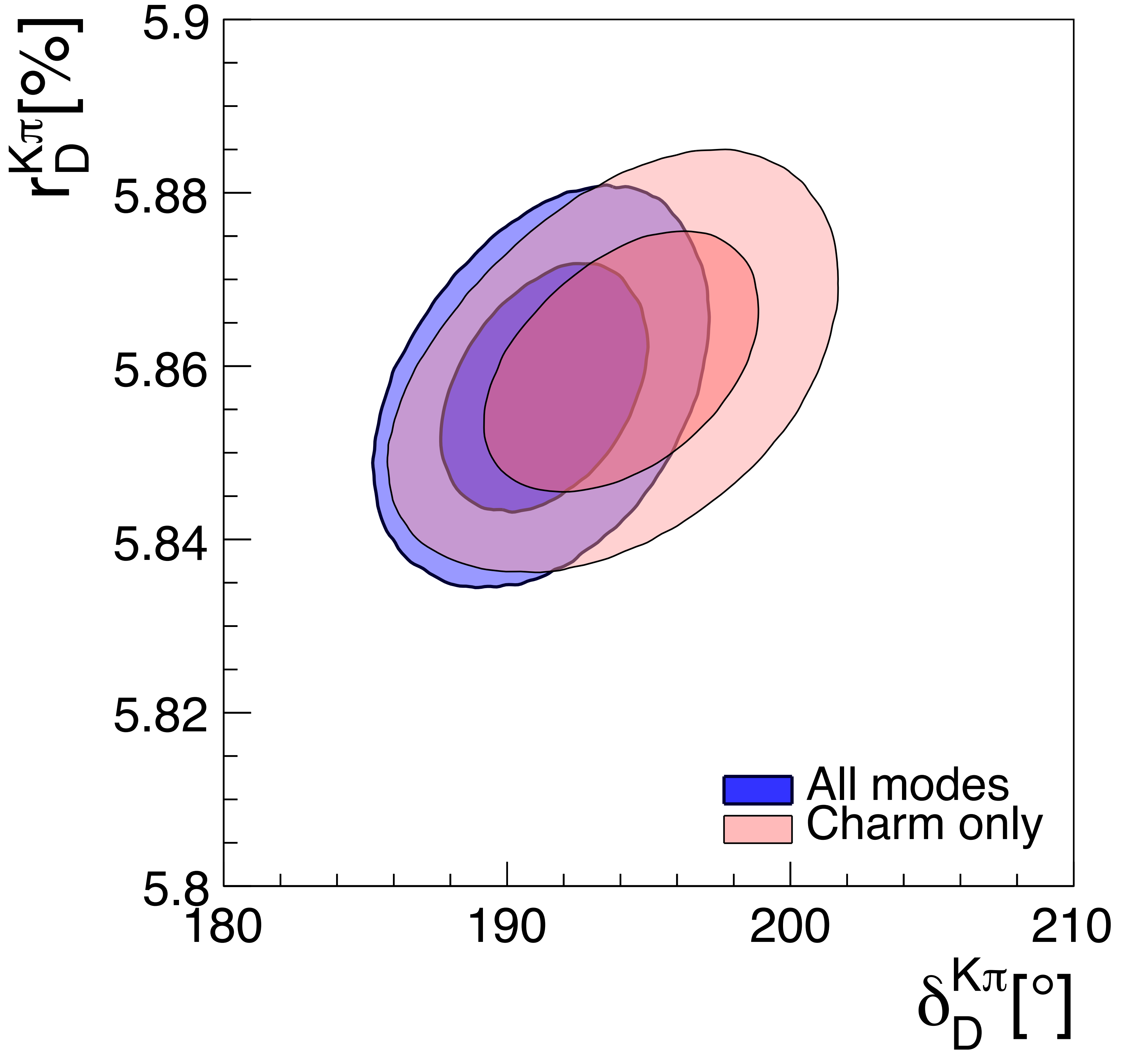


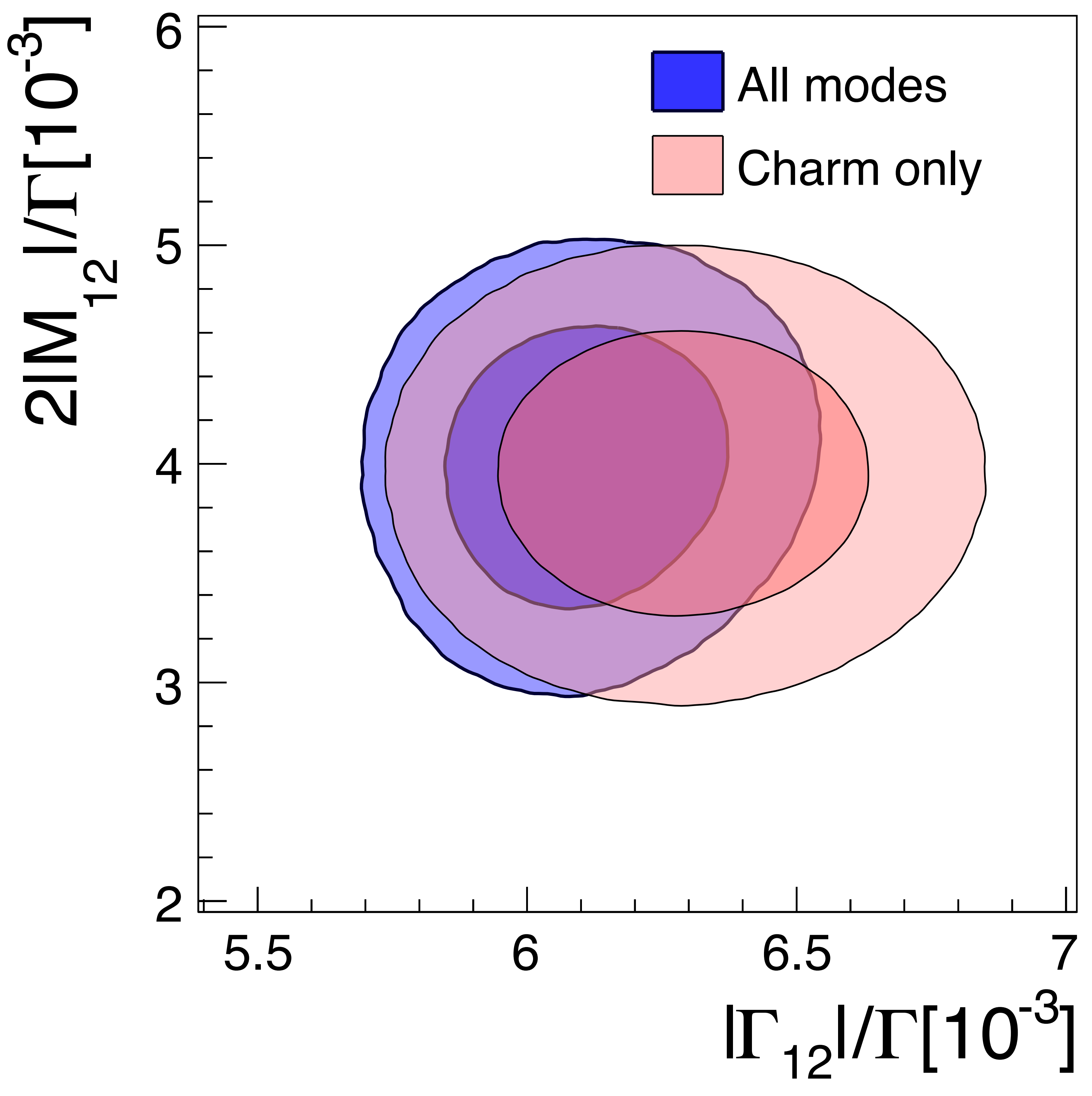


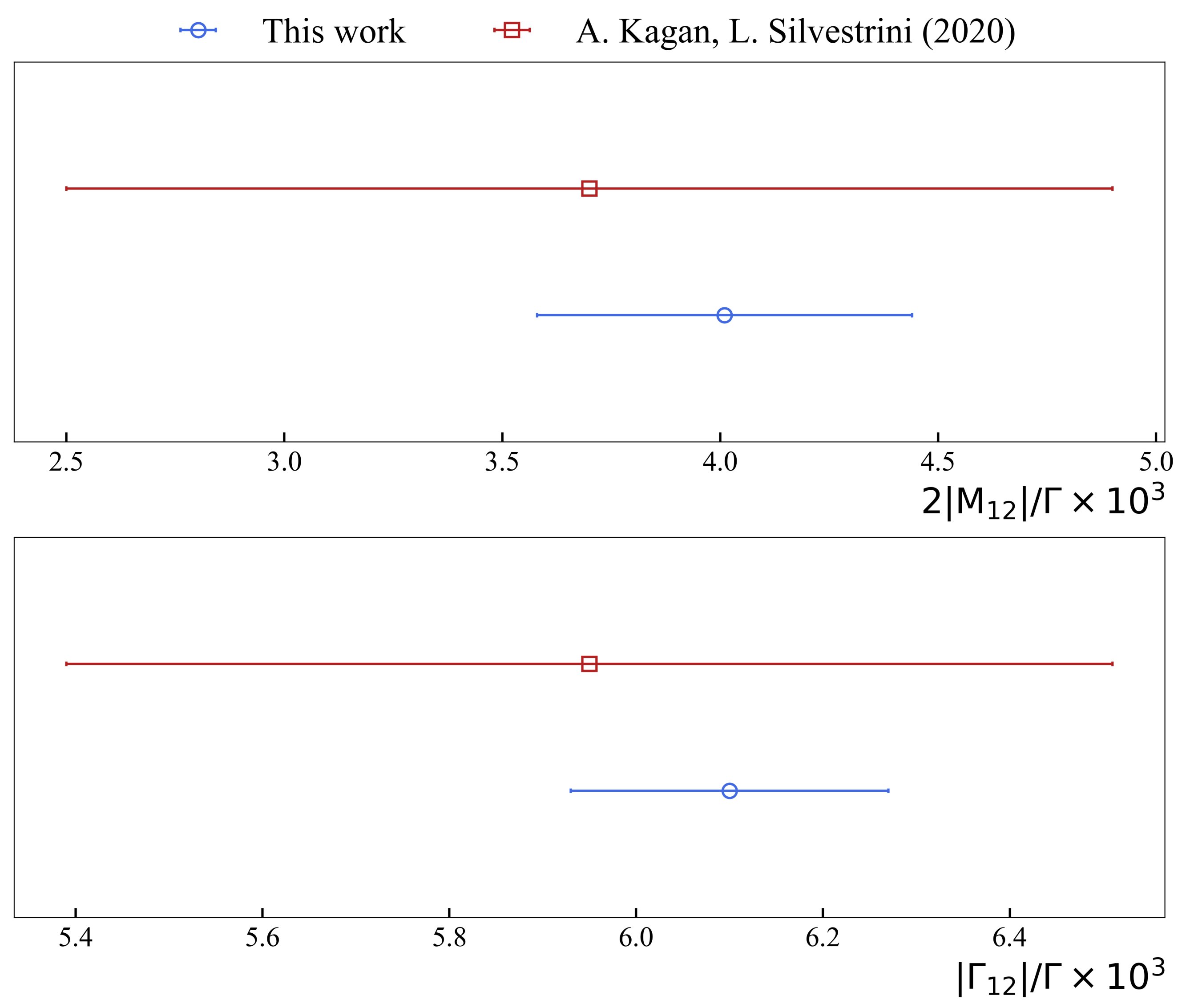
Impact of beauty observables

- Including beauty observables guarantees a better determination of $\delta_D^{K\pi}$, resulting in

$$\mathcal{O}(25\%)\ \text{improvement on}\ |\Gamma_{12}|$$

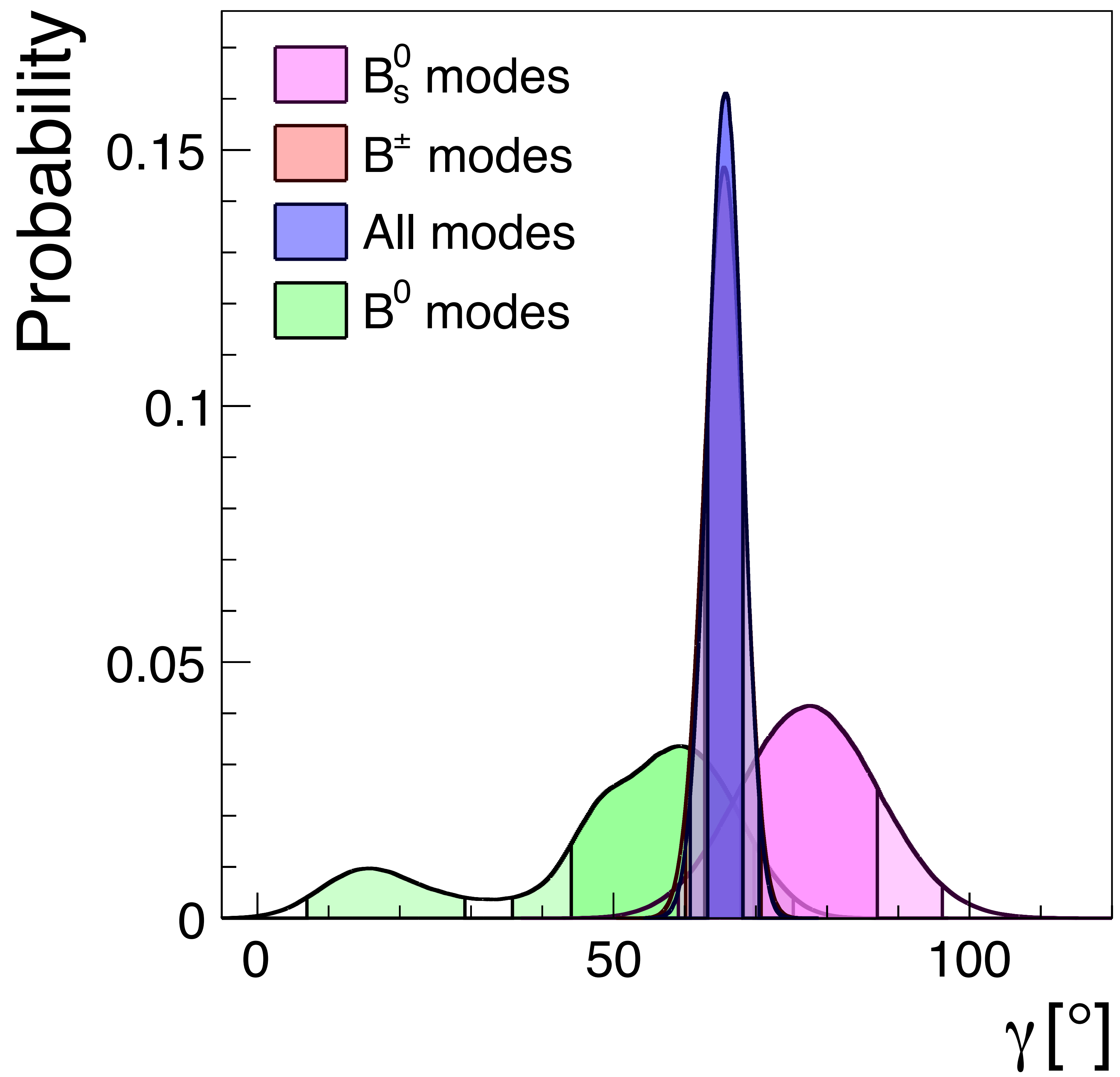






CKM angle γ

- We included observables coming from time-dependent analysis of $B^0 \to D^- \pi^+$ and $B^0_s \to D^-_s K^+$ to extract the CKM angle γ



All modes estimate

$\gamma = 65.7(2.5)^\circ$

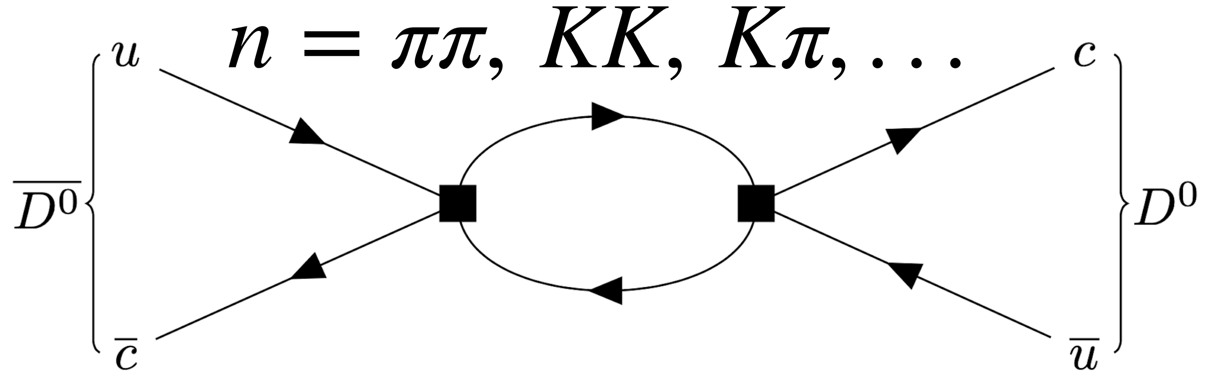
UT*fit* Indirect determination

$\gamma_{\rm UT} = 65.6(1.4)^\circ$

Next steps

- Computing the dominant LD from Lattice QCD?

$$\mathrm{LD} = i\int d^4x\ \langle D|\ \mathrm{T}\{\mathrm{H}_w^{\Delta C=1}(x)\mathrm{H}_w^{\Delta C=1}(0)\}\ |\overline{D}\rangle$$



- Light intermediate states obstruct the analytic continuation to Euclidean time $(t \to -i\tau)$

$$\mathrm{LD} = \sum_{n;\ \mathbf{p}_n=0} \langle D|\ \mathrm{H}_w^{\Delta C=1}(0)\,|n\rangle\,\langle n|\ \mathrm{H}_w^{\Delta C=1}(0)\,|\overline{D}\rangle \int_0^\infty d\tau\ e^{-(E_n - m_D)\tau} \qquad E_n > m_D$$

- The RM123 Coll. has proposed a novel method to circumvent this problem **M. T. Hansen, A. Lupo, N. Tantalo (2019)**
  - Spectral Function Reconstruction (SFR) method **R. Frezzotti, G. Gagliardi et al (2023)**
  - $B_s \to \mu^+\mu^-\gamma$ decay rate at large q^2 **R. Frezzotti, G. Gagliardi et al (2024)**
  - $K \to l\bar{\nu}_l l'^+ l'^-$ decay rate **In preparation**

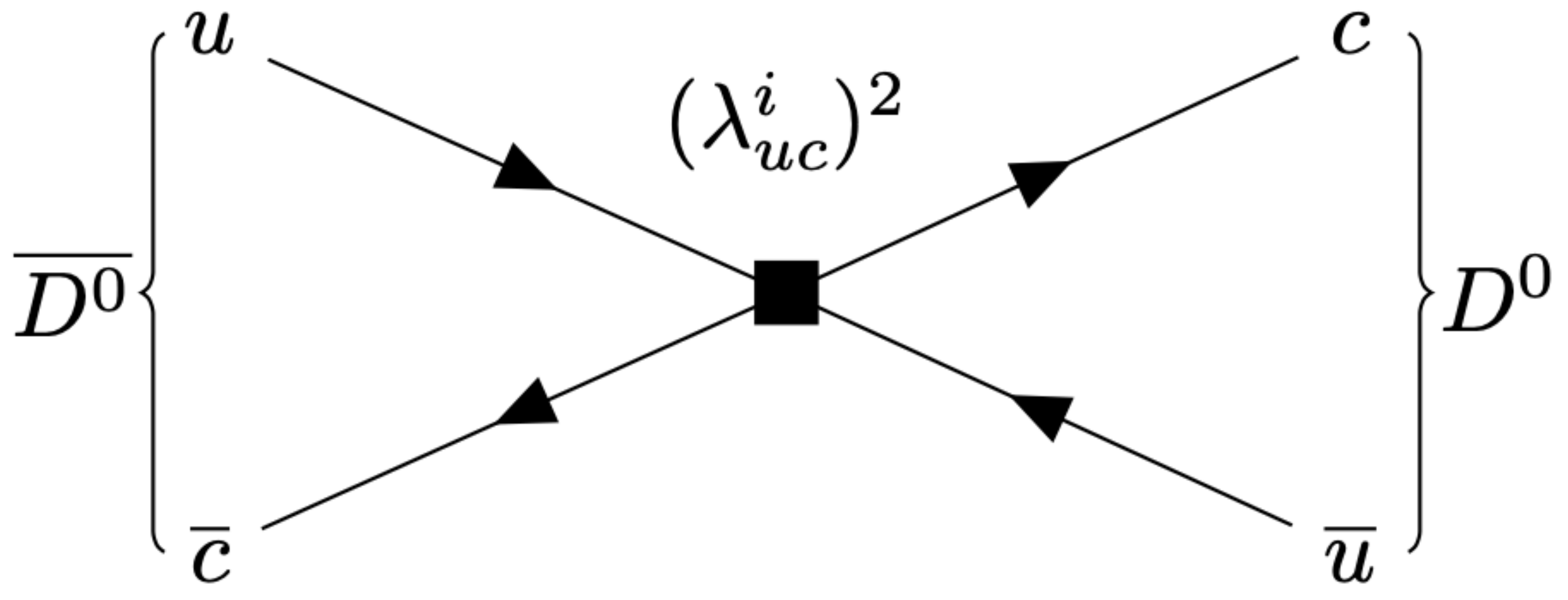
- A recent paper has proposed to apply the SFR method to $D - \overline{D}$

M. Di Carlo, F. Erben, M. T. Hansen (2025)

Backup

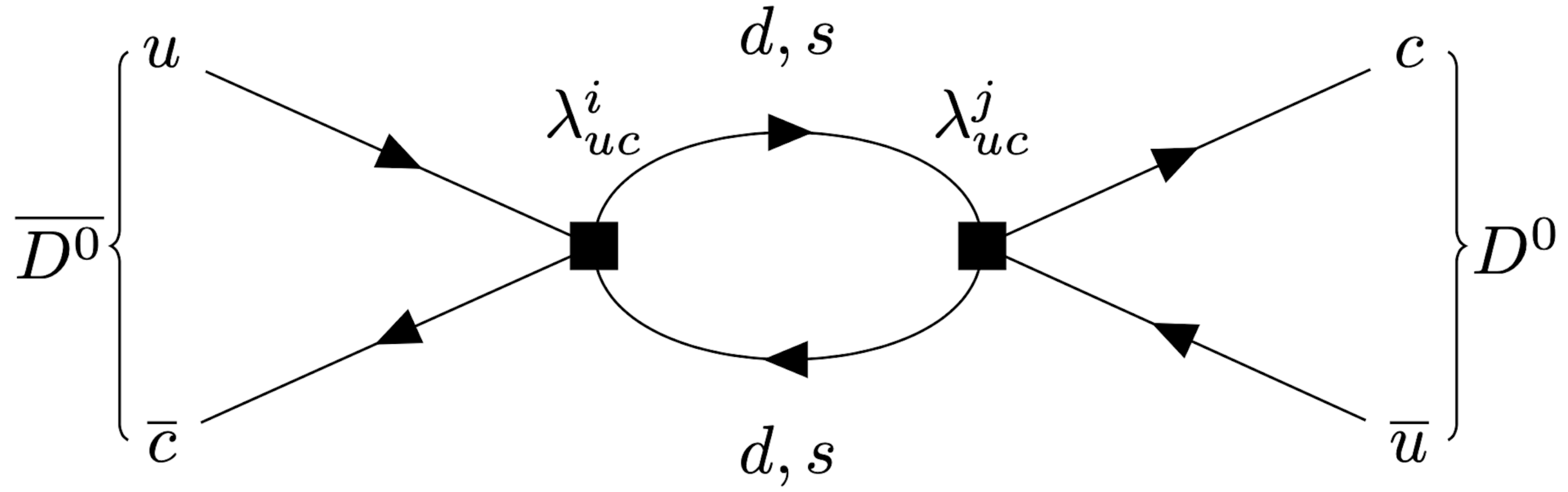
Short vs long distance

- The short distance contribution is given by



SM: $\propto (\lambda^b_{uc} m_b)^2 \approx (\theta_C^5 m_b)^2$

- The long distance contribution is given by



SM: $\propto (\lambda^s_{uc} m_s)^2 \approx (\theta_c m_s)^2 \approx 10^2 \times \mathrm{SD}$

Amplitude decomposition

- The absorptive part of the mixing hamiltonian reads

$$\Gamma_{12}^{\rm SM} = \frac{(\lambda_{uc}^s - \lambda_{uc}^d)^2}{4}\Gamma_2 + \frac{(\lambda_{uc}^s - \lambda_{uc}^d)\lambda_{uc}^b}{2}\Gamma_1 + \frac{(\lambda_{uc}^b)^2}{4}\Gamma_0$$

- The amplitudes and CKM matrix elements satisfy

$$\Gamma_0 = (\bar{s}s + \bar{d}d)^2 = O(1) \qquad \Gamma_1 = (\bar{s}s - \bar{d}d)(\bar{s}s + \bar{d}d) = O(\varepsilon) \qquad \Gamma_2 = (\bar{s}s - \bar{d}d)^2 = \mathcal{O}(\varepsilon^2)$$

$$\lambda_{uc}^s - \lambda_{uc}^d \approx 0.44 - i1.2 \times 10^{-4} \qquad \lambda_{uc}^b \approx (5.7 + i12) \times 10^{-5}$$

- We get the expansion

$$\Gamma_{12}^{\rm SM} = \frac{(\lambda_{uc}^s - \lambda_{uc}^d)^2}{4}\Gamma_2 \times \left[1 + (0.86 + i1.8) \times 10^{-3}\left(\frac{0.3}{\epsilon}\right) + (-6.4 + i7.8) \times 10^{-7}\left(\frac{0.3}{\epsilon}\right)^2\right]$$

Estimates

- Estimates of $\phi_2^{\mathrm{M},\Gamma}$ can be obtained by using the SM definitions

$$\phi_2^{\Gamma}\Big|_{SM} = \arg\left[1 + \frac{2\lambda_{uc}^b}{\lambda_{uc}^s - \lambda_{uc}^d}\frac{\Gamma_1}{\Gamma_2}\right] = \arg\left[1 - \frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} \times \left(\frac{2}{1 - \frac{V_{us}^* V_{cs}}{V_{ud}^* V_{cd}}}\right)\varepsilon^{-1}\right]$$

$$\simeq \left|\frac{\lambda_{uc}^b}{\lambda_{uc}^d}\right| \sin(\gamma)\varepsilon^{-1} \approx (2.2 \times 10^{-3}) \times \left[\frac{0.3}{\varepsilon}\right]$$

Upper bound

$$|\phi_2^\Gamma| = \left|\frac{\lambda_{uc}^b}{\lambda_{uc}^d}\right| \sin(\gamma)\frac{\Gamma_1}{\Gamma_2}$$

- Now, we have that $|\Gamma_2| = y_{12}\Gamma/(\lambda_{uc}^d)^2$ and

$$|\phi_2^\Gamma| = \left|\frac{\lambda_{uc}^b \lambda_{uc}^d}{y_{12}}\right| \sin(\gamma)\frac{|\Gamma_{sd}|}{\Gamma}\frac{|\Gamma_{ss} - \Gamma_{dd}|}{|\Gamma_{sd}|} \rightarrow O(\varepsilon)$$

$SU(3)_f$

$$< 1 + O(\varepsilon)$$

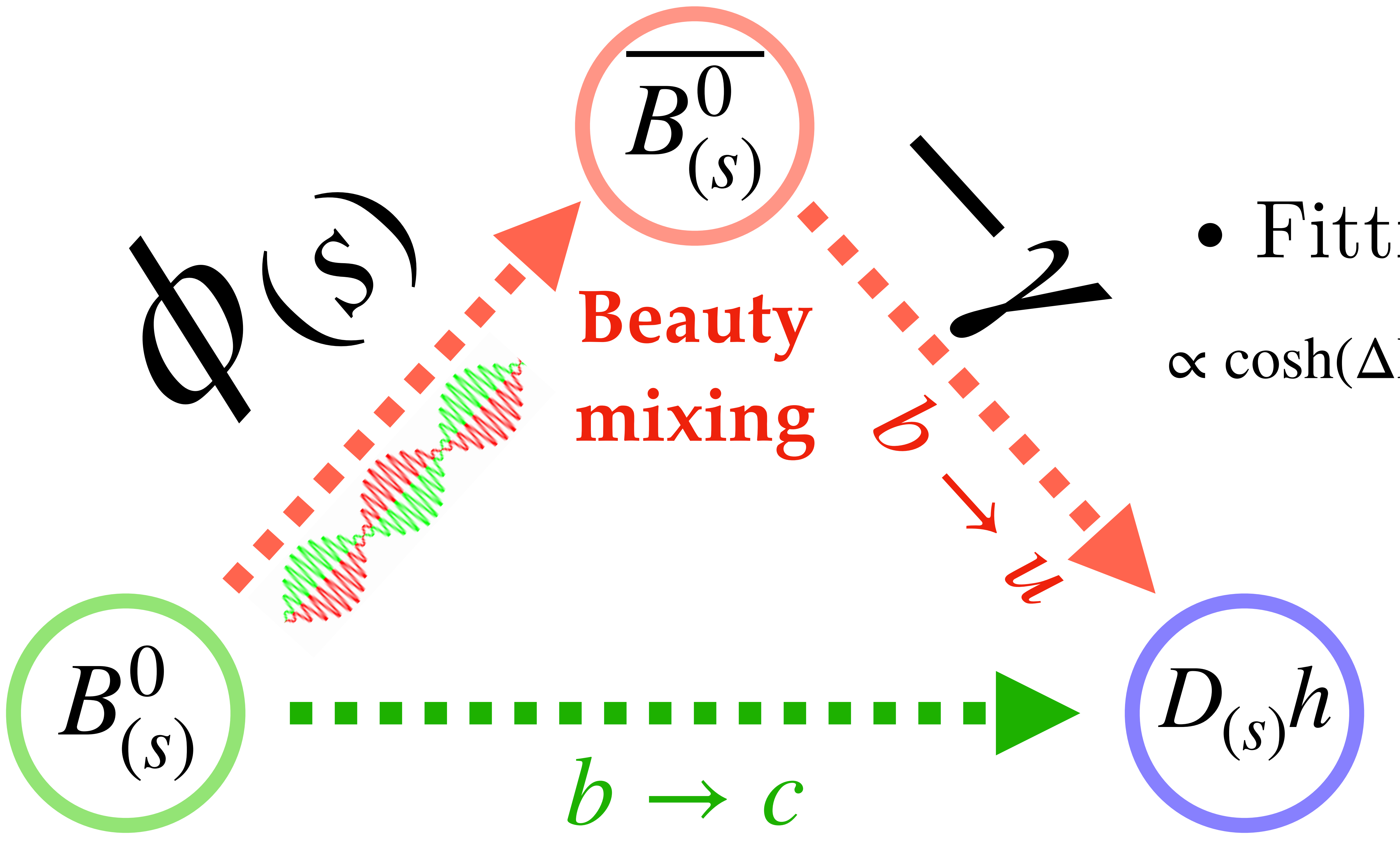
Also see M. Bobrowski, A. Lenz, J. Riedl, J. Rohrwild (2010)

$$|\phi_2^\Gamma| < 5 \times 10^{-3}\varepsilon(1 + O(\varepsilon)) < 0.3^\circ$$

Neutral B meson obs.

- Exploiting the **CPV phase of the interference** between $B^0_{(s)}$ mixing and decay to charmed mesons $D^{\mp}_{(s)}h^{\pm}$

Mixing phases $\phi = -2\beta$ $\phi_s = 2\beta_s$



- Fitting the time-dependent decay rates

$$\propto \cosh(\Delta\Gamma_{(s)}t/2) - G_f \sinh(\Delta\Gamma_{(s)}t/2) + C_f \cos(\Delta m_{(s)}t) - S_f \sin(\Delta m_{(s)}t)$$

Observables!!

C_f $G_f \propto \cos(\delta^f_{B^0_{(s)}} + (\phi_{(s)} - \gamma))$

$S_f \propto \sin(\delta^f_{B^0_{(s)}} + (\phi_{(s)} - \gamma))$

The spectral density

- We define a correlator

$$C(t) = e^{-im_D t} \int d^3x \, \langle D \left| \, \mathrm{H}_w^{\Delta C=1}(x) \mathrm{H}_w^{\Delta C=1}(0) \right| \overline{D} \rangle, \qquad \mathrm{LD} = i \int dt \; e^{im_D t} C(t)$$

- Introducing a spectral density

$$C(t) = \int_{E^*}^{\infty} \frac{dE'}{2\pi} \, e^{-iE't} \rho(E'), \qquad \rho(E') = \langle D \,|\, \mathrm{H}_w^{\Delta C=1}(0) (2\pi)^4 \delta^3(\hat{P}) \delta(\hat{H} - E') \mathrm{H}_w^{\Delta C=1}(0) \,|\, \overline{D} \rangle$$

- The LD contribution reads

$$\mathrm{LD}(\varepsilon) = \lim_{\varepsilon \to 0} \int_{E^*}^{\infty} \frac{dE'}{2\pi} \, \frac{\rho(E')}{E' - m_D - i\varepsilon} = \lim_{\varepsilon \to 0} \int_{E^*}^{\infty} \frac{dE'}{2\pi} \, K(E' - m_D; \varepsilon) \rho(E')$$

The SFR method

- The determination of the spectral density from the Euclidean $C(t)$ is ill-posed

$$C(t) = \int \frac{dE'}{2\pi}\, e^{-E't}\rho(E')$$

- The convolution of $\rho(E')$ with a Kernel at fixed ε can be computed as

$$\mathrm{LD}(\varepsilon) \simeq \sum_t g_t(E,\varepsilon)C(t), \qquad K(E'-m_D;\epsilon) = \sum_t g_t(E,\varepsilon)e^{-E't}$$

- The HLT method provides us with a method to determine the coefficients

$$W[\mathbf{g}] = A[\mathbf{g}]/A[\mathbf{0}] + \lambda B[\mathbf{g}]$$

$$A[\mathbf{g}] = \int_{E^*}^{\infty} dE' \left| \sum_t g_t(E,\varepsilon)e^{-E't} - K(E'-E;\varepsilon)\right|^2 \qquad B[\mathbf{g}] \propto \sum_{t_1,t_2} g_{t_1}(E,\varepsilon)g_{t_2}(E,\varepsilon)\mathrm{Cov}(t_1,t_2)$$