Koba-Nielsen-Olesen (KNO) scaling and jet substructure in QCD jets at the LHC

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in collaboration with

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Outline

- **KNO scaling and multiplicity distributions in QCD jets**
  - Multiplicity probability distributions in DLA
  - KNO scaling functions in DLA
  - KNO scaling functions in MDLA (energy conservation)
  - KNO scaling functions in NBD
  - Mean multiplicities: DLA vs. $\mathrm{N^3LO}$ vs. PYTHIA vs. ATLAS
- **KNO scaling in QCD jets via jet substructure techniques**
  - Two-point energy correlation functions (ECFs)
  - KNO scaling with quark-gluon jet discrimination
- **Summary**
- **References**
  - X.-P. Duan, L. Chen, G.-L. Ma, C. A. Salgado, and B. Wu, arXiv:2503.24200
  - X.-P. Duan, L. Chen, G.-L. Ma, C. A. Salgado, and B. Wu, work in progress

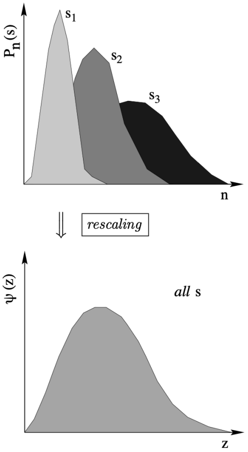
Introduction: Koba-Nielsen-Olesen (KNO) scaling

Koba, Nielsen, Olesen, NPB 40 (1972)

- **KNO scaling** describes a universal behavior of multiplicity distributions:

$$P_n(s) = \frac{1}{\bar{n}(s)} \Psi(\frac{n}{\bar{n}(s)})$$

- **KNO scaling support:**
  - e^+e^- collisions
  - Deep Inelastic Scattering (DIS)
- **KNO scaling violation:**
  - hadron-hadron collisions
- In pQCD, KNO scaling is predicted to emerge asymptotically in high-virtuality parton showers
- **QCD jets** provide a clean probe to test KNO scaling:
  - Initiated by high-virtuality partons
  - Less affected by soft physics than inclusive multiplicities



Hegyi, NPB 92 (2001)

Generating Functions in QCD jets within DLA

Dokshitzer, Khoze, Mueller, and Troian, (1991)

- The generating functions (GFs):

$$Z_a(u,Q) \equiv \sum_{n=0}^{\infty} u^n P_a(n,Q)$$

- Jet scale: $Q = p_T R$
- The generating functions within DLA with respect to Q:

$$\frac{\partial}{\partial \ln Q} Z_a(u,Q) = Z_a(u,Q) \times c_a \int \frac{dz}{z} \gamma_0^2 \left[Z_g(u,zQ) - 1 \right]$$

- Running coupling: $\gamma_0 = \sqrt{\frac{2N_c\alpha_s(k_\perp^2)}{\pi}}$
- Quark: $c_q \equiv C_F/N_c = 4/9$
- Gluon: $c_g \equiv C_A/N_c = 1$

Multiplicity probability distributions in QCD jets within DLA

- Restricting to the DLA phase space in $zQ = k^0\theta > k_\perp > Q_0$, GFs are given by:

$$Z_a(u,y) = u \exp\left\{c_a \int_0^y \mathrm{d}\bar{y}\,(y-\bar{y})\gamma_0^2[Z_g(u,\bar{y})-1]\right\}$$

- Infrared cutoff Q_0 for $y \equiv \ln(Q/Q_0)$, $\bar{y} \equiv \ln(k_\perp/Q_0)$
- Multiplicity probability distributions:

$$P_a(n,Q) = \frac{1}{n!}\frac{\partial^n}{\partial u^n} Z_a(u,Q)\bigg|_{u=0}$$

- Following recursive relation between $P_a(n)$'s:

$$P_a(1,Q) = \exp\left\{-c_a \int_0^y \mathrm{d}\bar{y}\,(y-\bar{y})\gamma_0^2\right\},$$

$$P_a(n+1,Q) = \sum_{k=1}^{n} \frac{k}{n} P_a(n+1-k,Q) \times c_a \int_0^y \mathrm{d}\bar{y}\,(y-\bar{y})\gamma_0^2 P_g(k,\bar{y}),$$

which satisfies the normalization condition $\sum_{n=0}^{\infty} P_a(n,Q) = 1$

Mean multiplicity distributions in QCD jets within DLA

- Mean multiplicity distributions derived by taking the first derivative of the GFs with respect to u at $u = 1$:

$$\bar{n}_a(Q) = 1 + c_a \int_0^y \mathrm{d}\bar{y}\,(y - \bar{y})\gamma_0^2 \bar{n}_g(\bar{y})$$

- Relation between quark and gluon jets:

$$\bar{n}_q(Q) - 1 = c_q[\bar{n}_g(Q) - 1]$$

- Mean multiplicity distributions from the second-order ODE for gluon jets:

$$\bar{n}_g = \begin{cases} \cosh(\gamma_0 y) & \text{for fixed coupling} \\ z_1\,\{I_1[z_1]K_0[z_2] + K_1[z_1]I_0[z_2]\} & \text{for running coupling} \end{cases} \tag{1}$$

- Mean multiplicity distributions from the multiplicity probability distributions:

$$\bar{n}_a(Q) = \sum_{n=1}^{\infty} n P_a(n, Q) \tag{2}$$

KNO scaling functions in QCD jets within DLA

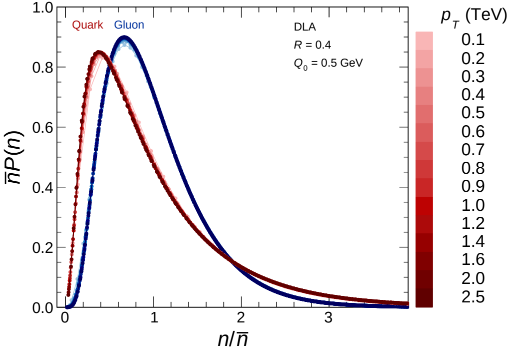
- KNO scaling functions:

$$\Psi_a(x) = \bar{n}_a P_a(n)$$

$$x = n/\bar{n}_a$$

  - $n = 1000$, $Q_0 = 0.5$ GeV, $R = 0.4$
  - p_T range: 0.1—2.5 TeV

- **KNO scaling emerges over a wide p_T range relevant to LHC energies**
- Minor discrepancies appear only at low p_T



- **Quark-jet scaling function lies above gluon-jet scaling function at both small and large x**
- **Quark-jet distribution peak is lower and shifts to the left compared to gluon jets, ensuring probability conservation**

KNO scaling functions in QCD jets within DLA and MDLA

- **Asymptotic** KNO scaling functions $\Psi_a(x)$ are given by inverse Laplace transform of $\Phi_a(\beta)$:

$$\Psi_a(n/\bar{n}_a) \equiv \lim_{Q\to\infty}[\bar{n}_a P_a(n,Q)] = \int \frac{d\beta}{2\pi i}\Phi_a(\beta)e^{\beta\frac{n}{\bar{n}_a}}$$

Bassetto, NPB 303 (1988)

Dokshitzer, PLB 305 (1993)

- Taking the limit $\boldsymbol{Q} \to \infty$ in Z_a yields from GFs:

$$\Phi_a(\beta) \equiv \lim_{Q\to\infty} Z_a(e^{-\frac{\beta}{\bar{n}_a}},Q) = \sum_{k=0}^{\infty}\frac{(-\beta)^k}{k!}f_a^{(k)}$$

- With additionally **energy conservation** in the DLA, the evolution equation can reduces to:

$$\frac{\partial}{\partial \ln Q}Z_a(u,Q) = c_a\int\frac{dz}{z}\gamma_0^2\left[Z_g(u,zQ)Z_a(u,(1-z)Q) - Z_a(u,Q)\right]$$

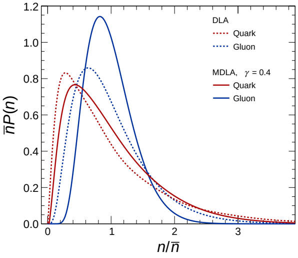
- Iterative equations within MDLA:

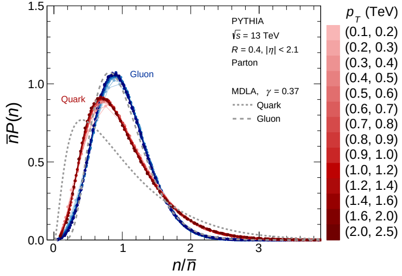
$$f_g^{(m)} = \frac{\gamma_0 m}{m^2-1}\sum_{k=1}^{m-1}\frac{m!}{k!(m-k)!}\frac{\Gamma(\gamma_0 k)\Gamma(\gamma_0(m-k)+1)}{\Gamma(\gamma_0 m+1)}f_g^{(k)}f_g^{(q-k)}$$

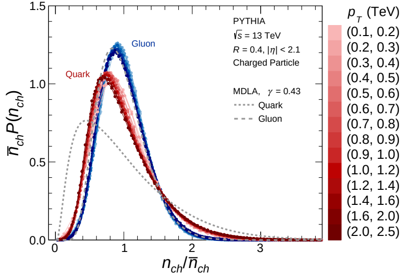
$$f_q^{(m)} = \frac{c_q^{1-m}}{m^2}f_g^{(m)} + \gamma_0\sum_{k=1}^{m-1}\frac{(m-1)!}{k!(m-k)!}\frac{\Gamma(\gamma_0 k)\Gamma(\gamma_0(m-k)+1)}{\Gamma(\gamma_0 m+1)}c_q^{1-k}f_g^{(k)}f_q^{(m-k)}$$

KNO in QCD jets: DLA vs. MDLA vs. PYTHIA

- KNO scaling functions are studied using DLA, MDLA, and PYTHIA
- **MDLA peaks shift to larger x compared to DLA**
- PYTHIA shows approximate KNO scaling for both quark and gluon jets at parton and charged-particle levels (with MPI and hadronization)
- In MDLA: $\gamma = 0.37$ for parton, $\gamma = 0.43$ for charged particle
  - **Similar to the gluon jet results in PYTHIA**
  - Large deviation to quark jet results in PYTHIA





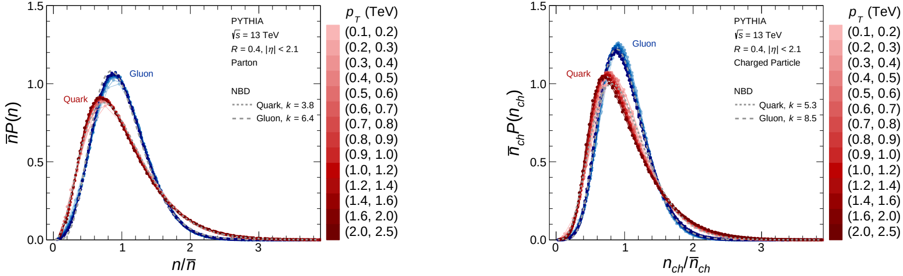


KNO in QCD jets: NBD vs. PYTHIA

- KNO scaling functions are modeled using the negative binomial distribution (NBD):

$$f(x) = x^{k-1} e^{-kx} \frac{k^k}{\Gamma(k)}$$

  with $f(x) = \Psi(x) = \bar{n}P(n)$, $x = n/\bar{n}$, and k as the fit parameter
- Parton level: $k = 3.8$ (quark), $k = 6.4$ (gluon)
- Charged-particle level: $k = 5.3$ (quark), $k = 8.5$ (gluon)
- **NBD fits are similar to the PYTHIA results for both quark and gluon jets**
- Small deviations appear near the distribution peak



Multiplicity distributions in inclusive jets within DLA

- Multiplicity distributions in inclusive jets expressed as:

$$P(n) = r_q P_q(n) + r_g P_g(n)$$

$$\bar{n} = r_q \bar{n}_q + r_g \bar{n}_g$$

$$r_a \equiv \frac{\mathrm{d}\sigma_a/\mathrm{d}p_T}{\mathrm{d}\sigma_q/\mathrm{d}p_T + \mathrm{d}\sigma_g/\mathrm{d}p_T}$$

- Leading-order (LO) differential cross section in pQCD:

$$\frac{\mathrm{d}\sigma^{\mathrm{LO}}}{\mathrm{d}p_{Tc}} = 2p_{Tc} \sum_{a,b,d} \int \mathrm{d}y_c \mathrm{d}y_d x_a f_{a/p}(x_a, \mu^2) x_b f_{b/p}(x_b, \mu^2) \frac{\mathrm{d}\hat{\sigma}_{ab\to cd}}{\mathrm{d}\hat{t}}$$

- KNO scaling functions for inclusive jets derived from quark/gluon contributions:

$$P(n) = r_q P_q(n) + r_g P_g(n) = r_q \frac{\Psi_q(x)}{\bar{n}_q} + r_g \frac{\Psi_g(x)}{\bar{n}_g}$$

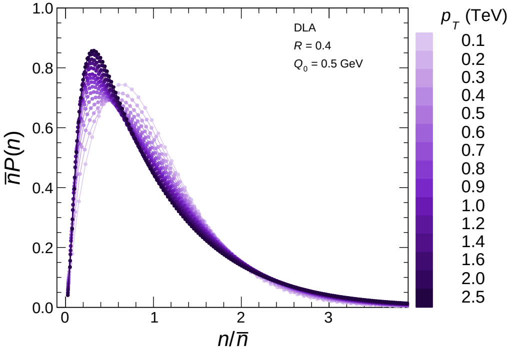
KNO in inclusive jets within DLA

- PDFs: CT18NLO via LHAPDF
- **No universal KNO scaling are observed across a wide p_T range**
  - Due to differing scaling behaviors of quark and gluon jets in inclusive samples
- At $p_T = 0.1$ TeV with $r_g \approx 0.78$:

$$\Psi(x) \approx 0.4\,\Psi_q(x_q) + 0.7\,\Psi_g(x_g)$$

- At $p_T = 2.5$ TeV with $r_q \approx 0.81$:

$$\Psi(x) \approx 1.0\,\Psi_q(x_q) + 0.1\,\Psi_g(x_g)$$

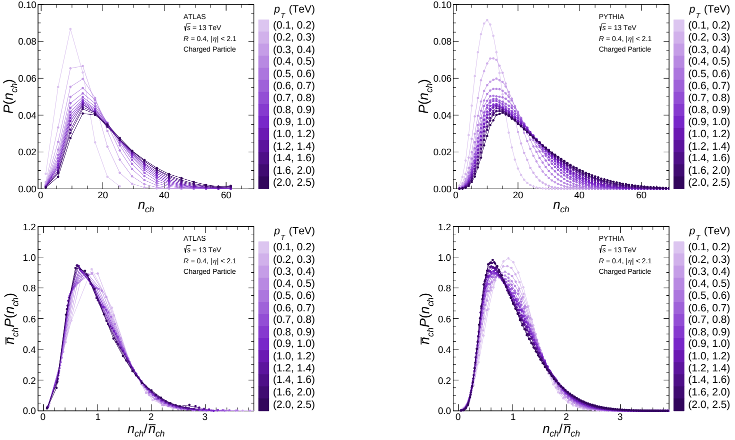


- **Gluon jets dominate at low p_T, while quark jets dominate at high p_T in hard scattering**
- **Inclusive KNO functions are shaped mainly by gluon jets at low p_T, and by quark jets at high p_T**

KNO in inclusive jets: ATLAS vs. PYTHIA

- Charged-particle multiplicity distributions from PYTHIA agree with the ATLAS data
- **ATLAS and PYTHIA show a p_T-dependent, similar to DLA trends**
- PYTHIA shows a more pronounced peak shift between low- and high-p_T regions

ALTAS, PRD 100 (2019)

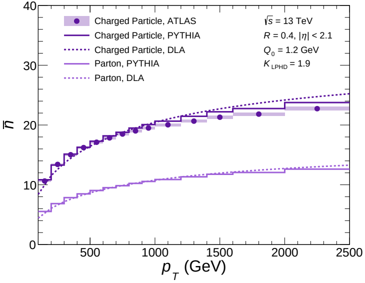


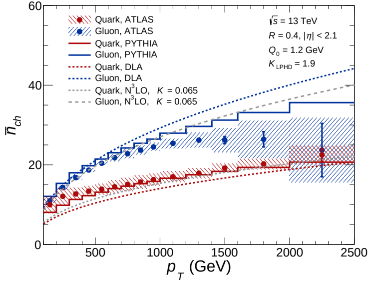
Mean multiplicities in QCD jets: ATLAS vs. PYTHIA vs. DLA vs. N^{3}LO

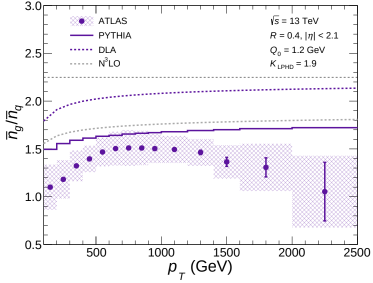
- Mean multiplicities in DLA:

$$\bar{n} = r_q \bar{n}_q + r_g \bar{n}_g, \quad \bar{n}_g = z_1 \left\{ I_1[z_1] K_0[z_2] + K_1[z_1] I_0[z_2] \right\}, \quad \bar{n}_q - 1 = c_q [\bar{n}_g - 1]$$

- DLA predictions with $Q_0 = 1.2$ GeV, LPHD with $K_{\text{LPHD}} = 1.9$
- **For leading dijets, DLA slightly underestimates $\bar{n}_{ch}$ at low p_T and overestimates $\bar{n}_{ch}$ at high p_T**
- DLA and N^{3}LO ($K = 0.065$) overestimate $\bar{n}_{ch}$ for gluon jets at high p_T, but systematically underestimate $\bar{n}_{ch}$ for quark jets across all p_T
- **In DLA, the constant ratio $r = 9/4$ no longer holds when accounting for a mother particle**
- **Mean multiplicity ratio increases with p_T in PYTHIA, DLA, and N^{3}LO** Capella et al, PRD 61 (2000)





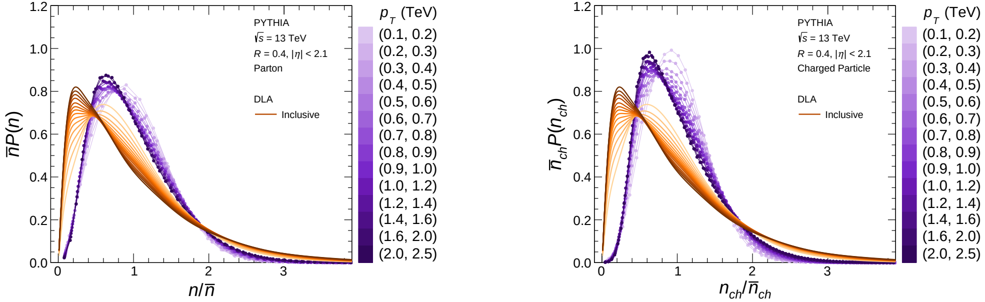


KNO in inclusive jets: DLA vs. PYTHIA

- KNO functions for inclusive multiplicity distributions:

$$\Psi(x) = \bar{n}P(n) = \bar{n}[r_q P_q(n) + r_g P_g(n)] = r_q \frac{\bar{n}}{\bar{n}_q}\Psi_q(x_q) + r_g \frac{\bar{n}}{\bar{n}_g}\Psi_g(x_g)$$

- **Notable differences observed between DLA and PYTHIA at both parton and charged-particle levels**

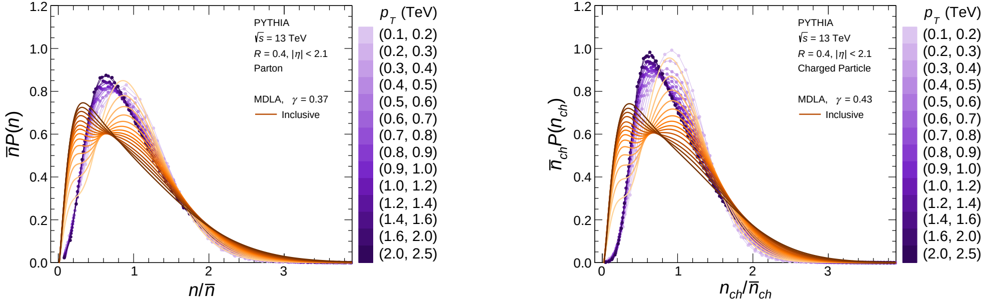


KNO in inclusive jets: MDLA vs. PYTHIA

- KNO functions for inclusive multiplicity distributions:

$$\Psi(x) = \bar{n}P(n) = \bar{n}[r_q P_q(n) + r_g P_g(n)] = r_q \frac{\bar{n}}{\bar{n}_q}\Psi_q(x_q) + r_g \frac{\bar{n}}{\bar{n}_g}\Psi_g(x_g)$$

- **MDLA functions show improved agreement with PYTHIA compared to DLA**
- **Differences become larger at high p_T, where quark jets dominate**

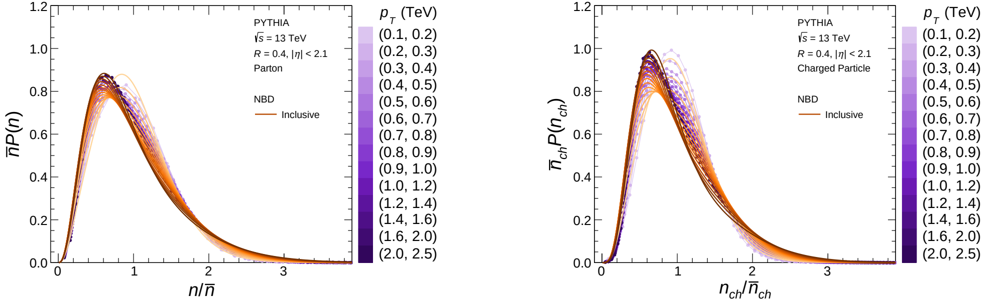


KNO in inclusive jets: NBD vs. PYTHIA

- KNO functions for inclusive multiplicity distributions:

$$\Psi(x) = \bar{n}P(n) = \bar{n}[r_q P_q(n) + r_g P_g(n)] = r_q \frac{\bar{n}}{\bar{n}_q}\Psi_q(x_q) + r_g \frac{\bar{n}}{\bar{n}_g}\Psi_g(x_g)$$

- **NBD functions show good agreement with PYTHIA across the full p_T range**
- NBD functions provide good fits for both quark and gluon jets



KNO scaling in QCD jets via jet substructure techniques

- **Two-point energy correlation functions (ECFs)** are defined as follows:

Larkoski, Salam, Thaler, JHEP 06 (2013)

$$\mathrm{ECF}(0,\beta) = 1$$
$$\mathrm{ECF}(1,\beta) = \sum_{i\in J} p_{Ti}$$
$$\mathrm{ECF}(2,\beta) = \sum_{i<j\in J} p_{Ti}p_{Tj}(R_{ij})^{\beta}$$

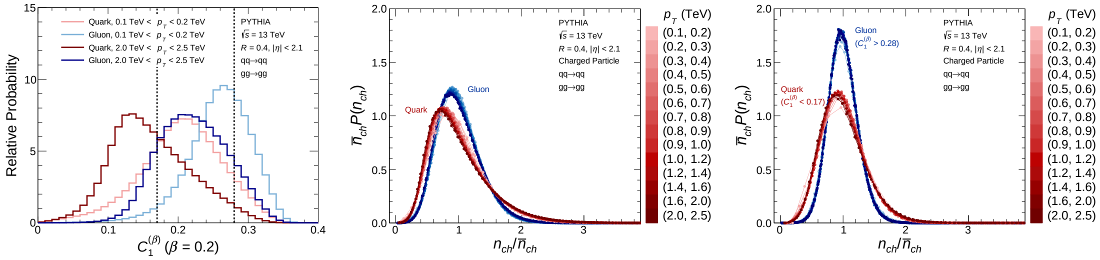
- Angular distance: $R_{ij} = \sqrt{(y_i - y_j)^2 + (\phi_i - \phi_j)^2}$
- **Energy correlation double ratio**:

$$C_1^{(\beta)} = \frac{\mathrm{ECF}(2,\beta)\mathrm{ECF}(0,\beta)}{\mathrm{ECF}(1,\beta)^2}$$

- Optimization parpmeter: $\beta = 0.2$
- Applying cuts on $C_1^{(\beta)}$ enhances quark-/gluon-initial jet purity in the samples

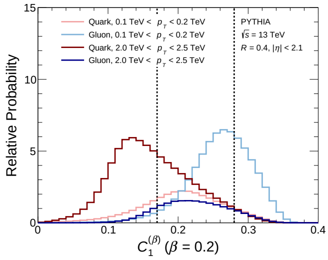
KNO scaling with quark-gluon jet discrimination: pure samples

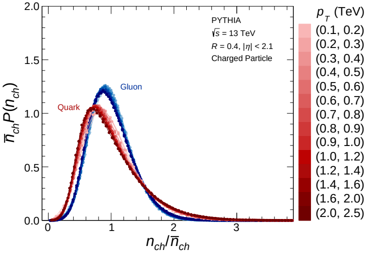
- Two independent hard processes: $qq \to qq$ and $gg \to gg$
- $C_1^{(\beta)}$ distributions of quark and gluon jets generated from independent hard processes
  - Quark: $C_1^{(\beta)} < 0.17$;   Gluon: $C_1^{(\beta)} > 0.28$
- KNO scaling observed for pure quark and gluon jets
- **$C_1^{(\beta)}$ selection makes the scaling peak sharper**
- Quark KNO scaling function lies above that of gluons at both low and high x

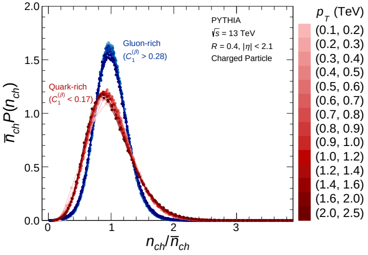


KNO scaling with quark-gluon jet discrimination: hybrid samples

- $C_1^{(\beta)}$ distributions weighted by quark/gluon jet cross sections from hard scattering
  - Quark-rich: $C_1^{(\beta)} < 0.17$
  - Quark purity increases from 76% to 94% with rising p_T
  - Gluon-rich: $C_1^{(\beta)} > 0.28$
  - Gluon purity decreases from 88% to 52% over the same p_T range
- **Quark-rich and gluon-rich jets show similar KNO scaling behavior as in pure samples**







Summary

- **KNO scaling and multiplicity distributions in QCD jets**
  - Observed in both quark and gluon jets at parton and hadron levels
  - Distinct scaling functions persist at LHC energies
  - Key for understanding charged-particle multiplicities in leading dijets
  - Mean multiplicity ratio increases with p_T in PYTHIA, DLA, and $\mathrm{N^3LO}$
- **Jet substructure techniques**
  - Two-point ECFs enable quark-gluon jet discrimination
  - $C_1^{(\beta)}$ cuts sharpen KNO scaling features
- **Outlook**
  - Precision experimental validation at the LHC
  - Extend jet substructure methods to test KNO scaling
  - Improve theoretical predictions beyond DLA/MDLA
  - Explore KNO scaling universality across collision systems

Thank you!