$\psi(2S)$ production at very high p_T

by J.P. Lansberg with **K. Lynch** and V. Bertone

Introduction

A bit of history: the $\psi(2S)$ anomaly

[1] pQCD + hadronisation w/o soft gluon emission

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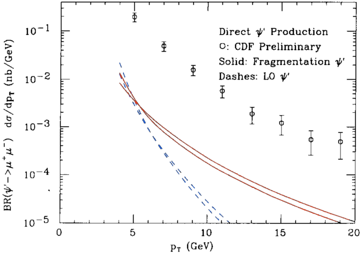
E. Braaten, M. A. Doncheski, S. Fleming, M. L. Mangano, PLB 333 (1994) 548



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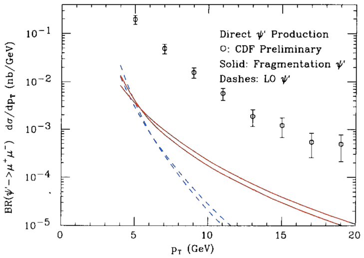

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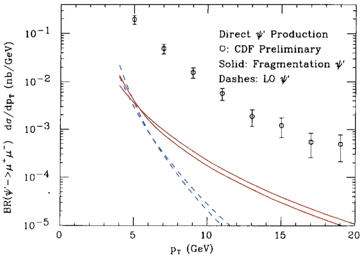

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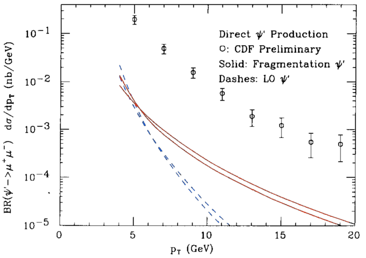

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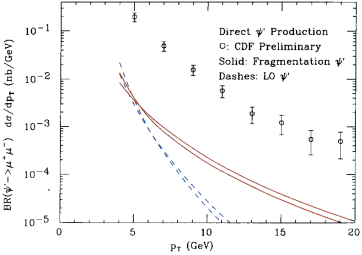
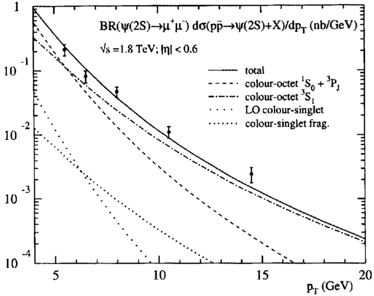


M. Krämer / Prog. Part. Nucl. Phys. 47 (2001) 141–201



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- Fragmentation-function- based CSM computations (red solid) show an agreement with the p_T shape but off by a factor 30
- Confirmed by the final measurements and subsequent RUN 2 data and then similar observation made with J/ψ after χ_c removal
- This triggered the introduction of the Colour Octet Mechanism (long-dash, dot-dash) from NRQCD (EFT valid at small v)

G. Bodwin, E. Braaten, G. Lepage, PRD 51 (1995) 1125; P. Cho, A. Leibovich PRD 53 (1996) 6203

- Reminder : **CSM** $\equiv$ ${}^3S_1^{[1]}$ for ψ; LO in v^2
  COM: ${}^3S_1^{[8]}, {}^1S_0^{[8]}, {}^3P_J^{[8]}$: v^4, i.e. NNLO in v^2 but enhanced in p_T at LO in α_s

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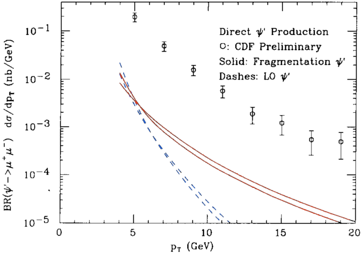

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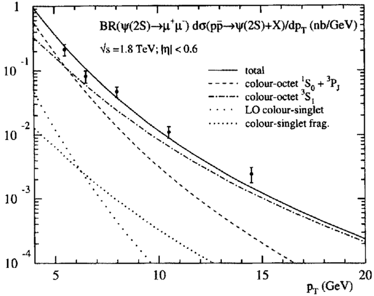

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See e.g. JPL Phys. Rept. 889 (2020) 1

- Approach valid for $p_T \not\gg m_Q$ (FFNS) [Usual approach]

$$\frac{d\sigma}{dp_{T,\mathcal{Q}}} = \sum_{i,j,n} f_{i/A}(\mu_{F_i}) \otimes f_{j/B}(\mu_{F_i}) \otimes \frac{d\hat{\sigma}_{ij\to Q\bar{Q}[n]X}}{dp_{T,Q\bar{Q}[n]}}(\mu_{F_i}, \mu_R, m_Q)\langle \mathcal{O}^{\mathcal{Q}}_{Q\bar{Q}[n]}\rangle$$

  - $\hat{\sigma}_{ij\to Q\bar{Q}[n]X}$ computed with NRQCD with $m_Q \neq 0$ at fixed order (FO) in α_s
  - QCD corrections growing with p_T for ${}^3S_1^{[1]}$, ${}^1S_0^{[8]}$, ${}^3P_J^{[8]}$ [see next slide]
  - Non-perturbative physics factorised out in Long Distance Matrix Elements (LDMEs)
  - $\langle \mathcal{O}^{\mathcal{Q}}_{{}^3S_1^{[1]}}\rangle$ from potential models, other LDMEs unknown and fit to the data

QCD corrections to the CSM for $\psi(2S)$ at colliders

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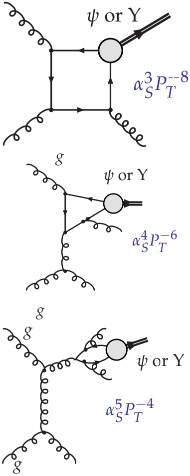

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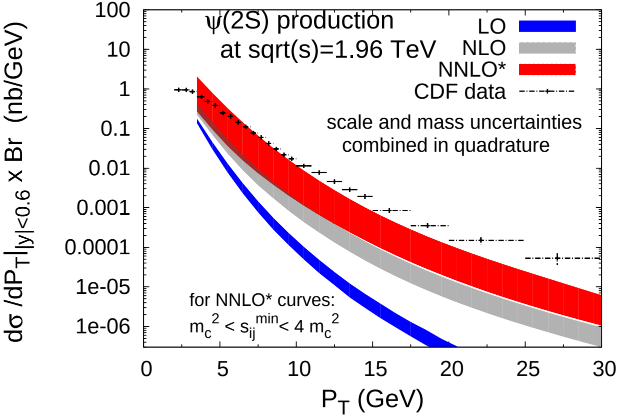

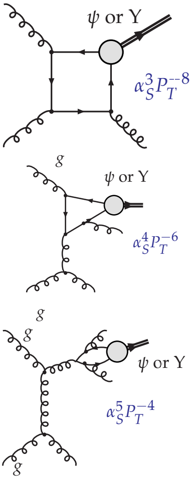

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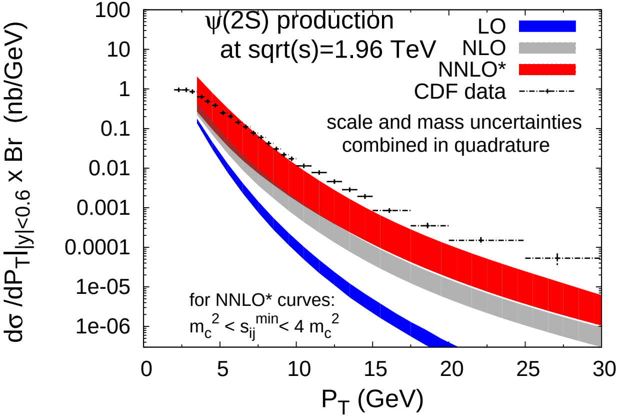

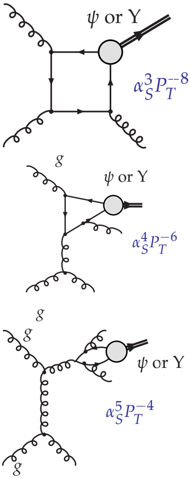

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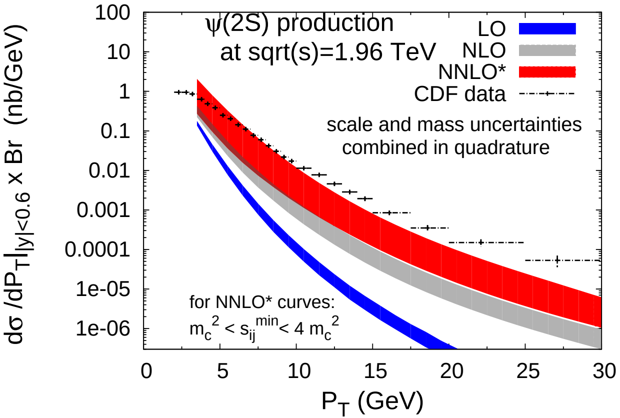

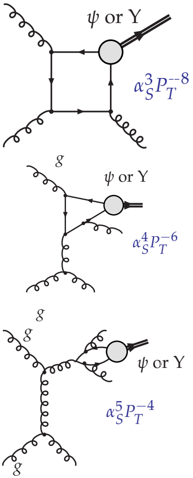

- NNLO* is only an approximate NNLO valid at large p_T
- It probably makes sense to focus first on the p_T scaling and then on α_s

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- Approach valid for $p_T \gg m_Q$ (ZM-VFNS) [with fragmention functions (FFs)]

$$\frac{d\sigma}{dp_{T,\mathcal{Q}}} \simeq \sum_{i,j,k} f_{i/A}(\mu_{F_i}) \otimes f_{j/B}(\mu_{F_i}) \otimes \frac{d\hat{\sigma}_{ij\to kX}}{dp_{T,k}}(\mu_{F_i}, \mu_{F_f}, \mu_R) \otimes D_k^{\mathcal{Q}}(\mu_{F_f}, \{\mu_0\})$$

  - Correspond to the leading-power (LP) contribution of an expansion in p_T
  - m_Q ($\ll p_T$) neglected in $\hat{\sigma}$, kept in the FFs
  - FFs DGLAP evolved to account for the resummation of $\alpha_s \ln(p_T/m_Q)$
  - Unlike other mesons, under NRQCD, FF z dependence is computable

$$D_i^{\mathcal{Q}}(z, \{\mu_0\}) = \sum_n D_i^{Q\bar{Q}[n]}(z, \{\mu_0\})\langle \mathcal{O}^{\mathcal{Q}}_{Q\bar{Q}[n]}\rangle$$

  - Note: fragmentation is not a new mechanism, just a subset of the usual approach !

What about higher-order contribution in $m_{\mathcal{Q}}^2/p_T^2$?

Y.Q. Ma et. al., PRL 113, 142002 (2014)

- The first term is valid up to corrections $\mathcal{O}(m_{\mathcal{Q}}^2/p_T^2)$

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- For ${}^3S_1^{[1]}$: LP first appears at $\mathcal{O}(\alpha_s^3)$ vs. NLP first appears at $\mathcal{O}(\alpha_s^1)$
- This is expected to compensate the p_T suppression of the NLP

FF vs FO for the CSM

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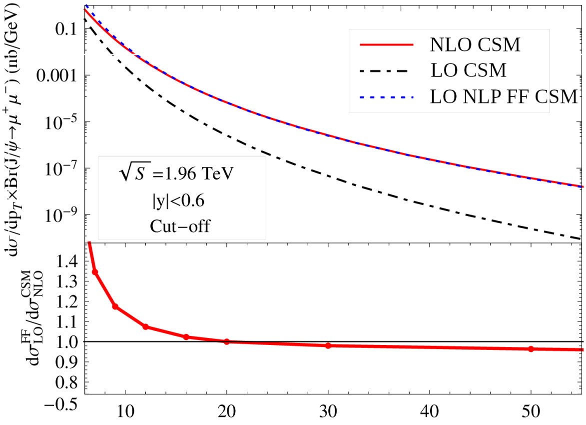

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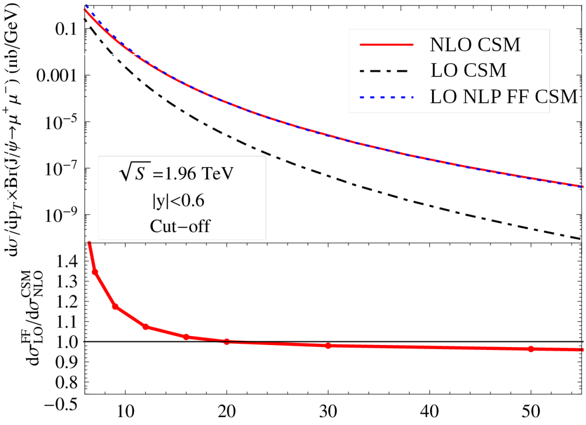

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- We found that NNLO* is larger than NLO

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- Kang et al. found that NLP reproduces NLO
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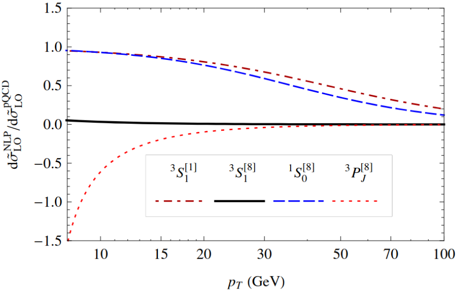

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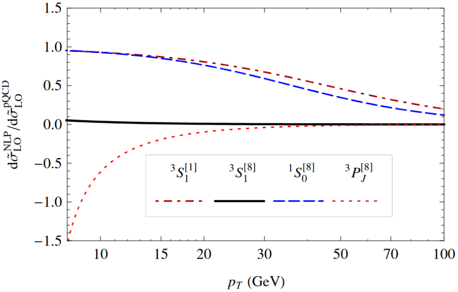

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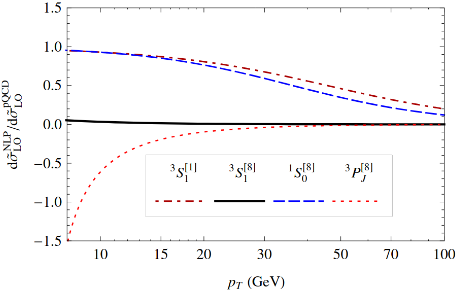

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Caveat:

- No evolution of FF here
- NLO α_s and v^2 corrections might affect the comparison

New very high p_T $\psi(2S)$ data at the LHC

- ATLAS(CMS) measured prompt $\psi(2S)$ up to $p_T = 140(110)$ GeV

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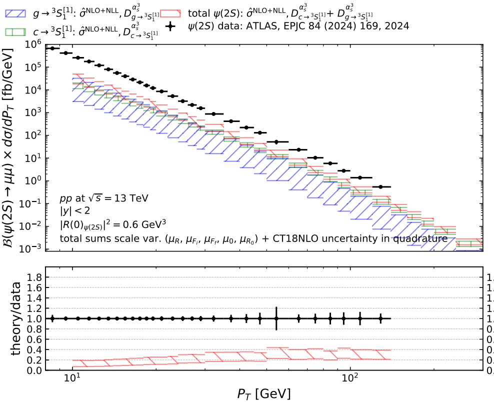

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- ATLAS(CMS) measured prompt $\psi(2S)$ up to $p_T = 140(110)$ GeV

ATLAS: EPJC 84 (2024) 169; CMS PLB 780 (2018) 251

- Perfect sample with minimal NLP corrections and no feed down
- Advance the FF CSM computation with state-of-the art theory, i.e.
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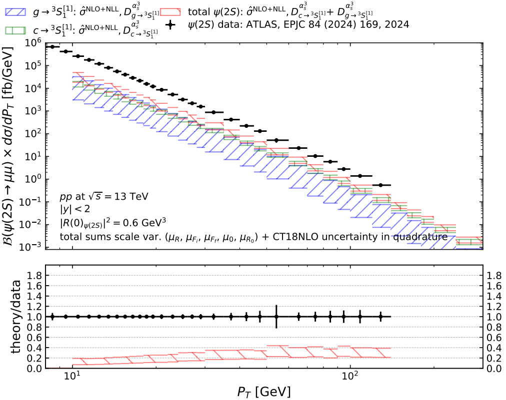
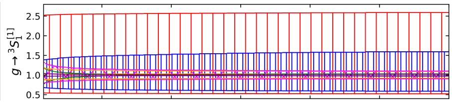


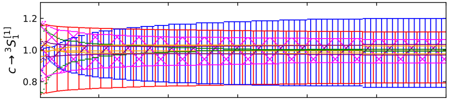
pp at $\sqrt{s} = 13$ TeV, $|y| < 2$

total sums scale var. ($\mu_R, \mu_{F_i}, \mu_{F_f}, \mu_0, \mu_{R_0}$) + CT18NLO uncertainty in quadrature

μ_{R_0} μ_0 μ_{F_i} μ_{F_f} μ_R PDF



Blue band above



Green band above

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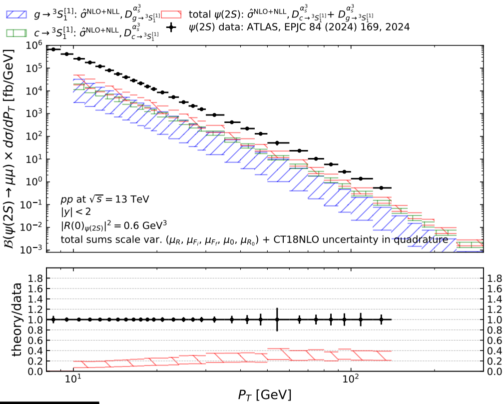

Summary:

- i) Charm frag. dominates, ii) large μ_R uncertainty from gluon frag. (LO at α_s^3)

  [i) known; ii) so far completely overlooked !]
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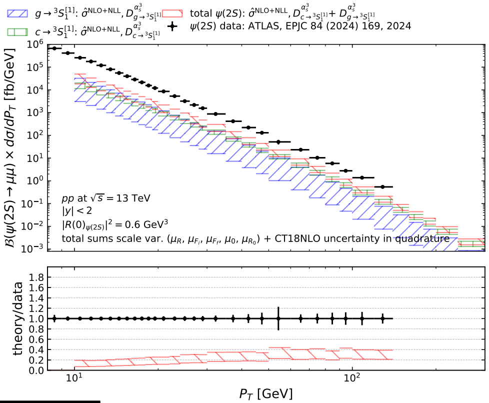

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Digression : importance of the higher moments

Fragmentation function enters cross section as a convolution with $d\tilde{\sigma}_k$

$$\frac{d\sigma}{dp_{T,\mathcal{Q}}} \simeq \sum_k \underbrace{\sum_{i,j} f_i(\mu_{F_i}) \otimes f_j(\mu_{F_i}) \otimes \underbrace{\frac{d\hat{\sigma}_{ij\to kX}}{dp_{T,k}}(\mu_{F_i},\mu_{F_f},\mu_R)}_{\propto p_{T,k}^{-4} \text{ at LO}}}_{d\tilde{\sigma}_k \propto p_{T,k}^{-n}} \otimes D_k^{\mathcal{Q}}(\mu_{F_f})$$

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- pp cross sections sensitive to the $\mathcal{O}(5)^{\text{th}}$ Mellin Moment of the fragmentation function **See e.g. J. Baines, hep-ph/0601164**

Relativistic corrections to the CSM FF

- NRQCD is based on a v^2 expansion

  [beside that of α_s and m_Q/p_T]
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$R_2 \equiv \langle 0|\mathcal{O}_2(^3S_1^{[1]})|0\rangle / \langle 0|\mathcal{O}_0(^3S_1^{[1]})|0\rangle$

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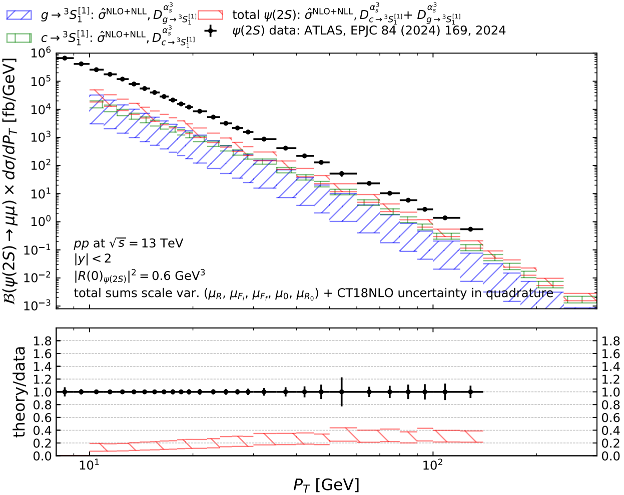

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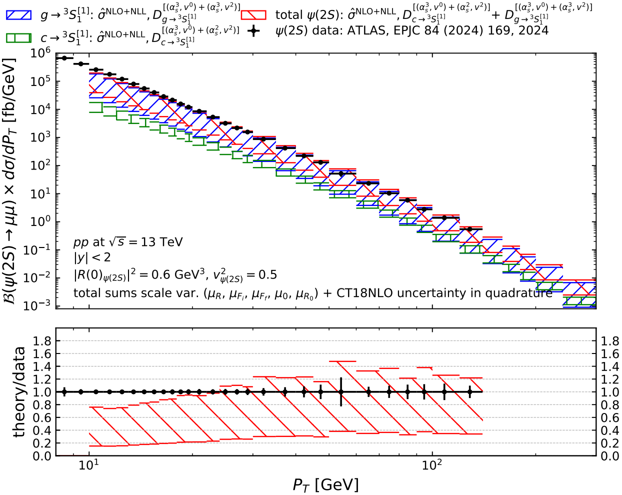

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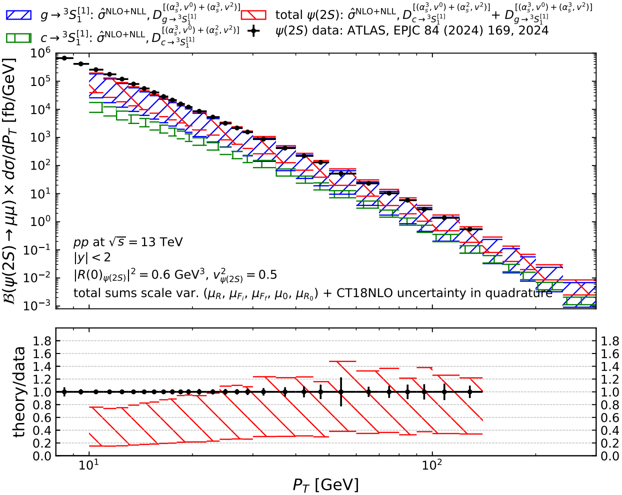

- The v^2 corrections boost the gluon frag. contribution by $\mathcal{O}(5)$ for $v^2_{\psi(2S)} = 0.5$
- Charm-contribution essentially unchanged [end-point behaviour less crucial]
- CMS data agree with ATLAS one, thus also with our calculation
- Main uncertainty from $v^2_{\psi(2S)}$ and μ_{R_0} [urgent to compute gluon FF at NLO]

A look back at the Tevatron data

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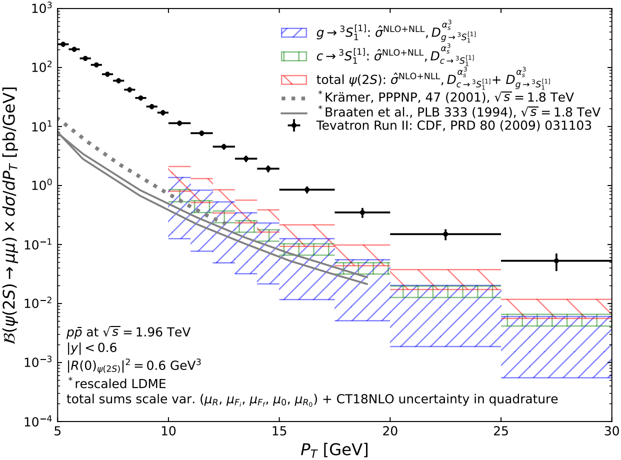

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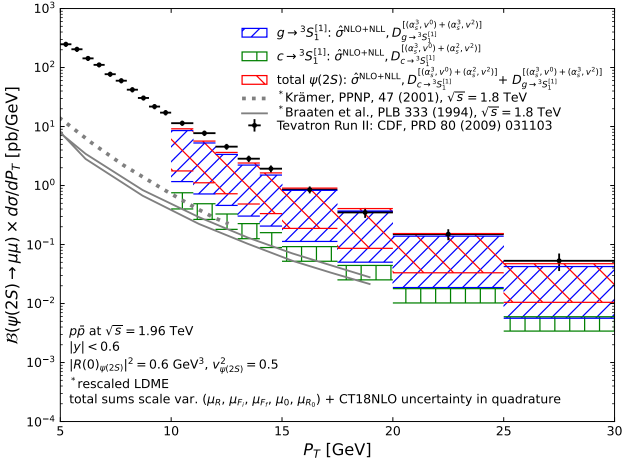

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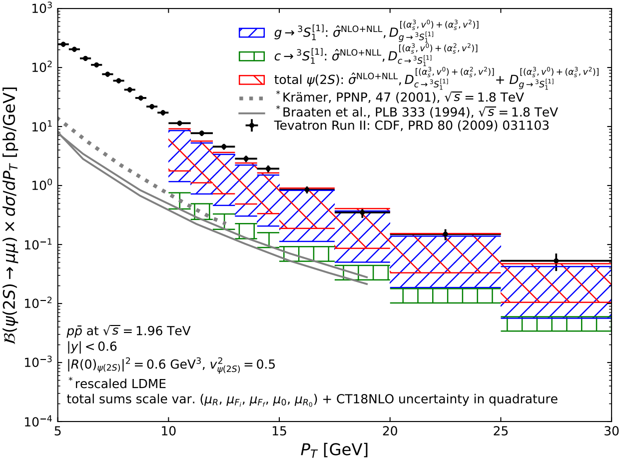

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- Below $p_T = 30$ GeV, NLP corrections might be significant for the gluon channel

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- Thank you for your attention !

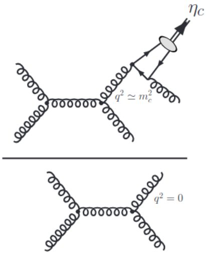
Backup

Computation of fragmentation functions

- From the decay of a virtual particle:
  - computed as the ratio of the cross sections
    - Example $g \to \eta_c$

Int.J.Mod.Phys.A 21 (2006) 3857-3916

$$\int_0^1 D(z, m_c)\,dz \simeq$$



- Using the Collins-Soper definition **Nucl. Phys. B 194 (1982) 445**
  - Gauge-invariant definition that includes an eikonal coupling in Feynman rules

Fragmentation functions at lowest order in α_s

⊗ $g \to c\bar{c}(^3S_1^8)$: **Phys. Rev. Lett. 74 (1995) 3327**

$$D_g^{J/\psi[^3S_1^{[8]}]}(z,\mu_0) = \delta(1-z)\frac{\pi\alpha_s(\mu_0)}{24m_Q^3}\langle\mathcal{O}_8^{J/\psi}(^3S_1)\rangle \quad (1)$$

⊗ $g \to c\bar{c}(^1S_0^8)$: **Phys. Rev. D 89 (2014) 094029, Phys. Rev. D 55 (1997) 2693, JHEP 11 (2012) 020**

$$D_g^{J/\psi[^1S_0^{[8]}]}(z,\mu_0) == \frac{(N_c^2-4)\alpha_s^2(\mu_0)}{4N_c m_Q^3}[2(1-z)\log(1-z)+3z-2z^2]\langle\mathcal{O}_8^{J/\psi}(^1S_0)\rangle \quad (2)$$

⊗ $g \to c\bar{c}(^3S_1^1)$: **Phys. Rev. Lett. 71 (1993) 1673, Phys. Rev. D 96, 094016 (2017)**

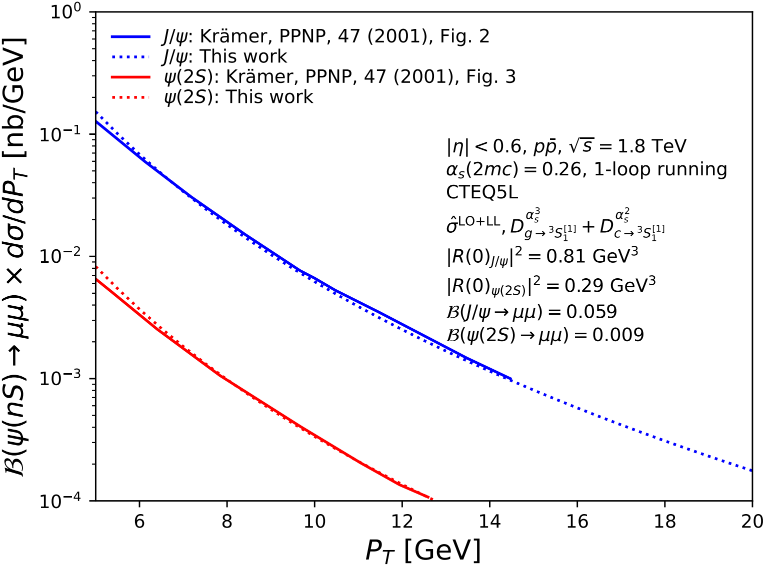
$$D_g^{J/\psi[^3S_1^{[1]}]}(z,\mu_0) = \frac{128(N_c^2-4)\pi^3\alpha_s^3(\mu_0)}{3N_c^2(2m_Q)^3}\left(CI_{13}+\sum_{i=0}^{11}C_i\,L_i\right)\frac{\langle\mathcal{O}_1^{J/\psi}(^3S_1)\rangle}{2N_c}$$

$$L_0=1\,,\ L_1=\ln z\,,\ L_2=\ln(1-z)\,,\ L_3=\ln(2-z)\,,\ L_4=\ln^2 z\,,\ L_5=\ln^2(1-z)\,,\ L_6=\ln^2(2-z)\,, \quad (3)$$

$$L_7=\ln z\ \ln(1-z)\,,\ L_8=\ln z\ \ln(2-z)\,,\ L_9=Li_2(1-z)\,,\ L_{10}=Li_2\left(\frac{z-1}{z-2}\right)\,,\ L_{11}=Li_2\left(\frac{2(z-1)}{z-2}\right)\ \ldots$$

- All LO expressions for g, q, c, Q to $^3S_1^{[1]}$, $^3S_1^{[8]}$, $^3P_J^{[8]}$, and $^1S_0^{[8]}$ collected in **Phys. Rev. D 89, 094029 (2014)**

Checks


J/ψ: Krämer, PPNP, 47 (2001), Fig. 2
J/ψ: This work
$\psi(2S)$: Krämer, PPNP, 47 (2001), Fig. 3
$\psi(2S)$: This work
$|\eta| < 0.6$, $p\bar{p}$, $\sqrt{s} = 1.8$ TeV
$\alpha_s(2mc) = 0.26$, 1-loop running
CTEQ5L
$\hat{\sigma}^{\mathrm{LO+LL}}, D^{\alpha_s^3}_{g\to {}^3S_1^{[1]}} + D^{\alpha_s^2}_{c\to {}^3S_1^{[1]}}$
$|R(0)_{J/\psi}|^2 = 0.81$ GeV^3
$|R(0)_{\psi(2S)}|^2 = 0.29$ GeV^3
$\mathcal{B}(J/\psi \to \mu\mu) = 0.059$
$\mathcal{B}(\psi(2S) \to \mu\mu) = 0.009$
$\mathcal{B}(\psi(nS) \to \mu\mu) \times d\sigma/dP_T$ [nb/GeV]
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
6
8
10
12
14
16
18
20
P_T [GeV]

Evolution of fragmentation function I

- The fragmentation function is computed at $\mu_0 \sim m_Q$ and is convoluted with the hard partonic cross section at $\mu_F \sim p_T$ where $p_T \gg m_{\mathcal{Q}}$

$$\frac{d\hat{\sigma}_{ij\to kX}}{dp_{T,k}}(\mu_F) \otimes D_k^{\mathcal{Q}}(\mu_F)$$

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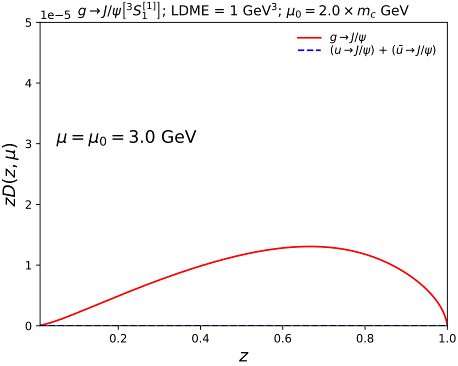
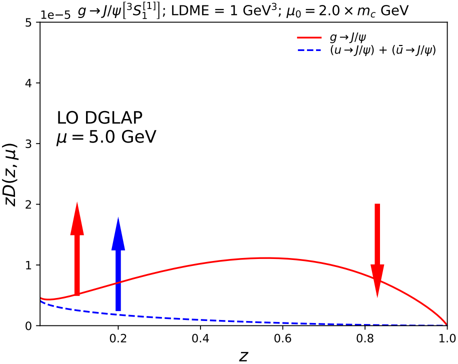


- Initial condition for $D_g^{\mathcal{Q}}(\mu_0)$ via $^3S_1^{[1]}$ chanel **Phys. Rev. Lett. 71 (1993) 1673, Phys. Rev. D 89 (2014) 094029**
- $D_k^{\mathcal{Q}}(\mu_0) = 0$ for $k \in \{q, \bar{q}, Q, \bar{Q}\}$

Evolution of fragmentation function II



- Effect of evolution:
  - Large-z gluon shrinks
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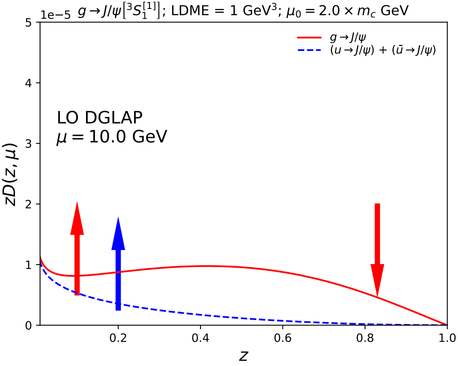
Evolution of fragmentation function III



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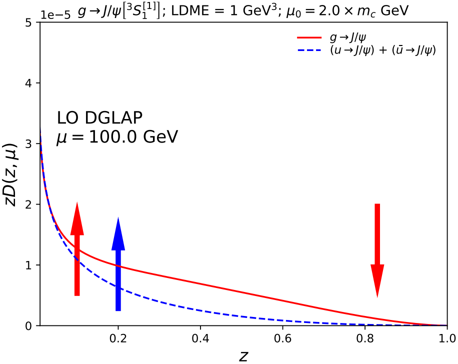

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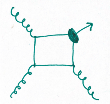
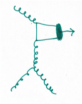
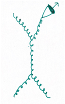
FFNS vs. ZM-VFNS: p_T hierarchy

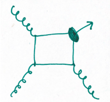
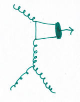
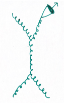
Fixed **F**lavour **N**umber **S**cheme:

${}^3S_1^{[1]}$	${}^1S_0^{[8]}$	${}^3S_1^{[8]}$
		
$\alpha_s^3 p_T^{-8}$	$\alpha_s^3 p_T^{-6}$	$\alpha_s^3 p_T^{-4}$

- v^2-supressed terms (${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$) are leading and subleading in p_T

FFNS vs. ZM-VFNS: p_T hierarchy

Fixed Flavour Number Scheme:

$^3S_1^{[1]}$	$^1S_0^{[8]}$	$^3S_1^{[8]}$
		
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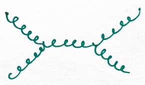
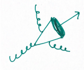
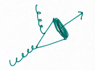
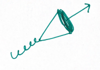
Zero Mass Variable Flavour Number Scheme:

$\alpha_s^2\, p_T^{-4}$ () $\otimes$ ...

$^3S_1^{[1]}$	$^1S_0^{[8]}$	$^3S_1^{[8]}$
		
$\alpha_s^3(\mu_0)$	$\alpha_s^2(\mu_0)$	$\alpha_s(\mu_0)$

- All contributions enter with same scaling in p_T
- Number of couplings modifies FF at μ_0

Matching Scheme Bodwin et. al.; Phys.Rev.D 93 (2016) 3, 034041, Phys.Rev.D 92 (2015) 7, 074042

- In order to describe the whole p_T region one should combine the FFNS and ZM-VFNS contributions
- However, there is a double counting between the FFNS and ZM-VFNS
- This double counting is removed by introducting a matching term

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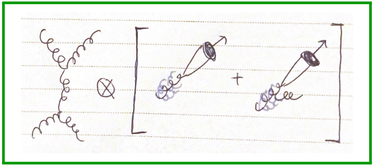
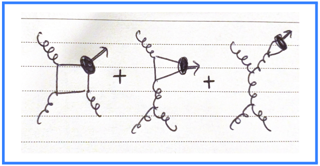
$$d\sigma^{\text{LP+NLO}} = \underbrace{d\sigma^{\text{ZM-VFNS}}}_{\alpha_s^2 \otimes \alpha_s^2} + \underbrace{d\sigma^{\text{FFNS}}}_{\alpha_s^3} - d\sigma_{\text{matching}}$$





Matching Scheme Bodwin et. al.; Phys.Rev.D 93 (2016) 3, 034041, Phys.Rev.D 92 (2015) 7, 074042

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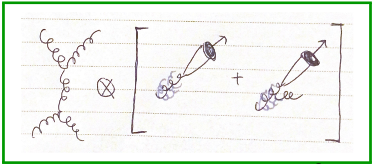
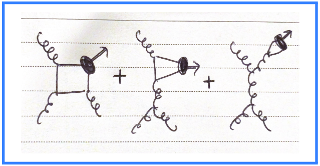
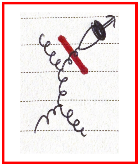




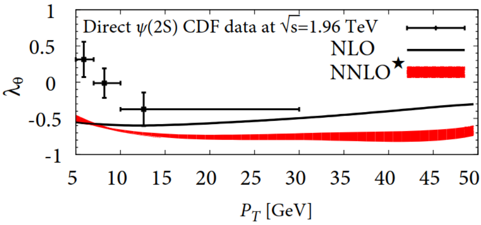


- Double counting is $\mathcal{O}(\alpha_s^3)$
- Matching term is the $\mathcal{O}(\alpha_s^3)$ component of the ZM-VFNS contribution without evolution

$\mathcal{Q}$ polarisation at large P_T

Phys.Rept. 889 (2020) 1-106, Phys. Rev. D 96, 094016 (2017)

- $\frac{dN}{d\cos\theta} \propto 1 + \lambda_\theta \cos^2\theta$ where $\lambda_\theta = \frac{1/2\sigma_T - \sigma_L}{1/2\sigma_T + \sigma_L}$
- $\lambda_\theta = +1$ transverse; $\lambda_\theta = -1$ longitudinal; $\lambda_\theta = 0$ unpolarised
- **F**ixed **F**lavour **N**umber **S**cheme results:
  - transversely polarised at LO
  - longitudinaly polarised at NLO, NNLO*



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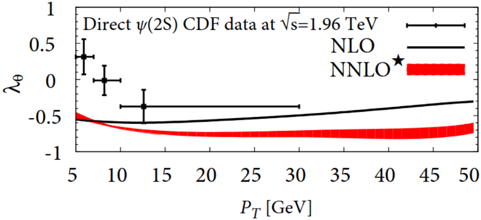

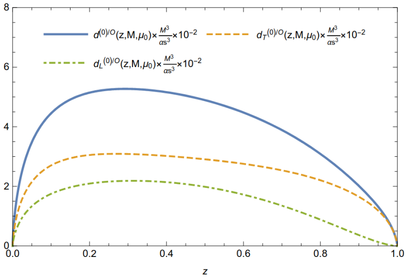

- What about FF?

${}^3S_1^{[1]}$ FF at μ_0

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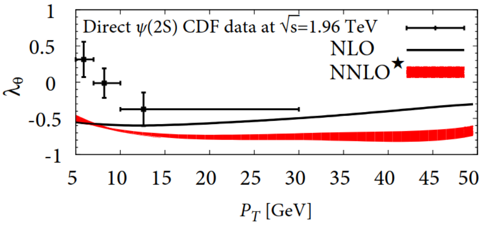
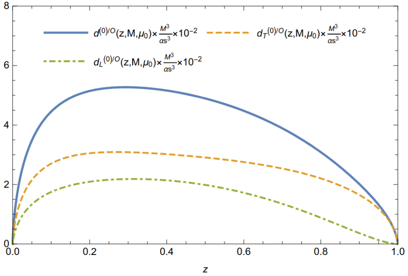




- What about FF?
  - $z = 0.1$: $\lambda_\theta \approx -0.1$ and $z = 0.9$: $\lambda_\theta \approx 0.4$

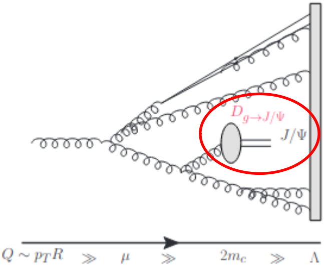
${}^3S_1^{[1]}$ FF at μ_0

$\mathcal{Q}$ in jet and fragmentation functions

See talk of Paul Caucal on Monday

Quarkonia in jets - formalism

- J/Ψ at high p_T is expected to predominantly come from jet fragmentation.
- Formalism based on the jet evolution outlined above + FF at the scale $\sim m_c$.



$$\frac{\mathrm{d}\sigma^{pp\to j_1+j_2(J/\Psi)+X}}{\mathrm{d}p_T \mathrm{d}z_{J/\Psi}} = H_{ab\to ij} \otimes f_a \otimes f_b \otimes J_j \otimes \mathcal{G}_i^{J/\Psi}(p_T, R, z, \mu)$$

$$\mathcal{G}_i^{J/\Psi} \sim C_{ij}(p_T, R, \mu) \otimes K_{\mathrm{DGLAP}}\left[D_{j\to J/\Psi}(2m_c)\right]$$

State κ	${}^3S_1^{[1]}$	${}^3S_1^{[8]}$	${}^1S_0^{[8]}$	${}^3P_J^{[8]}$
$g \to c\bar{c}(\kappa)$	α_s^3	α_s	α_s^2	α_s^2
LDME $\langle O_\kappa^{J/\Psi}\rangle$	$(v/c)^3$	$(v/c)^7$	$(v/c)^7$	$(v/c)^7$

$\Rightarrow$ competing orders of magnitudes between $g \to c\bar{c}(\kappa)$ and LDME in NRQCD.

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Available computing tools for the study of fragmentation functions

- Fragmentation function evolution (LHAPDF grid format):
  - APFEL++ (`https://github.com/vbertone/apfelxx`)
    - Input: $zD_i^{\mathcal{Q}}(z, \mu_0)$
    - Must be a continuous function
  - MELA (`https://github.com/vbertone/MELA`)
    - Input: $\tilde{D}_i^{\mathcal{Q}}(N, \mu_0)$
    - Can be discontinuous (e.g. contain δ functions/plus distributions)
- Tools for phenomenological studies:
  - INCNLO (`https://lapth.cnrs.fr/PHOX_FAMILY/readme_inc.html`)
  - FMNLO (`https://fmnlo.sjtu.edu.cn/`)

Heavy hadron (H_Q) production: $p_T \not\gg m_Q$

Nucl.Phys.B 421 (1994) 530-544; slides from Ingo Schienbein

H_Q production via **F**ixed **F**lavour **N**umber **S**cheme (FFNS):

$$\frac{d\sigma}{dp_{T,H_Q}} = \sum_{i,j,Q} f_{i/A}(\mu_F) \otimes f_{j/B}(\mu_F) \otimes \frac{d\hat{\sigma}_{ij\to QX}}{dp_{T,Q}}(\mu_F, \mu_R, m_Q) \otimes D_Q^{H_Q}$$

- $\otimes$ denotes a Mellin Convolution: $f \otimes g(x) = \int_0^1 dy \int_0^1 dz\, f(y)g(z)\delta(x - yz)$
- PDF:
  - Only light flavours in initial state: $i, j \in \{q, \bar{q}, g\}$, where $q = u, d, s$
  - **perturbative** μ_F evolution which absorbs initial-state collinear singularities
  - **non-perturbative** boundary condition: $f_{i/H}(x, \mu_0)$ at $\mu_0 = \mathcal{O}(1\ \mathrm{GeV})$
- Owing to m_Q, no final-state collinear singularities in $\hat{\sigma}$ or $D_Q^{H_Q}$!
- However, logs of the kind $\alpha_s \ln(p_T/m_Q)$ appear in $\hat{\sigma}$
- For $p_T \gg m_Q$, these logs are large and should be resummed

Heavy hadron (H_Q) production: $p_T \gg m_Q$

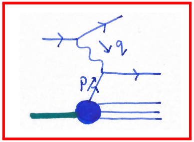
H_Q production via **Z**ero **M**ass **V**ariable **F**lavour **N**umber **S**cheme (ZM-VFNS):

- For large scale ($p_T \gg m_Q$) we can treat the quarks as massless in $\hat{\sigma}$ up to corrections $\mathcal{O}((m_Q/p_T)^2)$:

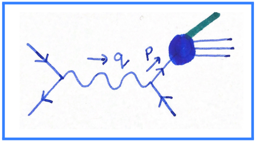
$$\frac{d\sigma}{dp_{T,H_Q}} \simeq \sum_{i,j,k} f_{i/A}(\mu_{F_i}) \otimes f_{j/B}(\mu_{F_i}) \otimes \frac{d\hat{\sigma}_{ij\to kX}}{dp_{T,k}}(\mu_{F_i}, \mu_{F_f}, \mu_R) \otimes D_k^{H_Q}(\mu_{F_f})$$

- In $\hat{\sigma}$ take $i, j, k \in \{q, \bar{q}, g, Q, \bar{Q}\}$ but consider them to be **massless**
- We introduce an additional scale, μ_{F_f}, and the large logs from the prevoious partonic cross section are effectively split into 2 terms $\ln(p_T/m_Q) = \ln(p_T/\mu_{F_f}) + \ln(\mu_{F_f}/m_Q)$:
  - $\ln(p_T/\mu_{F_f})$: contained within $\hat{\sigma}$, this is small provided $\mu_F \sim p_T$
  - $\ln(\mu_{F_f}/m_Q)$: resummed to all orders by evolution equations in $D_k^{H_Q}(\mu_{F_f})$
- The mass dependence is absorbed into the FF
- This results in a better control of the theortical uncertainty at large p_T

FFs: final-state counterpart to PDFs

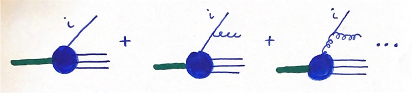


$e^- H \to e^- X$

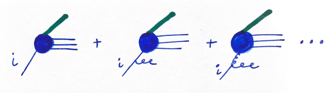


$e^+ e^- \to HX$

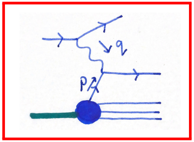
- **Parton Distribution Function:** $f_{i/H}(x, \mu^2)$
  parton i is emitted from hadron H carrying longitudinal momentum fraction x of H
  - DGLAP evolution amounts to resumming initial-state collinear divergences:



- **Fragmentation Function:** $D_i^H(z, \mu^2)$
  hadron H is emitted from parton i carrying longitudinal momentum fraction z of i
  - DGLAP evolution amounts to resumming final-state collinear divergences:



FFs: final-state counterpart to PDFs



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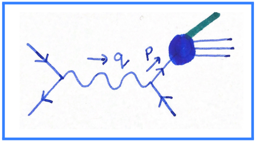


$e^+ e^- \to HX$

- **P**arton **D**istribution **F**unction: $f_{i/H}(x, \mu^2)$
  parton i is emitted from hadron H carrying longitudial momentum fraction x of H
  - Scale: $\mu^2 = -q^2$ [space-like]
  - DGLAP evolution with space-like (S) splitting kernels:

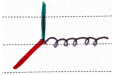
$$\frac{\partial}{\partial \ln \mu^2} f_{i/H}(x, \mu^2) = \sum_j \int_x^1 \frac{dx'}{x'} P^S_{ij}\left(\frac{x}{x'}, \alpha_s(\mu^2)\right) f_{j/H}\left(x', \mu^2\right)$$

- **F**ragmentation **F**unction: $D_i^H(z, \mu^2)$
  hadron H is emitted from parton i carrying longitudial momentum fraction z of i
  - Scale: $\mu^2 = q^2$ [time-like]
  - DGLAP evolution with time-like (T) splitting kernels:

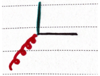
$$\frac{\partial}{\partial \ln \mu^2} D_i^H(z, \mu^2) = \sum_j \int_z^1 \frac{dz'}{z'} P^T_{ji}\left(z', \alpha_s(\mu^2)\right) D_j^H\left(\frac{z}{z'}, \mu^2\right)$$

Splitting kernels

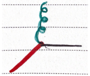
- The kernels $P_{ij}(x)$ describes the splitting of parton j into parton i carrying momentum fraction x of j
- At LO accuracy in α_s $P^{\mathrm{S}}_{ij} = P^{\mathrm{T}}_{ij} = P_{ij}$:



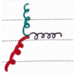
$$P_{qq}(x) = 2C_F\left(\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right)$$



$$P_{qg}(x) = 2T_R\left(x^2 + (1-x)^2\right)$$



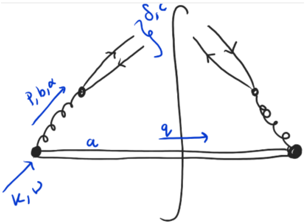
$$P_{gq}(x) = 2C_F\left(\frac{1+(1-x)^2}{x}\right)$$



$$P_{gg}(x) = 4C_A\left(\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right) + \delta(1-x)\frac{11C_A + 4N_f T_R}{3}$$

where $C_F = \frac{4}{3}$, $T_R = \frac{1}{2}$, and $C_A = 3$

Example: computation of $g \to J/\psi(^3S_1^{[8]})$ FF using Collins-Soper definition I



1. Compute Amplitude on LHS of cut line: [eikonal coupling]

$$\mathcal{A}_{\nu\alpha} = -i\delta^{ab}\left[g_{\nu\alpha}(n\cdot k) - p_\nu n_\alpha\right]\left(ig\mu^\epsilon\gamma^\alpha T^b\right)$$

Example: computation of $g \to J/\psi({}^3S_1^{[8]})$ FF using Collins-Soper definition II

2. Contract with colour and spin projector:

$$\mathrm{Tr}\left[\mathcal{A}_{\nu\alpha}\Pi_8^c\Pi_1^\delta\right],$$

$$\Pi_8^c = \sqrt{2}T^c, \quad \Pi_1^\delta = \frac{1}{4m_Q^2}\left(\frac{\not{p}_{\mathcal{Q}}}{2} - m_Q\right)\gamma_\delta\frac{(\not{p}_{\mathcal{Q}} + 2m_Q)}{4m_Q}\left(\frac{\not{p}_{\mathcal{Q}}}{2} + m_Q\right)$$

3. Compute amplitude square:

$$|\mathcal{A}|^2 = \mathrm{Tr}\left[\mathcal{A}_{\nu\alpha}\Pi_8^c\Pi_1^\delta\right]\left(\mathrm{Tr}\left[\mathcal{A}_{\nu'\alpha'}\Pi_8^{c'}\Pi_1^{\delta'}\right]\right)^\dagger \Pi_{\delta\delta'}\delta^{cc'}(-g_{\nu\nu'})\delta^{aa'}$$

- $\Pi_{\delta\delta'}\delta^{cc'}$: colour and spin polarisation of $Q\bar{Q}\left[{}^3S_1^{[8]}\right]$
- $(-g_{\nu\nu'})\delta^{aa'}$: contract eikonal indicies

Example: computation of $g \to J/\psi({}^3S_1^{[8]})$ FF using Collins-Soper definition III

4. Integrate over phase space and multiply by normalisation factors:

$$D_g^{J/\psi[{}^3S_1^{[8]}]}(z,\mu_0) = \frac{N_{\mathrm{CS}}}{k^4}|\mathcal{A}|^2 d\phi_0 \frac{\langle \mathcal{O}_8^{J/\psi}({}^3S_1)\rangle}{(D-1)(N_c^2-1)}$$

- $d\phi_0 = \frac{8\pi m_Q}{k\cdot n}\delta(1-z)$: normalisation of 0-body phase space
- $N_{\mathrm{CS}} = \frac{z^{D-3}}{(N_c^2-1)(k\cdot n)2\pi(D-2)}$: Collins-Soper normalisation
- $k^4 = (2m_Q)^4$: off-shellness of fragmenting gluon
- $\langle \mathcal{O}_8^{J/\psi}({}^3S_1)\rangle$: LDME
- $(D-1)(N_c^2-1)$: spin and colour averaging

to obtain final expression at $\mu_0 \sim 2m_c$:

$$D_g^{J/\psi[{}^3S_1^{[8]}]}(z,\mu_0) = \delta(1-z)\frac{\pi\alpha_s(\mu_0)}{24m_Q^3}\langle \mathcal{O}_8^{J/\psi}({}^3S_1)\rangle$$