A precise determination of α_s from the heavy jet mass distribution

VNiVERSiDAD D SALAMANCA

Vicent Mateu

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Taskforce A. Bhattacharya, M. Benitez, A.H. Hoang, M.D. Schwartz, I.W. Stewart and X. Zhang based on 2502.12253

July 7th, 2025 EPS-HEP 2025, Marseille

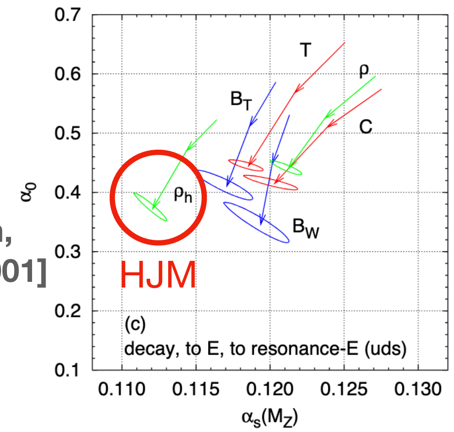
Outline

- A story on the strong coupling
- Event shapes and heavy jet mass
- Theoretical considerations
- Fitting procedure and results
- Summary

α_s from Event Shapes

[See also prior work of Catani, Trentadue, Turnock, Webber 1993]

[Salam, Wicke 2001]



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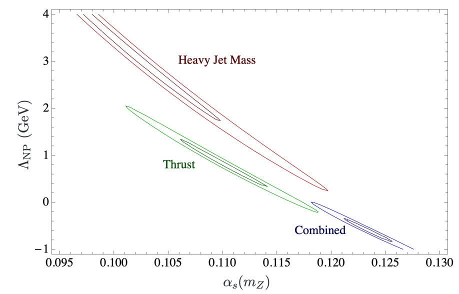
[See also Dissertori et al 2007]

N³LL resummation with NNLO matching



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Thrust	0.1101	0.821	66.9/47
Heavy Jet Mass	0.1017	3.17	60.4/43
Combined	0.1236	-0.621	453/92

[Chien, Schwartz 2010]

These studies used, in parts, only 25% of data bins:
$0.08 < \rho < 0.18$

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2001-2010

Prior to SCET, only NLL resummation possible

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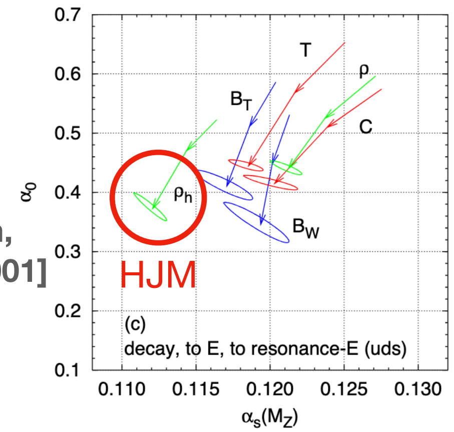

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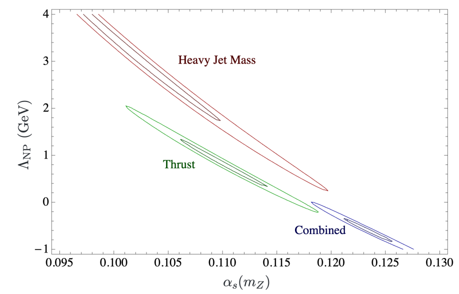

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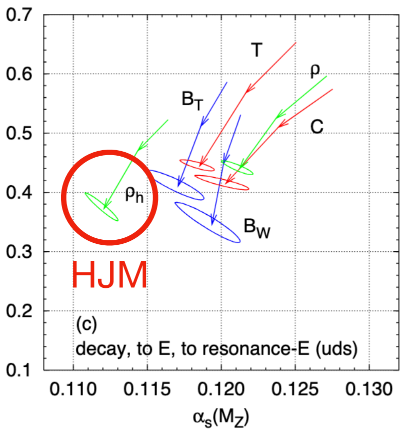

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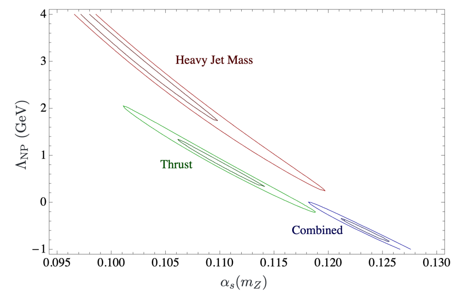

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At that time, fixed-order $\mathcal{O}(\alpha_s^3)$ results became available

Impossible to have consistent results for HJM!

α_s from Event Shapes

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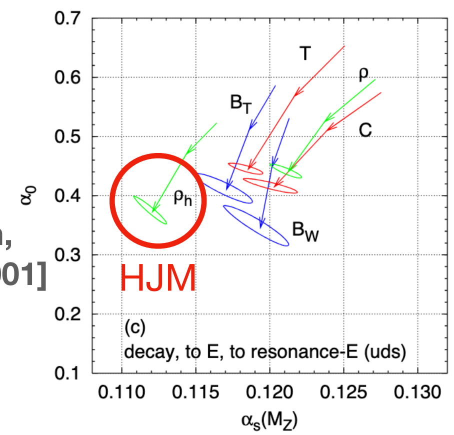

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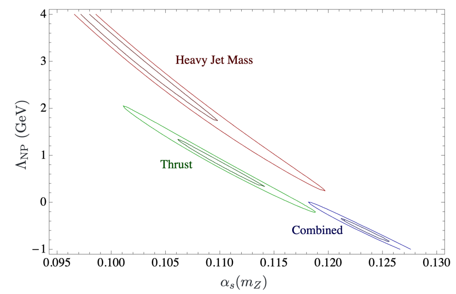

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very precise value, but well below the world average

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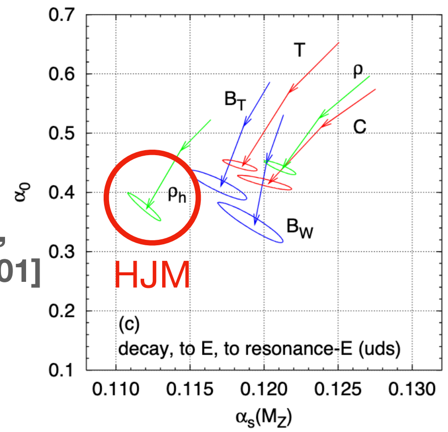

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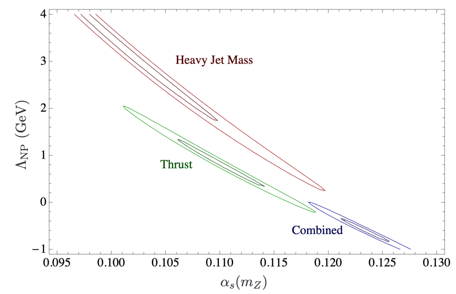

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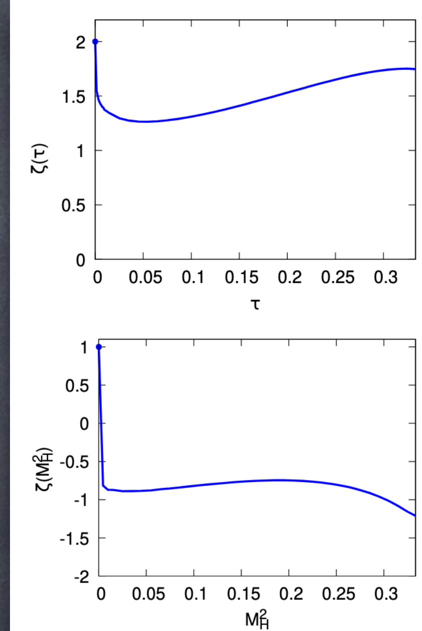

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doubts raised on 3-jet power corrections

renormalon-based “computation” of those

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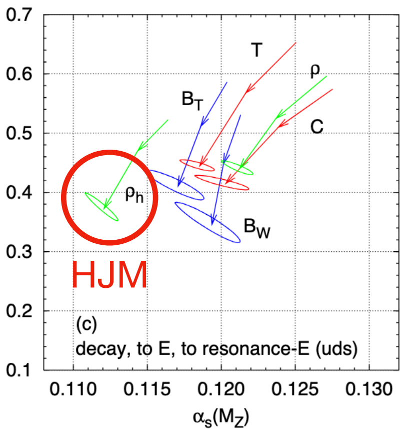

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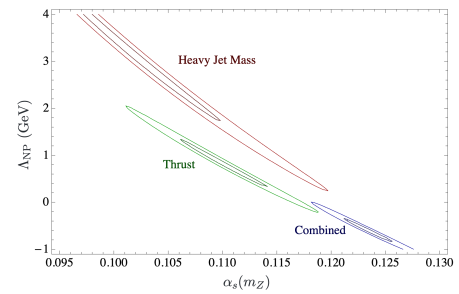

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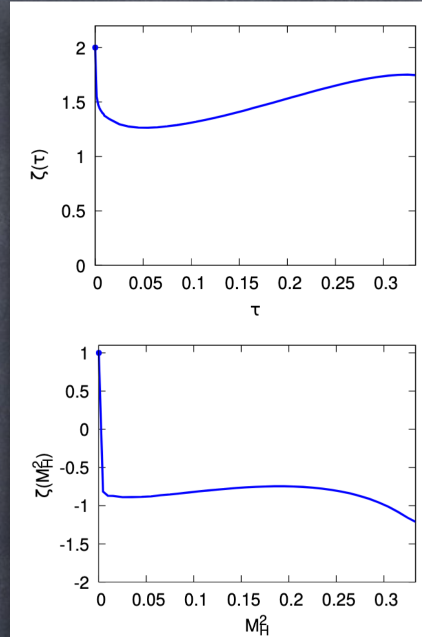

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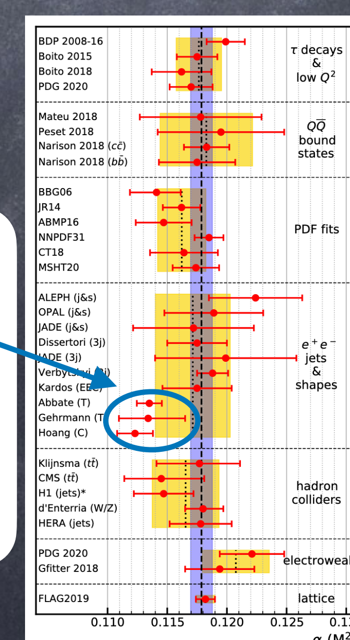

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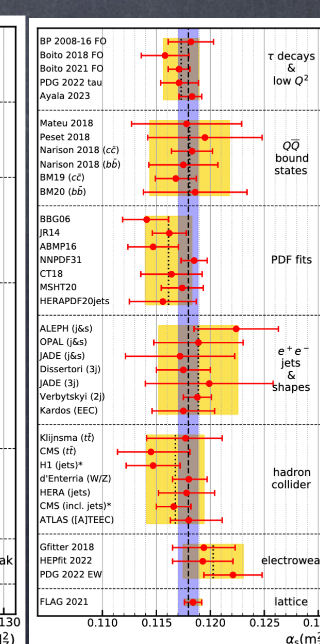

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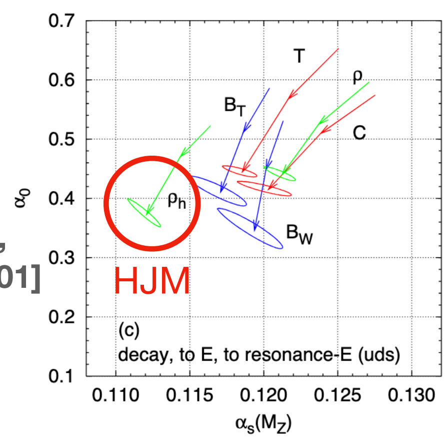

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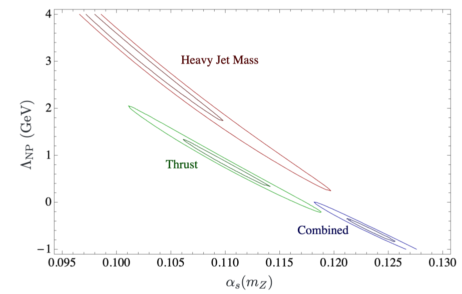

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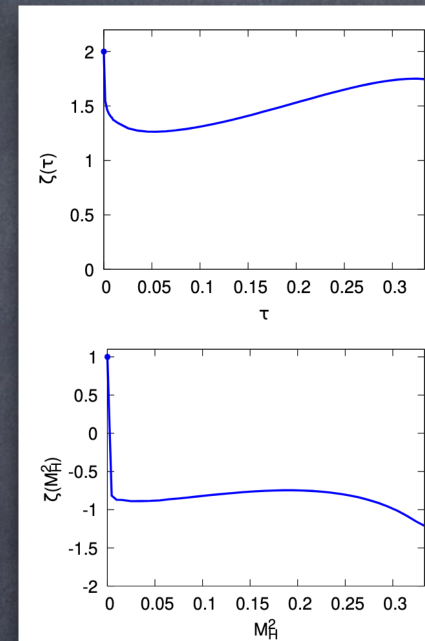

2024

On Determining $\alpha_s(m_Z)$ from Dijets in e^+e^- Thrust

Miguel A. Benitez[a], André H. Hoang[b], Vicent Mateu[a], Iain W. Stewart[b,c] and Gherardo Vita[d]

pure dijet analysis still yields small α_s

power corrections were not the issue!

2001-2010 → 2010-2015 → 2023 → 2023 → 2024-2025 → t

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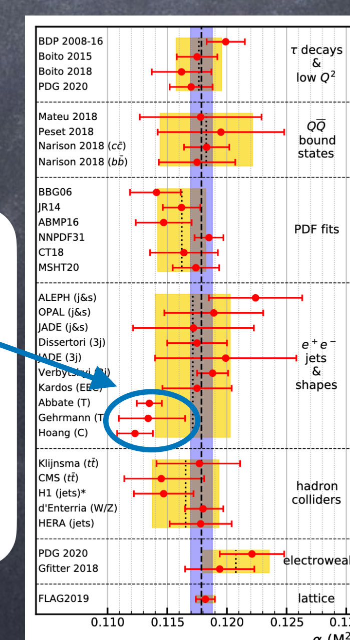

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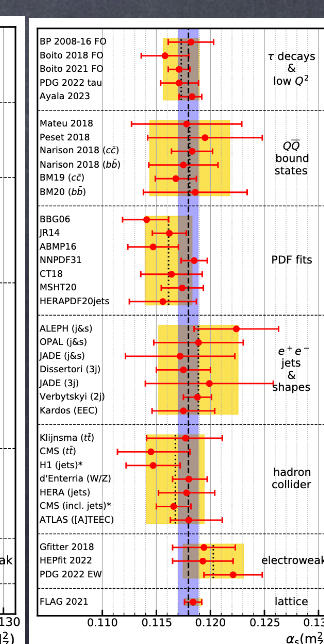

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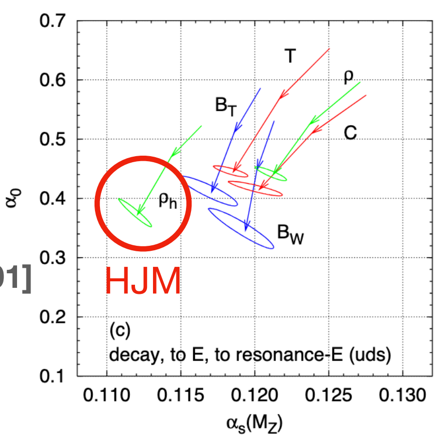

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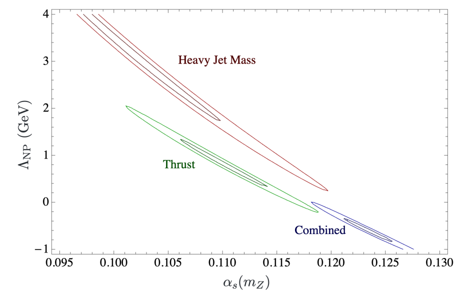

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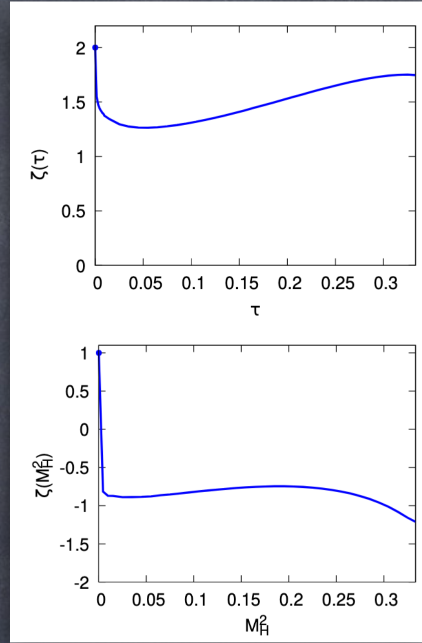

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2025

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Miguel A. Benitez[1], Arindam Bhattacharya[2], André H. Hoang[3], Vicent Mateu[1], Matthew D. Schwartz[2], Iain W. Stewart[3,4] and Xiaoyuan Zhang[2]

This talk

2001-2010 | 2010-2015 | 2023 | 2023 | 2024-2025 | t

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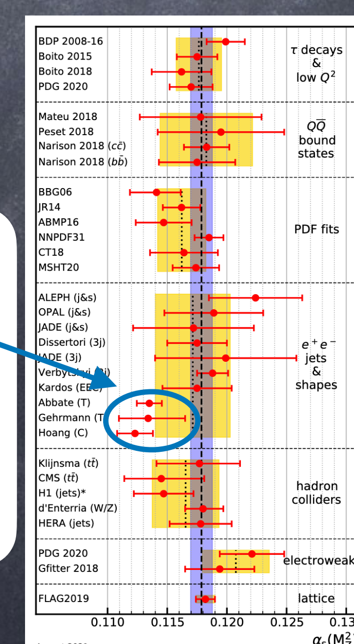

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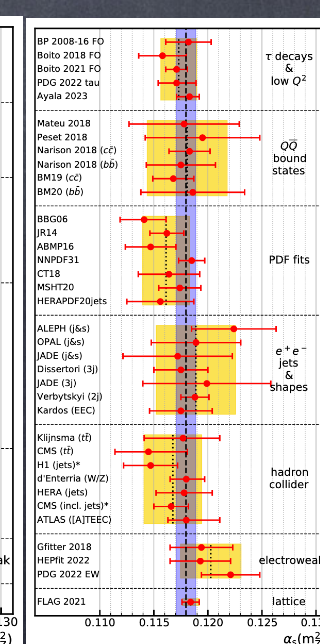
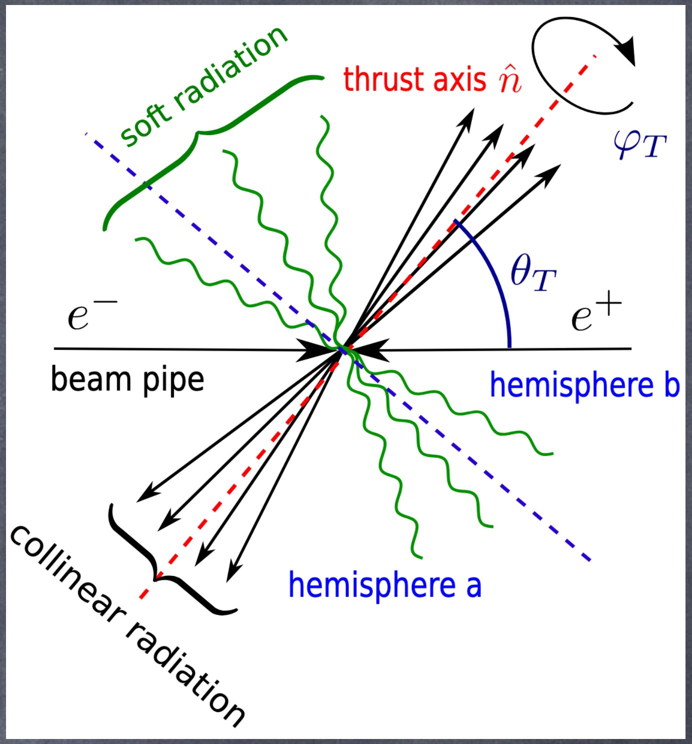


[PDG 2023]

The HJM distribution

Event shapes describe geometric properties of final-state particles

Anatomy of dijet event



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plane normal to $\hat{n}$ defines two hemispheres

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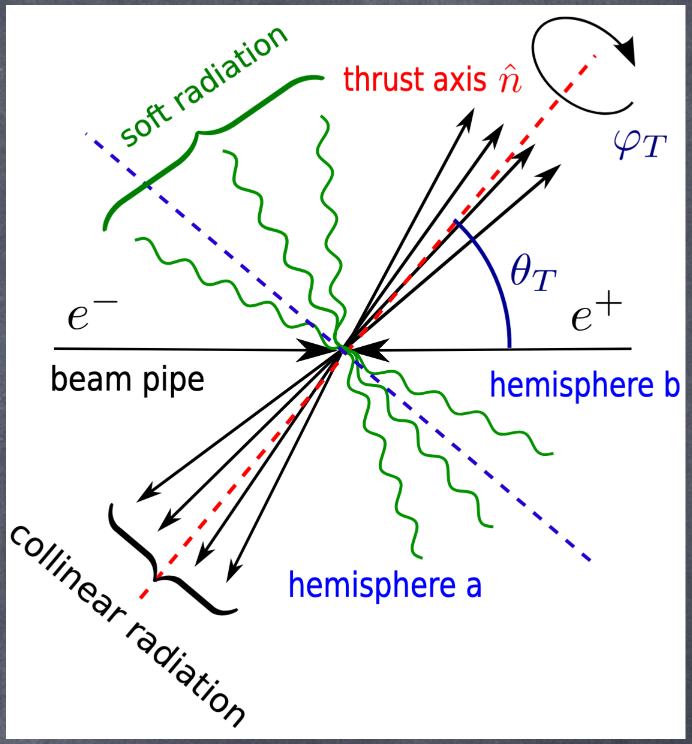

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hemispheres invariant masses $s_j = \left(\sum_{i \in j} p_i^\mu \right)^2$

heavy jet mass $\rho = \frac{1}{Q^2} \max(s_a, s_b)$

[Clavelli 1979] [Chandramohan, Clavelli 1981] [Clavelli, Wyler 1981]

$Q = \sqrt{(p_{e^+} + p_{e^-})^2}$ c.o.m. energy

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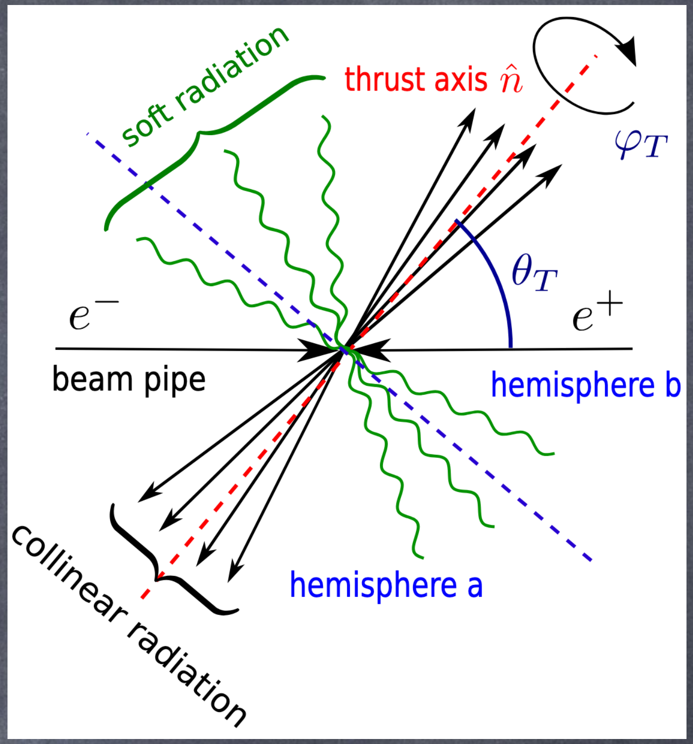

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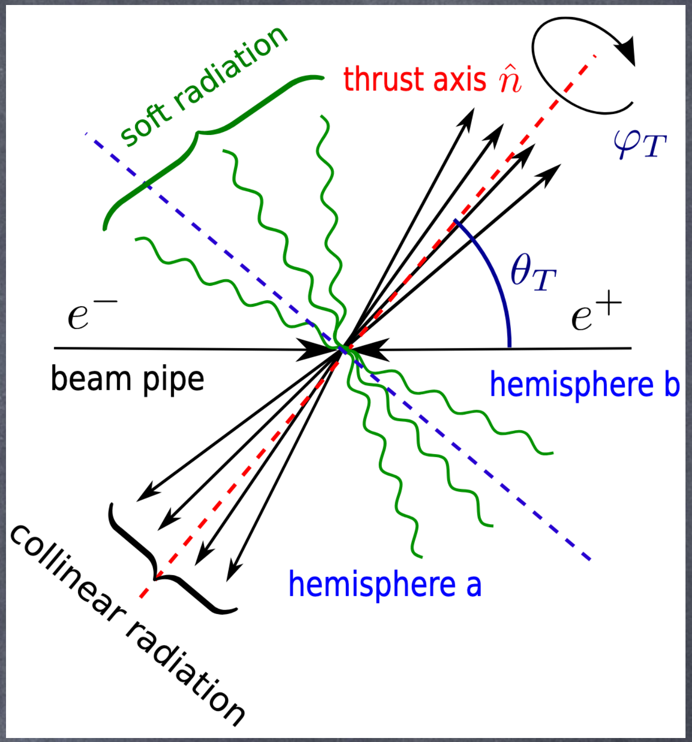
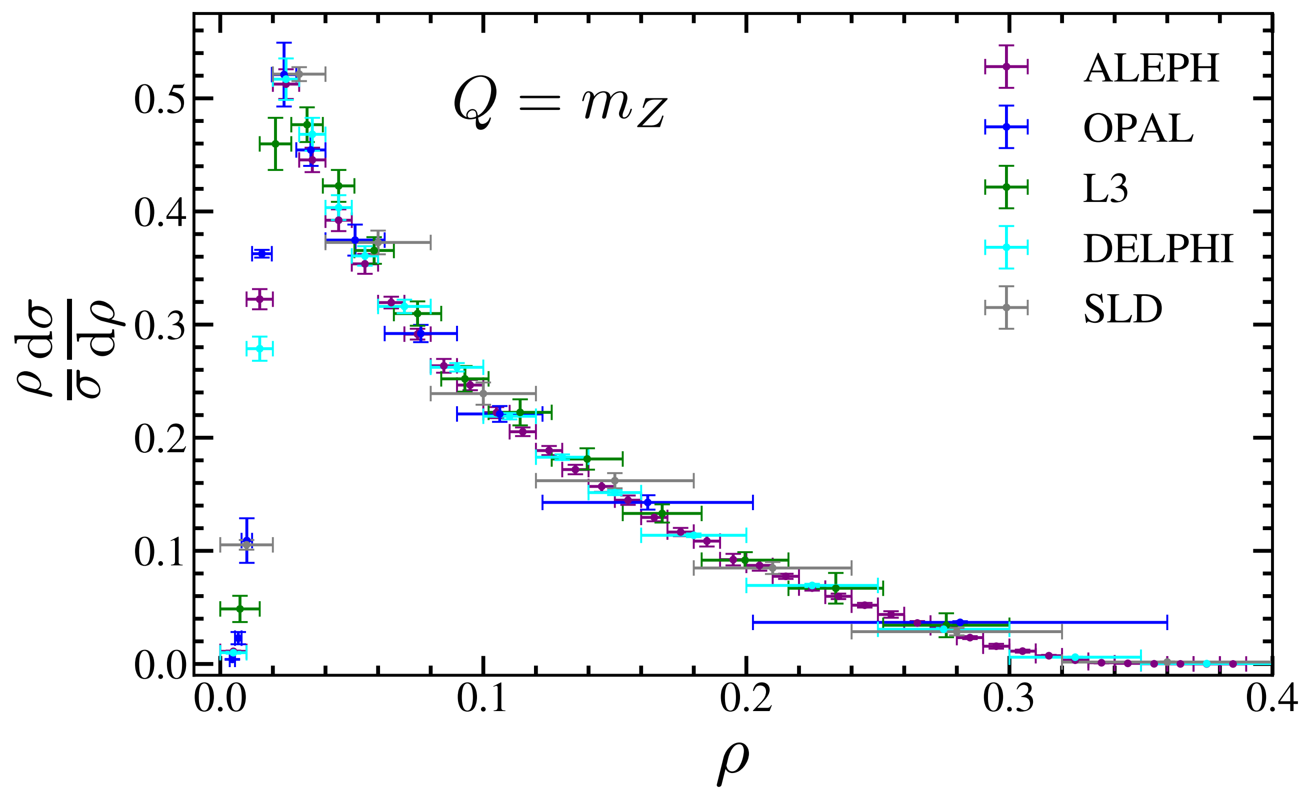


experimental distribution



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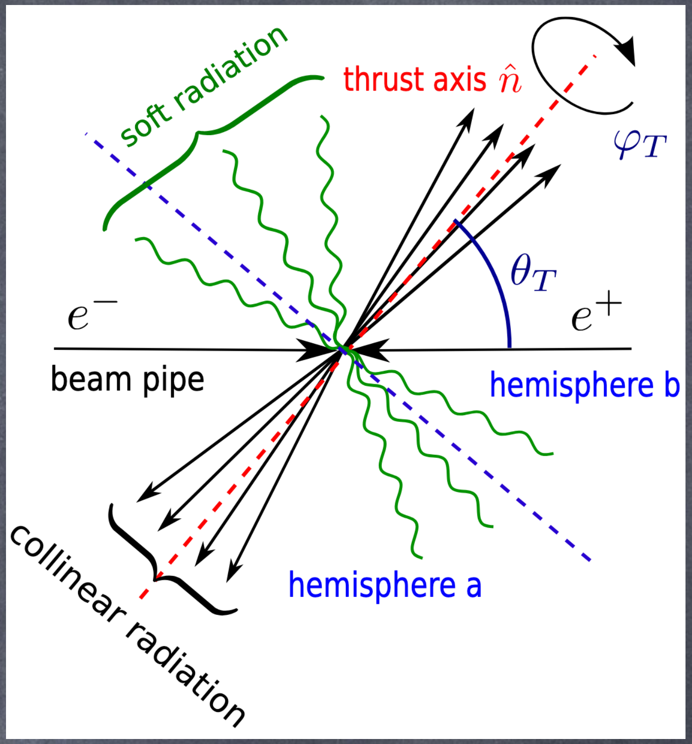

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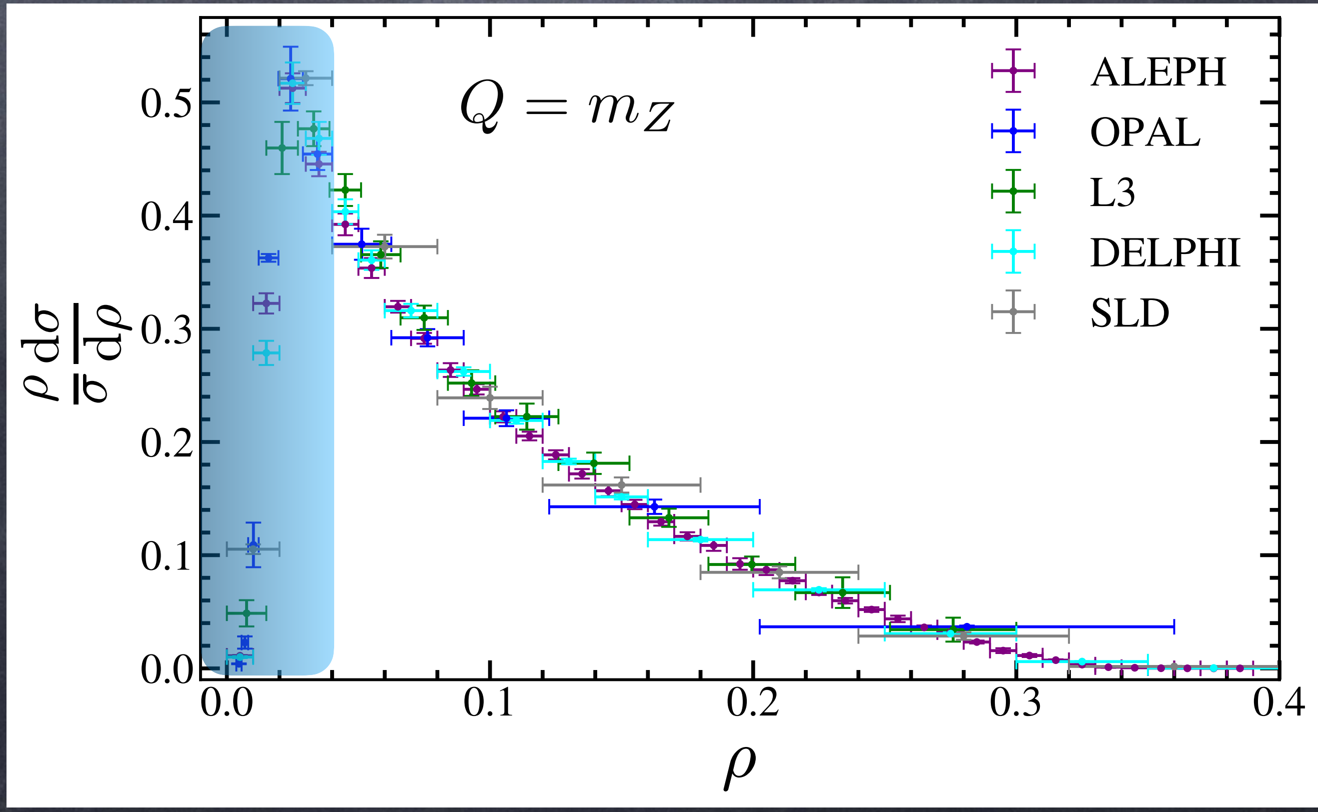

Peak region

$\frac{s_i}{Q} \sim \Lambda_{\text{QCD}}$

Heavily affected by non-perturbative dynamics

The HJM distribution

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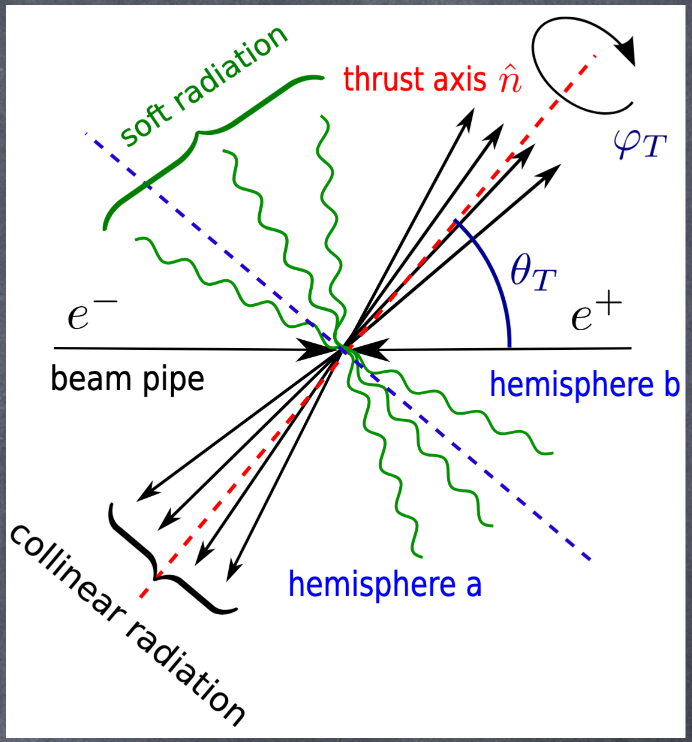

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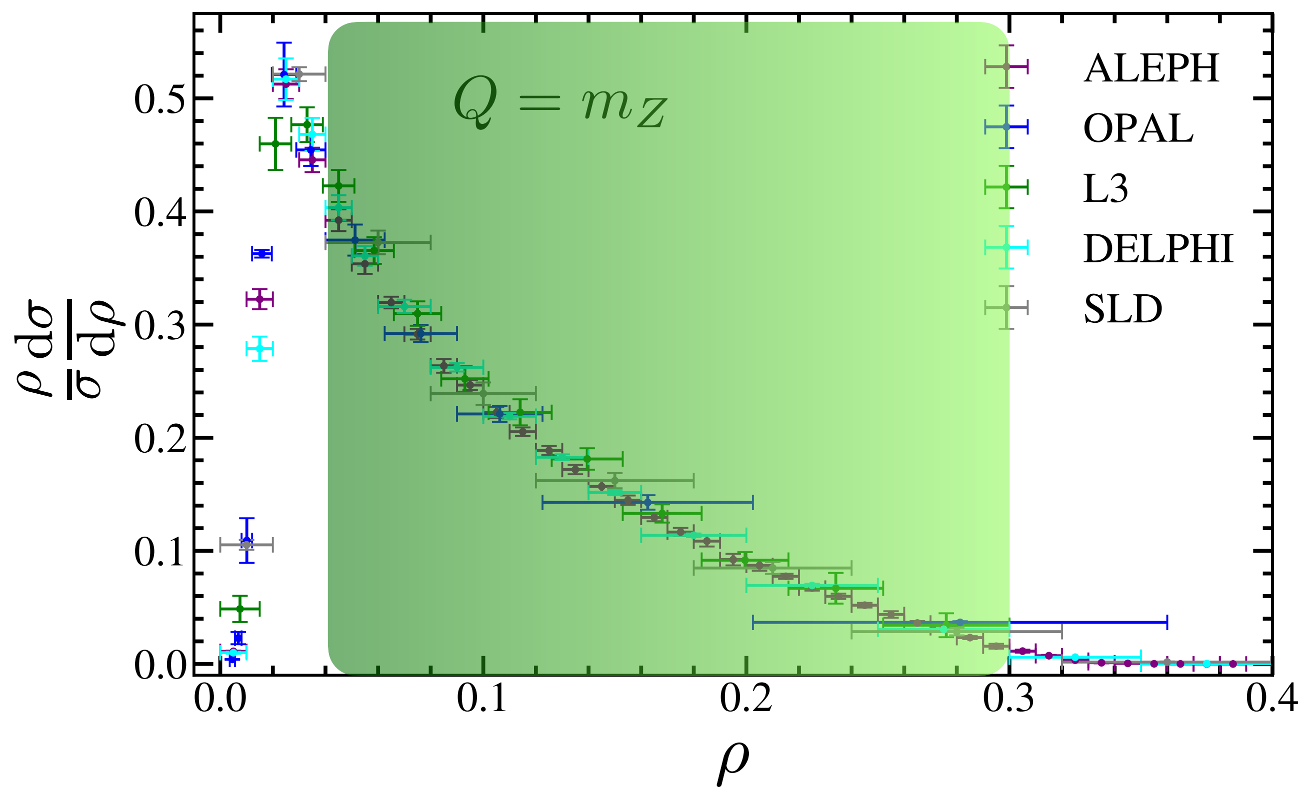
Tail region

$$\Lambda_{\text{QCD}} \ll \frac{s_i}{Q} \ll Q$$

Soft and collinear radiation dominate

clear separation between perturbative and non-perturbative modes

experimental distribution



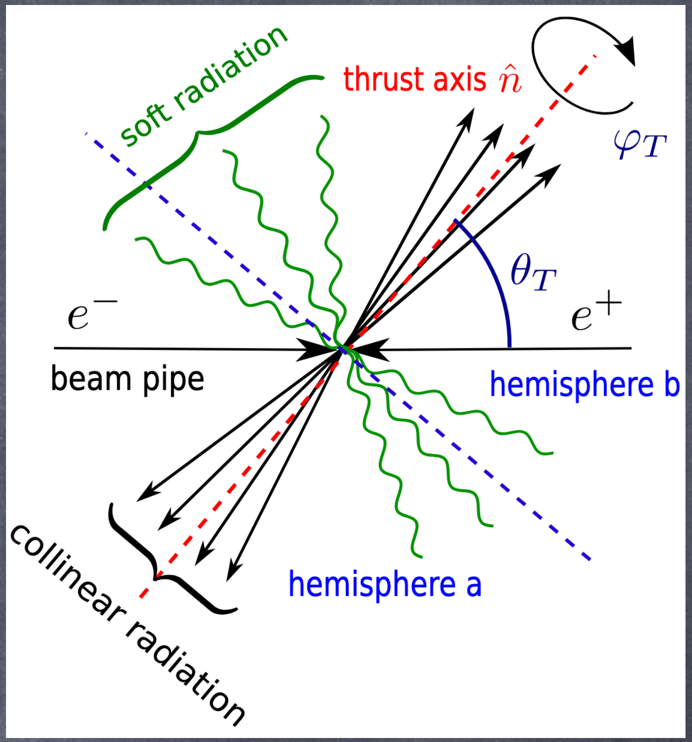
The HJM distribution

Event shapes describe geometric properties of final-state particles

thrust axis defined as vector that maximises $\sum_{i\in\text{final}} |\hat{n}\cdot\vec{p}_i|$

plane normal to $\hat{n}$ defines two hemispheres

Anatomy of dijet event



Tail region

$$\Lambda_{\text{QCD}} \ll \frac{s_i}{Q} \ll Q$$

Soft and collinear radiation dominate

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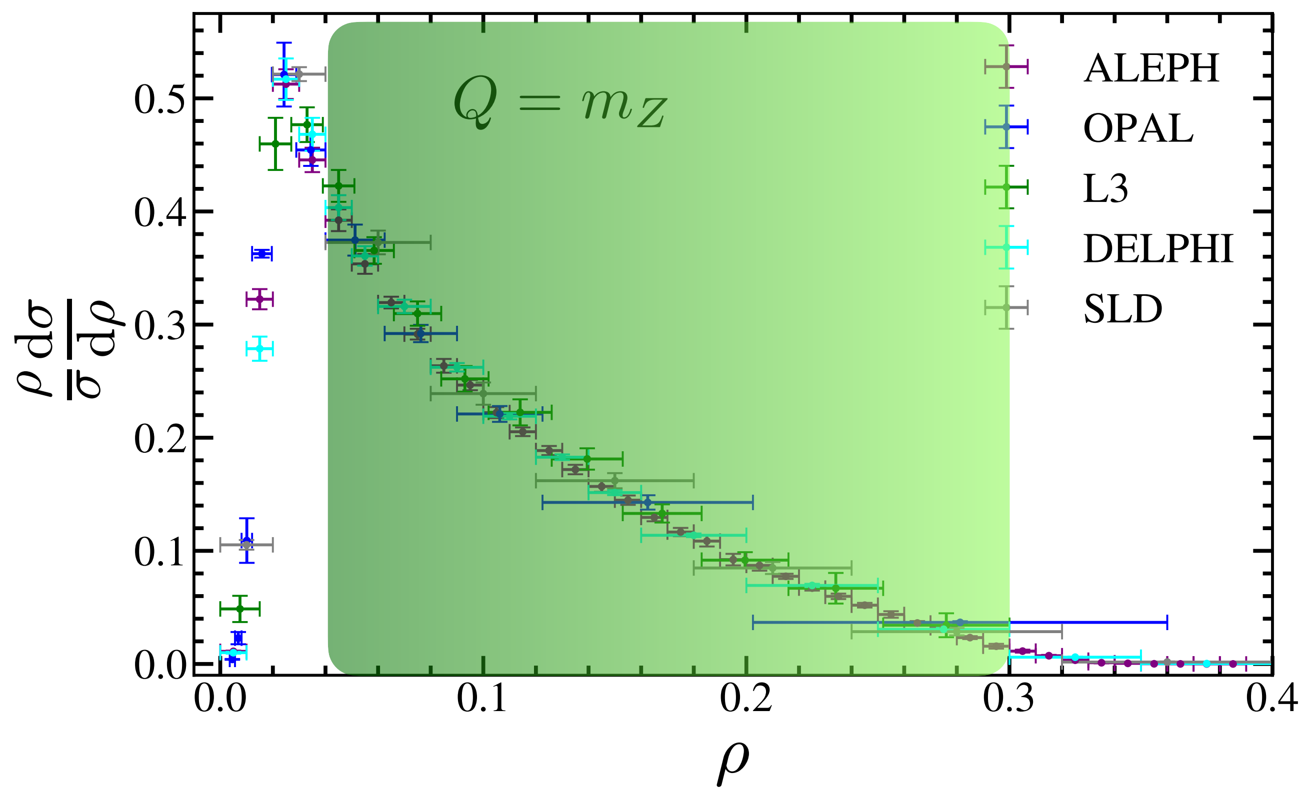

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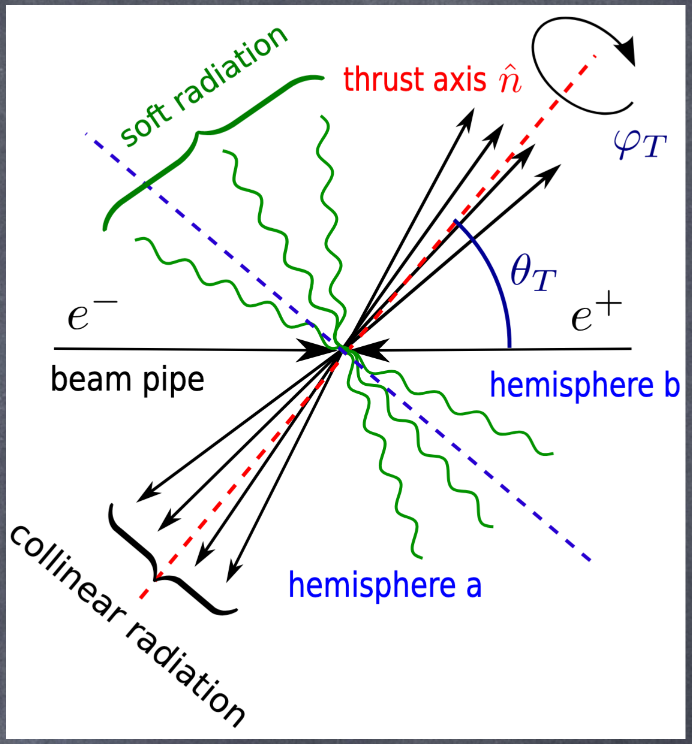

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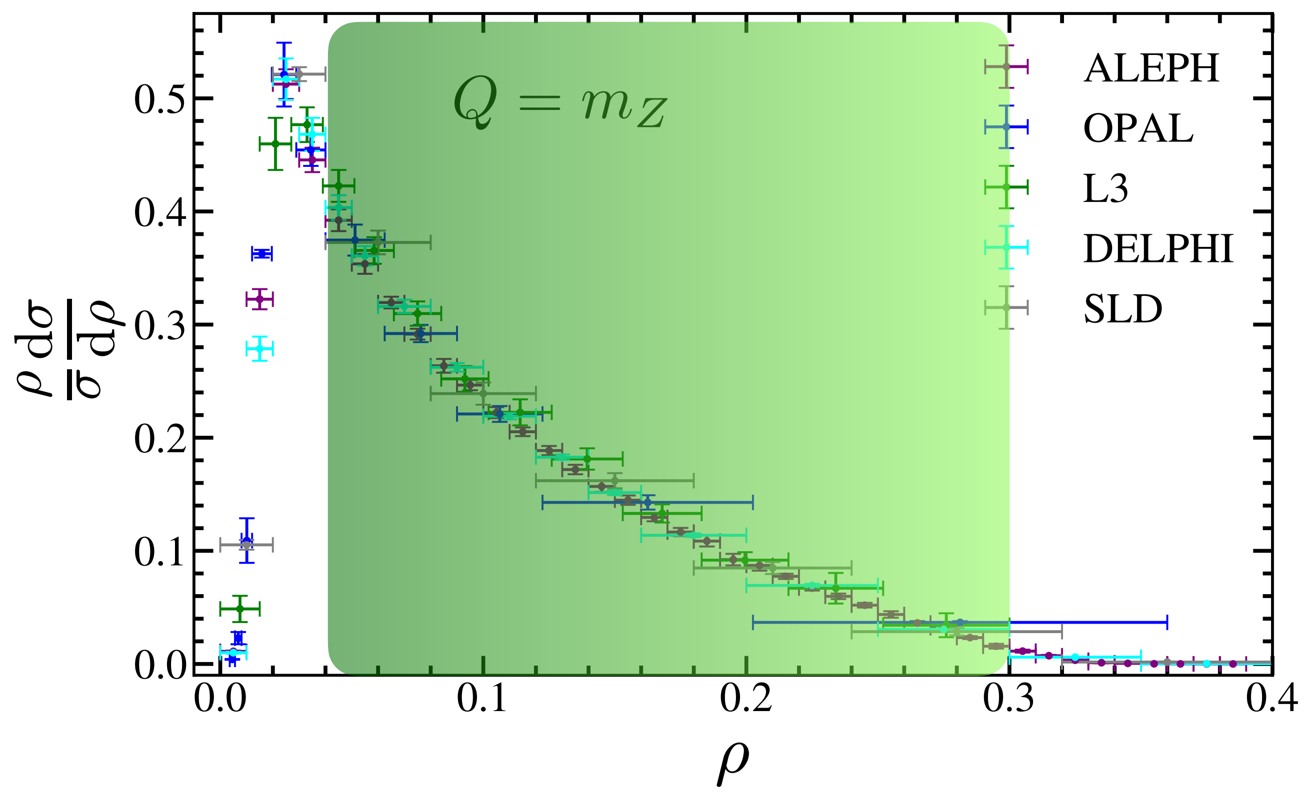

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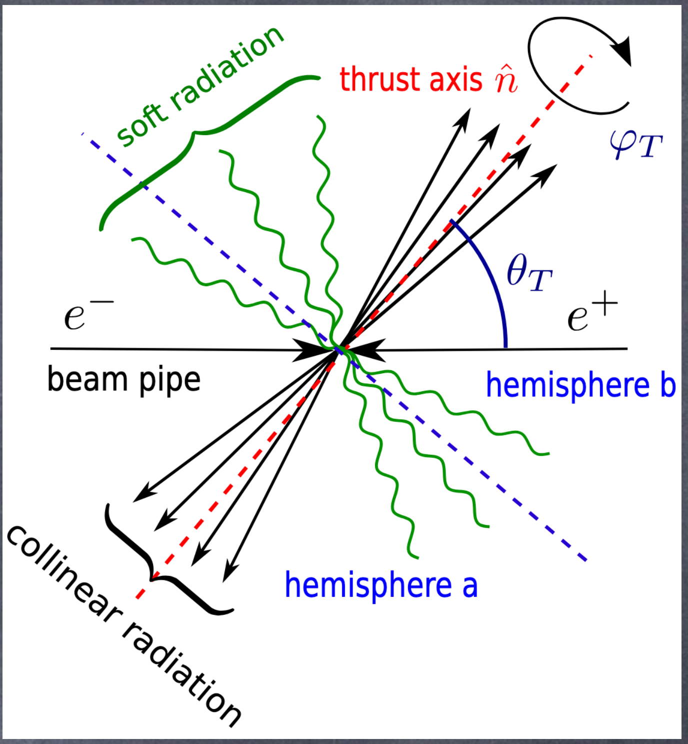


Far-tail region

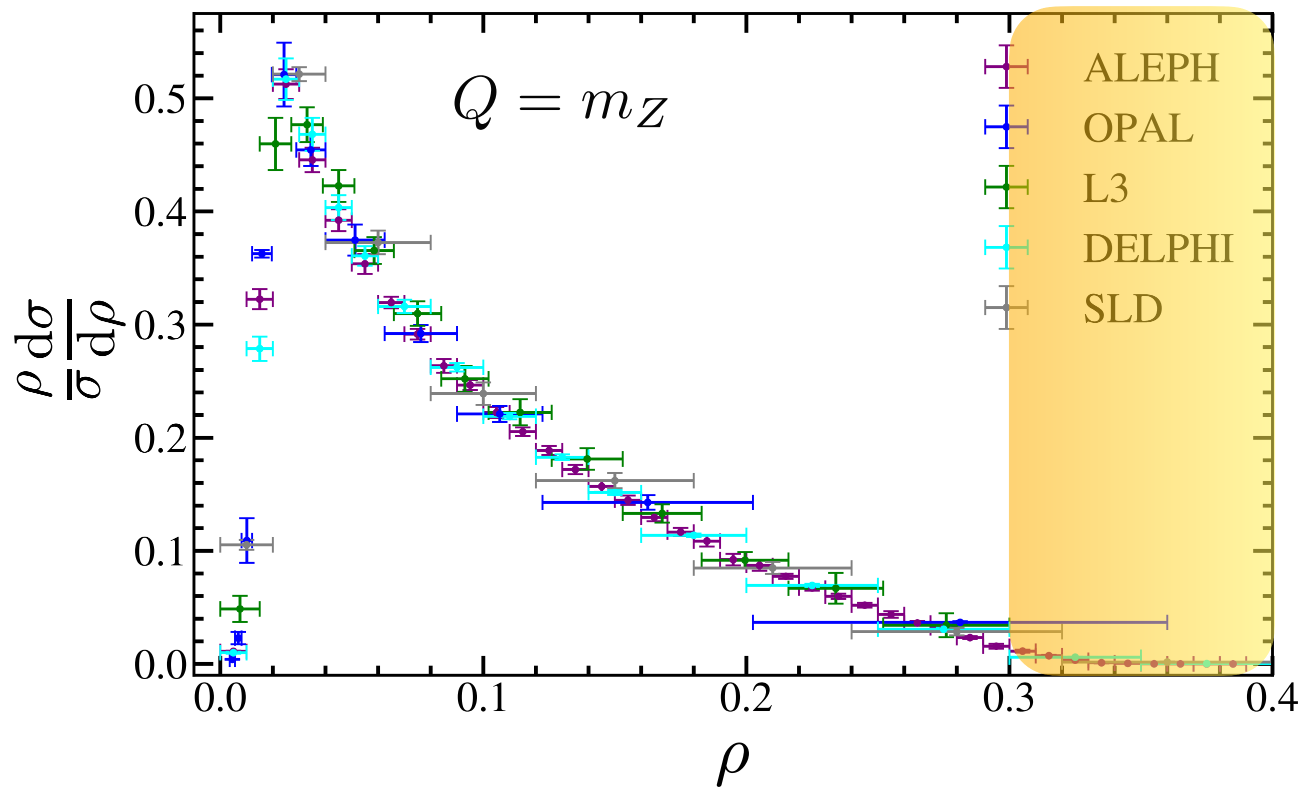
$s_i \sim Q^2$

No clear distinction between three scales

right description: FO

small hadronization corrections

experimental distribution



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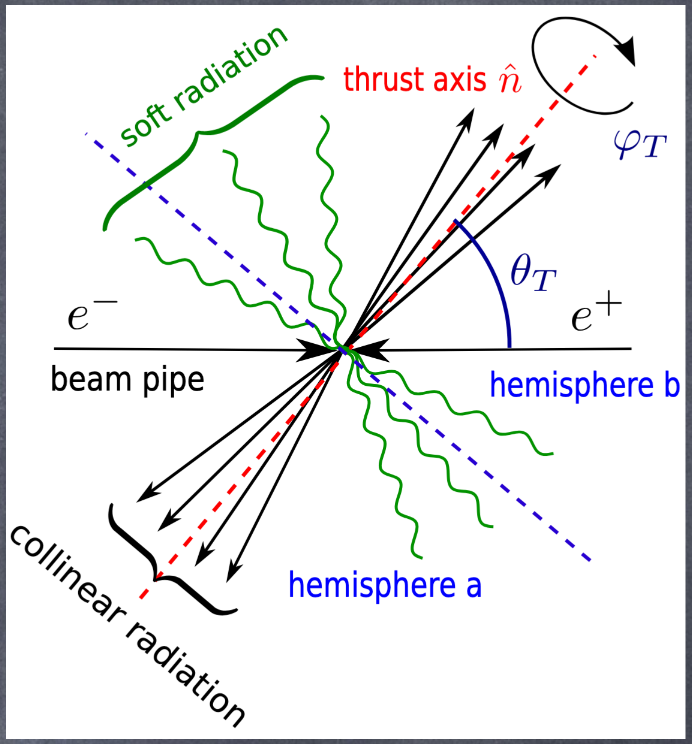

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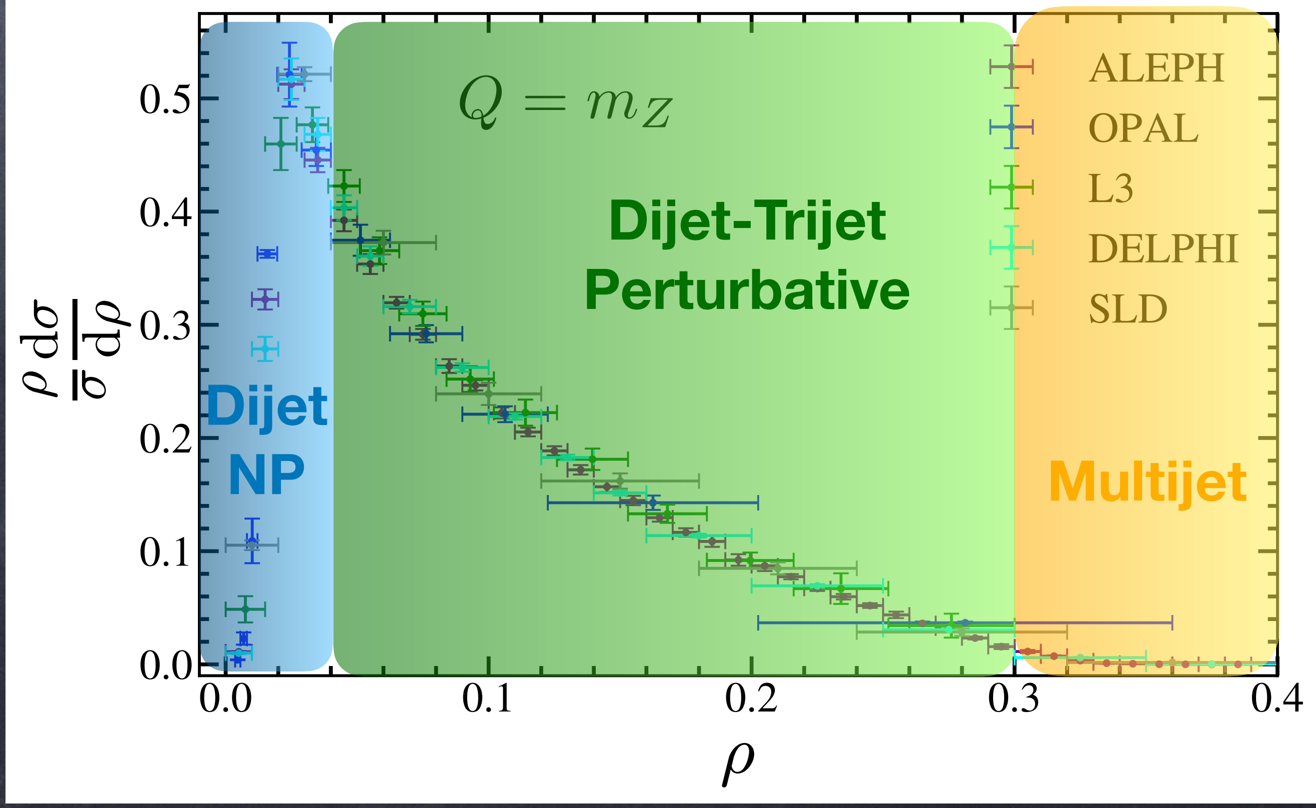

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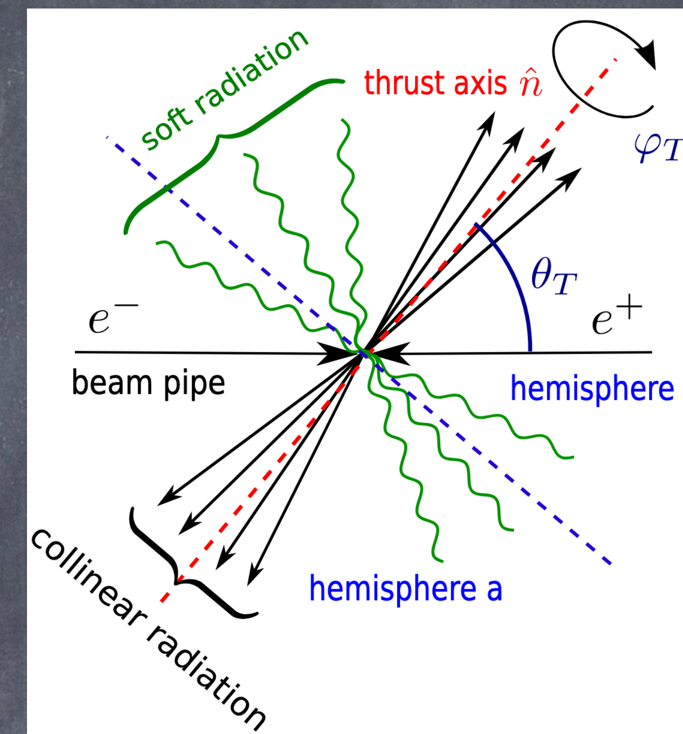

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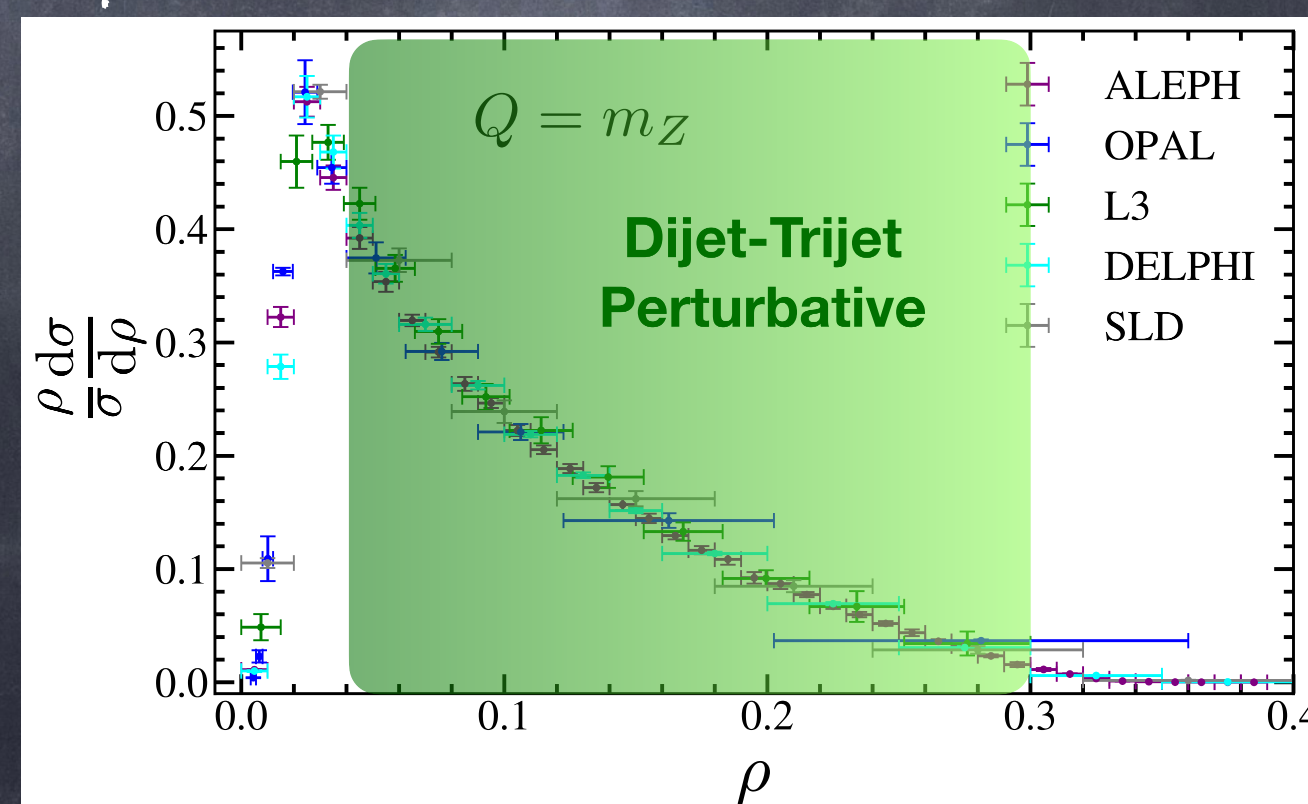

Ideal place for fits

HJM distribution – dijet

Dijet resummation @ N³LL: [A.H. Hoang, VM, I.W. Stewart & M.D. Schwartz, 2025] along with first-principles power corrections

HJM obtained as marginalisation of dihemisphere mass distro

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\rho} = 2Q^2 \int_0^{Q^2\rho} \mathrm{d}s \, \frac{\mathrm{d}^2\sigma}{\mathrm{d}s_1 \mathrm{d}s_2}(s_1 = Q^2\rho, s_2 = s) \equiv 2Q^2 \Xi(Q^2\rho, Q^2\rho)$$

semi-cumulative

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partonic semi-cumulative

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In tail region gives raise to non-trivial OPE in terms of

$$\Omega_{ij} = \Omega_{ji} = \int \mathrm{d}k_1 \, \mathrm{d}k_2 \, k_1^i \, k_2^j \, F(k_1, k_2)$$

Leading power: $\frac{\mathrm{d}\sigma_{\mathrm{dij}}}{\mathrm{d}\rho} = \frac{\mathrm{d}\hat{\sigma}_{\mathrm{dij}}}{\mathrm{d}\rho}\left(\rho - \frac{\Omega_{10}}{Q}\right)$

HJM distribution – dijet factorization

Factorisation of singular partonic distribution is also 2D

$$\frac{1}{\sigma_0}\frac{\mathrm{d}^2\hat{\sigma}_{\mathrm{sing}}}{\mathrm{d}s_1\mathrm{d}s_2} = H(Q,\mu)\int \mathrm{d}\ell_1\mathrm{d}\ell_2\, J(s_1 - Q\,\ell_1 - Q\,\bar{\Delta}(R,\mu),\mu)$$

$$\times\, J(s_2 - Q\,\ell_2 - Q\,\bar{\Delta}(R,\mu),\mu)\, e^{\delta(R,\mu)\left(\frac{\partial}{\partial\ell_1}+\frac{\partial}{\partial\ell_2}\right)}\,\widehat{S}(\ell_1,\ell_2,\mu)$$

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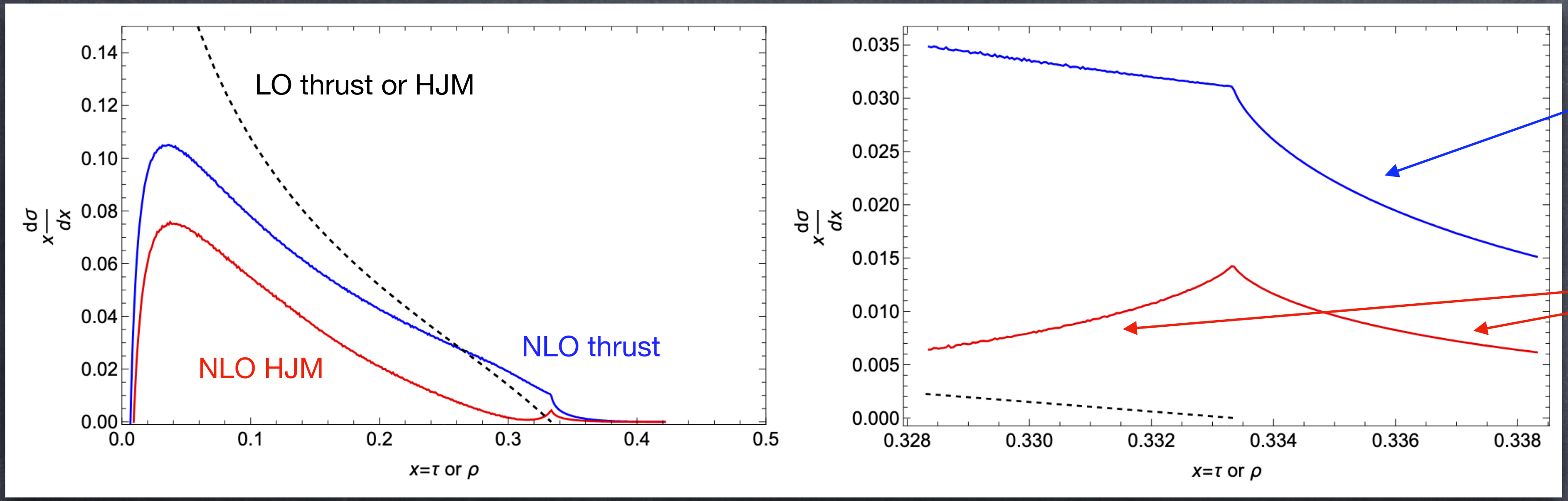
RGE between different matrix elements sums up large logs

Non-global logs in soft function makes this non-trivial

HJM distribution – dijet + shoulder resummation



[Bhattacharya, Schwartz, Zhang 2022]

Thrust has right shoulder only

HJM has left and right shoulders

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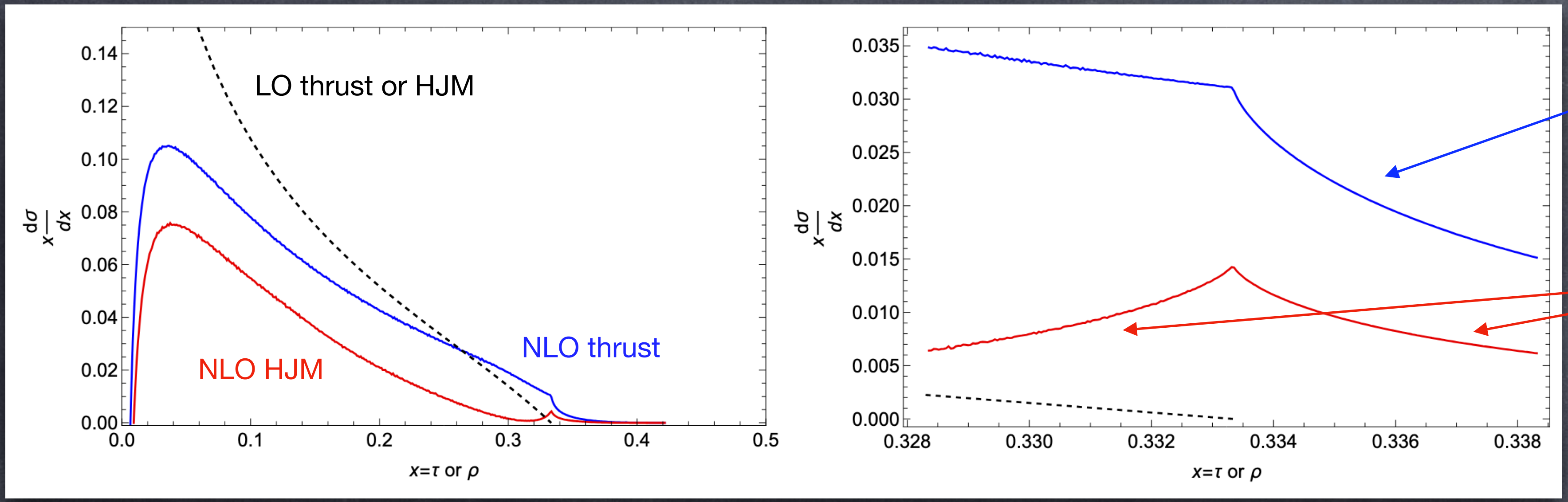

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Around symmetric trijet limit $\rho \to 1/3$, distribution factorizes as

$$\mathrm{d}\hat{\sigma}_{\mathrm{sh}} = H_{\mathrm{sh}} \times J_1 \otimes J_2 \otimes J_3 \otimes S_{1,2,3}$$

[Bhattacharya, Michel, Schwartz, Stewart, Zhang 2023]

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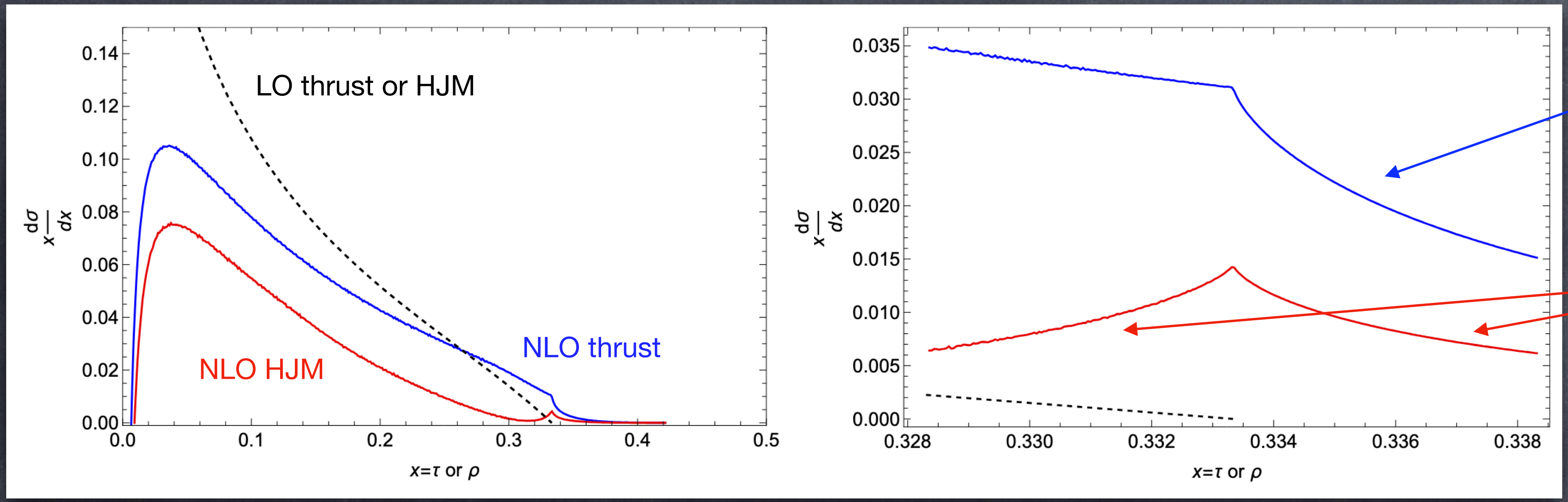

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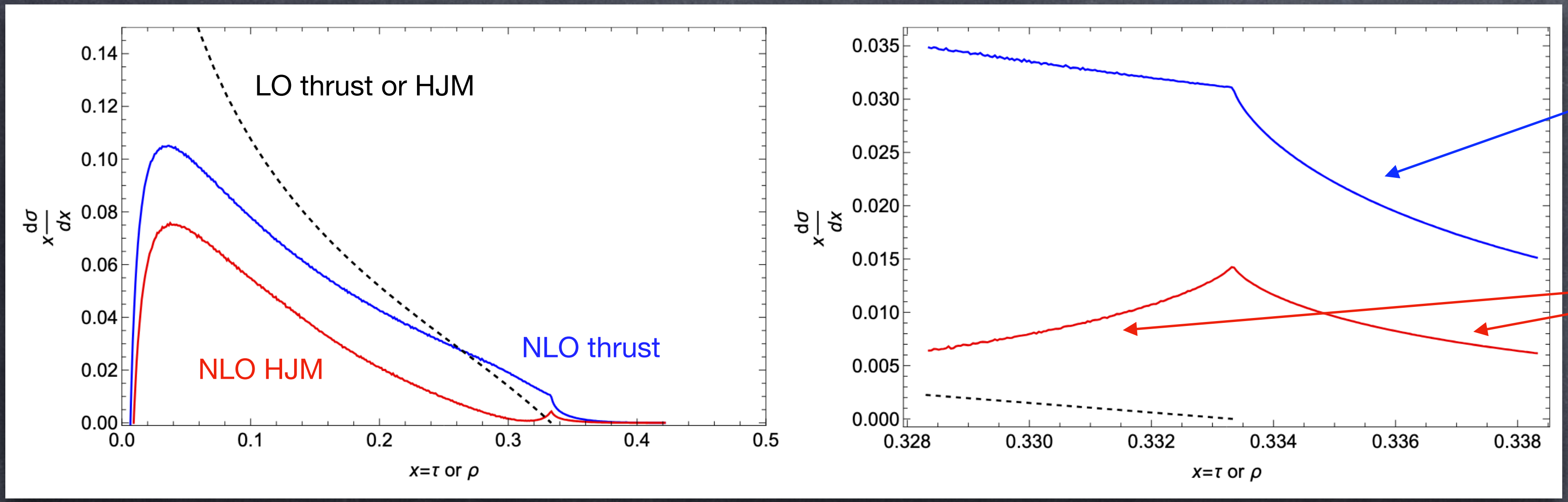
Matching between dijet, FO & shoulder writing full cross section as

$$\mathrm{d}\hat{\sigma} = \left[\mathrm{d}\hat{\sigma}_{\mathrm{dij}} - \mathrm{d}\hat{\sigma}_{\mathrm{dij}}^{\mathrm{sing}}\right] + \left[\mathrm{d}\hat{\sigma}_{\mathrm{sh}} - \mathrm{d}\hat{\sigma}_{\mathrm{sh}}^{\mathrm{sing}}\right] + \mathrm{d}\sigma_{\mathrm{FO}}$$

HJM distribution - dijet + shoulder resummation



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Parametrize 3-jet power corrections as

$$\frac{\mathrm{d}\sigma_{\mathrm{sh}}}{\mathrm{d}\rho} = \frac{\mathrm{d}\hat{\sigma}_{\mathrm{sh}}}{\mathrm{d}\rho}\left(\rho - \frac{\Theta_1}{Q}\right)$$

Fit procedure

Use χ^2 function including theoretical and experimental uncertainties

Experiment

35 GeV < Q < 207 GeV
(700 datapoints)

Minimal Overlap Model for systematic uncertainties

$$\sigma_{ij}^{\rm exp} = \delta_{ij}(\Delta_i^{\rm stat})^2 + \delta_{D_i D_j}\min(\Delta_i^{\rm sys}, \Delta_j^{\rm sys})^2$$

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Theory

pert. uncertainties assessed with renormalization scale variation
not Gaussian + highly correlated

flat random scan: 5000 sets of 17 parameters, yielding theory & uncertainty for each data-point x_i

$$\bar{x}_i = \frac{x_i^{\rm max} + x_i^{\rm min}}{2} \qquad \Delta_i^{\rm theo} = \frac{x_i^{\rm max} - x_i^{\rm min}}{2}$$

correlation $$r_{ij}^{\rm theo} = \frac{\langle (x_i - \bar{x}_i)(x_j - \bar{x}_j)\rangle}{\sqrt{\langle (x_i-\bar{x}_i)^2\rangle}\sqrt{\langle (x_j-\bar{x}_j)^2\rangle}}$$

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total covariance matrix

$$\sigma_{ij}^{\rm tot} = \sigma_{ij}^{\rm exp} + \Delta_i^{\rm theo}\,\Delta_j^{\rm theo}\,r_{ij}^{\rm theo}$$

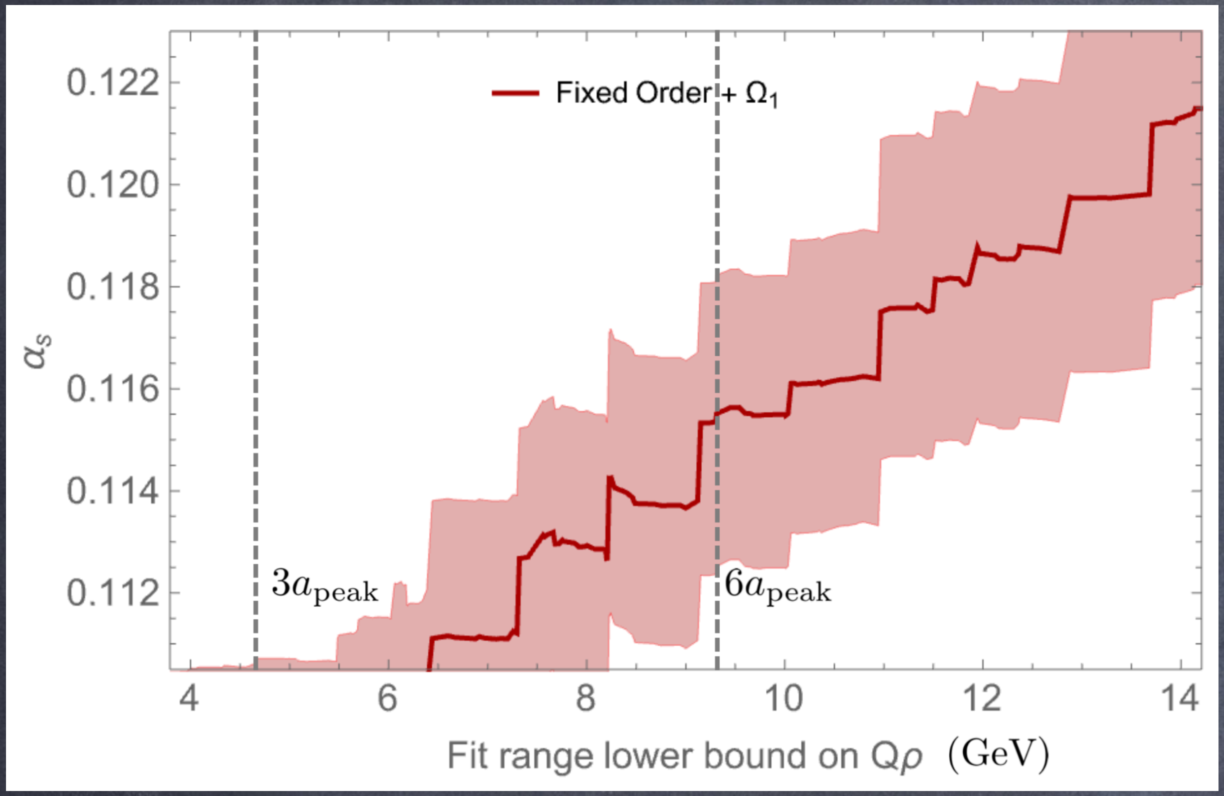
$\longrightarrow$

$$\chi^2 = \sum_{i,j=1}^{N_{\rm bins}} (\bar{x}_i - x_i^{\rm exp})\,(\bar{x}_j - x_j^{\rm exp})\,(\sigma_{\rm tot}^{-1})_{ij}$$

Fit results – Fixed Order

Results for α_s using fit range $\frac{a}{Q} < \rho < 0.3$ for different a

$a_{\text{peak}} \equiv$ peak position



high-sensitivity to fit range

large fit uncertainty

pick whichever value you like!

Model	$\alpha_s(m_Z)$	th+exp	Ω_1^ρ	Θ_1	fit range	χ^2/dof	Ω_1^ρ [GeV]	Θ_1 [GeV]
Fixed Order 2D	0.1166 ± 0.0034	± 0.0014	± 0.0027	–	± 0.0015	1.108	0.06 ± 0.13	–

Fit results – dijet resummation

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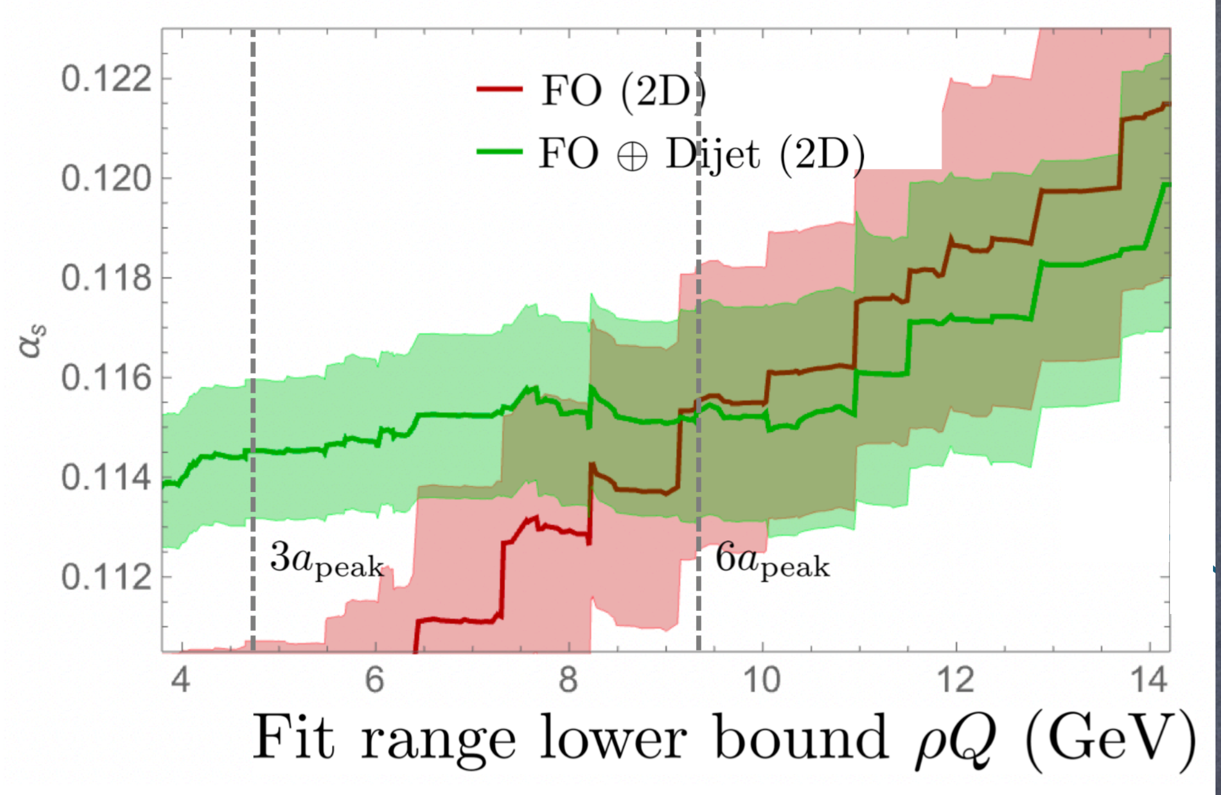

low-sensitivity to fit range

small fit uncertainty

$a \in [3a_{\mathrm{peak}}, 6a_{\mathrm{peak}}]$

data prefers $\Omega_{10} > 0$

as expected by theory

Model	$\alpha_s(m_Z)$	th+exp	Ω_1^ρ	Θ_1	fit range	χ^2/dof	Ω_1^ρ [GeV]	Θ_1 [GeV]
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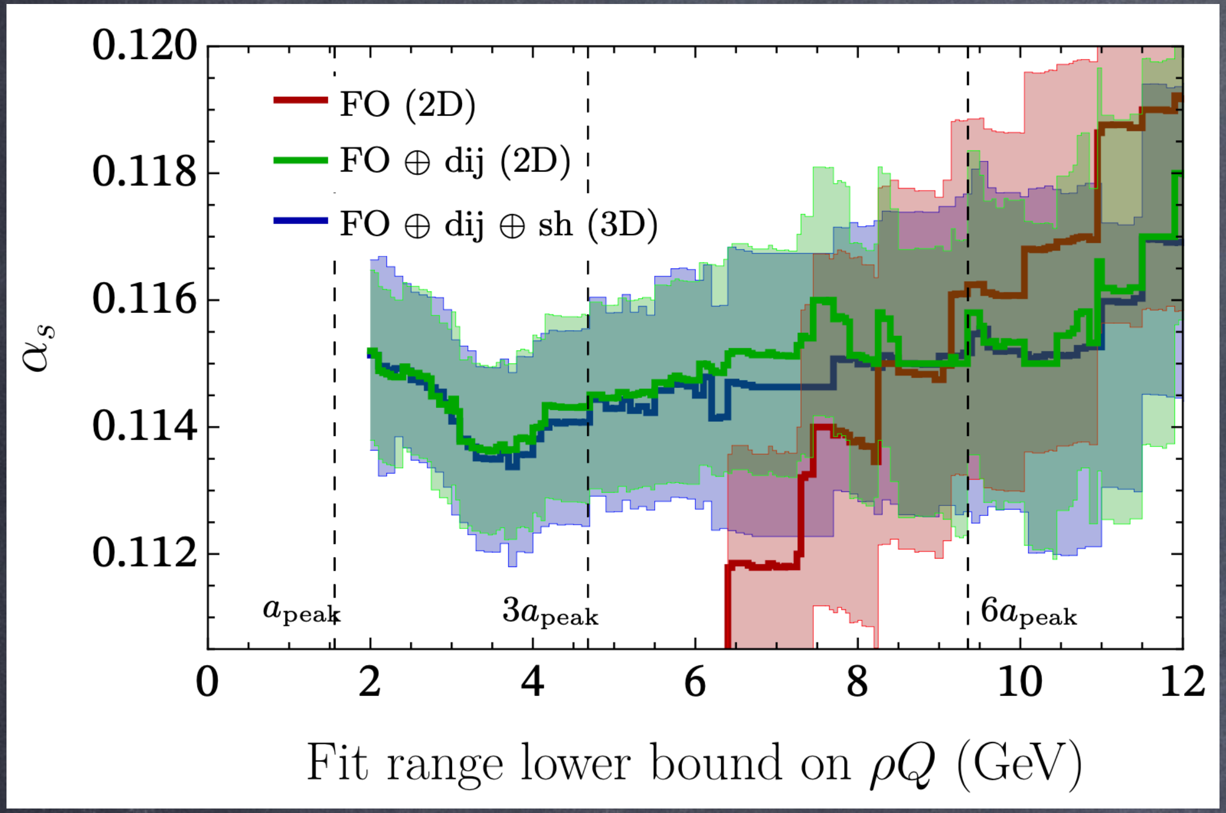

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FO + dijet 3D	0.1156 ± 0.0024	± 0.0010	± 0.0021	± 0.0004	± 0.0007	1.052	0.52 ± 0.08	0.53 ± 0.13
FO + dijet + shoulder 3D	0.1145 ± 0.0020	± 0.0009	± 0.0018	± 0.0001	± 0.0003	1.043	0.57 ± 0.09	-0.50 ± 0.17

Fit results – dijet + shoulder resummation

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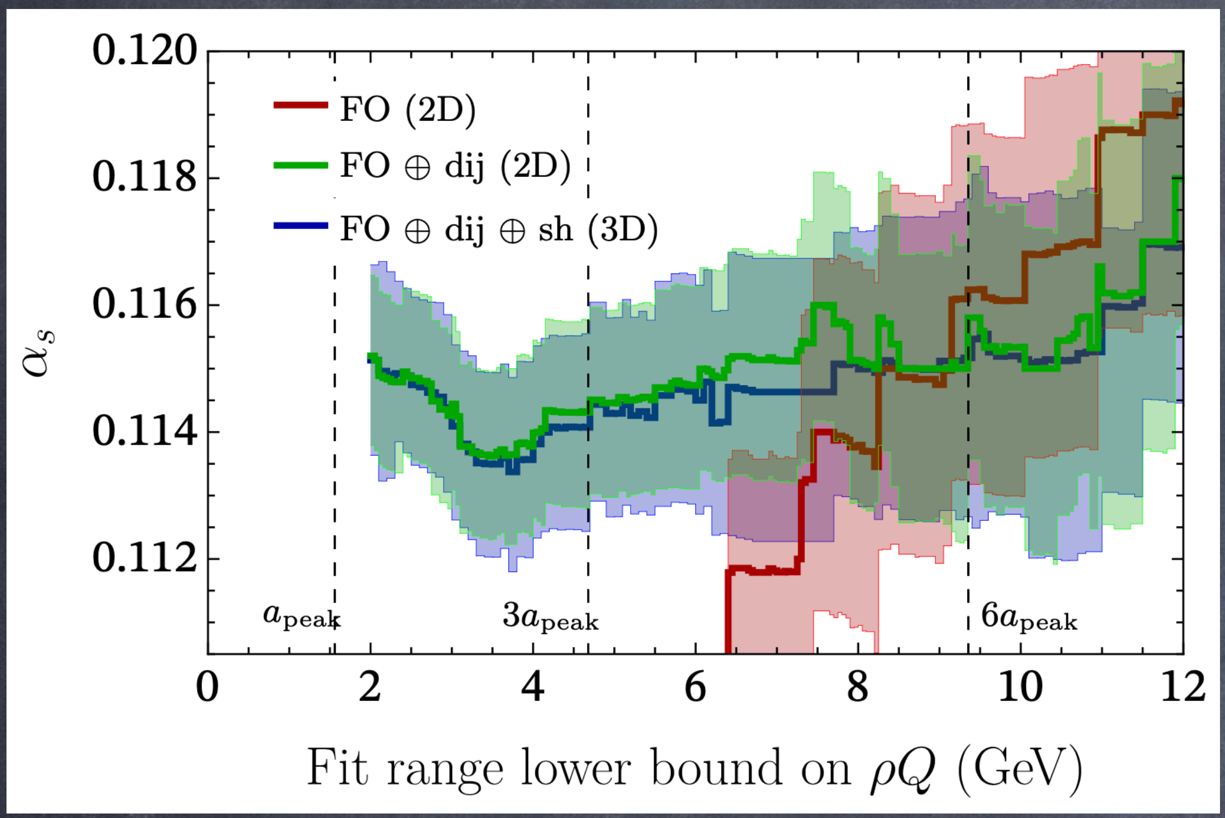

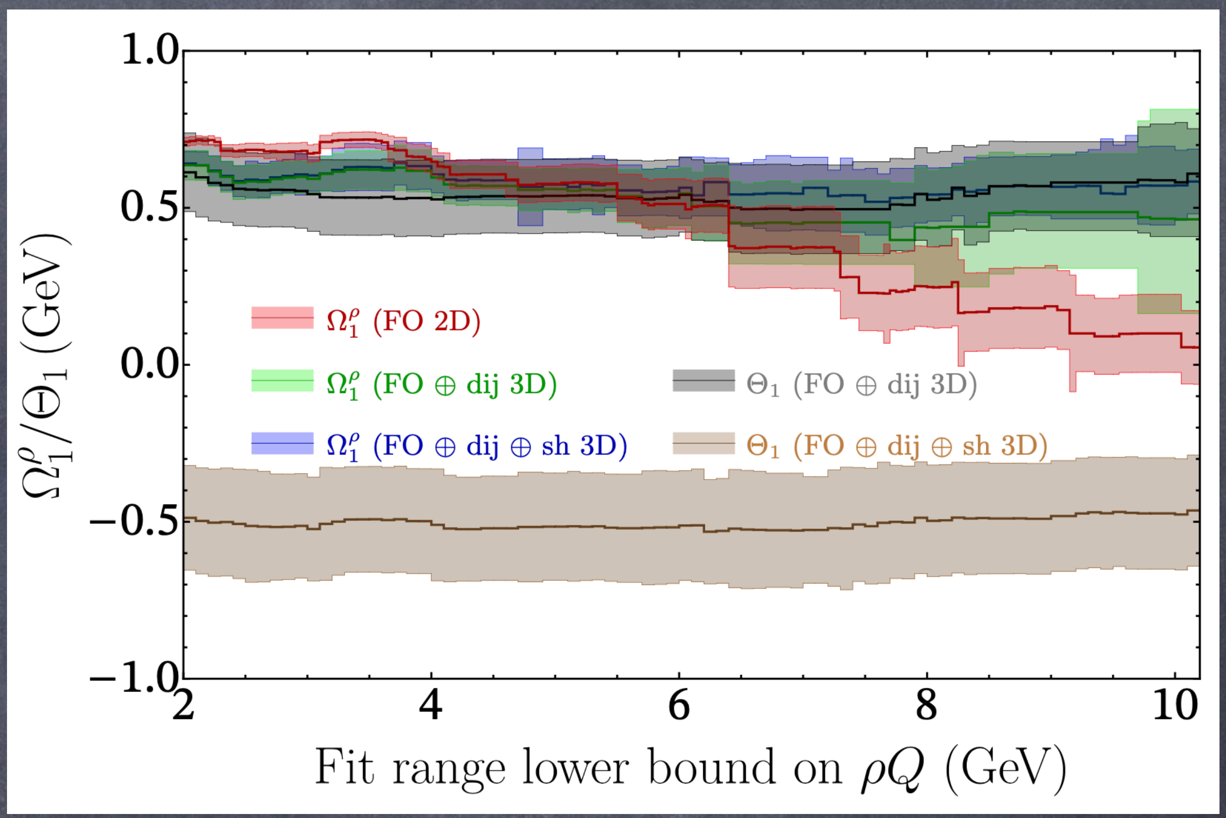

negative 3-jet power correction only with shoulder resummation!

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as expected by theory

Summary

- Innovations of our analysis
  - Improved dijet/OPE and trijet/shoulder region
  - Inclusion of theory correlations in fits
- Dijet resummation crucial for robust results
- Shoulder resummation crucial for $\Theta_1 < 0$

final results $\chi^2/\mathrm{dof} = 1.04$

$\alpha_s(m_Z) = 0.1145^{+0.0021}_{-0.0019}$ $\Omega_{10} = 0.57 \pm 0.09\,\mathrm{GeV}$ $\Theta_1 = -0.50 \pm 0.17\,\mathrm{GeV}$

compatible with thrust and C-parameter

comparison to data