Precise determination of the strong coupling from hadronic τ decays including $\tau \to \pi^-\pi^0\nu_\tau$ from Belle

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With: A. Eiben, M. Golterman, K. Maltman, L. Mansur, and S. Peris

2502.08147 DB, Eiben, Golterman, Maltman, Mansur, and Peris, *Phys. Rev. D* 111 (2025) 7, 074010
2012.10440 DB, Golterman, Maltman, Peris, Rodrigues, and Schaaf, *Phys. Rev. D* 103 (2021) 3, 034028

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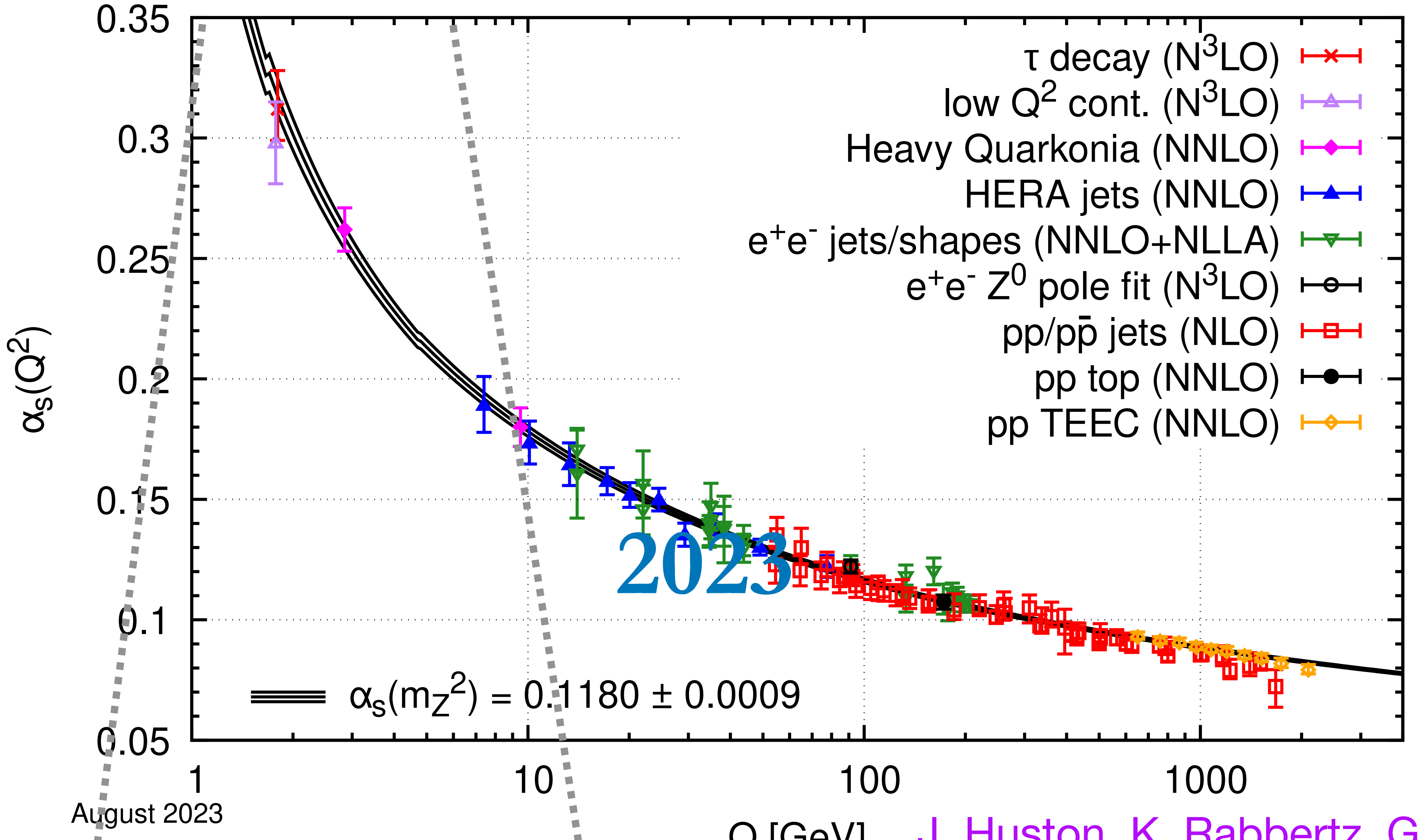
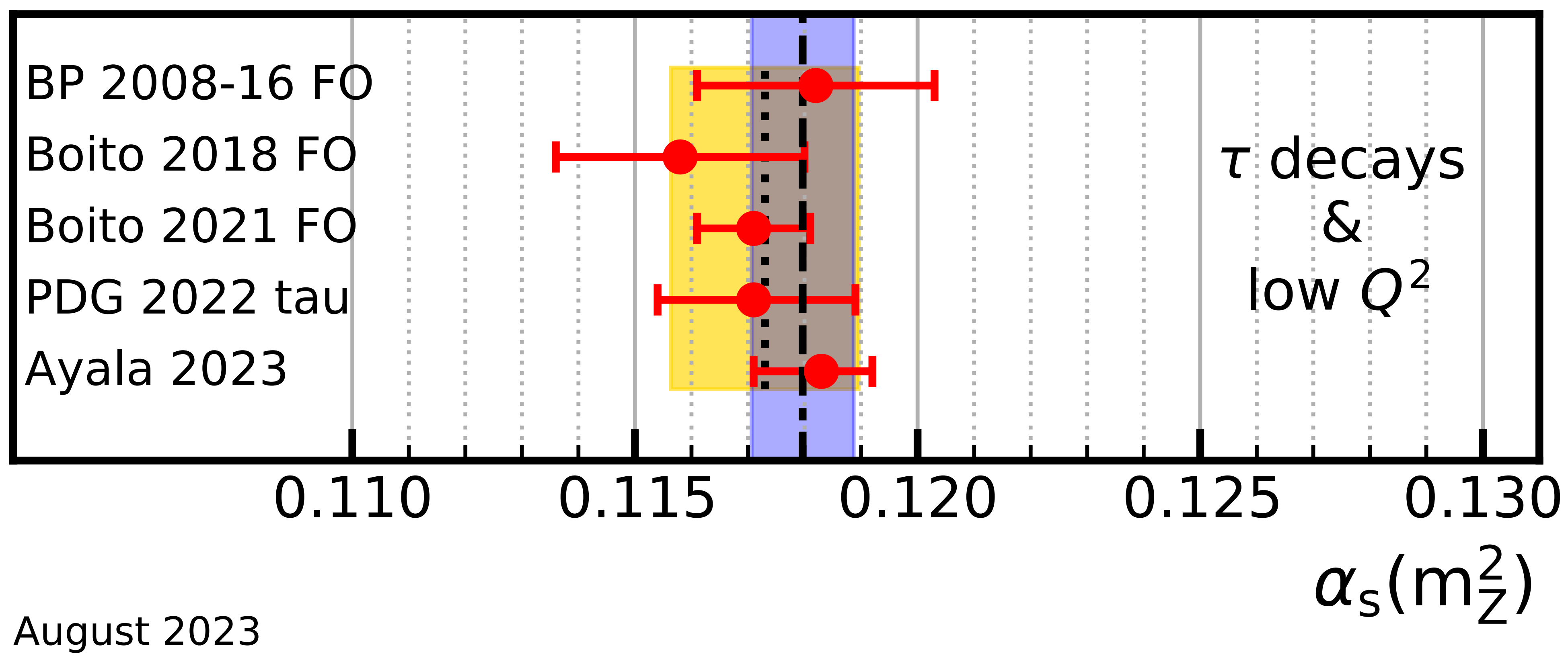
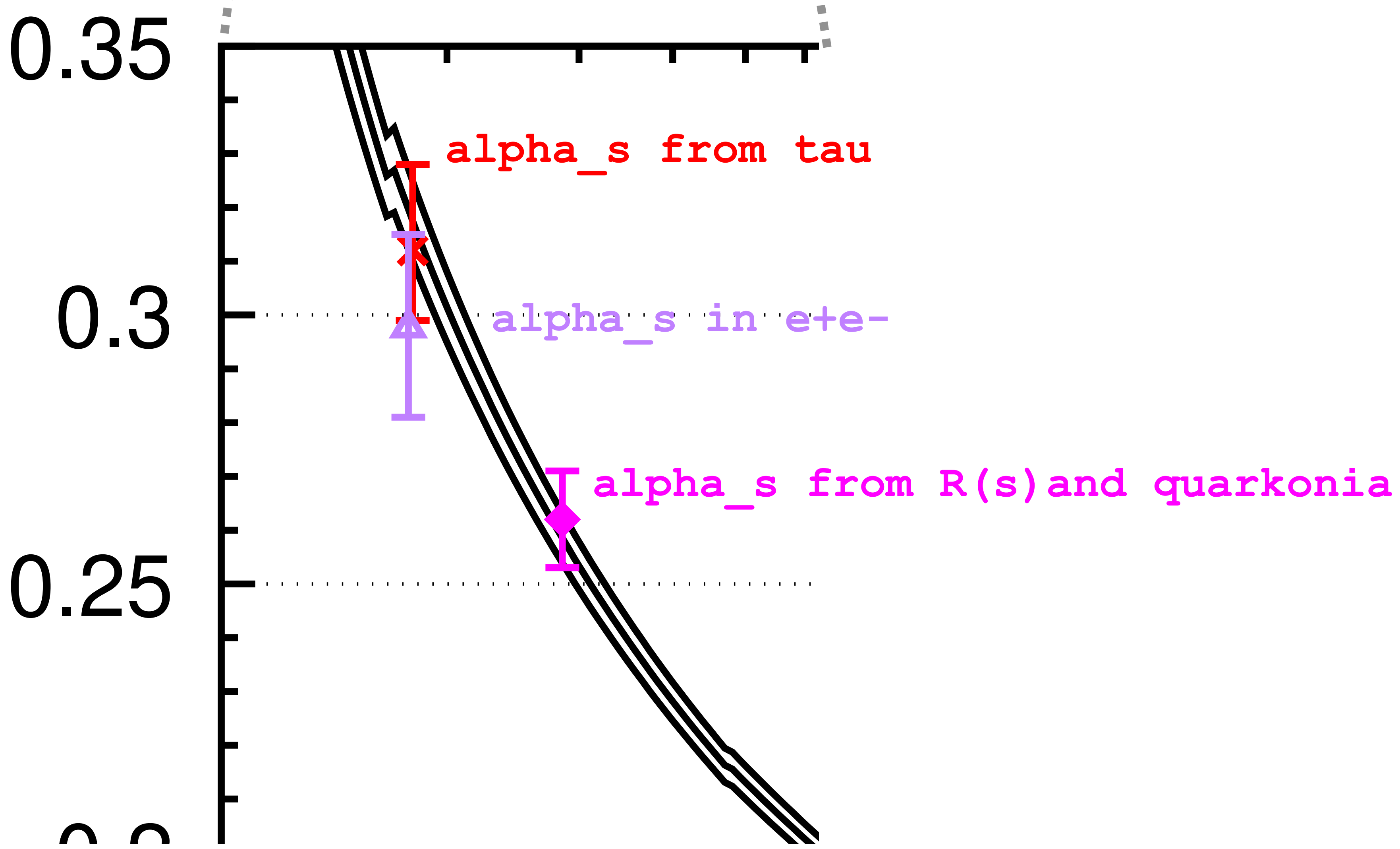
Strong coupling from hadronic tau decays



J. Huston, K. Rabbertz, G. Zanderighi, Particle Data Group, (2023)





- Important test of asymptotic freedom
- Larger coupling at lower energies: more sensitivity to QCD corrections, less precision required from experimental data

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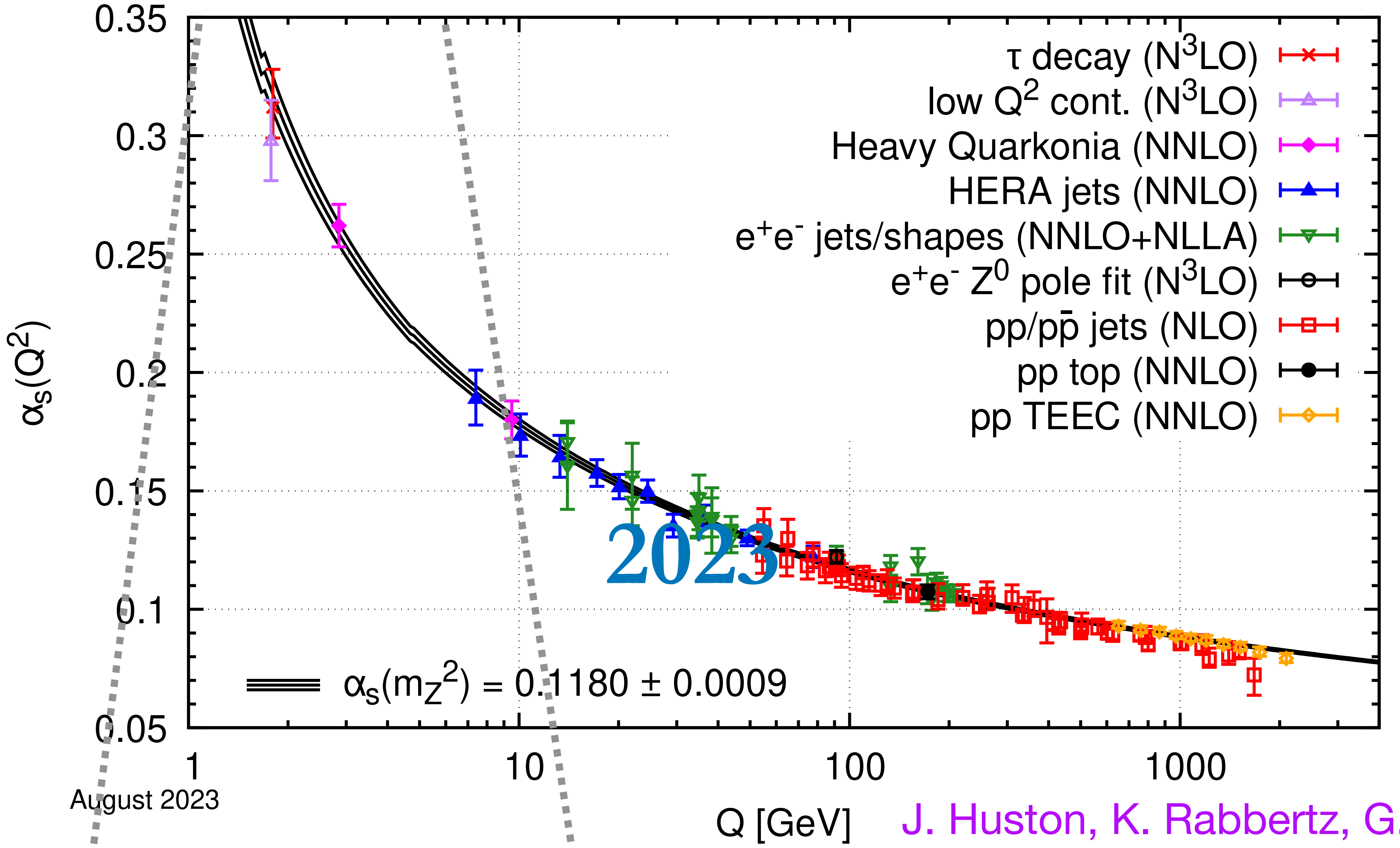

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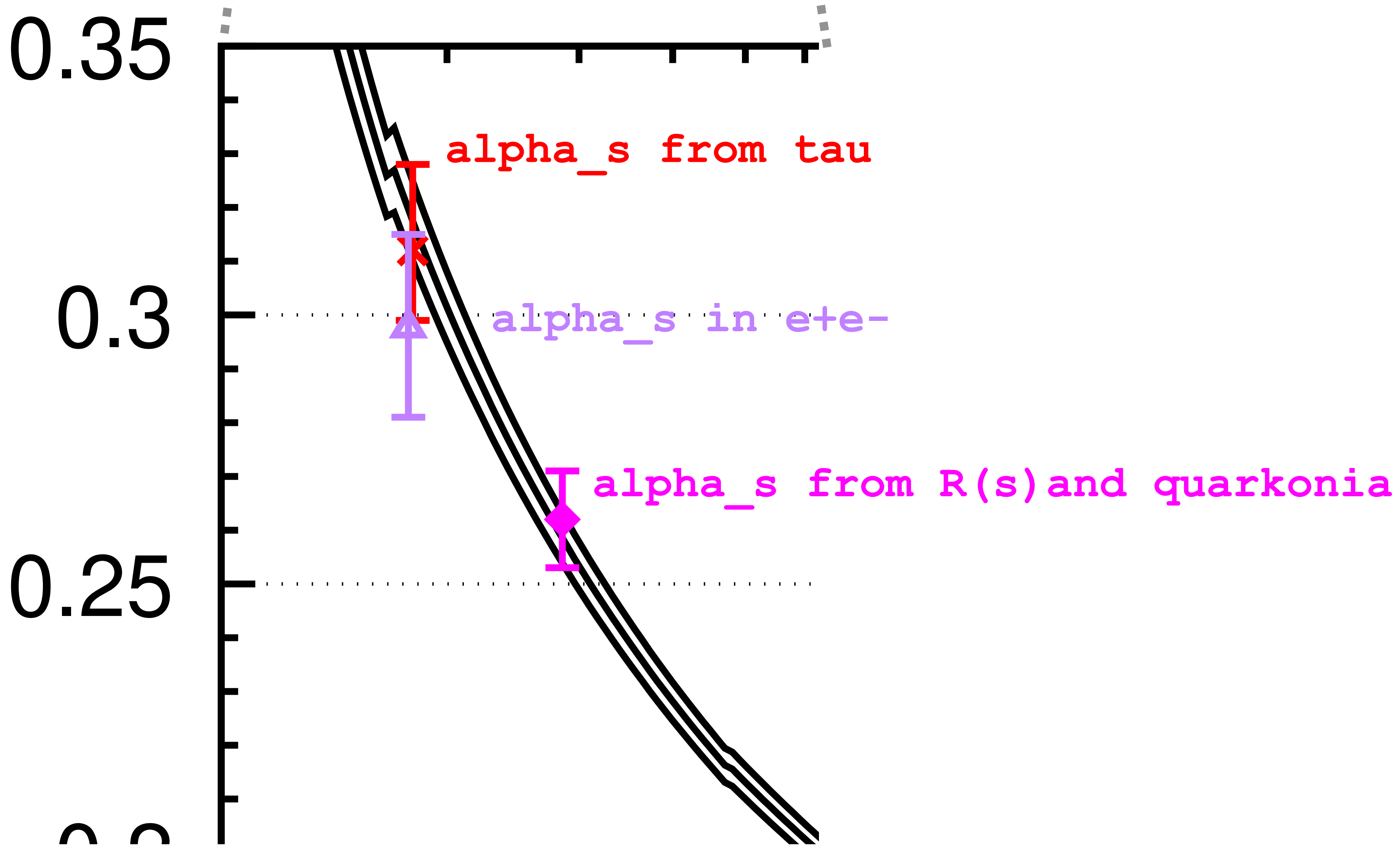
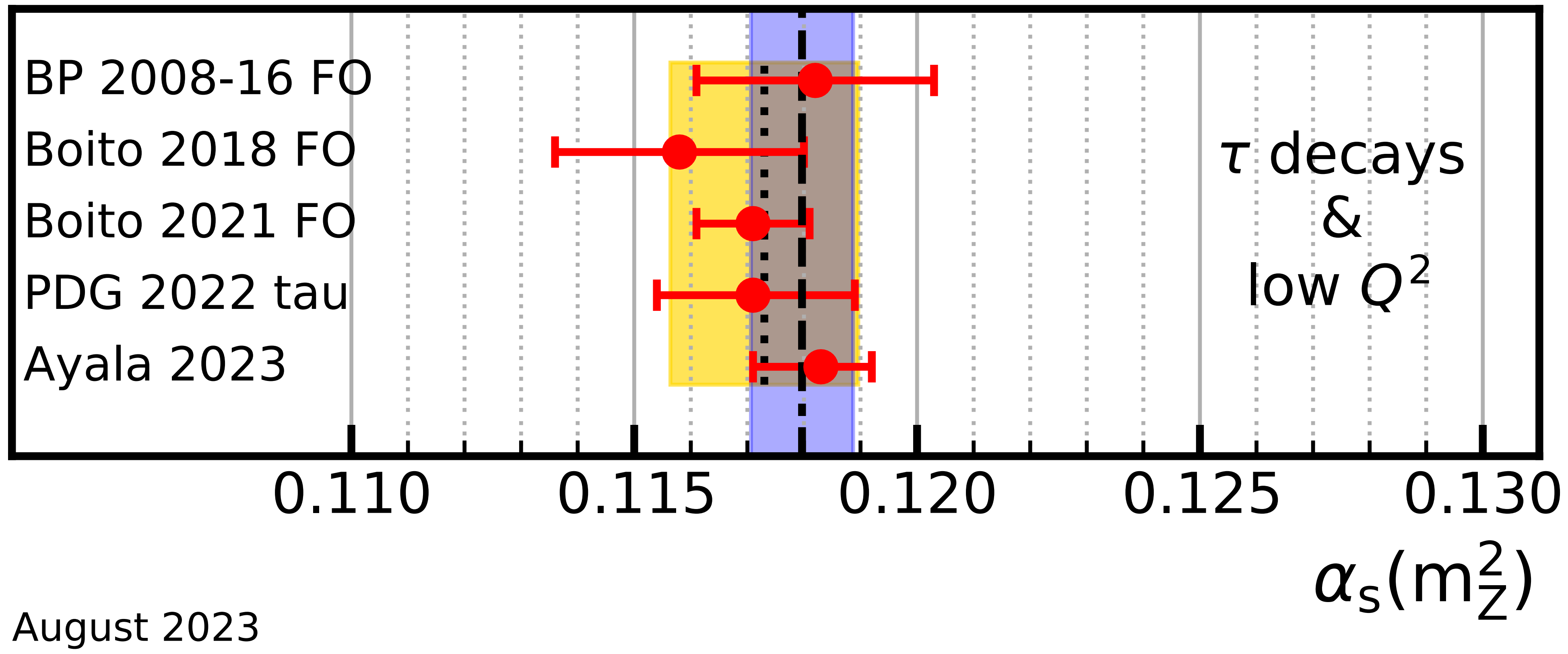
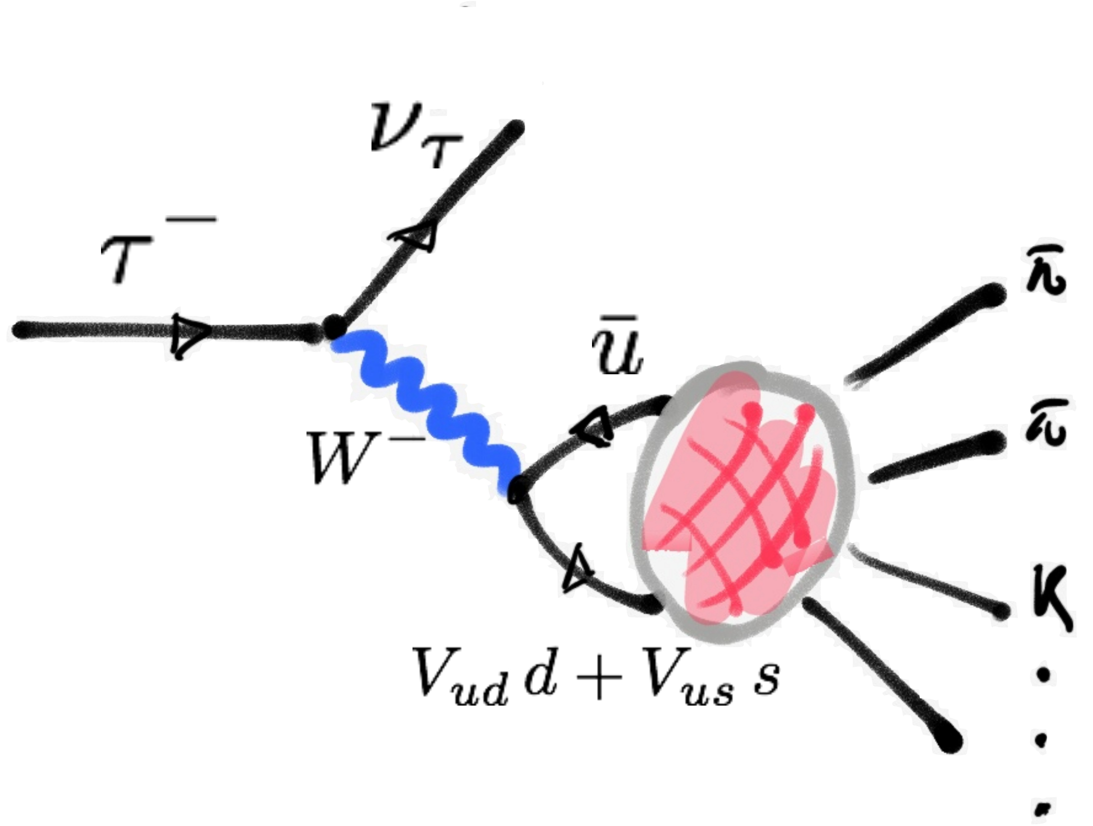




- Important test of asymptotic freedom
- Larger coupling at lower energies: more sensitivity to QCD corrections, less precision required from experimental data

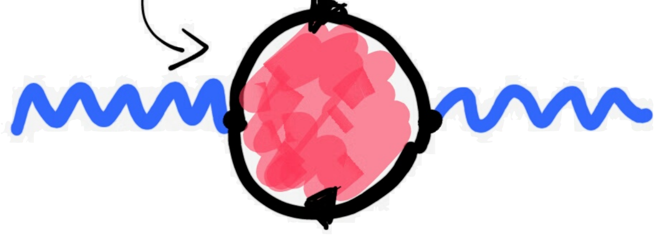
- Larger non-perturbative contributions (OPE, Duality Violations)
- Potential convergence issues in perturbation theory are enhanced (renormalons)

Strong coupling from hadronic tau decays



Massless (V & A) correlators

$$\Pi_{\mu\nu}(q) = i \int d^4x\, e^{iqx} \langle 0|T\{J_\mu(x) J_\nu(0)^\dagger\}|\rangle$$



Braaten, Narison, and Pich '92

Sum rules (using Cauchy's theorem)

$$\frac{1}{s_0}\int_0^{s_0} ds\, w(s)\,\underbrace{\frac{1}{\pi}\mathrm{Im}\Pi(s)}_{\text{experiment}} = -\frac{1}{2\pi i s_0}\oint_{|z|=s_0} dz\, \underbrace{w(z)\Pi(z)}_{\text{theory}}$$

- Here we focus on the light-quark current. Tiny mass corrections can then be neglected, m_u, $m_d \sim 0$. No net strangeness in final states.
- We extract the coupling from the vector-isovector channel (to be discussed in the next slides)
- Experimental results should be fully inclusive

Theory overview

$$\frac{-1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\,\Pi(z) = \frac{1}{4\pi^2} S_{\rm EW}\left(\delta_w^{\rm tree} + \boxed{\delta_w^{(0)}} + \delta_{{\rm OPE},w}^{D\geq 4} + \delta_{\rm DVs}^{(w)} + \delta_{\rm EW}^{(w)}\right)$$

theory

Perturbation theory

$$\delta_w^{(0)} = \sum_{n=1}^{\infty}\left(\frac{\alpha_s(\mu^2)}{\pi}\right)^n \sum_{k=1}^{n} k\, c_{n,k} \frac{1}{2\pi i}\oint_{|x|=1} \frac{dx}{x} W(x) \ln^{k-1}(-s_0 x/\mu^2)$$

$W(x) = 2\int_x^1 dz\, w(z)$

$$R_\tau = \frac{\Gamma(\tau\to({\rm had})+\nu_\tau)}{\Gamma(\tau\to e\bar\nu_e\nu_\tau)} = 3.634(8)$$

Banerjee et al. (HFLAV), 2411.18639

$$\approx N_c(1+\delta_{w_\tau}^{(0)}) = 3(1+0.2) \sim 3.6$$

Perturbative series for the decay width

Gorishnii, Kataev, Larin '91
Surguladze&Samuel '91

Baikov, Chetyrkin, Kühn '08

$$\begin{array}{ccccc} & \alpha_s^1 & \alpha_s^2 & \alpha_s^3 & \alpha_s^4 \\ \delta_{w_\tau}^{(0)} = & 0.1001 + & 0.0521 + & 0.0264 + & 0.0127 = 0.1914 \end{array}$$

pQCD correction is ~20%

(fixed order perturbation theory for R_τ, $\alpha_s(m_\tau^2) = 0.3144$)

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$$c_{5,1} = 283 \pm 140$$

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Duality Violations

$$\rho_{\rm DV}(s) = e^{-\delta-\gamma s}\sin(\alpha+\beta s)$$

Ansatz based on widely accepted assumptions about QCD: asymptotic Regge behavior and large-N_c. Main expected corrections: logarithmic and powers of *1/s*.

DB, Caprini, Golterman, Maltman, Peris, PRD '18

Theory: Fixed Order vs Contour Improved

- For a long time the question of renormalization scale fixing led to an ambiguity
- Fixed Order Perturbation Theory (FOPT, fixed scale) leads to smaller values for the coupling than Contour Improved Perturbation Theory (CIPT, running scale on the integration contour)

$$\begin{array}{rcccc} & \alpha_s^1 & \alpha_s^2 & \alpha_s^3 & \alpha_s^4 & \\ \delta^{(0)}_{\rm FO} = & 0.1001 + & 0.0521 + & 0.0264 + & 0.0127 & = 0.1914 \\ \delta^{(0)}_{\rm CI} = & 0.1353 + & 0.0255 + & 0.0100 + & 0.0070 & = 0.1779 \end{array}$$

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Benitez-Rathgeb, DB, Hoang, Jamin '22

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Benitez-Rathgeb, DB, Hoang, Jamin '22

We will not quote results from CIPT in this talk

Analysis strategy

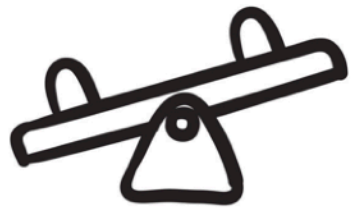
Any analytical weight function can be used in the FESRs (normally polynomials are used)

$$\frac{-1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\, \Pi(z) = \frac{1}{4\pi^2} S_{\rm EW}\left(\delta^{w}_{\rm tree} + \delta^{(0)}_{w} + \delta^{D\geq 4}_{{\rm OPE},w} + \delta^{(w)}_{\rm DVs} + \delta^{(w)}_{\rm EW}\right)$$

Desired properties from the choice of weights:

1. Good perturbative behavior
2. Small condensate contributions
3. Suppression of DVs

Suppression of DVs comes with the price of additional (unknown) higher dim. contributions from the OPE: ***see-saw effect***



DV strategy (this work)

DB, Eiben, Golterman, Maltman, Mansur, and Peris, 2502.08147.
DB, M. Golterman, K. Maltman, S. Peris, M. V. Rodrigues and W. Schaaf, 2012.10440

-Accept some DVs (oscillations evident in data)

-Strongly suppress contamination on the OPE side

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$$\rho_{\rm DV}(s) = e^{-\delta-\gamma s}\sin(\alpha+\beta s)$$

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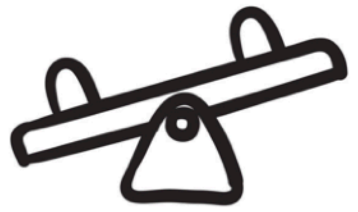

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Truncated OPE strategy

A Pich, A. Rodriguez-Sanchez 1605.06830
Davier, Höcker, Malaescu, Yuan, Zhang 1312.1501

- **Suppress DVs by using w(z) with zeros at z=0 (pinching) but need to ignore the higher order contributions on the OPE side (too many parameters)**
- **Contrary to previous understanding, α_s obtained from perturbative QCD, disregarding all non-perturbative contributions**

see DB, M. Golterman, K. Maltman, S. Peris ’24

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Choice of weights

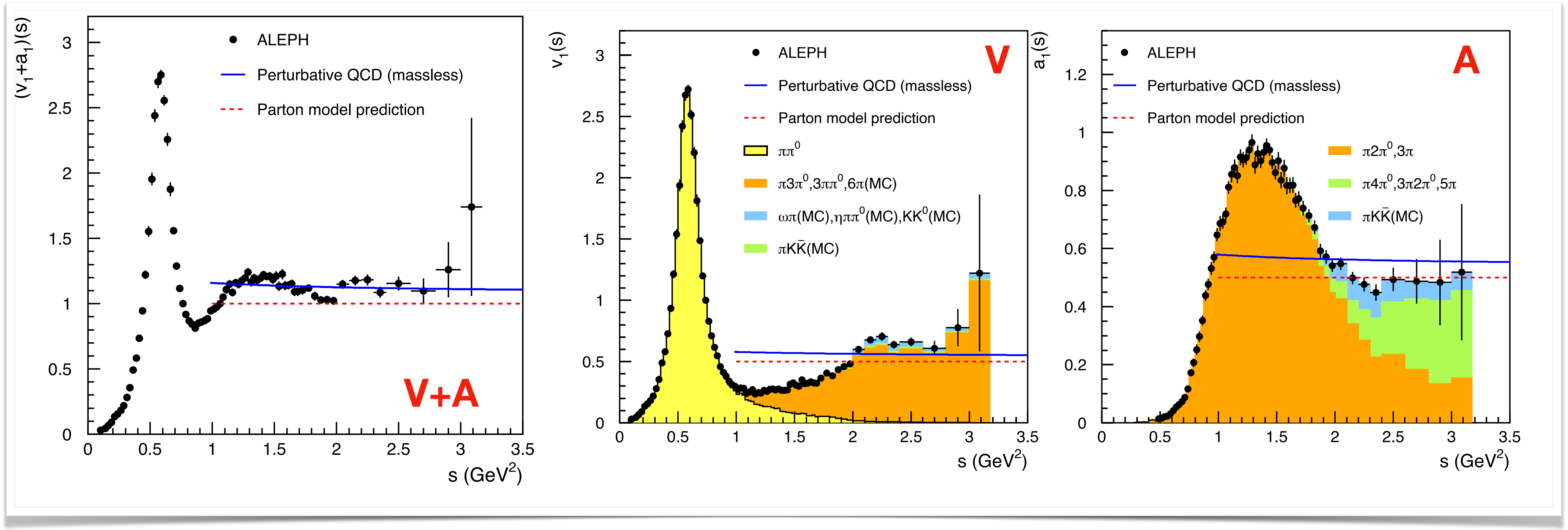
$w_0(y) = 1$	Tiny condensate contributions, sensitive to DVs
$w_2(y) = 1 - y^2$	$D = 6$
$w_3(y) = (1-y)^2(1+2y)$	$D = 6$ and 8: Tau kinematical Moment (R_τ)
$w_4(y) = (1-y^2)^2$	$D = 6$ and 10

DB, Cata, Golterman, Jamin, Maltman ’11,
Beneke, DB, Jamin ’12,
DB, M. Golterman, K. Maltman, S. Peris ’16
DB F Oliani ’20
DB, Golterman, Maltman, and Peris ’24

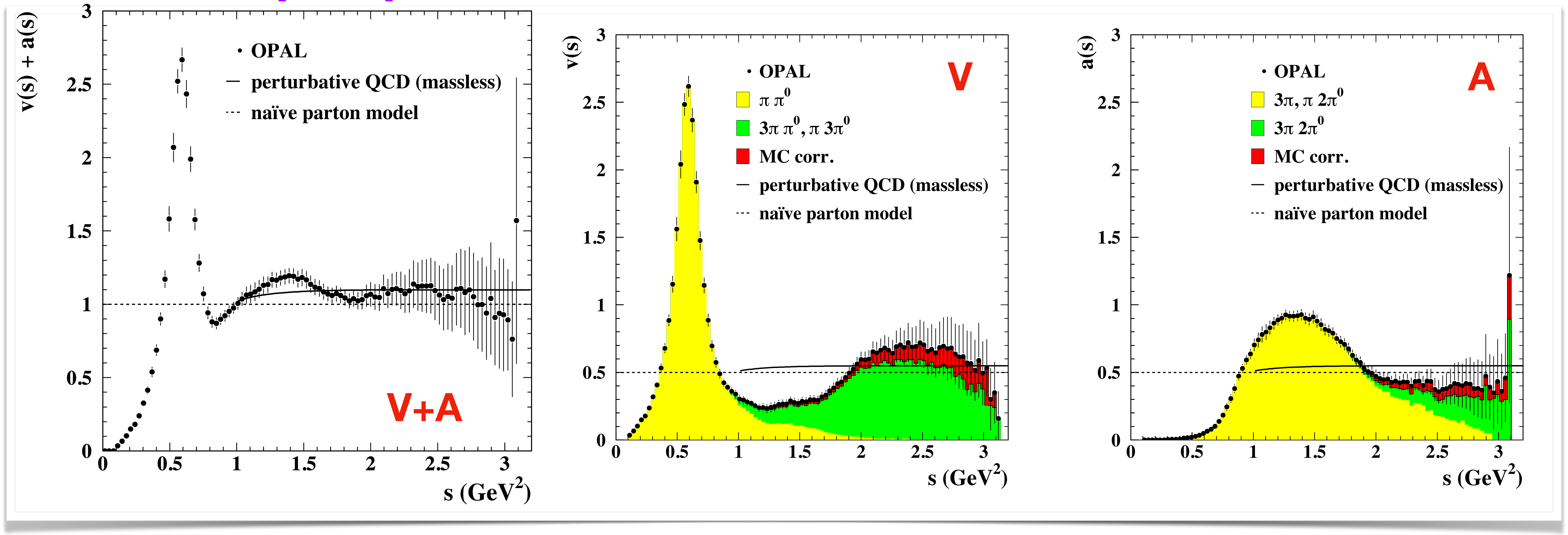
Main results are based on the simplest weight function $w_0(y) = 1$

Experimental data

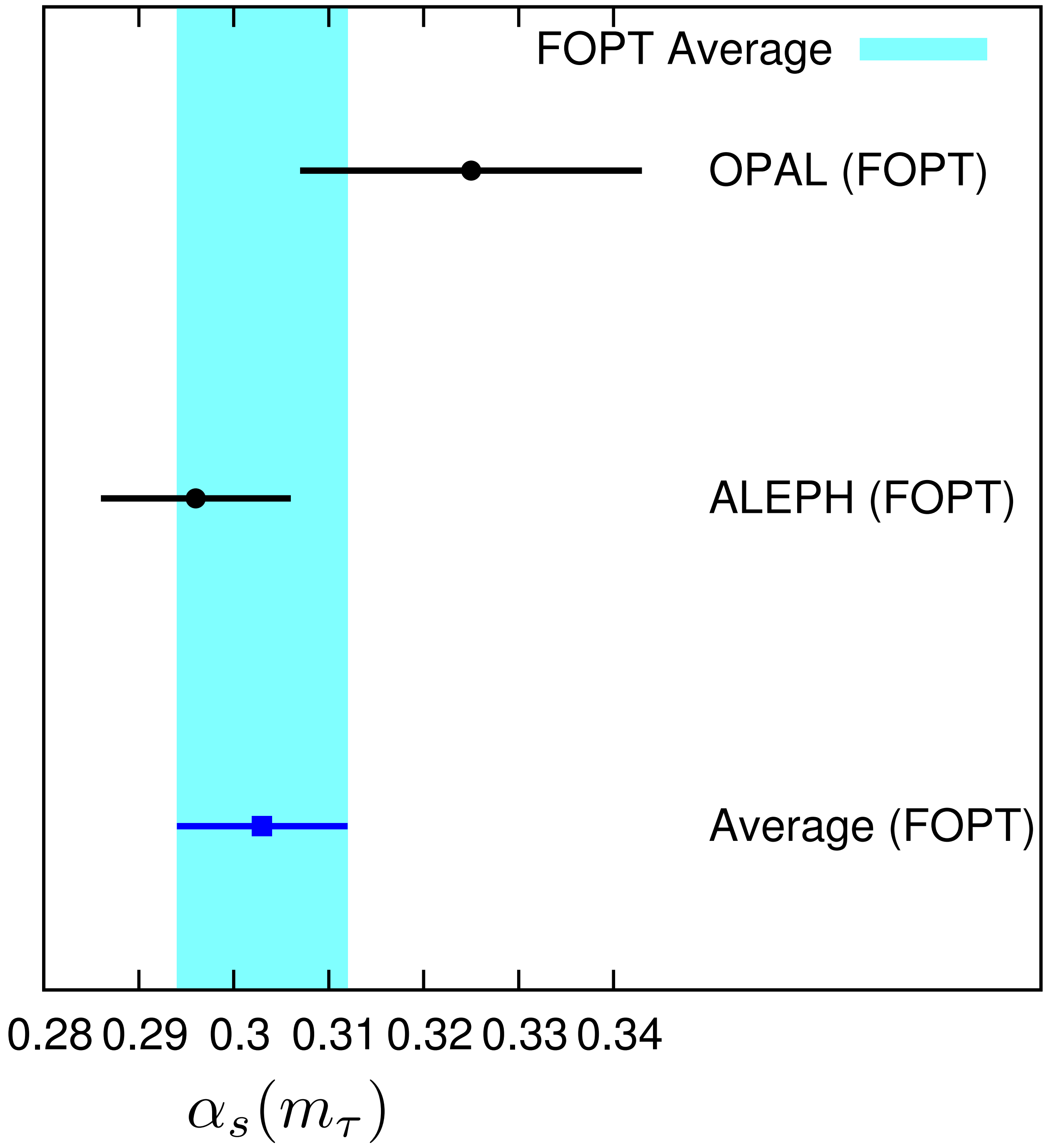
Davier et al [ALEPH] '14



Ackerstaff et al [OPAL] '98 updated for recent values of branching fractions Boito et al '11, '21



The two data sets led to compatible values for $\alpha_s(m_\tau)$ for the DV strategy



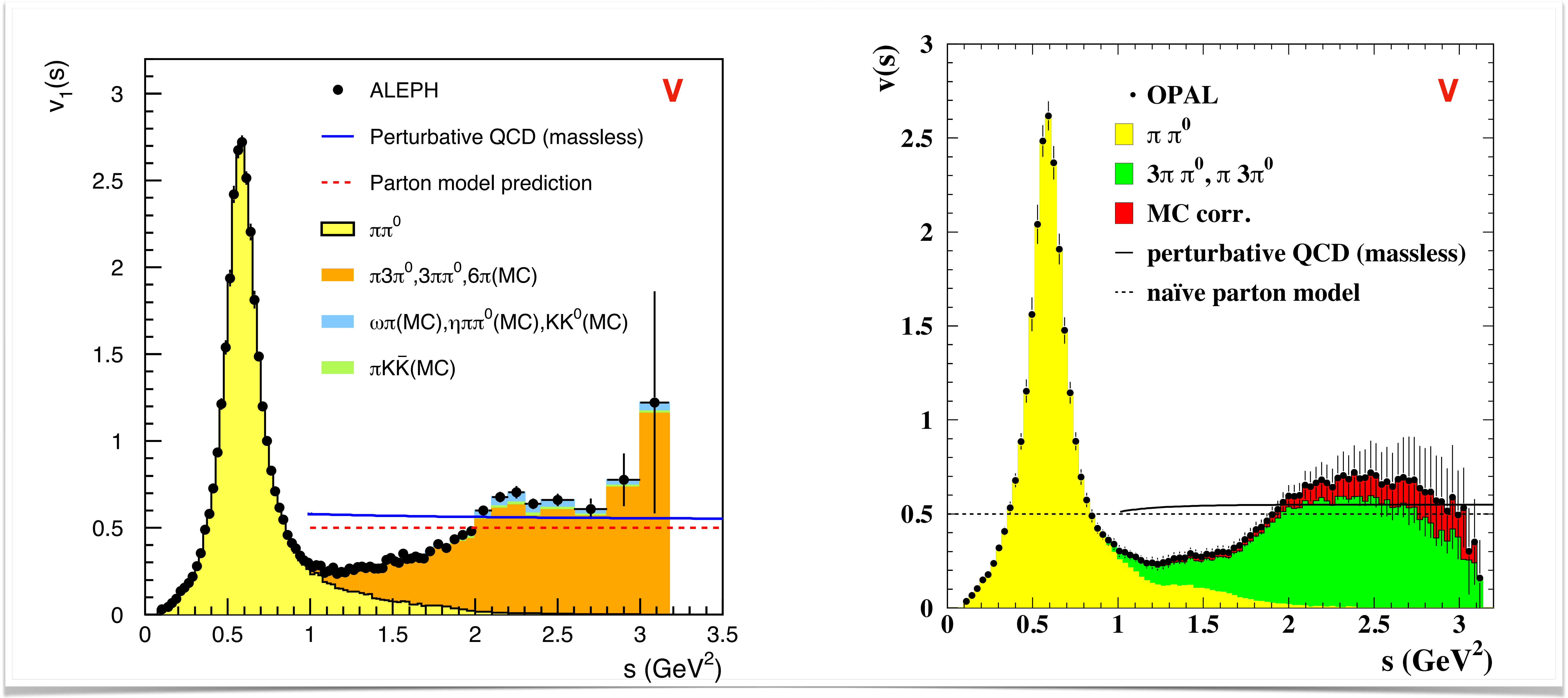
DB, Golterman, Jamin, Mahdavi, Maltman, Osborne, and Peris, '12

DB, Golterman, Maltman, Osborne, and Peris, '15

- Can we combine the data sets (as done, e.g., for g—2 of the muon)?
- Are they (locally) compatible?
- Can we improve on the LEP-based data sets with recent experimental results?

Anatomy of the data sets

- V channel dominated by $\tau \to 2\pi + \nu_\tau$ and $\tau \to 4\pi + \nu_\tau$
- Residual channels are subdominant (but very important for α_s!)
- ALEPH and OPAL used Monte Carlo input for several of the subdominant (residual) modes (**not real data**)



Recently measured channels in e^+e^- can be used to improve the vector channel using conserved vector current (CVC) for residual modes in regions where isospin breaking is small

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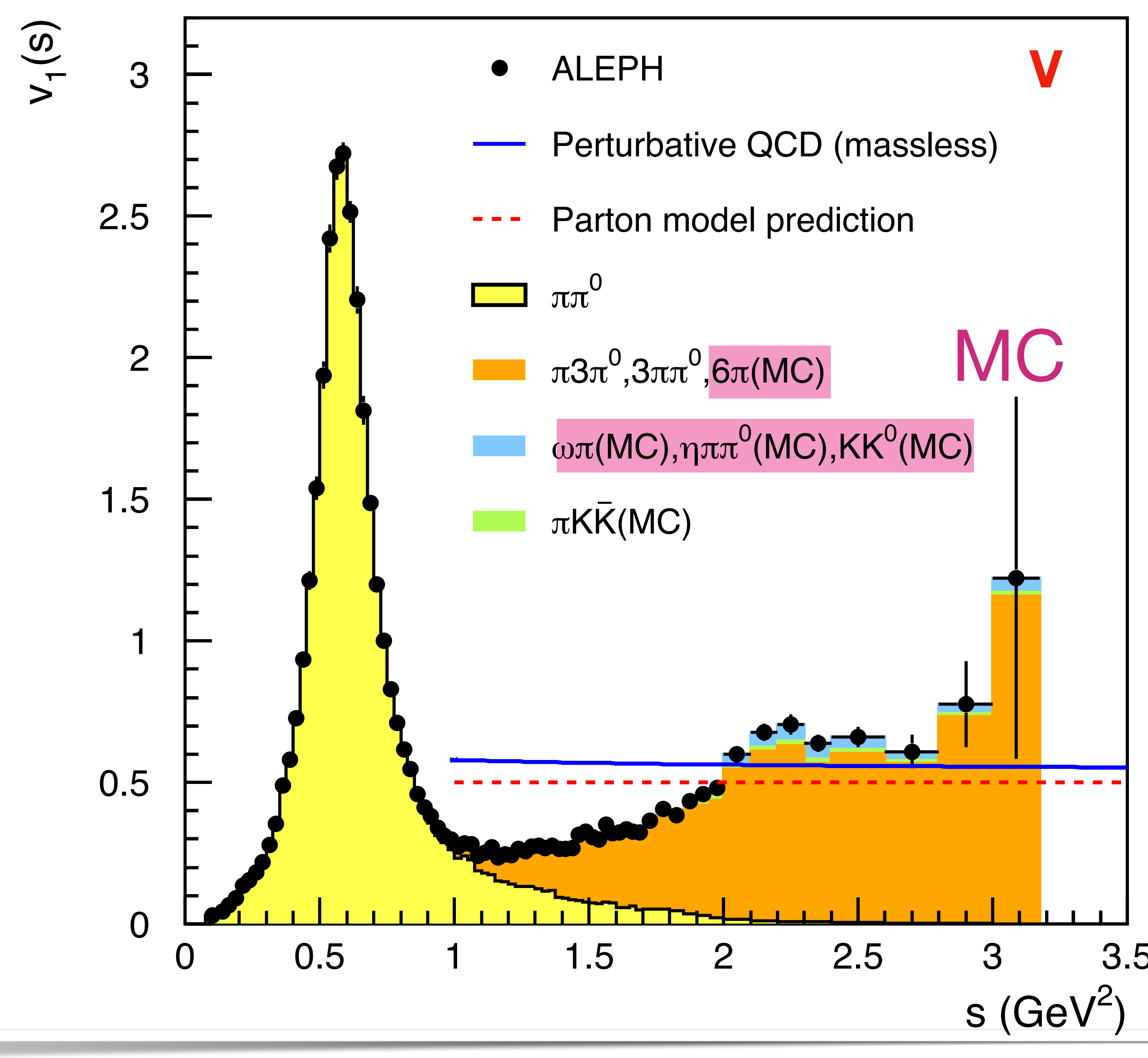

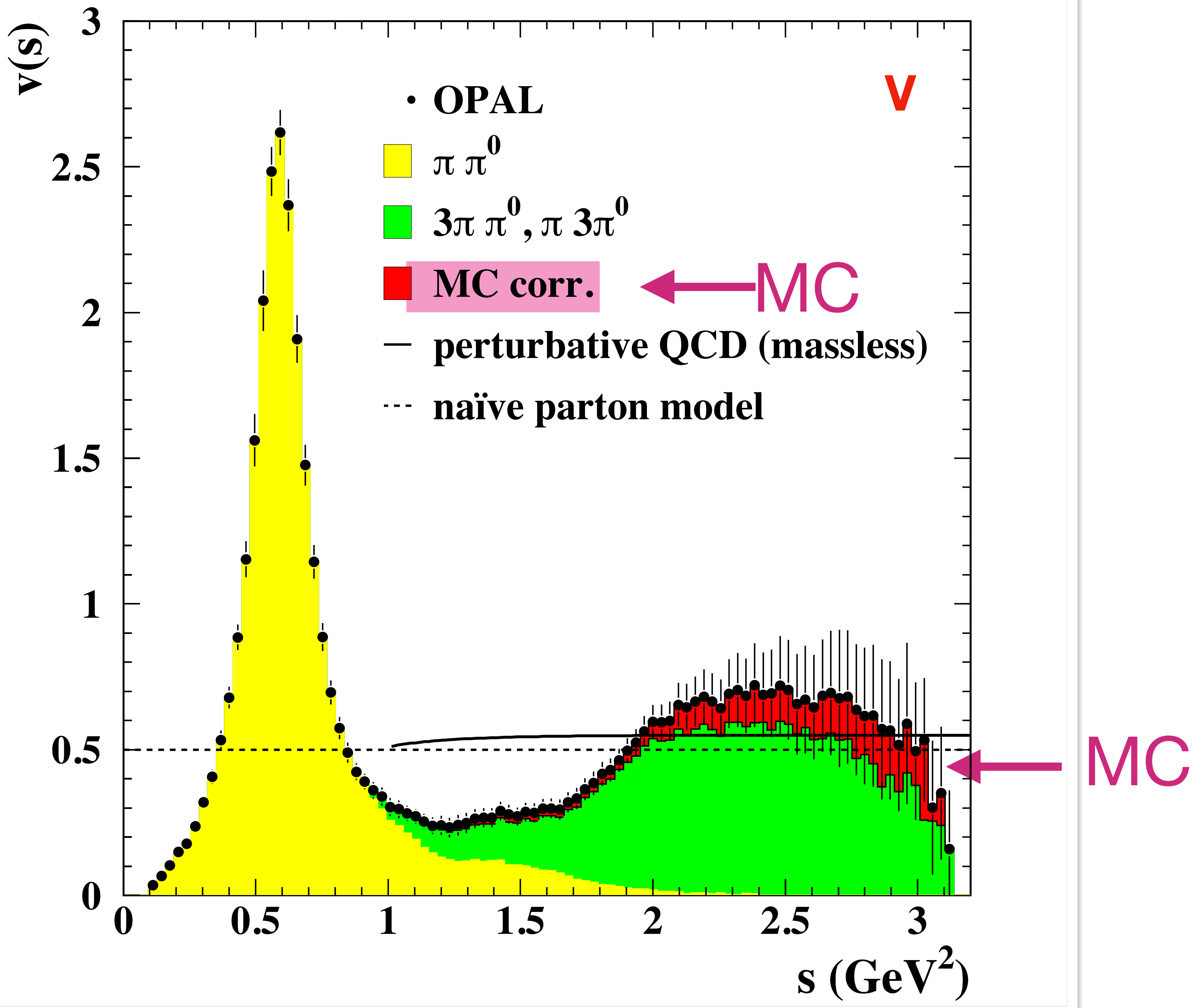

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Improved vector-isovector spectral function

- We introduced in 2020 a method to combine existing data and improve on previous spectral functions

DB, M. Golterman, K. Maltman, S. Peris, M. V. Rodrigues and W. Schaaf, 2012.10440

  - Combined 2π + 4π data from ALEPH and OPAL
  - Results based **solely** on experimental data: 7 residual channels obtained from recent e^+e^- data (no need for MC inputs)
  - All results updated for recent branching ratio measurements

WHAT'S NEW?

DB, Eiben, Golterman, Maltman, Mansur, and Peris, 2502.08147

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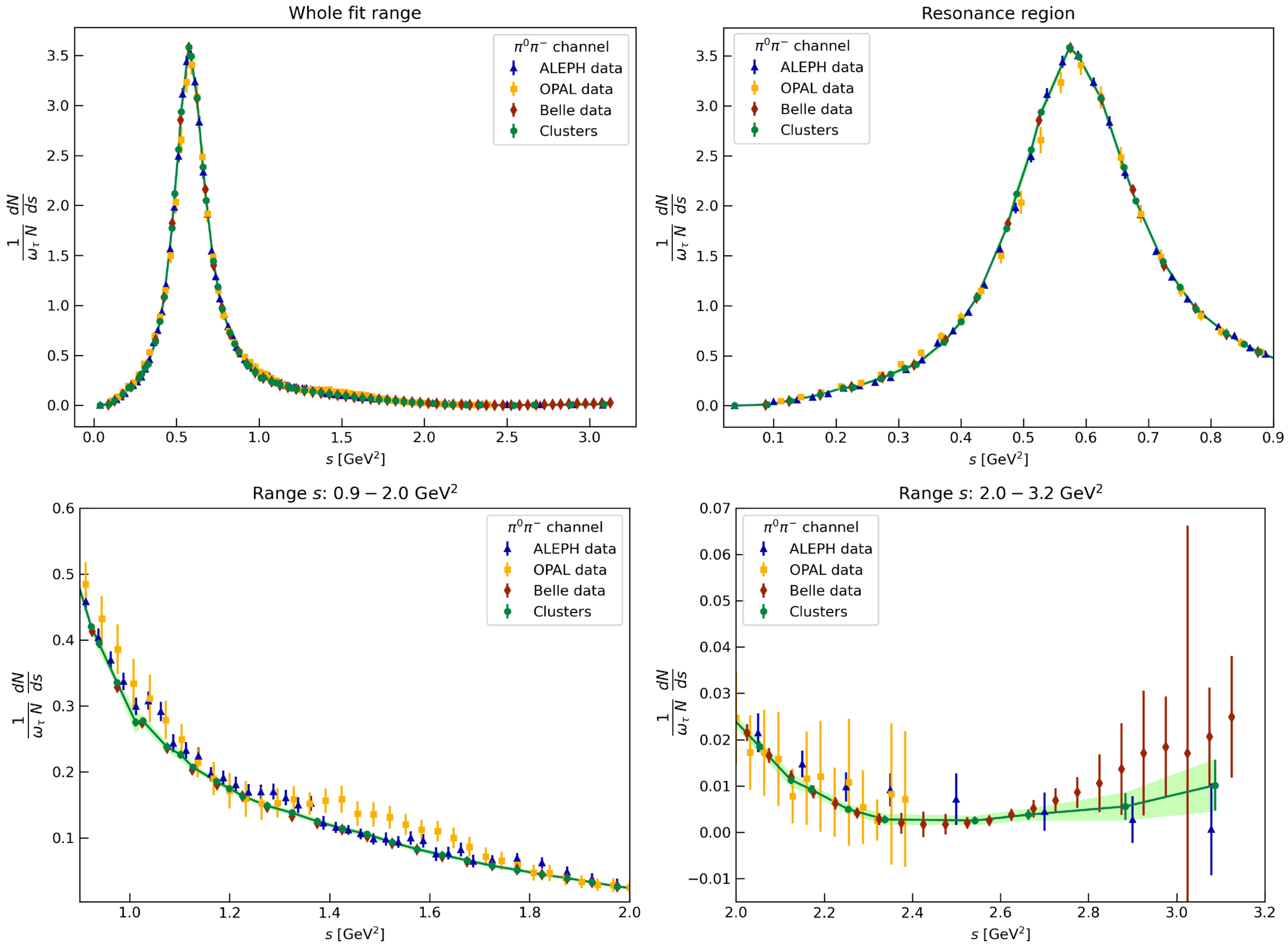
WHAT'S NEW? DB, Eiben, Golterman, Maltman, Mansur, and Peris, 2502.08147

- New data combination algorithm to deal with strong correlations in 4π spectra from ALEPH and OPAL
  - Based on the algorithm of Keshavarzi, Nomura, Teubner '18, '19 (used in the context of g—2 of the muon)
  - Statistical treatment of non-χ^2 fits from Bruno & Sommer, '23
- Inclusion of high statistics Belle spectrum for $\tau \to \pi^-\pi^0\nu_\tau$ for the first time in inclusive hadronic tau decays
- Results including the CLEO spectrum for $\tau \to \pi^-\pi^0\nu_\tau$ also given

  (but not enough information about correlations exist to include CLEO in the final results)
- New HFLAV branching ratios used Banerjee et al. (HFLAV) '24
- Algorithm to avoid d'Agostini bias in multiplication by global factors (branching fractions) Ball et al. (NNPDF) '10

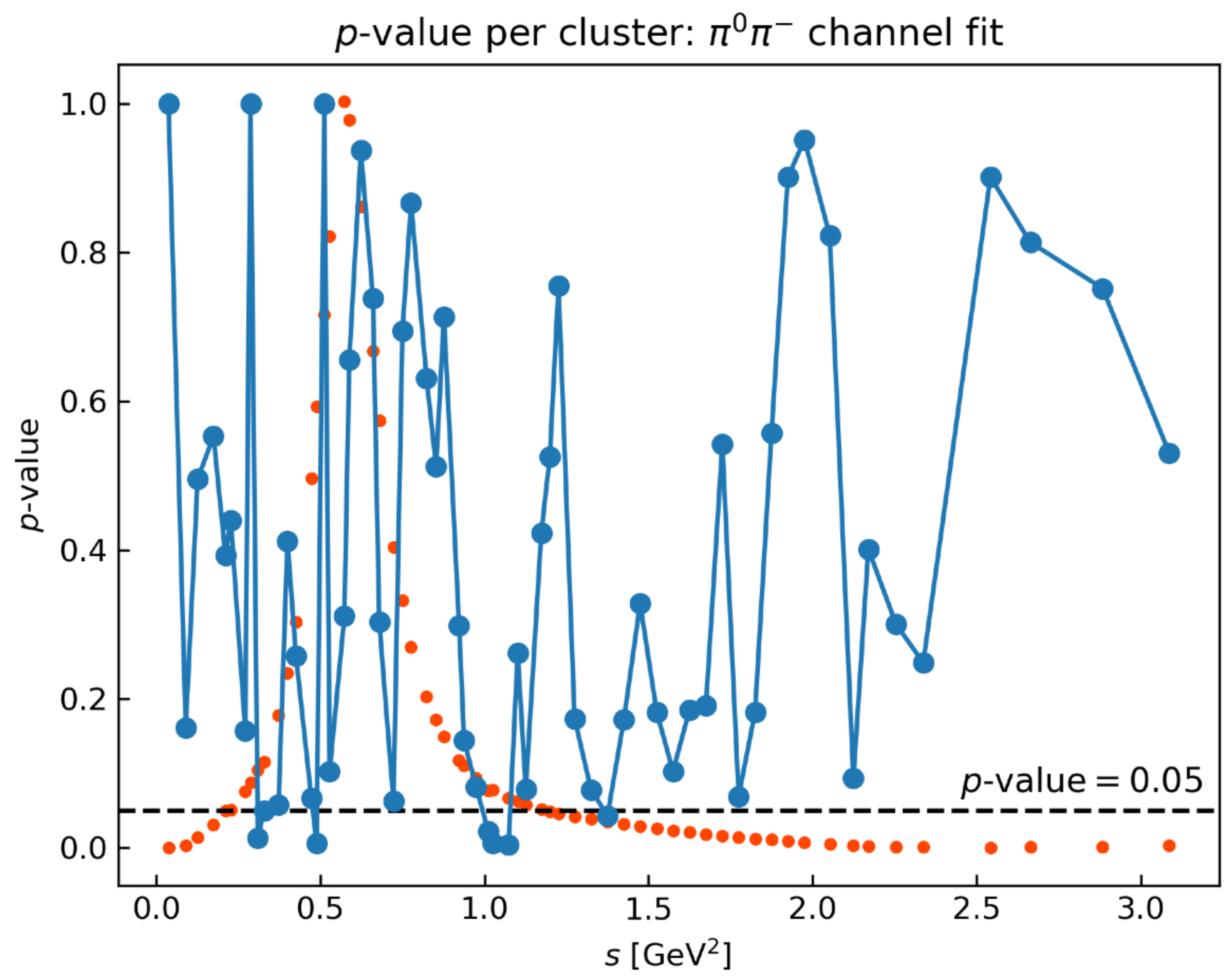
Improved vector-isovector spectral function: 2π channel

Combination of 2π spectra (combined results in green)



Good χ^2 both locally and globally

Effects of error inflation are minimal



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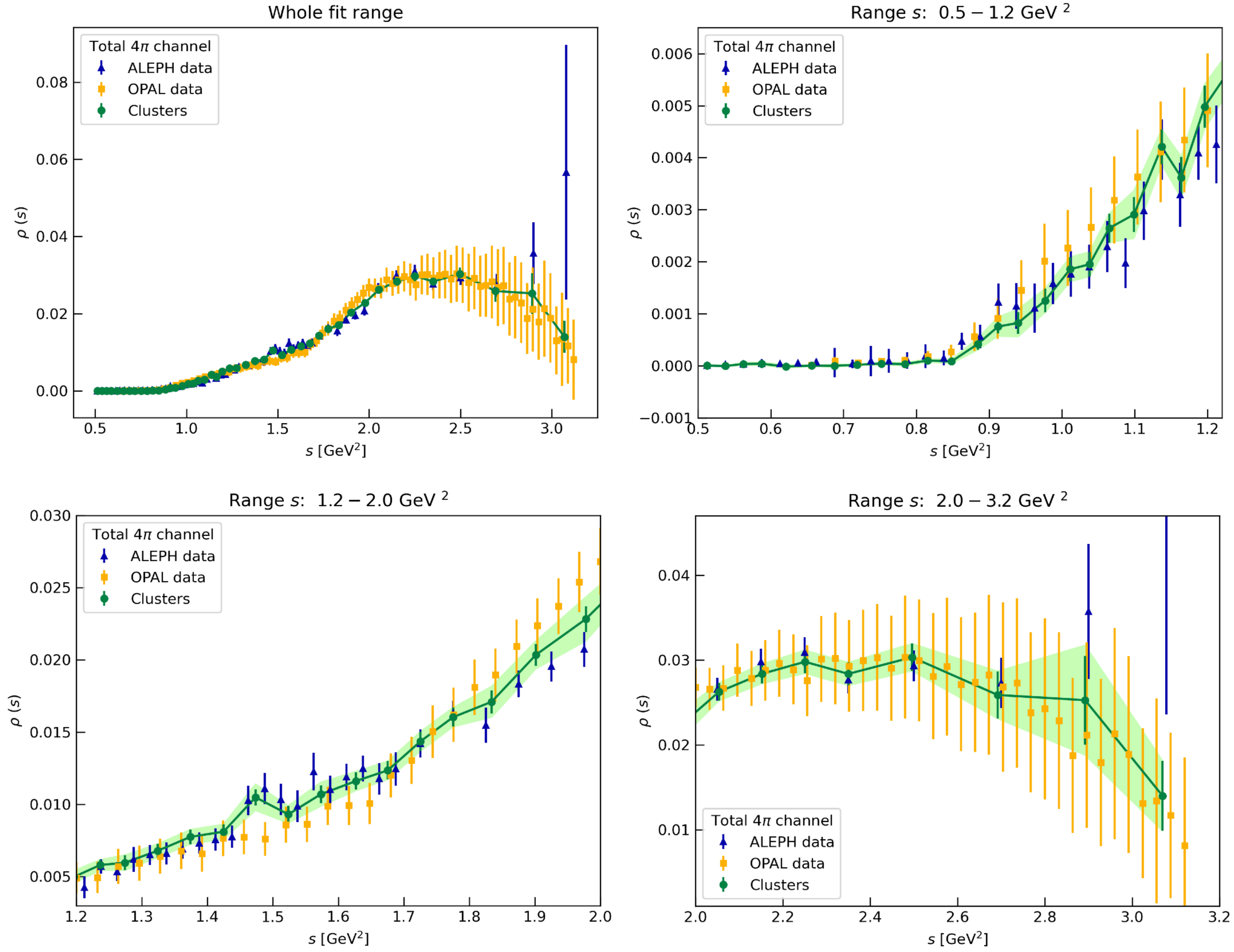
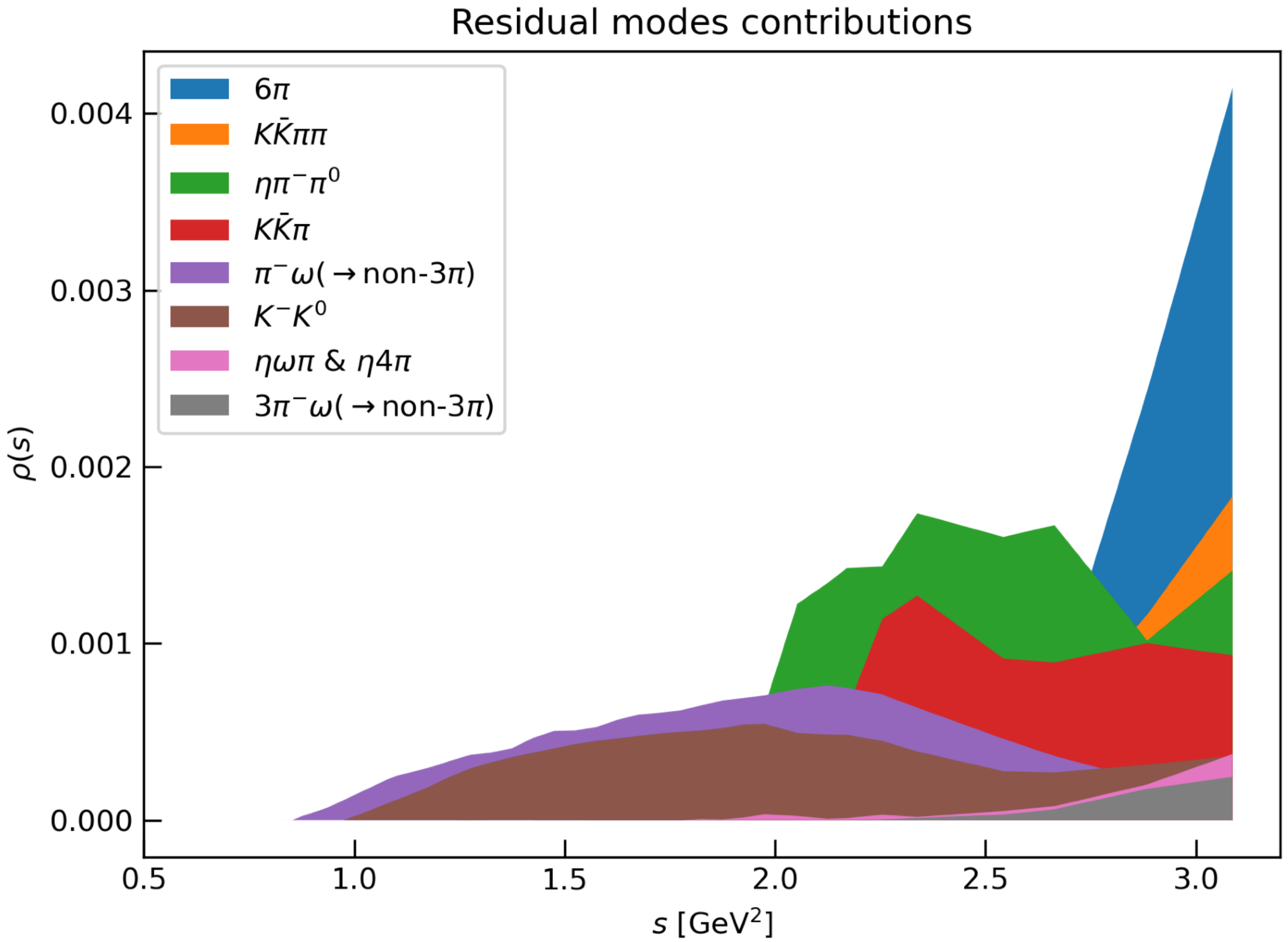


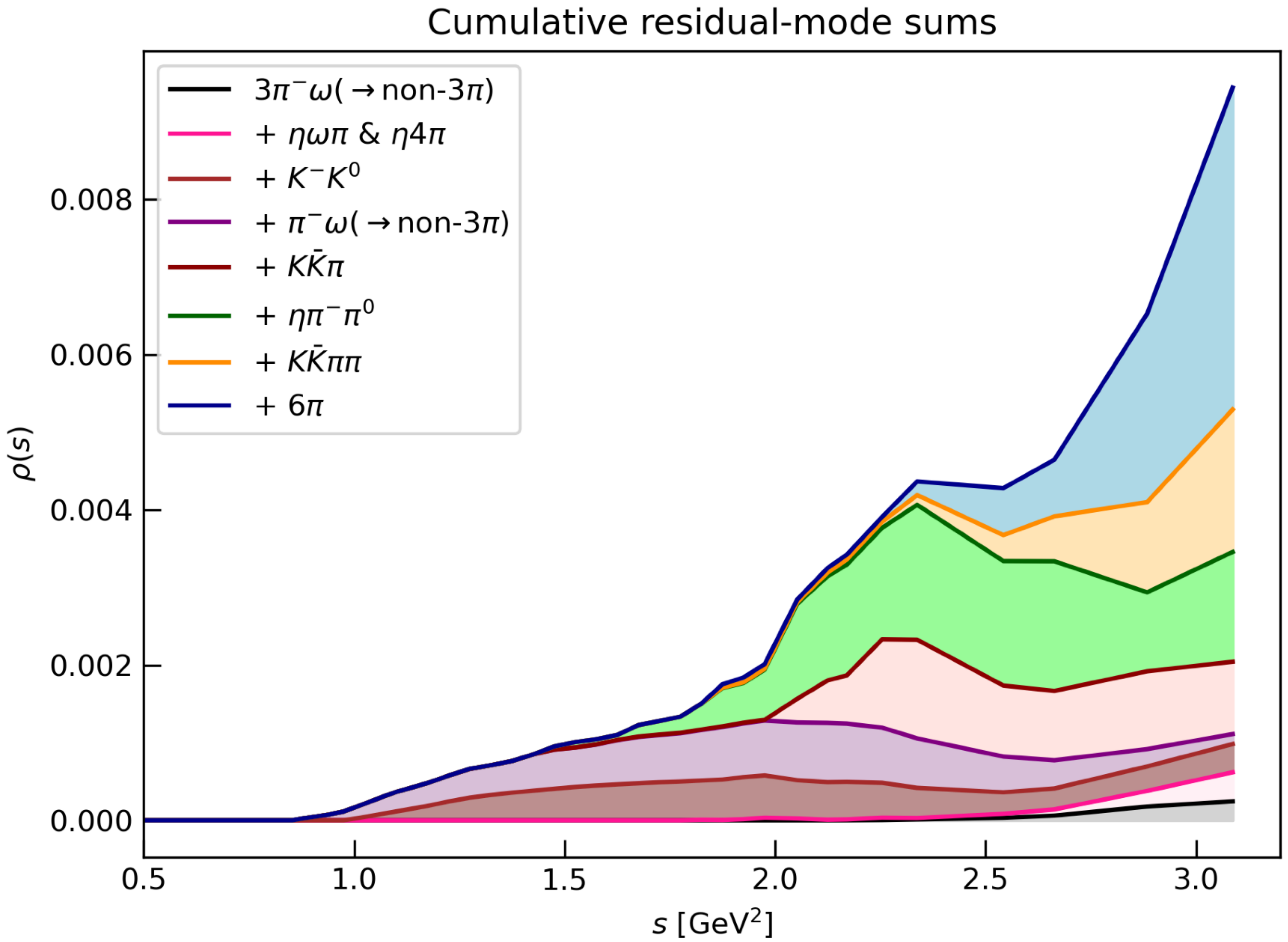
- Correlation matrices for 4π exclusive channels are ill-behaved (negative eigenvalues)
- Adding up the 4π channels softens the problem, but strong correlations in OPAL results lead to poor fits
- The main issue are the strong, but poorly known, correlations in the $\pi^- 3\pi^0$ OPAL channel
- Perform fits where the strong correlations in OPAL $\pi^- 3\pi^0$ channel are not included in the minimization, but are still used in the error propagation
- General *p*-value analysis for fits of this type well understood. Implemented with a Monte Carlo (see Bruno & Sommer, '23)

Residual channels

- 7 residual channels extracted from e^+e^- data + BaBar data for $\tau \to K K_S \nu_\tau$
- Dramatic improvement in errors for higher multiplicity modes (near end point)
- Isospin breaking corrections on already small residual modes (1—2%) would not affect final results

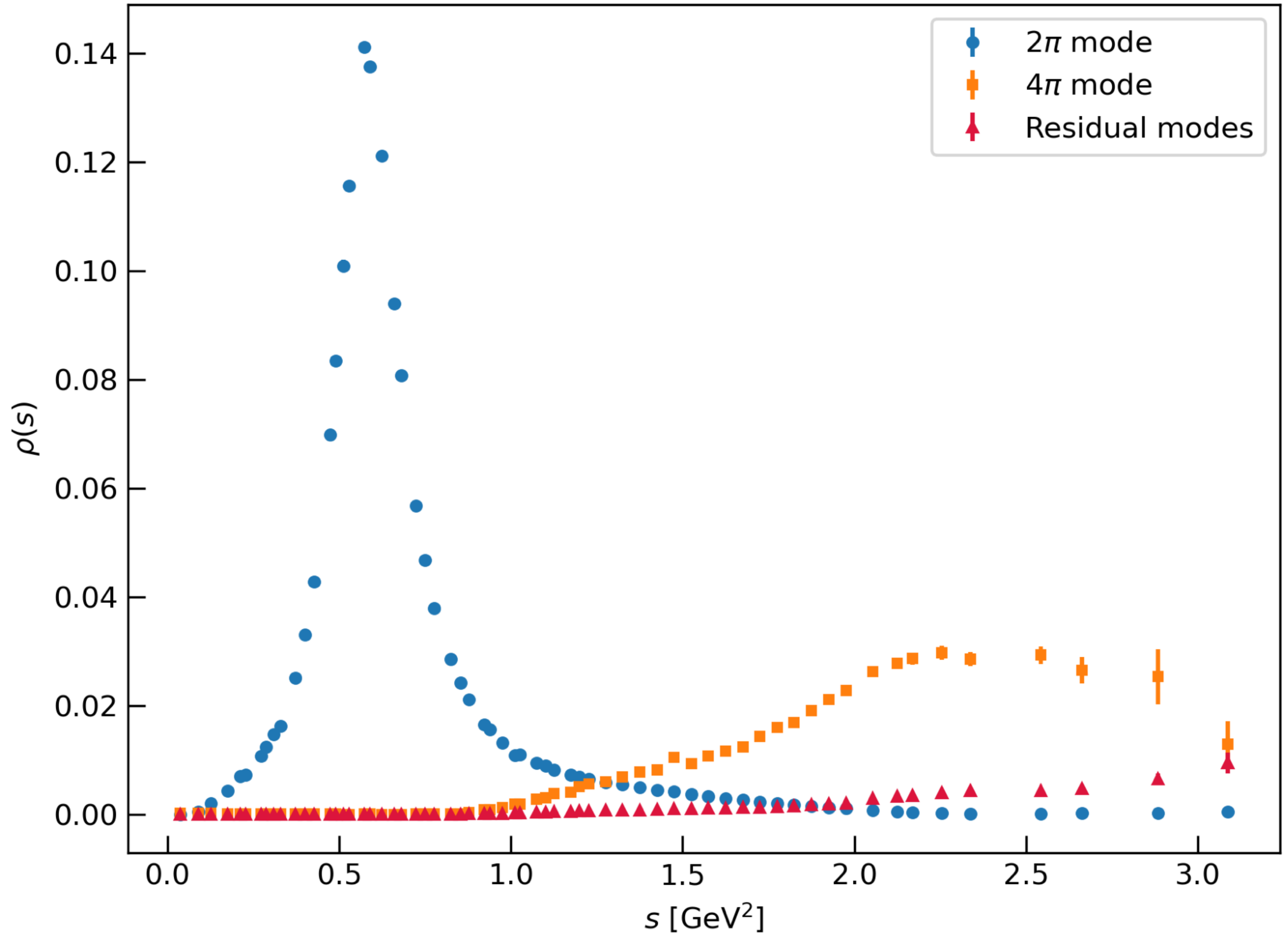
No Monte Carlo input



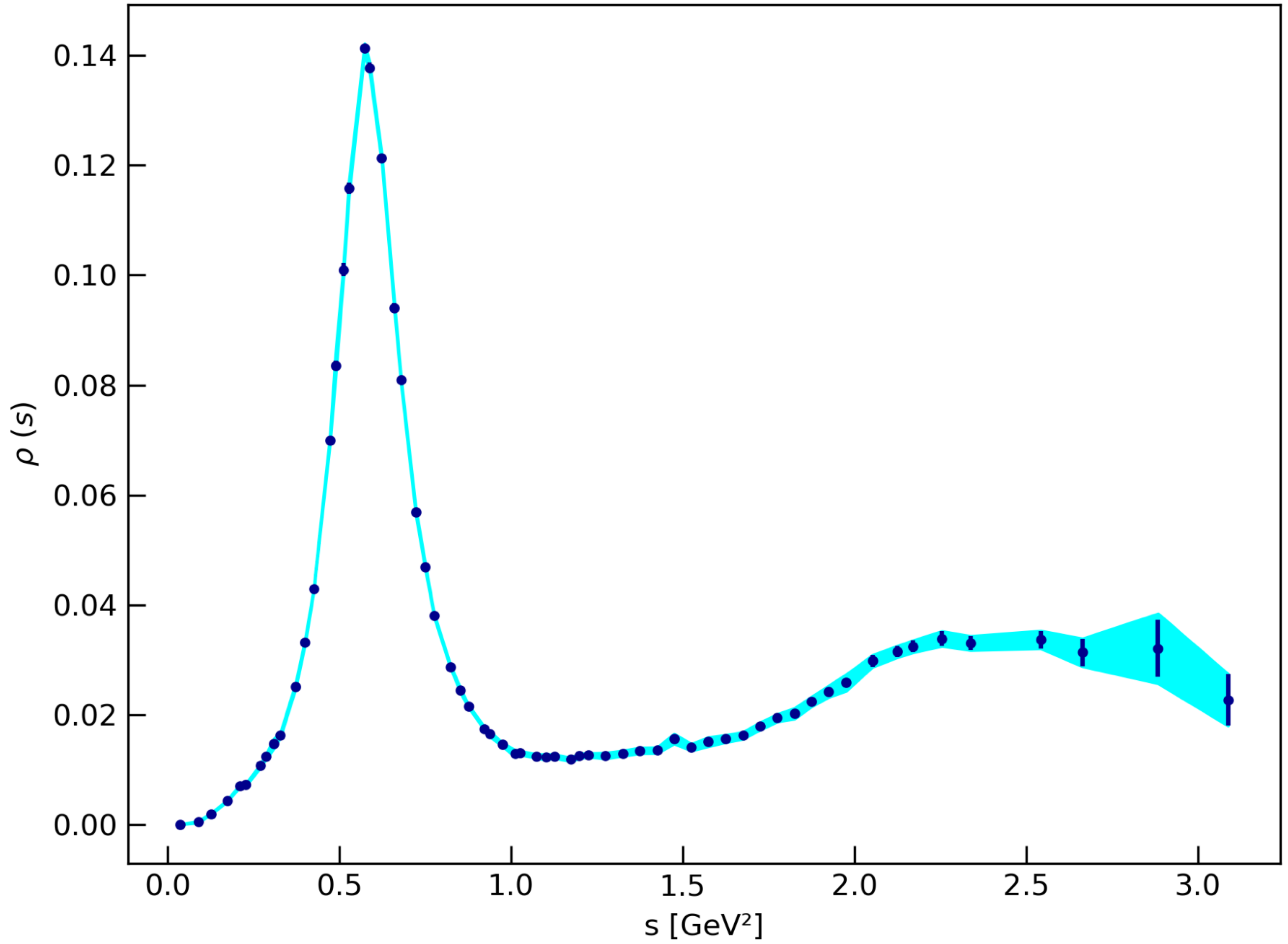


Final spectral function

Contributions



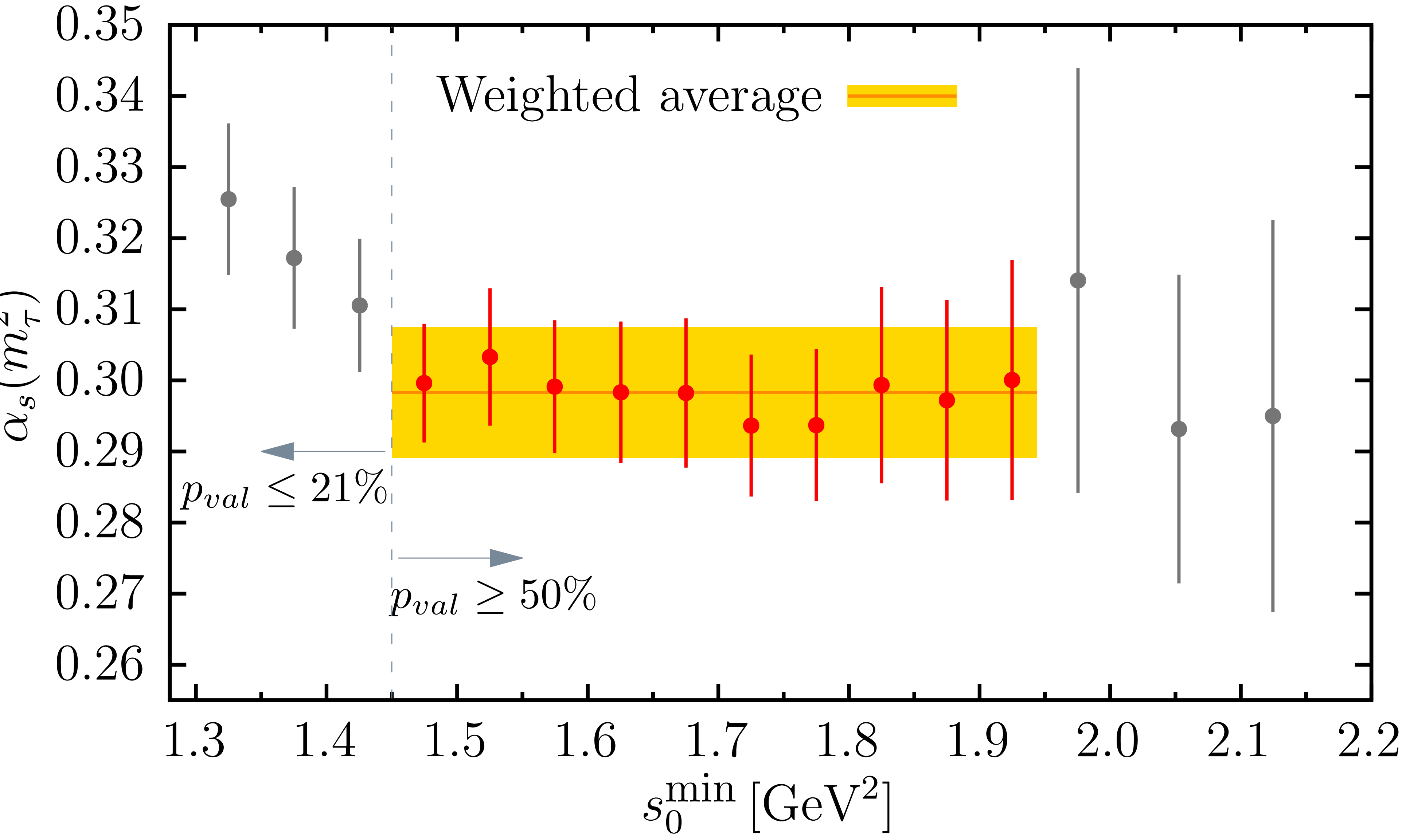
Total vector-isovector spectral function



DB, Eiben, Golterman, Maltman, Mansur, and Peris, 2502.08147

Strong coupling from the new spectral function

- Several fits, single moments or in combination
- Many fit windows: $[s_{\min}, m_\tau^2]$
- Consistency between different fits (α_s, condensates, DV params.)



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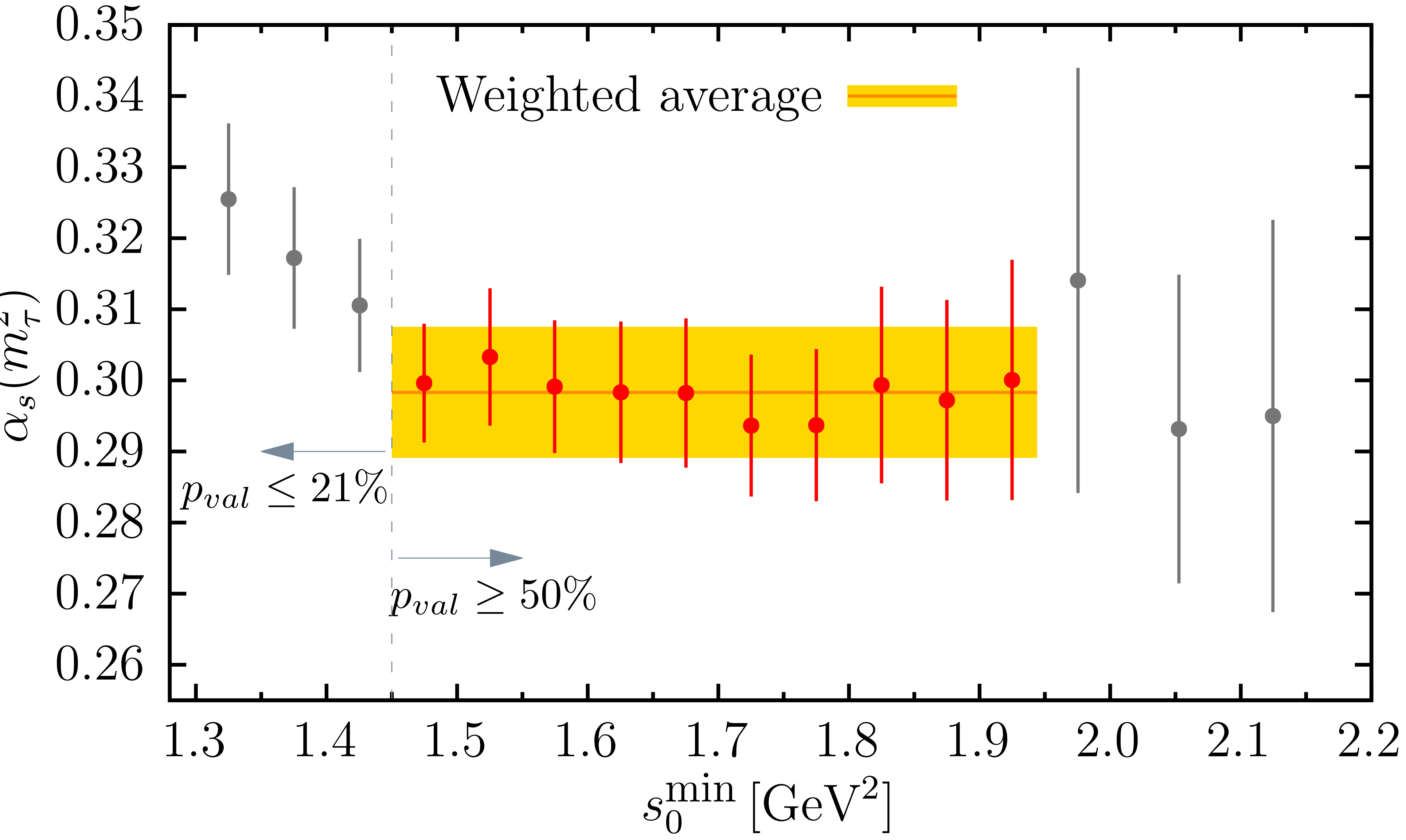
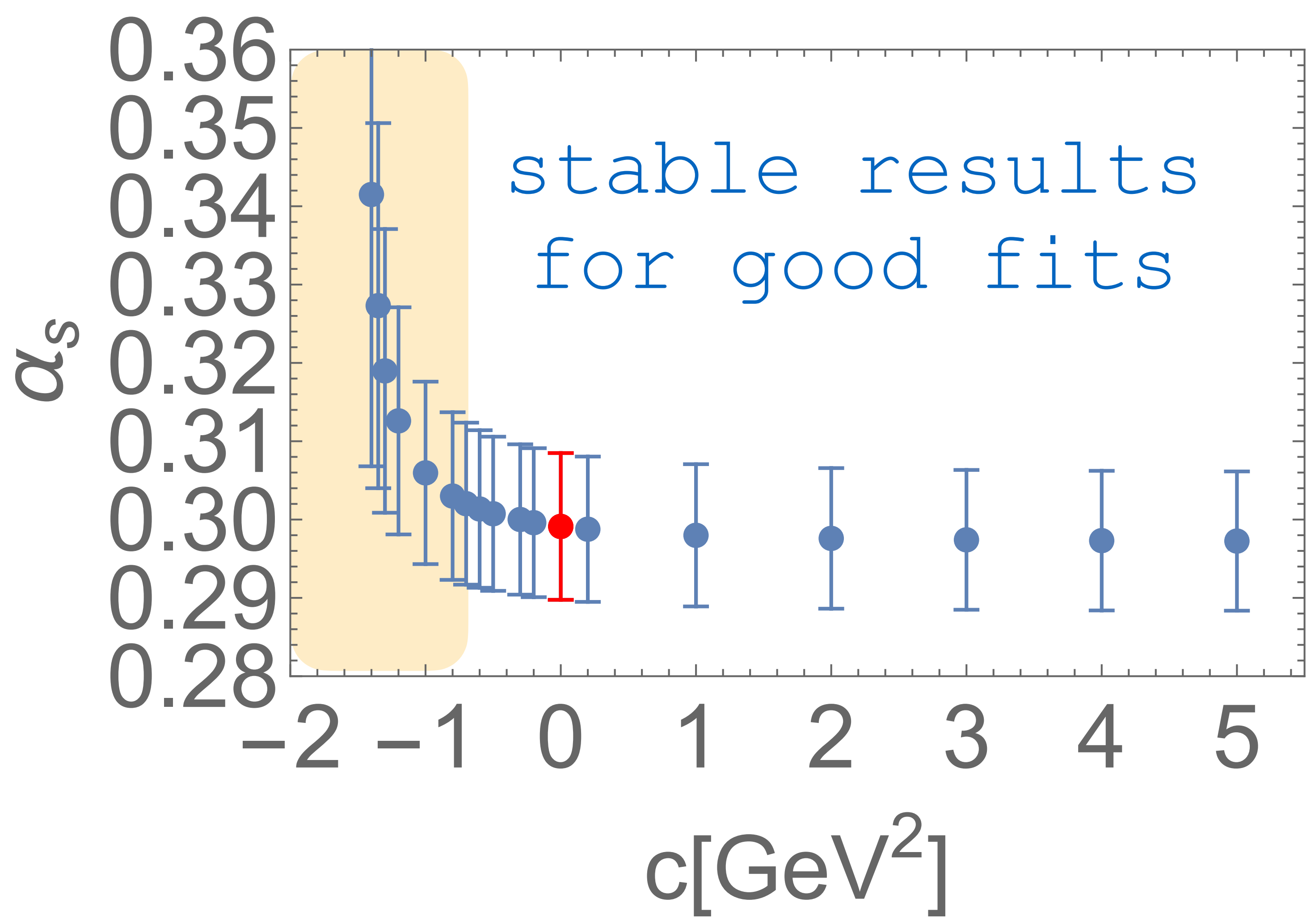
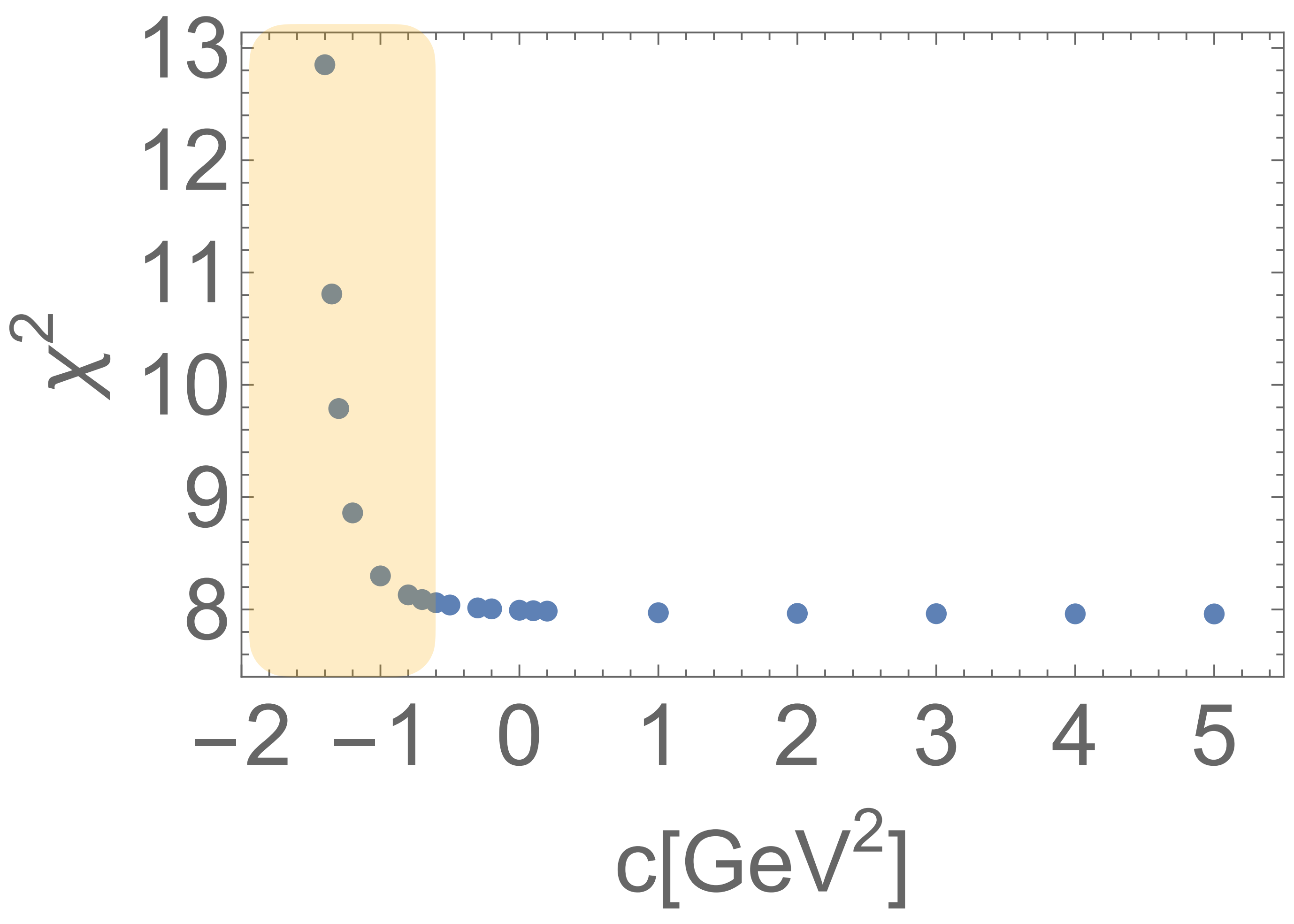
DB, Caprini, Golterman, Maltman, Peris, PRD '18

Pich & Rodriguez-Sanchez '22

$$\rho^{\rm DV}(s) = \left(1 + \frac{c}{s}\right) e^{-\delta-\gamma s}\sin(\alpha+\beta s)$$







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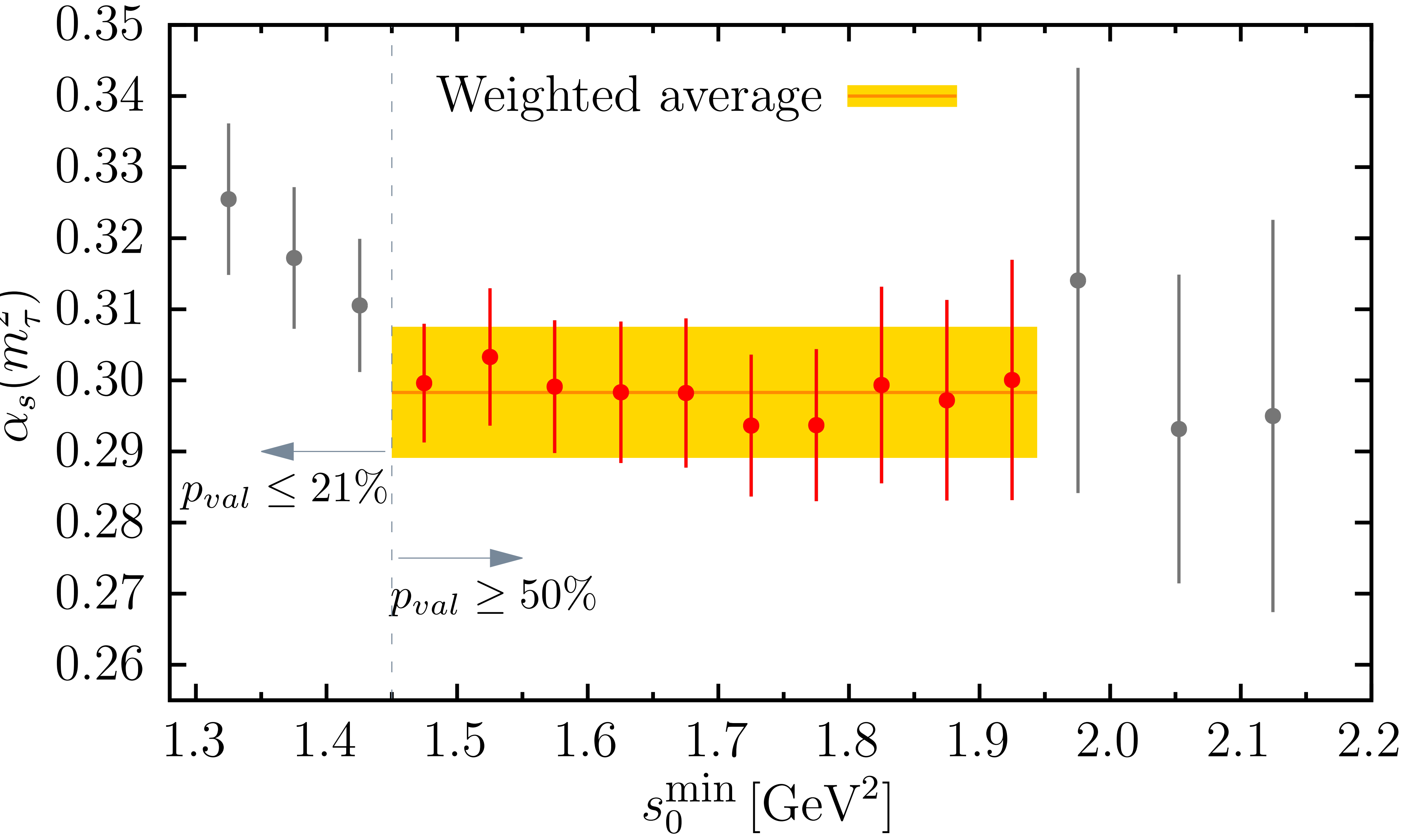

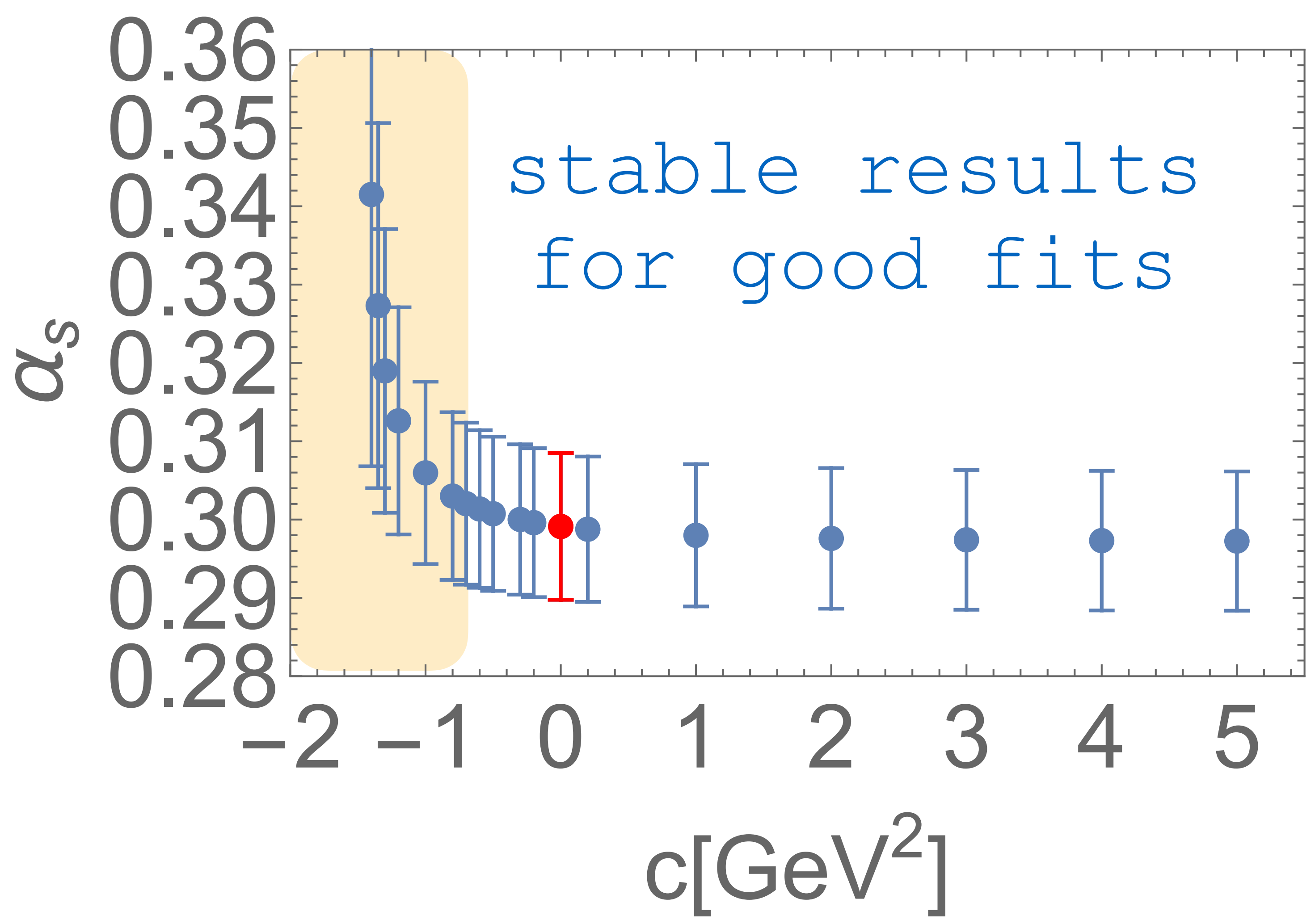

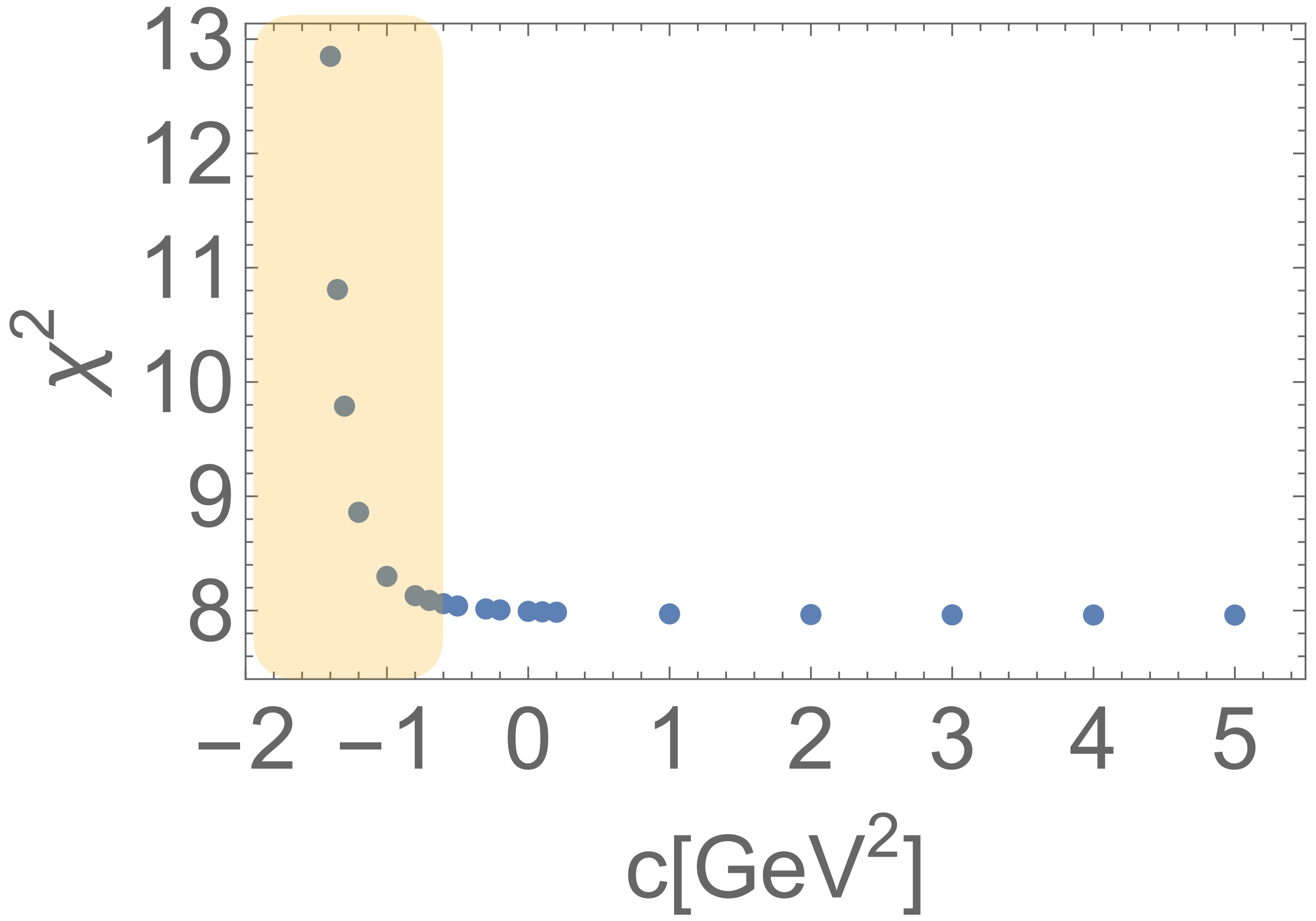

DB, Eiben, Golterman, Maltman, Mansur, and Peris, 2502.08147

$$\alpha_s(m_\tau^2) = 0.2983 \pm 0.0092_{\rm stat} \pm 0.0026_{\rm fit} \pm 0.0022_{\rm pert} \pm 0.0025_{\rm DVs}$$
$$= 0.2983 \pm 0.0101 \qquad (n_f = 3)$$

$$\alpha_s(m_Z^2) = 0.1159 \pm 0.0014 \qquad (n_f = 5)$$

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Our previous result from 2020 was significantly larger

$$\alpha_s(m_\tau^2) = 0.3077 \pm 0.0065_{\rm stat} \pm 0.0038_{\rm pert}$$
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DB, Eiben, Golterman, Maltman, Mansur, and Peris, '25

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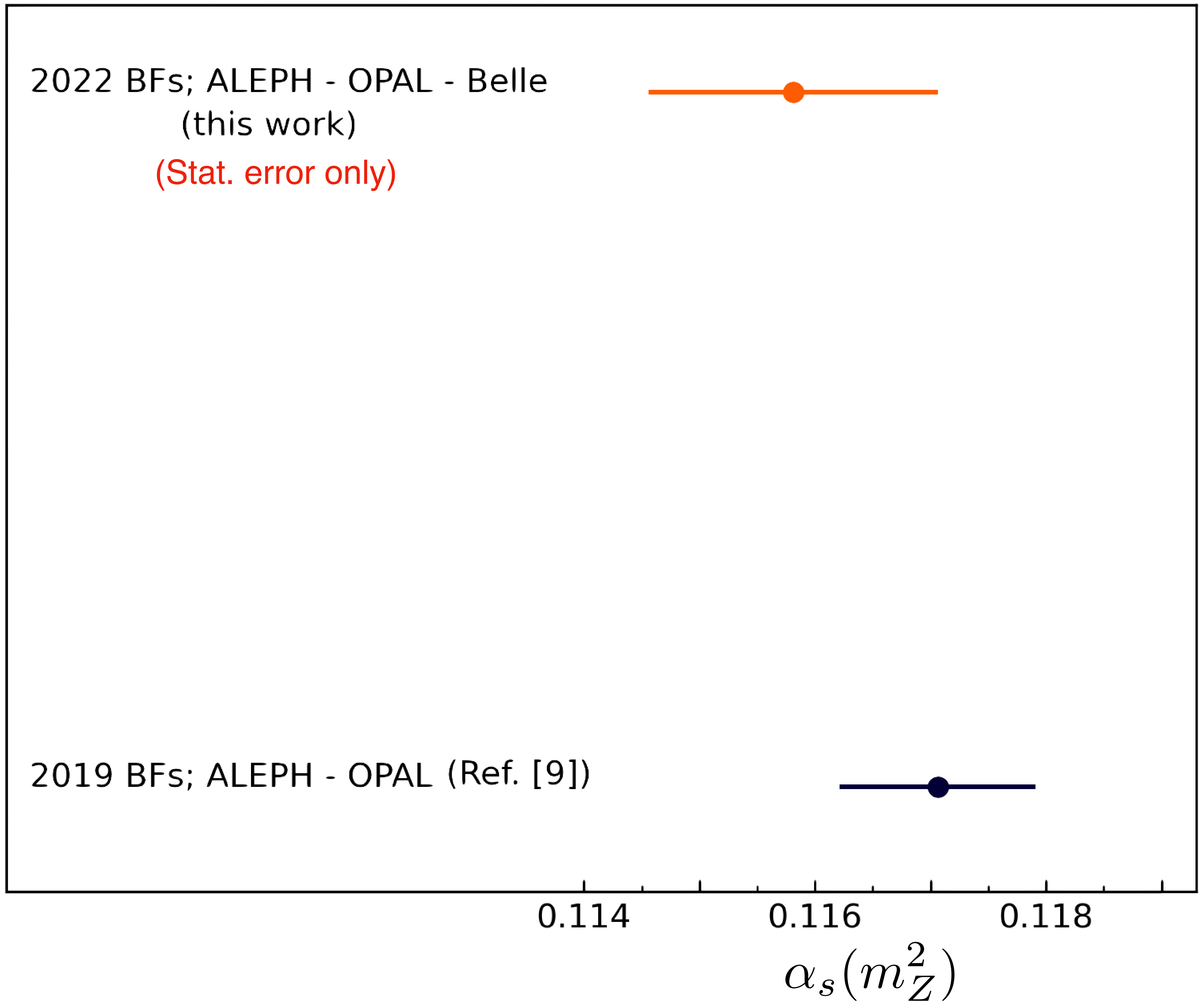

DB, M. Golterman, K. Maltman, S. Peris, M. V. Rodrigues and W. Schaaf, '20

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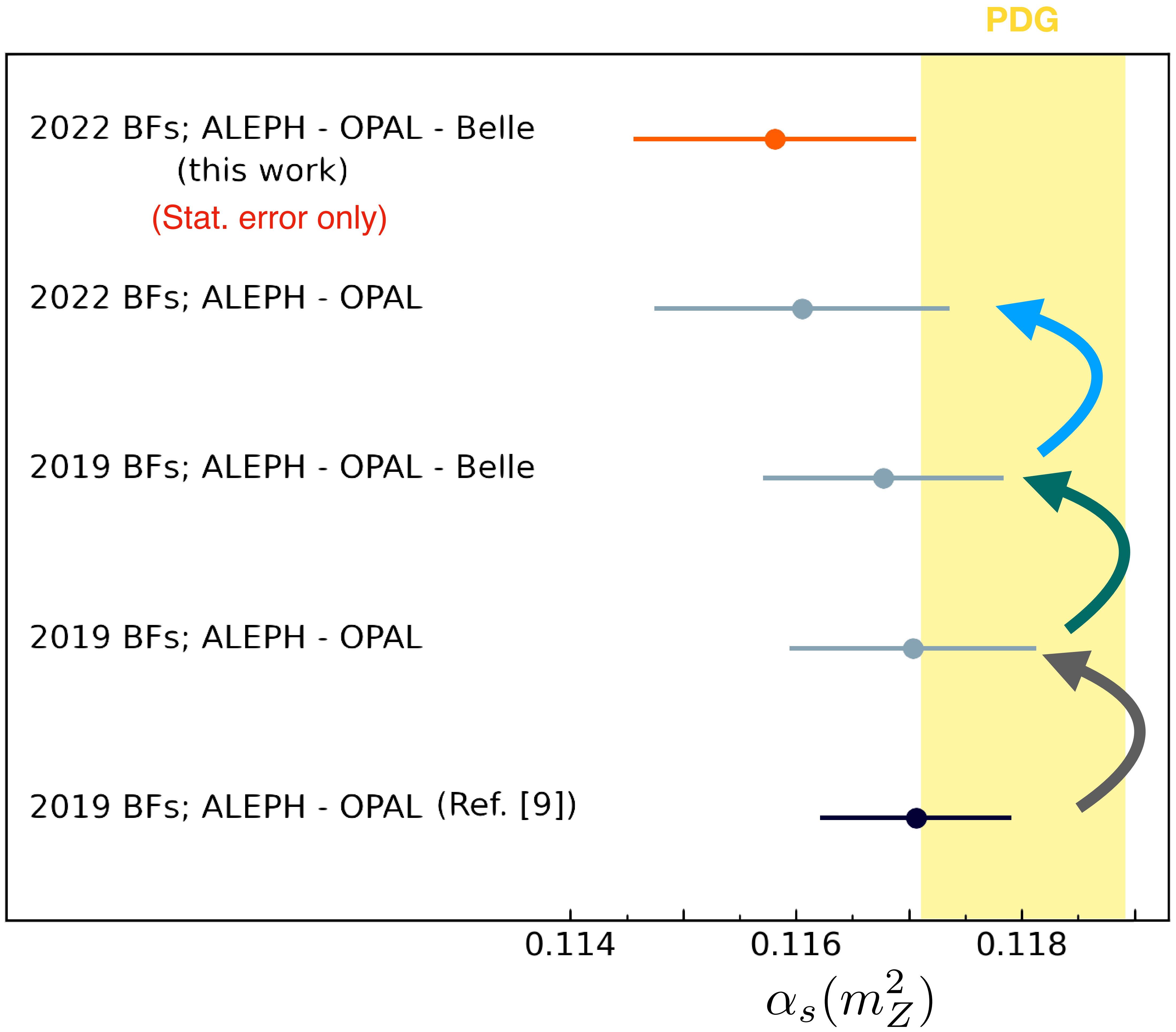


Final result with Belle, new BFs and new algorithm

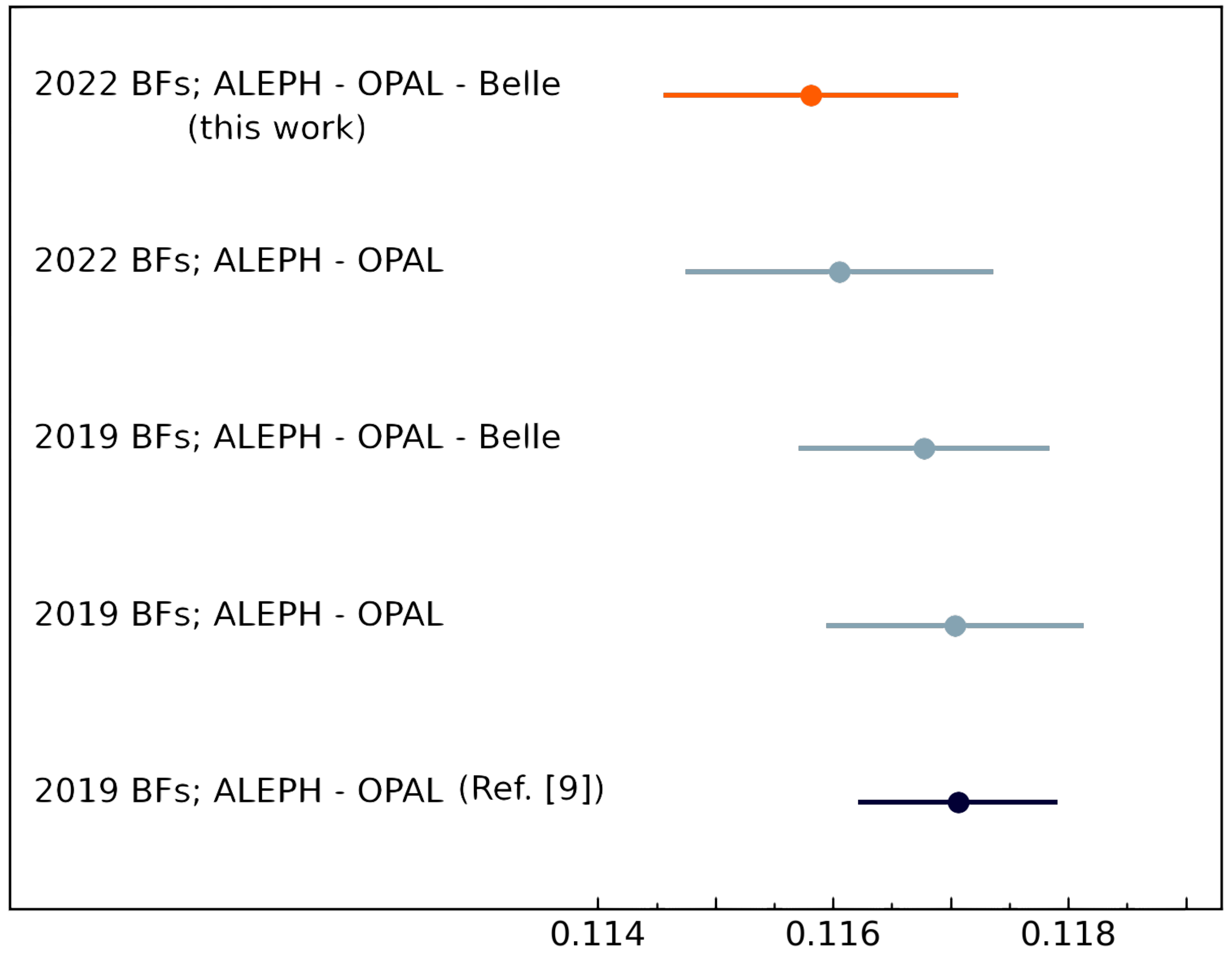
Largest shift due to new values of the 4π branching fractions!

Inclusion of the Belle spectrum leads to a small shift towards smaller values.

Changes in our data combination algorithm lead to slightly larger errors but no shift

Conclusions

- Vector channel in hadronic tau decays is special: e^+e^- data + CVC allows for improvement near kin. end point
- New vector-isovector spectral function purely based on exp. data, no MC input needed
- Inclusion, for the first time, of high-statistics Belle 2π spectrum in an inclusive hadronic tau decay analysis
- Our analysis can immediately incorporate any new spectrum for 2π or 4π tau decay channels (Belle II?)
- Improvements of this type not possible for the axial channel
- Final vector spectral function is competitive
- Final value for the strong coupling lower than before mainly due to changes in exp. BFs



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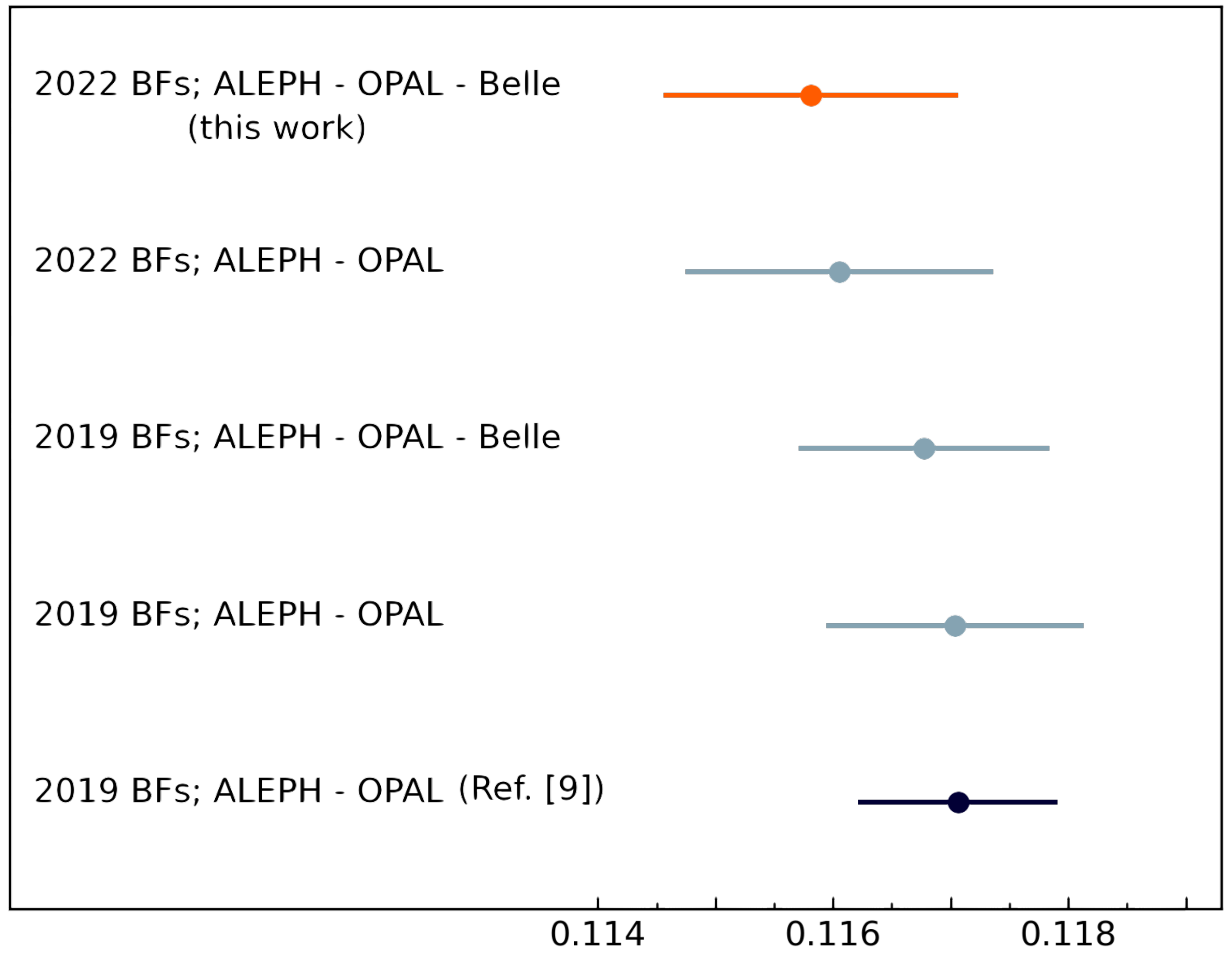
DB, Eiben, Golterman, Maltman, Mansur, and Peris, 2502.08147



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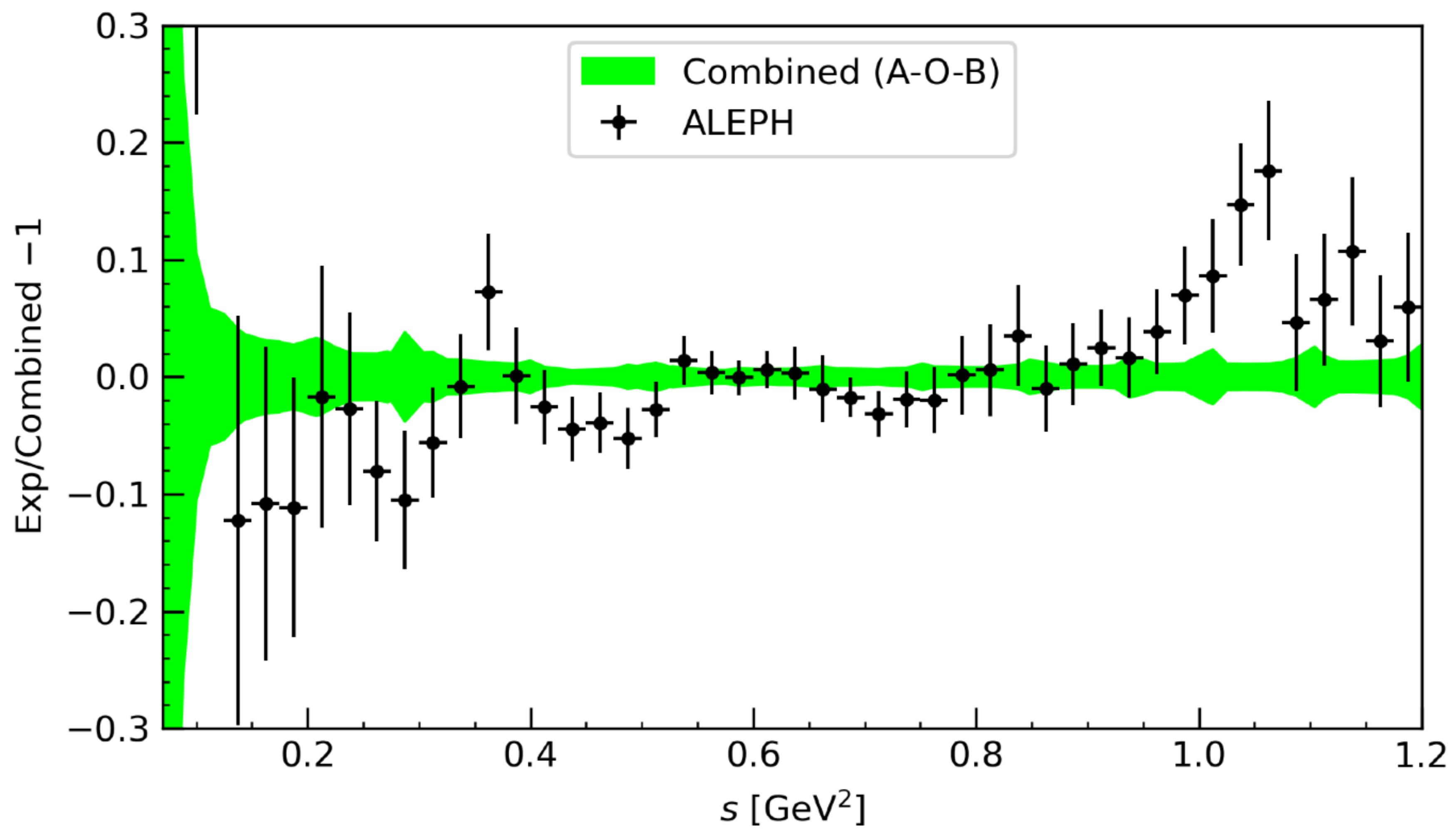
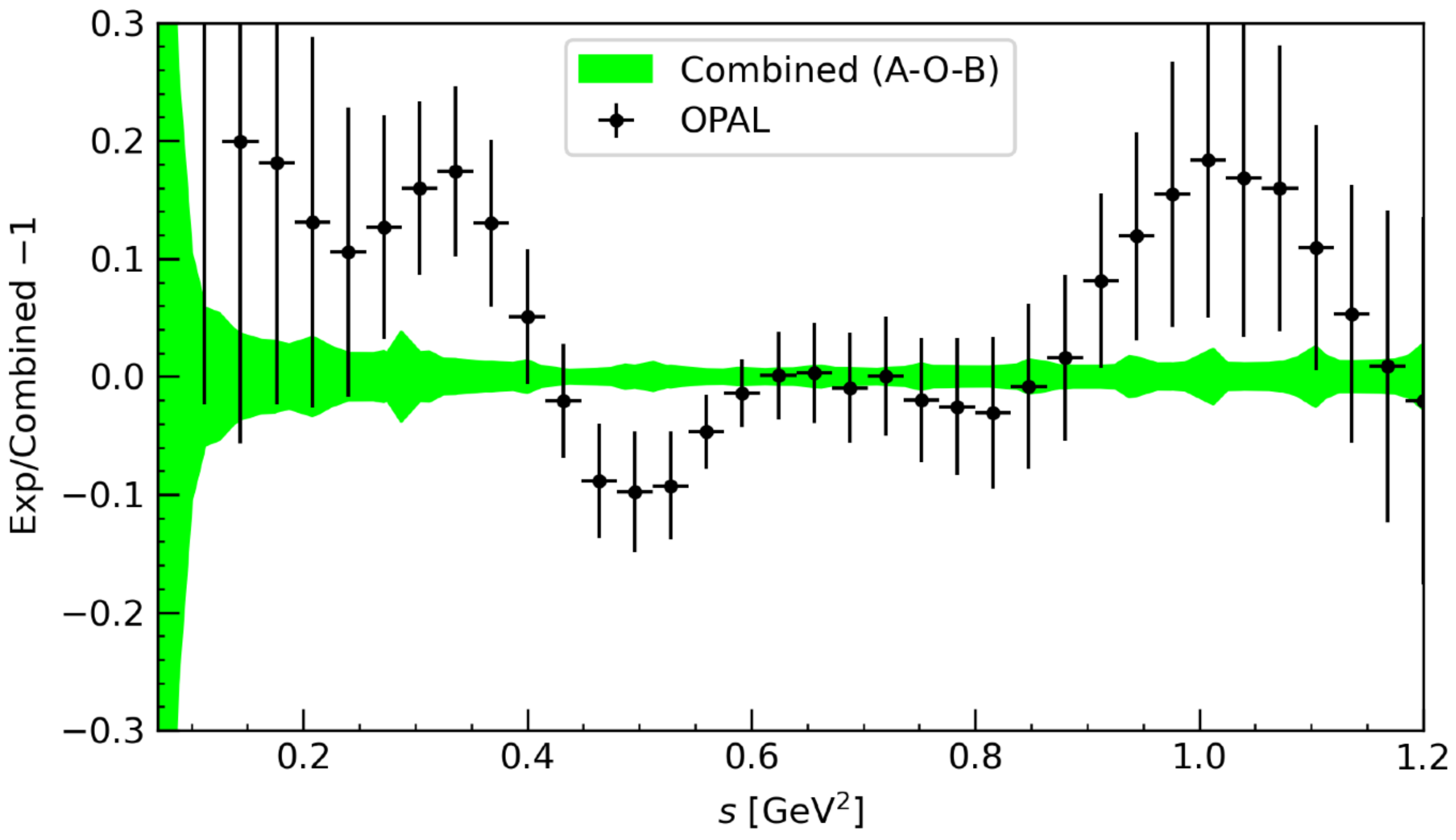
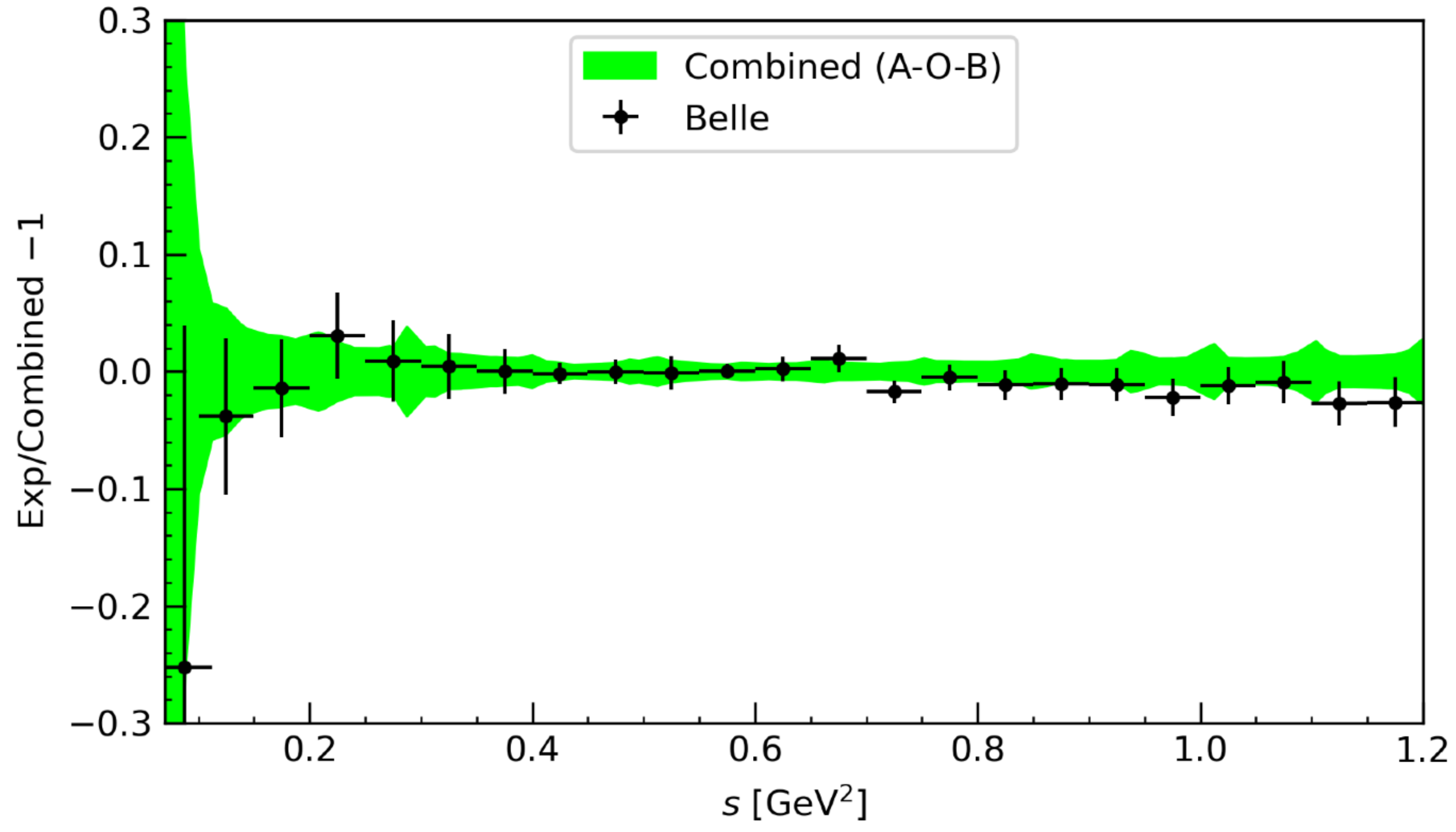
Extra

Improved vector-isovector spectral function: 2π channel

Combination of 2π spectra (combined results in green, A-O-B = ALEPH, OPAL, and Belle)

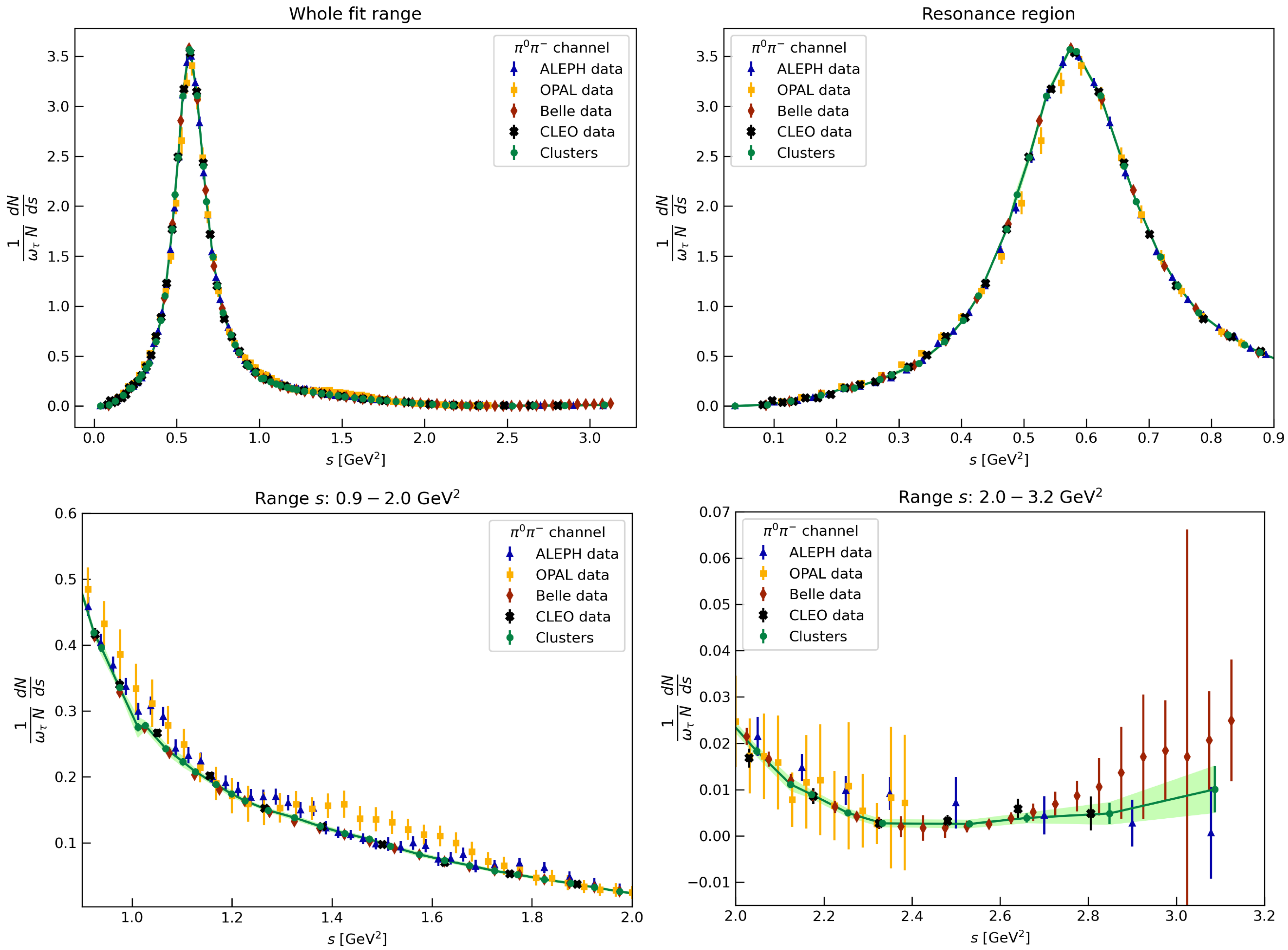






2π data combination dominated by Belle, but Belle spectrum systematically lower for large invariant masses

inclusion of CLEO 2π data



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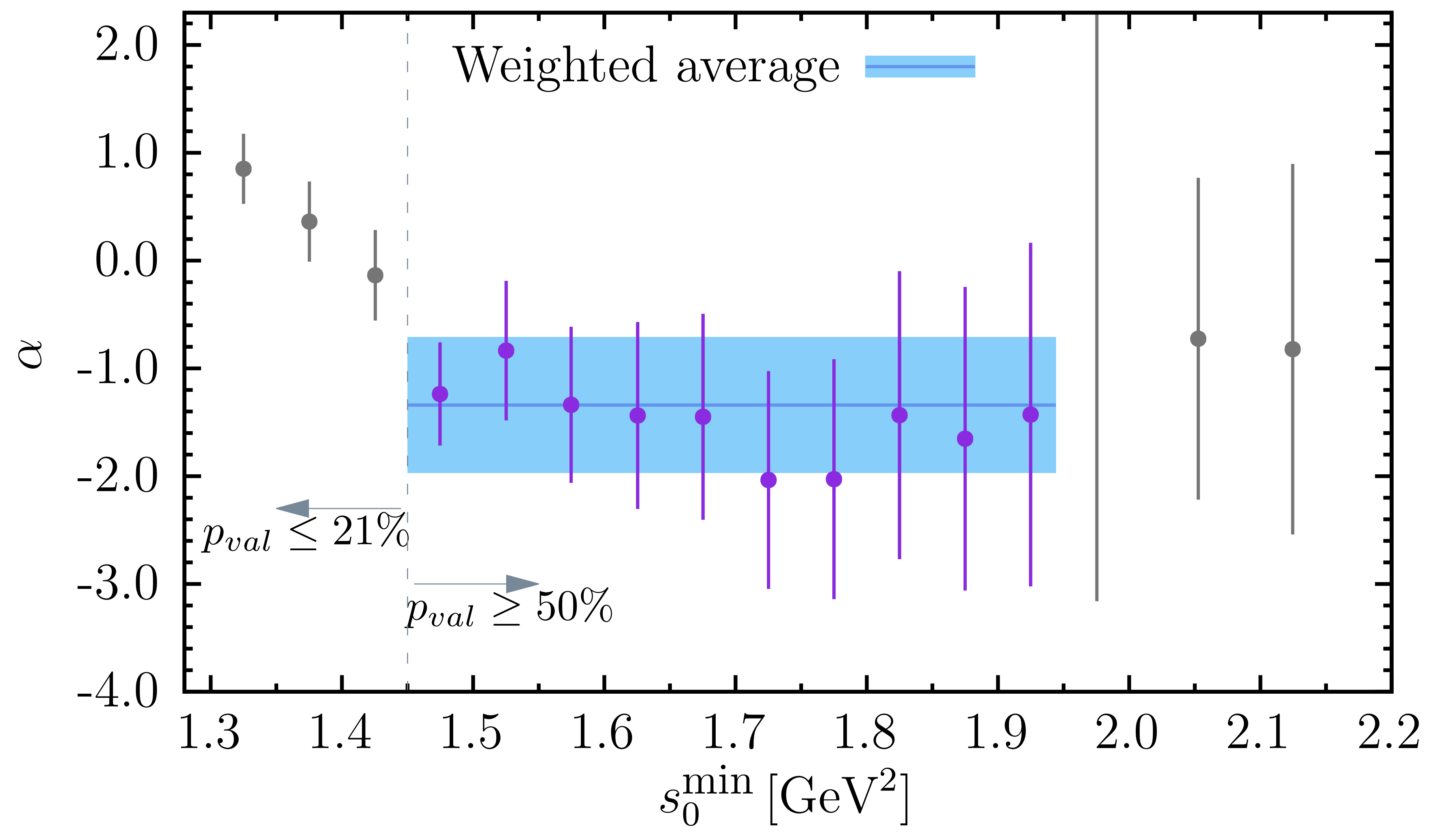
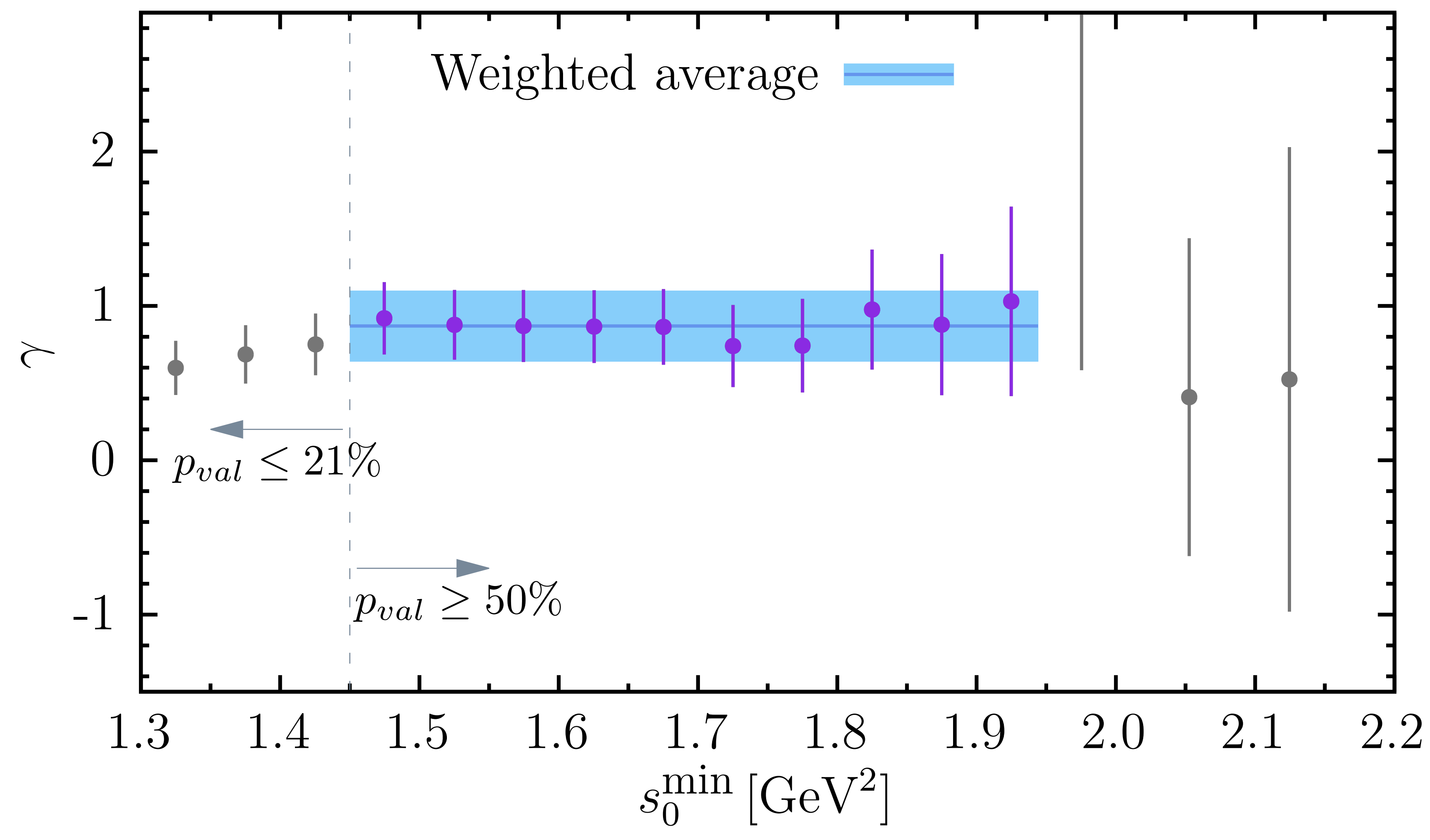
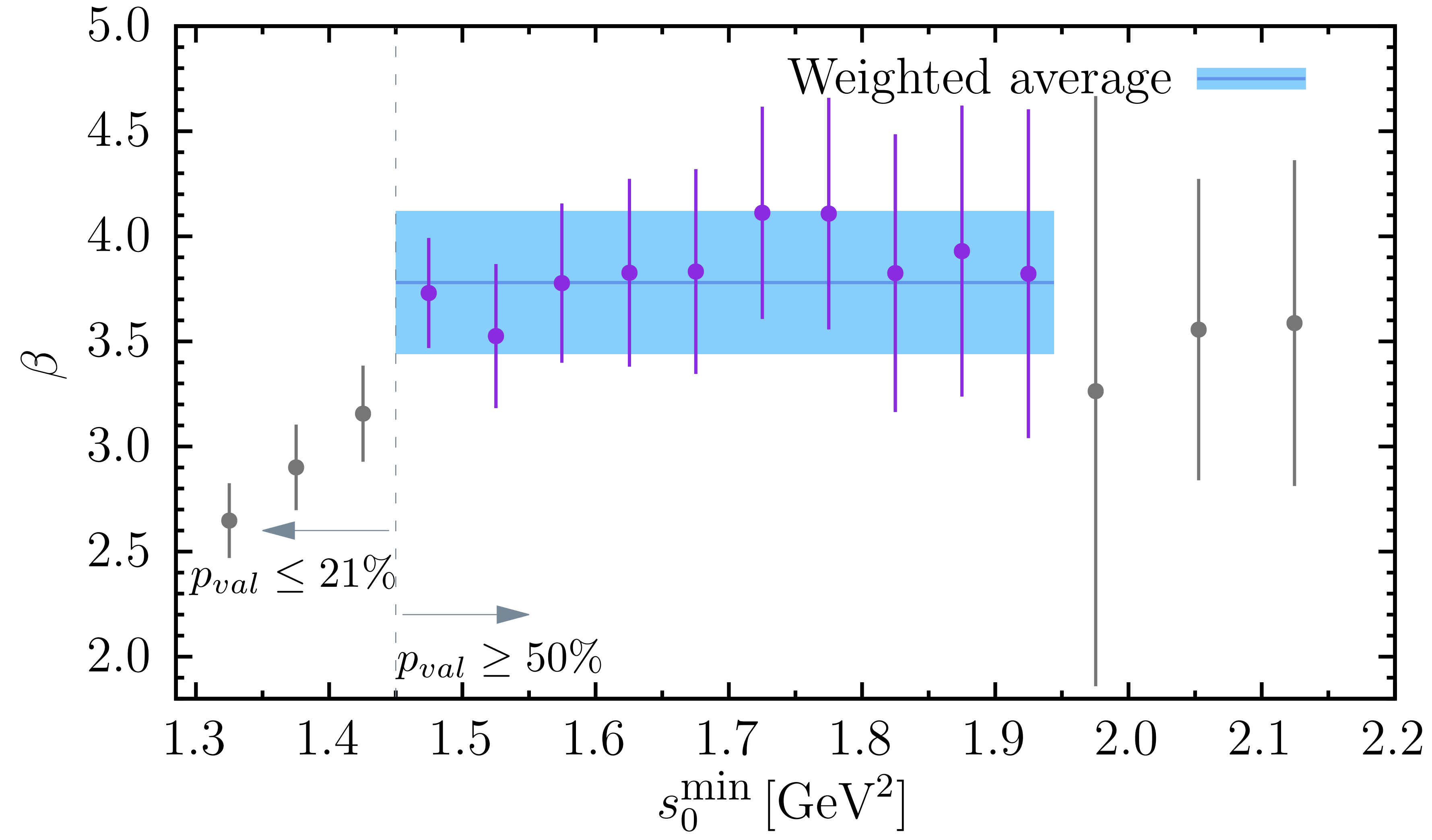
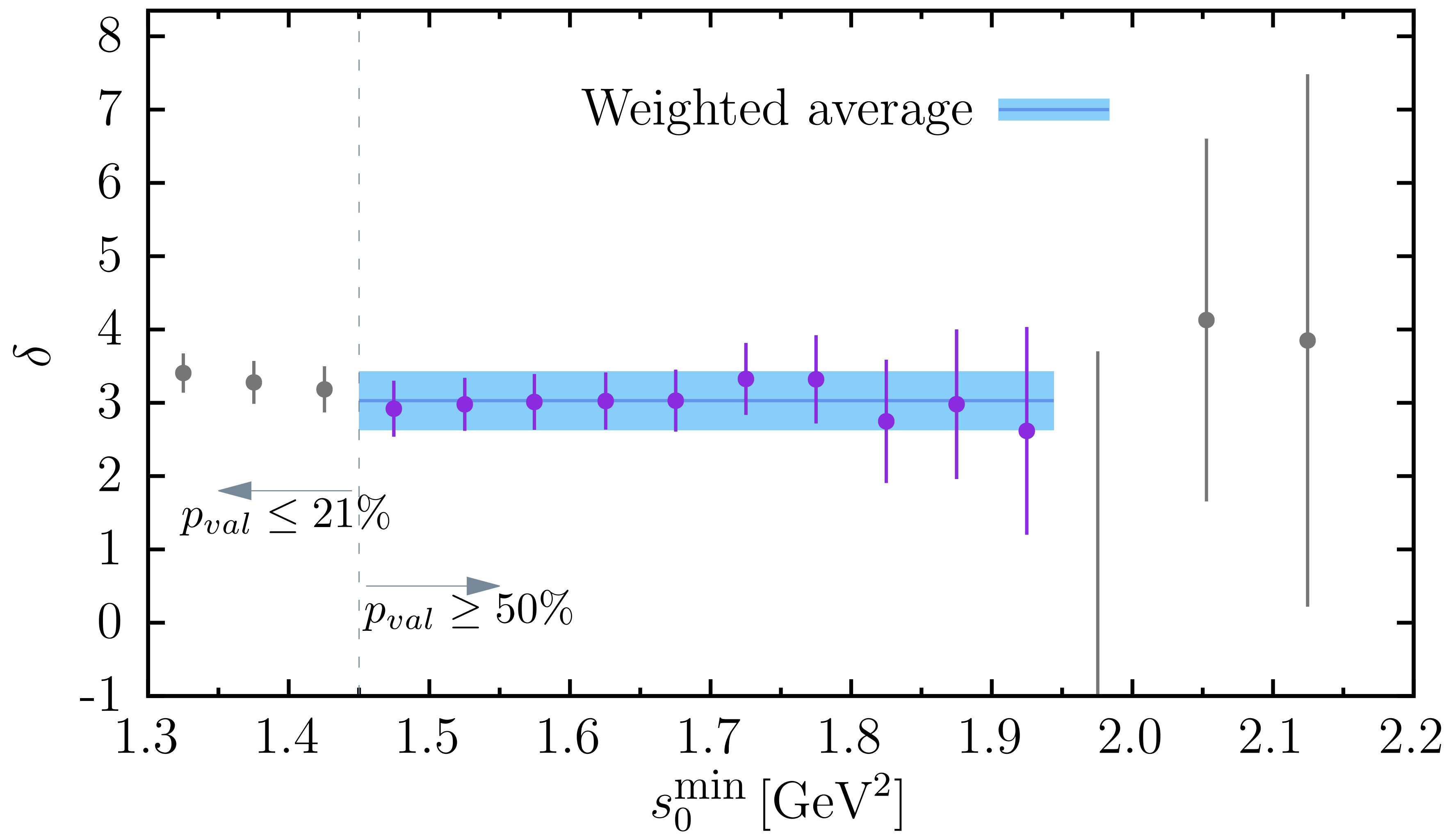
Duality Violation parameters

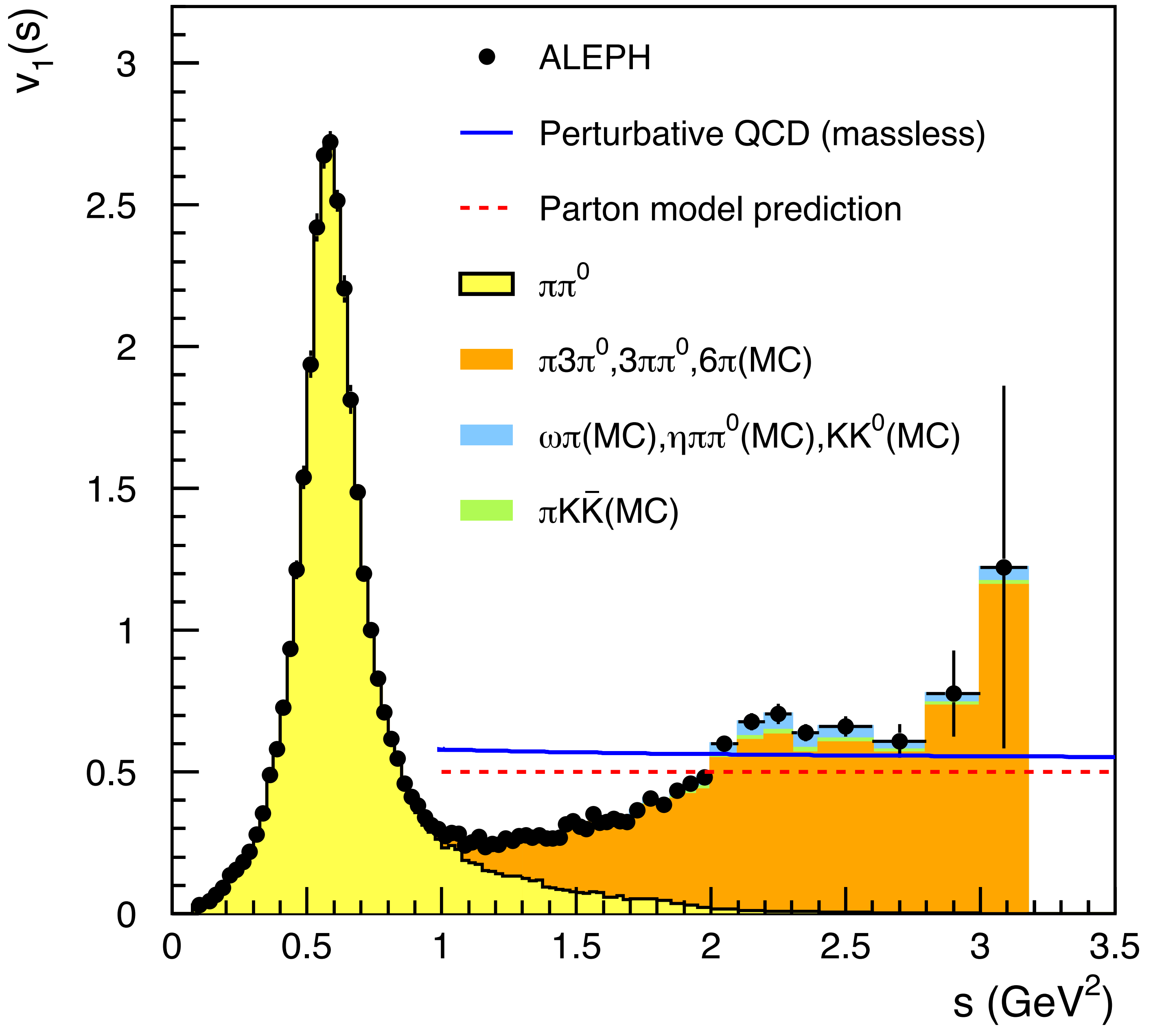


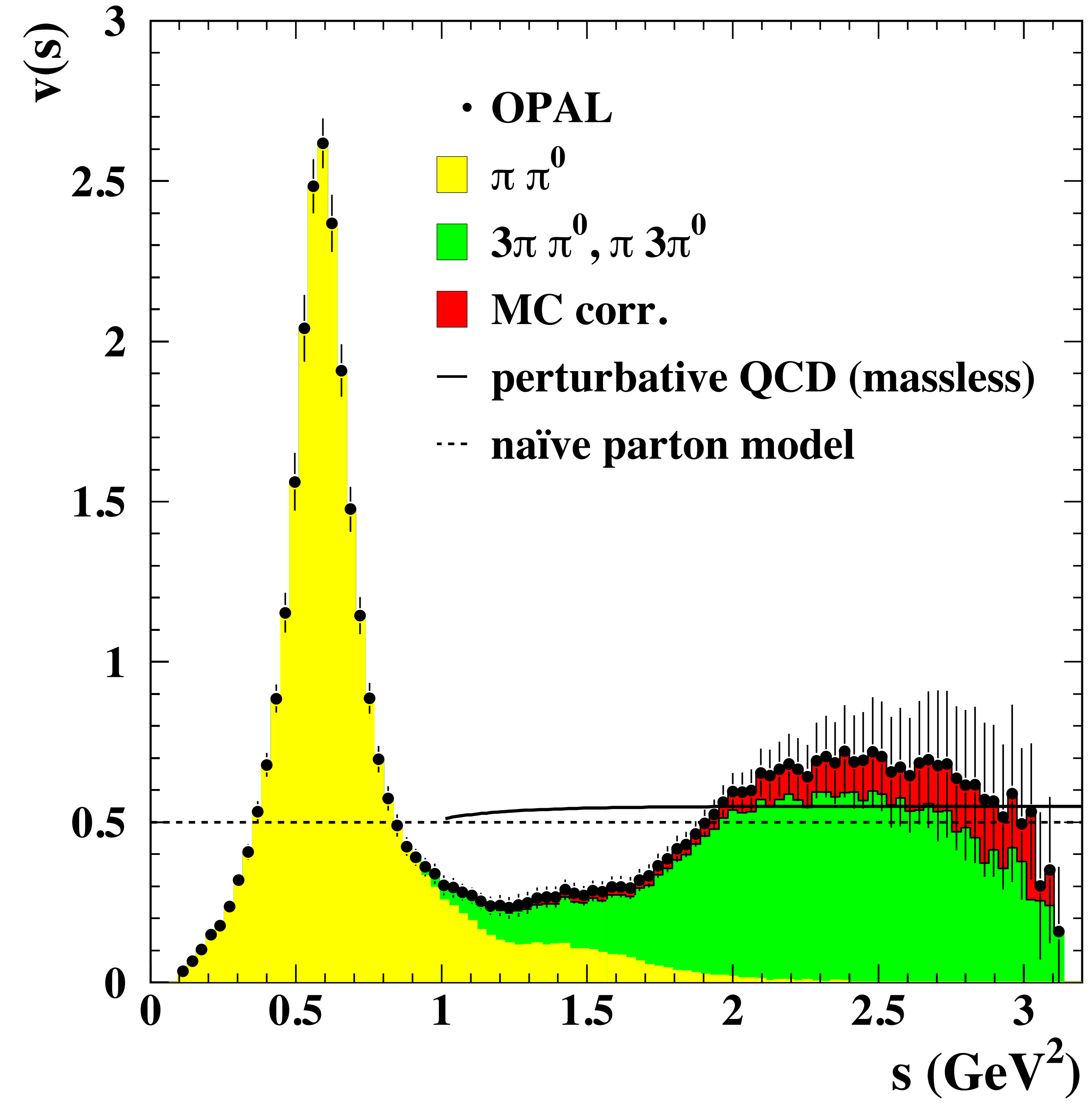


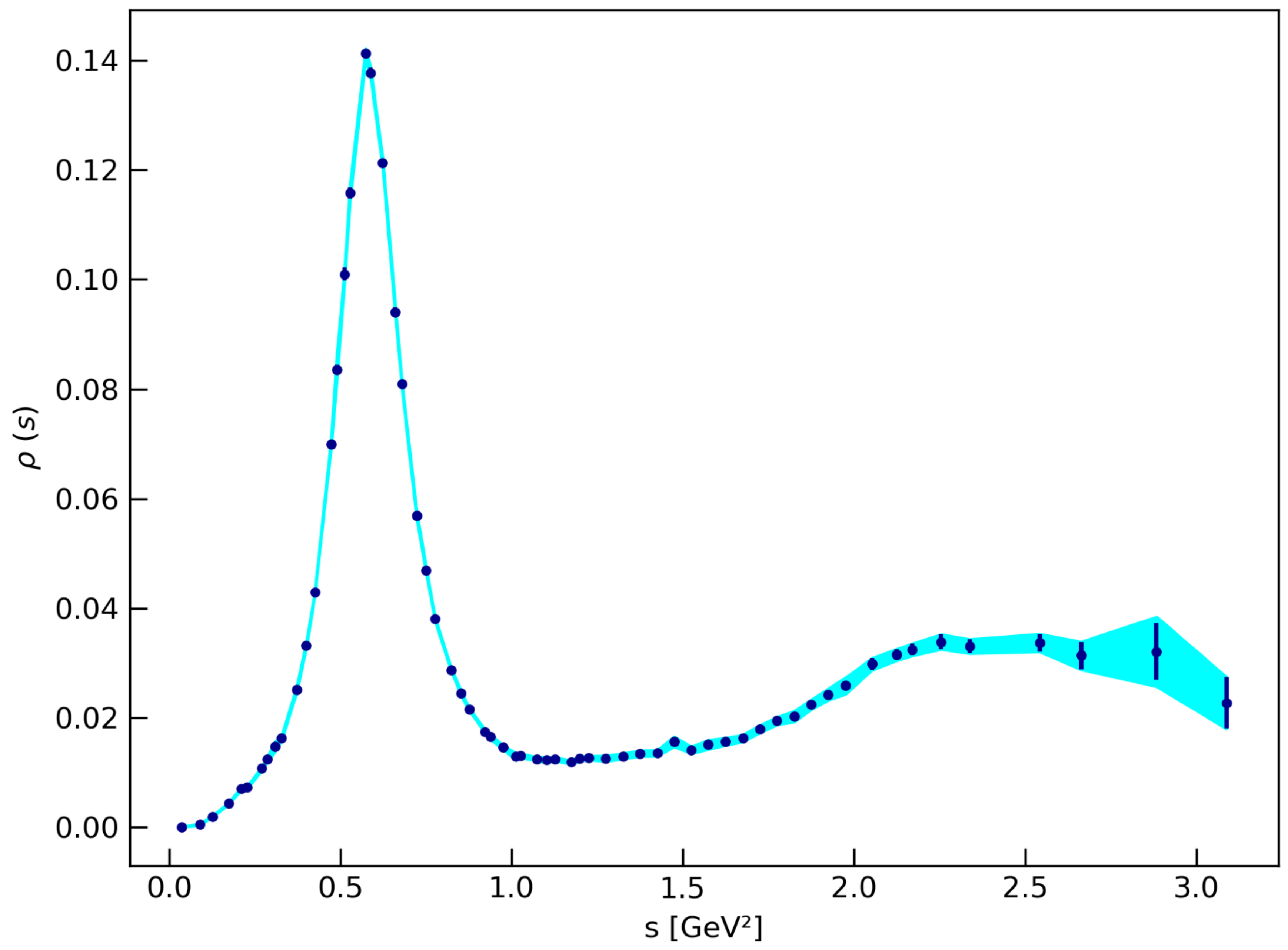




Vector-isovector spectral functions







(this work) DB, Eiben, Golterman, Maltman, Mansur, and Peris, '25

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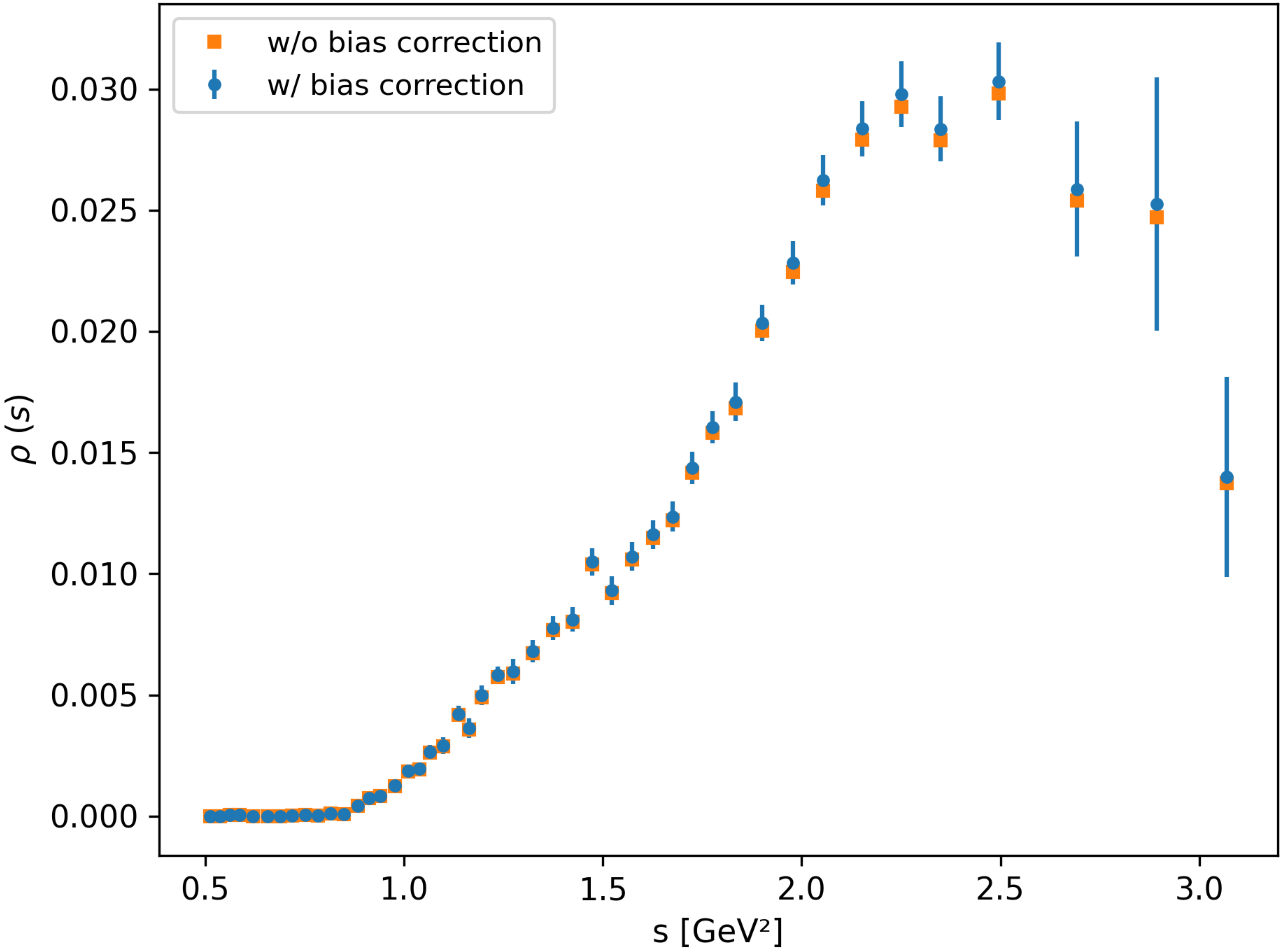
d'Agostini bias correction



- We correct for the d'Agostini bias that arise from the multiplication by the Branching Fractions.
- This done with (an approximate version of) NNPDF's iterative algorithm

Ball et al. (NNPDF) '10

- The effect turns out to be small