FROM 1D TO 3D TOMOGRAPHY OF THE PROTON PDFS WITH SMALL-X RESUMMATION

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Marseille
- T05 -
QCD & Hadronic Physics



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Madrid UAH

talento cm Programa de atracción de talento investigador Comunidad de Madrid

EUGLOH

PDFs & Resummation

The multidimensional structure of the proton

Wigner distributions $\rho\left(x, \mathbf{k}_T, \mathbf{b}_T\right)$ **5D**

$\int d^2\mathbf{k}_T$

$\int d^2\mathbf{b}_T$

Generalized Parton Distributions **GPDs** **3D**



Transverse Momentum Distributions **TMDs** **3D**

$\int d^2\mathbf{k}_T$

Parton Distribution Functions **PDFs** **1D**

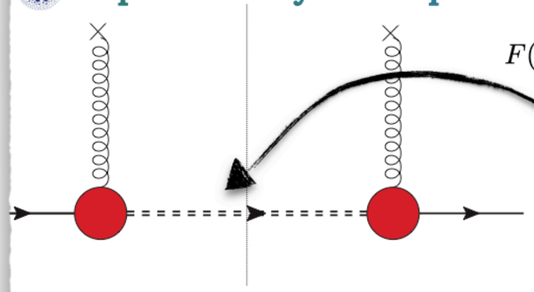
slide adapted from C. Bissolotti

Spectator-model gluon TMD PDFs

Our model at a glance

Spectator-system spectral-mass function

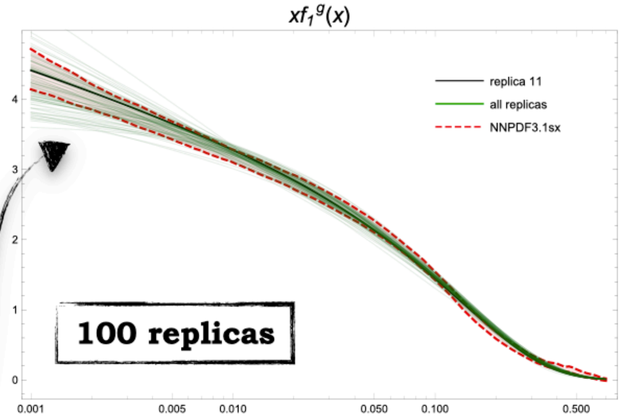
$$F(x, \boldsymbol{p}_T^2) = \int_M^\infty \mathrm{d}M_X \, \rho_X(M_X) \, \hat{F}(x, \boldsymbol{p}_T^2; M_X)$$



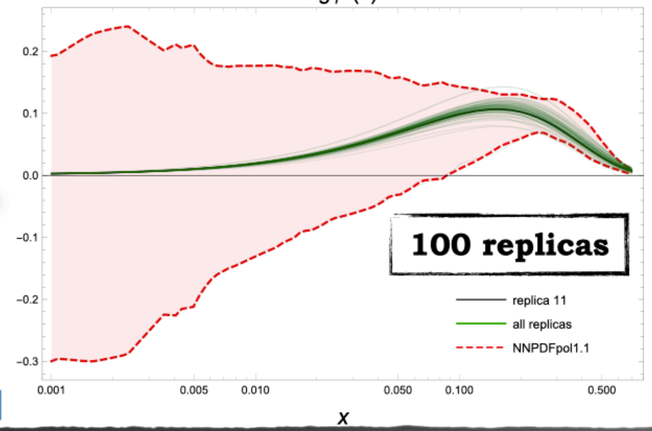
Instead of a single on-shell spectator, a continuum of spectators

Link with collinear factorization

p_T-integrated TMDs **have to** reproduce PDFs at the lowest scale (Q_0) *before* evolution



Spectral function **learns** small- and moderate-x info encoded in **NNPDF** collinear parametrizations (`NNPDF3.1sx + NNPDFpol1.1`)



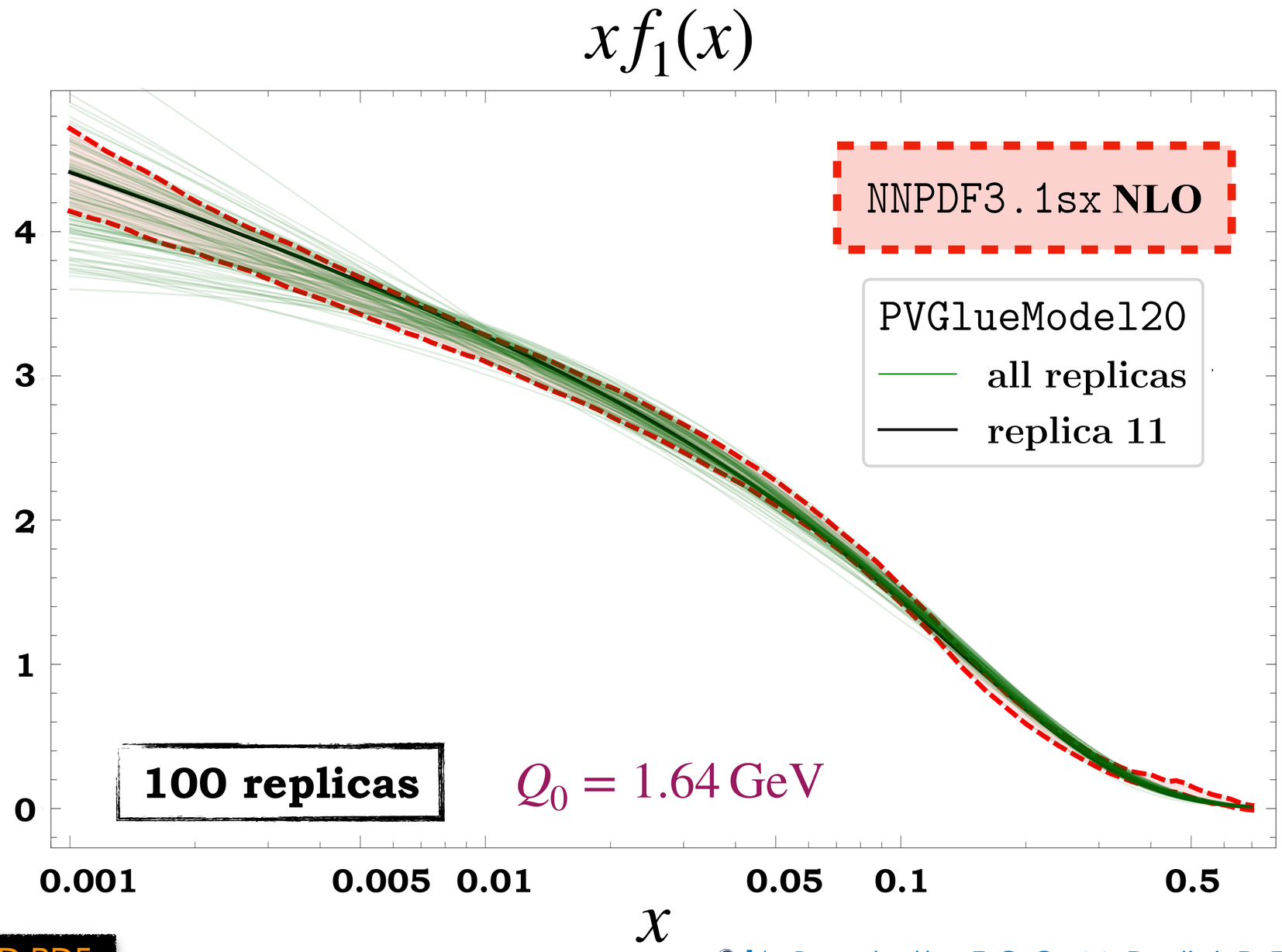
- ☑ **Simultaneous fit** of f_1 and g_1 PDFs
- ☑ Inclusion of small-x resummation effects (**BFKL**)
- ☑ Calculation of all leading-twist T-even gluon TMDs

2.3 Modeling gluon TMDs 🔗 [A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]

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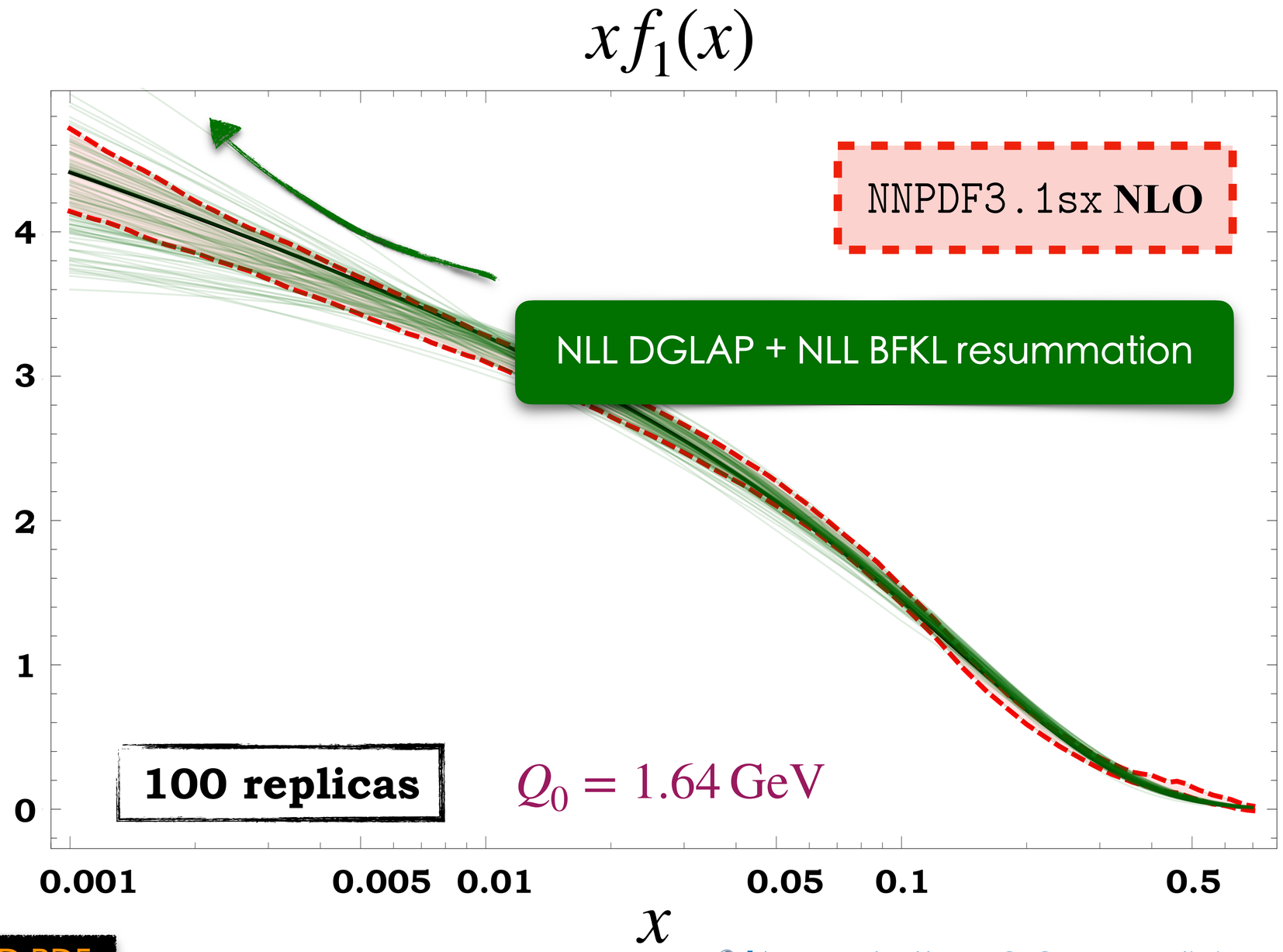
🔗 [my talk at the EIC User Group Early Career Workshop 2022]

Unpolarized gluon collinear PDF



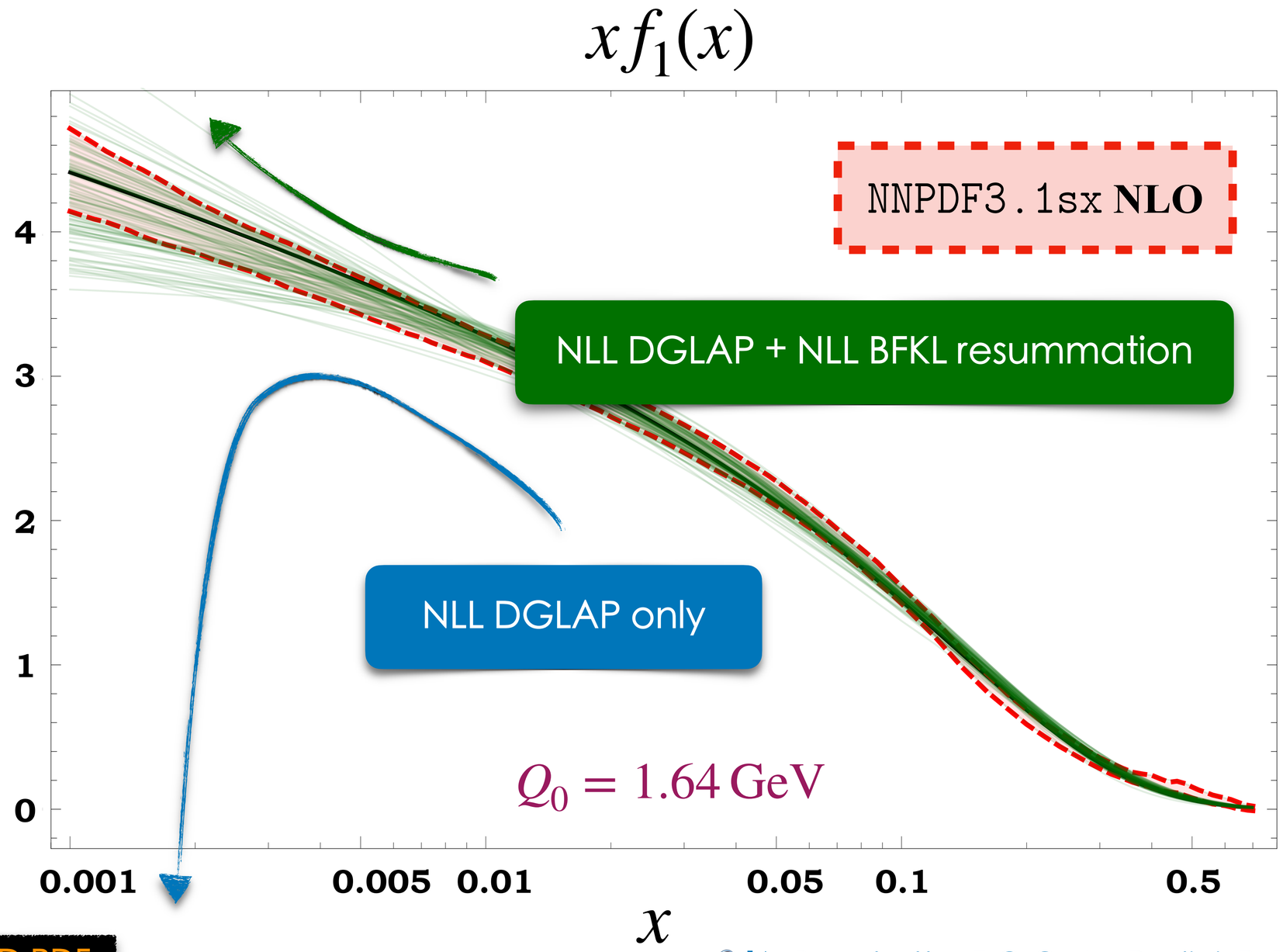
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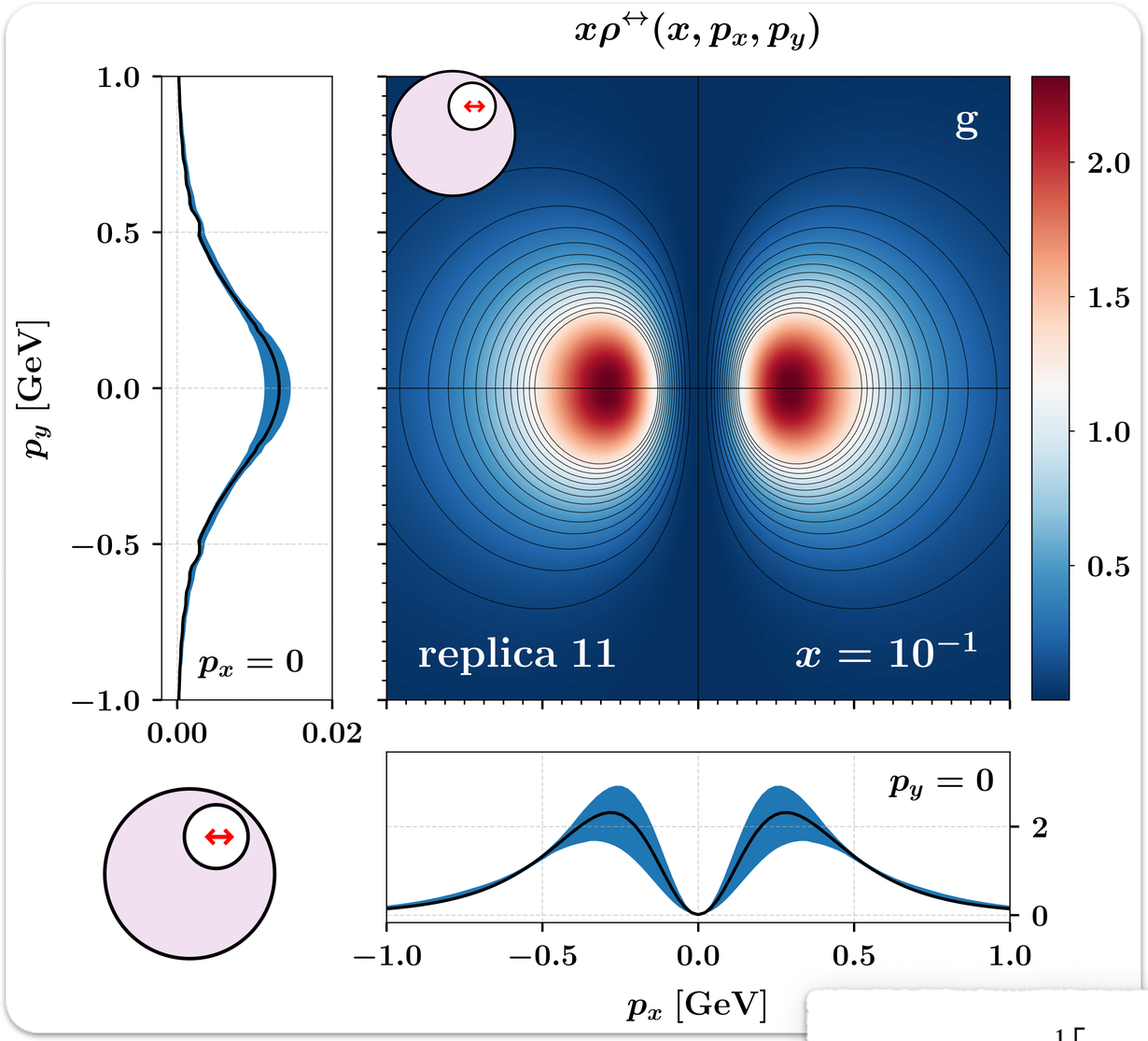
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Gluon Boer-Mulders effect

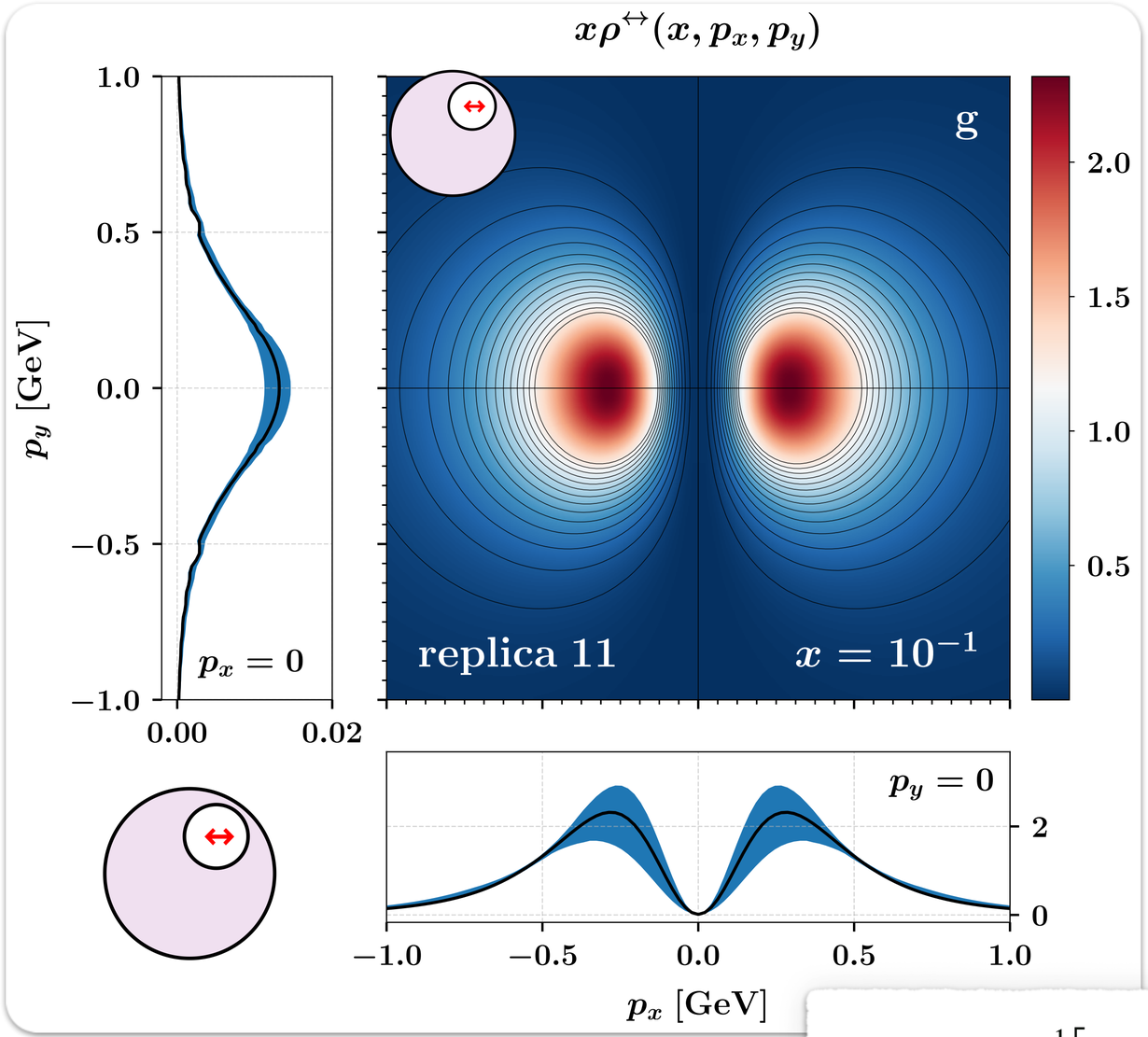
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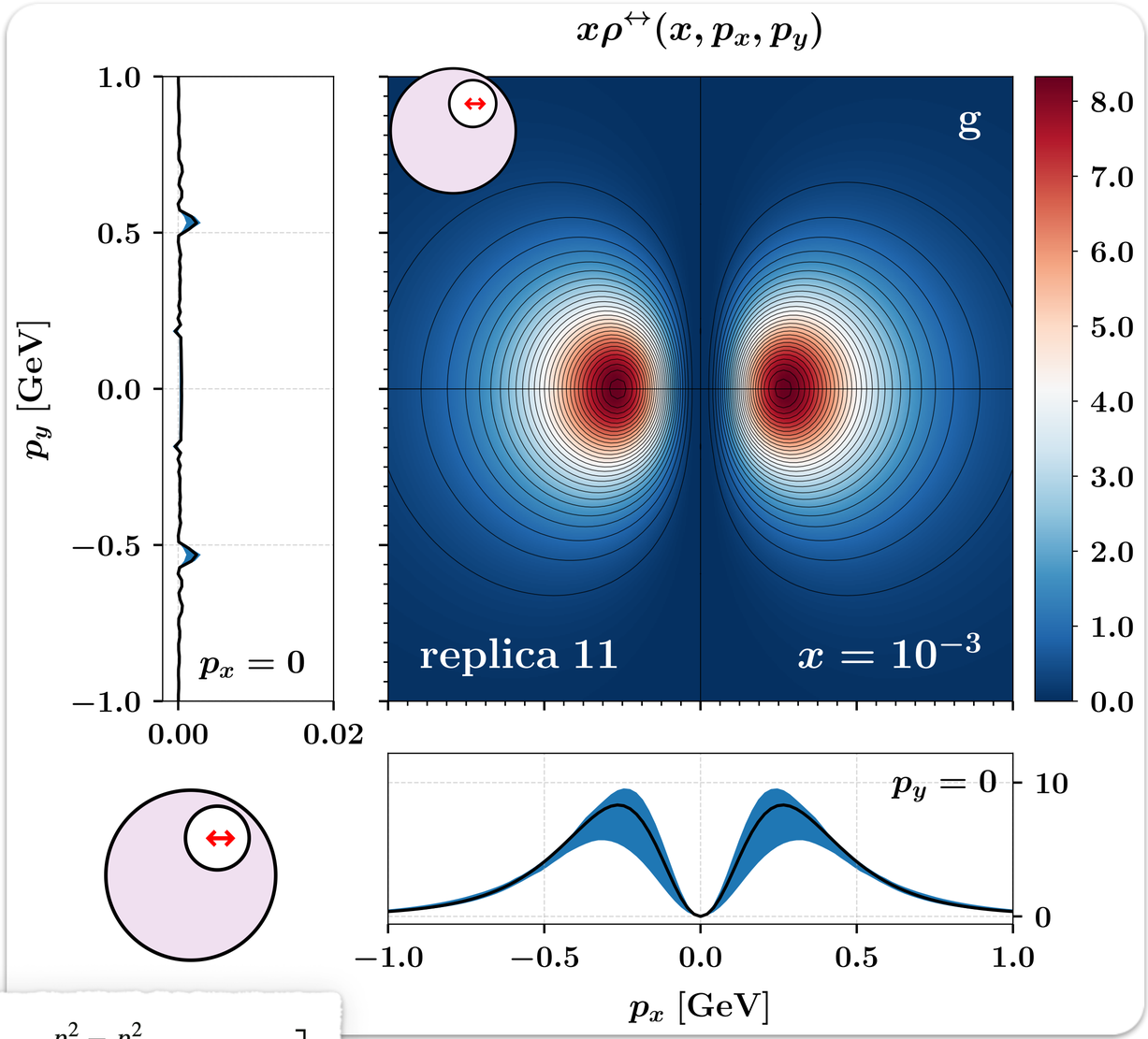


$$x\rho^{\leftrightarrow}(x, p_x, p_y) = \frac{1}{2}\left[x f_1^g(x, \boldsymbol{p}_T^2) + \frac{p_x^2 - p_y^2}{2M^2}\, x h_1^{\perp g}(x, \boldsymbol{p}_T^2)\right]$$

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UGDs from HELL

Small-x resummation from ABF-HELL

Example: DIS cross section within QCD collinear factorization

Collinear factorization

DGLAP *evolution*

$$\sigma(x,Q^2) = \int_x^1 \frac{dz}{z}\, C\left(z,\alpha_s(Q^2)\right) f\left(\frac{x}{z},Q^2\right)$$

$$\mu^2 \frac{d}{d\mu^2} f(x,\mu^2) = \int_x^1 \frac{dz}{z}\, P\left(z,\alpha_s(\mu^2)\right) f\left(\frac{x}{z},\mu^2\right)$$

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High-energy (small-x) *log enhancement* in ***coefficient functions*** & ***splitting functions***

Small-x logs are *single* logs

$$\alpha_s^n \frac{1}{x} \log^k \frac{1}{x} \quad (0 \le k \le n-1)$$

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¿ How to encode the small-x resummation ?

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¡ Let us make us of k_t factorization !

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$$-x\frac{d}{dx}\mathcal{F}(x,k_t^2) = \int_0^\infty \frac{dq_t^2}{k_t^2}\, \mathcal{K}\left(\frac{k_t^2}{q_t^2},\alpha_s(\cdot)\right) \mathcal{F}\left(x,q_t^2\right)$$

k_t factorization

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UGD

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Problem: NLL/LL corrections to the BFKL kernel are large and negative ⇒ ***strong instabilities***

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- **Running-coupling resummation** to turn back the small-N singularity to a simple pole

Small-x resummation from ABF-HELL

Small-x & large-Q^2 regime:

Collinear factorization

k_t factorization

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Resummed collinear coeff. function

Off-shell coeff. function

$$C(N,\alpha_s) = \int_0^\infty dk_t^2\, \mathcal{C}\left(N,\frac{k_t^2}{Q^2},\alpha_s\right) \mathcal{U}\left(N,\frac{k_t^2}{Q^2}\right)$$

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UGD **Evolutor** **PDF**

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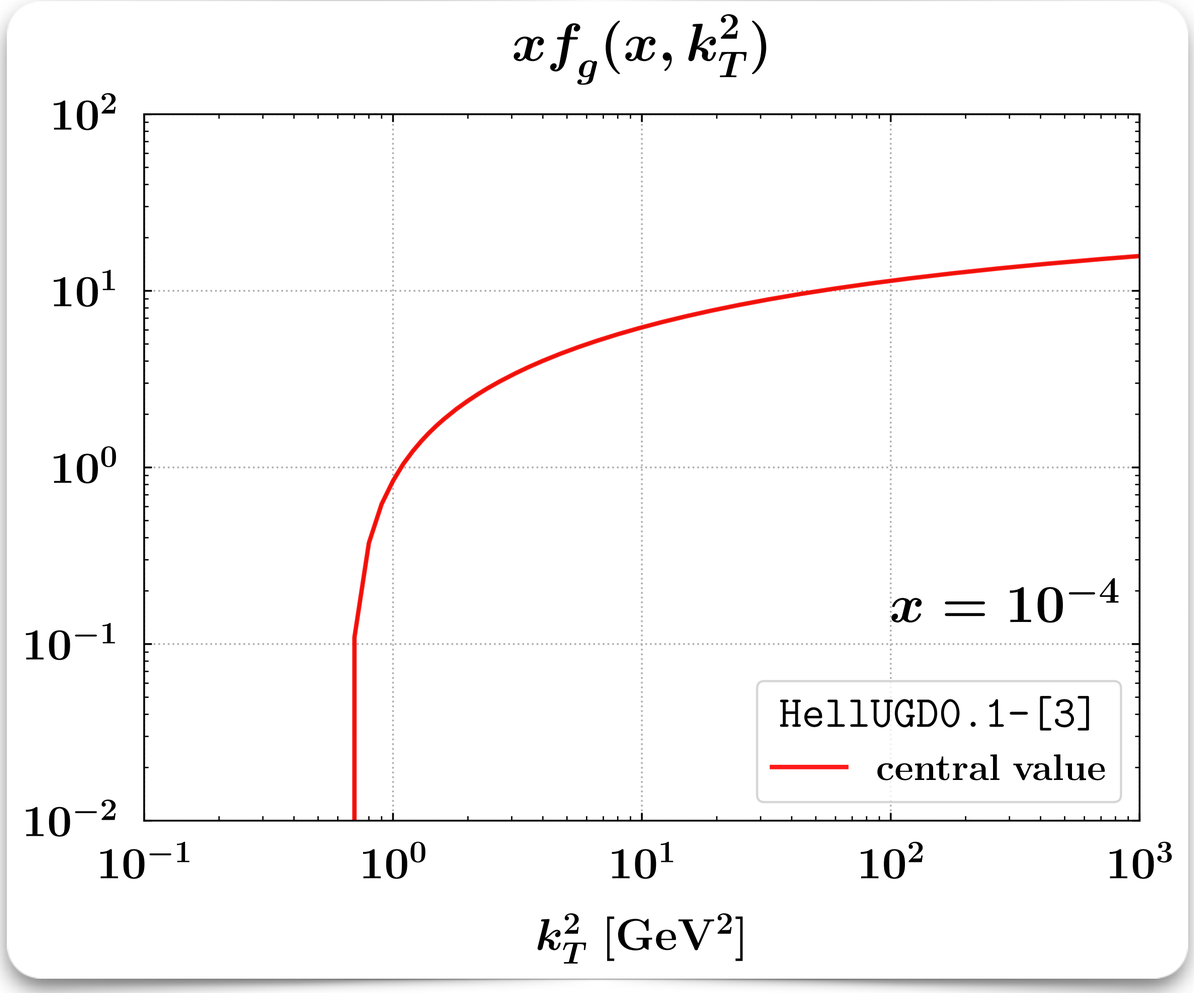
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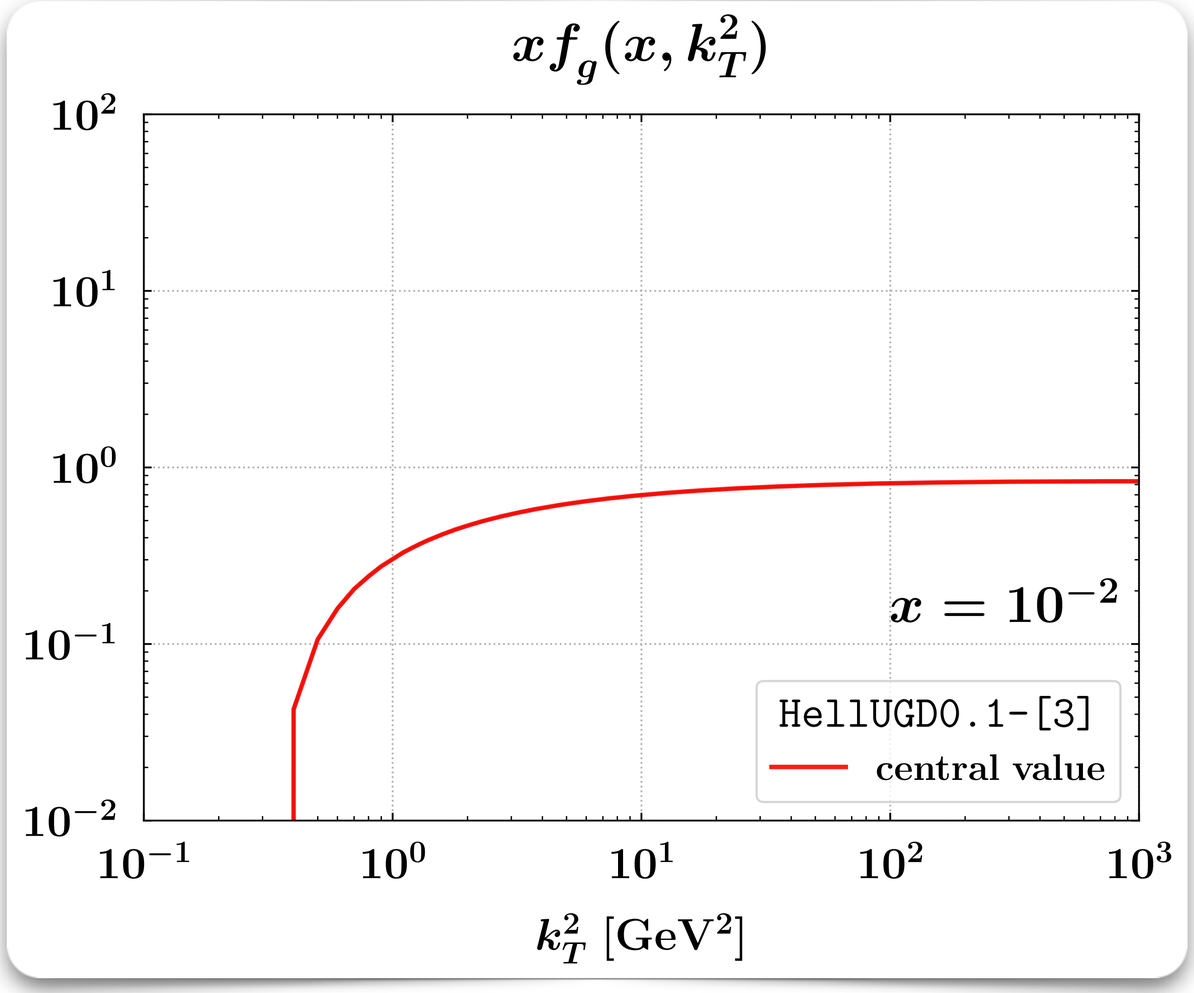
$$C(N,\alpha_s) = \int_0^\infty dk_t^2\, \mathcal{C}\left(N,\frac{k_t^2}{Q^2},\alpha_s\right) \mathcal{U}\left(N,\frac{k_t^2}{Q^2}\right)$$

$$\mathcal{U}\left(N,\frac{k_t^2}{Q^2}\right) = \frac{d}{dk_t^2} \exp \int_{Q^2}^{k_t^2} \frac{d\nu^2}{\nu^2} \gamma(N,\alpha_s(\nu^2))$$

Resummed anomalous dimension

The HELL UGDs

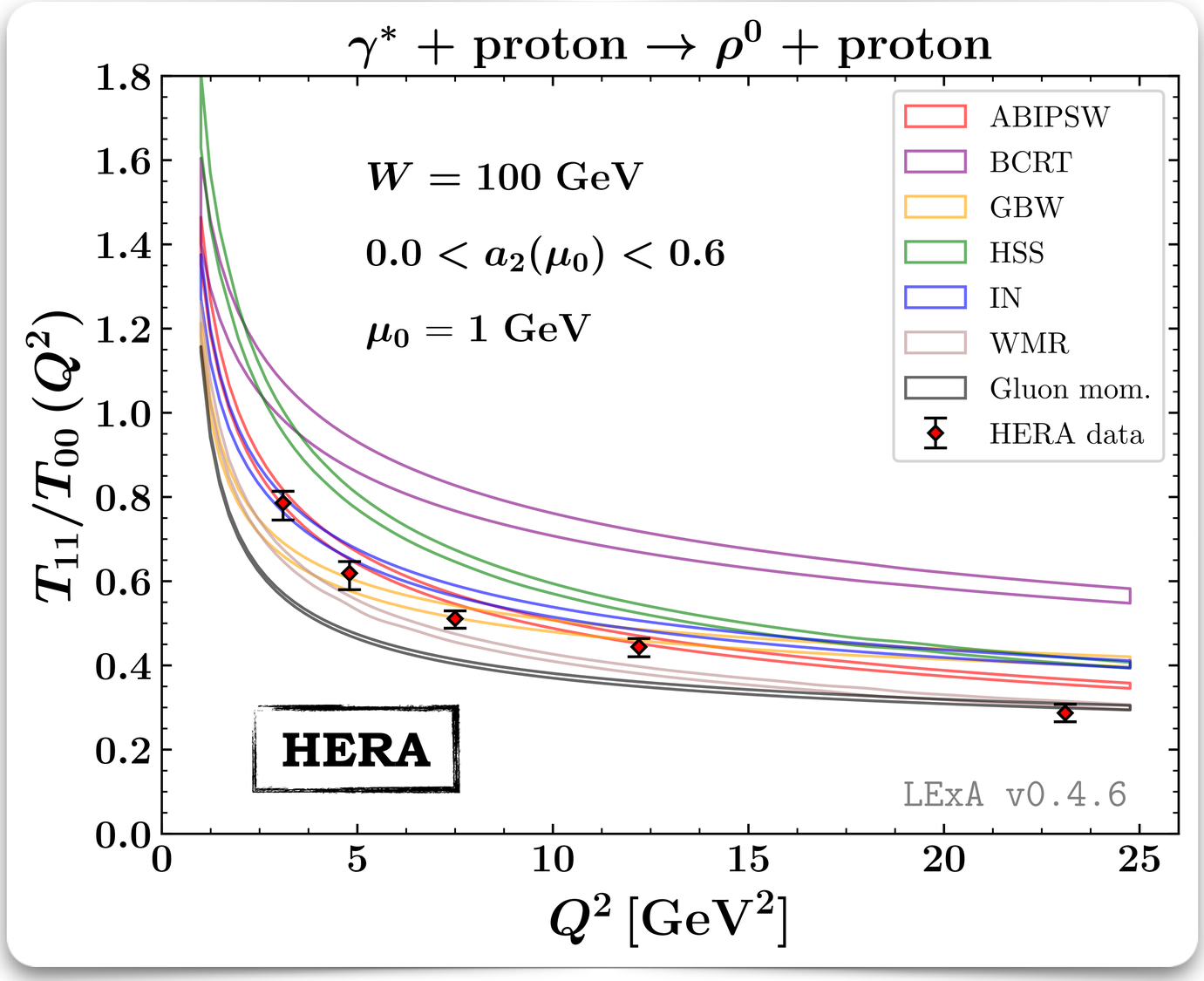




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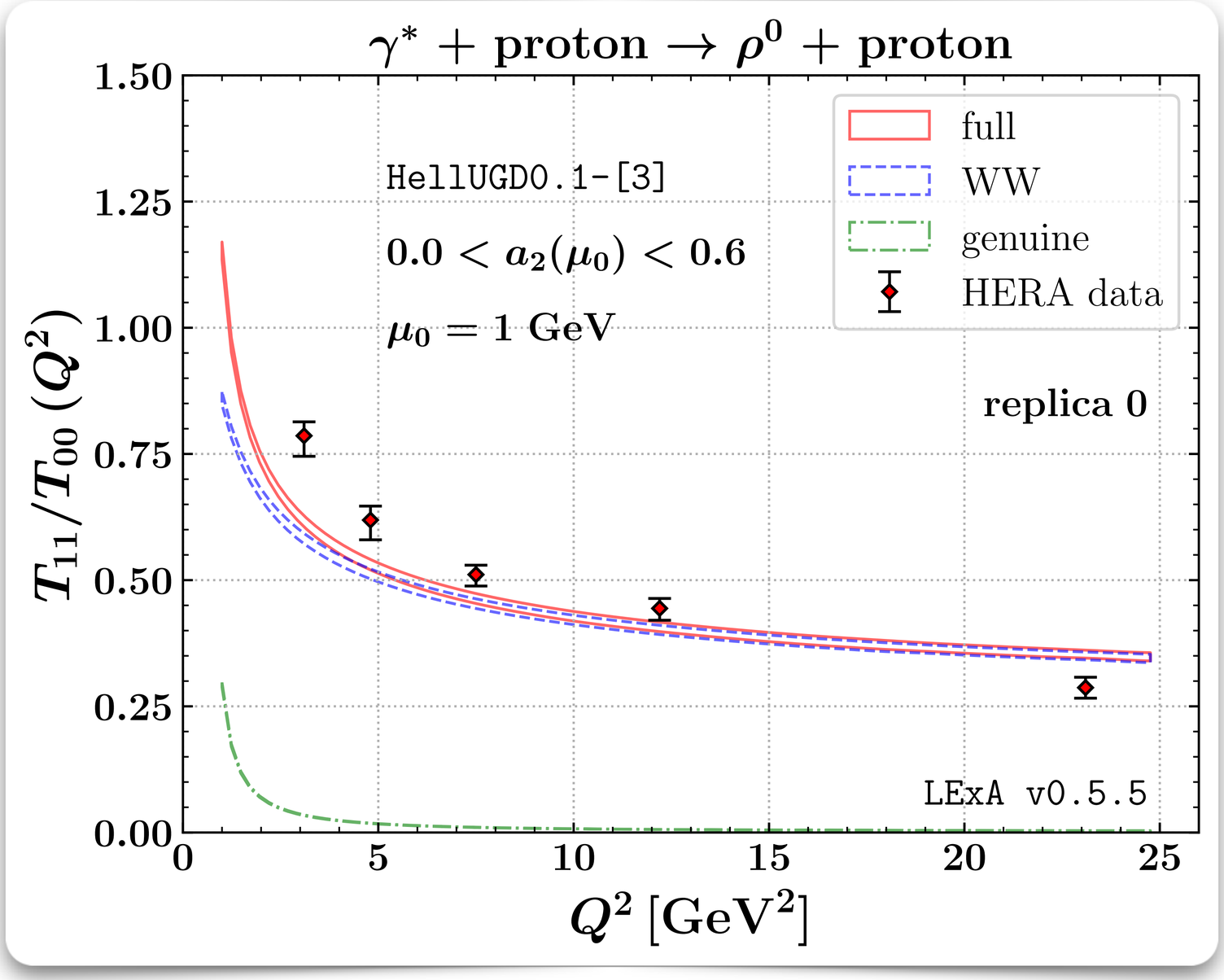
Applications with JETHAD

Exclusive forward ρ-meson: Helicity amplitudes



🔗 [A.D. Bolognino, F.G.C., D.Yu. Ivanov, A. Papa (2018)]
[A.D. Bolognino, PhD Thesis (2021)]

Helicity amplitudes and the HELL UGD



Road to the HELL UGD

Checkpoints & perspectives

Road to the HELL UGD



BFKL + DGLAP resummations ⇒ Proton structure at small-x

Road to the HELL UGD



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ABF-HELL UGD ⇒ solid methodology to hunt for precision

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Formal development & pheno applications



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¿ Possible extension to 3D tomography (TMD) ?

EXTRAS

The HELL methodology

HELL1.0

ABF for DGLAP resum

Coefficient function resum

M. Bonvini, S. Marzani, T. Peraro

[Eur. Phys. J. C 76 (2016) 11, 597]

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High Energy Large Logarithms

A small-x resummation code

$$\alpha_s \ln(1/x) \lesssim 1$$

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Improved β_0 resum

Full DIS(NC, CC, masses)

...road to HELL2.0

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M. Bovini

[Eur. Phys. J. C 78 (2018) 10, 834]

F. Silvetti, M. Bovini

[Eur. Phys. J. C 83 (2023) 4, 267]

Inclusive Higgs @LHC

Differential $Q\bar{Q}$ @LHC

HELL3.0

The JETHAD technology

High-energy resummation

Hunting BFKL

in semi-hard reactions

Mueller-Navelet, light hadrons

`ERIS` super-module

$$\alpha_s \ln(s) \lesssim 1$$



🔗 [Eur. Phys. J. C 81 (2021) 8, 691]

🔗 [Phys. Rev. D 105 (2022) 11, 114008]

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Δναμις, LExA, HATHOR super-modules

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Gluon TMD PDFs

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Quarkonium studies

from HF-NRevo

NRFF1.0 onium FFs

Vectors & pseudoscalars

`JETHAD` + DGLAP evolution operators

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JETHAD

v0.5.5

python + F

[Eur. Phys. J. C 81 (2021) 8, 691]

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Unintegrated parton densities

A (hybrid) high-energy factorization established

- **BFKL UGD**: pure small-x evolution, Reggeons
- CCH HEF, CCFM, PRA **uPDFs**; 🔗 small-x **PB TMDs**

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Small-x improved collinear PDFs

DGLAP description improved via BFKL

- **ABF approach**: PDFs + small-x resummed splitting

🔗 [R.D. Ball, V. Bertone, M. Bonvini, S. Marzani, J. Rojo, L. Rottoli (2018)]

Omnes viae small-x ducunt

Incomplete list of small-x formalisms → *linear* (BFKL) or *saturation* (BK/JIMWLK) effects embodied

Unintegrated parton densities

A (hybrid) high-energy factorization established

- **BFKL UGD**: pure small-x evolution, Reggeons
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DGLAP description improved via BFKL

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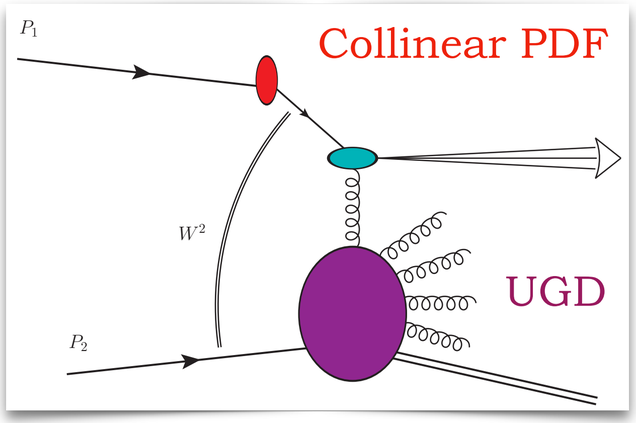
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Hybrid or pure factorization?

Forward emissions

* *Asymmetric* config. ↔ fast parton + small-x gluon

* Hybrid **high-energy**/**collinear** factorization



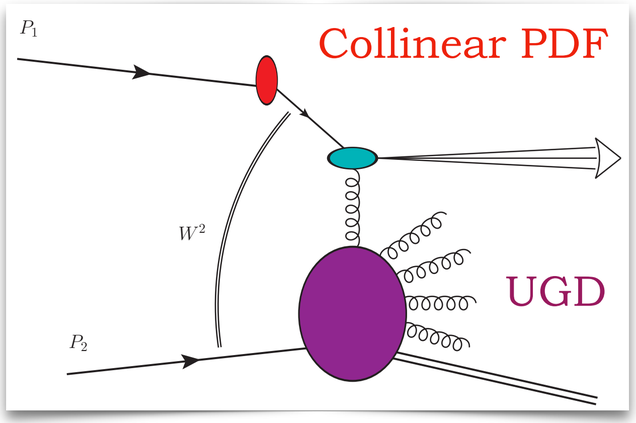
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* Phenomenology:
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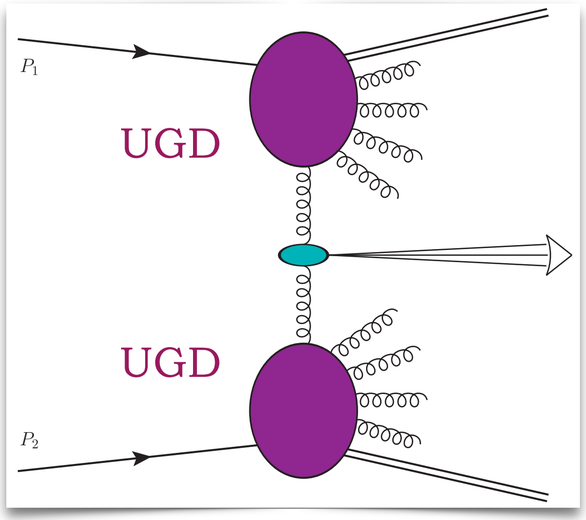
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Central emissions

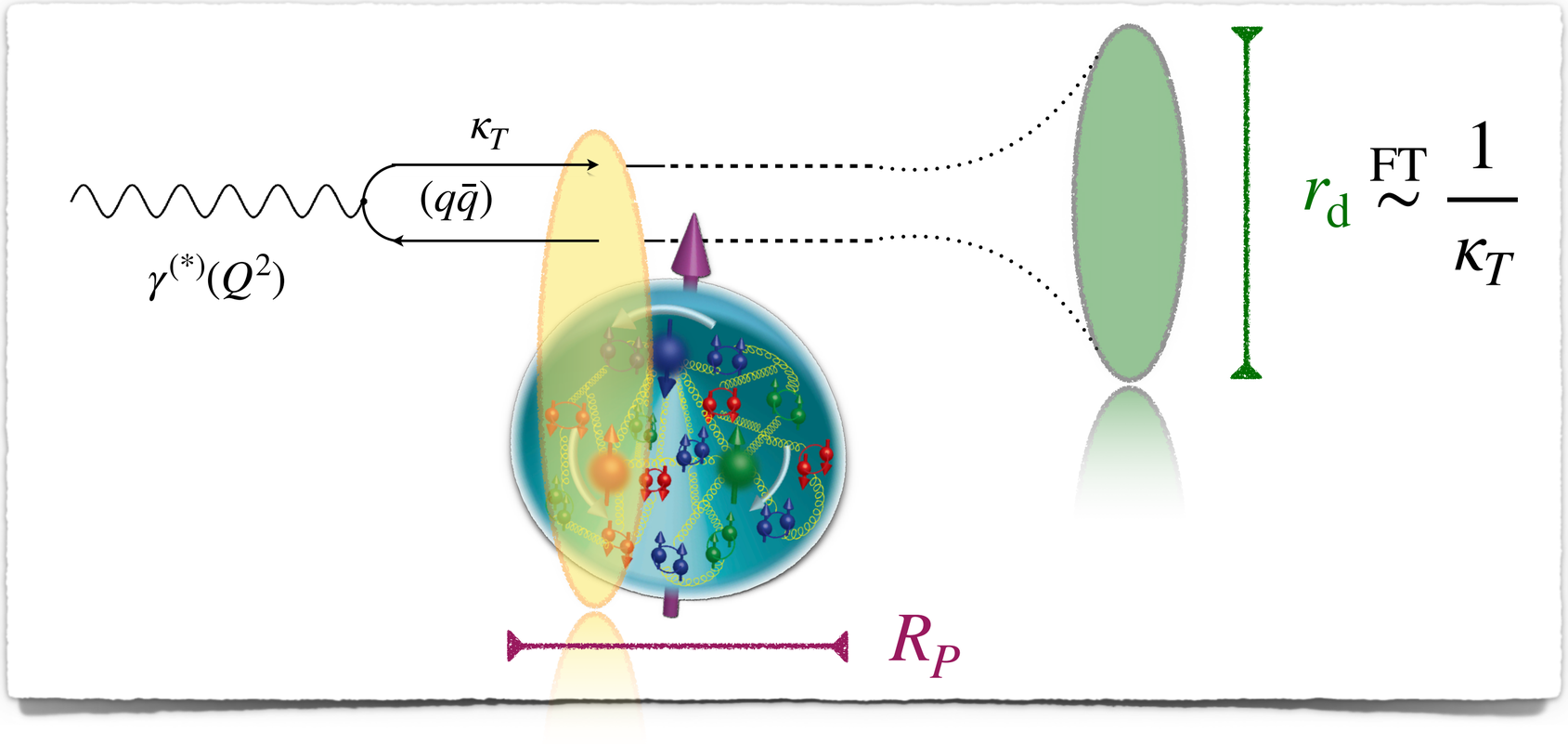
* *Gluon induced* ↔ small-x gluons
* Pure **high-energy** factorization



* Small-x dynamics to **enhance** f.o. description
* Phenomenology:
  central jet, Higgs or vector meson

Exclusive counterparts and ***lepto-hadronic*** channels

Forward scatterings & color dipoles



$$W_{\mu\nu} \propto \mathrm{Im}\left\{ i \int \mathrm{d}^4x\, e^{i\,q\cdot x} \langle P | \mathrm{T}\,[J_\mu(x)\, J_\nu(0)] | P \rangle \right\}$$

* Small-x $\Rightarrow$ ***Ioffe time*** $\gg$ R_P
* At least one J_μ <u>outside</u> proton…
* **…color dipole picture!**

Inclusive forward Drell-Yan dilepton production

- LHC, **forward region** $\rightarrow$ $(l^+ l^-)$ produced in the fragmentation region of h_2
  - ◇ Asymmetric configuration: $x_1 \gg x_2$, down to $x_2 \simeq 10^{-6}$

  $\Longrightarrow$ **possible small-x resummation effects expected!**

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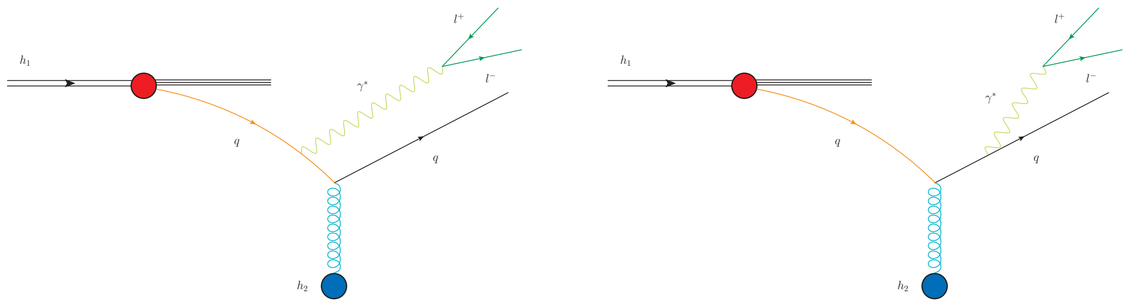
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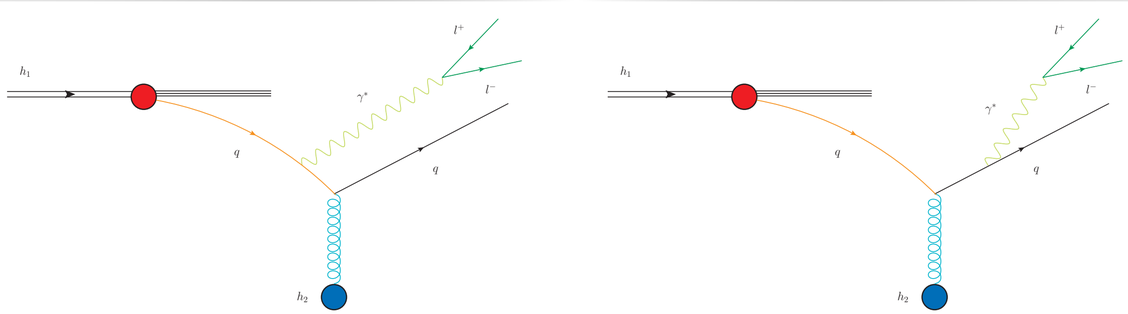
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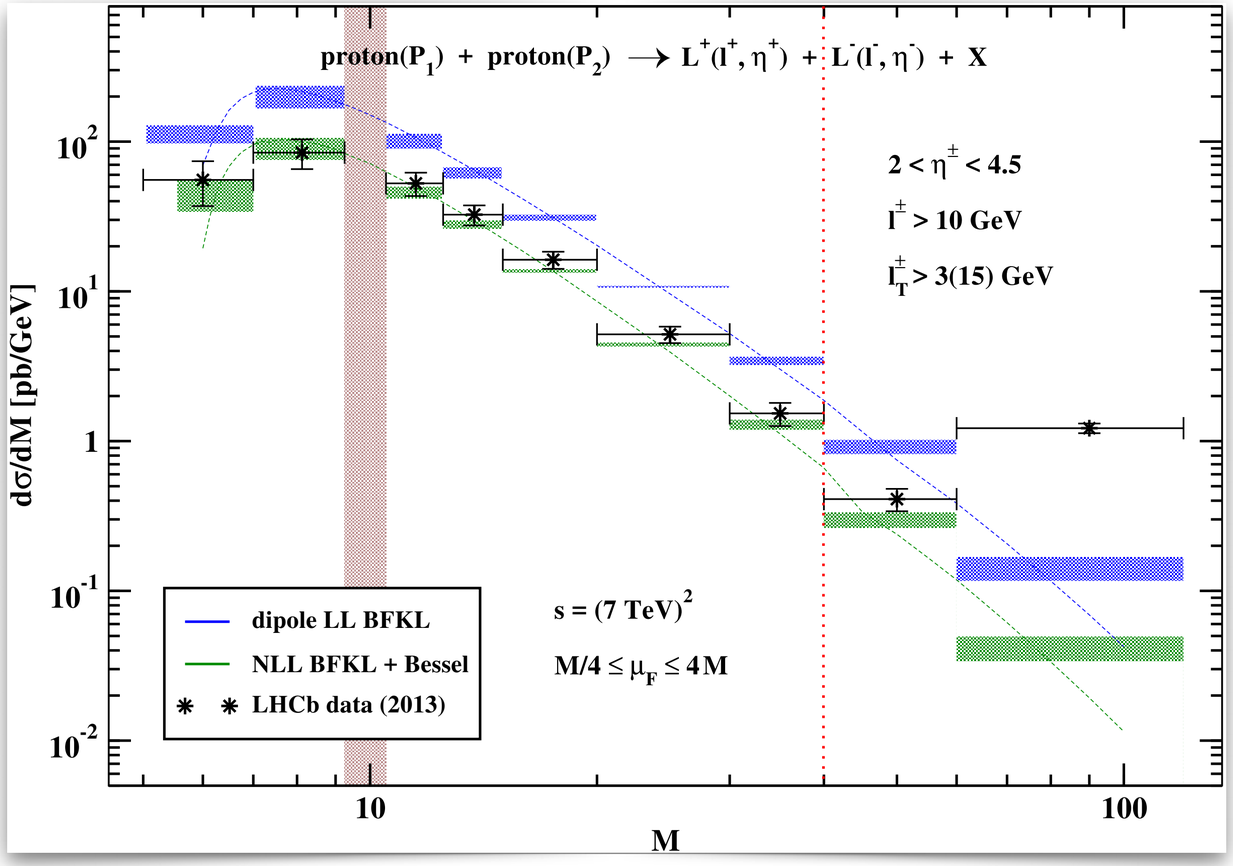
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[D. Brzeminski, L. Motyka, M. Sadzikowski, T. Stebel (2017)]

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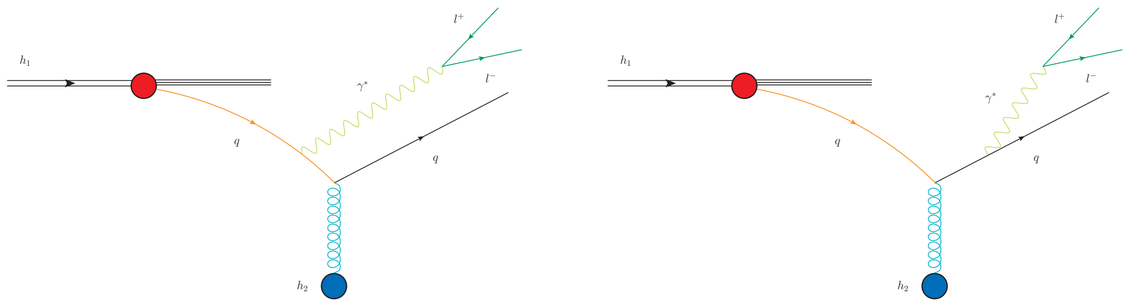
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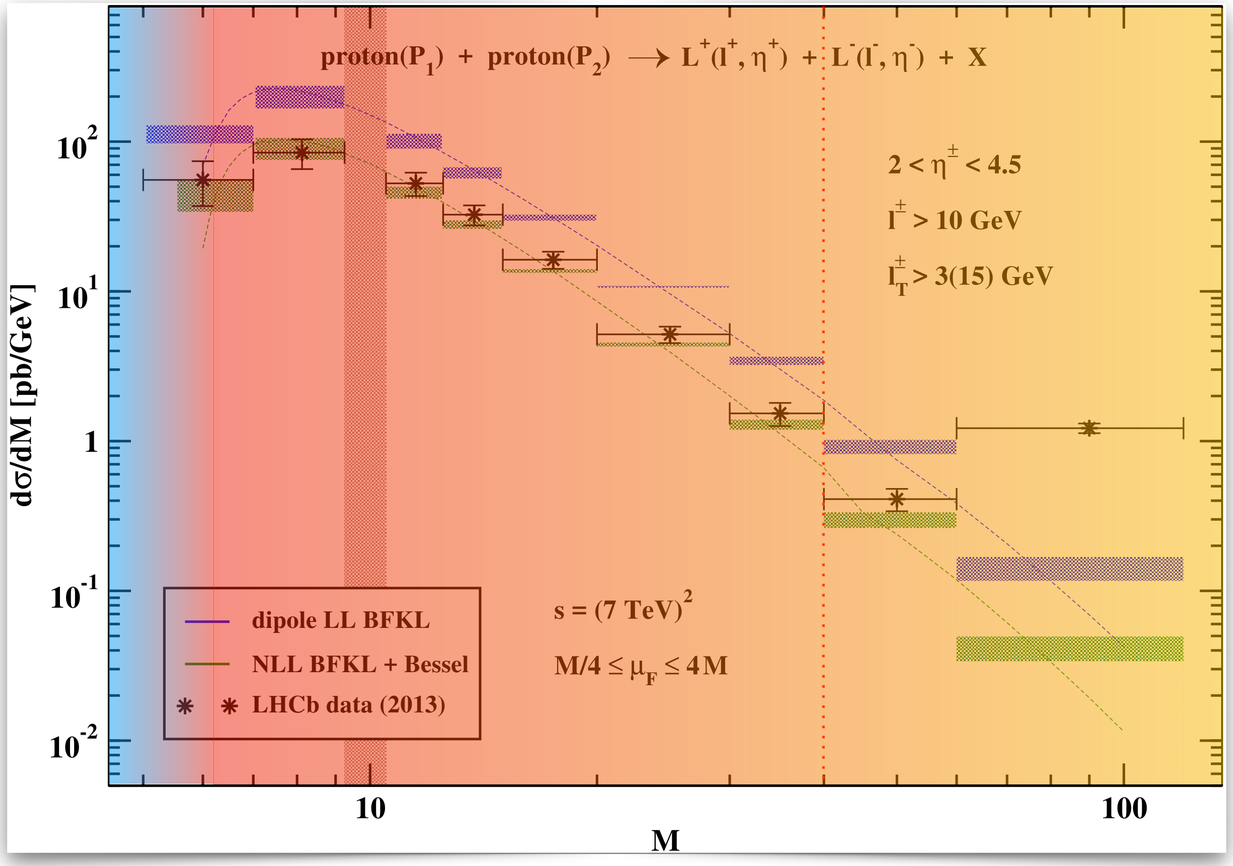
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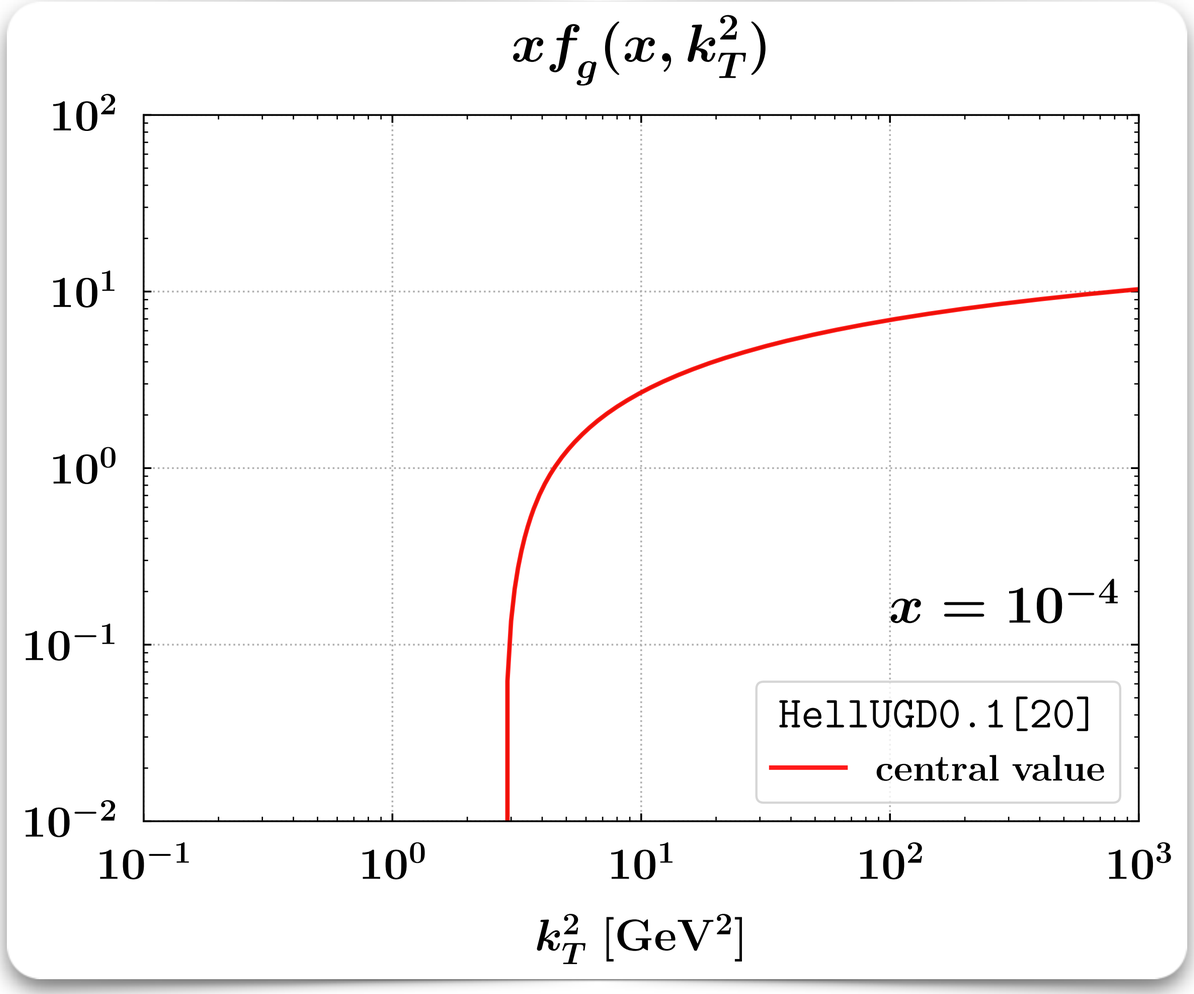
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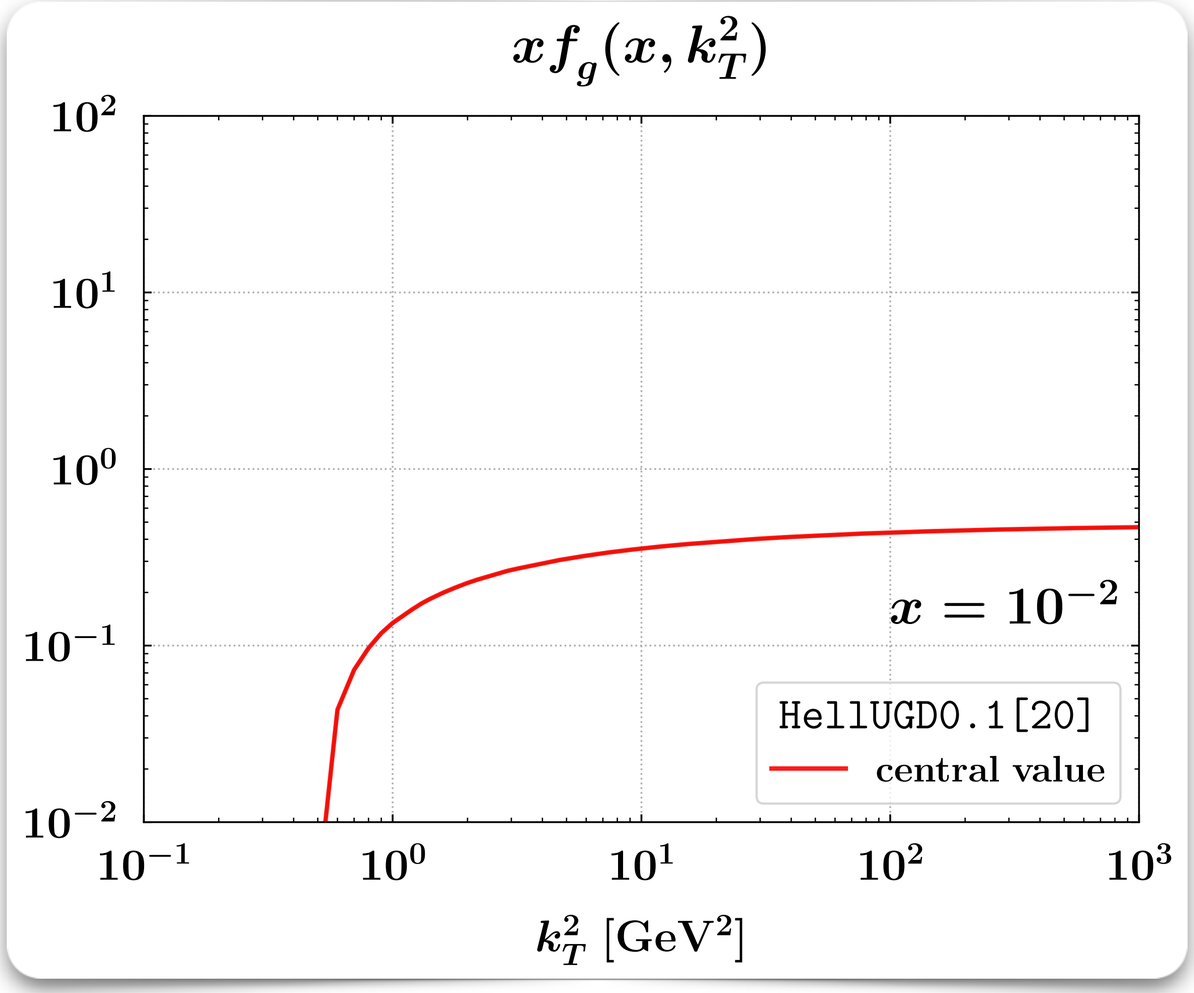
approaching limits of semi-hard regime



Z^0 contribution becoming relevant

The HELL UGDs (II)





Higgs production from LHC to FCC

PHYSICAL REVIEW LETTERS **120,** 202003 (2018)

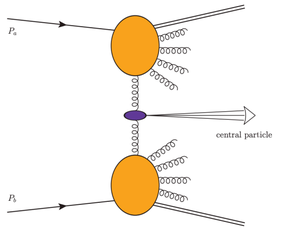
Double Resummation for Higgs Production

Marco Bonvini[1,*] and Simone Marzani[2,†]
[1]*INFN, Sezione di Roma 1, Piazzale Aldo Moro 5, 00185 Roma, Italy*
[2]*Dipartimento di Fisica, Università di Genova and INFN, Sezione di Genova, Via Dodecaneso 33, I-16146 Genova, Italy*

(Received 26 February 2018; published 16 May 2018)

We present the first double-resummed prediction of the inclusive cross section for the main Higgs production channel in proton-proton collisions, namely, gluon fusion. Our calculation incorporates to all orders in perturbation theory two distinct towers of logarithmic corrections which are enhanced, respectively, at threshold, i.e., large x, and in the high-energy limit, i.e., small x. Large-x logarithms are resummed to next-to-next-to-next-to-leading logarithmic accuracy, while small-x ones to leading logarithmic accuracy. The double-resummed cross section is furthermore matched to the state-of-the-art fixed-order prediction at next-to-next-to-next-to-leading accuracy. We find that double resummation corrects the Higgs production rate by 2% at the currently explored center-of-mass energy of 13 TeV and its impact reaches 10% at future circular colliders at 100 TeV.

DOI: 10.1103/PhysRevLett.120.202003



(*b*) Single central

High-energy resummation (BFKL) ⇒ PDFs at small-x

Altarelli-Ball-Forte to stabilize the $\mathrm{NLL_{sx}}$ BFKL kernel

$\mathrm{N^3LL_{lx}}$/$\mathrm{LL_{sx}}$/$\mathrm{N^3LO}$ rapidity-inclusive coefficient functions

$$C_{ij}(x,\alpha_s) = C_{ij}^{\mathrm{fo}}(x,\alpha_s) + \Delta C_{ij}^{\mathrm{lx}}(x,\alpha_s) + \Delta C_{ij}^{\mathrm{sx}}(x,\alpha_s)$$

Higgs production from LHC to FCC

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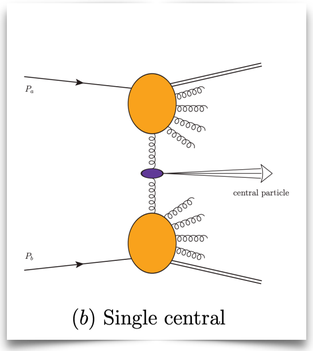
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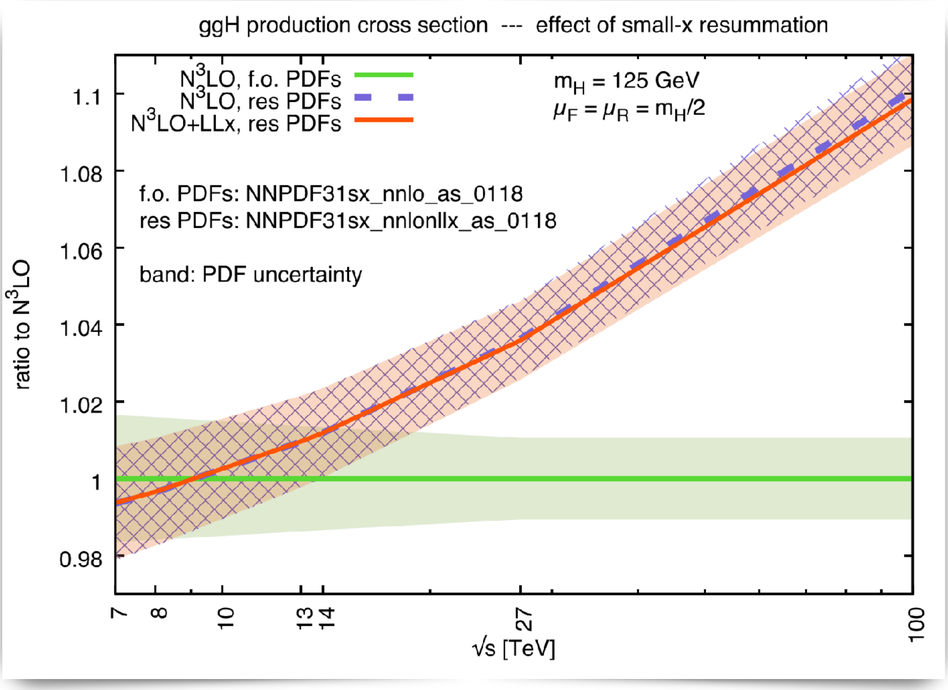
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(¡!) 100 TeV EW physics ⇔ small-x physics!

(¿?) Can LHC physics be resummation physics?

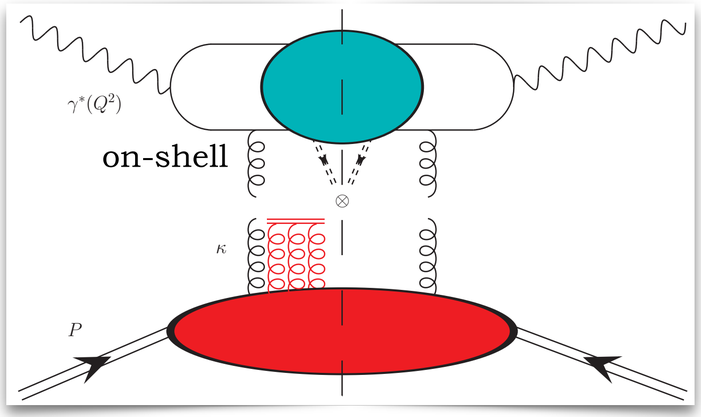
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HIGH-ENERGY FACTORIZATION AND THE UGD

TMD versus HEF

TMD

* Semi-inclusive processes
* $\kappa_T \ll$ hardest scale
* Language of **parton correlators**
* Diagram: **SIDIS onium**

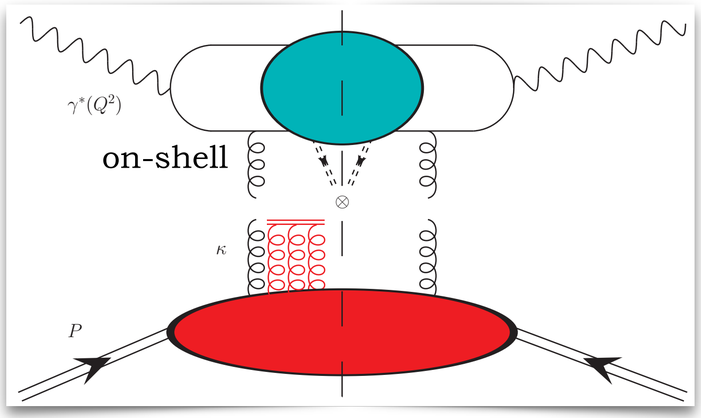


TMD
PDF

TMD versus HEF

TMD

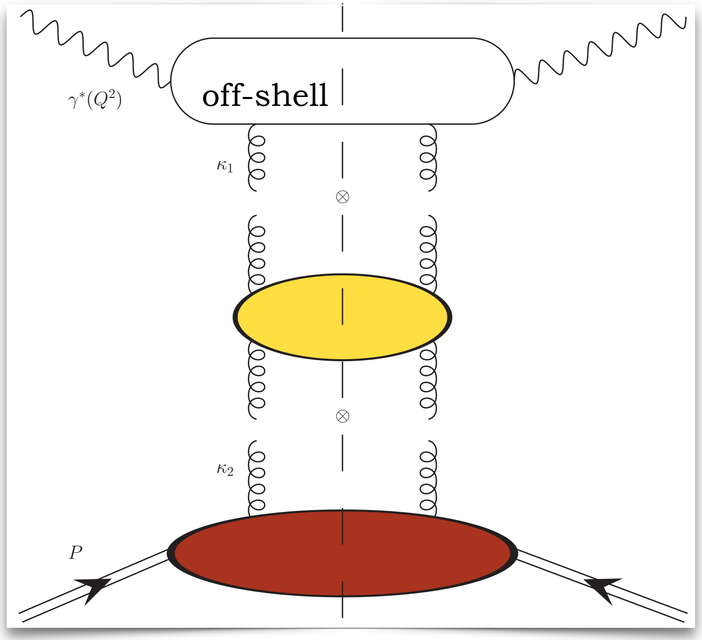
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TMD PDF

HEF

* Inclusive or exclusive processes **(!)**
* Small x, large κ_T
* Language of **Reggeized gluons**
* Diagram: **DIS**



$$\Phi^{\gamma^*\to\gamma^*} \otimes \mathcal{G}_{\text{BFKL}} \otimes \Phi^P_{[\text{NP}]}$$

TMD versus HEF

TMD

IR-safe colorless $\{\Phi^{i\to o}\}$

(Fadin-Martin theorem)

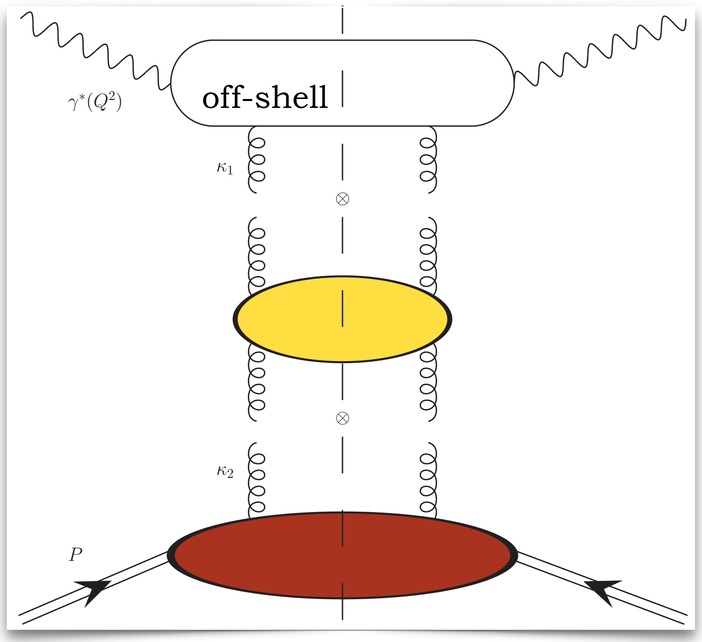
[V.S. Fadin, A.D. Martin (1999)]

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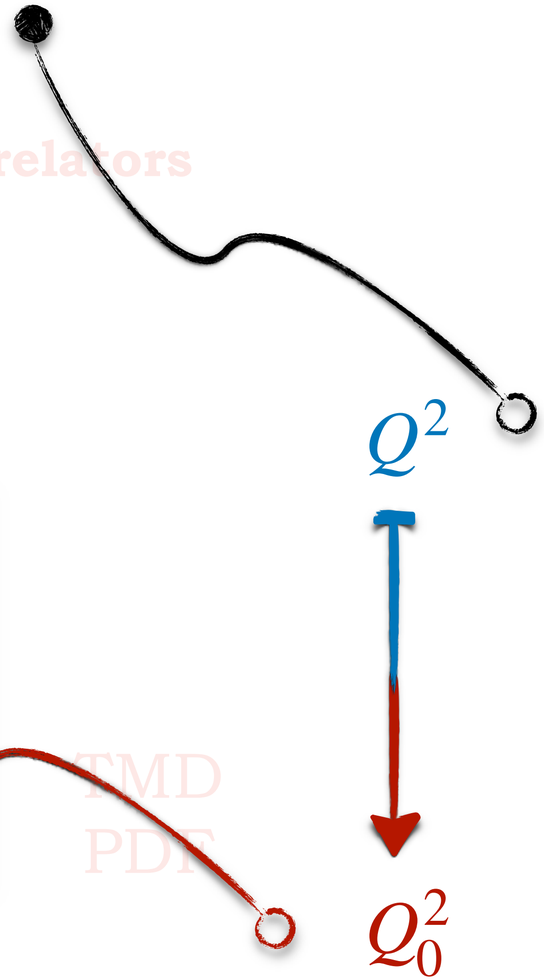
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IR diffusion pattern

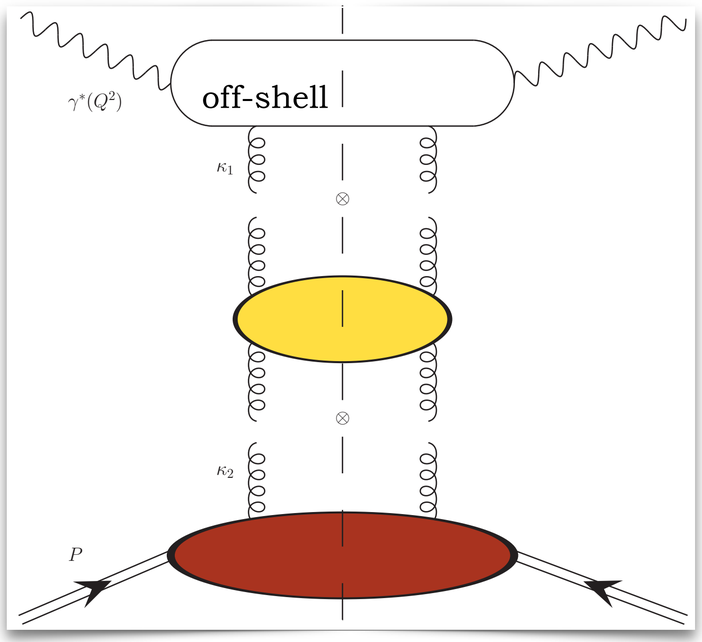
(Bartels' cigar)

[J. Bartels, H. Lotter (1993)]



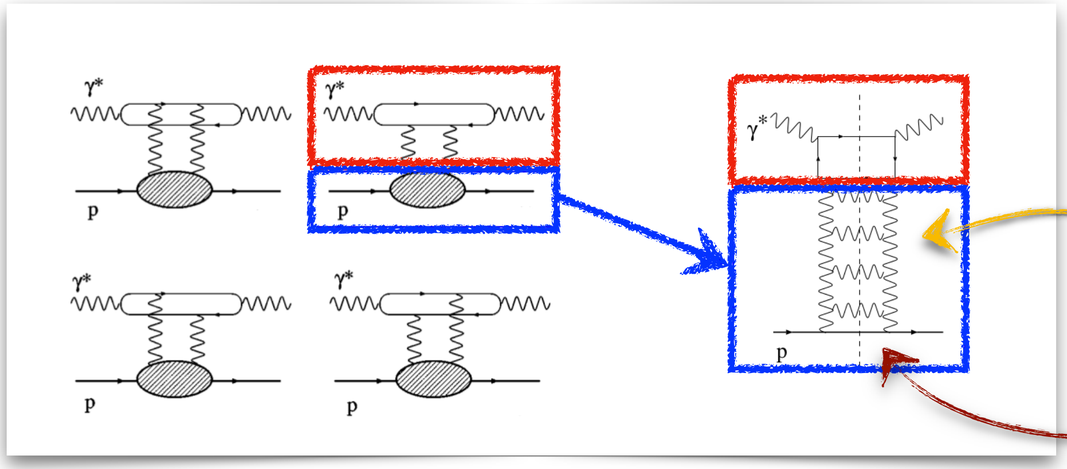
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High-energy factorization and the UGD



- example: **virtual photoabsorption** in **high-energy factorization**

$$\sigma_{\text{tot}}(\gamma^* p \to X) \propto \mathcal{I}m_s\{\mathcal{A}(\gamma^* p \to \gamma^* p)\} \equiv \Phi_{\gamma^* \to \gamma^*} \circledast \mathcal{F}(x, \kappa^2)$$

◇ $\mathcal{F}(x, \kappa^2)$ is the **unintegrated gluon distribution** (**UGD**) in the proton

▶ Small-x limit: **UGD** = [**BFKL gluon ladder**] $\circledast$ [**proton impact factor**]

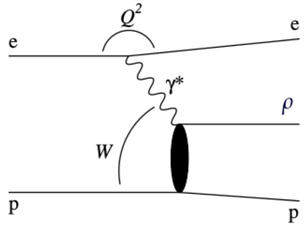
◇ Takes into account the resummation of high-energy logs

◇ Describes the coupling of the gluon Green's function to the proton

▶ Proton impact factor is non-perturbative $\Longrightarrow$ UGD needs to be modeled!

EXCLUSIVE VECTOR MESONS

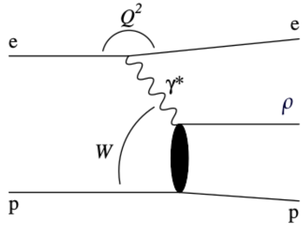
Exclusive forward ρ-meson leptoproduction @HERA



- High-energy regime:
  $s \equiv W^2 \gg Q^2 \gg \Lambda^2_{\mathrm{QCD}} \implies$ small $x = \frac{Q^2}{W^2}$
- photon virtuality Q is the **hard scale** of the process

▶ **Process solved in helicity** $\implies$ so far **unexplored testfield** for UGD

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  $s \equiv W^2 \gg Q^2 \gg \Lambda^2_{\rm QCD} \implies$ small $x = \frac{Q^2}{W^2}$
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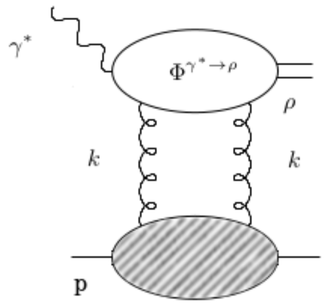
▶ **Process solved in helicity** $\implies$ so far **unexplored testfield** for UGD

Leading **helicity amplitudes** are known

Assumption:

- $\mathcal{I}m_s\{\mathcal{A}(\gamma^* p \to \rho\, p)\}$
- same W- and t-dependence for T_{11} and T_{00} $\implies$ high-energy factorization
  $\to$ same physical mechanism, scattering of small transverse size of dipole on the proton target, at work $\Rightarrow$ high-energy factorization

$$T_{\lambda_\rho \lambda_\gamma}(s; Q^2) = is \int \frac{d^2\kappa}{(\kappa^2)^2}\, \Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)}(\kappa^2, Q^2)\, \mathcal{F}(x, \kappa^2)\,, \quad x = \frac{Q^2}{s}$$

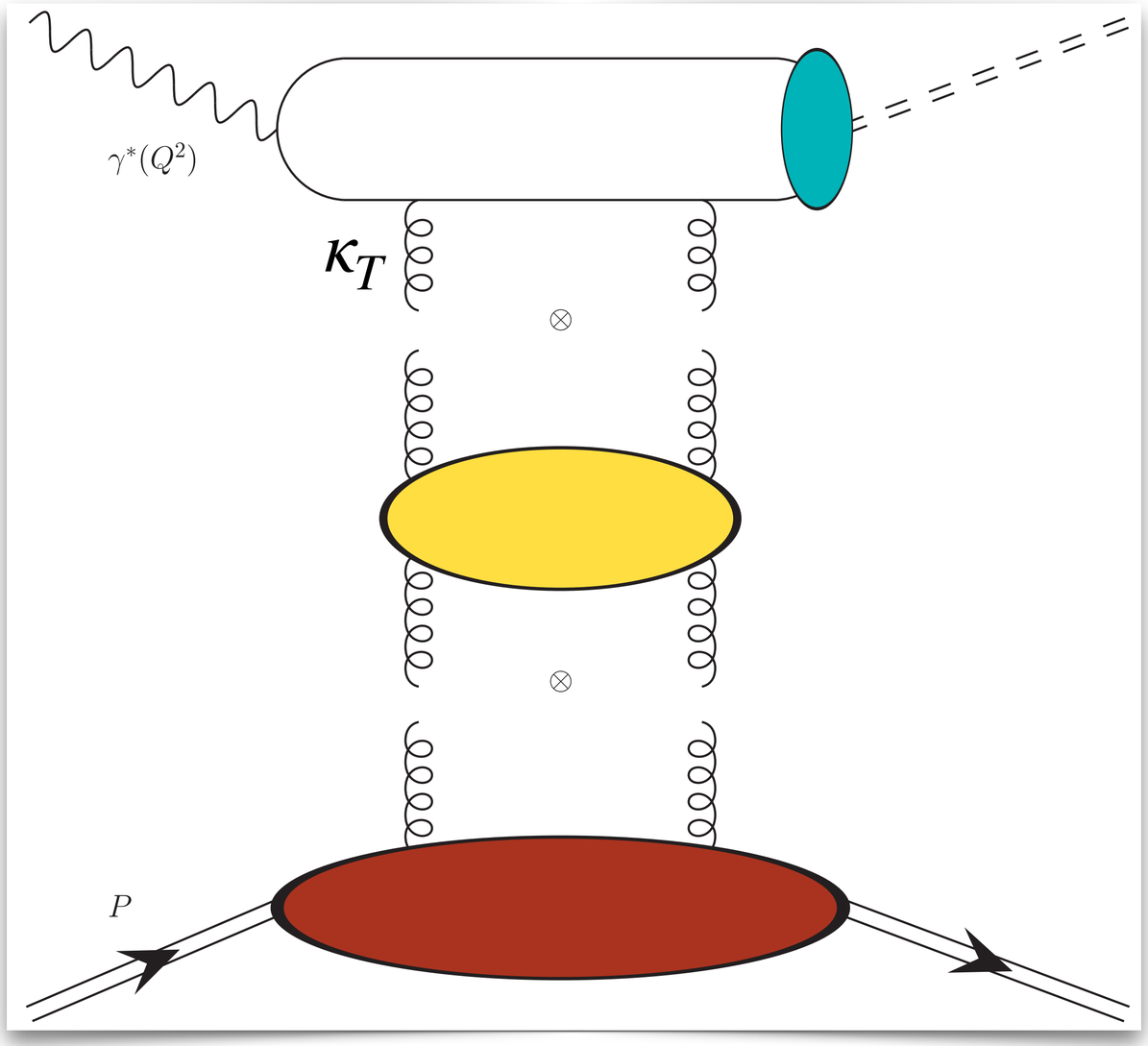


Interesting transitions:

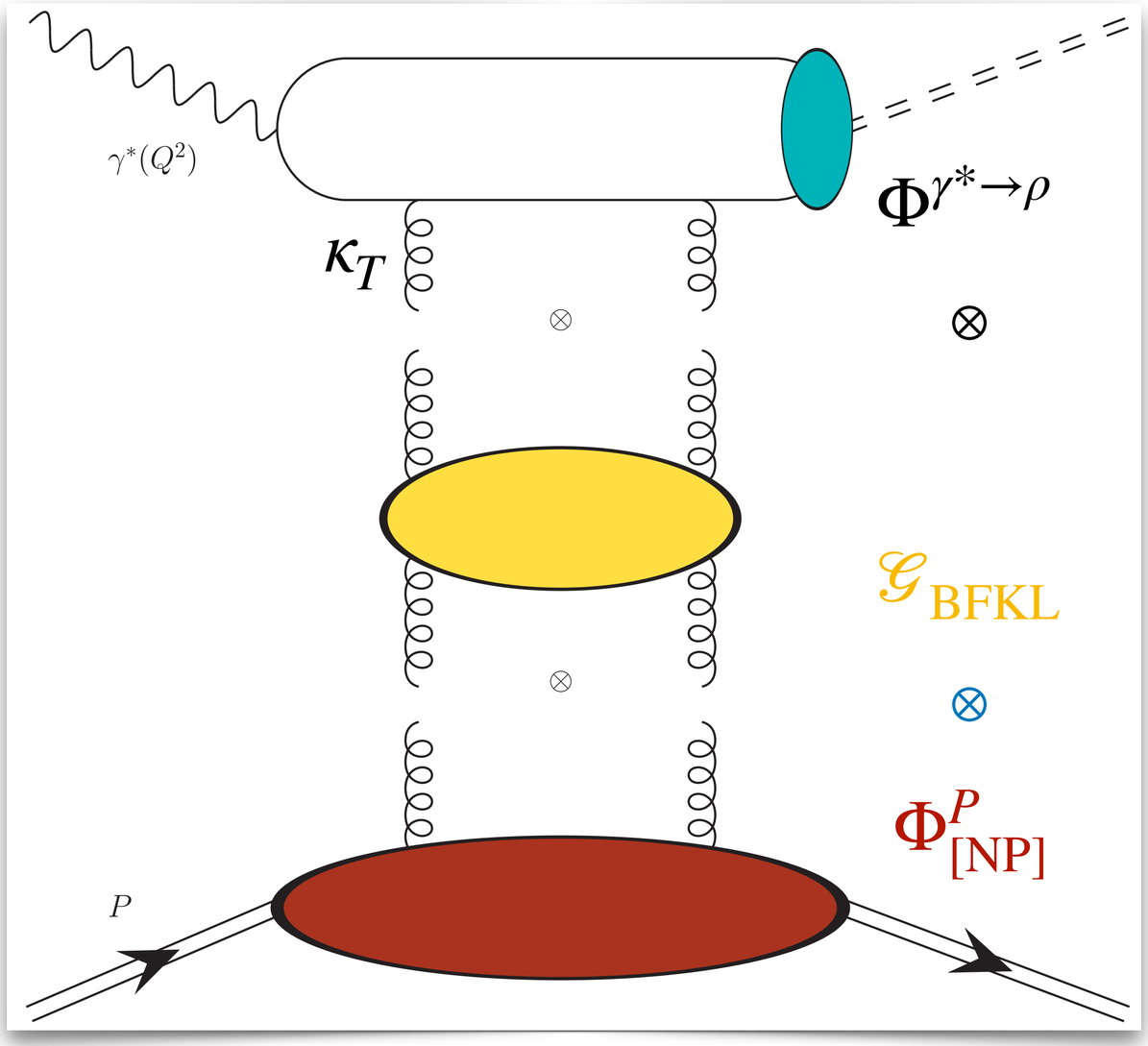
- $\gamma^*_L \to \rho_L \xrightarrow{\text{encoded by}} \Phi^{\gamma^*_L \to \rho_L}$
- $\gamma^*_T \to \rho_T \xrightarrow{\text{encoded by}} \Phi^{\gamma^*_T \to \rho_T}$

$\implies$ **DAs** enter in $\Phi^{\gamma^* \to \rho}$

A factorization…of factorizations

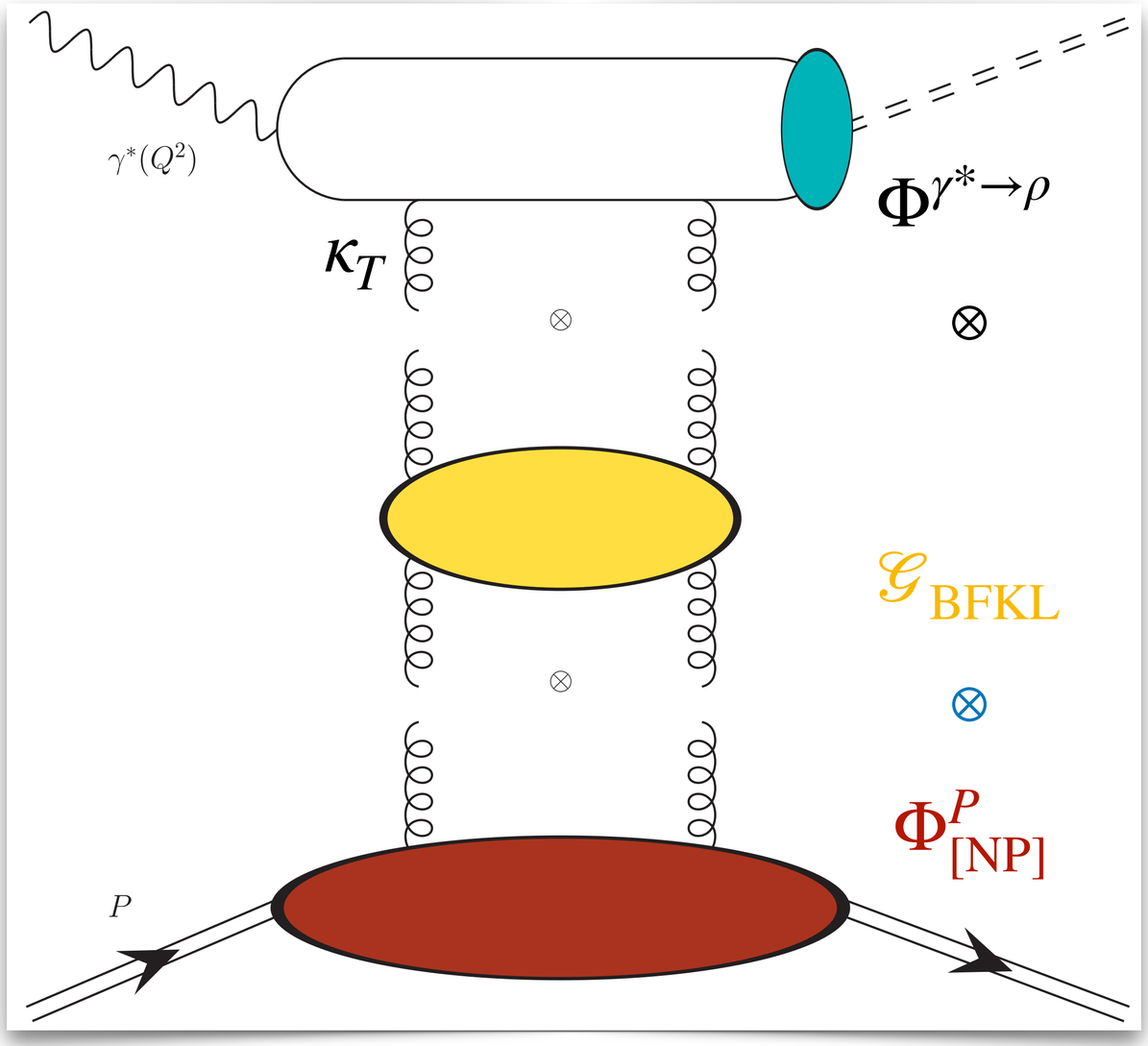


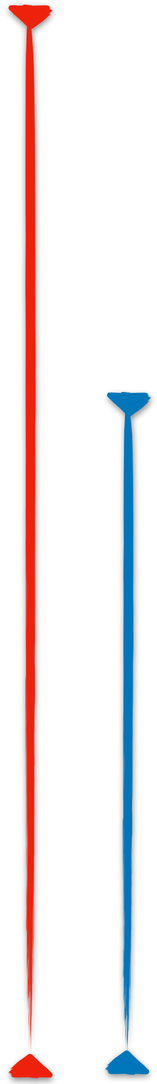
A factorization…of factorizations



BFKL factorization

A factorization…of factorizations

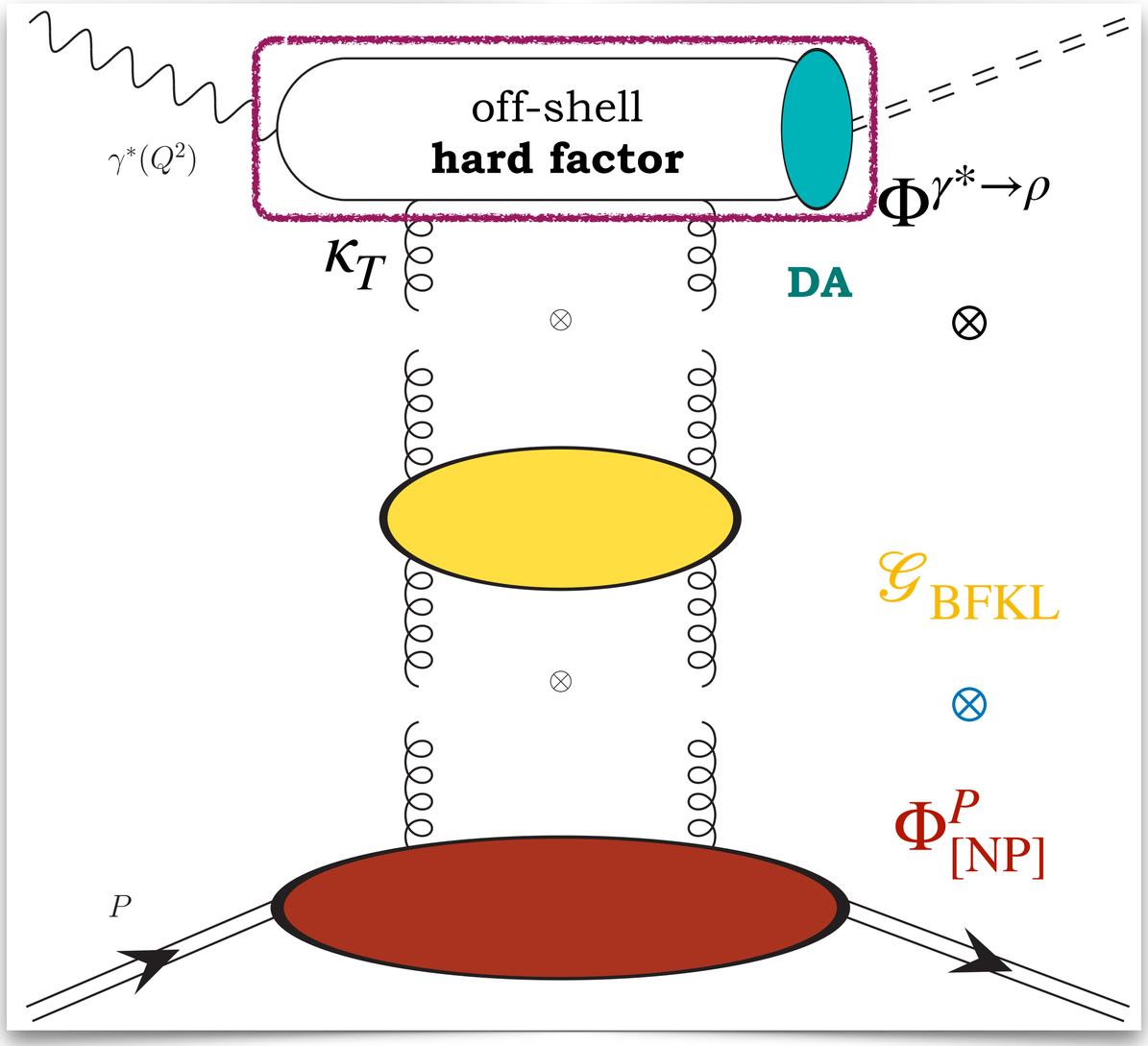




UGD

BFKL factorization

A factorization…of factorizations



$\gamma \to \rho$ impact factor

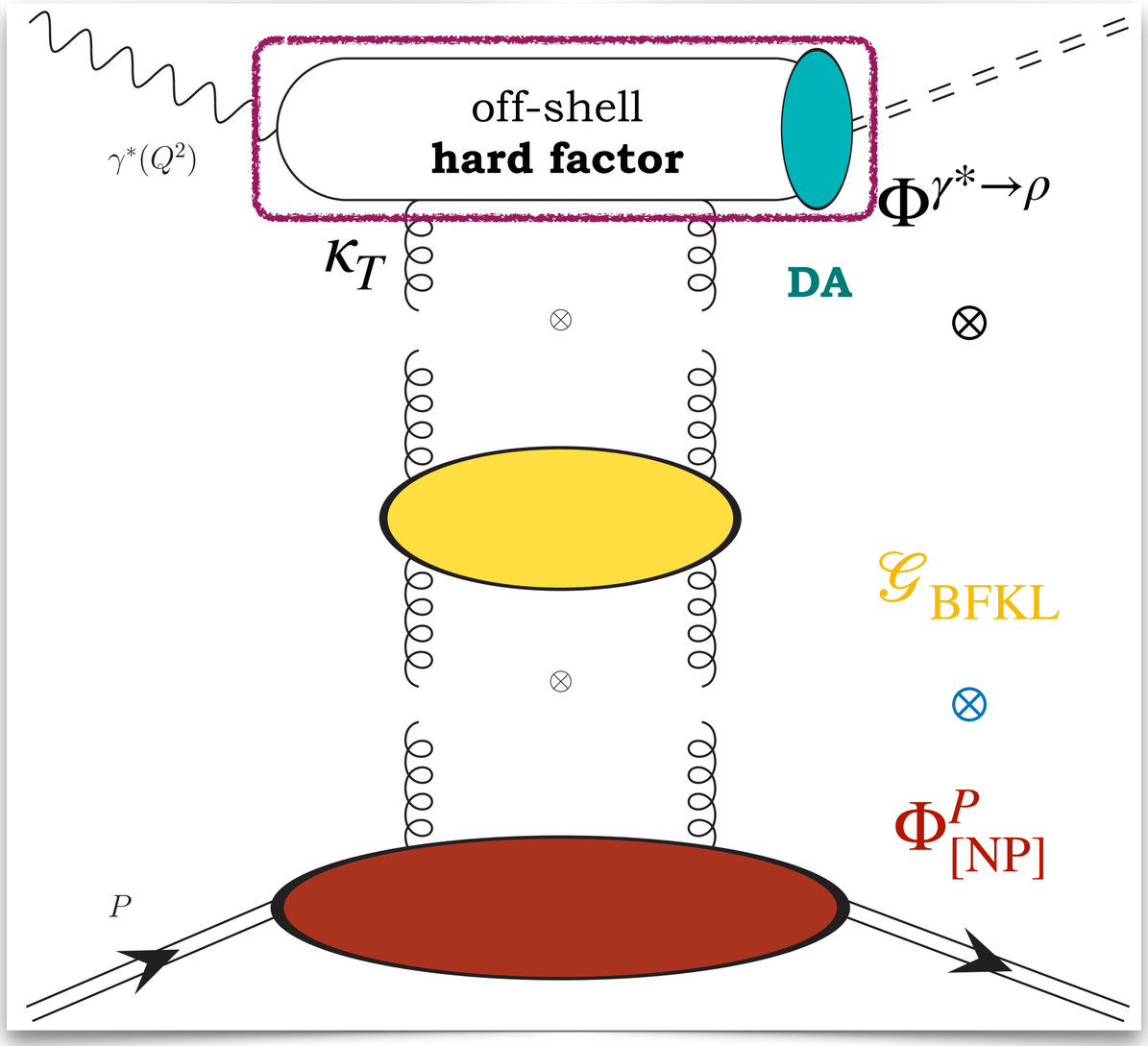
$$\Phi^{\gamma^*\to\rho} \propto \int_0^1 \mathrm{d}z\, T_H^{\gamma^*\to\rho}(z, \kappa_T, Q, \mu_R, \mu_F)\, \phi^{\lambda_\rho}(z, \mu_F)$$

Collinear convolution ⇔ large κ_T (!)

UGD

BFKL factorization

A factorization…of factorizations



$\gamma \to \rho$ impact factor

$$\Phi^{\gamma^*\to\rho} \propto \int_0^1 \mathrm{d}z\, T_H^{\gamma^*\to\rho}(z, \kappa_T, Q, \mu_R, \mu_F)\, \phi^{\lambda_\rho}(z, \mu_F)$$

Collinear convolution $\Leftrightarrow$ large κ_T (!)

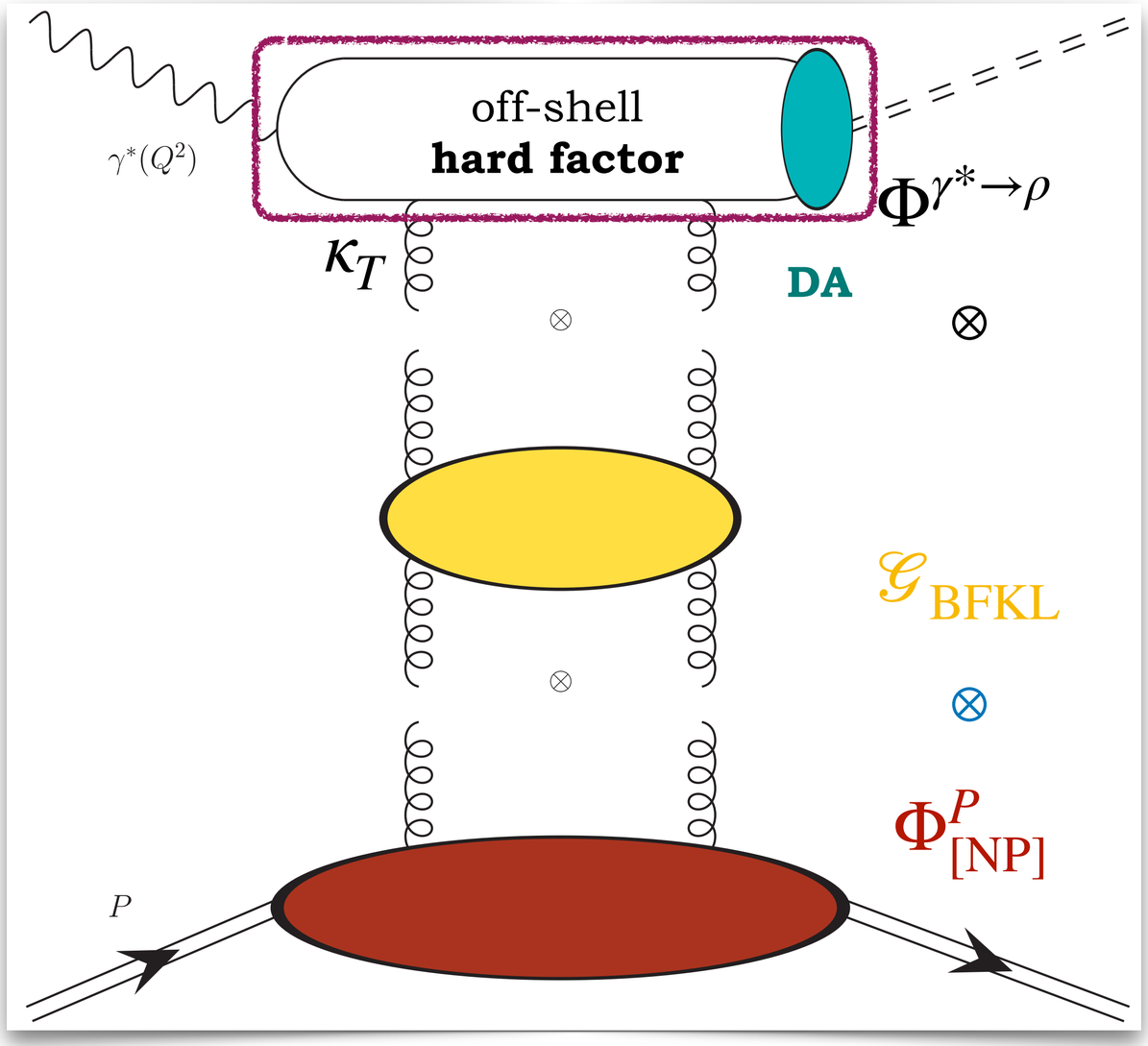
* $\gamma_L^* \to \rho_L$ transition [twist-2]

$$\frac{1}{\kappa_T^2}\Phi^{\gamma_L^*\to\rho_L} \underset{\kappa_T\to 0^+}{\sim} \text{constant}$$

UGD

BFKL factorization

A factorization...of factorizations



$\gamma \to \rho$ impact factor

$$\Phi^{\gamma^*\to\rho} \propto \int_0^1 \mathrm{d}z\, T_H^{\gamma^*\to\rho}(z, \kappa_T, Q, \mu_R, \mu_F)\, \phi^{\lambda_\rho}(z, \mu_F)$$

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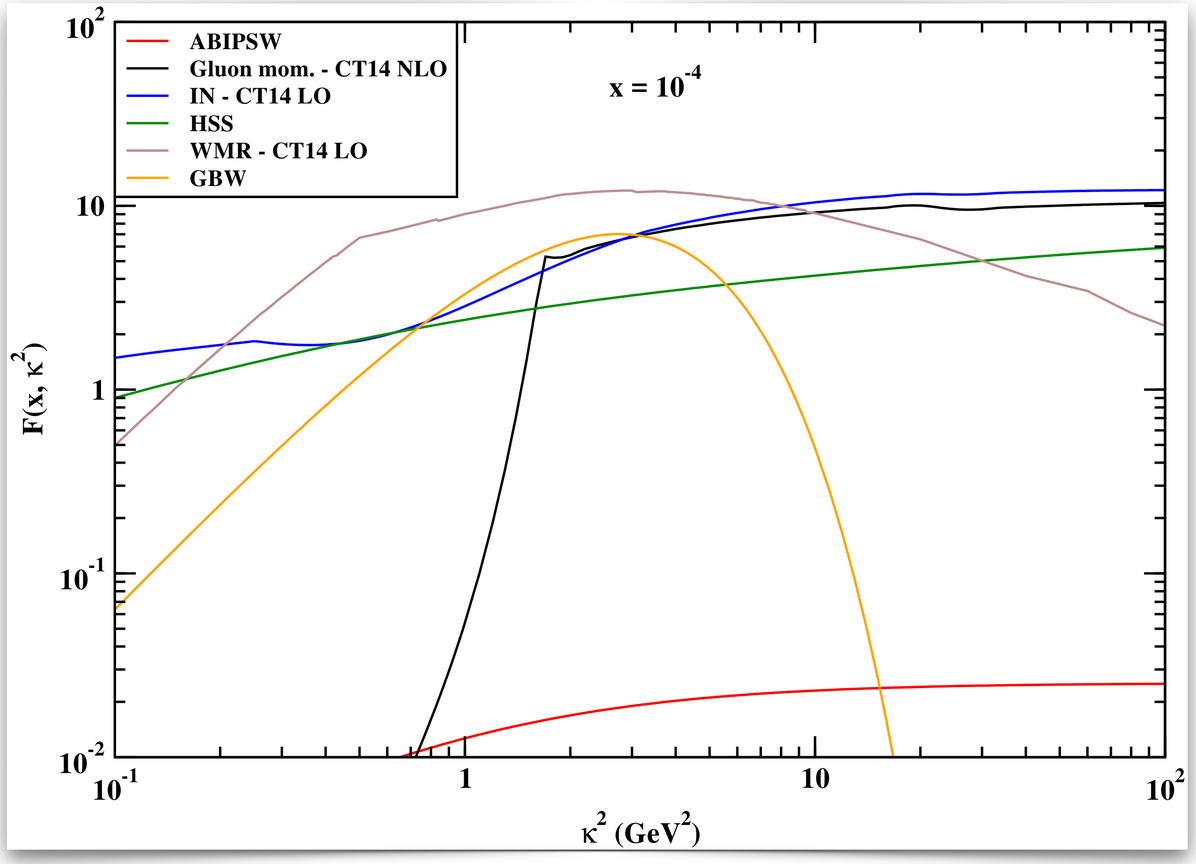
* $\gamma_T^* \to \rho_T$ transition [twist-3]

$$\frac{1}{\kappa_T^2}\Phi^{\gamma_T^*\to\rho_T} \underset{\kappa_T\to 0^+}{\sim} \ln\frac{\kappa_T^2}{Q^2}$$

UGD

BFKL factorization

UGD models



- **ABIPSW**: x-independent model

  [I. V. Anikin et al., *Phys. Rev. D* **84** (2011)]

- **Gluon mom. derivative**:

$$\mathcal{F}(x, \kappa^2) = \frac{d\, xg(x, \kappa^2)}{d \ln \kappa^2}$$

- **IN**: soft + hard component $\xrightarrow{\text{to probe}}$ different regions of κ

  [I. P. Ivanov and N. N. Nikolaev, *Phys. Rev. D* **65** (2002)

- **HSS**: $\mathcal{G}_{\text{BFKL}} \otimes$ [proton IF]

  [I. Bautista, A. Fernandez Tellez, M. Hentschinski, *Phys. Rev. D* **94** (2016) no.5, 054002]
  [G. Chachamis, M. Deák, M. Hentschinski, G. Rodrigo and A. Sabio Vera, *JHEP* **1509** (2015) 123]

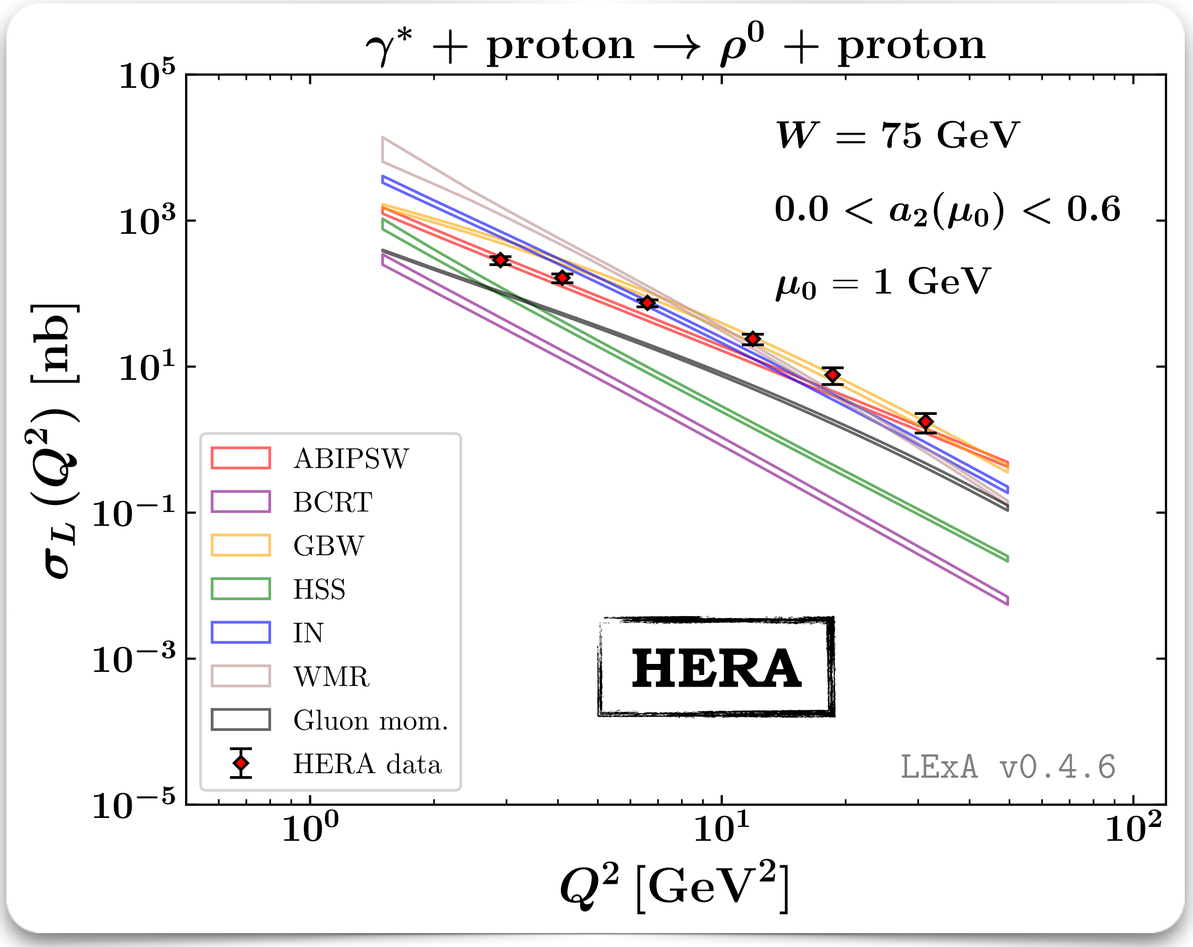
- **WMR**: angular ordering of gluon emissions

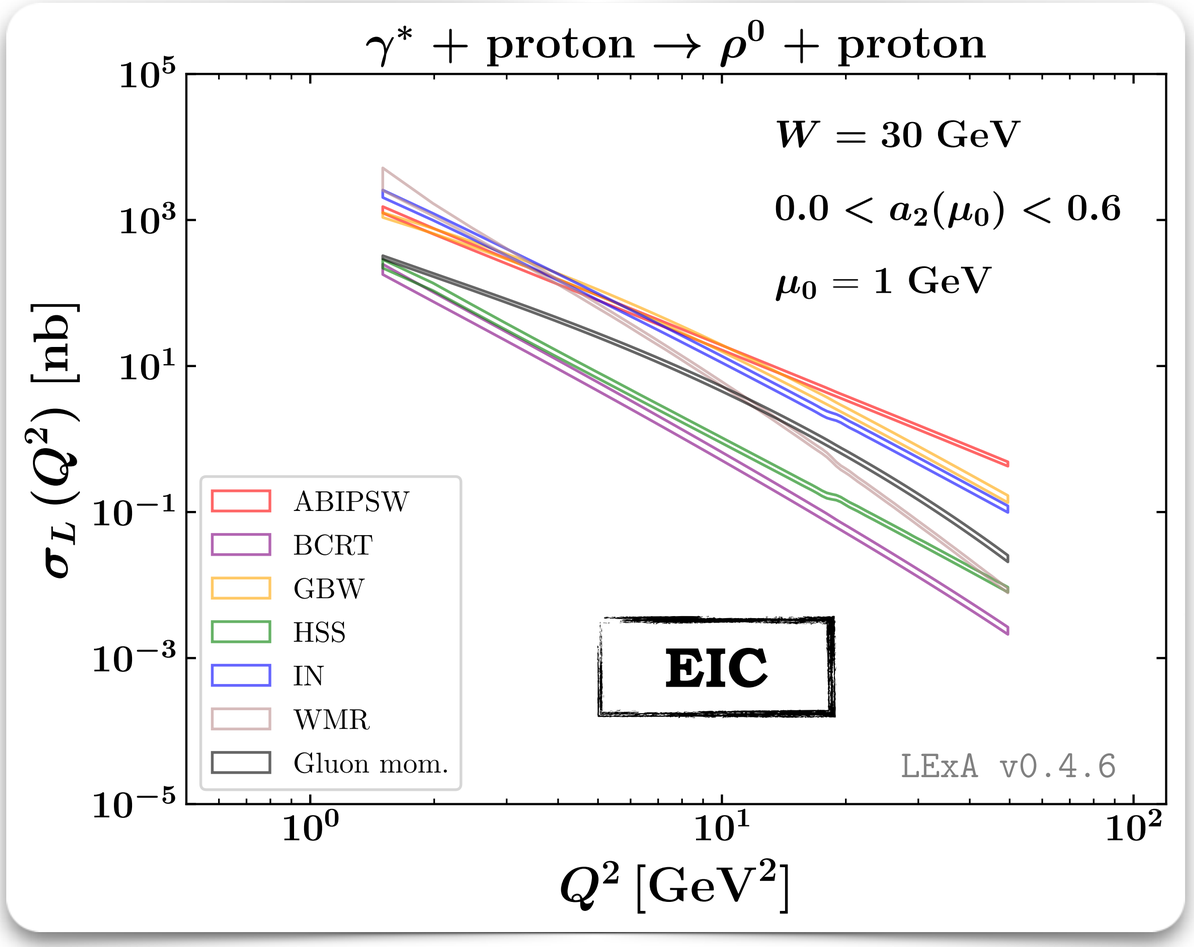
  [G. Watt, A.D. Martin, M.G. Ryskin, *Eur. Phys. J. C* **31** (2003) 73]

- **GBW**: FT of dipole cross section

  [K.J. Golec-Biernat, M. Wüsthoff, *Phys. Rev. D* **59** (1998) 014017]

Exclusive forward ρ-meson: $\sigma_L(Q^2)$





(saturation effects in ρ-meson polarization @HERA) [A. Besse, L. Szymanowski, S. Wallon (2013)]
(ρ-meson helicity amplitudes @HERA) [A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa (2018)]
(extension to ϕ-meson emissions) [A.D. Bolognino, A. Szczurek, W. Schäfer (2020)]
(in this slide) [A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa, W. Schäfer, A. Szczurek (2021)]

Diffractive slope

Empirical parametrization → introduces *smaller* uncertainties than UGD ones

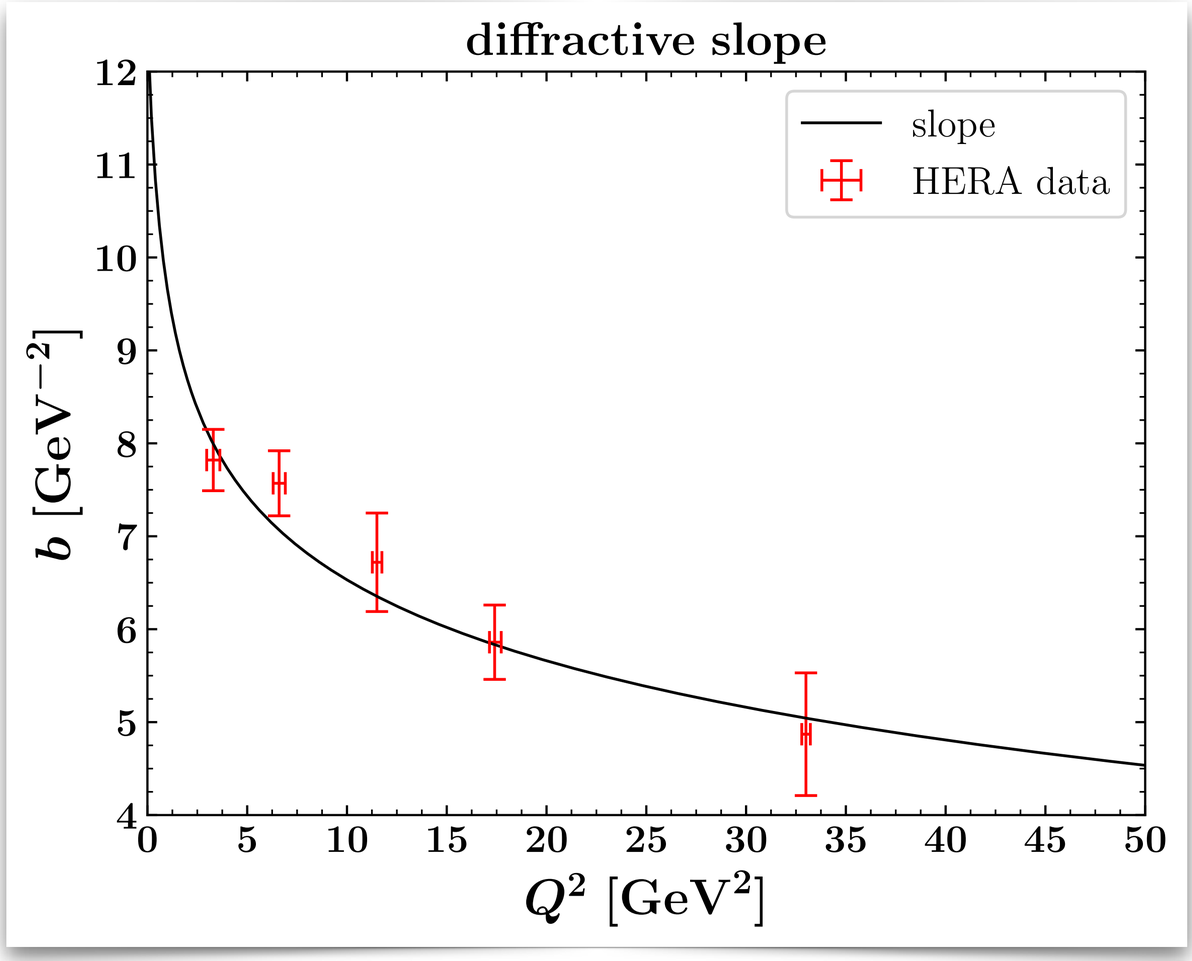
[J. Nemchik, N.N. Nikolaev, E. Predazzi, B.G. Zakharov, V. R. Zoller (1998)]

$$b(Q^2) = \beta_0 - \beta_1 \log\left[\frac{Q^2 + m_\rho^2}{m_{J/\psi}^2}\right] + \frac{\beta_2}{Q^2 + m_\rho^2}$$

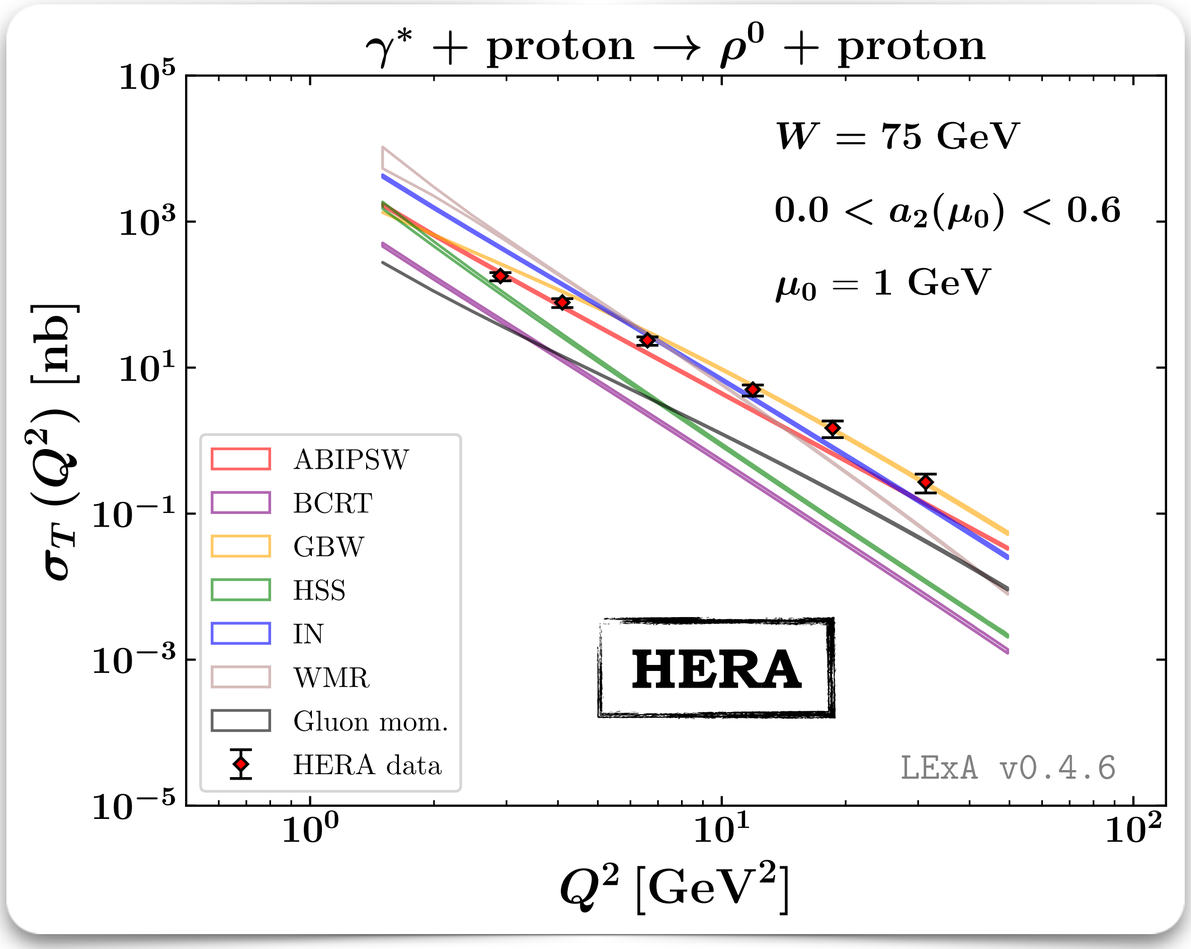
$$\sigma_L\left(\gamma^* p \to \rho\, p\right) = \frac{1}{16\pi b(Q^2)}\left|\frac{T_{00}(s, t=0)}{W^2}\right|^2$$

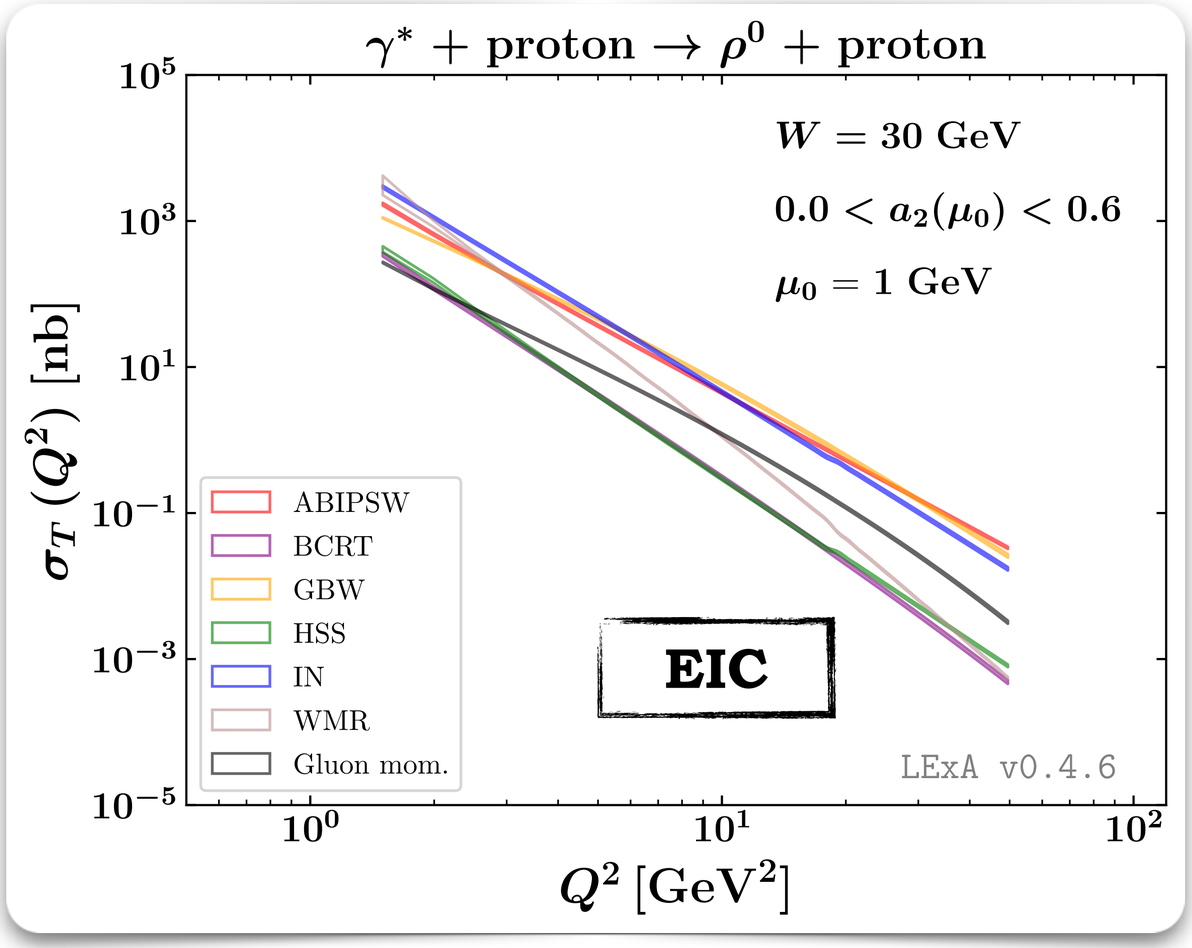
$$\sigma_T\left(\gamma^* p \to \rho\, p\right) = \frac{1}{16\pi b(Q^2)}\left|\frac{T_{11}(s, t=0)}{W^2}\right|^2$$

$\beta_0 = 6.5\ \text{GeV}^{-2}$, $\beta_1 = 1.2\ \text{GeV}^{-2}$ and $\beta_2 = 1.6$



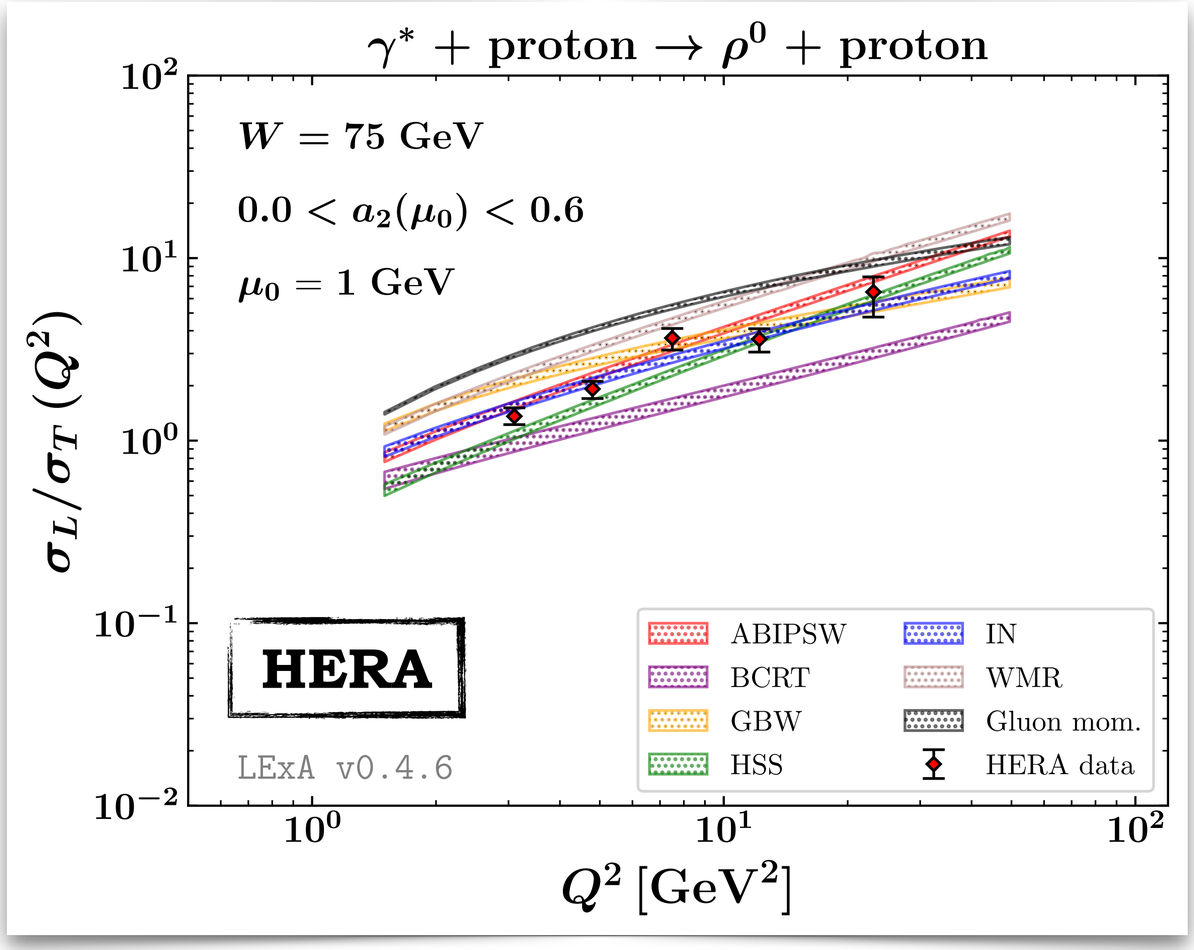
Exclusive forward ρ-meson: $\sigma_T(Q^2)$

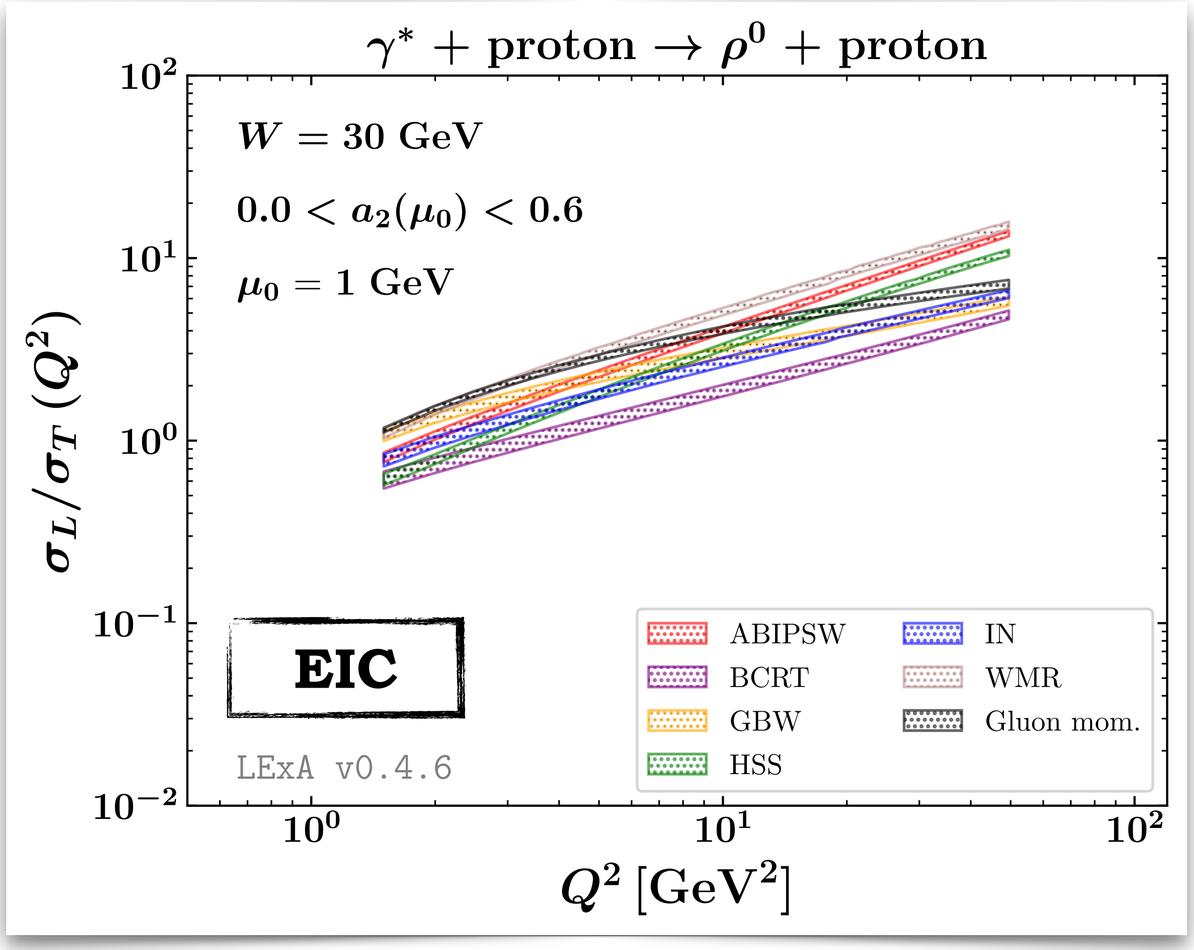




(saturation effects in ρ-meson polarization @HERA) 🔗 [A. Besse, L. Szymanowski, S. Wallon (2013)]
(ρ-meson helicity amplitudes @HERA) 🔗 [A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa (2018)]
(extension to ϕ-meson emissions) 🔗 [A.D. Bolognino, A. Szczurek, W. Schäfer (2020)]
(in this slide) 🔗 [A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa, W. Schäfer, A. Szczurek (2021)]

Exclusive forward ρ-meson: $\sigma_L/\sigma_T(Q^2)$





(saturation effects in ρ-meson polarization @HERA) 🔗 [A. Besse, L. Szymanowski, S. Wallon (2013)]
(ρ-meson helicity amplitudes @HERA) 🔗 [A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa (2018)]
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Single forward emissions

Exclusive light VM: ρ^0, ω, ϕ

* *Small-size* dipoles $\Rightarrow$ large κ_T
* Collinear description: twist-2/-3 LVM NP **DAs**

$$\Phi^{\gamma^* \to \rho} \propto \int_0^1 \mathrm{d}z \, T_H^{\gamma^* \to \rho}(z, \kappa_T, Q, \mu_R, \mu_F) \, \phi^{\lambda_\rho}(z, \mu_F)$$

* Significance of small κ_T under investigation…
* HERA indication: no large-r_d dynamics
* LVMs as tools: discrimination among UGD models
* LVMs as tools: UGD *extraction* $\Leftarrow$ HERA + **EIC** *fits*

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* LVMs as tools: UGD *extraction* ⇐ HERA + **EIC** *fits*

Quarkonia

* Size of dipoles ⇒ wide range of κ_T
* Description: **NRQCD** (combined with LFWFs)

$$\left[\mathrm{LFWF} \otimes \mathscr{A}_{\mathrm{dip.}} \right] \overset{\text{dilute}}{\longleftrightarrow} \left[\Phi^{\gamma^* \to J/\Psi} \otimes \mathrm{UGD} \right]$$

* Validity of *small-size* dipoles questionable…
* NRQCD: large-r_d dynamics for $\Psi(2s)$ ($\Upsilon(2s)$?)

🔗 [K. Suzuki *et al.* (2000)]; 🔗 [J. Cepila *et al.* (2019)]; 🔗 [M. Hentschinski *et al.* (2020)]

* Onia as tools: *scan* of TMD/HEF *intersection* range

FORWARD DRELL-YAN

Forward Drell-Yan production

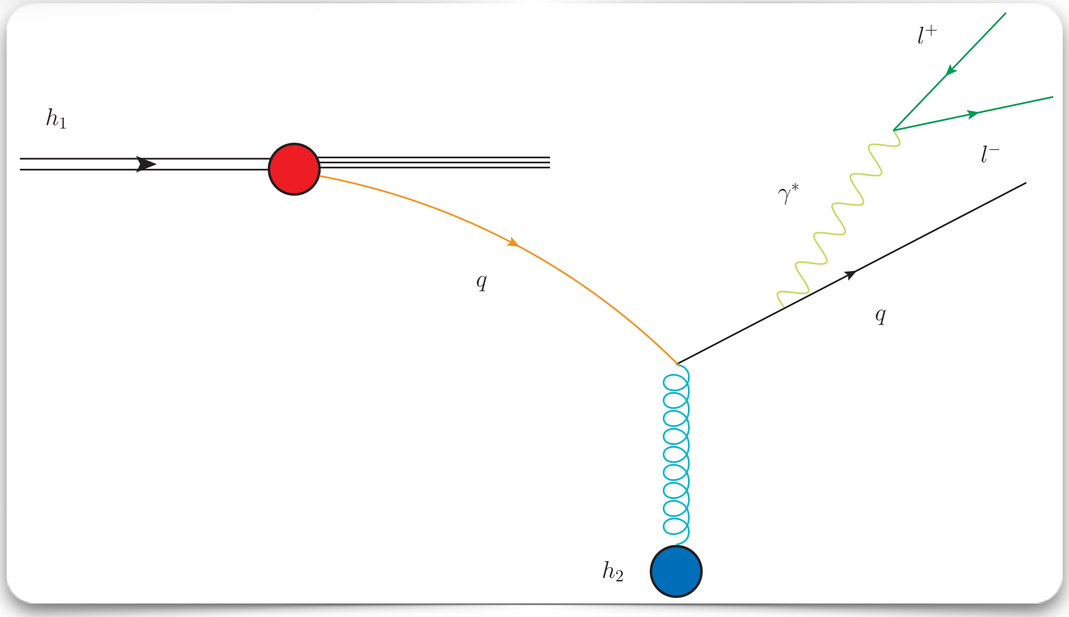
- LHC, **forward region** $\rightarrow$ $(l^+ l^-)$ produced in the fragmentation region of h_2
  - Asymmetric configuration: $x_1 \gg x_2$, down to $x_2 \simeq 10^{-6}$

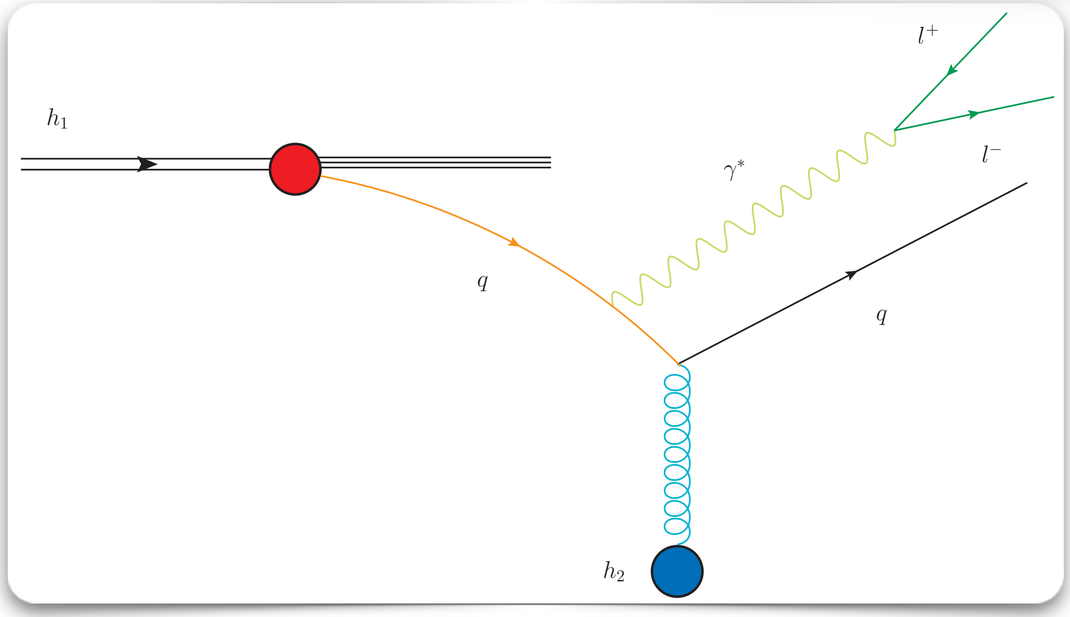
$\Longrightarrow$ **possible small-x resummation effects expected!**

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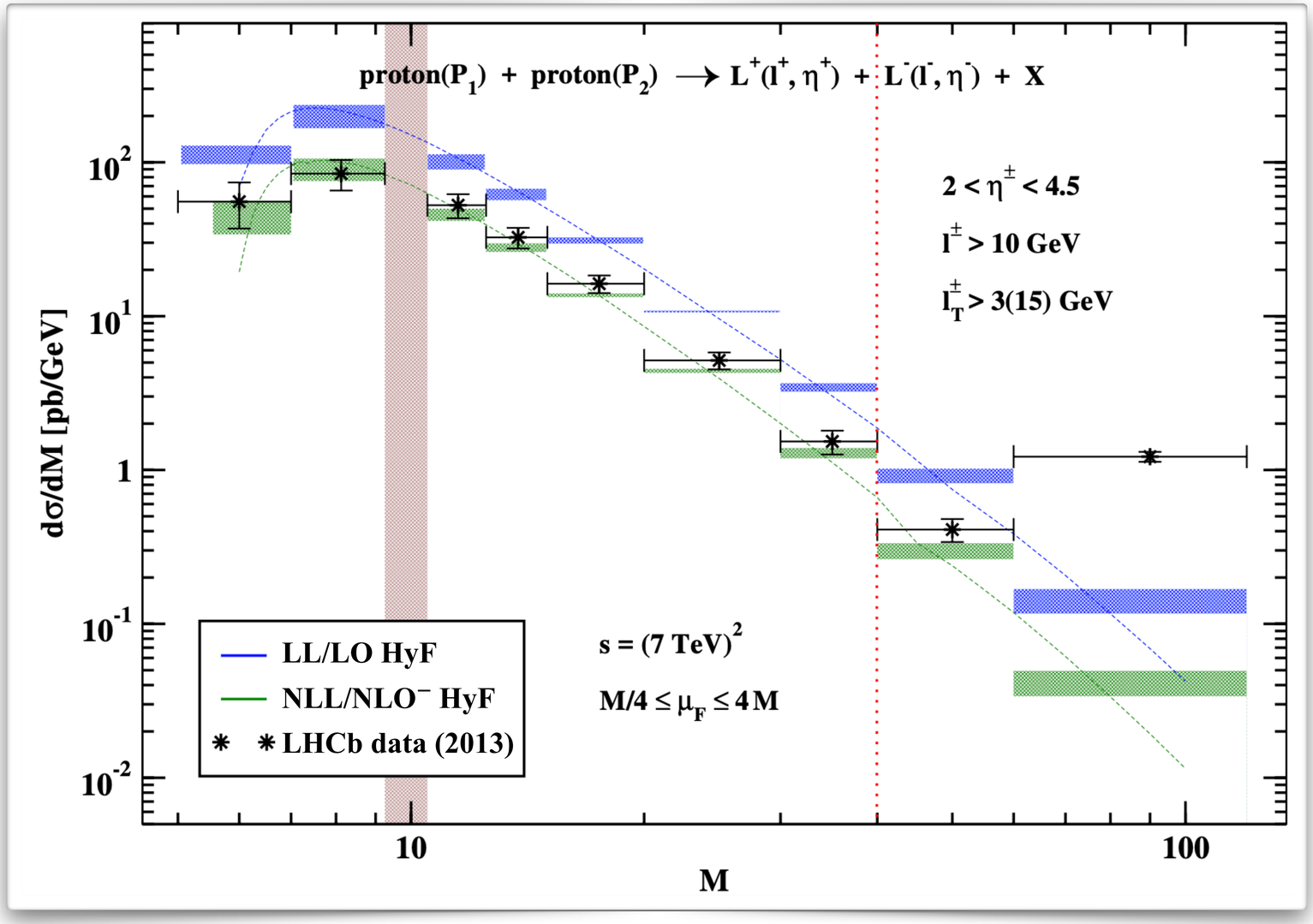


(LO DY emission functions + Mellin) [G. Golec-Biernat, E. Lewandowska, A. M. Stasto, Phys. Rev. D 82 (2010) 094010]

(LO DY emission functions + Mellin) [L. Motyka, M. Sadzikowski, T. Stebel, JHEP 05 (2015) 087]

(Twist decomposition + pheno) [D. Brzemiński, L. Motyka, M. Sadzikowski, T. Stebel, JHEP 01 (2017) 005]

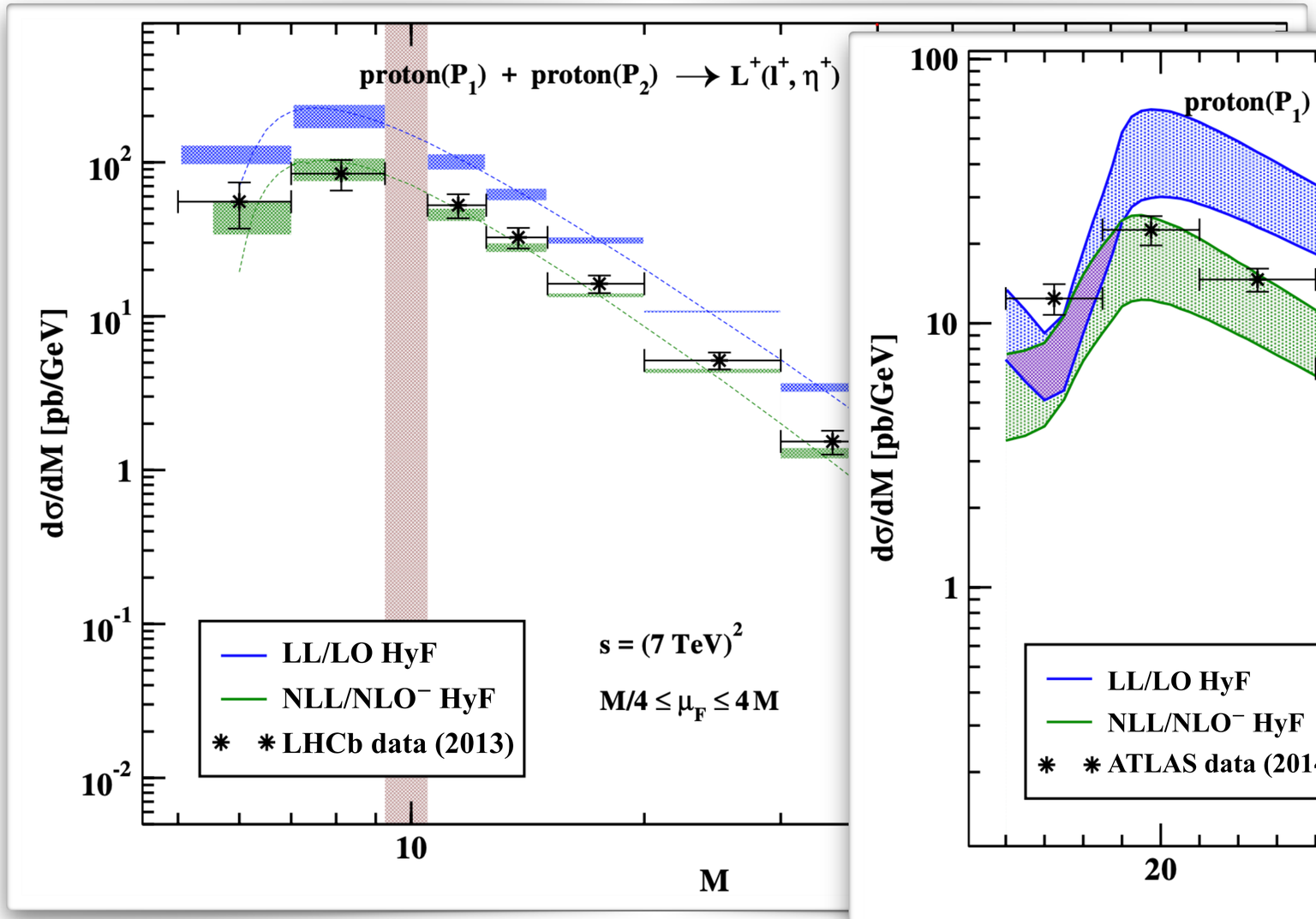
Forward Drell-Yan production @LHC (2018)



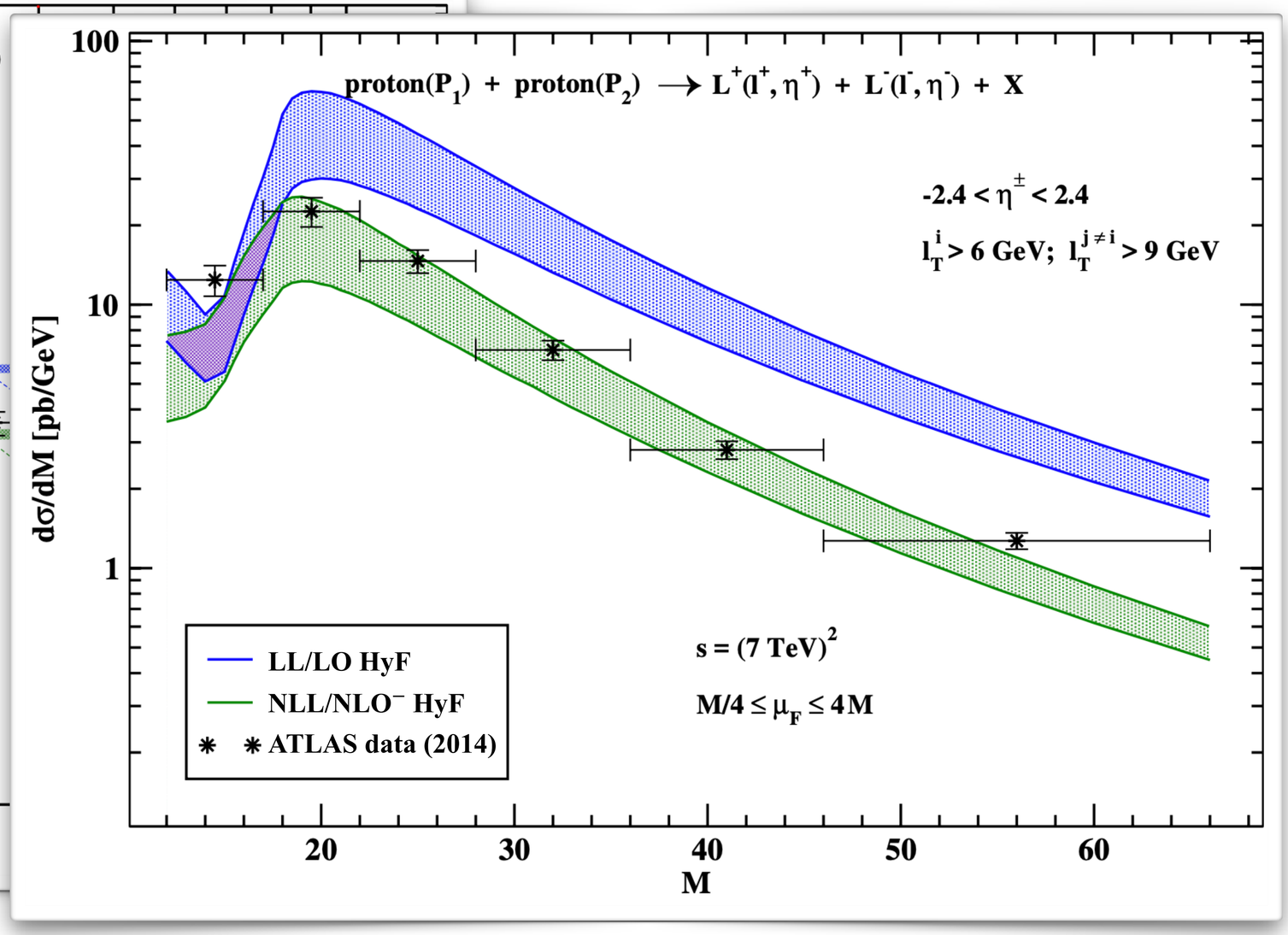
NLL/NLO^{-} studies @7TeV LHCb

🔗 [F. G. C., D Gordo Gómez, A. Sabio Vera, Phys. Lett. B 786 (2018) 201-206]

Forward Drell-Yan production @LHC (2018)



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NLL/NLO^{-} studies @7TeV ATLAS

🔗 [F. G. C., D Gordo Gómez, A. Sabio Vera, Phys. Lett. B 786 (2018) 201-206]

UGD pheno: Partial conclusions

Summary...

Exclusive polarized ρ-leptoproduction

[A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa, arXiv:1808.02395, to appear in *Eur. Phys. J. C*]

- T_{11}/T_{00} helicity-amplitude ratio to constrain the **UGD** in the HERA range
- High sensitivity to **distinct UGD models**
- Low sensitivity to **region of small-κ_T values**

  effect of $\longrightarrow$ dominance of **small-size dipoles**

Forward Drell–Yan dilepton production

[F.G. C., D. Gordo Gómez, A. Sabio Vera, *Phys. Lett. B* **786** (2018) 201]

- Good description of $\mathrm{d}\sigma/\mathrm{d}M$ in the **BFKL approach** at the LHC
- Same observable well described also by **fixed-order** calculations

  [LHCb Coll.: LHCb-CONF-2012-013, CERN-LHCb-CONF-2012-013]; [ATLAS Coll.: G. Aad et al., *JHEP* **1406** (2014) 112]

- Future data for Drell–Yan production in **forward directions**

  peerless to $\longrightarrow$ gauge need for **high-energy resummation**

SMALL-X GLUON TMDS

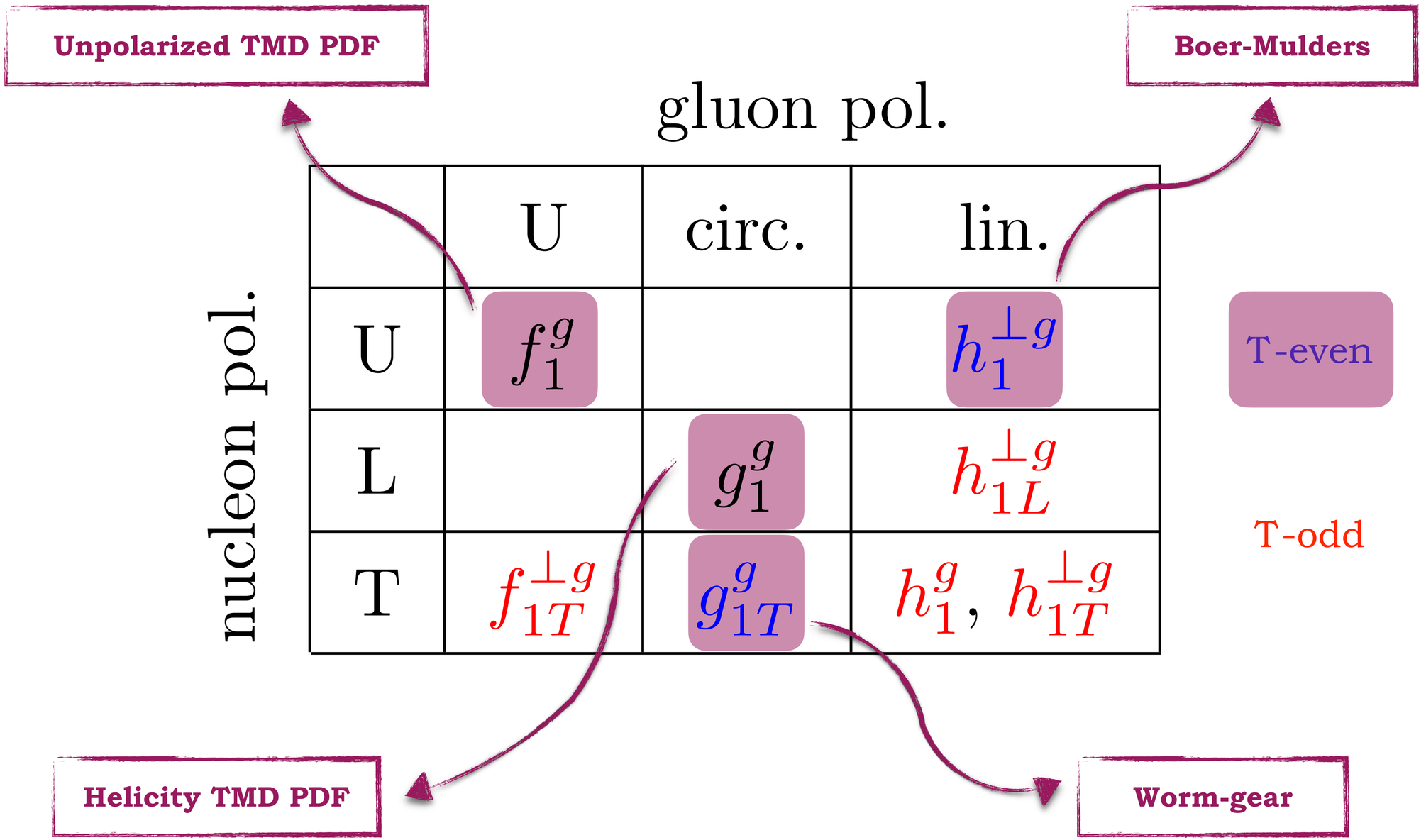
Gluon TMD PDFs at leading twist

nucleon pol. \ gluon pol.	U	circ.	lin.
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g,\ h_{1T}^{\perp g}$

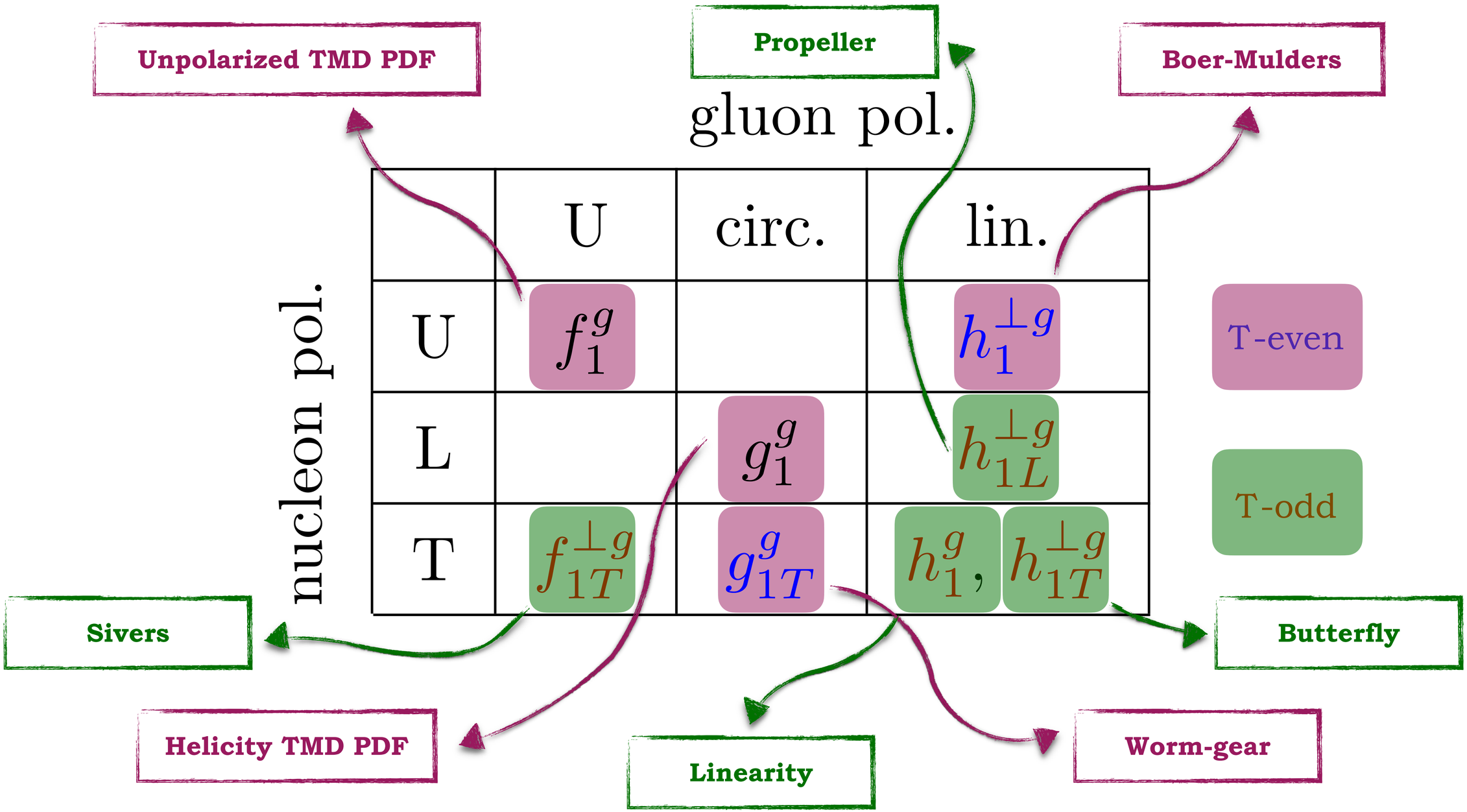
T-even

T-odd

Gluon TMD PDFs at leading twist



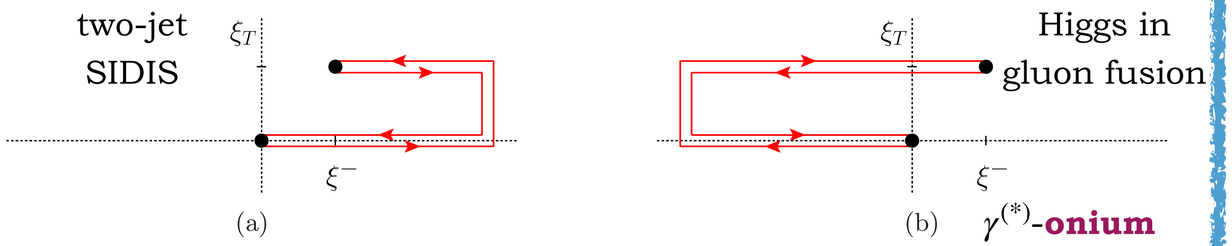
Gluon TMD PDFs at leading twist



Accessing f-type and d-type gluon TMD PDFs

f-type (WW)

(a) $[+,+]$ or (b) $[-,-]$



Higgs in
gluon fusion

$\gamma^{(*)}$-**onium**
hadroproduction

- Color flow annihilated within final/initial state
- *f*-type gluon TMDs $\rightarrow$ f^{abc} color structure
- Modified universality:

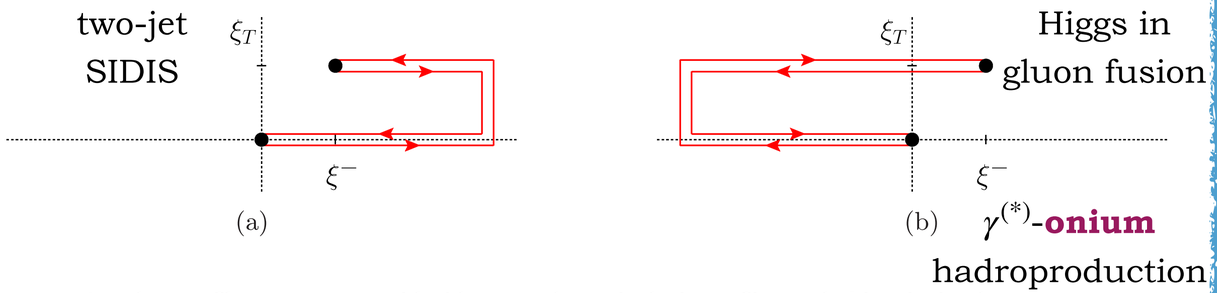
$$f_1^{[+,+]} = f_1^{[-,-]},$$
$$f_{1T}^{\perp[+,+]} = -f_{1T}^{\perp[-,-]}$$

- Phenomenology: Higgs, **quarkonia** or $\gamma\gamma$ in pp, two-jet SIDIS, heavy-quark pair SIDIS

Accessing f-type and d-type gluon TMD PDFs

f-type (WW)

(a) [+ , +] or (b) [− , −]



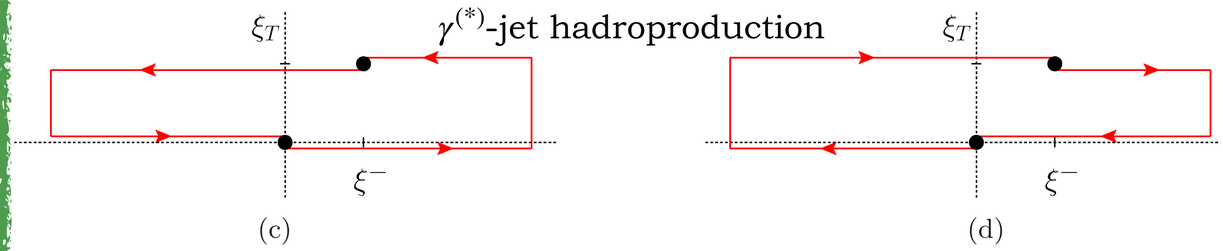
- Color flow annihilated within final/initial state
- f-type gluon TMDs → f^{abc} color structure
- Modified universality:

$$f_1^{[+,+]} = f_1^{[-,-]},$$
$$f_{1T}^{\perp[+,+]} = -f_{1T}^{\perp[-,-]}$$

- Phenomenology: Higgs, **quarkonia** or $\gamma\gamma$ in pp, two-jet SIDIS, heavy-quark pair SIDIS

d-type (DP)

(c) [+ , −] or (d) [− , +]



- Color flow involving both initial and final states
- d-type gluon TMDs → d^{abc} color structure
- Modified universality:

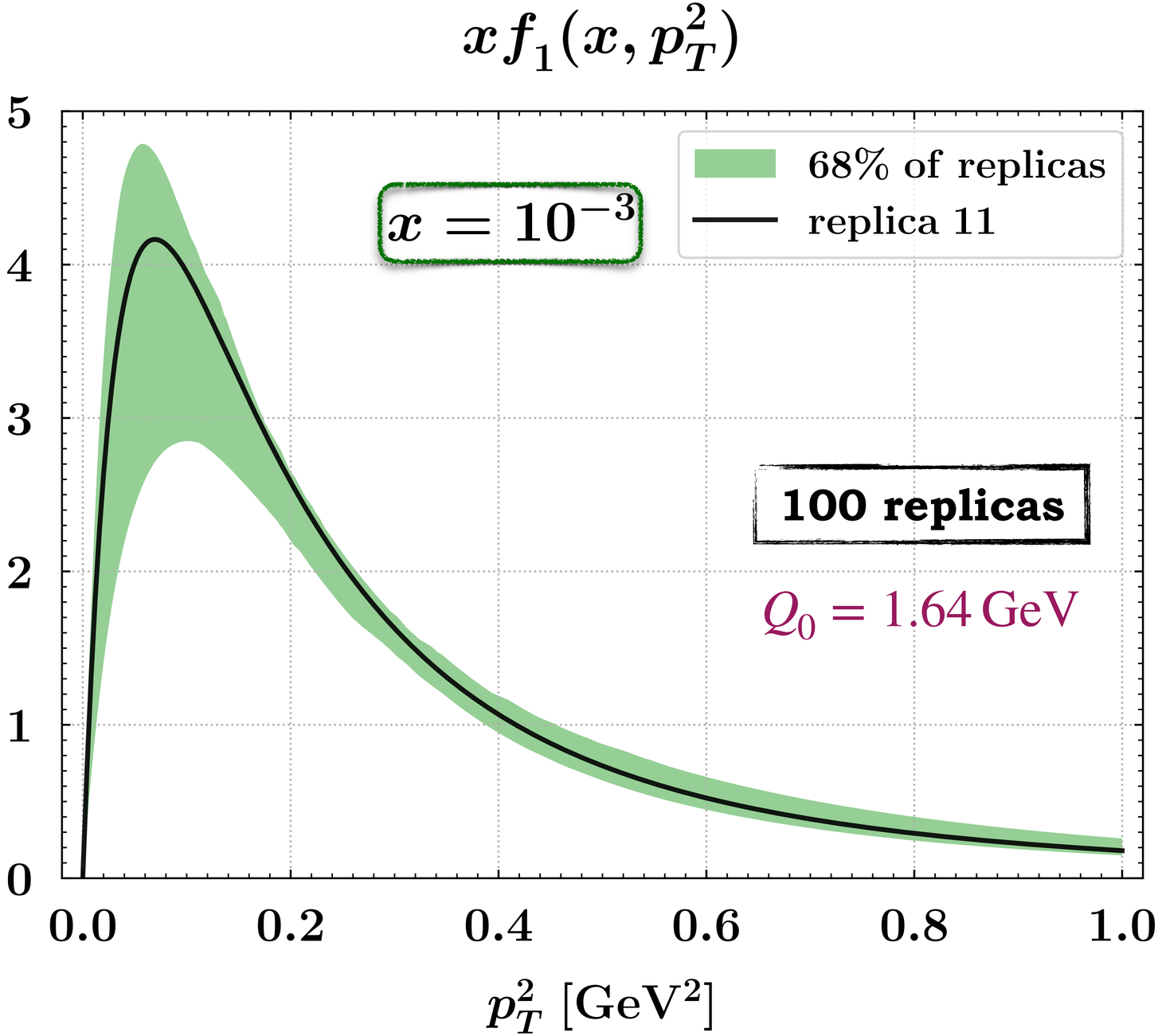
$$f_1^{[+,-]} = f_1^{[-,+]},$$
$$f_{1T}^{\perp[+,-]} = -f_{1T}^{\perp[-,+]}$$

- Phenomenology: single hadron or $\gamma^{(*)}$-jet hadroproduction, SIDIS or Drell-Yan (subleading)

¡ Gauge link → two main **independent** sets of TMD PDFs, **not related** to each other !

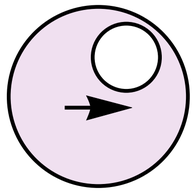
Unpolarized gluon TMD PDF

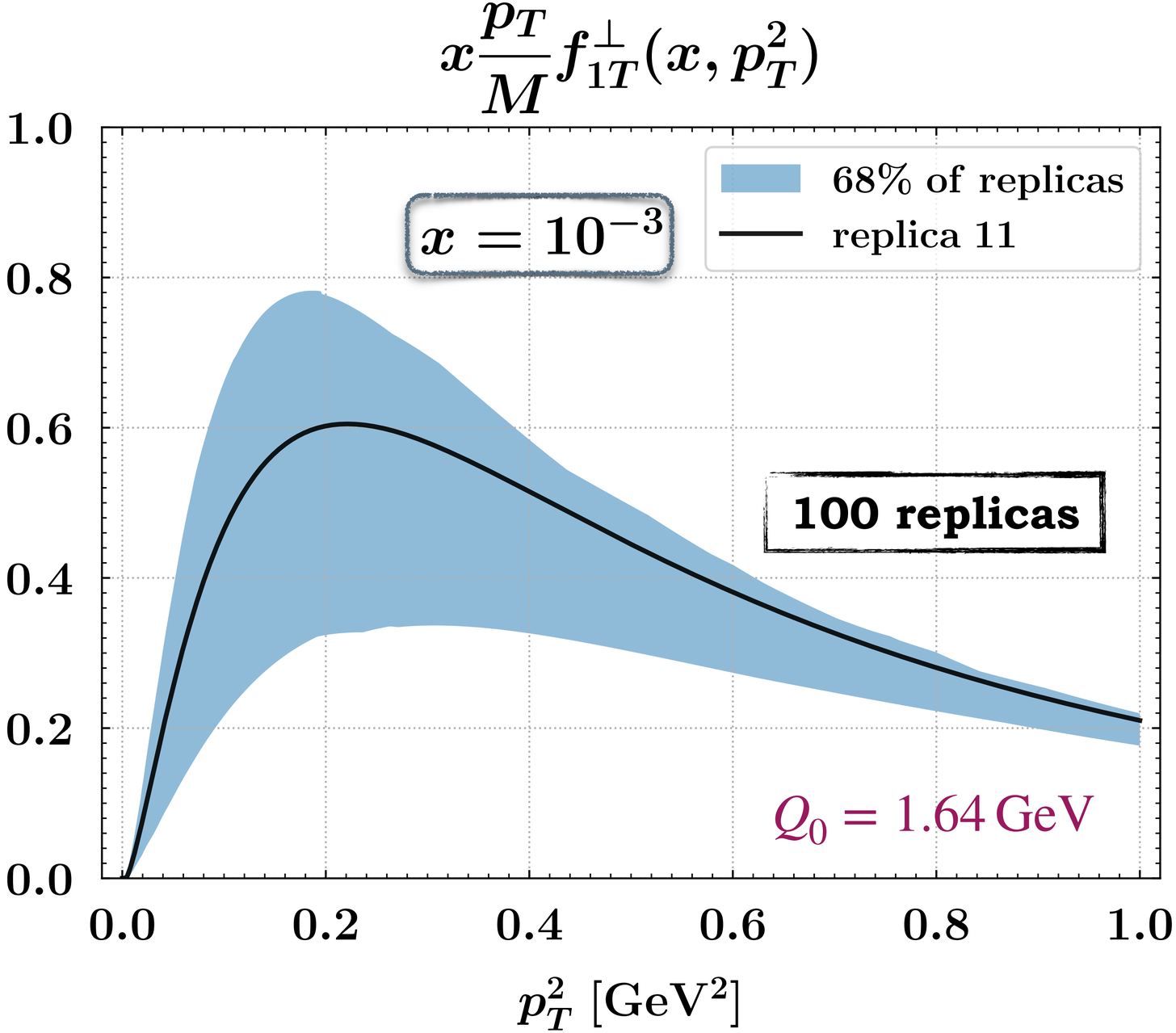
[A. Bacchetta, F.G.C., M. Radici, P. Taels (EPJC 2020)]



f-type Sivers gluon TMD

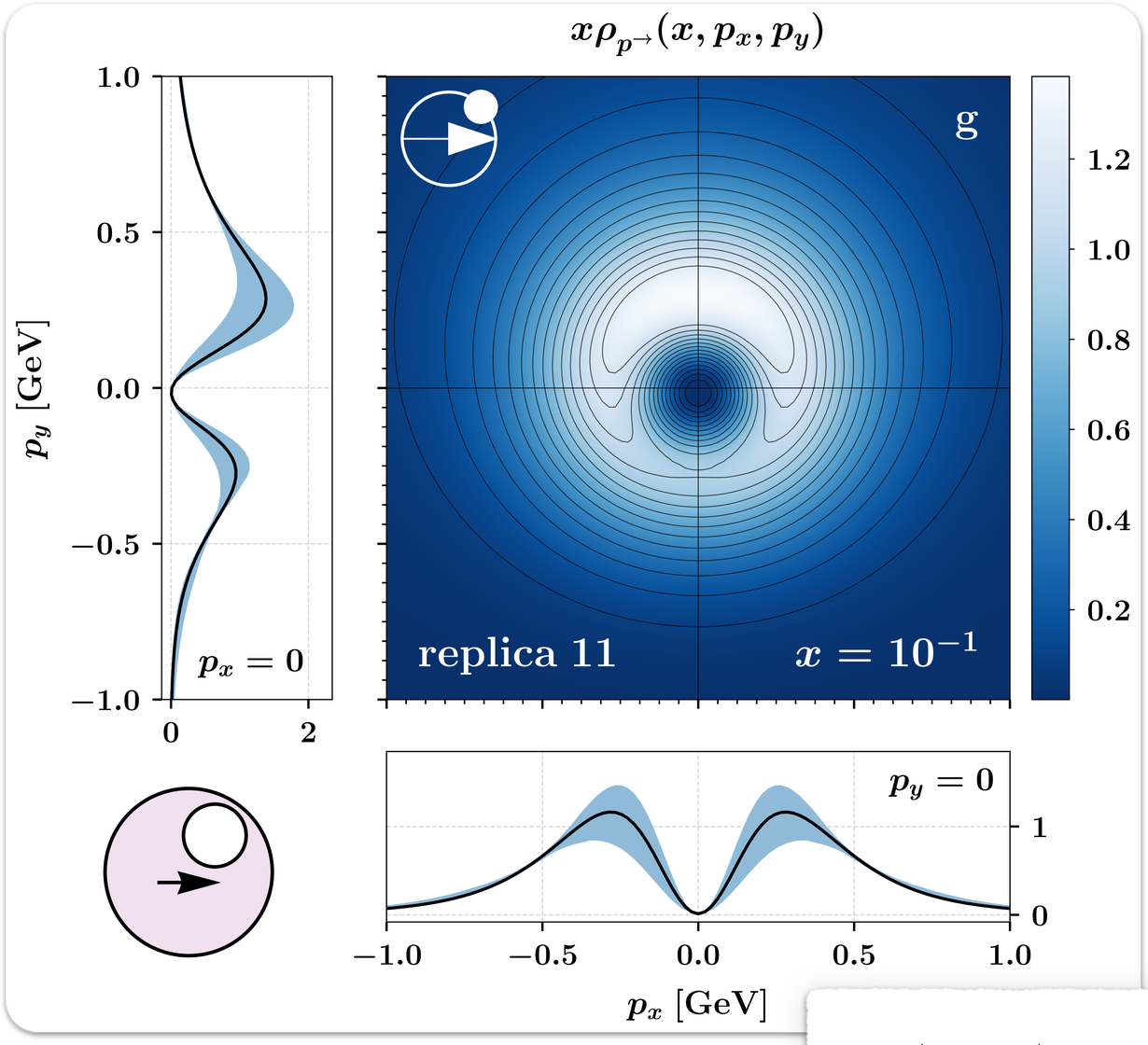
[A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]





Gluon Sivers effect

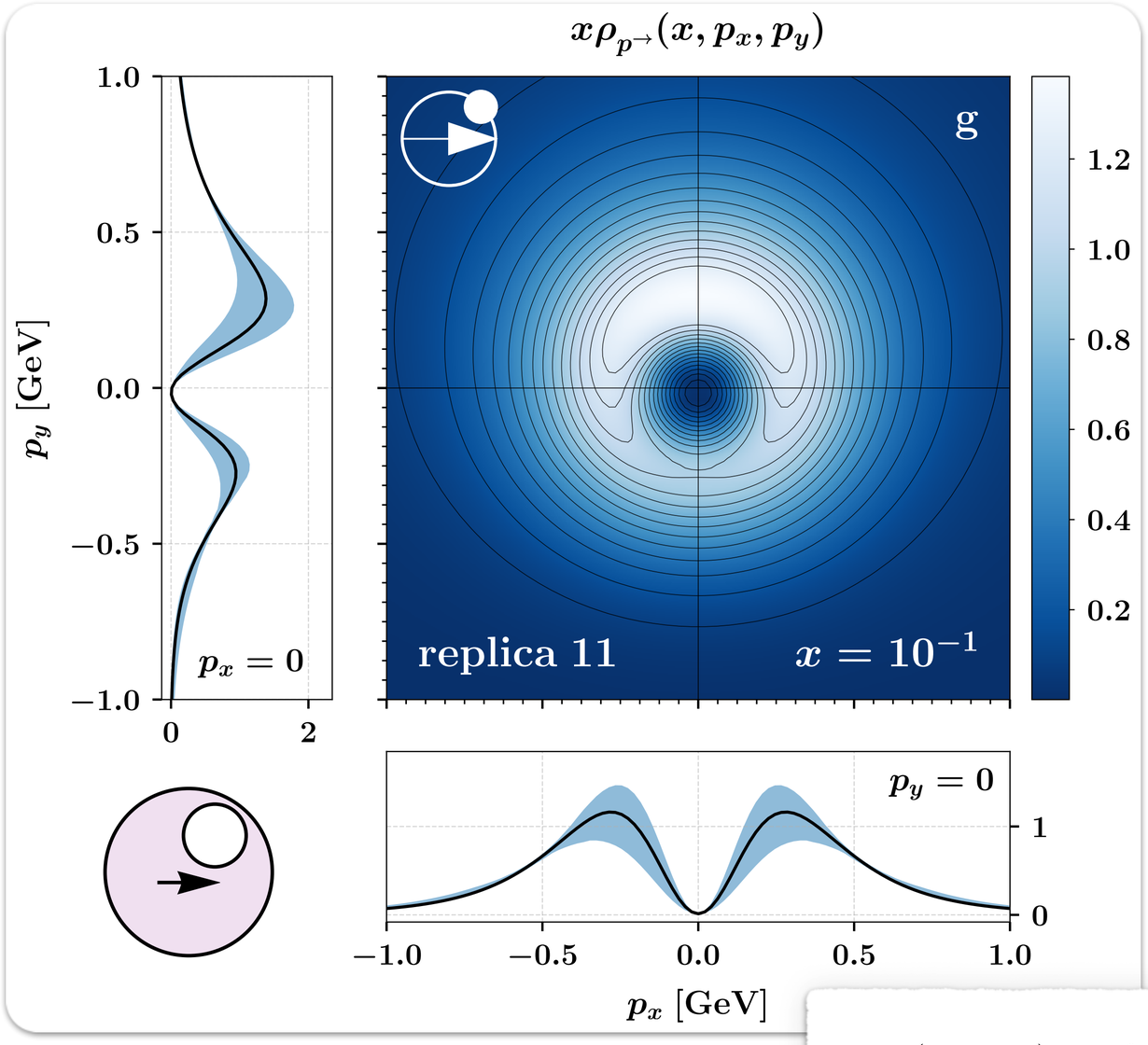
🔗 [A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]

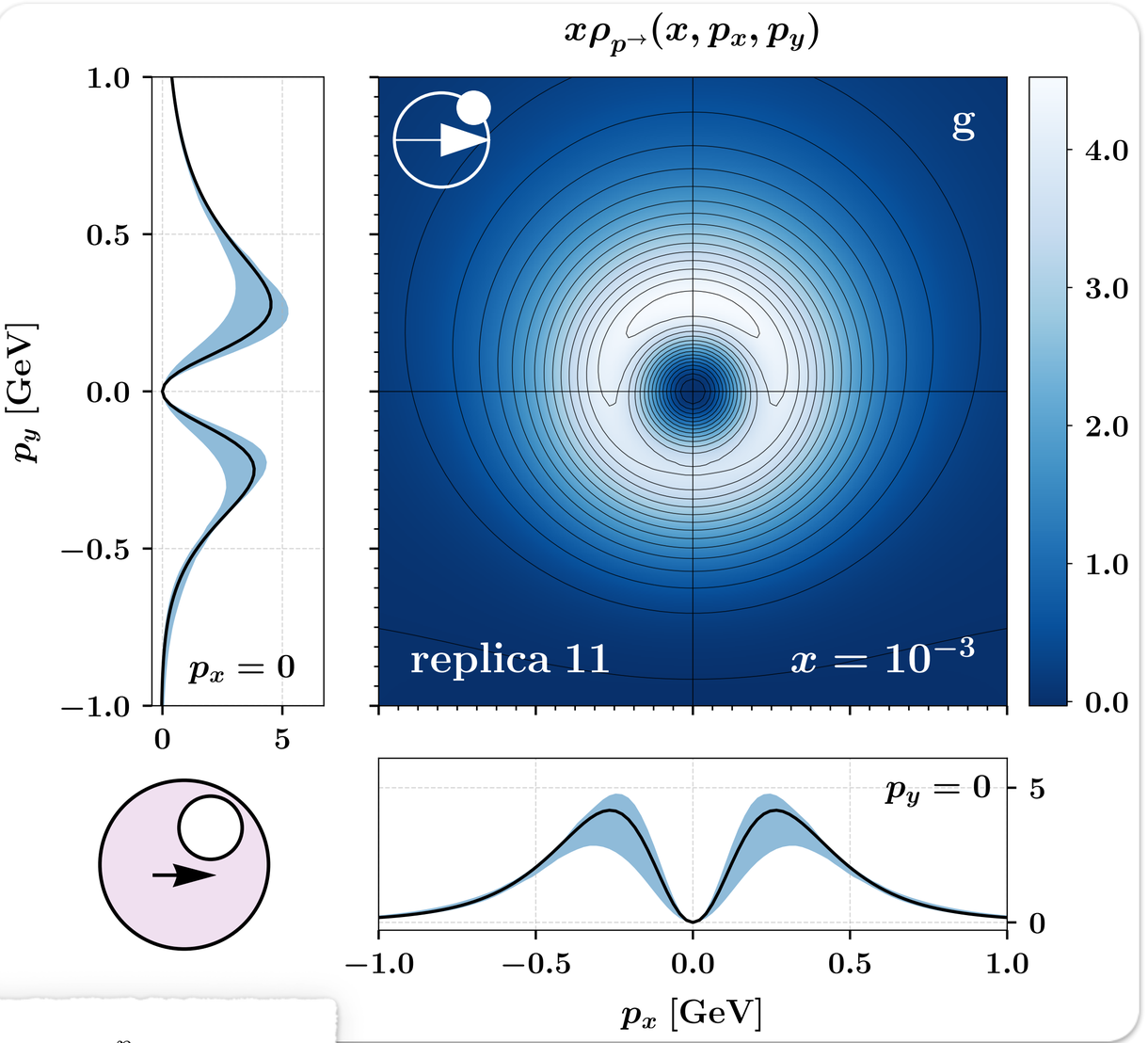


$$x\rho_{p^\rightarrow}(x, p_x, p_y) = x\, f_1(x, \boldsymbol{p}_T^2) + x\frac{p_y}{M}\, f_{1T}^\perp(x, \boldsymbol{p}_T^2)$$

Gluon Sivers effect

[A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]

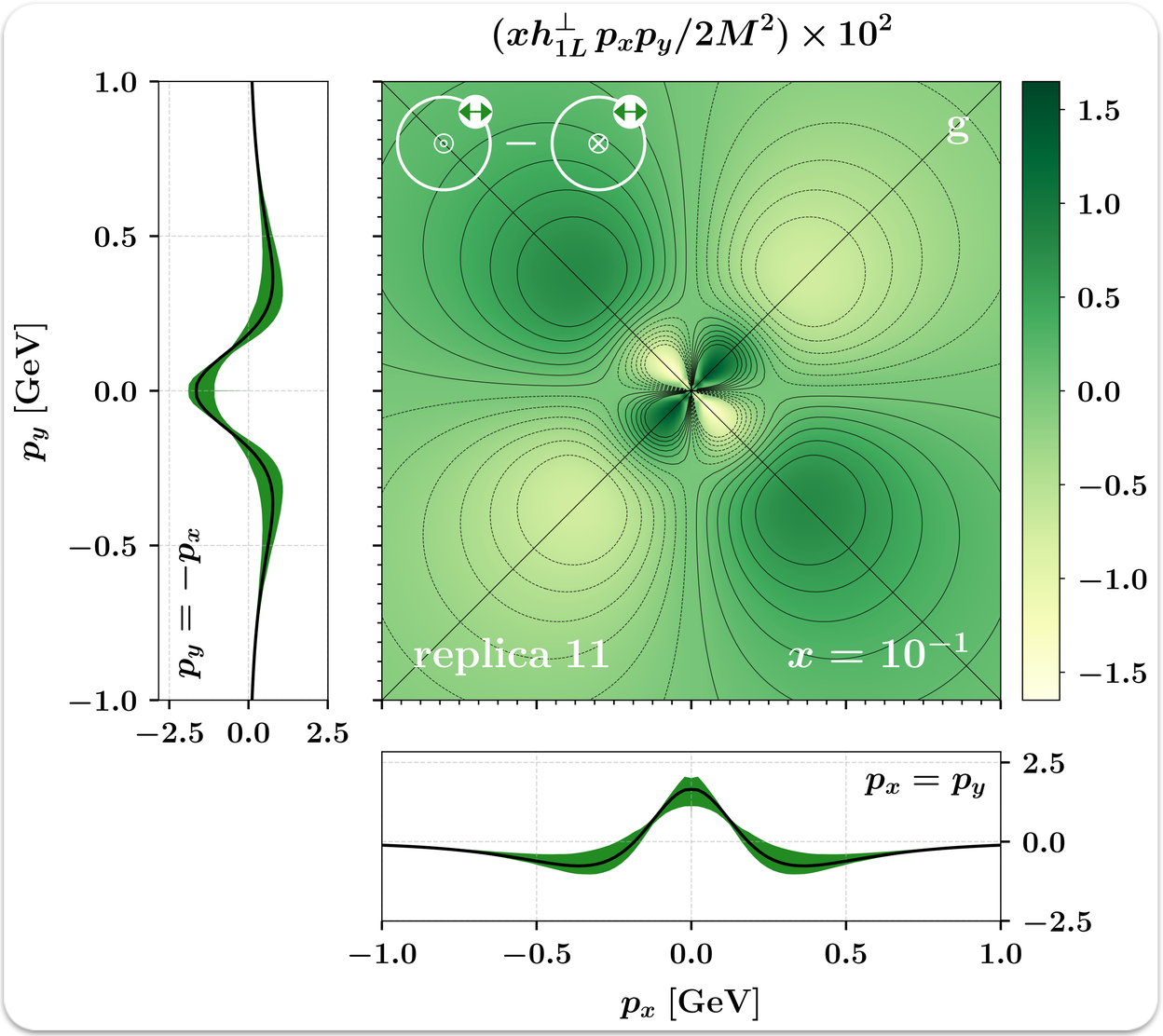


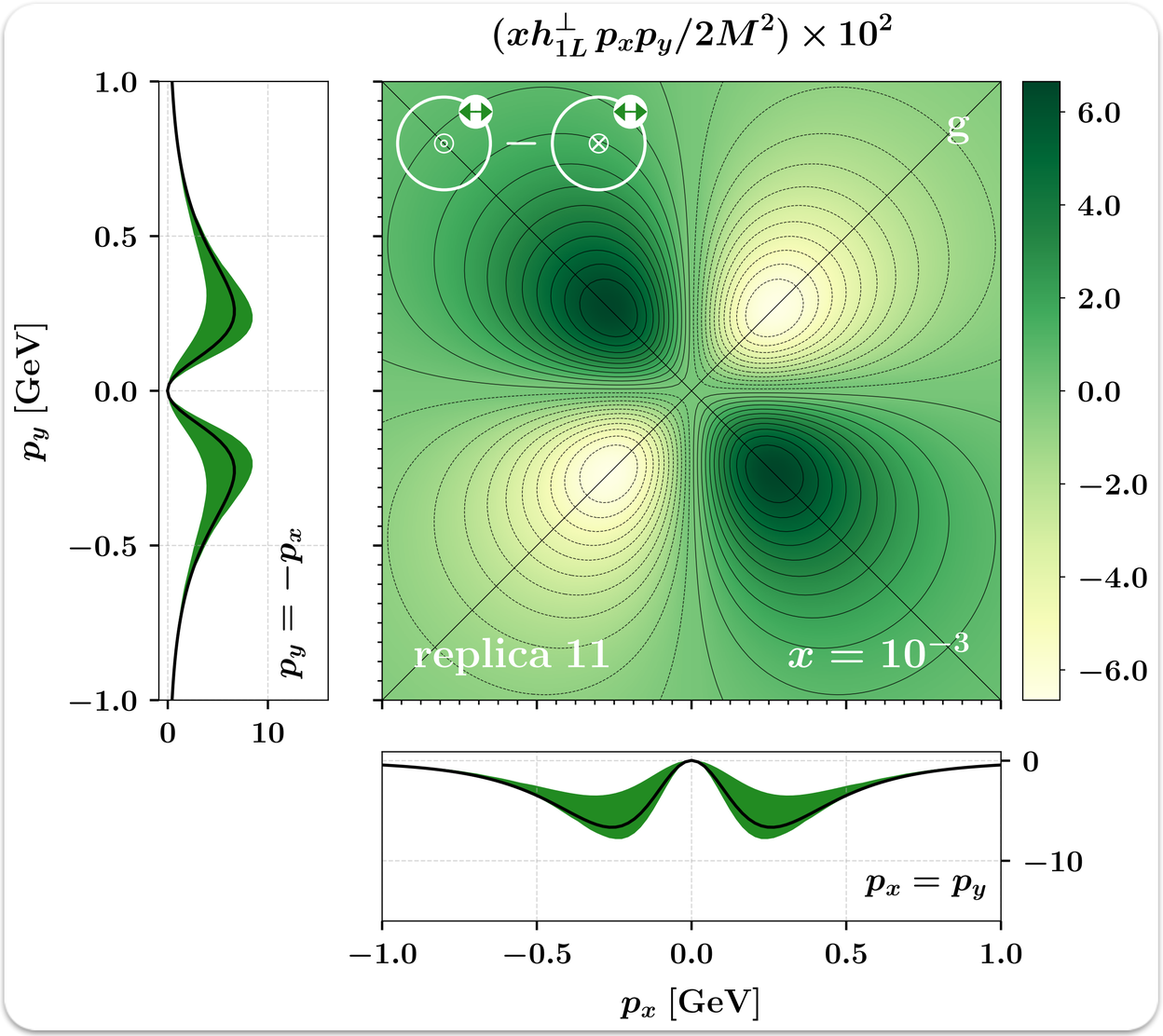


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Gluon propeller effect

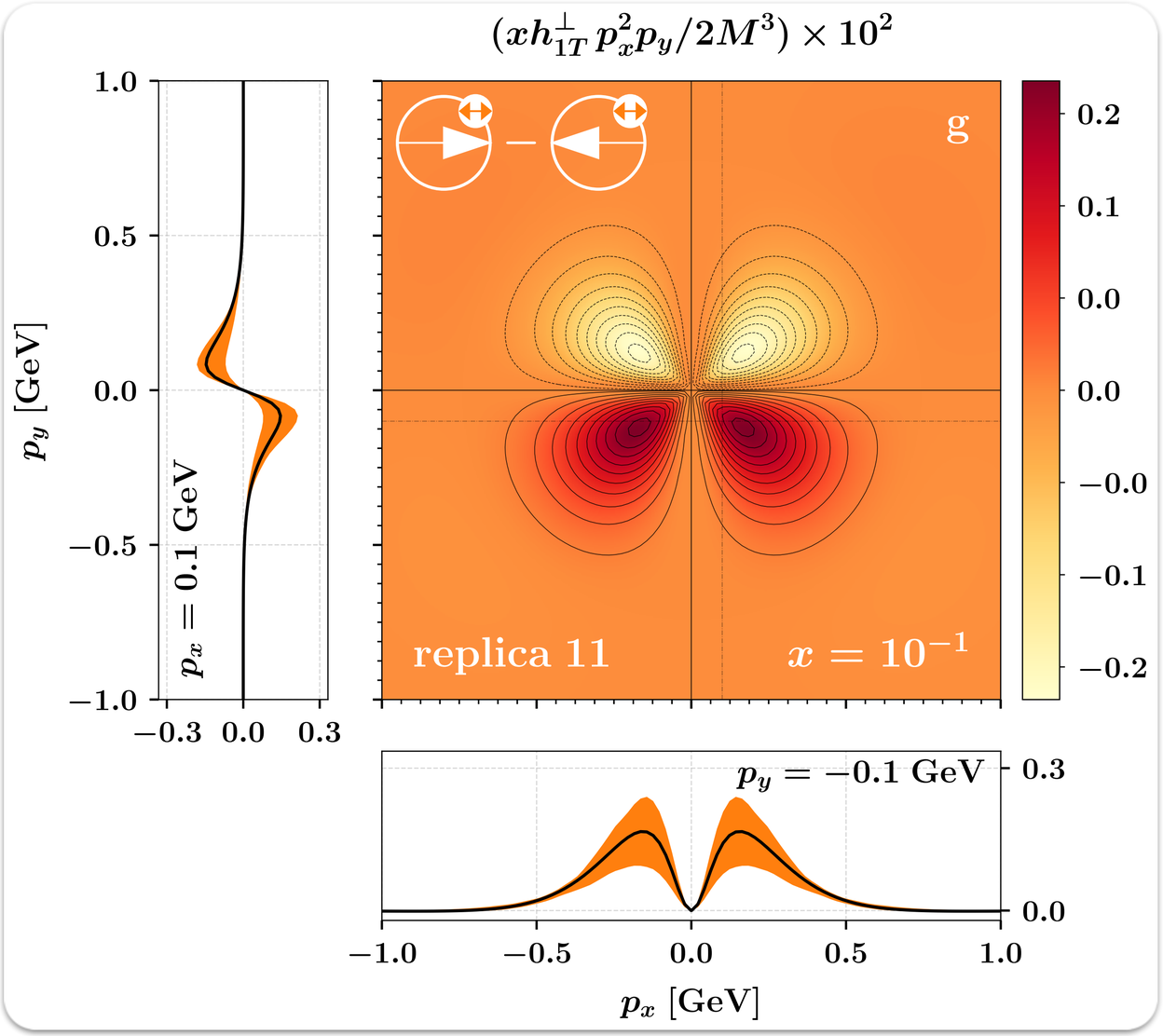
[A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]

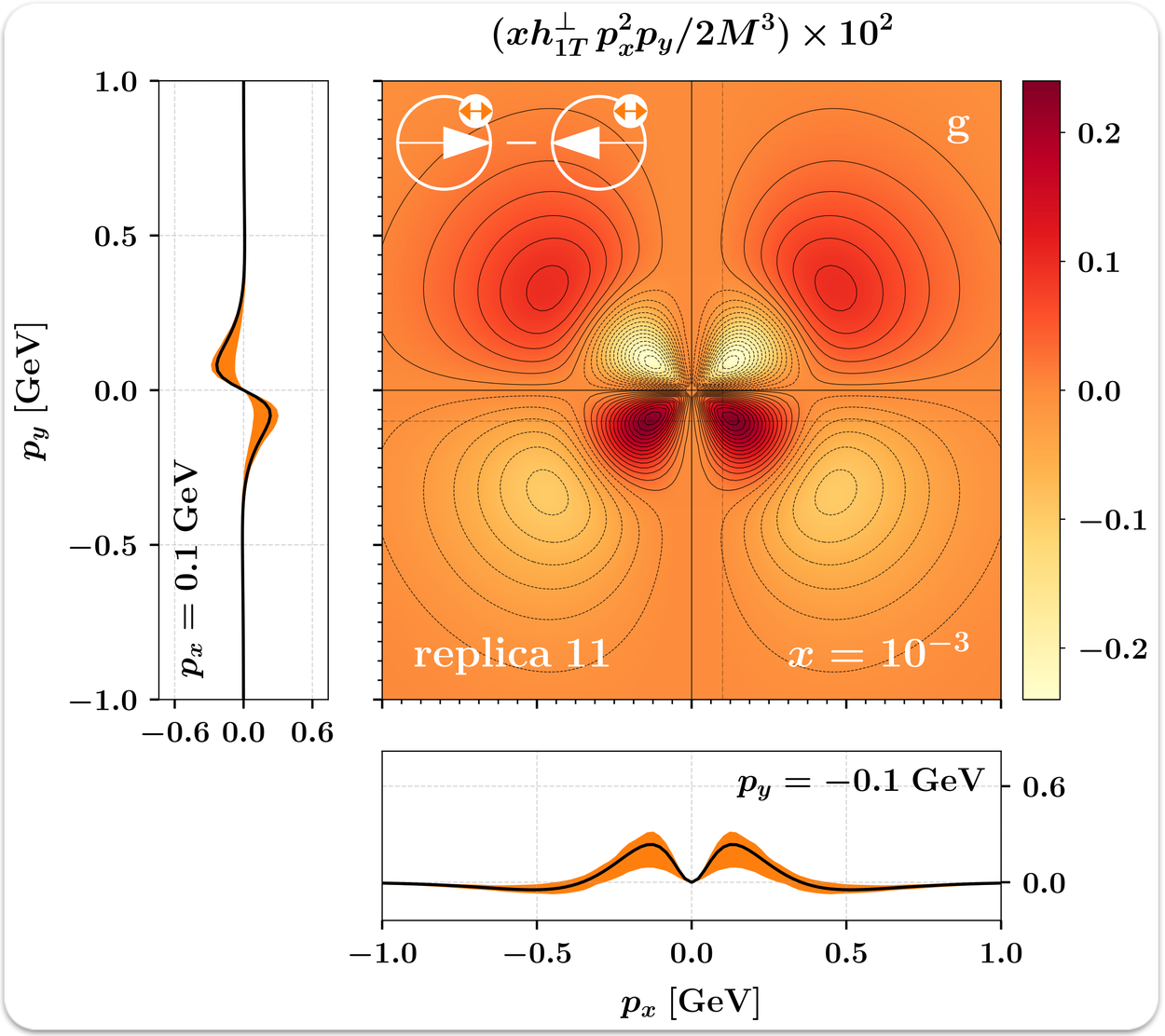




Gluon butterfly effect

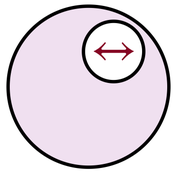
[A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]





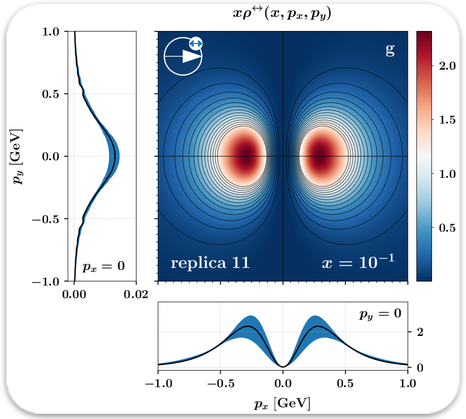
Quarkonia: Precision & Exploration

TMD exploration: Assets



EIC, LHCb, LHCspin

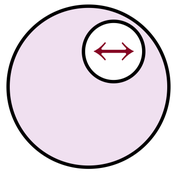
Boer-Mulders



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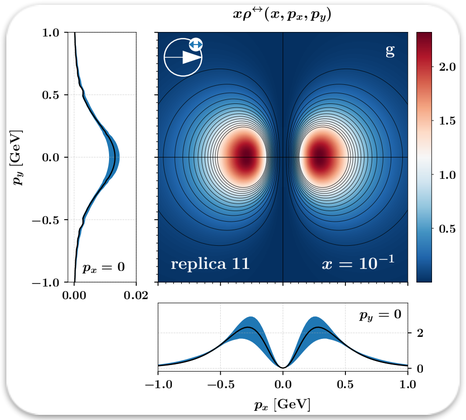
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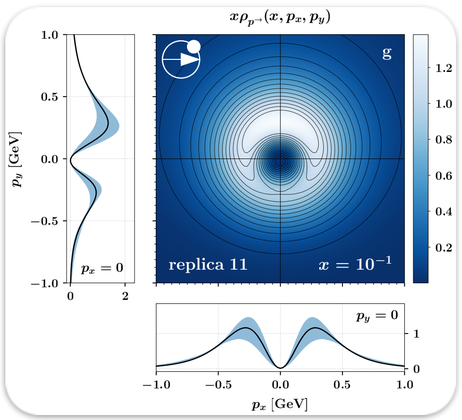
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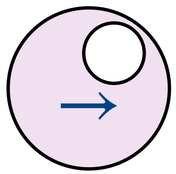
Boer-Mulders



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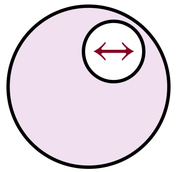


EIC, LHCspin

Sivers

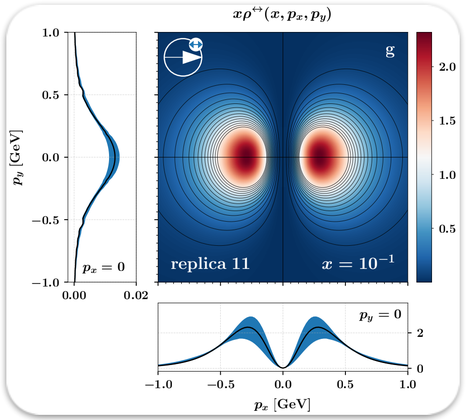
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TMD exploration: Assets

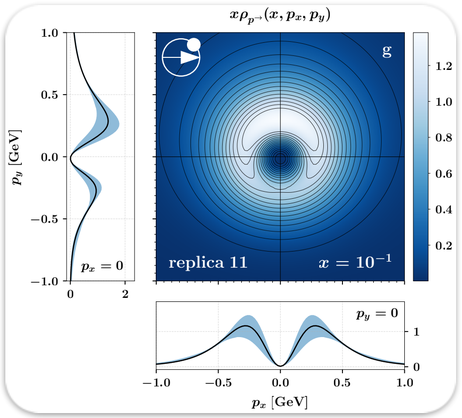


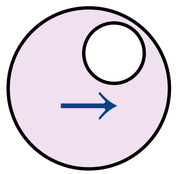
EIC, LHCb, LHCspin

Boer-Mulders



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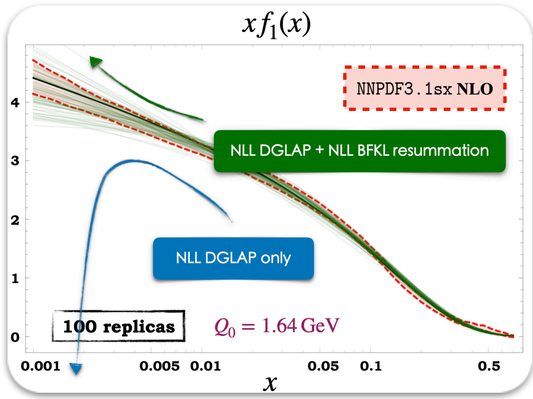


EIC, LHCspin

Sivers

Precision QCD: Challenges

Forward Onia ⇒ Gluon PDF & TMD Positivity

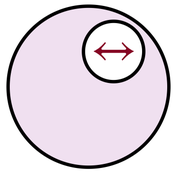


Low x and low Q^2

🔗 [G. Altarelli et al. (1998)]
🔗 [A. Candido et al. (2020)]
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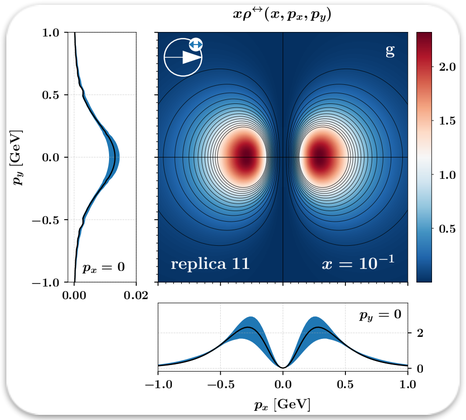
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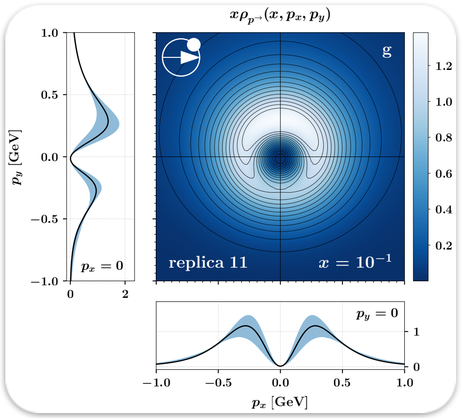


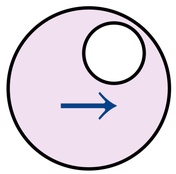
EIC, LHCb, LHCspin

Boer-Mulders



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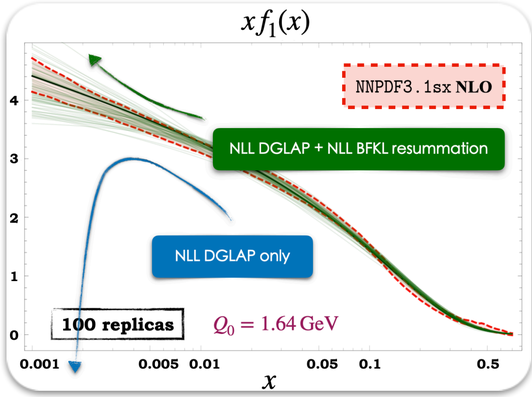


EIC, LHCspin

Sivers

Precision QCD: Challenges

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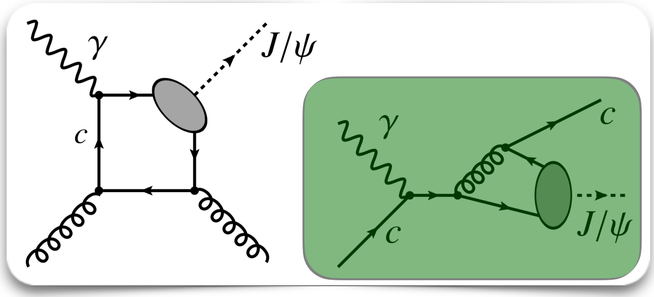


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$\gamma + g \rightarrow J/\psi + c$ ⇒ Valence Intrinsic Charm @EIC

Large x
Moderate to large p_T



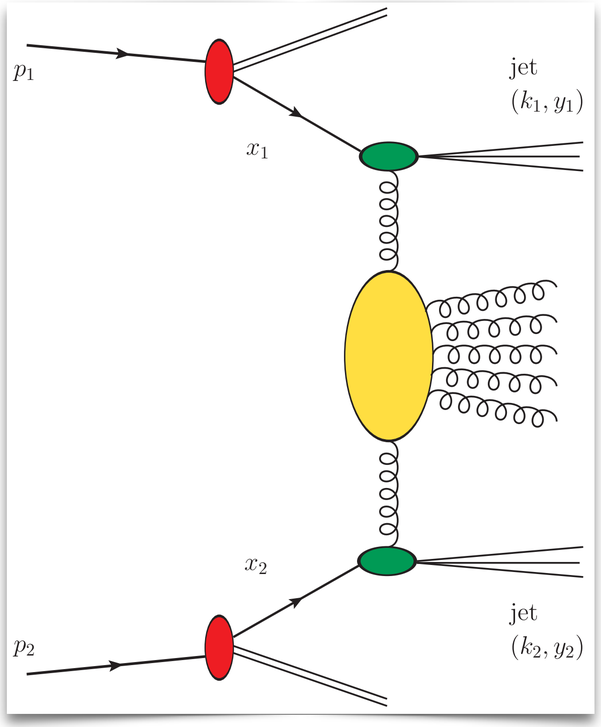
(J/ψ @EIC) 🔗 [C. Flore, J.-P. Lansberg, H.-S. Shao, Y. Yedelkina (2020)]
(Intrinsic Charm + valence) studies 🔗🔗 [NNPDF Collaboration (2022, 2023)]

MUELLER-NAVELET JETS

Mueller-Navelet jets @LHC & resummation instabilities

- Inclusive hadroproduction of two jets with high p_T and large rapidity separation, ΔY
- Moderate x (collinear PDFs), but t-channel p_T (BFKL resummation) $\Rightarrow$ hybrid factorization (HyF)

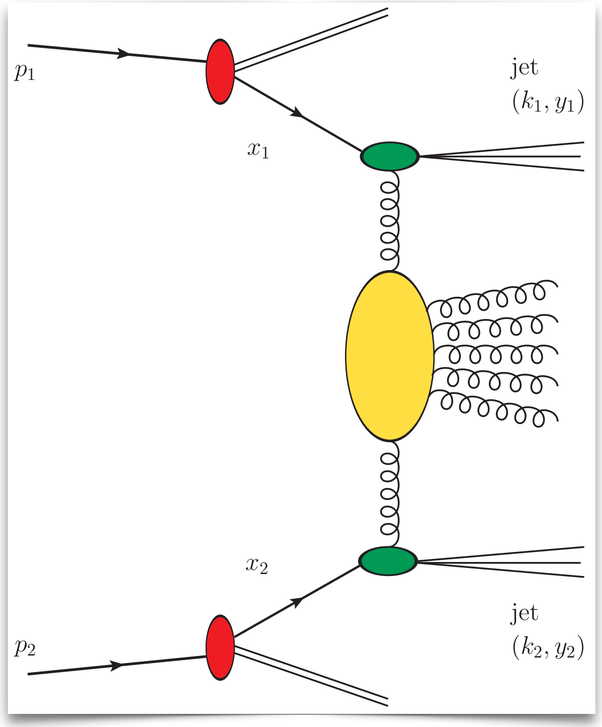
$$\frac{d\sigma}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2} = \sum_{r,s=q,g} \int_0^1 dx_1 \int_0^1 dx_2\, f_r(x_1, \mu_F)\, f_s(x_2, \mu_F)\, \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu_F)}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2}$$



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NLO(+)

NLL

NLO(+)

$$\frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu)}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2} = \frac{1}{(2\pi)^2} \times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\, \mathcal{V}_J^{(r)}(\vec{q}_1, s_0, x_1, \vec{k}_1) \times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2) \times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}}\, \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{k}_2)$$

jet vertices (off-shell coefficient functions)

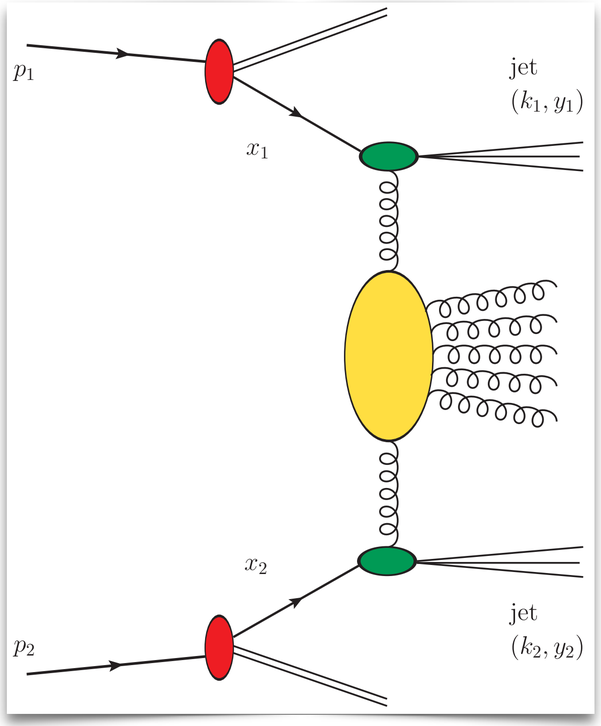
BFKL Green's function

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NLO(+)

NLL

NLO(+)

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BFKL Green's function



- NLL/LL instabilities + NLO missing threshold ⇒ precision studies hampered

Mueller-Navelet jets @LHC & resummation instabilities

- Strong manifestation of higher-order instabilities via scale variation (¡!)
- ¡ At natural scales: NLL/LL ratio large, no agreement with data, unphysical values !
- BLM scales, theory vs experiment: CMS @7TeV with symmetric p_T-ranges, only

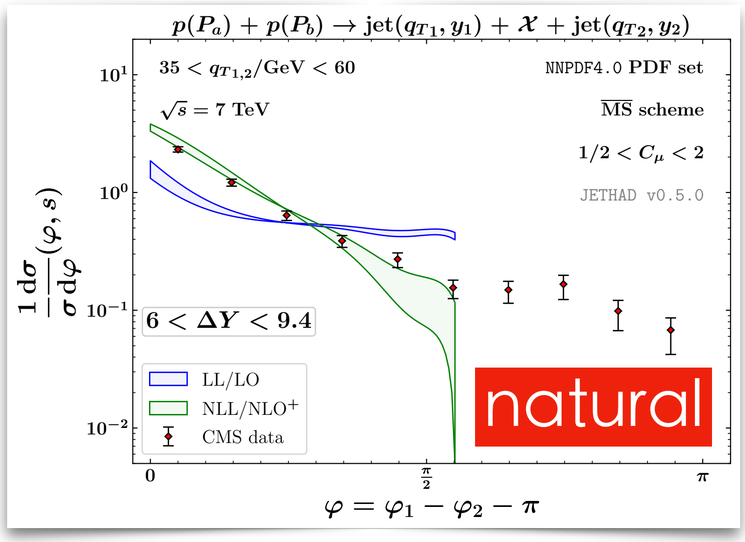
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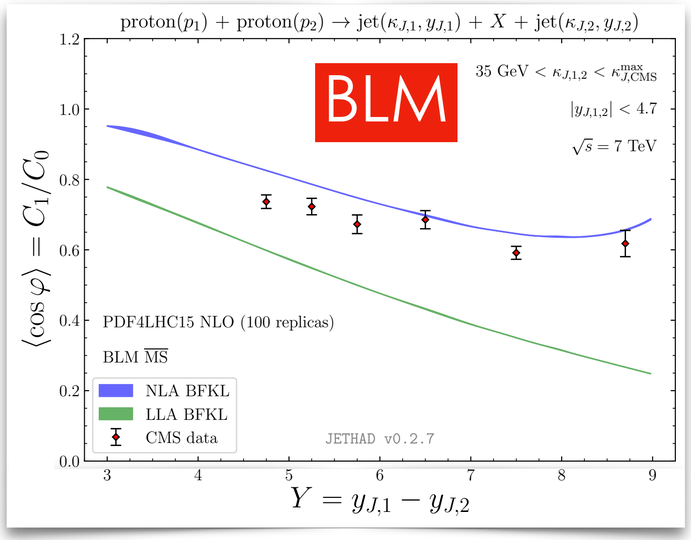
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🔗 [CMS Collaboration, JHEP 08 (2016) 139]

🔗 [B. Ducloué et al., Phys. Rev. Lett. 112 (2014) 082003]

🔗 [F. Caporale et al., Eur. Phys. J. C 74 (2014) 10, 3084]





(left figure) 🔗 [F. G. C., A. Papa, Phys. Rev. D 106 (2022) 11, 114004]

(right figure) 🔗 [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]

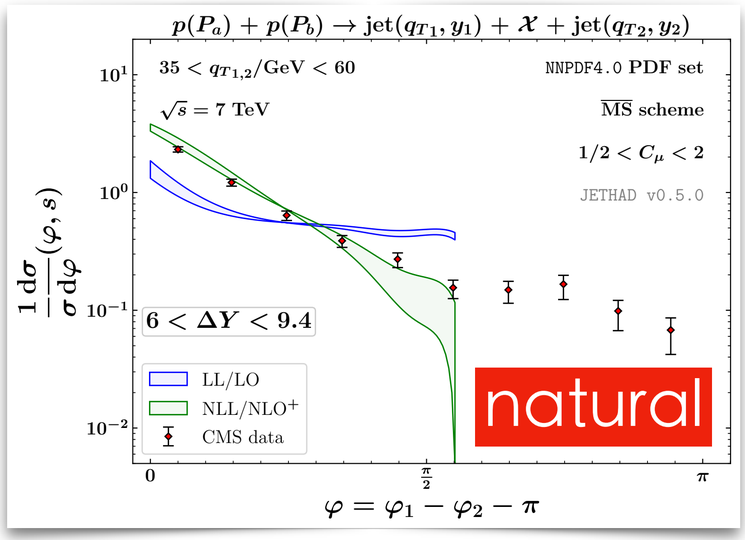
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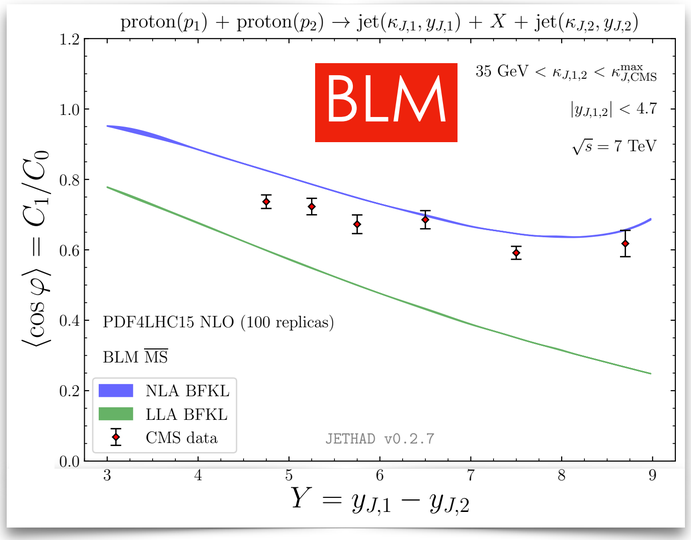
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- $\mu_R^{\mathrm{BLM}} \gg \mu_R^{\mathrm{nat.}} \Rightarrow d\sigma^{\mathrm{BLM}}/d\sigma^{\mathrm{nat.}} \sim 10^{-(1\div 2)} \Rightarrow$ precision studies hampered



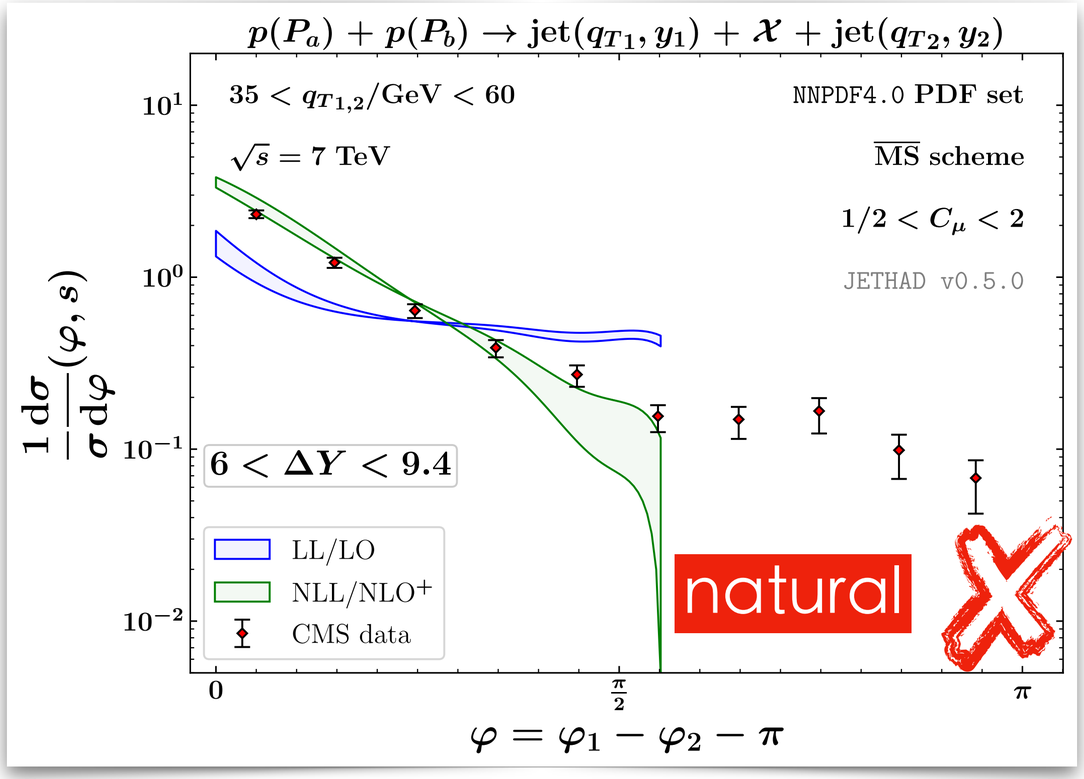
- Unsuccessful scale optimization → processes featuring natural stability (¿?)

Azimuthal-angle multiplicity

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma(\Delta Y, s)}{\mathrm{d}\varphi} = \frac{1}{2\pi}\left\{1 + 2\sum_{n=1}^{\infty}\cos(n\varphi)\langle\cos(n\varphi)\rangle\right\}$$

MUELLER-NAVELET JETS

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(figure below) 🔗 [F. G. C., A. Papa, Phys. Rev. D 106 (2022) 11, 114004]



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MUELLER-NAVELET JETS

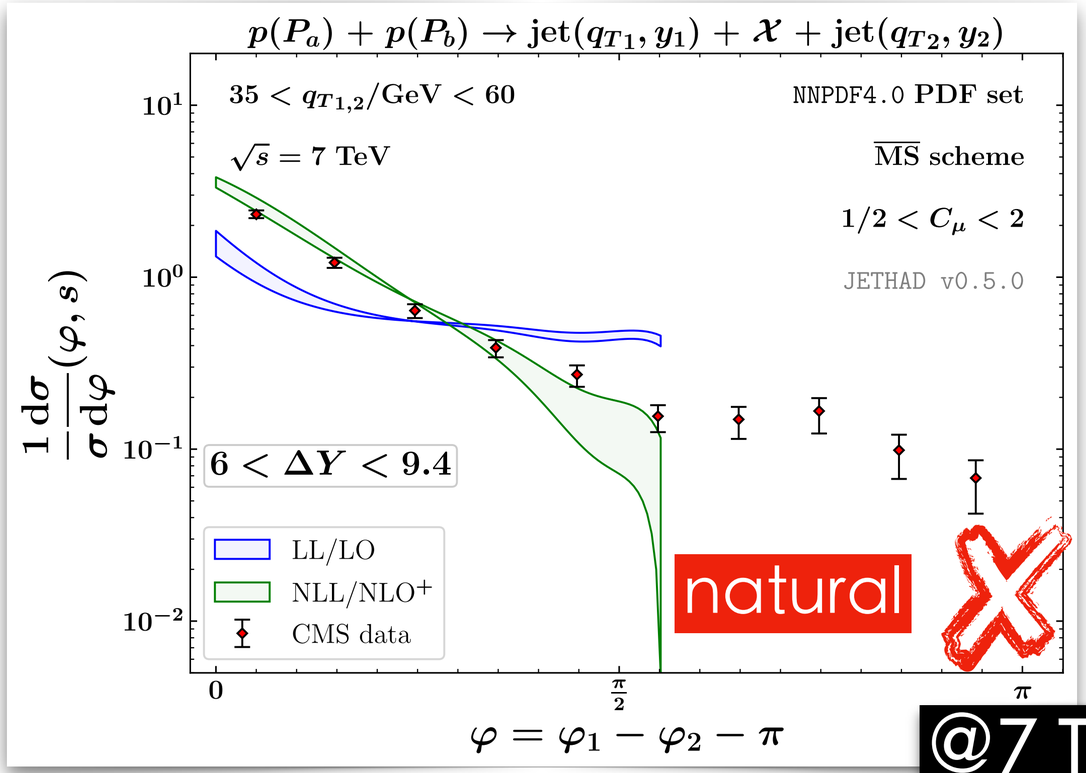
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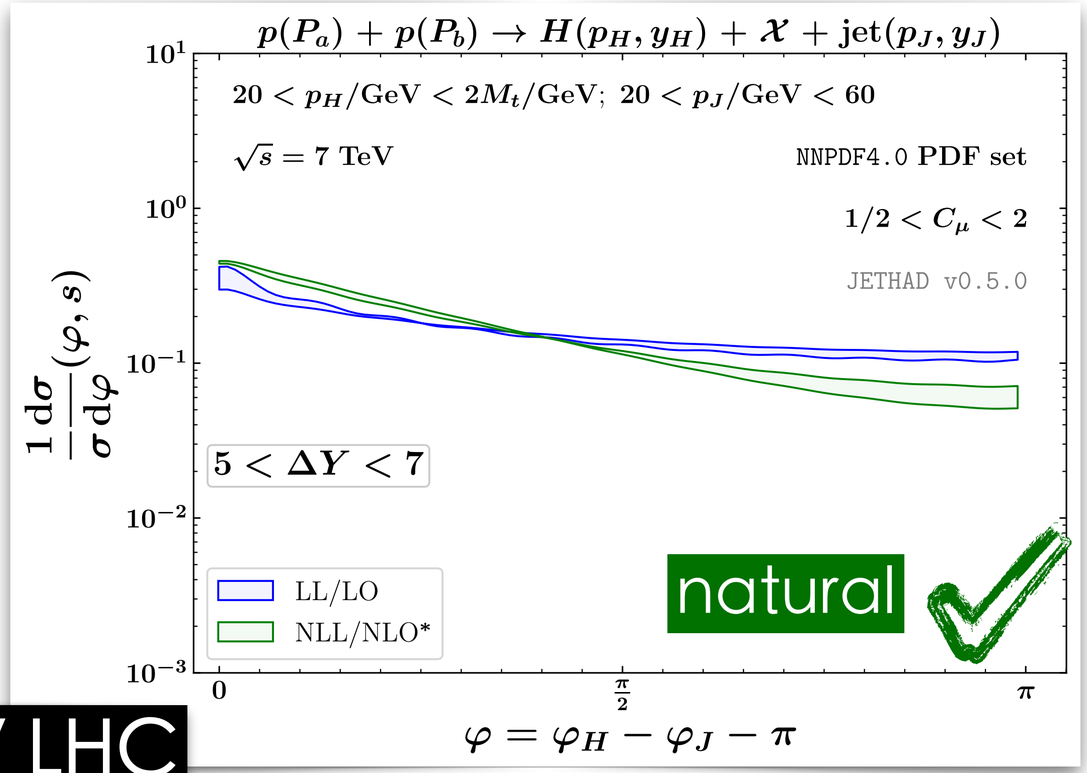
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HIGGS + JET

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@7 TeV LHC

Hybrid factorization via the JETHAD code

🔗 [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]. 🔗 [F. G. C. , Phys. Rev. D 105 (2022) 11, 114008]

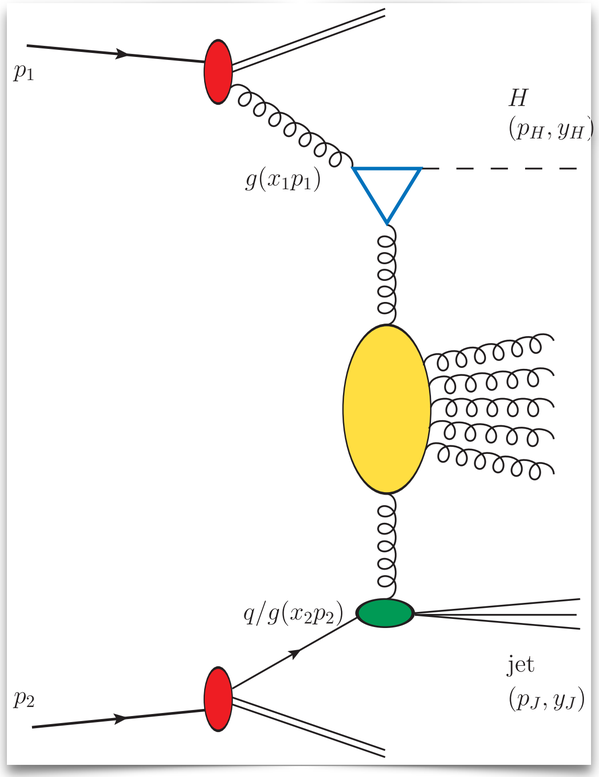
HIGGS + JET DISTRIBUTIONS

From Mueller-Navelet to Higgs and heavy flavor

Pheno path: hunt for channels leading to a NLL stabilization pattern at natural scales (¡!)

HIGGS BOSON

Stabilizers ⇔ large Higgs transverse masses



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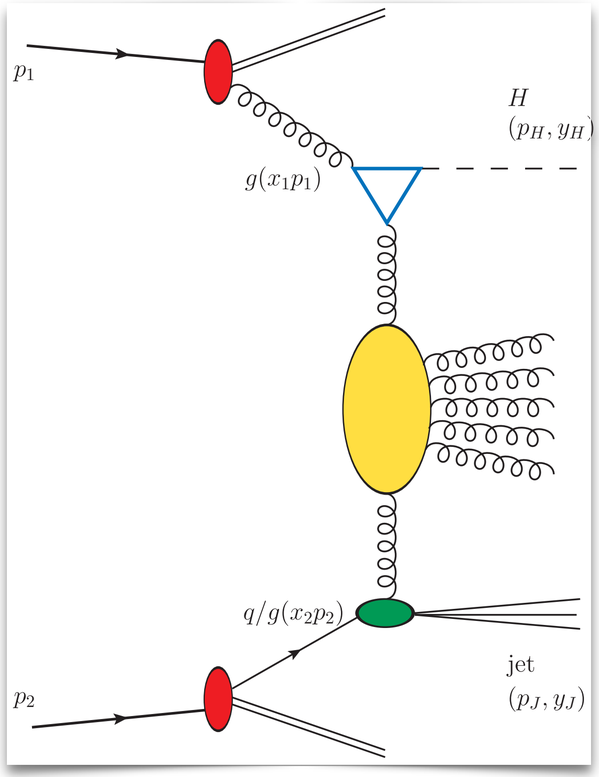
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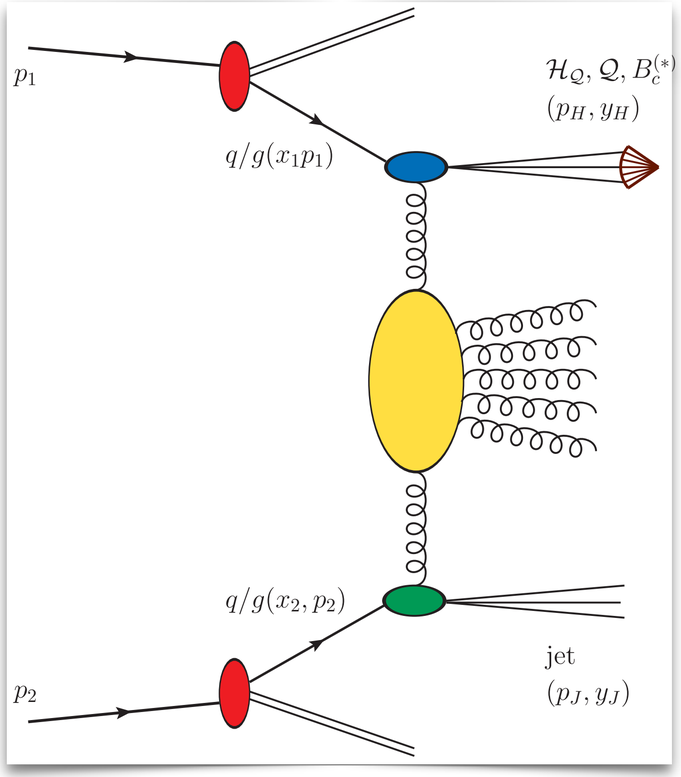


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HEAVY FLAVOR AT LARGE P_T

Stabilizers ⇔ gluon fragmentation channels



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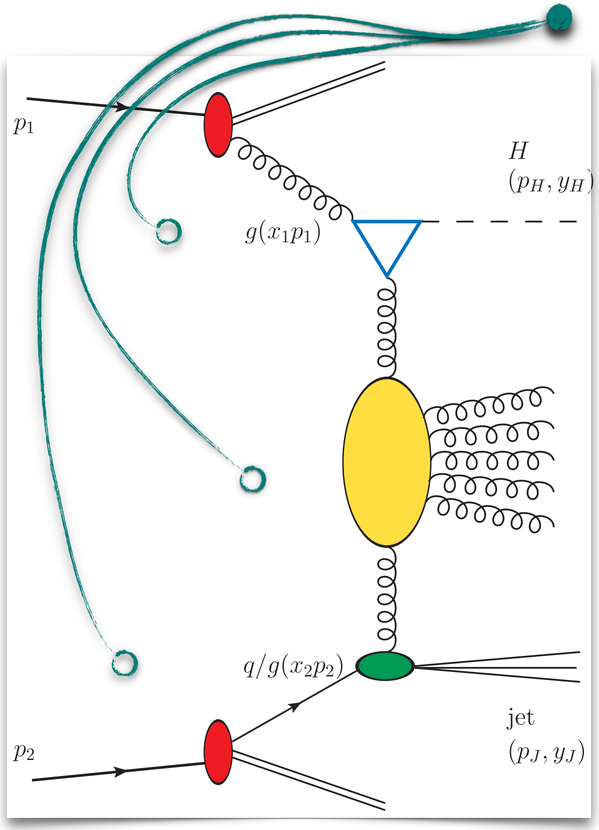
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$\mu_{F,R} \sim M_{H,\perp}$

NLO*

= LO + NLO^{RGE}

NLL

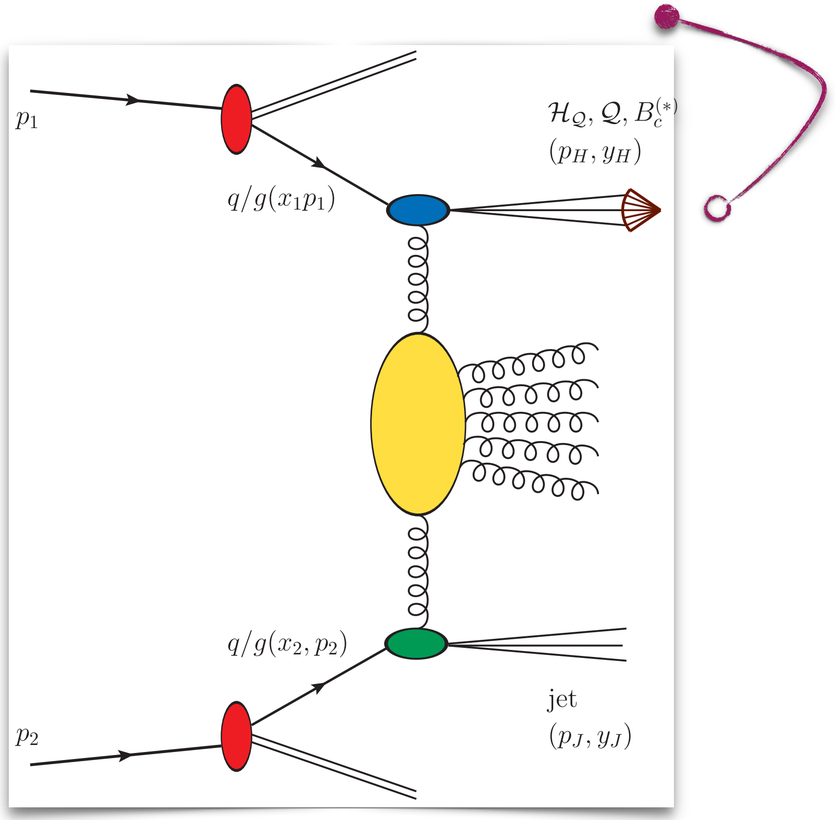
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NLL

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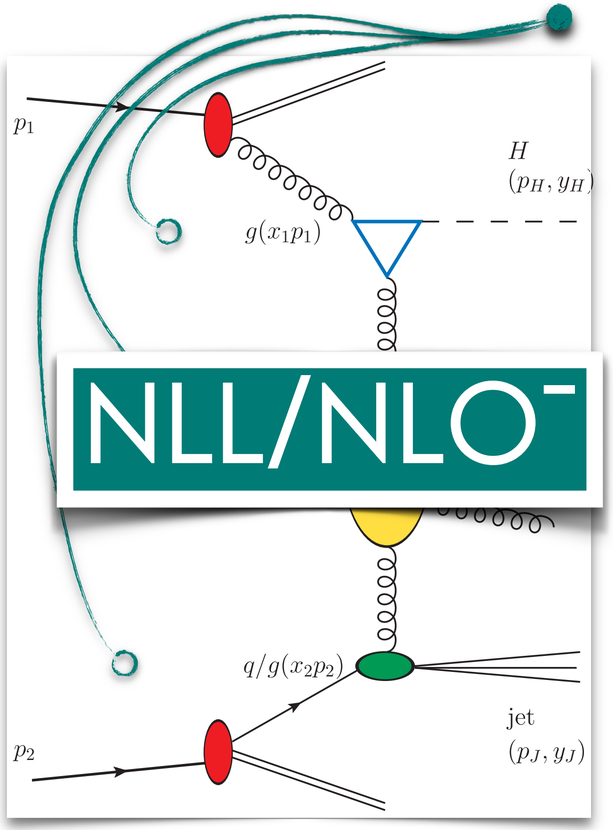
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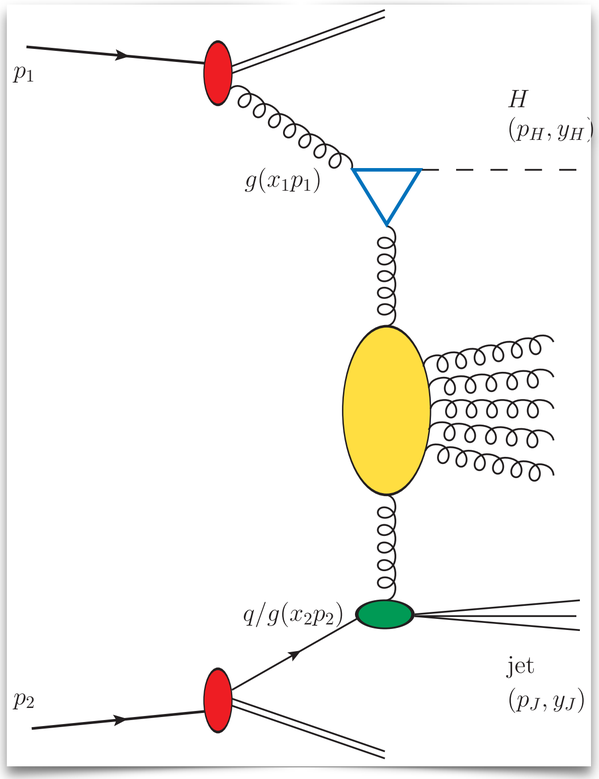
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Inclusive Higgs + jet at the LHC

- Inclusive h.p. of a Higgs + jet system with high p_T and large rapidity separation, ΔY
- Large energy scales expected to stabilize the high-energy resummed series

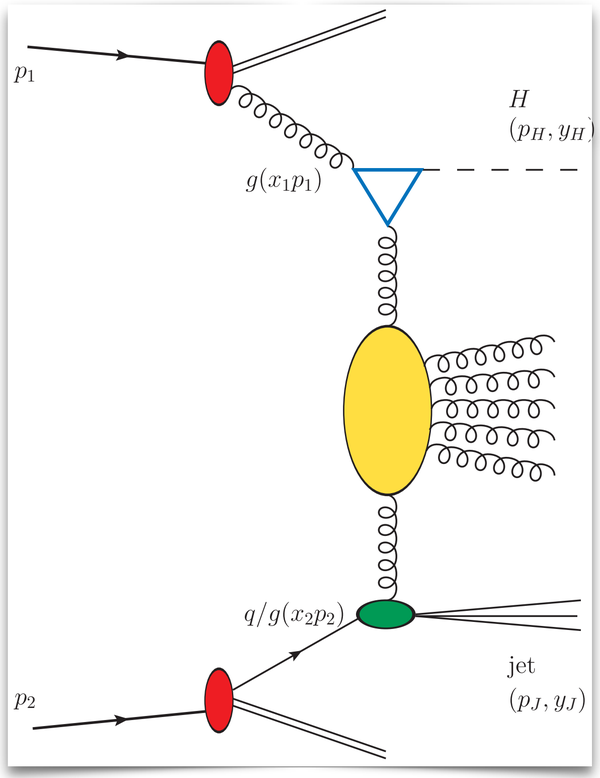
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$$\varphi = \varphi_H - \varphi_J - \pi$$



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Higgs vertex (off-shell coefficient function)

jet vertex (off-shell coefficient function)

NLO*

NLL

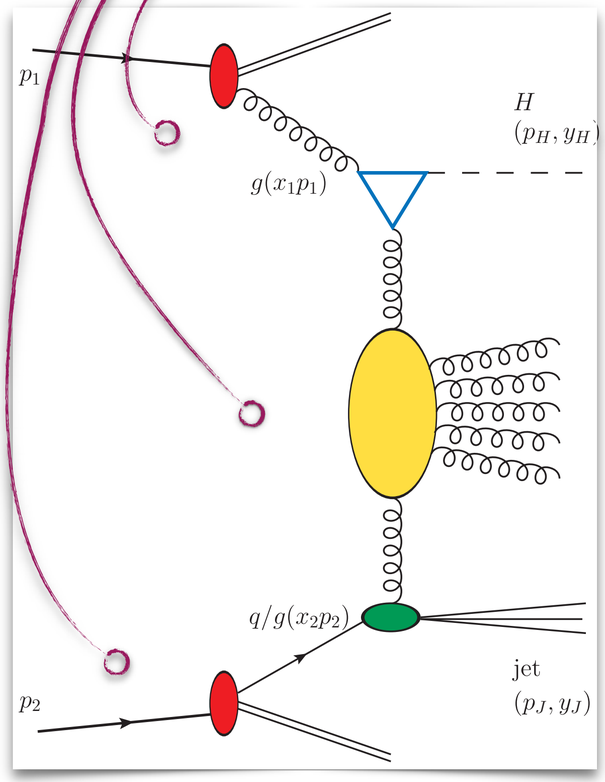
NLO*

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BFKL Green's function

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$\mu_R \sim \sqrt{M_{H,\perp}\, p_J}$

NLL

$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\, \mathcal{V}_H^{(r)}(\vec{q}_1, s_0, x_1, \vec{p}_H)$$

$$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2)$$

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The high-energy QCD dynamics from Higgs+jet correlations at FCC

Francesco G. Celiberto [1,2,3] and Alessandro Papa [4,5]

FCC Week 2022, Sorbonne Université, France

Hors d'œuvre

- Higgs sector → SM benchmarks, BMS portals
- Gluon fusion → key ingredient for precision QCD
- Fixed-order ← improved by resummations
- FCC energies ↔ high-energy (HE) resummation
- Higgs+jet → golden channel to hunt for HE signals

NLL/NLO differential cross section

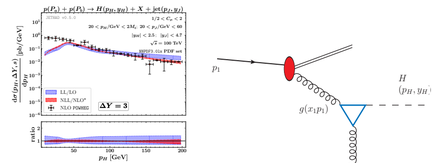
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PDFs with threshold

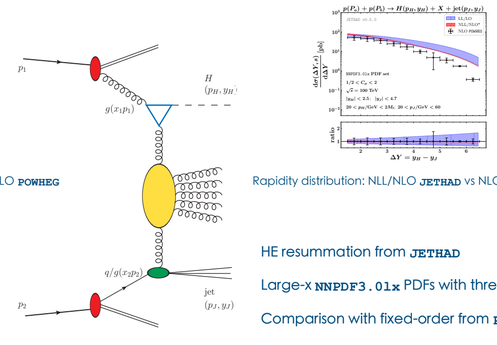
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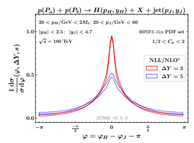
Hybrid high-energy and collinear factorization at work



Higgs p_T distribution: NLL/NLO JETHAD vs NLO POWHEG



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Azimuthal distribution at NLL/NLO

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- ☑ Way toward precision studies of HE QCD (¡!)
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Further information

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Contact: fceliberto@ectstar.eu

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ECT* · FBK Future Built on Knowledge · HAS QCD Hadronic Structure and Quantum Chromodynamics · Future Circular Collider · Università della Calabria Dipartimento di Fisica · INFN Istituto Nazionale di Fisica Nucleare · TIFPA

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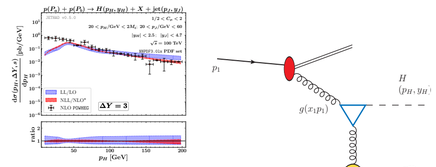
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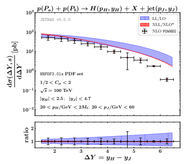
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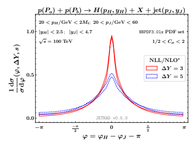
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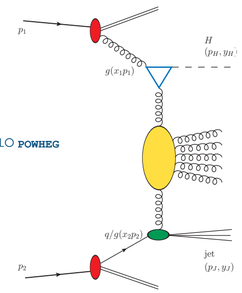
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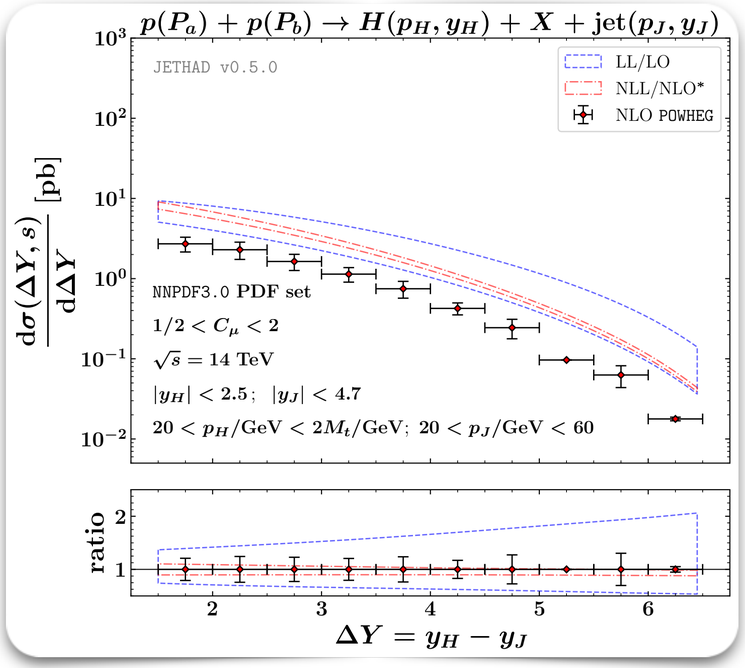
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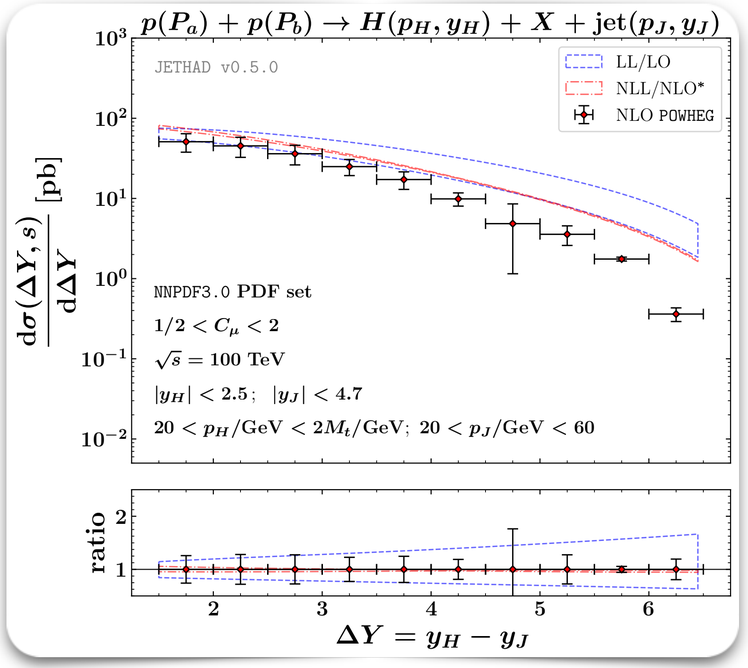
ECT* · FBK · HAS QCD · FUTURE CIRCULAR COLLIDER · UNIVERSITÀ DELLA CALABRIA DIPARTIMENTO DI FISICA · INFN · TIFPA

$$C_n(\Delta Y, s) = \int_{p_H^{\min}}^{p_H^{\max}} d|\vec{p}_H| \int_{p_J^{\min}}^{p_J^{\max}} d|\vec{p}_J| \int_{y_H^{\min}}^{y_H^{\max}} dy_H \int_{y_J^{\min}}^{y_J^{\max}} dy_J\, \delta\left(y_H - y_J - \Delta Y\right) \mathcal{C}_n$$

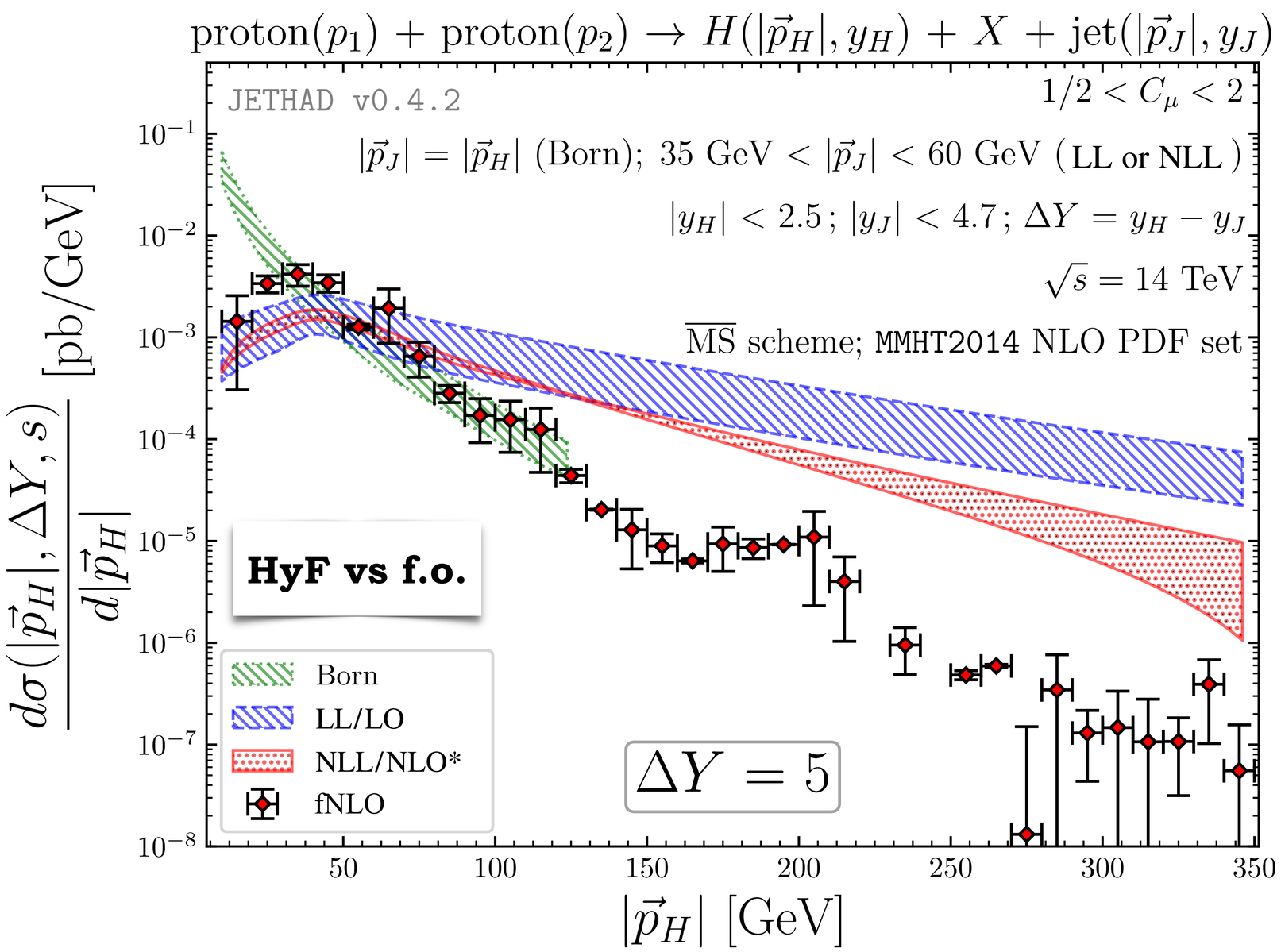
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14 TeV LHC

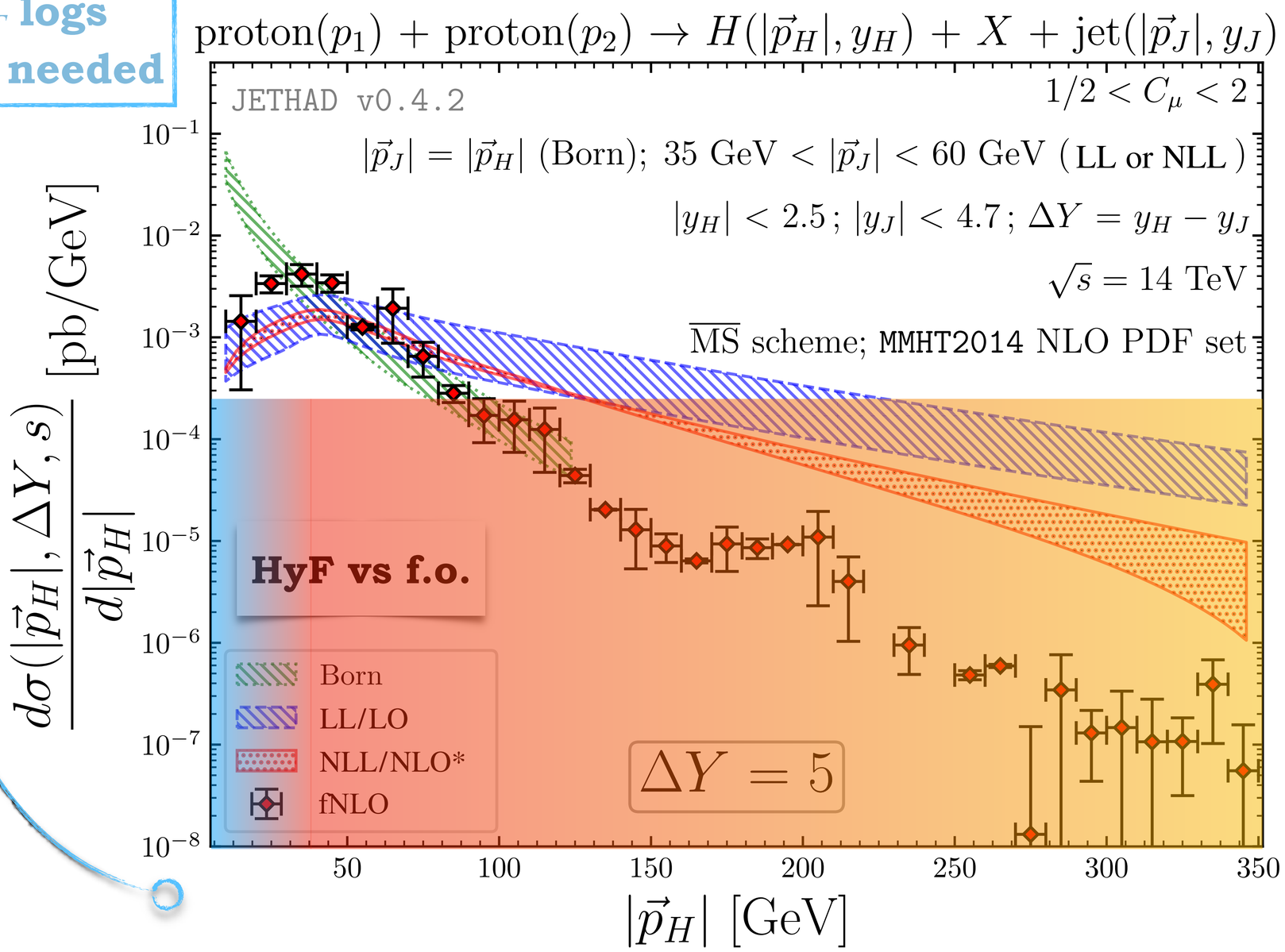


100 TeV FCC



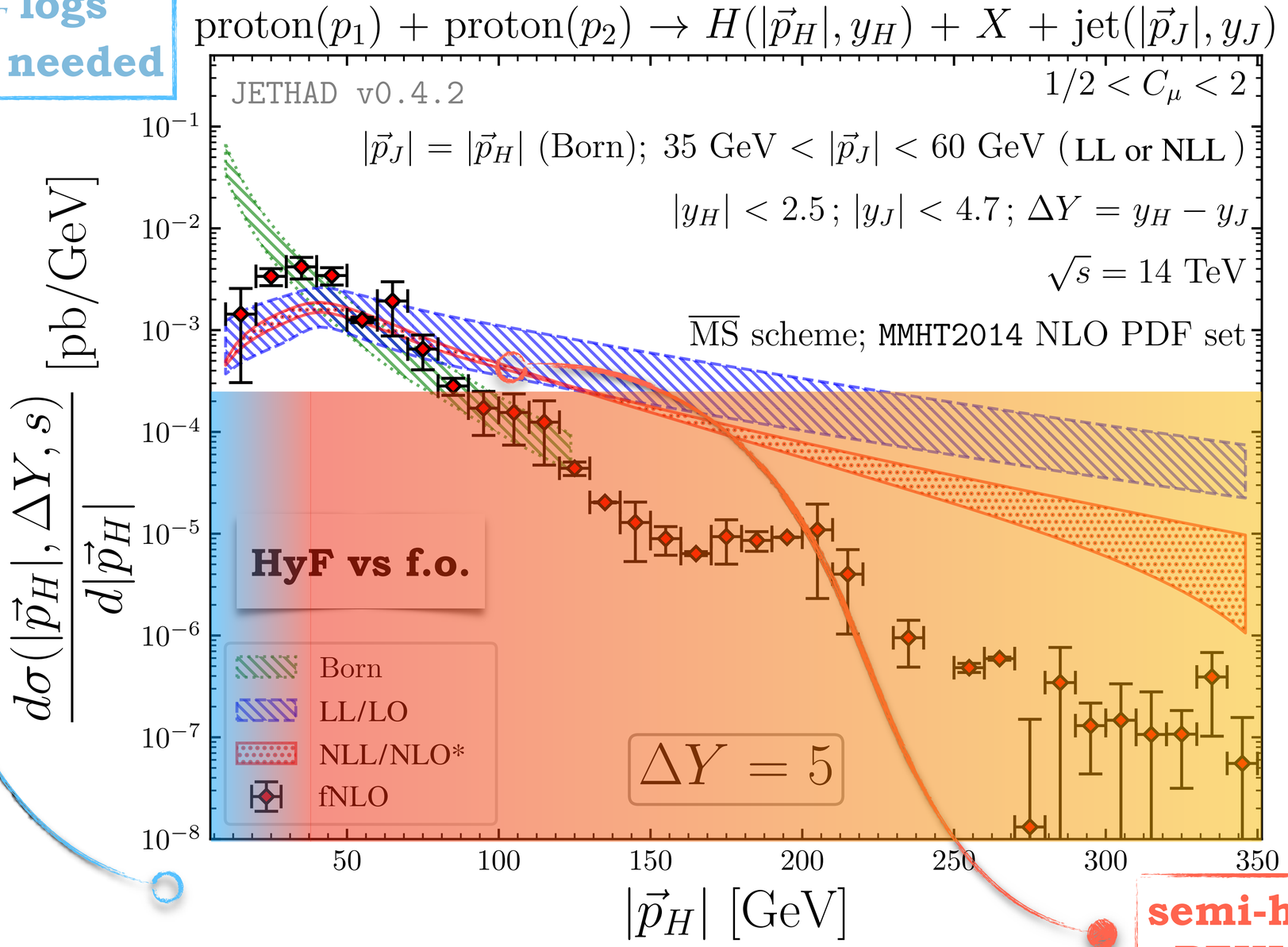
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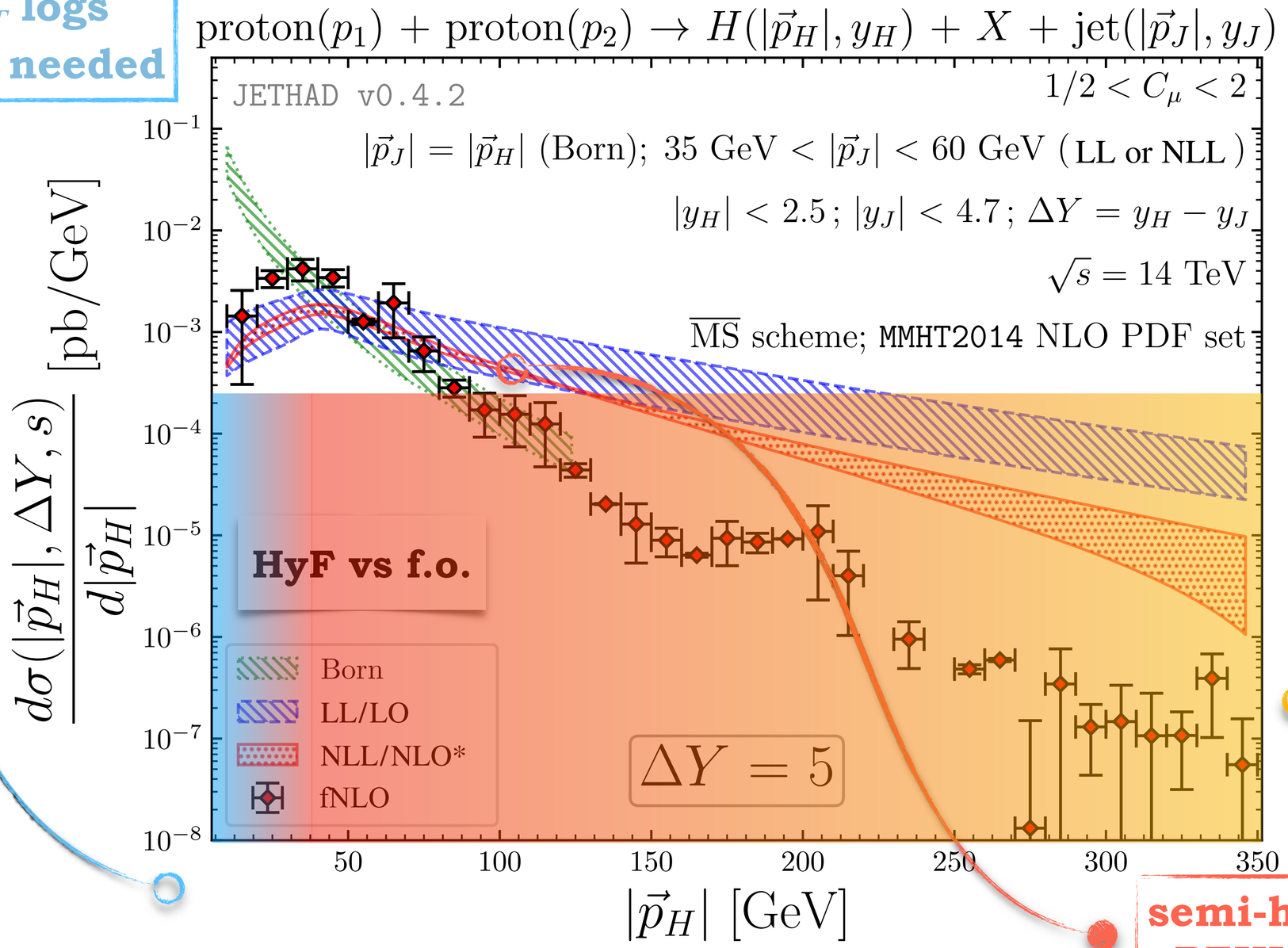


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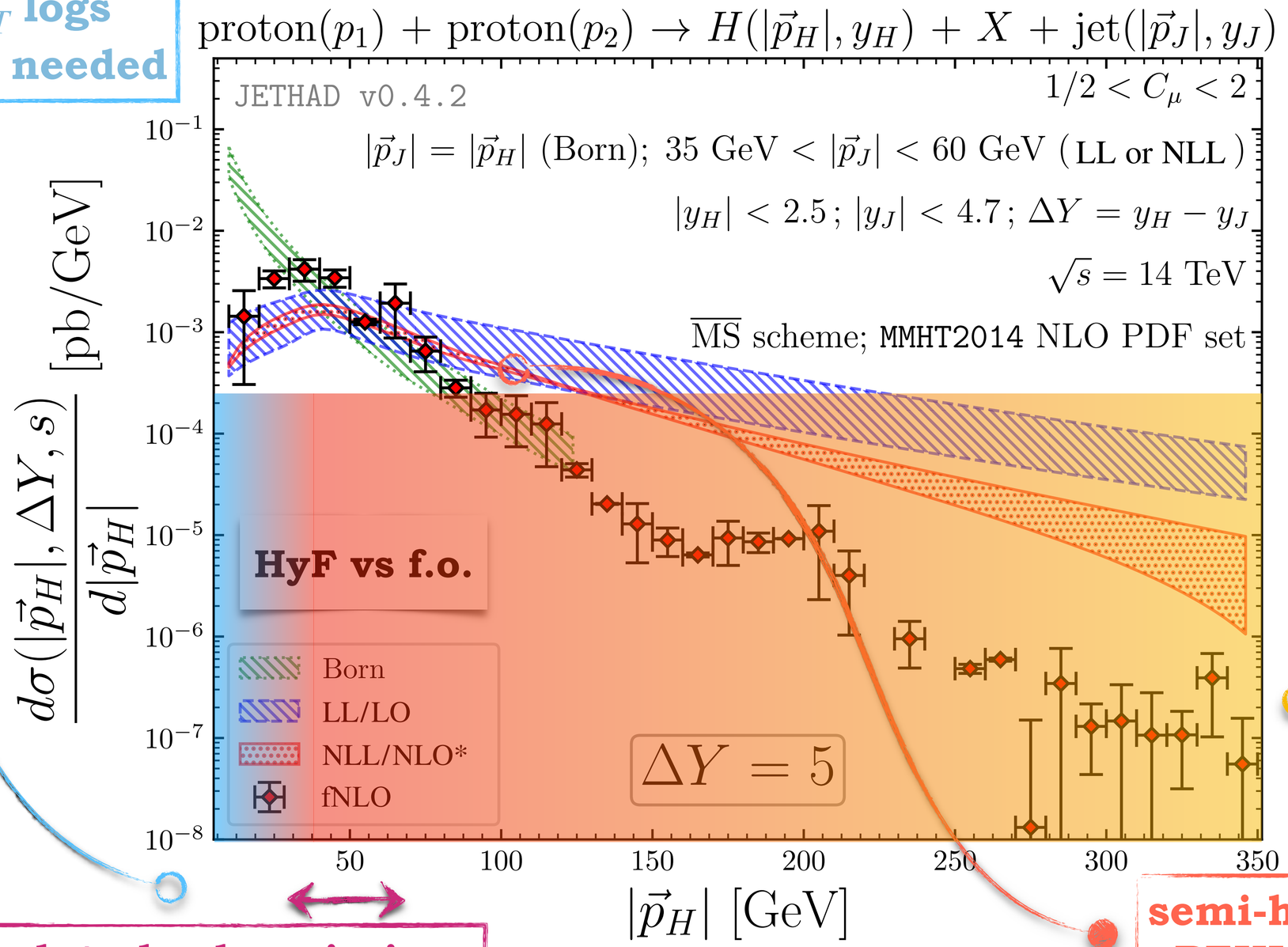


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large p_T logs
p_T-resum. needed

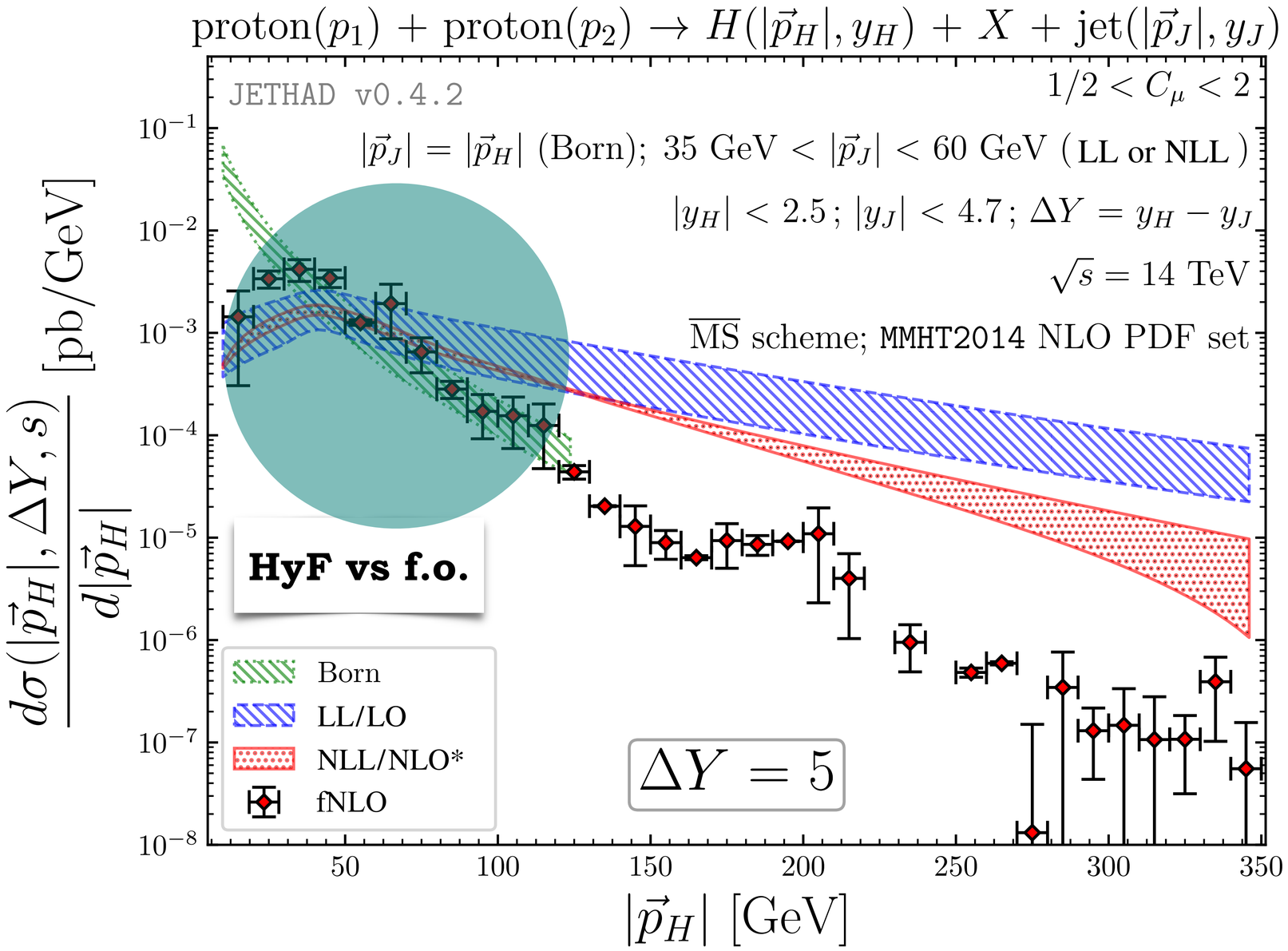


semi-hard regime
BFKL expected

almost back-to-back emissions
large imbalance double logs

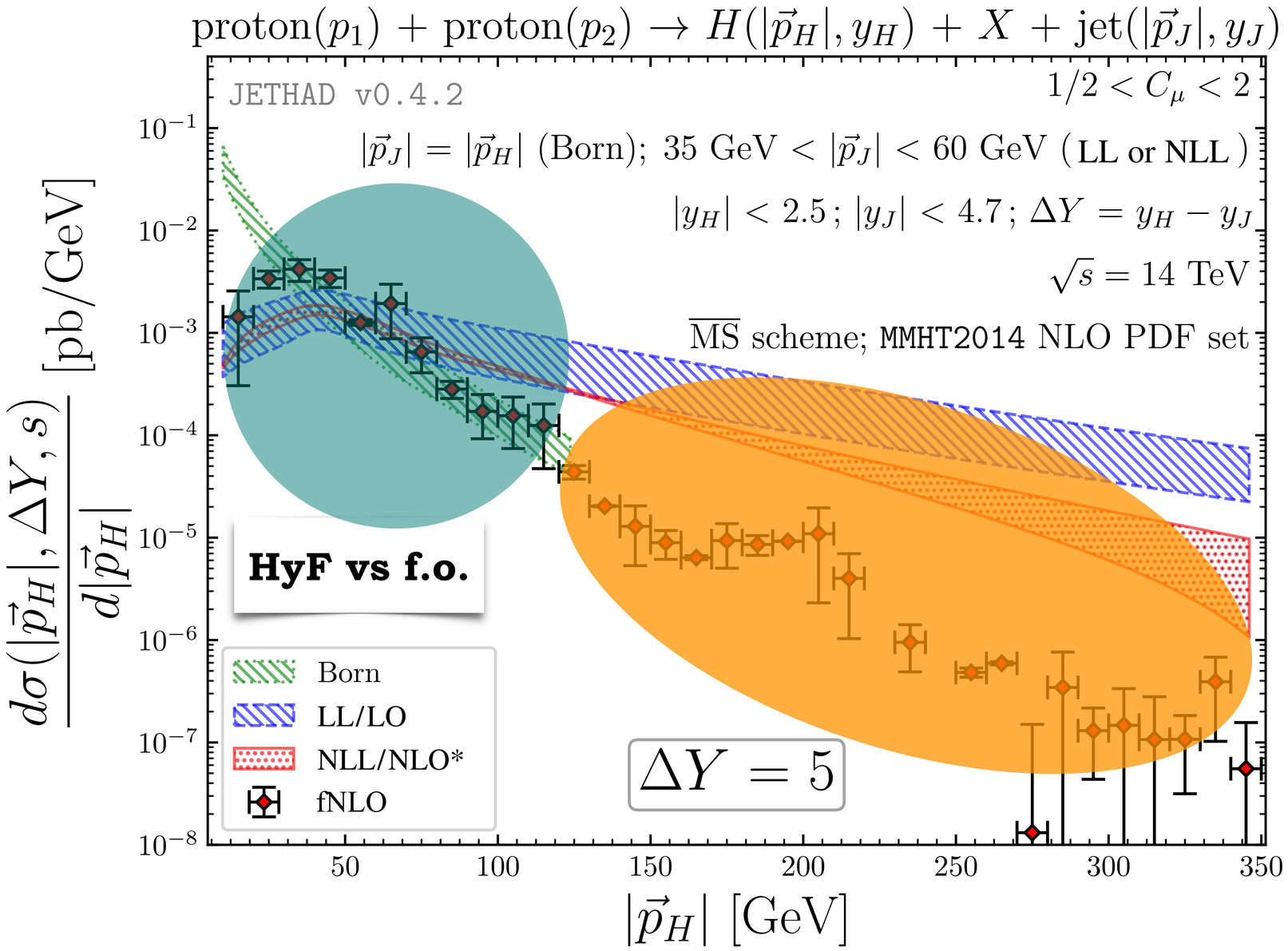
(Higgs + jet, NLL/NLO* HyF) [F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]

(Higgs + jet, NLL/NLO* HyF) 🔗 [F. G. C., D Yu. Ivanov, M, M. A. Mohammed, A. Papa, Eur. Phys. J. C (2021) 8, 780]



¡ **Precision corrections** *expected*

(Higgs + jet, NLL/NLO* HyF) 🔗 [F. G. C., D Yu. Ivanov, M, M. A. Mohammed, A. Papa, Eur. Phys. J. C (2021) 8, 780]



¡ Precision corrections *expected*, but **HyF** *predicts* **large deviations** from **f.o.** **!**

NLL accurate predictions
matched to NLO
via the JETHAD Method

Matching NLL to NLO with JETHAD

¡ **Precision corrections** *expected* ⇔ *need* for an accurate NLL-to-NLO **Matching procedure** !

JETHAD Method → NLL/NLO Additive Matching (analytic: BFKL kernel + coefficient functions)

$$\underbrace{\mathrm{d}\sigma^{\mathrm{NLL/NLO^{-}}}(\Delta Y,\varphi,s)}_{\text{NLO}+\text{NLL}^{-}} = \underbrace{\mathrm{d}\sigma^{\mathrm{NLO}}(\Delta Y,\varphi,s)}_{\text{NLO fixed order}} + \mathrm{d}\sigma^{\mathrm{NLL^{-}}}(\Delta Y,\varphi,s) - \underbrace{\Delta\mathrm{d}\sigma^{\mathrm{NLL/NLO^{-}}}(\Delta Y,\varphi,s)}_{\text{NLO POWHEG w/o PS}}$$

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$$\underbrace{\mathrm{d}\sigma^{\mathrm{NLL/NLO^-}}(\Delta Y,\varphi,s)}_{\text{NLL/NLO}^-} = \underbrace{\mathrm{d}\sigma^{\mathrm{NLO}}(\Delta Y,\varphi,s)}_{\text{NLO fixed order}} + \underbrace{\mathrm{d}\sigma^{\mathrm{NLL^-}}(\Delta Y,\varphi,s)}_{\text{NLL}^- \text{ resum (HyF)}} - \underbrace{\Delta\mathrm{d}\sigma^{\mathrm{NLL/NLO^-}}(\Delta Y,\varphi,s)}_{\text{NLL}^- \text{ expanded at NLO}}$$

NLO POWHEG w/o PS

NLL⁻ JETHAD w/o NLO⁻ double counting

Matching NLL to NLO with JETHAD

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$$\underbrace{\mathrm{d}\sigma^{\mathrm{NLL/NLO^{-}}}(\Delta Y,\varphi,s)}_{\text{NLL/NLO}^{-}\text{ matched}} = \underbrace{\mathrm{d}\sigma^{\mathrm{NLO}}(\Delta Y,\varphi,s)}_{\text{NLO fixed order}} + \underbrace{\mathrm{d}\sigma^{\mathrm{NLL^{-}}}(\Delta Y,\varphi,s)}_{\text{NLL}^{-}\text{ resum (HyF)}} - \underbrace{\Delta\mathrm{d}\sigma^{\mathrm{NLL/NLO^{-}}}(\Delta Y,\varphi,s)}_{\text{NLL}^{-}\text{ expanded at NLO}}$$

$$\underbrace{\text{NLO POWHEG} \oplus \text{NLL}^{-}\text{ JETHAD}}_{\text{NLO}^{-}\text{ matched}} \qquad \underbrace{\text{NLO POWHEG w/o PS}}_{\text{NLO fixed order}} \qquad \underbrace{\text{NLL}^{-}\text{ JETHAD w/o NLO}^{-}\text{ double counting}}_{\text{NLL}^{-}\text{ resum (HyF)} \;-\; \text{NLL}^{-}\text{ expanded at NLO}}$$

HELL + ggHiggs

$N^3LL_{lx}/LL_{sx}/N^3LO$

Inclusive Higgs

🔗 [M. Bonvini, S. Marzani (2018)]

Matching NLL to NLO with JETHAD

¡ **Precision corrections** *expected* ⇔ *need* for an accurate NLL-to-NLO **Matching procedure** !

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NLO POWHEG ⊕ NLL⁻ JETHAD — NLO POWHEG w/o PS — NLL⁻ JETHAD w/o NLO⁻ double counting

HELL + ggHiggs

$N^3LL_{lx}/LL_{sx}/N^3LO$

Inclusive Higgs

🔗 [M. Bonvini, S. Marzani (2018)]

HEJ framework

NLL_{sx}^{-}/NLO

Higgs + jet(s)

🔗 [J. R. Andersen et al. (2022)]

Matching NLL to NLO with JETHAD

¡ Precision corrections *expected* ⇔ *need* for an accurate NLL-to-NLO **Matching procedure !**

JETHAD Method → NLL/NLO Additive Matching (analytic: BFKL kernel + coefficient functions)

$$\underbrace{\mathrm{d}\sigma^{\mathrm{NLL/NLO^{-}}}(\Delta Y,\varphi,s)}_{\textbf{NLL/NLO}^{-}\text{ matched}} = \underbrace{\mathrm{d}\sigma^{\mathrm{NLO}}(\Delta Y,\varphi,s)}_{\textbf{NLO}\text{ fixed order}} + \underbrace{\mathrm{d}\sigma^{\mathrm{NLL^{-}}}(\Delta Y,\varphi,s)}_{\textbf{NLL}^{-}\textbf{ resum (HyF)}} - \underbrace{\Delta\mathrm{d}\sigma^{\mathrm{NLL/NLO^{-}}}(\Delta Y,\varphi,s)}_{\textbf{NLL}^{-}\textbf{ expanded at NLO}}$$

$$\underbrace{\textbf{NLL/NLO}^{-}\text{ matched}}_{\textbf{NLO}\text{ POWHEG}\oplus\textbf{NLL}^{-}\text{ JETHAD}} = \underbrace{\textbf{NLO}\text{ fixed order}}_{\textbf{NLO}\text{ POWHEG w/o PS}} + \underbrace{\textbf{NLL}^{-}\textbf{ resum (HyF)} - \textbf{NLL}^{-}\textbf{ expanded at NLO}}_{\textbf{NLL}^{-}\text{ JETHAD w/o NLO}^{-}\text{ double counting}}$$

HELL + ggHiggs

N^3LL_{lx}/LL_{sx}/N^3LO

Inclusive Higgs

🔗 [M. Bonvini, S. Marzani (2018)]

HEJ framework

NLL_{sx}^{-}/NLO

Higgs + jet(s)

🔗 [J. R. Andersen et al. (2022)]

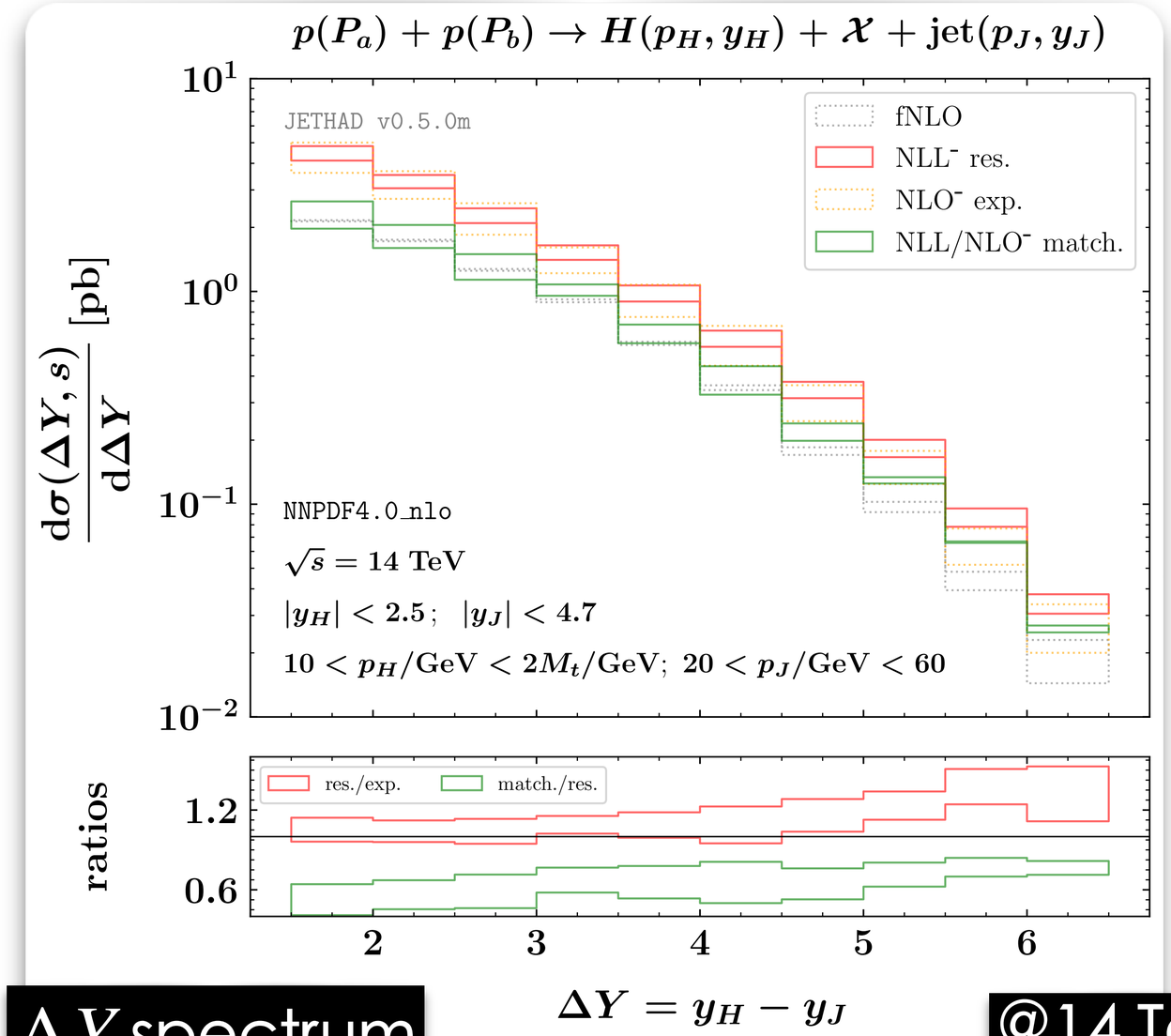
RadISH + MCFM-8.3

$NNLL_{TM}$/NLO

Higgs + jet

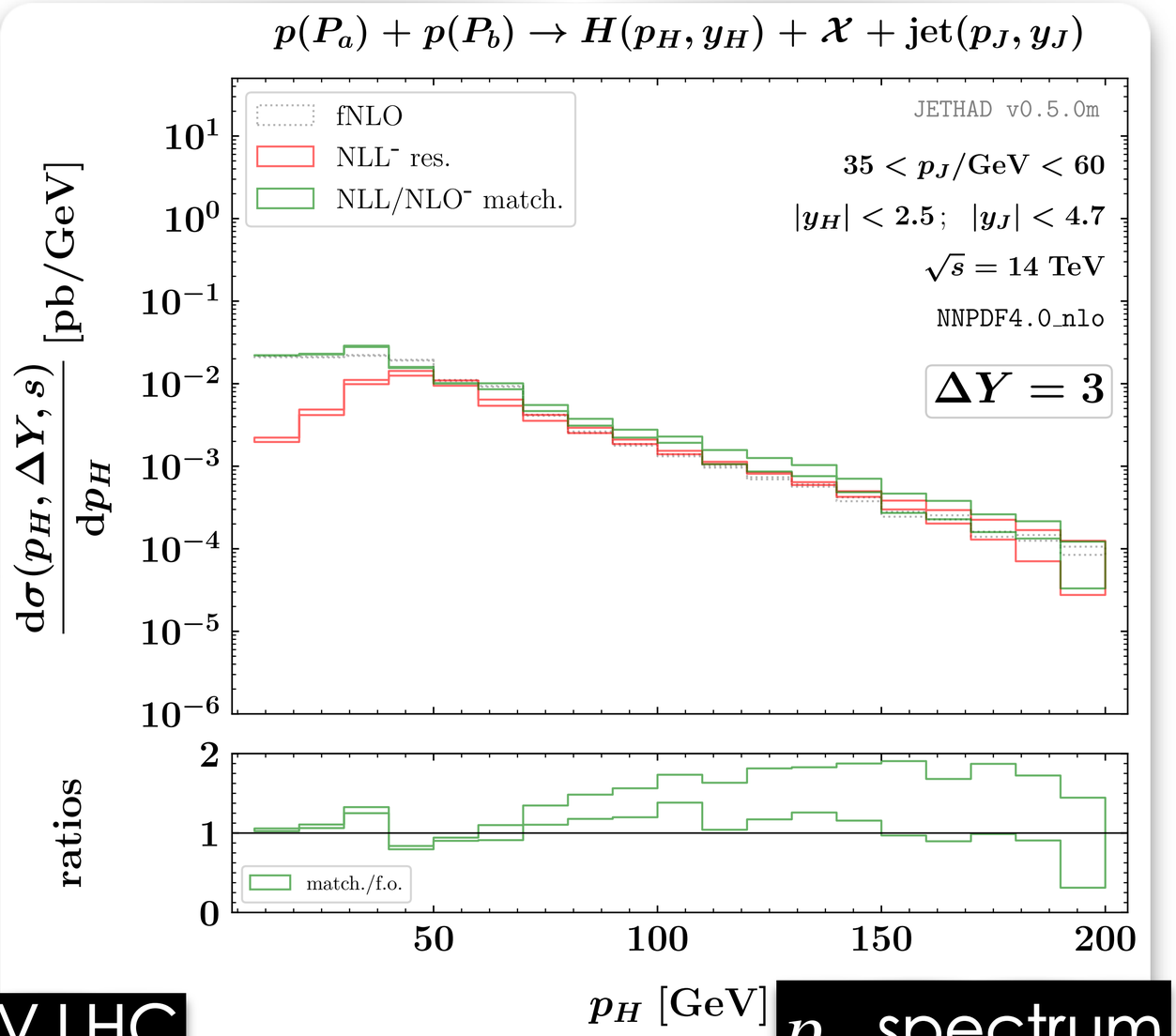
🔗 [P.F. Monni et al. (2020)]

The Higgs + jet spectrum from POWHEG + JETHAD



ΔY spectrum

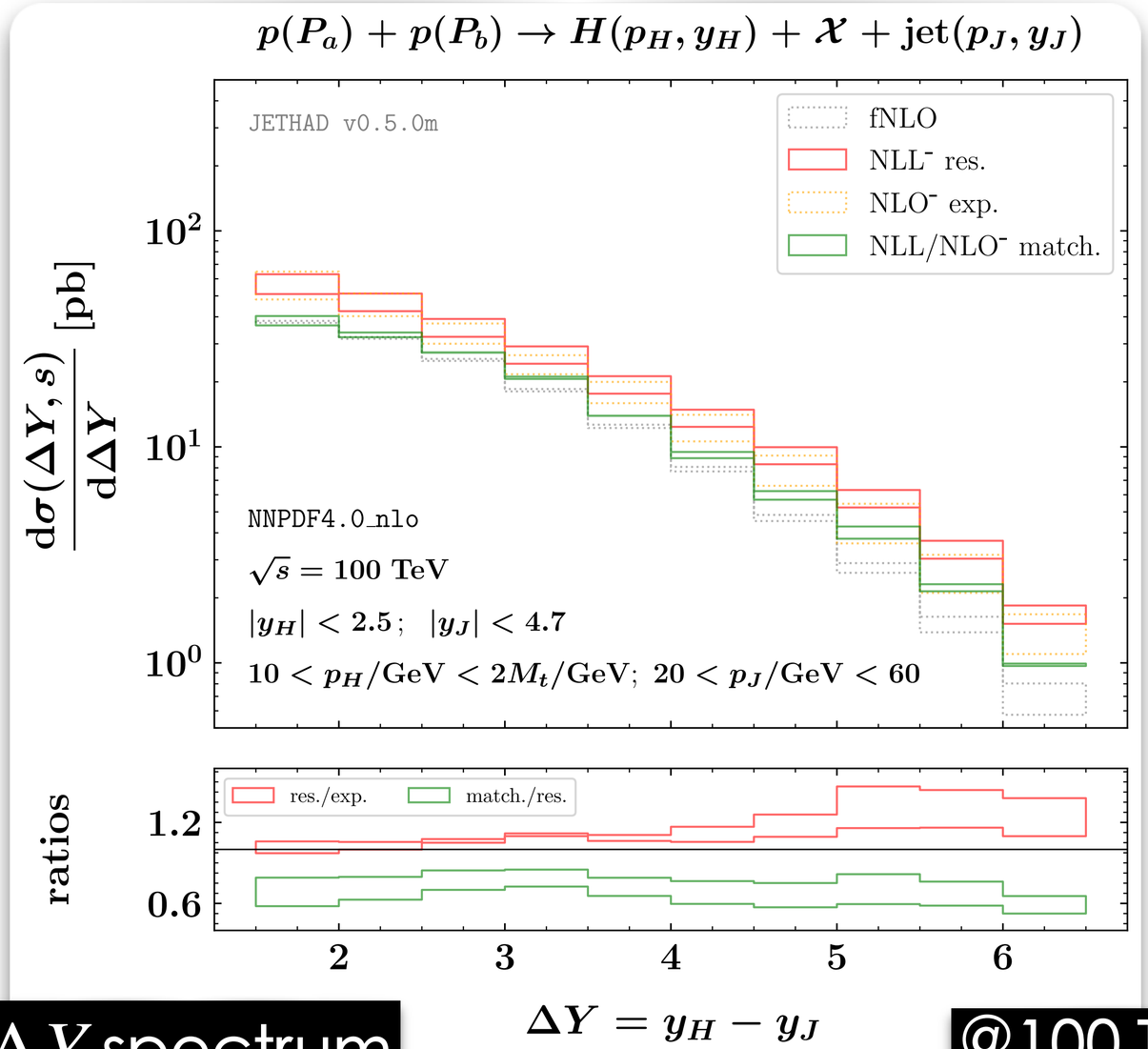
@14 TeV LHC



p_H spectrum

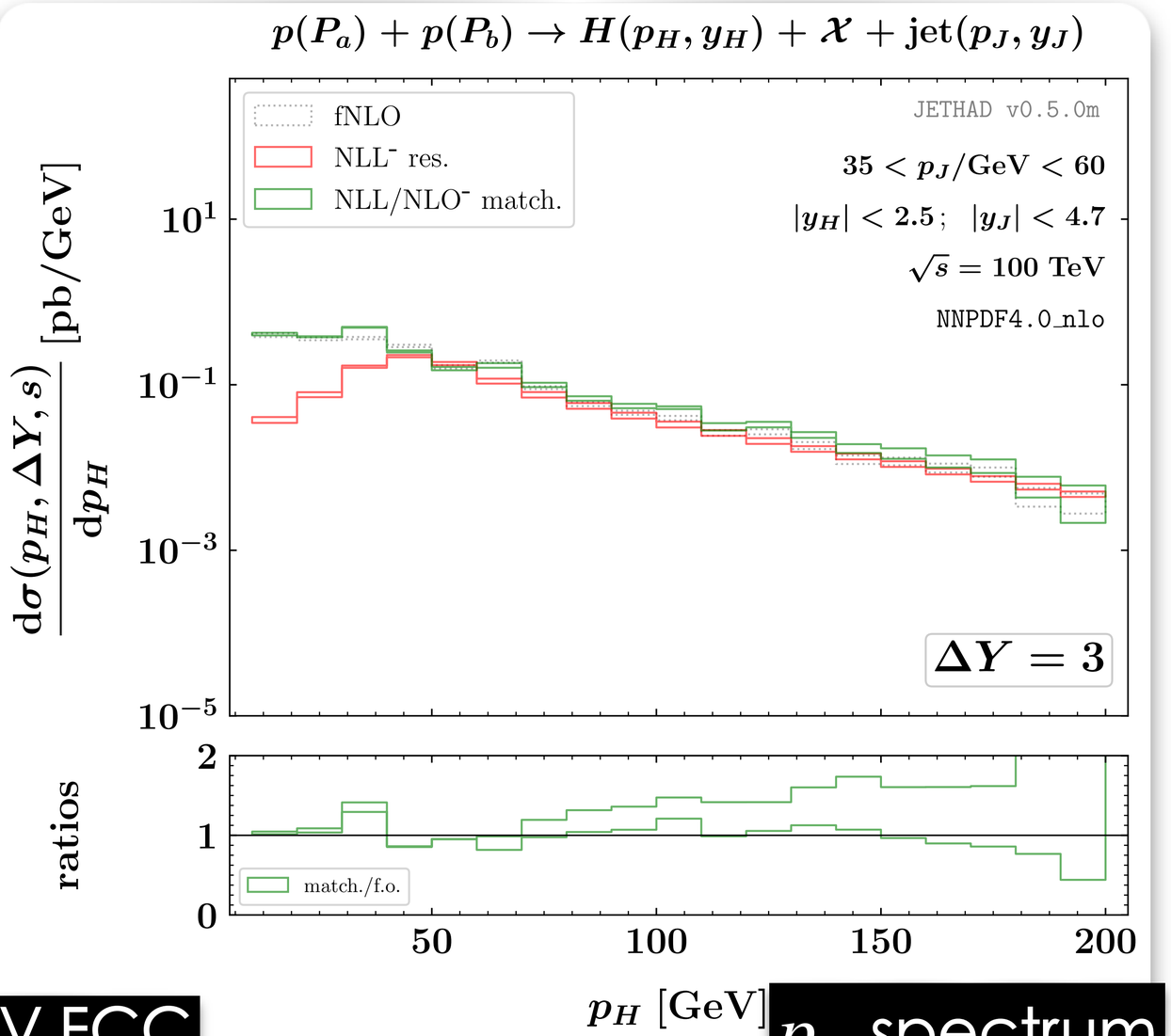
NLL matched to NLO fixed-order POWHEG + JETHAD (in progress)

The Higgs + jet spectrum from POWHEG + JETHAD



ΔY spectrum

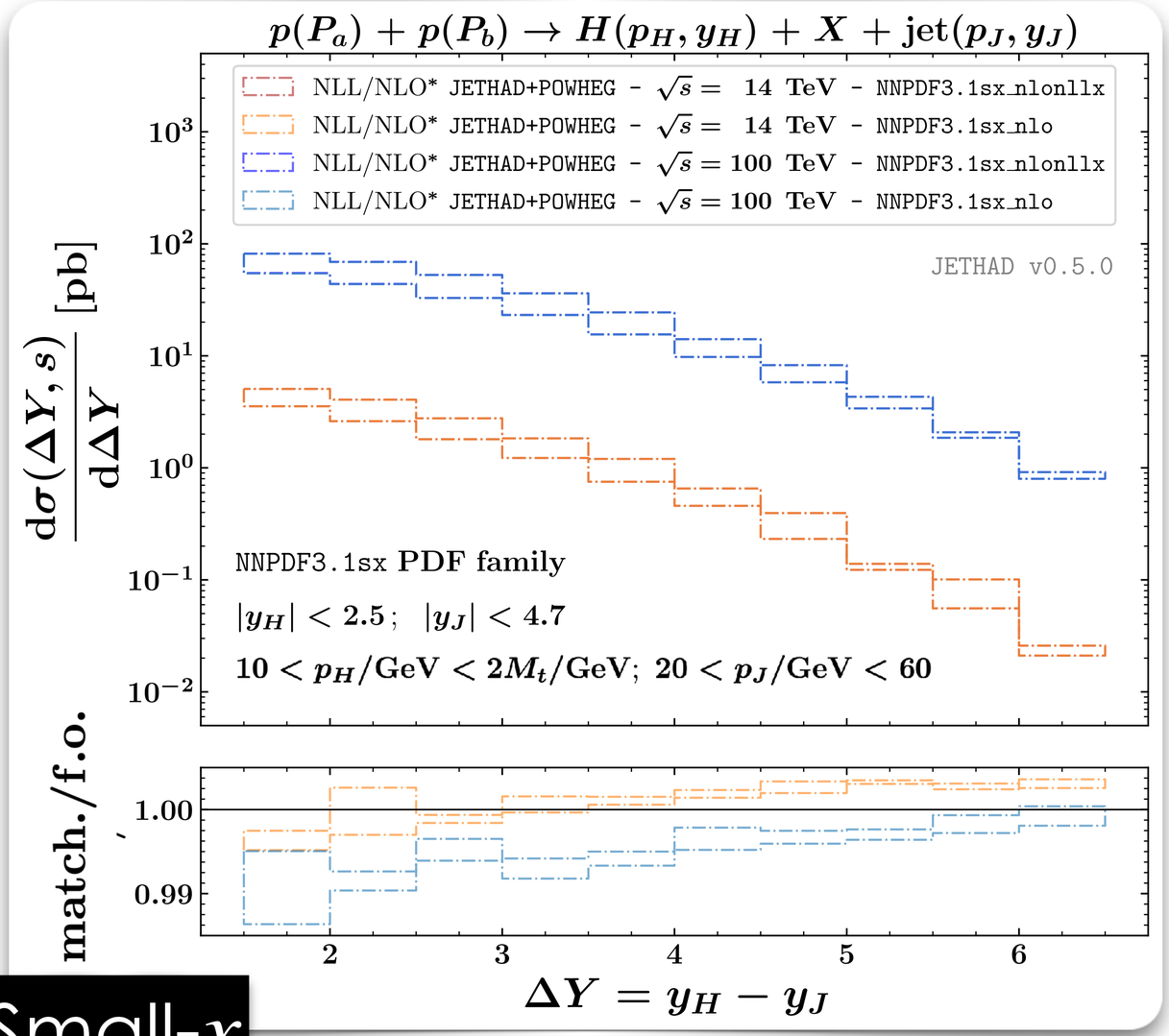
@100 TeV FCC



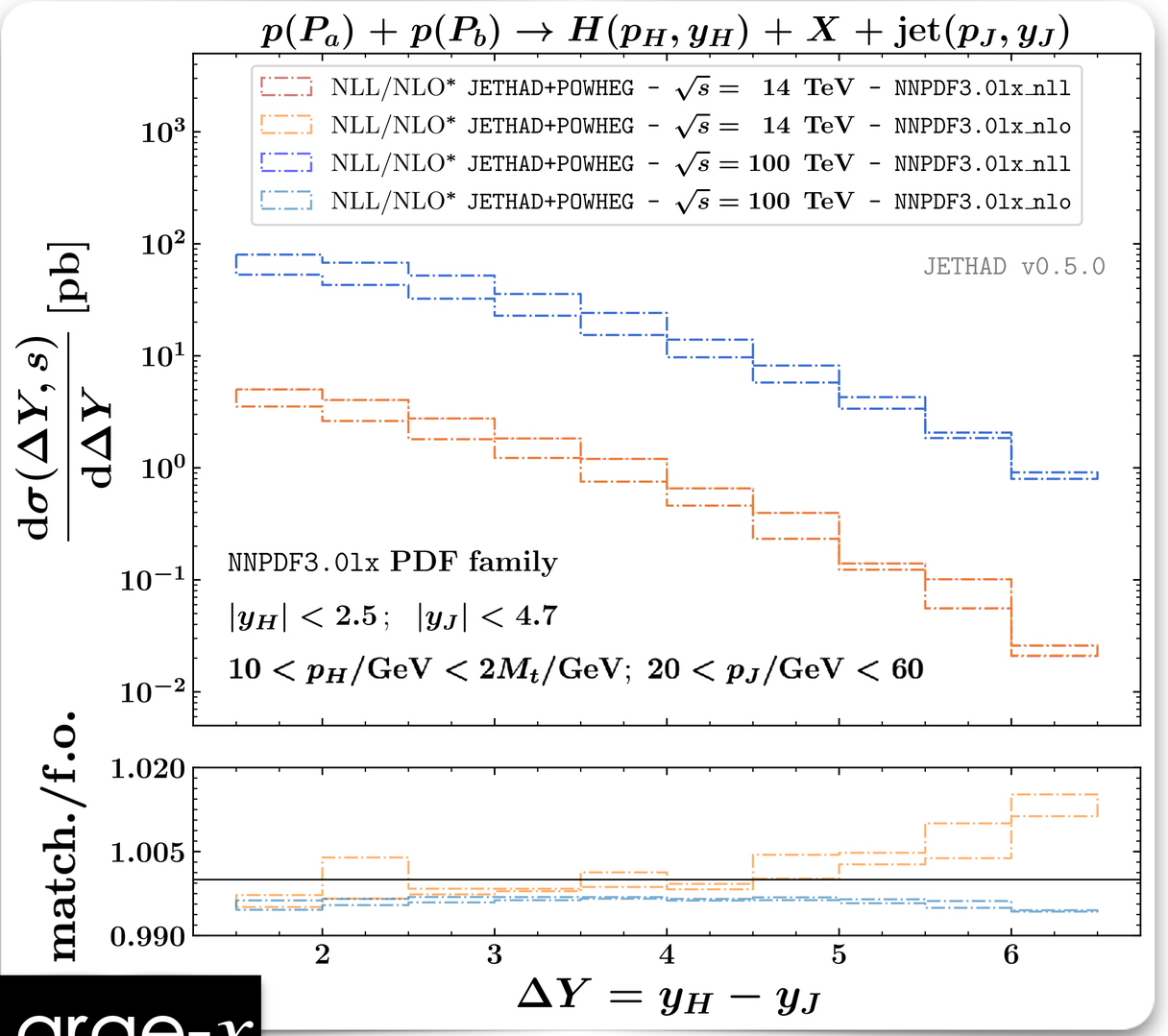
p_H spectrum

NLL matched to NLO fixed-order JETHAD + POWHEG (in progress)

Small-x and large-x enhancement from PDFs



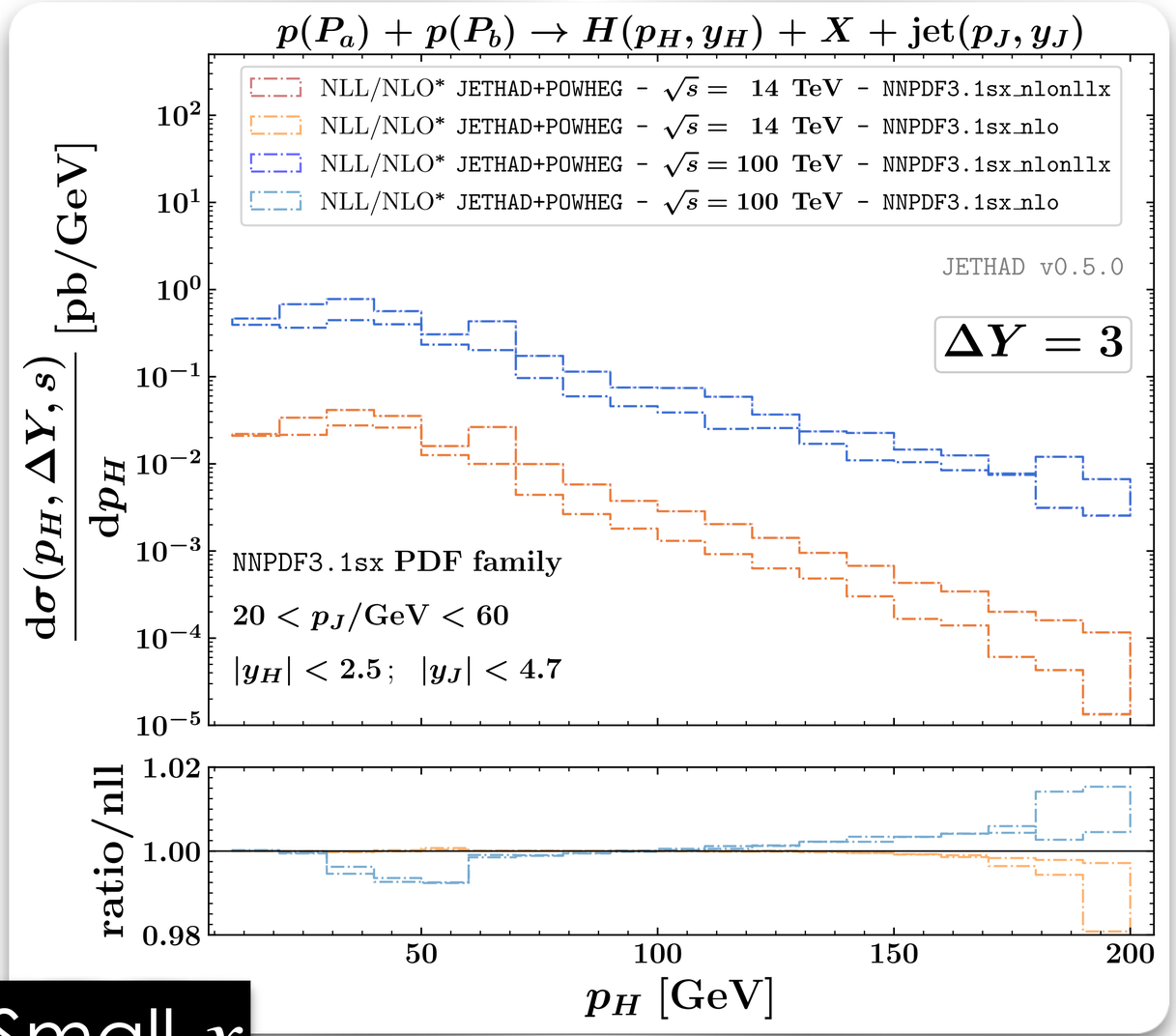
Small-x



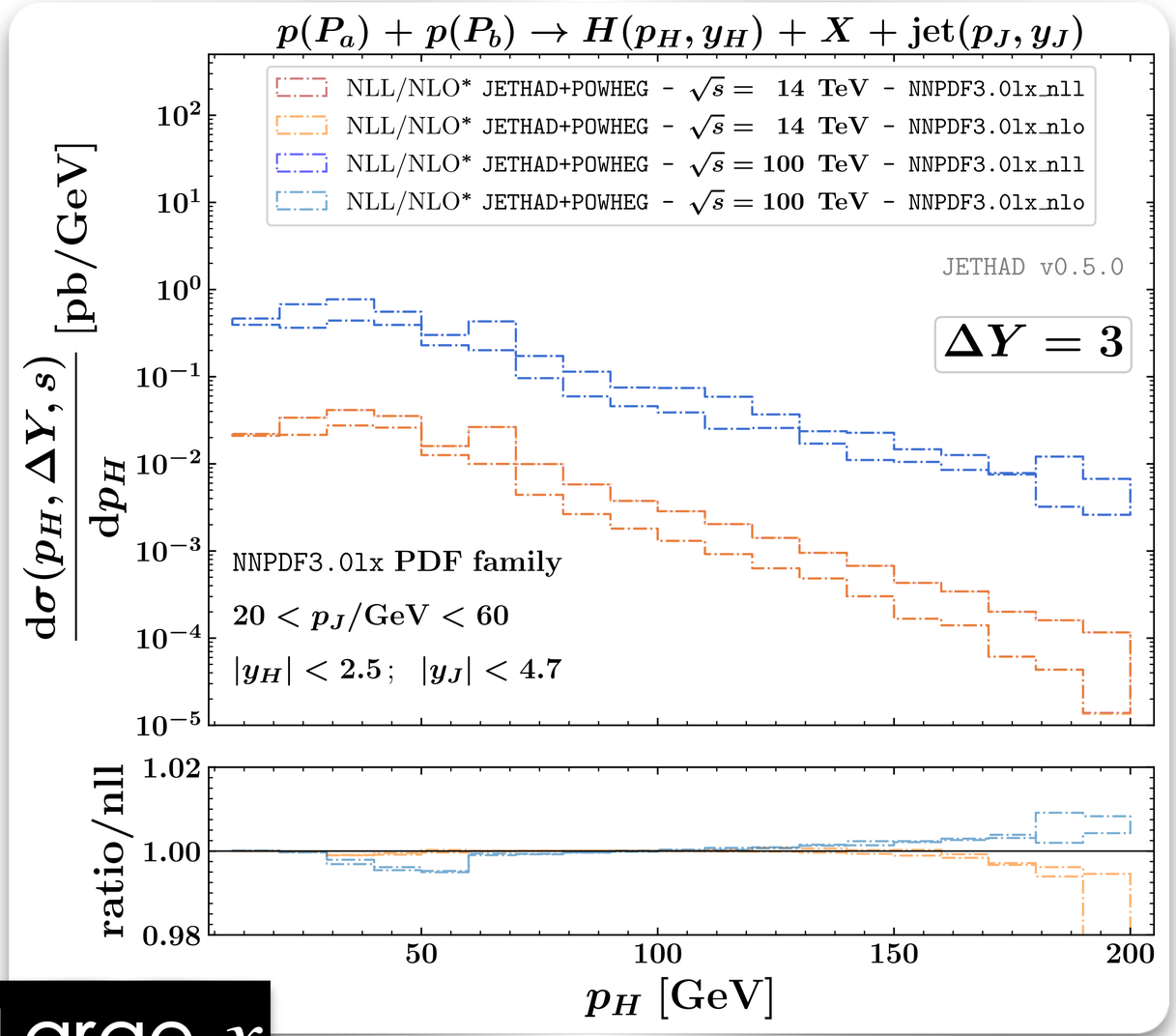
Large-x

Impact of large-x threshold logs on NLO emission functions to be gauged

Small-x and large-x enhancement from PDFs



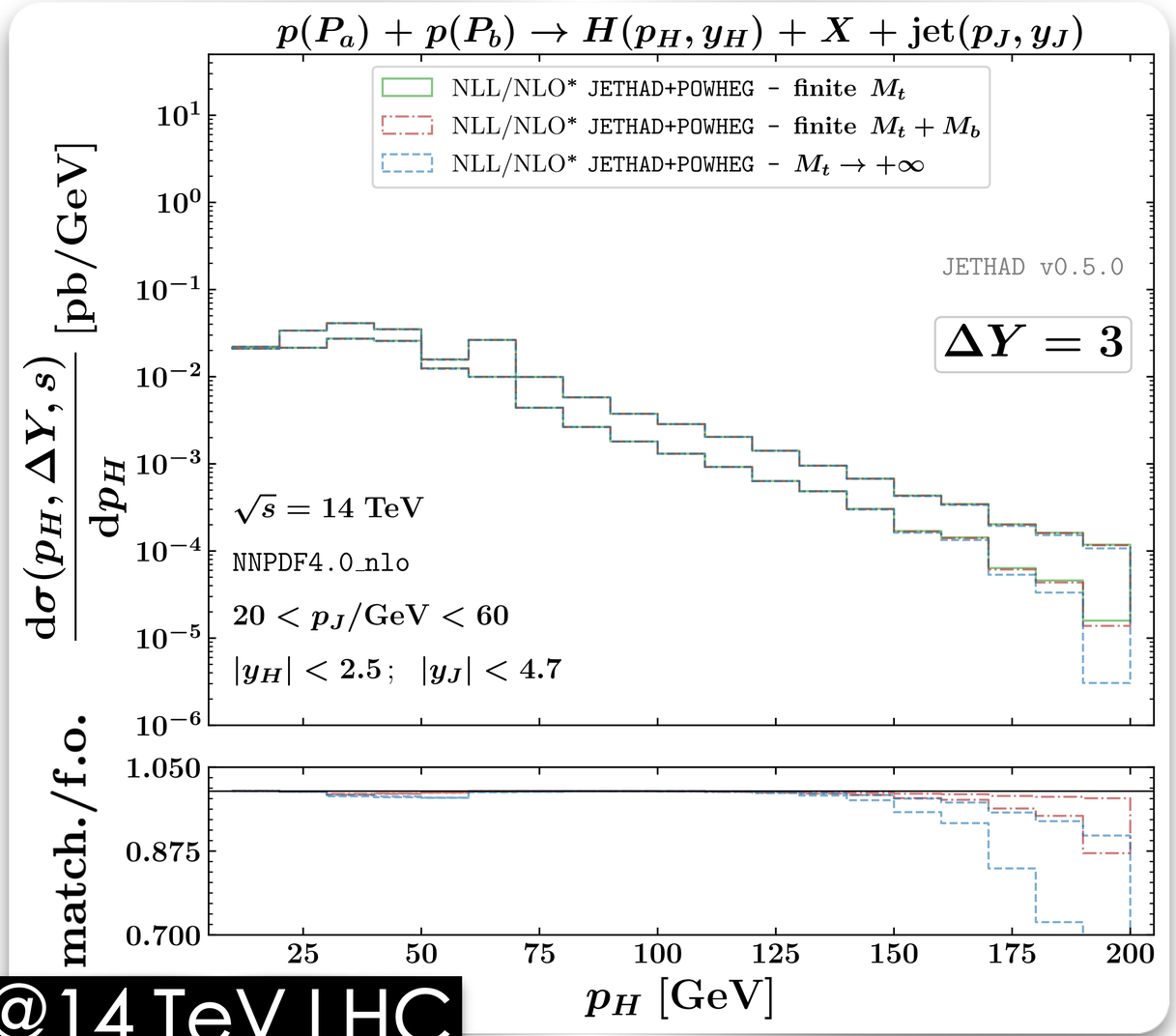
Small-x



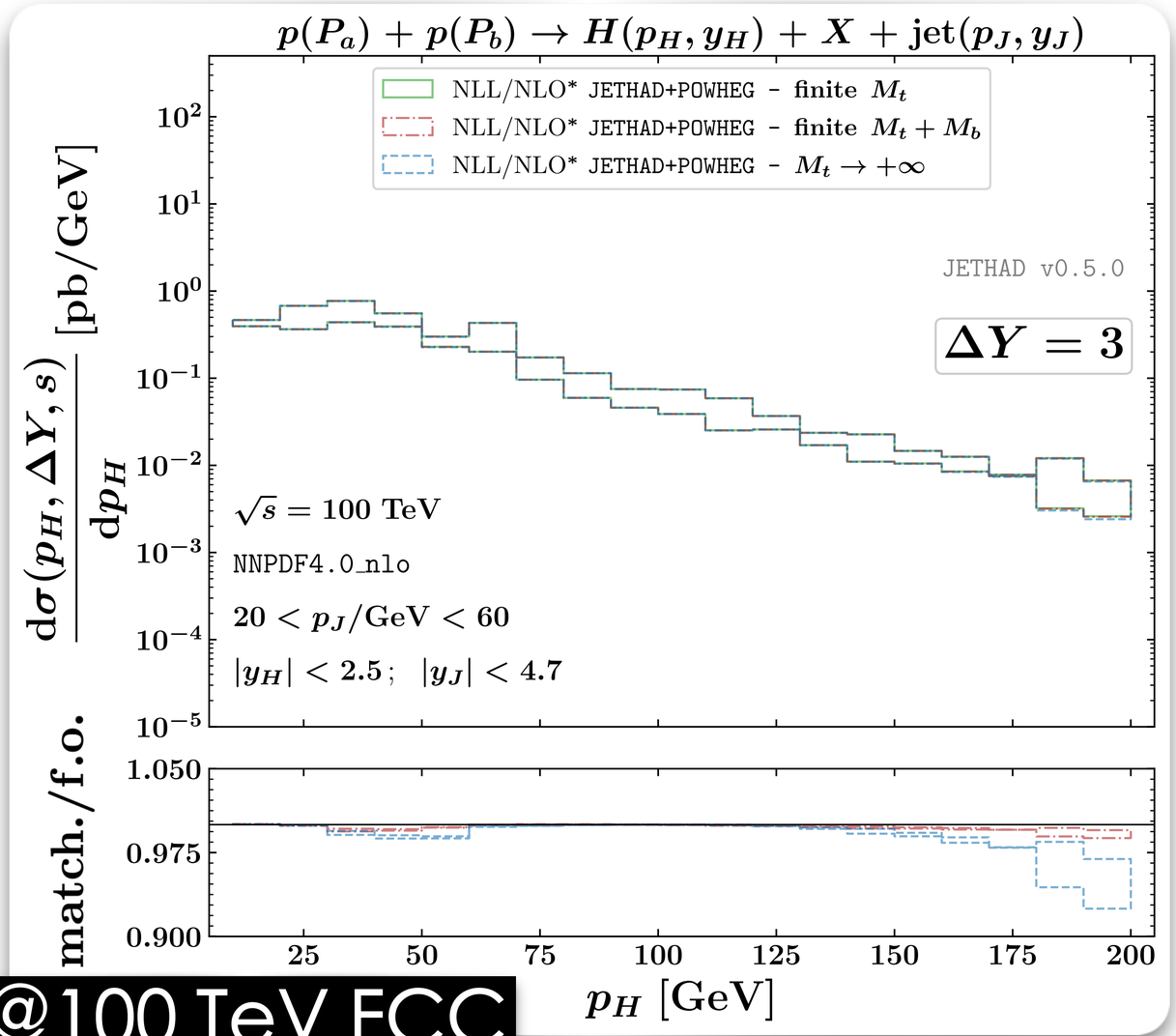
Large-x

Impact of large-x threshold logs on NLO emission functions to be gauged

Finite top- and bottom-mass corrections



@14 TeV LHC

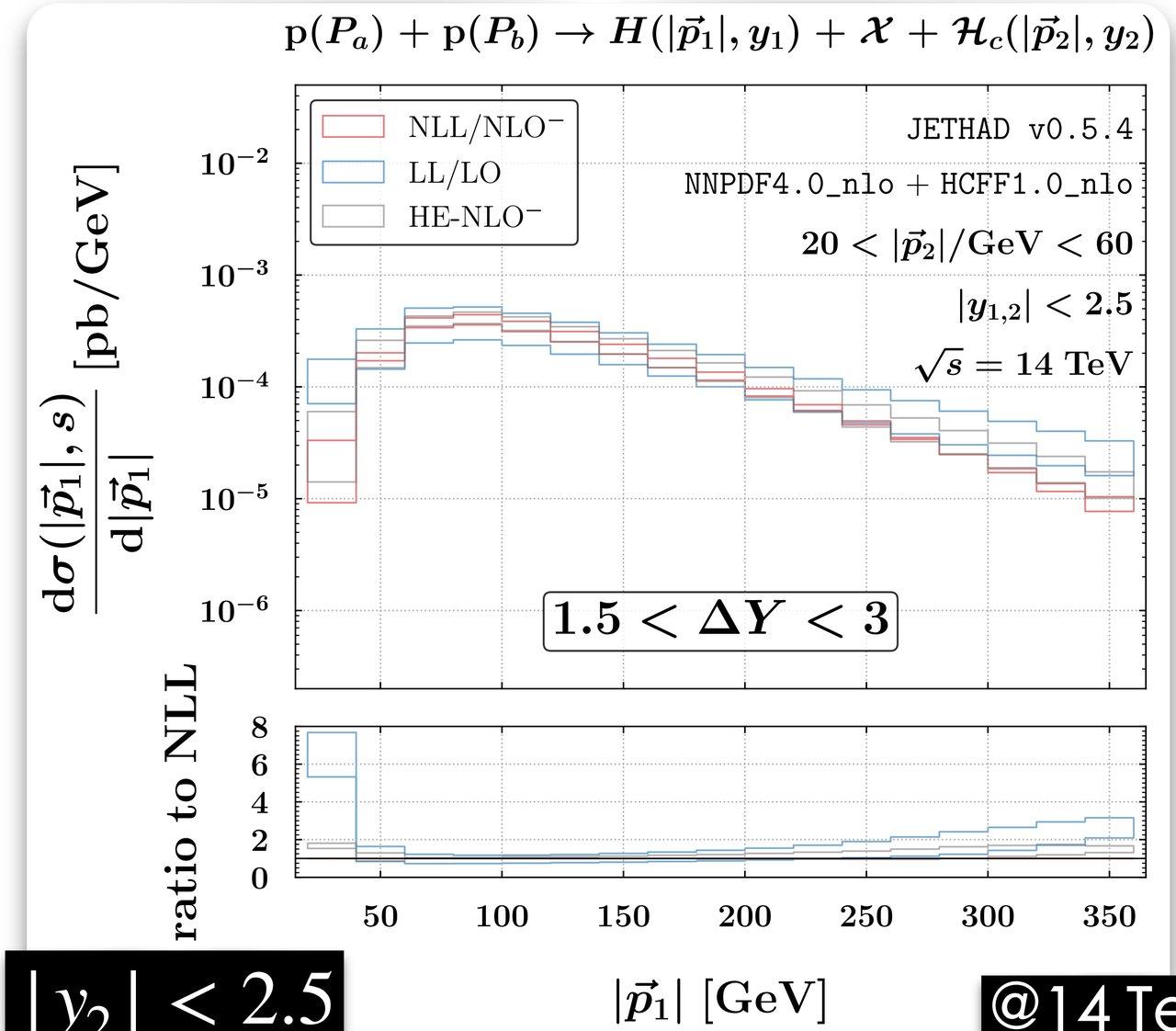


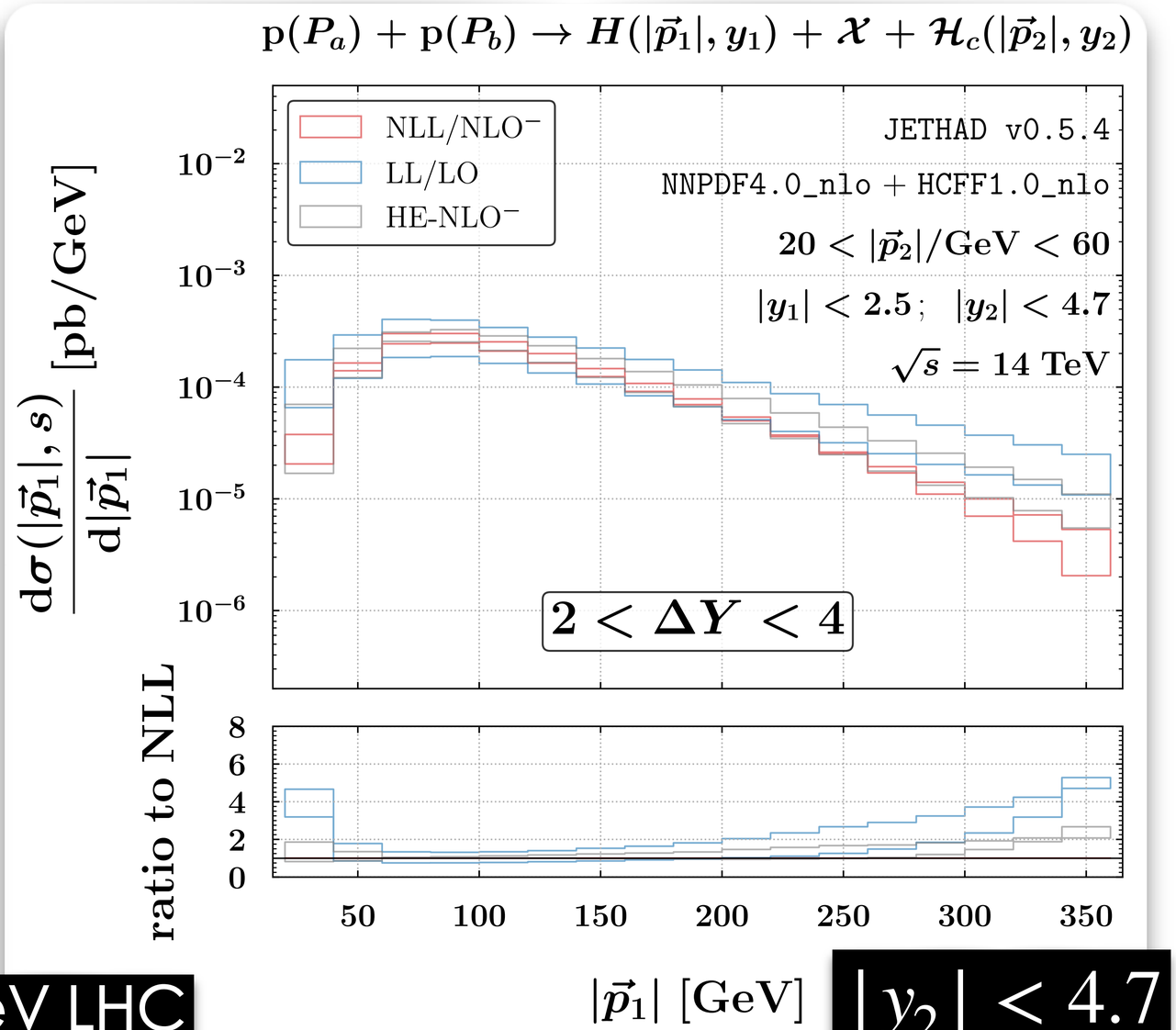
@100 TeV FCC

(Physical masses at NLO) [F. G. C., L. Delle Rose, M. Fucilla, G. Gatto, A. Papa (2024)]

The NLL/NLO$^-$ spectrum of Higgs + charm

The Higgs + charm spectrum at NLL/NLO$^-$ HyF





$|y_2| < 2.5$ @14 TeV LHC $|y_2| < 4.7$

(NLL/NLO$^+$ HyF) [F. G. C., L. Delle Rose, M. Fucilla, G. Gatto, A. Papa (in progress)]

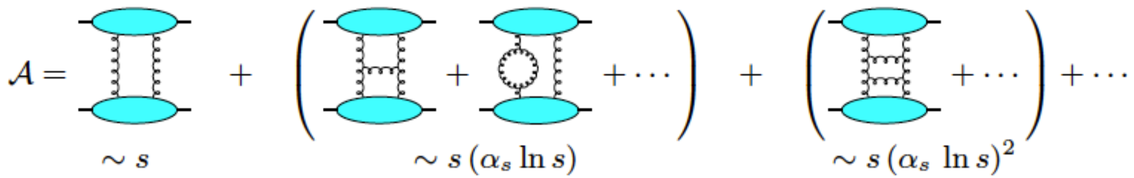
BASICS OF BFKL

The high-energy resummation

- **BFKL resummation**: [V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977); Y.Y. Balitskii, L.N. Lipatov (1978)]

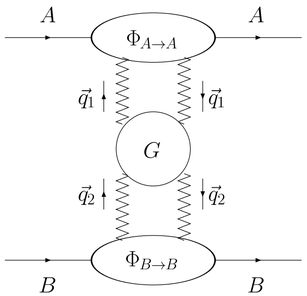
  based on ⟶ **gluon Reggeization**

  leading logarithmic approximation (LL): $\alpha_s^n \left(\ln s\right)^n$



  next-to-leading logarithmic approximation (NLL): $\alpha_s^{n+1} \left(\ln s\right)^n$

Total cross section for $A + B \to X$: $\sigma_{AB}(s) = \dfrac{\mathcal{I}m_s\{\mathcal{A}_{AB}^{AB}\}}{s}$ ⇐ ***optical theorem***



- $\mathcal{I}m_s\left\{\mathcal{A}_{AB}^{AB}\right\}$ factorization:

  convolution of the **Green's function** of two interacting Reggeized gluons with the **impact factors** of the colliding particles

Green's function is **process-independent**, describes energy dependence and obeys BFKL equation; impact factors are known in the **NLL just for few processes**

The high-energy resummation

$$\mathrm{Im}_s\left(\mathcal{A}\right) = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}}\, \Phi_A\left(\vec{q}_1, s_0\right) \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}}\, \Phi_B\left(-\vec{q}_2, s_0\right) \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega\left(\vec{q}_1, \vec{q}_2\right)$$

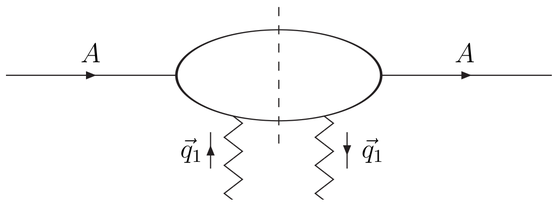
- **Green's function** is **process-independent** and takes care of the **energy dependence**

  $\longrightarrow$ determined through the **BFKL equation**

  [Ya.Ya. Balitskii, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

- **Impact factors** are **process-dependent** and depend on the hard scale, but not on the energy

  $\longrightarrow$ known in the NLA just for few processes



- Successful tests of NLA BFKL in the **Mueller–Navelet** channel with the advent of the LHC; nevertheless, *new BFKL-sensitive observables* as well as *more exclusive final-state reactions* are needed (**di-hadron**, **hadron-jet**, **heavy-quark pair**, **multi-jet**, production processes,...)

(**MN jets**) [B. Ducloué, L. Szymanowski, S. Wallon (2014); F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015, 2016)]
(**di-hadron**) [F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]
(**four-jet**) [F. Caporale, F.G.C., G. Chachamis, A. Sabio Vera (2016)]
(**multi-jet**) F. Caporale, F.G.C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016, 2017, 2017)]
(**heavy-quark pair**) [F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2018); A.D. Bolognino, F.G.C., D.Yu. Ivanov, M. Fucilla, A. Papa (2018)]
(**hadron-jet**) [M.M.A. Mohammed, MD thesis (2018); A.D. Bolognino, F.G.C., D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

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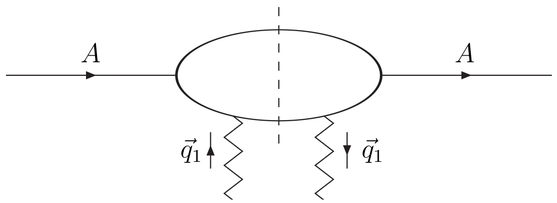
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(**hadron-jet**) [M.M.A. Mohammed, MD thesis (2018); A.D. Bolognino, F.G.C., D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

¡ NEW ! NLO HIGGS

(κ_T space) 🔗 [M. Hentschinski et al. (2021)]

(κ_T & Mellin) 🔗 [F.G.C. et al. (2022)]

Gluon Reggeization in perturbative QCD

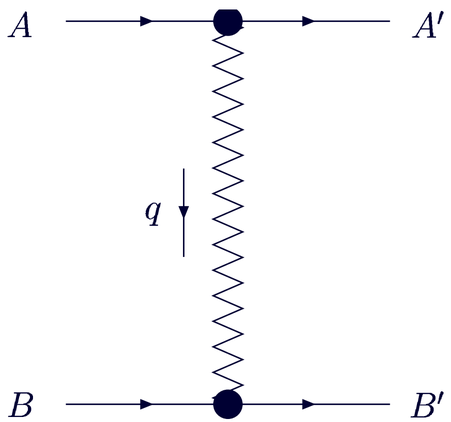
- ◇ Gluon quantum numbers in the t-channel: 8^- representation
- ◇ Regge limit: $s \simeq -u \to \infty$, t not growing with s

$\to$ amplitudes governed by **gluon Reggeization** $\to$ $D_{\mu\nu} = -i\frac{g_{\mu\nu}}{q^2}\left(\frac{s}{s_0}\right)^{\alpha_g(q^2)-1}$

$\xrightarrow{\text{feature}}$ all-order resummation: **LLA** $[\alpha_s^n (\ln s)^n]$ + **NLA** $[\alpha_s^{n+1} (\ln s)^n]$

$\xrightarrow{\text{consequence}}$ factorization of elastic and real part of inelastic amplitudes

$\xrightarrow{\text{example}}$ Elastic scattering process: $A + B \longrightarrow A' + B'$



$$(\mathcal{A}_8^-)_{AB}^{A'B'} = \Gamma_{A'A}^c \left[\left(\frac{-s}{-t}\right)^{j(t)} - \left(\frac{s}{-t}\right)^{j(t)}\right] \Gamma_{B'B}^c$$

$$j(t) = 1 + \omega(t)\,, \qquad j(0) = 1$$

$\omega(t) \to$ Reggeized gluon trajectory

$\Gamma_{A'A}^c = g\langle A'|T^c|A\rangle \Gamma_{A'A} \to$ PPR vertex

$T^c \to$ fundamental (q) or adjoint (g)

- QCD is the unique SM theory where all elementary particles reggeize
- Possible extensions: N=4 SYM, AdS/CFT,...

The high-energy resummation

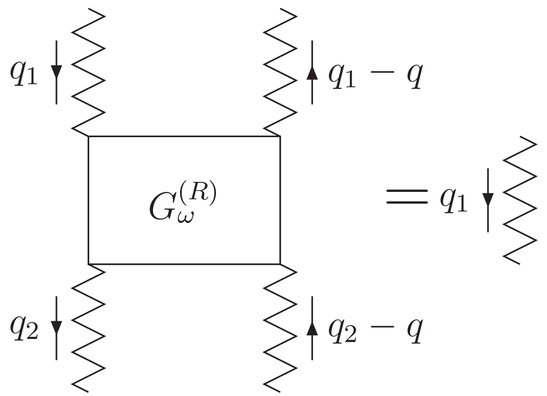
$$\mathcal{I}m_s\{\mathcal{A}\} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}}\, \Phi_A(\vec{q}_1, s_0) \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}}\, \Phi_B(-\vec{q}_2, s_0) \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

- **Green's function** is **process-independent** and takes care of the **energy dependence**

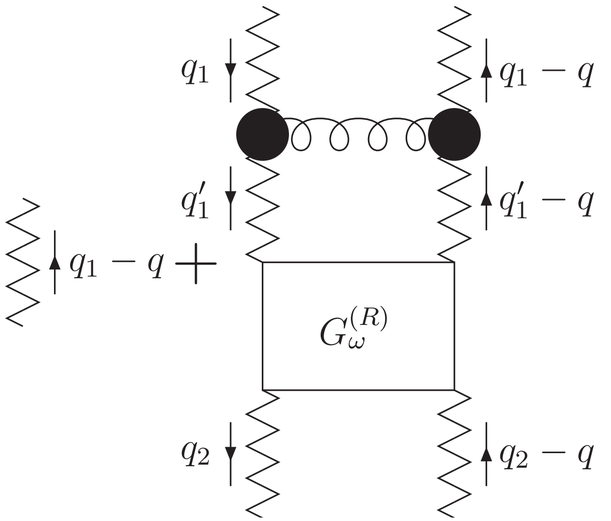
  $\longrightarrow$ determined through the **BFKL equation**

  [Ya.Ya. Balitskii, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

$$\omega\, G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{D-2}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2}q\, K(\vec{q}_1, \vec{q})\, G_\omega(\vec{q}, \vec{q}_1)\,.$$

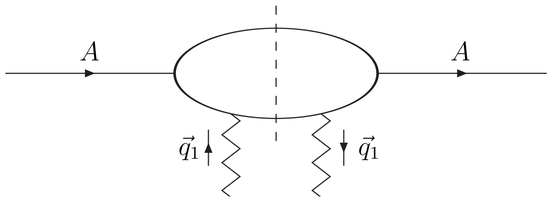


$=$



The high-energy resummation

- **Impact factors** are **process-dependent** and depend on the hard scale, but not on the energy
  $\longrightarrow$ known in the NLA just for few processes



◇ **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
[M. Ciafaloni, G. Rodrigo (2000)]

◇ $\gamma^* \longrightarrow V$, with $V = \rho^0,\ \omega,\ \phi$, forward case

[D.Yu. Ivanov, M.I. Kotsky, A. Papa (2004)]

◇ forward jet production

[J. Bartels, D. Colferai, G.P. Vacca (2003)]
(exact IF) [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, A. Perri (2012)]
(small-cone IF) [D.Yu. Ivanov, A. Papa (2012)]
(several jet algorithms discussed) [D. Colferai, A. Niccoli (2015)]

◇ forward identified hadron production

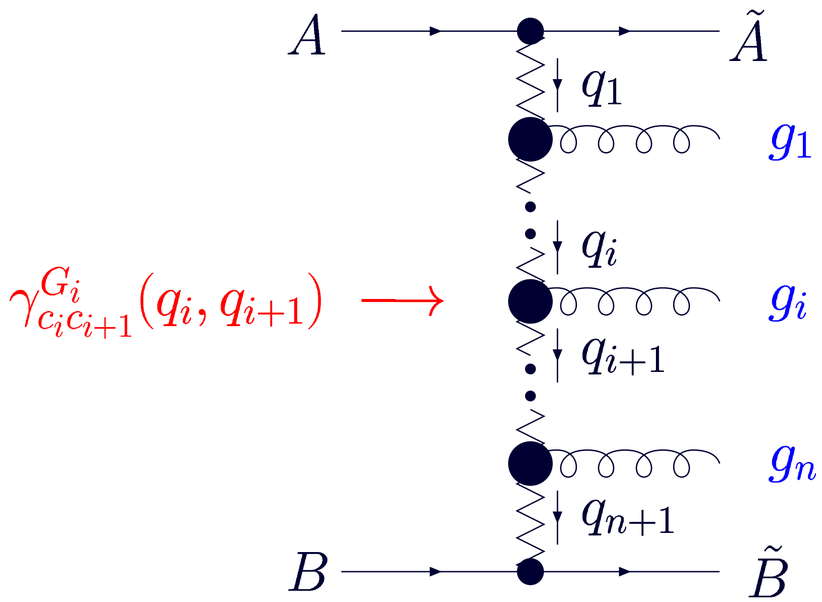
[D.Yu. Ivanov, A. Papa (2012)]

◇ $\gamma^* \longrightarrow \gamma^*$

[J. Bartels *et al.* (2001), I. Balitsky, G.A. Chirilli (2011, 2013)]

BFKL in the LLA (I)

Inelastic scattering process $A + B \;\to\; \tilde{A} + \tilde{B} + n$ in the LLA



$\gamma^{G_i}_{c_i c_{i+1}}(q_i, q_{i+1}) \longrightarrow$ multi-Regge kinematics

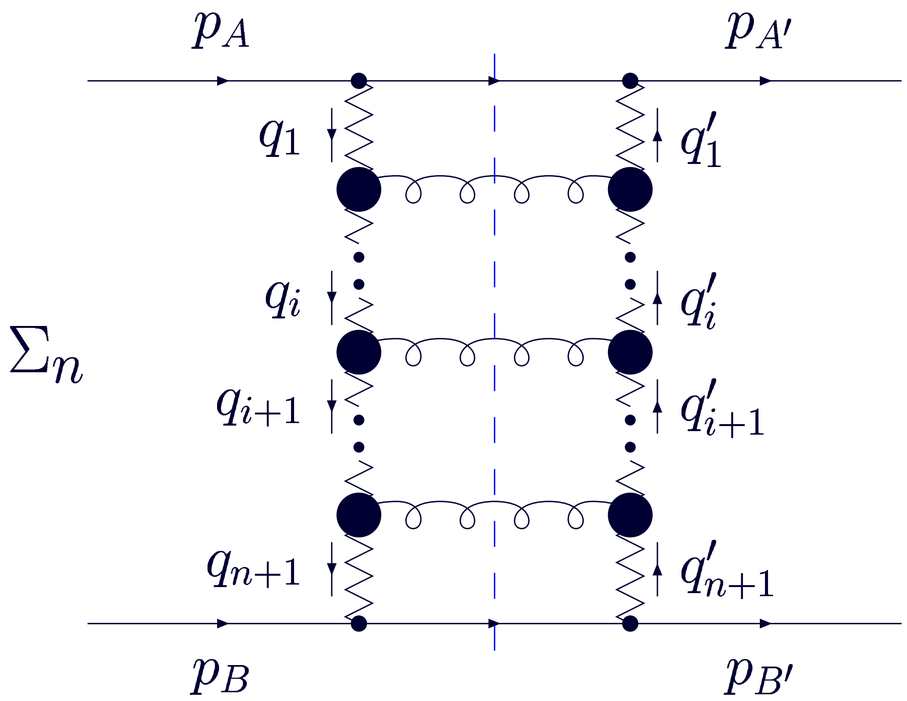
$$\mathrm{Re}\mathcal{A}^{\tilde{A}\tilde{B}+n}_{AB} = 2s\,\Gamma^{c_1}_{\tilde{A}A}\left(\prod_{i=1}^{n}\gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1})\left(\frac{s_i}{s_R}\right)^{\omega(t_i)}\frac{1}{t_i}\right)\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_R}\right)^{\omega(t_{n+1})}\Gamma^{c_{n+1}}_{\tilde{B}B}$$

$\gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1}) \;\to$ RRG vertex

$s_R \;\to$ energy scale, irrelevant in the LLA

BFKL in the LLA (II)

Elastic amplitude $A + B \longrightarrow A' + B'$ in the LLA via s-channel unitarity



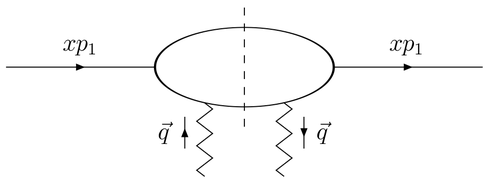
$$\mathcal{A}_{AB}^{A'B'} = \sum_{\mathcal{R}} (\mathcal{A}_{\mathcal{R}})_{AB}^{A'B'} , \qquad \mathcal{R} = 1 \text{ (singlet)}, 8^- \text{ (octet)}, \dots$$

The 8^- color representation is important for the bootstrap, i.e. the consistency between the above amplitude and that with one Reggeized gluon exchange

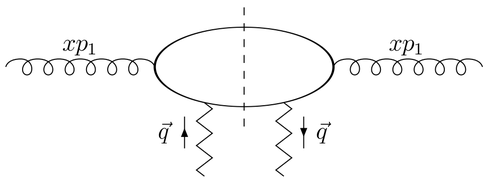
Forward-jet impact factor

- take the impact factors for **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
[M. Ciafaloni and G. Rodrigo (2000)]

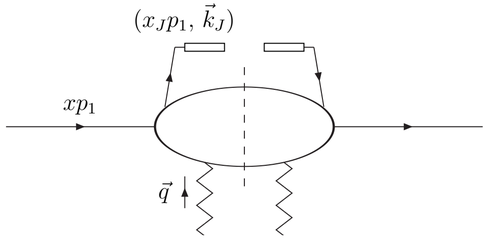


quark vertex

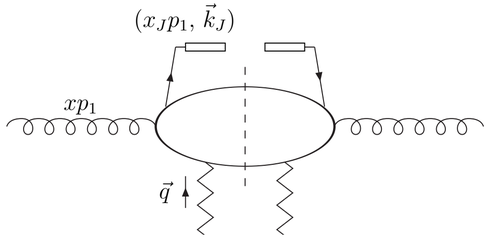


gluon vertex

- "open" one of the integrations over the phase space of the intermediate state to allow one parton to generate the jet



quark jet vertex



gluon jet vertex

- use QCD collinear factoriz.: $\sum_{s=q,\bar{q}} f_s \otimes$ [quark vertex] $+ f_g \otimes$ [gluon vertex]