A precise α_s determination from the R-improved QCD Static Energy

José Manuel Mena Valle V.Mateu P.G.Ortega

Universidad de Salamanca

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Consejería de Educación

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Introduction

- α_s most important parameter in QCD $\rightarrow$ should be determined with high precision.
- Our strategy $\rightarrow$ comparing the QCD Static Energy obtained in lattice simulations with highly accurate perturbative results.
- $V_{\rm QCD} \propto \alpha_s \rightarrow$ very sensitive.
- We improve this method building on previous analyses [PRD, 86 (2012) 114031, PRD, 90 (2020) 074038] in several ways:
  - Leading renormalon subtraction $\rightarrow$ short-distance scheme (MSR).
  - Resummation of associated large logs with R-evolution.
  - Profiles functions for the renormalization scales.
- We can fit (for the first time) lattice data up to $r \sim 1\,\text{fm} \rightarrow E \sim 200$ MeV.

Static Energy and Static Potential

- The Static Energy is defined as the potential energy between an infinitely massive quark anti-quark pair at a distance r, corrected by ultra-soft effects. In pNRQCD

$$E_{\rm s}(r) = V_{\rm s}(r,\mu) + \delta_{\rm us}(r,\mu).$$

- The Static Potential is the basic object to understand the behavior of non-relativistic QCD:

$$V_{\rm s}(r,\mu) = V_{\rm s}^{\rm soft}(r,\mu) + V_{\rm s}^{\rm us}(r,\mu),$$

$$V_{\rm s}^{\rm soft}(r) = -C_F \frac{\alpha_s(\mu)}{r} \sum_{i=0}^{3} \left[\frac{\alpha_s(\mu)}{4\pi}\right]^i \sum_{j=0}^{i} a_{ij} \log^j\left(r\mu e^{\gamma_E}\right).$$

- Coefficients a_{i0} are known to four loops. $a_{ij\geq 0}$ obtained with RGE. $\alpha_s(\mu) = \alpha_s^{(n_\ell=3)}(\mu)$.

- $V_{\rm s}$ known in perturbation theory up to $\mathcal{O}(\alpha_s^4) \to$ ultra-soft contributions show up for the first time

$$V_{\rm s}^{\rm us}(r,\mu) = -\frac{C_A^3 C_F}{12\pi} \frac{\alpha_s^4(\mu)}{r} \log\left(\mu r e^{\gamma_E}\right).$$

Ultra-soft term and resummation

- μ_{us} appears both in the ultrasoft static potential $V_{\mathrm{s}}^{\mathrm{us}}$ and in the matrix element δ_{us}

$$\delta_{\mathrm{us}}\left(\mu_s,\mu_{\mathrm{us}}\right)=-\frac{C_A^3 C_F}{12\pi}\frac{\alpha_s^3\left(\mu_s\right)\alpha_s\left(\mu_{\mathrm{us}}\right)}{r}\log\left[\frac{C_A\alpha_s\left(\mu_s\right)e^{-5/6}}{\mu_{\mathrm{us}}r}\right]$$

$$V_{\mathrm{s}}^{\mathrm{us}}(r,\mu_s,\mu_{\mathrm{us}})=-\frac{C_A^3 C_F}{12\pi}\frac{\alpha_s^3(\mu_s)\alpha_s(\mu_{\mathrm{us}})}{r}\log\left(\mu_{\mathrm{us}}re^{\gamma_E}\right)$$

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- One can see the necessity of resummation with these two (incompatible) choices that minimize logs.
  - $\mu_{us} \sim \alpha_s/r$ [use this one: no large logs in δ_{us}]
  - $\mu_{us} \sim 1/r$ [discarded: sum up logs in V_s^{us}]

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$$\mu\frac{\mathrm{d}V_s\left(r,\mu_s,\mu\right)}{\mathrm{d}\mu}=-\frac{2C_F C_A^3}{24r}\frac{\alpha_s(\mu)}{\pi}\left[1+B\frac{\alpha_s(\mu)}{\pi}\right]\alpha_s^3\left(\mu_s\right)\left\{1+3\frac{\alpha_s\left(\mu_s\right)}{4\pi}\left[a_{1,0}+2\beta_0\log\left(r\mu_s e^{\gamma E}\right)\right]\right\}.$$

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- Integrating from $\mu=\mu_s\sim 1/r$ to $\mu=\mu_{us}\sim\alpha_s/r$

$$\begin{aligned}V_s\left(r,\mu_s,\mu_{\mathrm{us}}\right)=&V_s\left(r,\mu_s\right)+U_{\mathrm{us}}\left(r,\mu_s,\mu_{\mathrm{us}}\right),\\ U_{\mathrm{us}}\left(r,\mu_s,,\mu_{us}\right)=&\frac{C_A^3 C_F}{6\beta_0 r}\alpha_s^3\left(\mu_s\right)\left\{\left(1+3\frac{\alpha_s\left(\mu_s\right)}{4\pi}\left[a_{1,0}+2\beta_0\log\left(r\mu_s e^{\gamma_E}\right)\right]\right)\log\left[\frac{\alpha_s\left(\mu_{\mathrm{us}}\right)}{\alpha_s\left(\mu_s\right)}\right]\right.\\ &\left.+\left(B-\frac{\beta_1}{4\beta_0}\right)\left[\frac{\alpha_s\left(\mu_{\mathrm{us}}\right)}{\pi}-\frac{\alpha_s\left(\mu_s\right)}{\pi}\right]\right\}\end{aligned}$$

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- Replacing in the static energy $\rightarrow$ no large logs for $\mu_s \sim 1/r$ and $\mu_{us} \sim \alpha_s/r$

$$\begin{aligned} E_\mathrm{s}(r) = & V_\mathrm{s}^\mathrm{soft}(r,\mu_s) + \frac{C_A^3 C_F}{12}\frac{\alpha_s^3(\mu_s)}{r}\left\{\frac{2}{\beta_0}\left(B-\frac{\beta_1}{4\beta_0}\right)\left[\frac{\alpha_s(\mu_\mathrm{us})}{\pi}-\frac{\alpha_s(\mu_s)}{\pi}\right]\right. \\ & +\frac{2}{\beta_0}\left(1+3\frac{\alpha_s(\mu_s)}{4\pi}\left[a_{1,0}+2\beta_0\log\left(r\mu_s e^{\gamma_E}\right)\right]\right)\log\left[\frac{\alpha_s(\mu_\mathrm{us})}{\alpha_s(\mu_s)}\right] \\ & \left.-\frac{\alpha_s(\mu_\mathrm{us})}{\pi}\log\left[\frac{C_A\alpha_s(\mu_s)e^{-5/6}}{r\mu_\mathrm{us}}\right]-\frac{\alpha_s(\mu_s)}{\pi}\log(r\mu_s e^{\gamma_E})\right\}. \end{aligned}$$

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- To cancel the renormalon we need a short-distance mass scheme $\rightarrow$ MSR mass [JHEP 04 (2018) 003]

$$\delta m_Q^{\rm MSR}(\mu, R) \equiv m_Q^{\rm pole} - m_Q^{\rm MSR}(R) = R\sum_{n=1}^{\infty}\delta_n^R\left[\frac{\alpha_s(R)}{4\pi}\right]^n$$

$$= R\sum_{n=1}^{\infty}\left[\frac{\alpha_s(\mu)}{4\pi}\right]^n\sum_{j=0}^{n-1}\delta_{nj}^R\log^j\left(\frac{\mu}{R}\right).$$

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- New type of large logs $\rightarrow$ resummation using R-evolution [PRL, 101 (2008) 151602].

Previous works

- To see the advantages of R-evolution we go back to the first work that used the static energy to obtain α_s [PRD, 86 (2012) 114031]

$$E_s(r) = V_s(r, \mu_s, \mu_{us}) + \delta_{US}(r, \mu_s, \mu_{us}) + RS(\rho),$$

  - $V_s(r, \mu_s, \mu_{us}) \to \ln(r\mu_{us})$
  - $RS(\rho) \to \ln(\rho/\mu_{us})$
- There is no right choice for μ_{us} $\to$ fits only posible for small values of r.
- The force was used in [PRD, 100 (2019) 114511]

$$F_{\mathrm{s}}(r) = \frac{\mathrm{d}E_{\mathrm{s}}(r)}{\mathrm{d}r}$$

- Force avoids explicit substraction.
- Using fully canonical scales $\sim 1/r$ $\to$ limits fit-range to $r \sim 0.076$ fm.

R-improvement, renormalon substractions

- The next goal is to obtain a renormalon-free potential.

$$E_{Q\overline{Q}}(r) = 2m_Q^{\rm MSR}(R_0) + 2\delta m_Q^{\rm MSR}(R_0,\mu) + E_s(r) \equiv 2m_Q^{\rm MSR}(R_0) + V_s^{\rm MSR}(r,\mu,R_0).$$

- We sum up large logs of μ/R_0.

$$\begin{aligned}\delta m_Q^{\rm MSR}(R_0) &= \delta m_Q^{\rm MSR}(R_0) + \delta m_Q^{\rm MSR}(R) - \delta m_Q^{\rm MSR}(R)\\ &= \delta m_Q^{\rm MSR}(R) + m_Q^{\rm MSR}(R) - m_Q^{\rm MSR}(R_0)\\ &= \delta m_Q^{\rm MSR}(R) + \Delta^{\rm MSR}(R,R_0).\end{aligned}$$

- $\Delta^{\rm MSR}(R,R_0)$: solution to MSR mass R-RGE $\rightarrow$ sums up logs of R/R_0.
- We have $\delta m_Q^{\rm MSR}(R) \sim \log(\mu/R)$. By choosing $\mu = R \rightarrow$ no large logs.
- We define the R-improved static potential:

$$V_s^{\rm MSR}(r,\mu,R_0) = V_s(r,\mu) + 2\delta^{\rm MSR}(R,\mu) + 2\Delta^{\rm MSR}(R,R_0)\,.$$

Subtraction schemes: RS scheme

- It is defined from the pole mass by subtracting its leading asymptotic behavior [JHEP 06 (2001) 022].

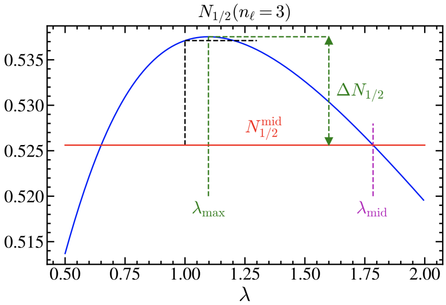
$$m_Q^{\rm pole} - m_Q^{\rm RS}(R) = \frac{2\pi}{\beta_0} R N_{1/2} \sum_{n=1}^{\infty} \left[\frac{\beta_0 \alpha_s(R)}{2\pi}\right]^n \sum_{\ell=0}^{\infty} g_\ell \left(1+\hat{b}_1\right)_{n-1-\ell},$$

- $N_{1/2}$ is the normalization of the leading renormalon

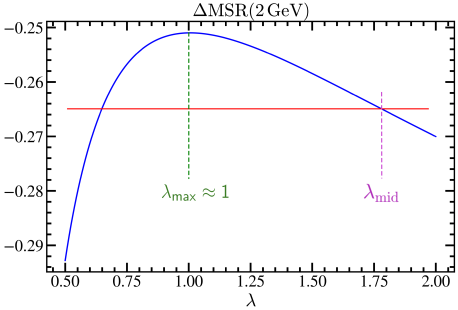
$$N_{1/2}^{(n)} = \frac{\beta_0}{2\pi} \sum_{k=0}^{n} \frac{S_k}{\left(1+\hat{b}_1\right)_k} \qquad S_k = \sum_{k=0}^{j} \tilde{\gamma}_k^R \sum_{i=0}^{j-k} (-1)^i \tilde{b}_i^N \tilde{g}_{j-i-k}^N,$$

- $N_{1/2}$ depends on λ, it reshuffles higher perturbative orders. We vary it to estimate $N_{1/2}$ (similar to scale variation).
- λ is also used to estimate R-evolution uncertainty.

Dependence on λ



[JHEP 04 (2018) 003]



- The canonical value of $\lambda = 1$ is clearly biased.
- It is necessary to vary it to estimate the uncertainty coming from higher order missing terms.
- We pick $\lambda_{mid} = 1.784$ and vary it from 1.5 to 2.1.

Profiles

- The static potential depends on $\log(r\mu_s) \rightarrow$ should be small.

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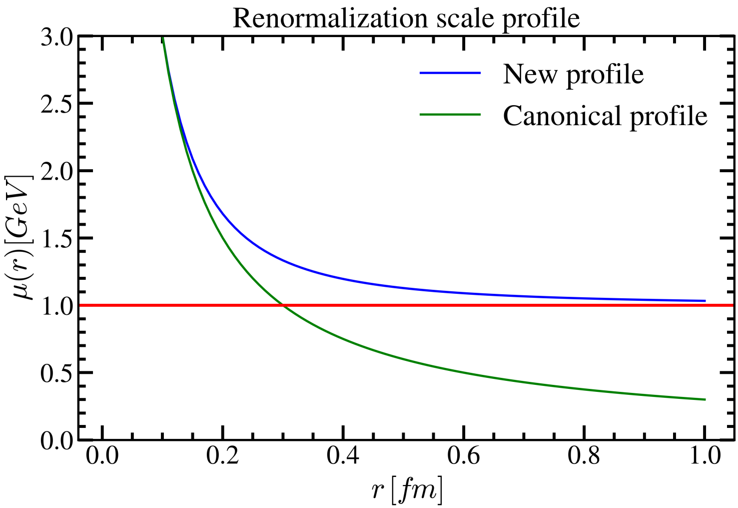
- Choosing $\mu_s \sim 1/r$ doesn't always work $\to \mu_s < \Lambda_{\mathrm{QCD}}$ for $r \gtrsim 0.2$ fm ~ 1 GeV.
- Solution: use profile functions that ensure series convergence:

$$\mu_s = \sqrt{\left(\frac{\xi}{r}\right)^2 + \frac{b}{r} + a^2} + \mu_0 - a = \begin{cases} \frac{\xi}{r} & \text{for } r \to 0 \\ \mu_0 & \text{for } r \to \infty \end{cases}$$

with $\xi = \mathcal{O}(1)$ and $\mu_0 \sim 1\,\mathrm{GeV}$. This makes sure the series is stable and convergent over the entire spectrum.

Profiles



Renormalization scale profile
New profile
Canonical profile
$\mu(r)\,[GeV]$
$r\,[fm]$
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0

Fits

- Lattice QCD data from HotQCD. $(2+1)$ flavor simulations. We have 9 lattices sets with 2512 points between all of them.
- The static potential is defined up to an additive constant $\rightarrow$ each experiment n has a different offset A_n.
- Minimization of χ^2 function.

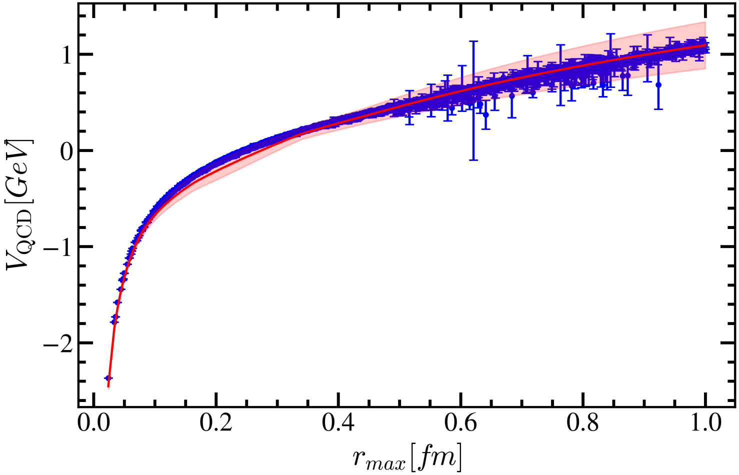
$$\chi^2 = \sum_{n=1}^{N_{\mathrm{exp}}} \sum_{i=1}^{N_i} \left(\frac{V_{n,i}^{\mathrm{pert}} + A_n - V_{n,i}^{\mathrm{exp}}}{\sigma_{n,i}^{\mathrm{exp}}} \right)^2 .$$

- We can marginalize analytically first with respect to the offsets.

$$\frac{\partial \chi^2}{\partial A_n} = 0 \Longrightarrow \tilde{A}_n = -\frac{\sum_i \frac{\left(V_{n,i} - V_{n,i}^{\mathrm{exp}}\right)}{\sigma_{n,i}^2}}{\sum_i \frac{1}{\sigma_{n,i}^2}} \Longrightarrow \tilde{\chi}^2 = \sum_{n,i} \frac{\left(V_i - V_i^{\mathrm{exp}}\right)^2}{\sigma_i^2} - \frac{\left[\sum_{n,i} \frac{\left(V_i - V_i^{\mathrm{exp}}\right)}{\sigma_i^2}\right]^2}{\sum_{n,i} \frac{1}{\sigma_i^2}}$$

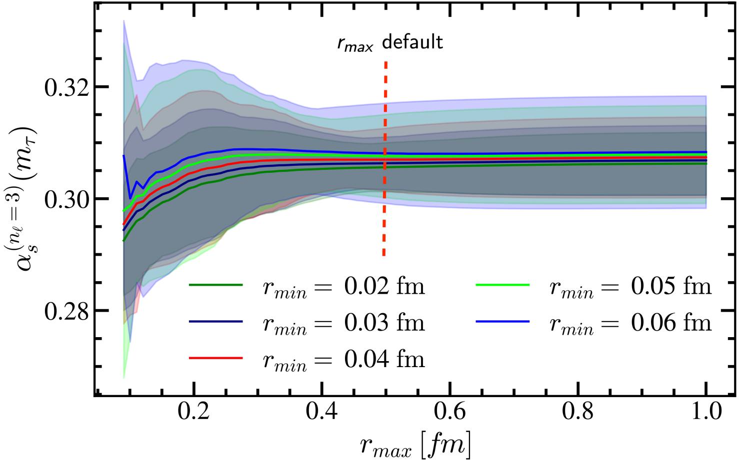
- We perform the fit with 500 random profiles.

Potential

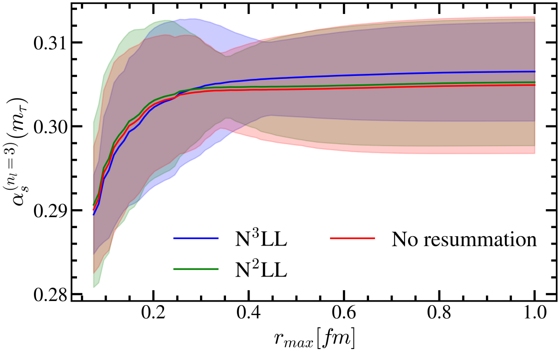


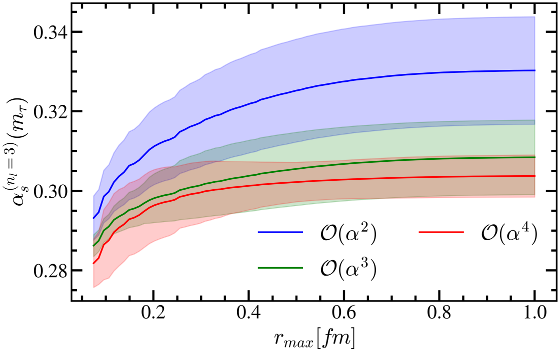
Study of the dataset

Fit for $r \in [r_{\min}, r_{\max}]$. $r_{\text{data}} \geq 0.023$ fm.

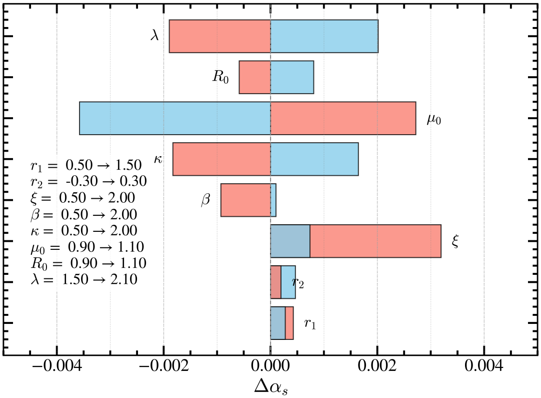


Order-by-order agreement





Variation of profiles parameters one at a time



$$\mu_s = \sqrt{\left(\frac{\xi}{r}\right)^2 + \frac{b}{r} + a^2} + \mu_0 - a$$

$$R = \sqrt{\left(\frac{\beta}{r}\right)^2 + \frac{b}{r} + a^2} + R_0 - a$$

$$\mu_{us} = \kappa \frac{C_A}{2} \left\{\mu_s \alpha_s(\mu) - \mu_0 \alpha_s(\mu_0)\right\} + \mu_0 .$$

Preliminary results

- We perform a fit with $r \in [0.03, 0.5]$ fm. Error includes: perturbative (dominant), lattice (negligible) and dataset dependence (also negligible).

All data $\rightarrow \alpha_s^{(n_f=3)}(m_\tau) = 0.3107 \pm 0.00001_{\text{lattice}} \pm 0.053_{\text{pert}} \pm 0.0001_{\text{r}_{\text{max}}} \pm 0.0007_{\text{r}_{\text{min}}}$

$$\downarrow$$

$$\alpha_s^{(n_f=3)}(m_\tau) = 0.311 \pm 0.053$$

$$\alpha_s^{(n_f=5)}(m_Z) = 0.1175 \pm 0.0007$$

Competitive with the w.a. $\alpha_s^{(5)}(m_Z) = 0.1180 \pm 0.0009$ and compatible at 0.5-σ level. Comparing with previous analyses:

$$\alpha_s^{(n_f=5)}(m_Z) = 0.11660^{+0.00110}_{-0.00056} \text{ [PRD 100 (2019) 11]}$$

$$\alpha_s^{(n_f=5)}(m_Z) = 0.1181 \pm 0.0009 \text{ [JHEP 09 (2020) 016]}$$

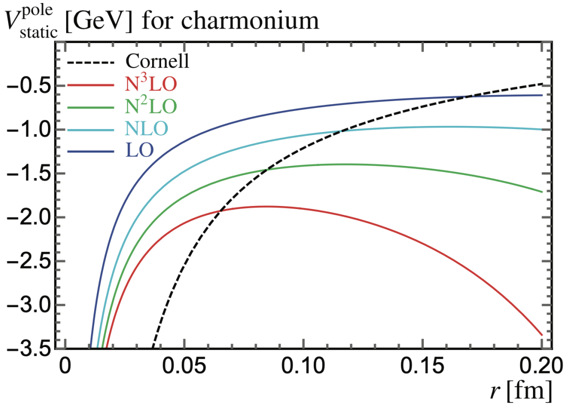
Conclusions

- Used the static energy to determine α_s, building (and improving) on previous analyses.
- Performed ultra-soft large-log resummation up to $\mathrm{N^3LL}$.
- Employed the MSR mass and R-evolution to improve the static potential.
- Designed profile functions to increase the validity of the potential up to $r \sim 1\,\mathrm{fm} \sim 200$ MeV $\rightarrow$ lowest value reached.
- Carried out fits to lattice data to obtain a very competitive result for α_s.

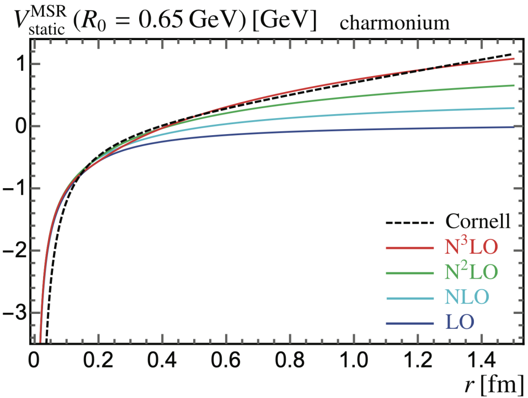
$$\alpha_s^{(n_f=5)}(m_Z) = 0.1175 \pm 0.0007 \quad \text{[preliminary]}$$

BACKUP

Renormalon substractions



[Eur. Phys. J. C 79 (2019) 4, 323]



[Eur. Phys. J. C 79 (2019) 4, 323]

Force-type subtractions

- Integrating the Force is equal to perform R-evolution.
- The renormalon doesn't depend on r $\rightarrow$ we can subtract the potential at r_0.

$$E_{\rm s}^F(r, r_0) \equiv E_{\rm s}(r) - E_{\rm s}(r_0) = E_{\rm s}(r) - E_{\rm s}(r_1) + [E_{\rm s}(r_1) - E_{\rm s}(r_0)]$$
$$= E_{\rm s}(r) - E_{\rm s}(r_1) + \int_{r_0}^{r_1} {\rm d}r' F_{\rm s}(r') \equiv E_{\rm s}(r) - E_{\rm s}(r_1) + \Delta_F(r_0, r_1)$$

- [PRD, 90(7), 074038] chooses $r_1 = r \rightarrow$ only Δ_F left.
- Connecting with R-evolution, the substraction term is (choosing $R = 1/r_1$ and $\mu = R$):

$$E_s(r_1) \equiv \delta_{\rm soft}^F(R) = \frac{1}{2} V_{\rm s}^{\rm soft}\left(\frac{1}{R}\right) = -2\pi C_F R \sum_{i=1} \left[\frac{\alpha_s(R)}{4\pi}\right]^i \sum_{j=0}^{i-1} a_{i-1,j}\gamma_E^j \equiv R \sum_{i=1} \left[\frac{\alpha_s(R)}{4\pi}\right]^i \delta_i^F .$$

- We can express Δ_F as an R-evolution integral:

$$\gamma_{\rm soft}^F(R) = -\frac{1}{2}\left[r^2 F_{\rm s}^{\rm soft}(r)\right]_{r=1/R},$$
$$\Delta_F^{\rm soft}(r_0, r_1) = \int_{r_0}^{r_1} \frac{{\rm d}r'}{(r')^2}\left[(r')^2 F_{\rm s}^{\rm soft}(r')\right] = \frac{1}{2}\int_{1/r_0}^{1/r_1} {\rm d}R' \gamma_{\rm soft}^F(R') .$$

- It inherits the infrared sensitivity of the Static Potential.

α_s dependence on ξ