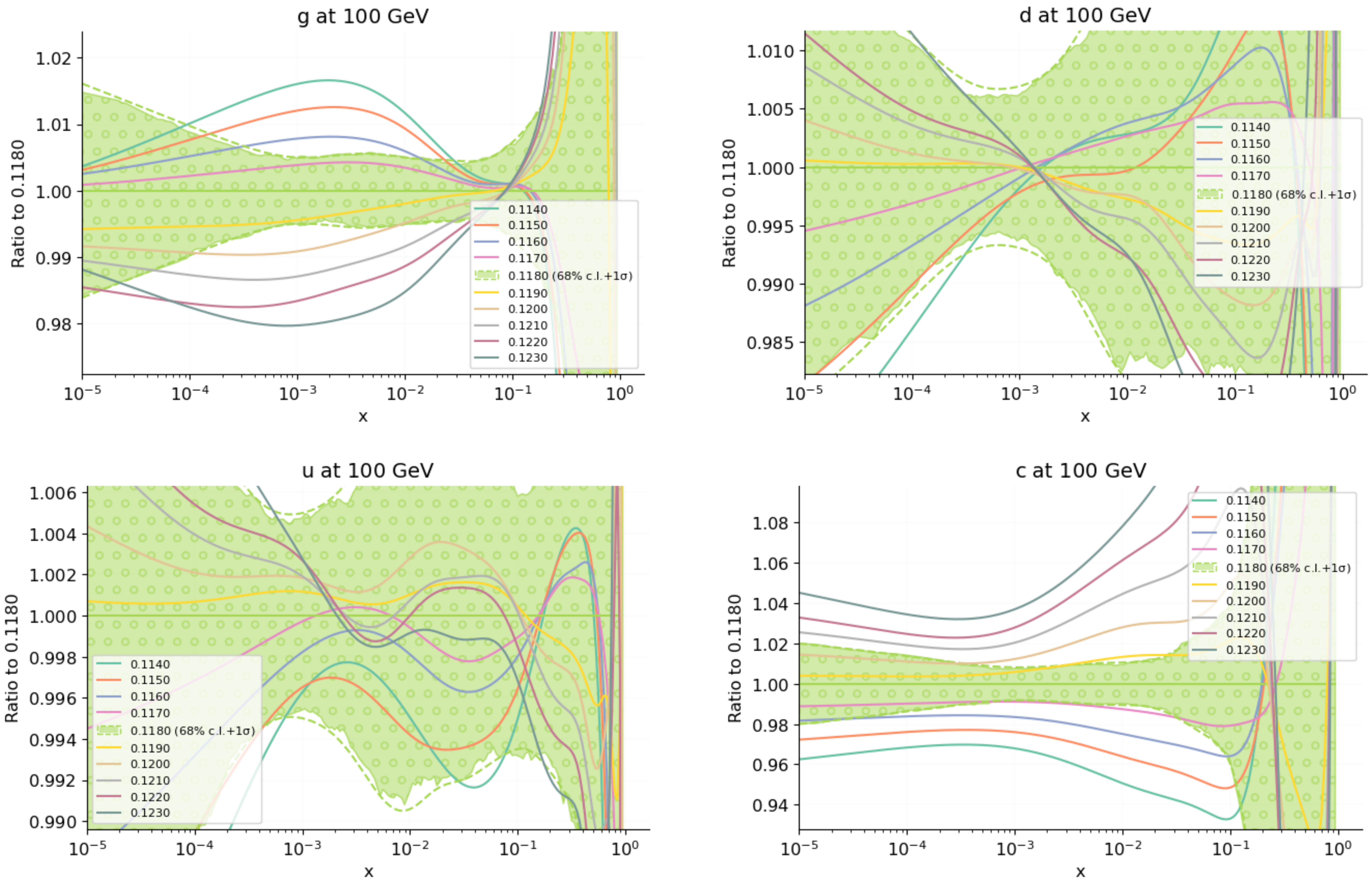
$\alpha_s(m_Z)$ at approximate N³LO QCD and NLO QED from a global PDF analysis

Based on 2506.13871 with the NNPDF collaboration

Roy Stegeman
The University of Edinburgh

EPS-HEP, Marseille, 7 July 2025

PDFs and α_s are correlated



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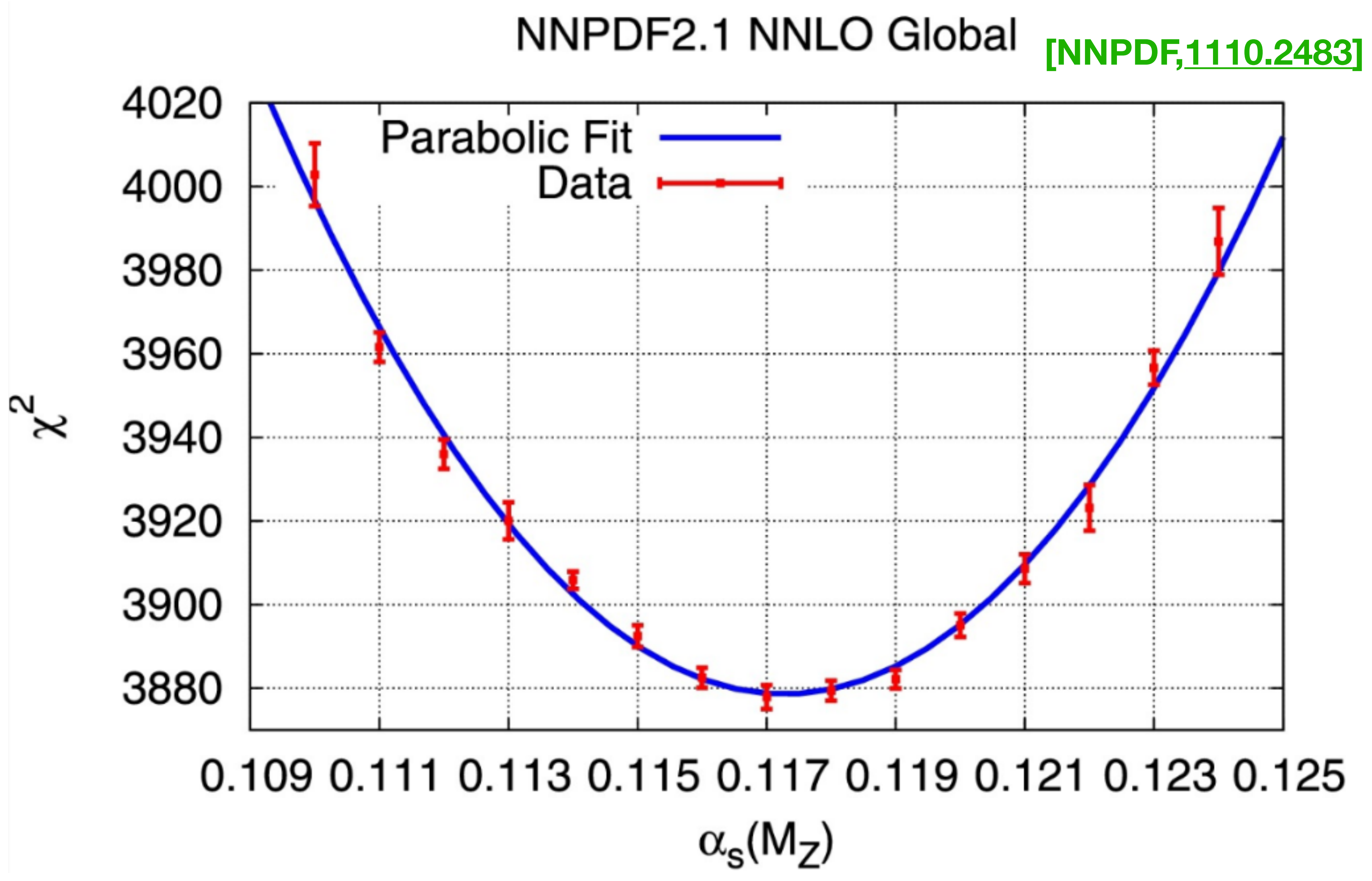

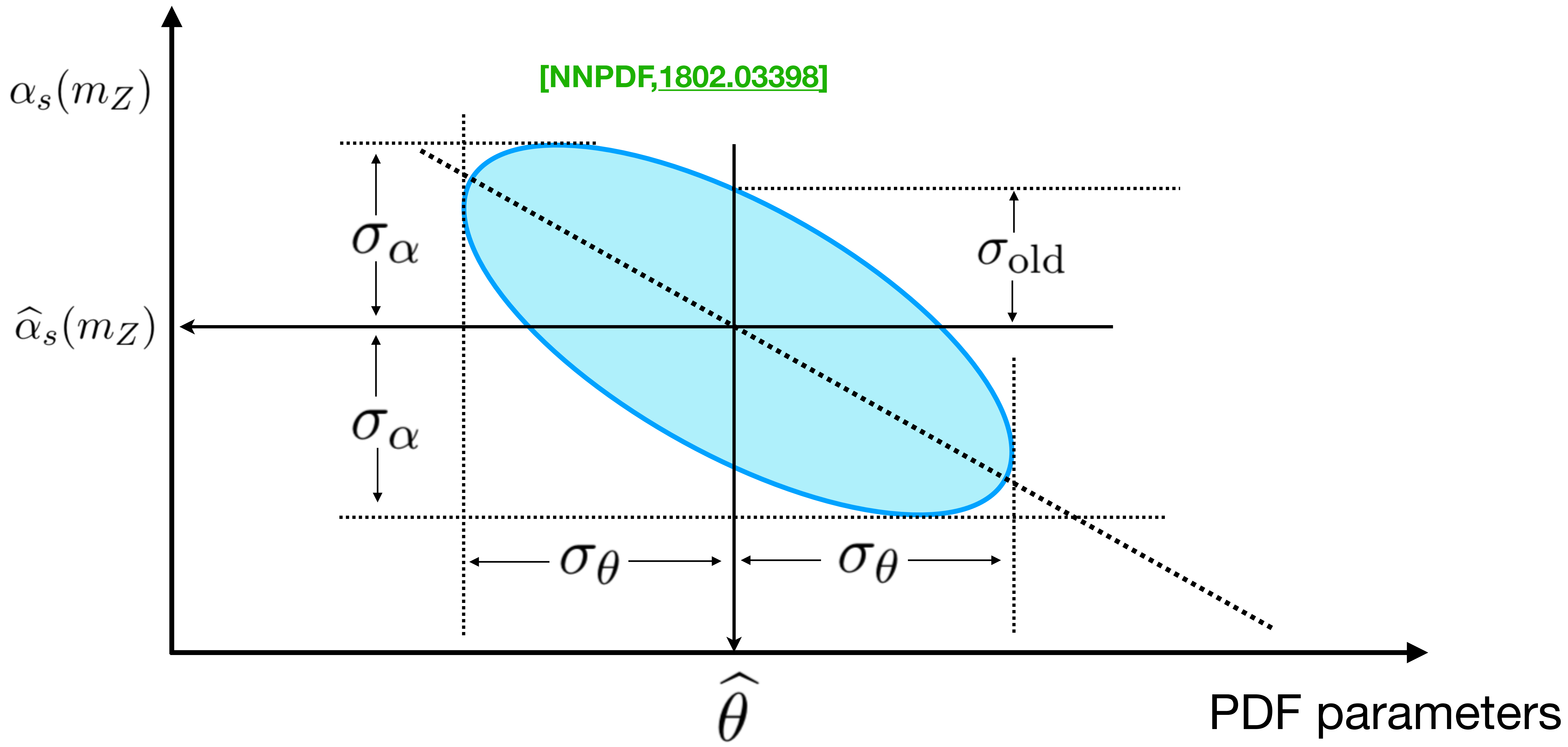

✘ In many cases α_s is determined by extracting it from a parabolic fit to the χ^2 profile

In this way correlations between PDF parameter fluctuations and α_s are not fully taken into account

✔ Minimise α_s and PDF simultaneously

How to account for correlations between PDFs and α_s?

NNPDF can't (easily) treat α_s as another trainable parameter

Rerunning DGLAP evolution at every training step is not feasible, therefore it is precomputed at fixed α_s

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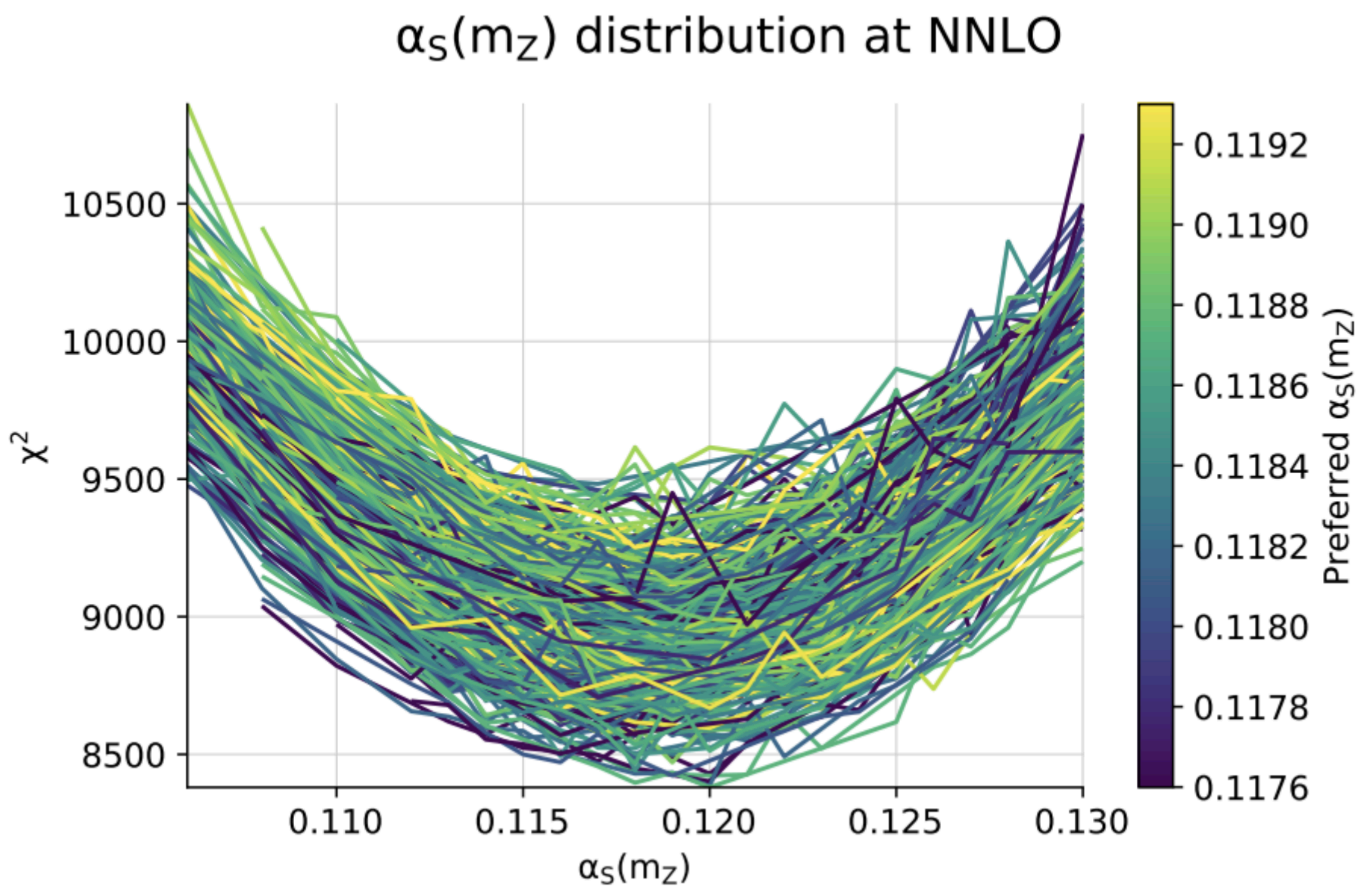
Two methods have been developed to avoid this limitation:

1) **Multiple fits** of the same data replica, changing only the value of $\alpha_s(m_Z)$, thereby **correlating PDFs** at different $\alpha_s(m_Z)$
   [NNPDF, 1802.03398]
2) Based on **a single fit** with an $\alpha_s(m_Z)$ **theory covmat,** and computing the fit's preferred value for alphas a posteriori in a Bayesian framework
   [Ball, Pearson, 2105.05114]

The results shown in this talk correspond to the theory covmat method, but agreement is always within 1 per-mille



Correlated replicas fitted to the same data replica at different α_s

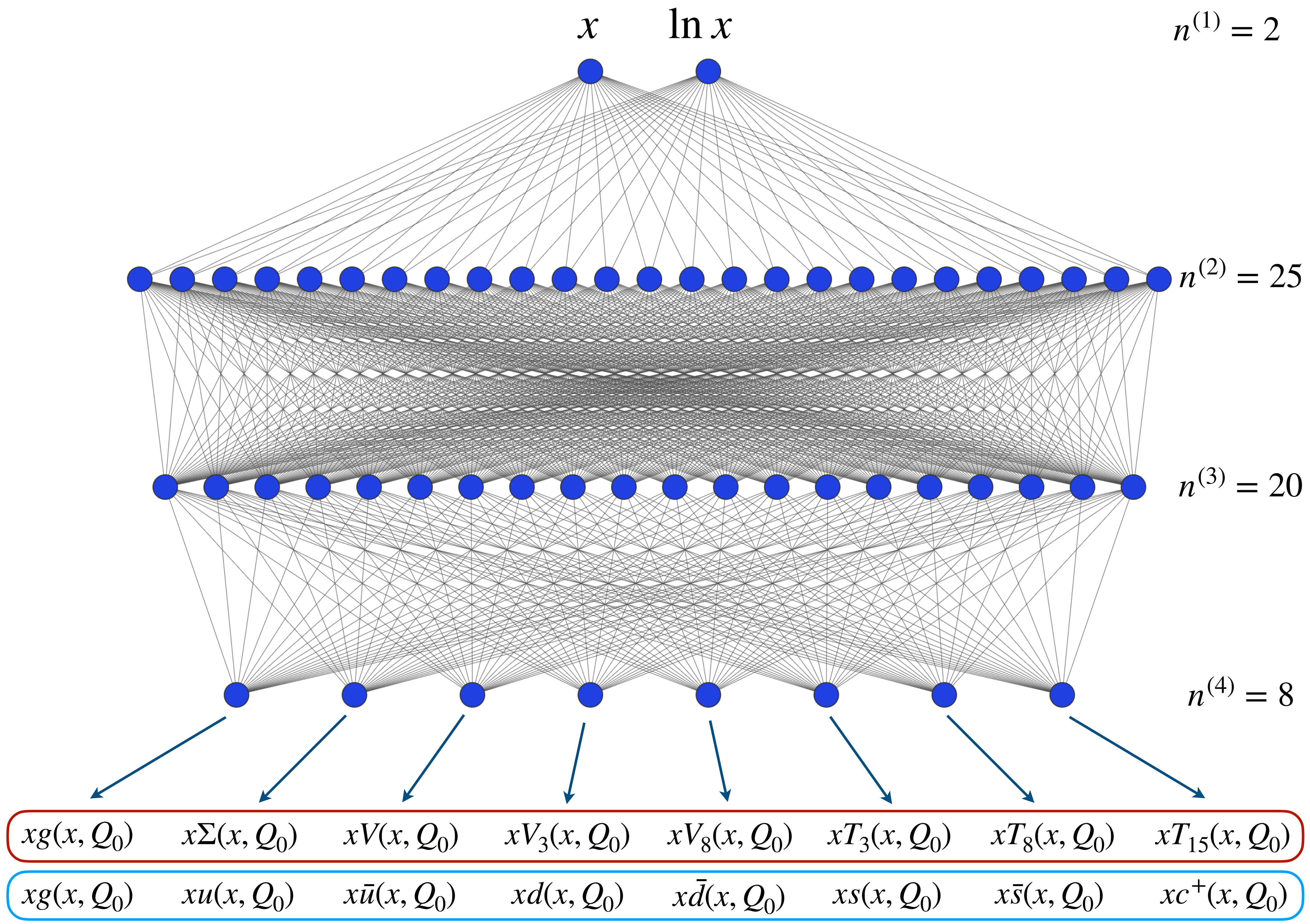
α_s from NNPDF4.0

Last determination was based on NNPDF3.1

$\alpha_s(M_Z) = 0.1185(5)^{\mathrm{PDF}}$ **[NNPDF, 1802.03398]**

Lots of progress since then:

- Updated Machine Learning methodology **[NNPDF:2109.02653]**
- NNPDF4.0 global dataset **[NNPDF:2109.02653]**
- Missing higher order uncertainties at the level of the fit **[NNPDF, 2401.10319]**
- NLO QED and photon PDF **[NNPDF:2401.08749]**
- aN3LO QCD **[NNPDF:2402.18635]**



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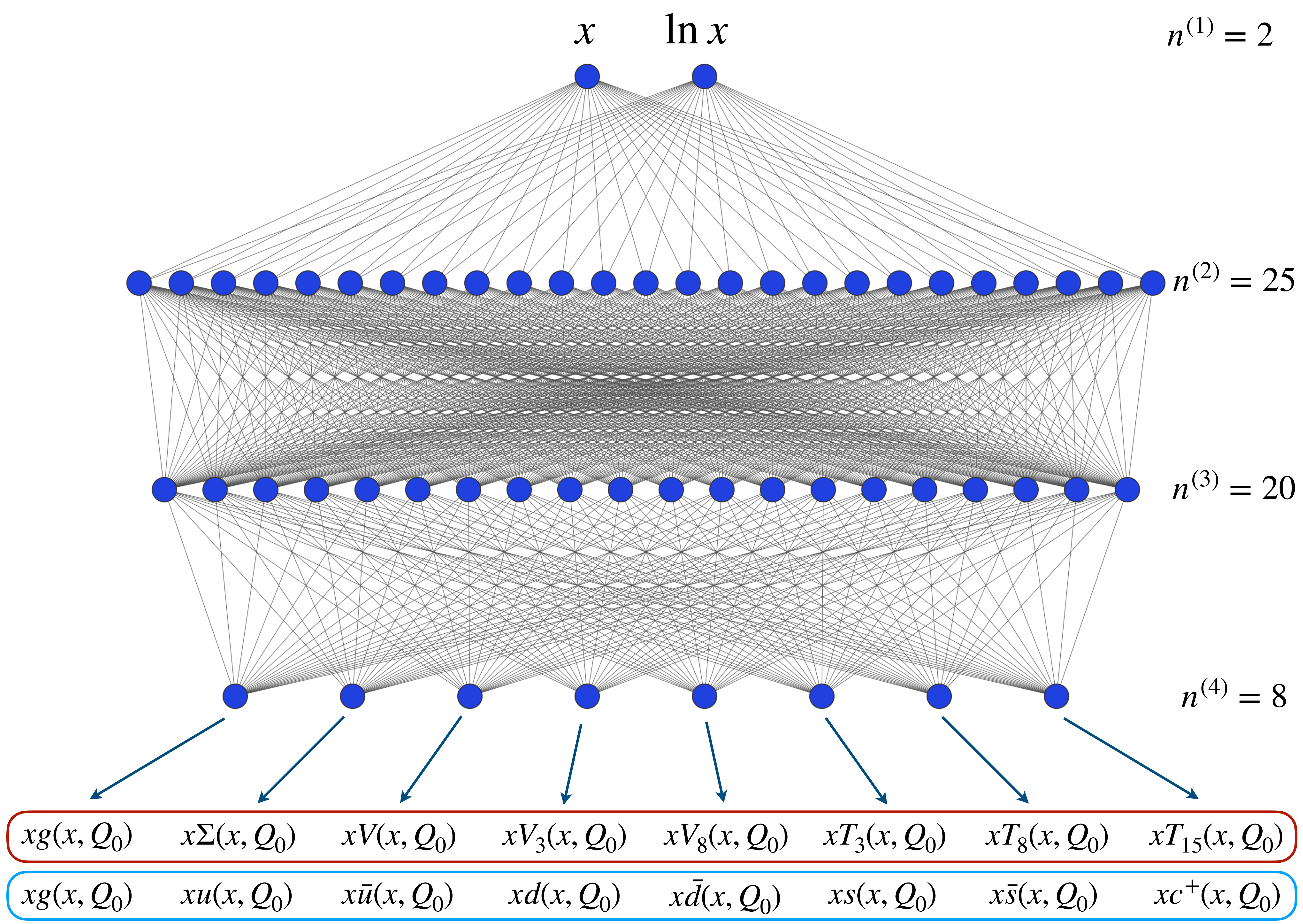


Let's first look at the **methodology**:

NNPDF4.0 methodology, NNPDF3.1-like dataset:
$\alpha_s(M_Z) = 0.1188(5)^{\text{PDF}}$

Consistent with the NNPDF3.1 result!

- Impact of theory parameters
- Validation

New treatment of theory uncertainty

NNPDF3.1

- In NNPDF3.1 the **dominant source of uncertainty** was from missing higher orders (MHOU):

$$\alpha_s(m_Z) = 0.1185(5)^{\rm PDF}(1)^{\rm meth}(11)^{\rm MHOU} = 0.1185(12)$$

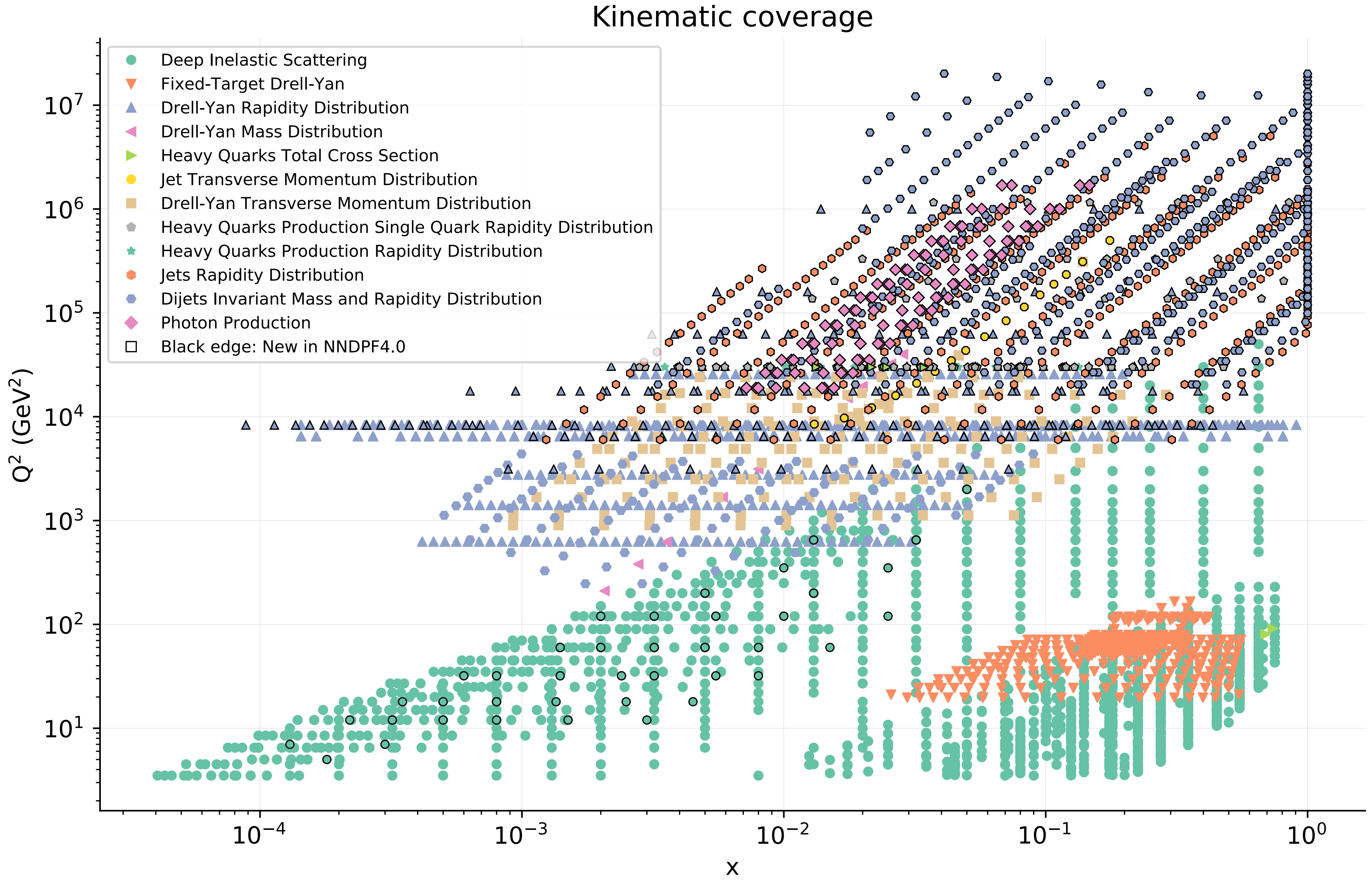
- Obtained from NNLO-NLO shift

$$\Delta\alpha_s^{\rm MHOU} \equiv \frac{1}{2}\left|\alpha_s^{\rm NNLO} - \alpha_s^{\rm NLO}\right| = 0.0011$$

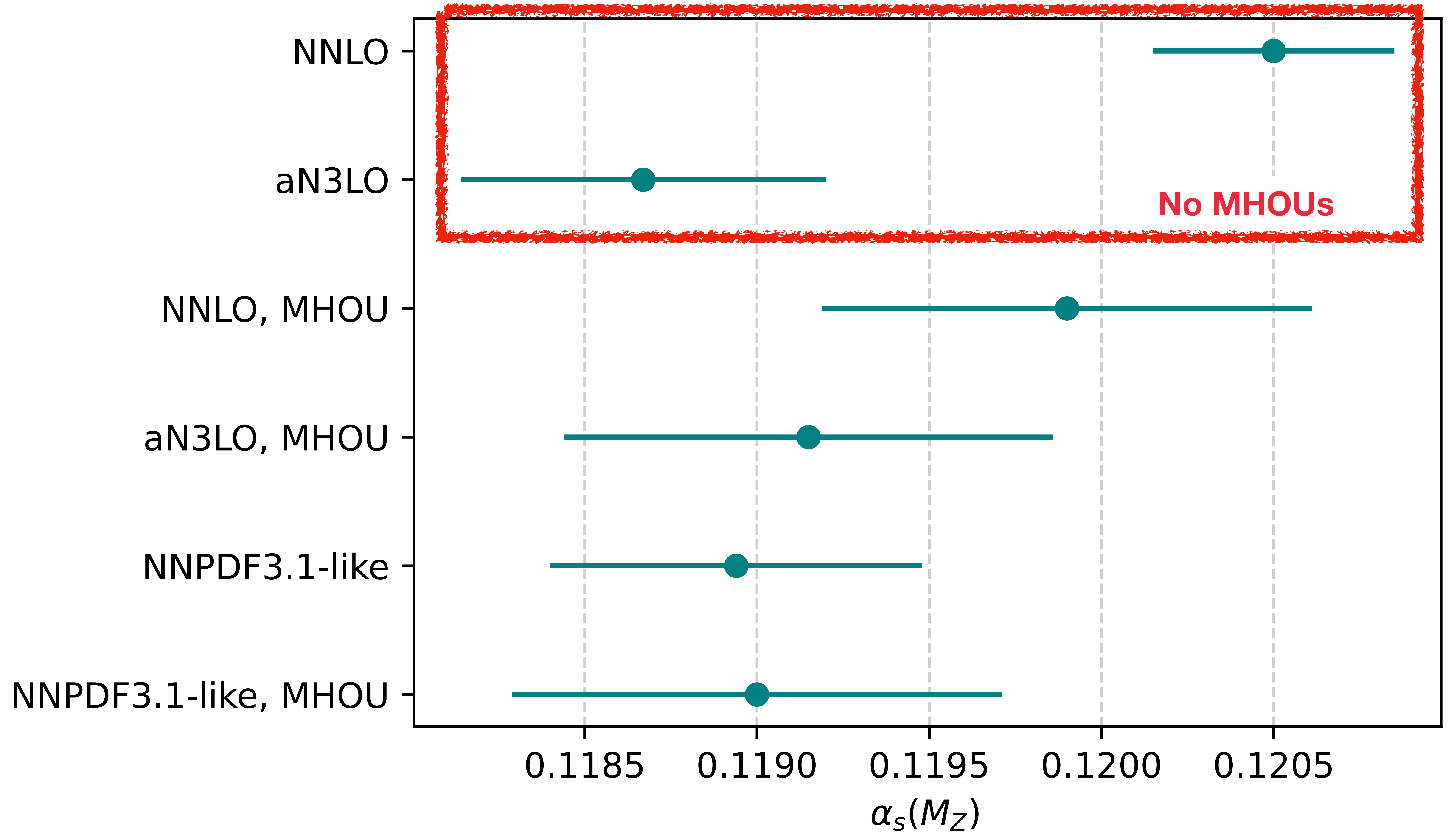
NNPDF4.0 (this work)

- Including a theory covariance matrix **[NNPDF, 2401.10319]** from scale variations at the level of the fit leads to much **reduced uncertainty**:

NNPDF4.0 methodology, NNPDF3.1-like data: $\alpha_s(m_Z) = 0.1188(6)^{\rm PDF+MHOU}$

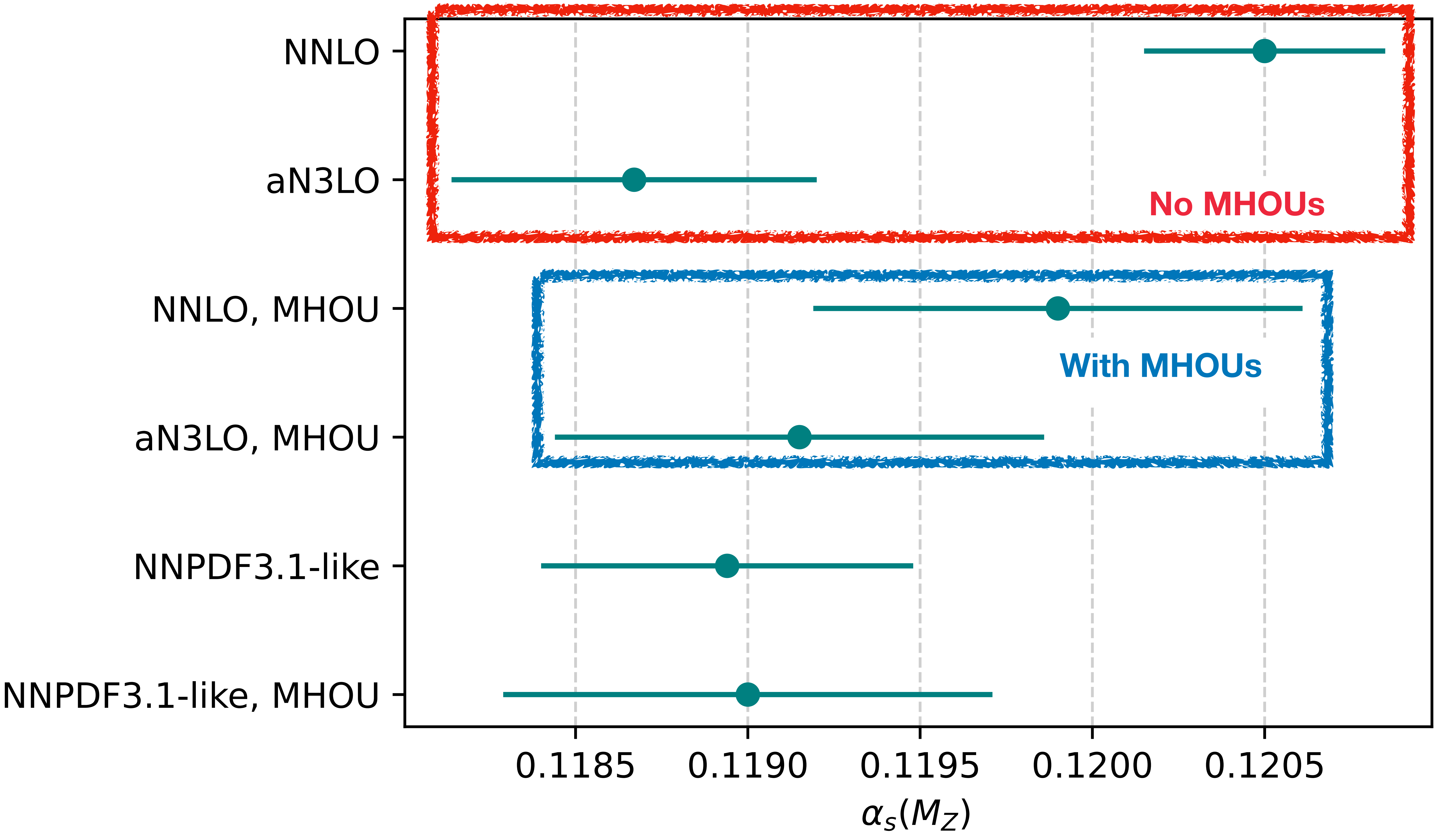


Impact of missing higher order uncertainties (MHOUs) and aN^{3}LO [NNPDF, 2402.18635]



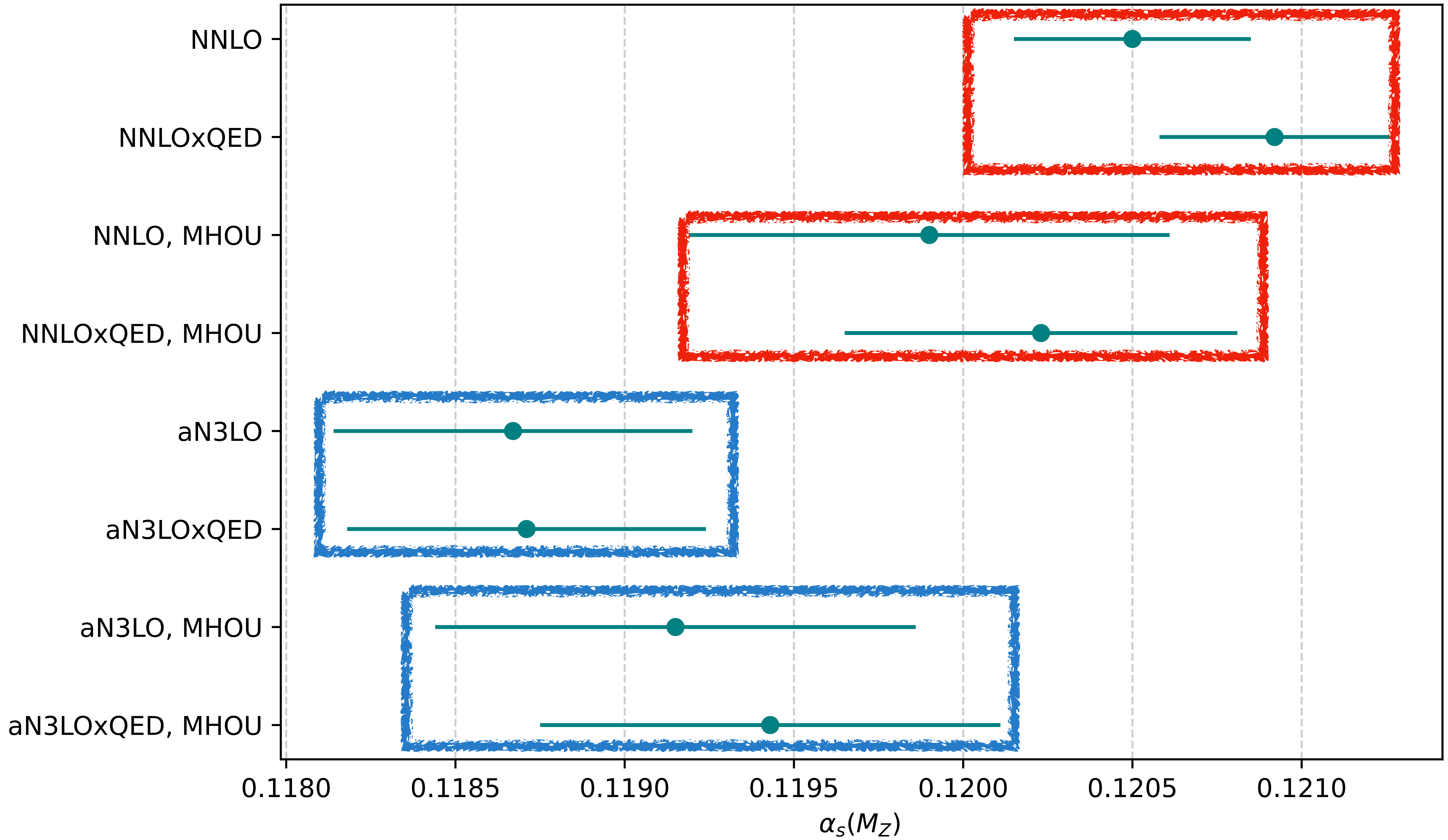
These and following results for the NNPDF4.0 dataset

Impact of missing higher order uncertainties (MHOUs) and aN^{3}LO [NNPDF, 2402.18635]



These and following results for the NNPDF4.0 dataset

Impact of QED corrections and the photon PDF [NNPDF, 2401.08749]

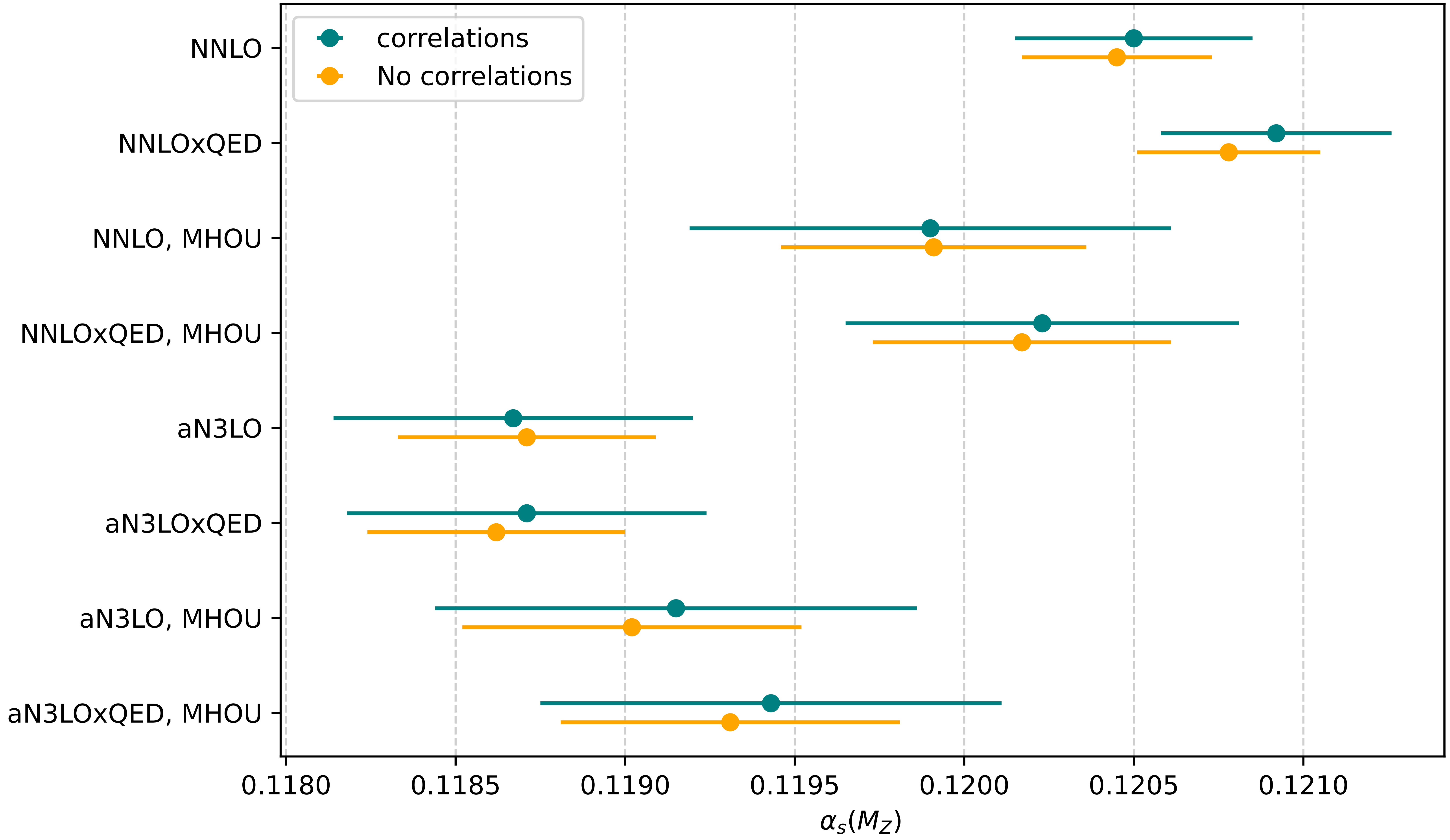


QED means

- NLO QED corrections to DGLAP evolution
- Determine also the photon PDF

QED has a bigger impact at **NNLO** than at **aN3LO**

Impact of PDF-α_s correlations



Correlations increase the uncertainty by 25% to 60%

$\alpha_s(m_Z)$ at different values of m_t pole mass

m_t [GeV]	NNLO	NNLO, MHOU
175	0.1208(4)	0.1200(6)
172.5	0.1204(4)	0.1200(7)
170	0.1200(4)	0.1198(6)

PDG value is $m_t = 172.4(7)$ GeV

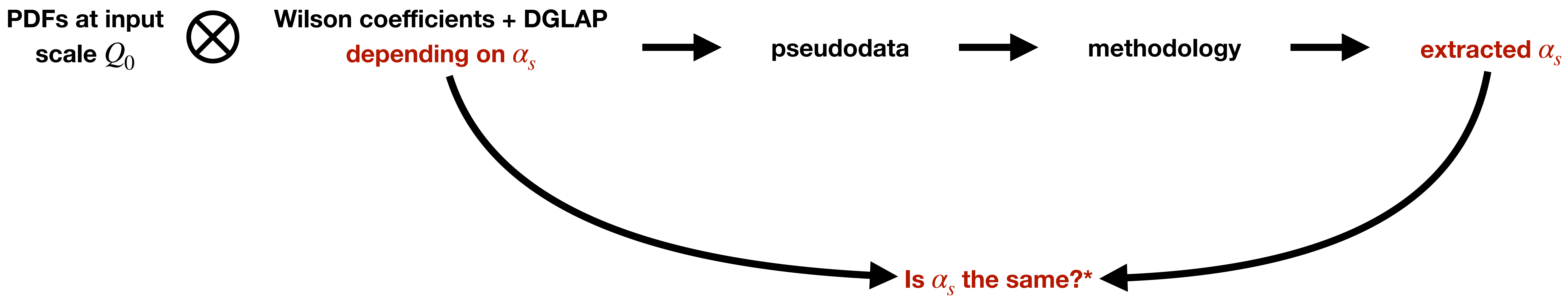
We find the impact of the top mass to be negligible

- Results
- Validation

Q: How to validate the methodologies?

A: Closure tests [Del Debio, Giani, Wilson, 2111.05787]

Basic idea: generate a global pseudo dataset from theory predictions and extract α_s from this



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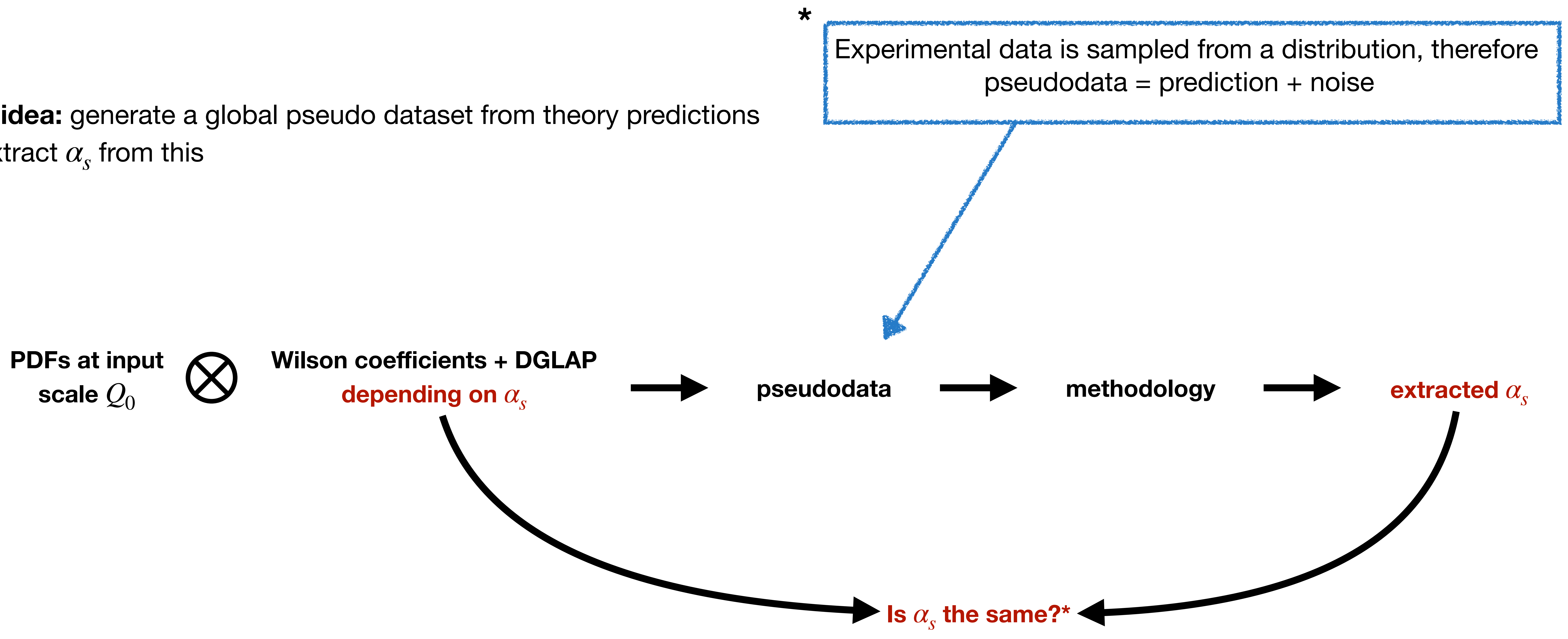
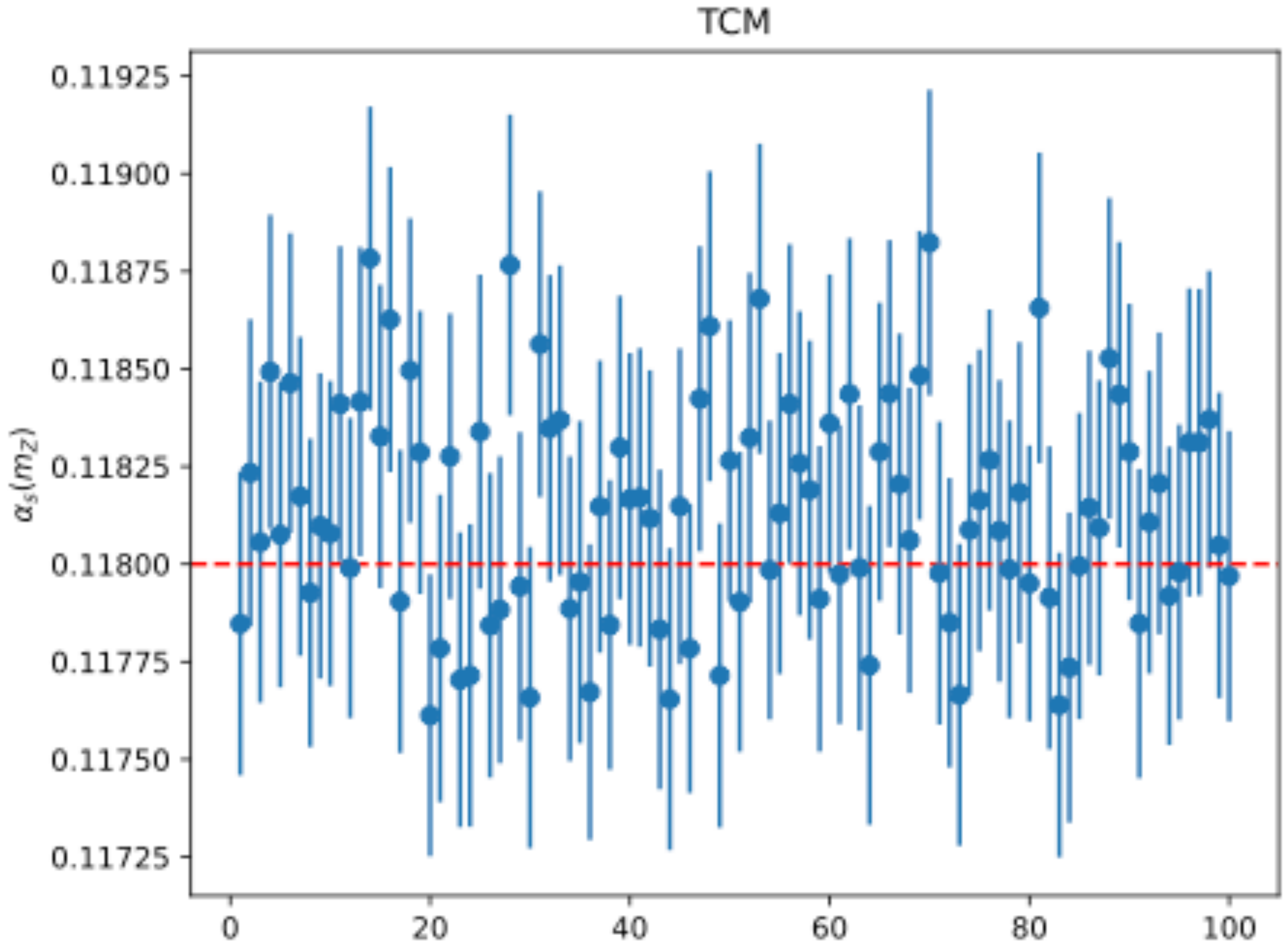


The closure test suggests a bias…

1) Generate pseudodata samples around $\alpha_s(m_Z) = 0.118$
2) Extract $\alpha_s(m_Z)$ for each pseudodata sample
3) Check if our method returns the correct answer ✗

We find a three-sigma indication for a bias!

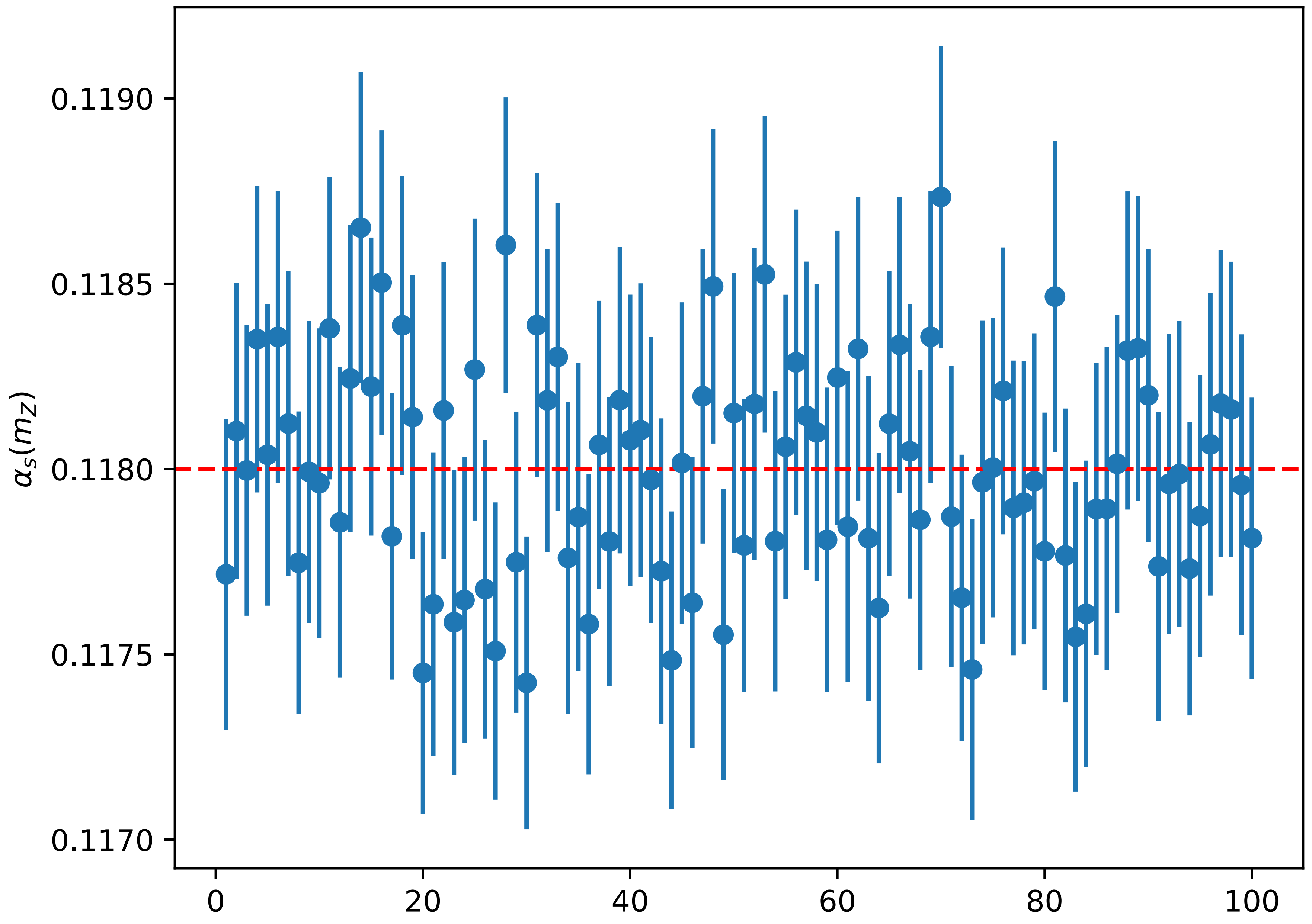
$\alpha_s(m_Z) = 0.11813(4)$



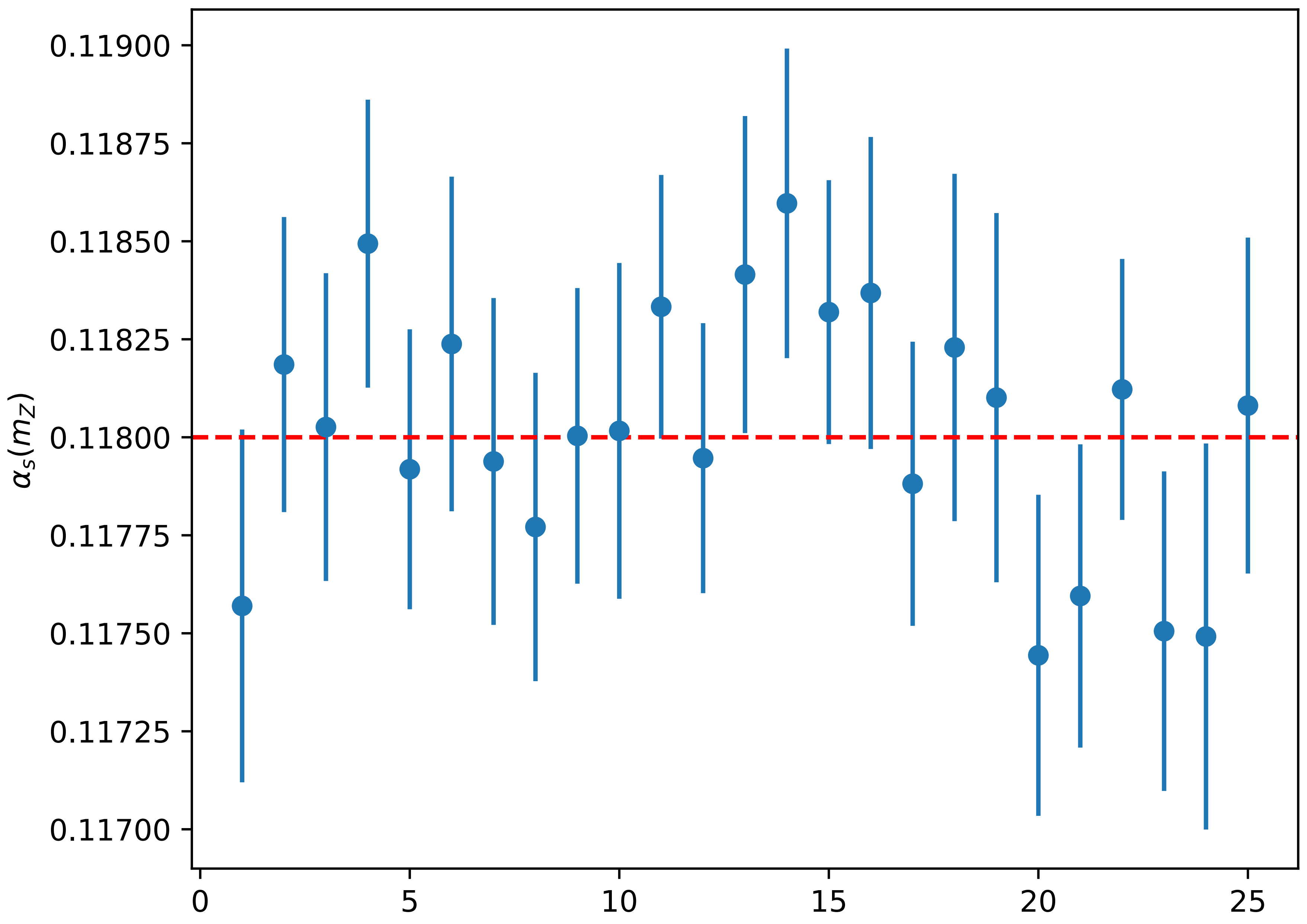
…but we can understand the origin!

In NNPDF4.0 we enforce positivity of observables and PDFs

Theory covmat method



Correlated replicas method



$\alpha_s(m_Z) = 0.11798(4)$ ✓

$\alpha_s(m_Z) = 0.11804(8)$ ✓

Both methodologies pass the closure test!

But how do we account for this bias?

Accounting for the positivity bias

- Simple solution: remove positivity from the fit
- Actually not so simple: the two methods no longer agree
- Conservative option: add shift due to bias as a linear correction to the uncertainty
  In this case 0.1194-0.1187=0.0007

	Theory covmat	Correlated replicas
with positivity	0.1194(7)	0.1193(7)
W/o positivity	0.1187(9)	0.1191(9)

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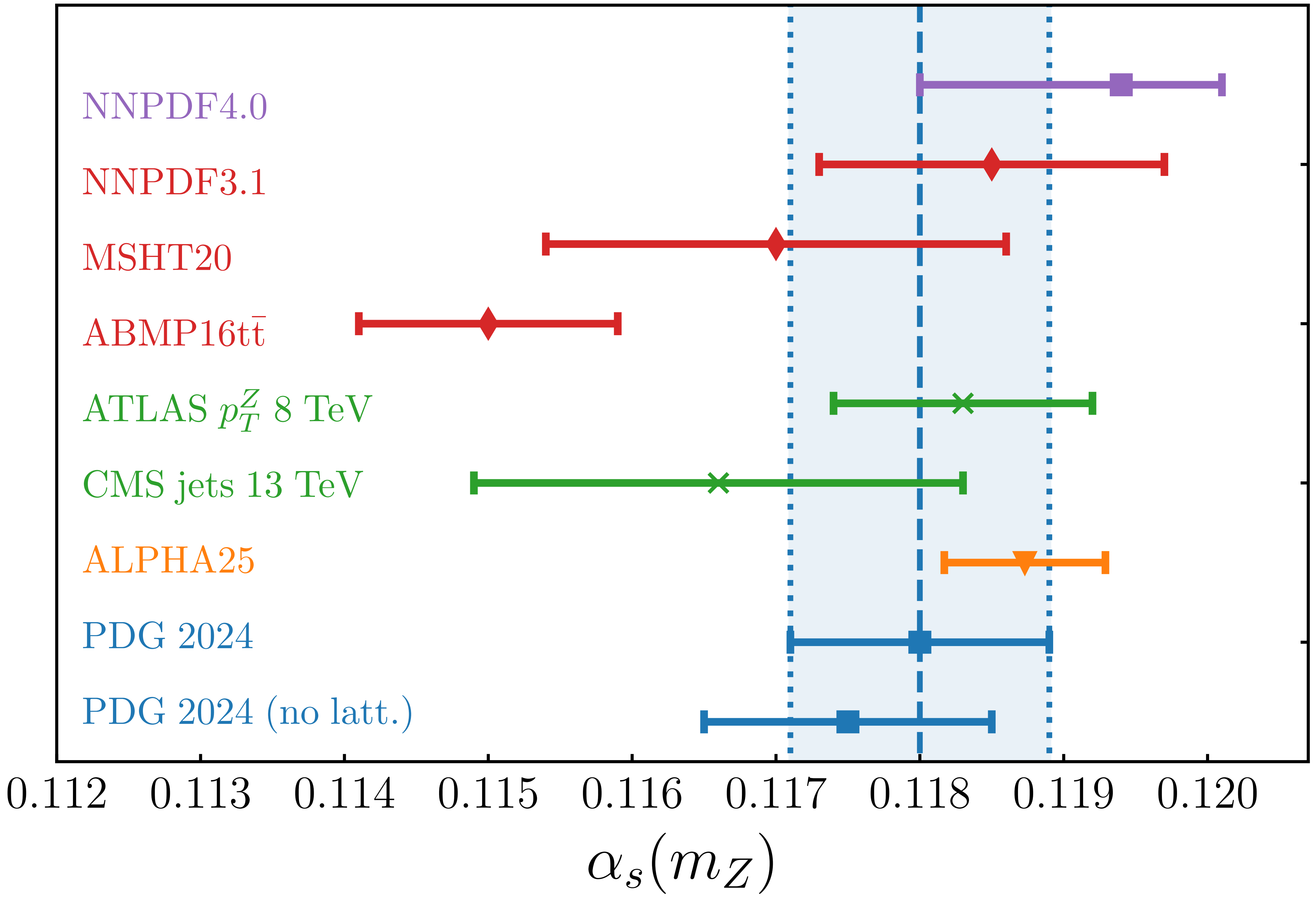
$$\alpha_s(M_Z)^{\mathrm{aN3LO,QED,MHOU}} = 0.1194^{+0.0007}_{-0.0007}{}^{\mathrm{PDF}}{}_{-0.0007}{}^{\mathrm{positivity}} = 0.1194^{+0.0007}_{-0.0014}$$

Summary and Outlook

- Strong correlations between the PDFs and α_s means that a simultaneous determination is needed
- aN3LO, MHOU, QED each have a significant impact on the value of $\alpha_s(m_Z)$
- Impact of top mass is negligible
- Our methodologies have been validated by means of closure testing
- Account for bias due to positivity constraint
- $\alpha_s(M_Z)^{\mathrm{aN3LO,QED,MHOU}} = 0.1194^{+0.0007}_{-0.0014}$
- Next: extend to other parameters

  See Jaco's talk from this morning!



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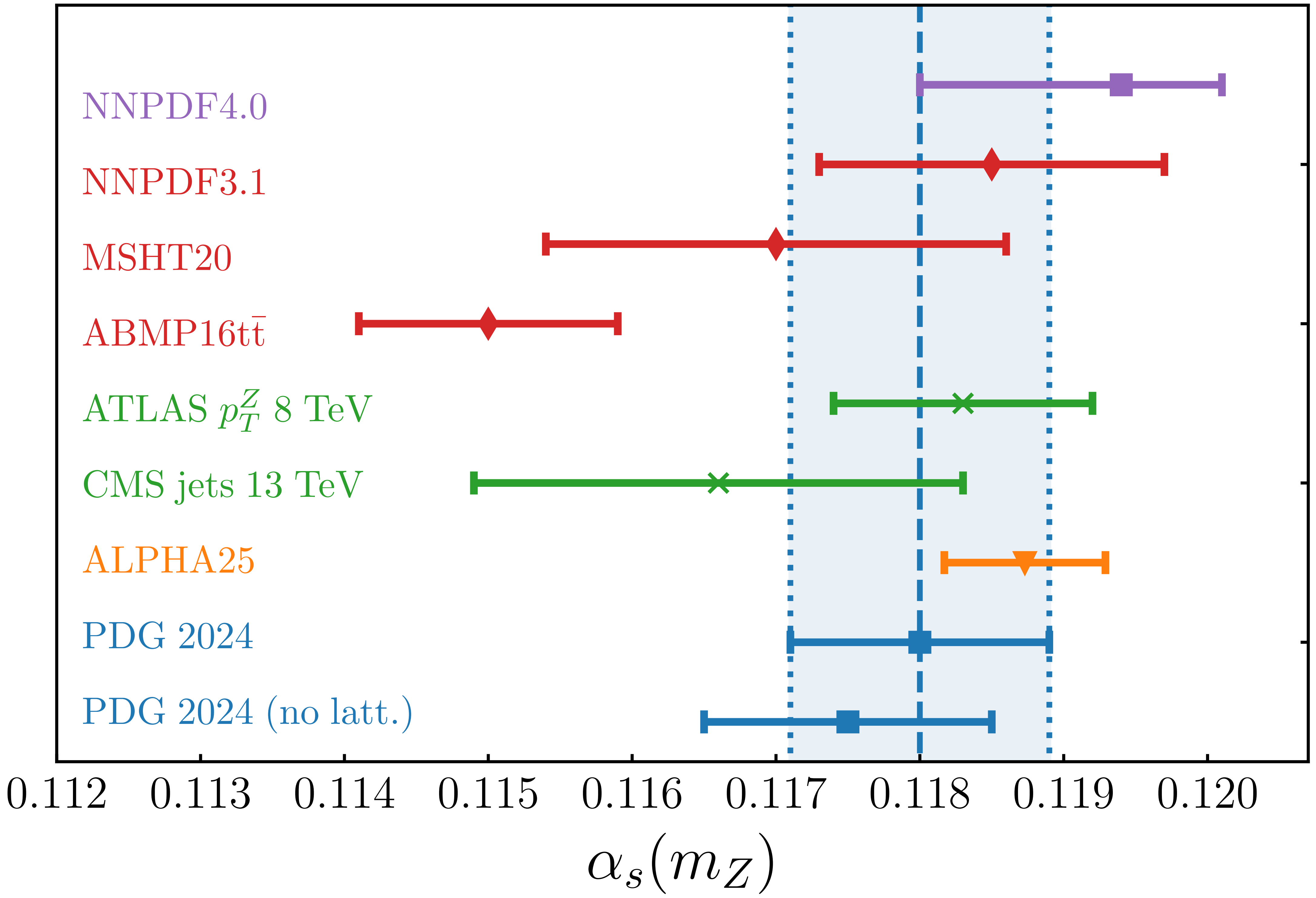


Thank you for your attention!

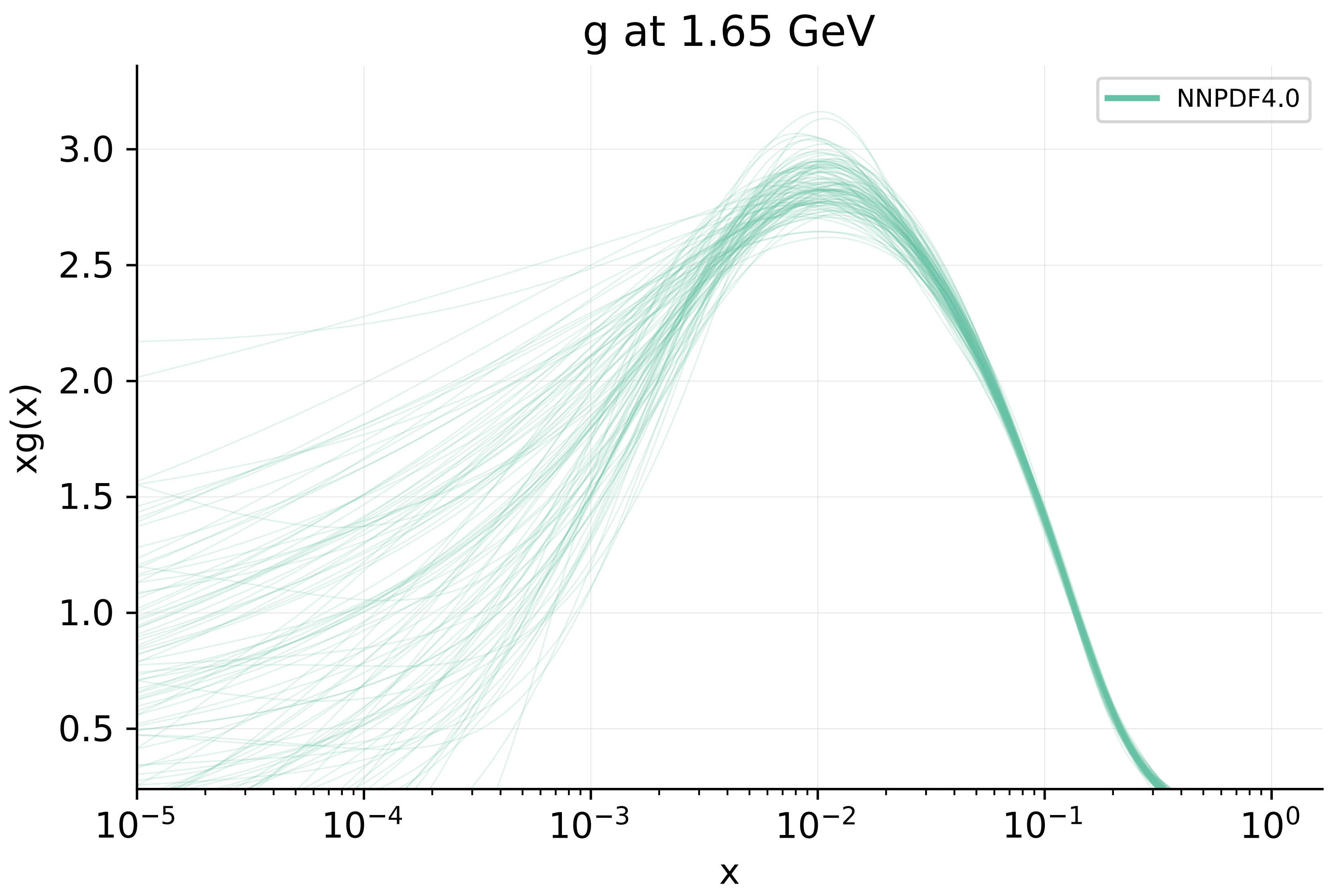
Backup slides

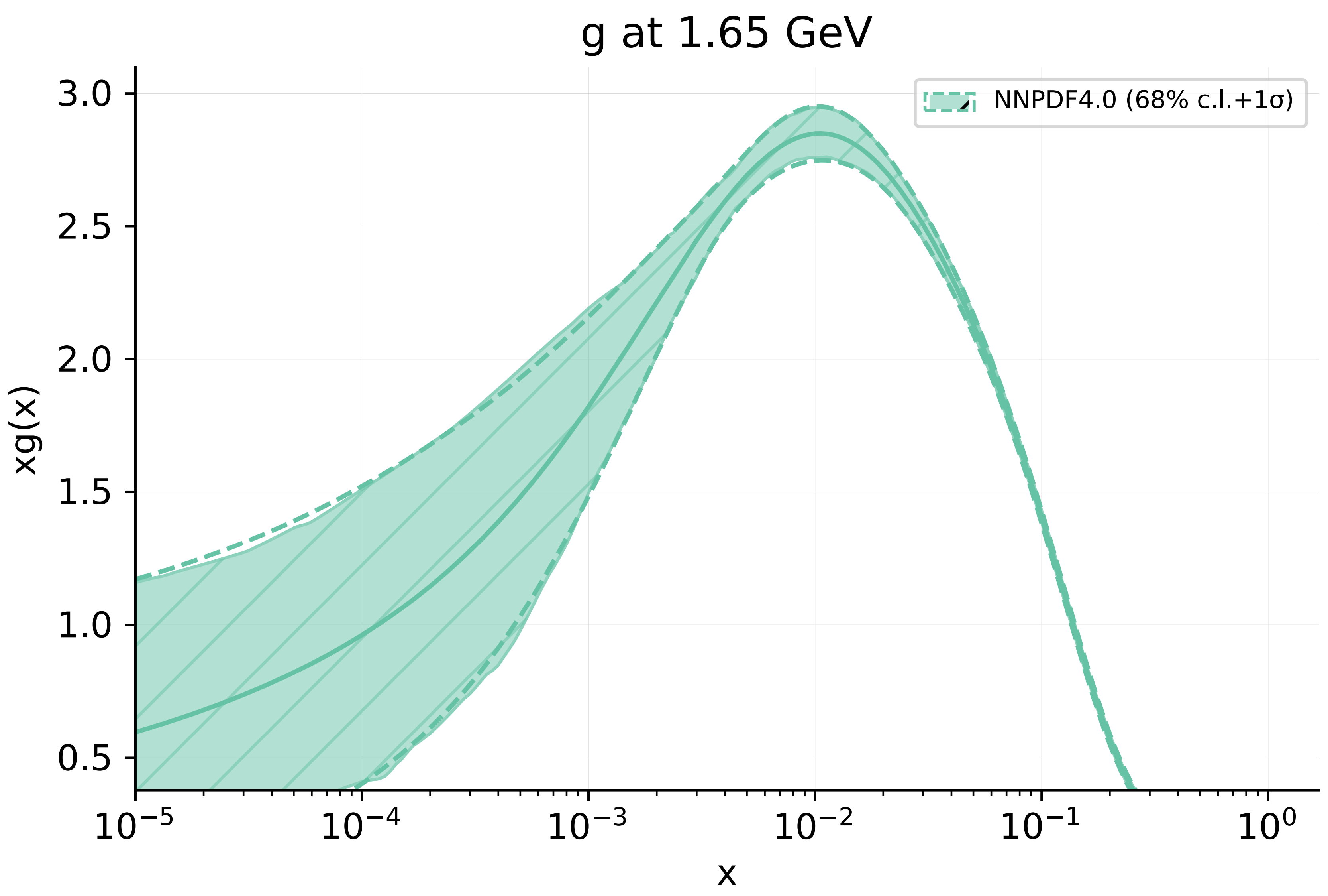
Propagating experimental uncertainty to PDFs

An NNPDF set (usually) consists of 100 PDF replicas produced as follows:

1. Assume experimental data is **defined** by a vector of central values and a covariance matrix
2. Sample this distribution to create 100 Monte Carlo replicas in data space
3. Perform a fit to each of the data replicas

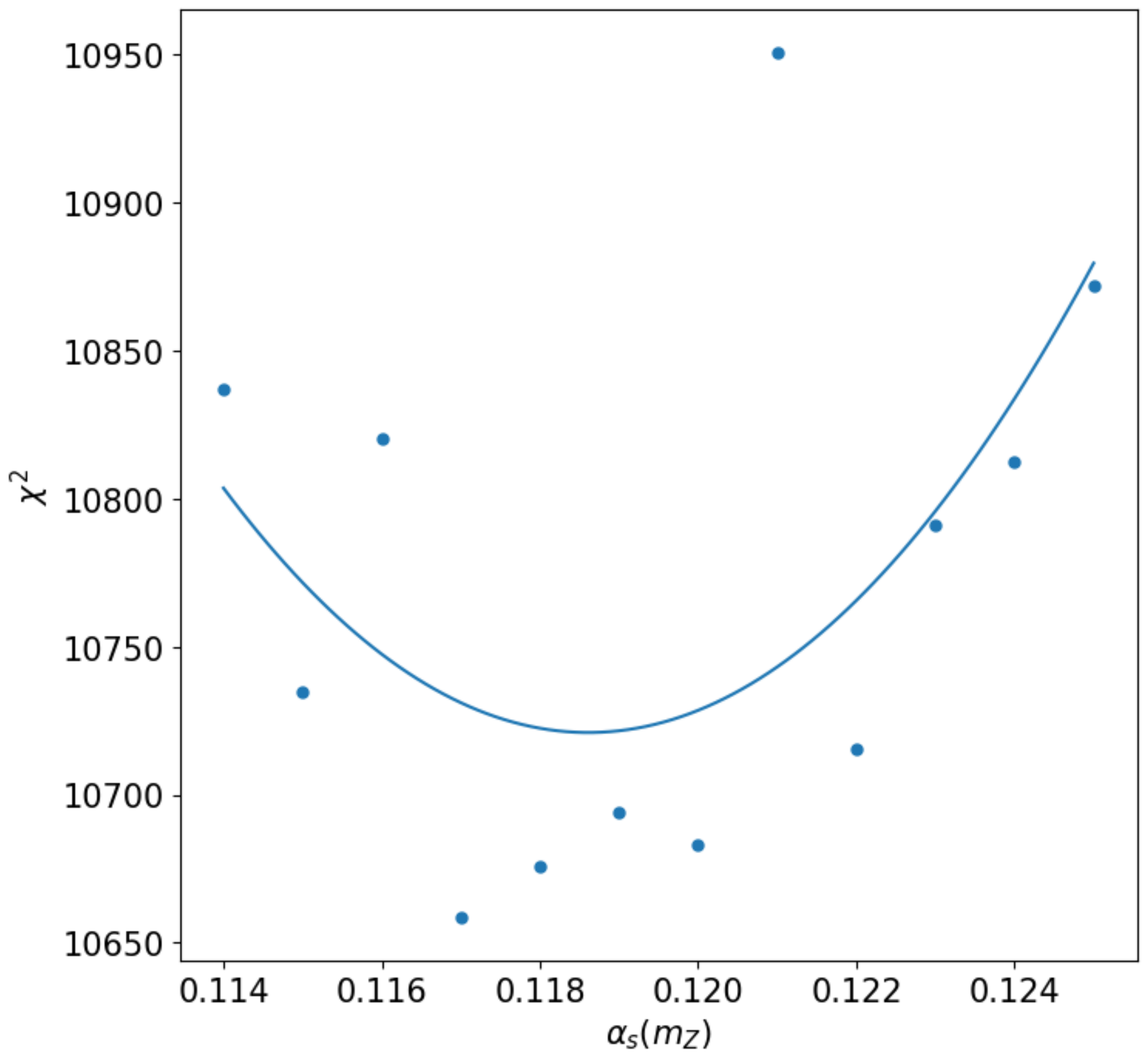
➡A PDF set encoding experimental uncertainties





Simultaneous minimization of PDF and α_s

Correlated replicas method



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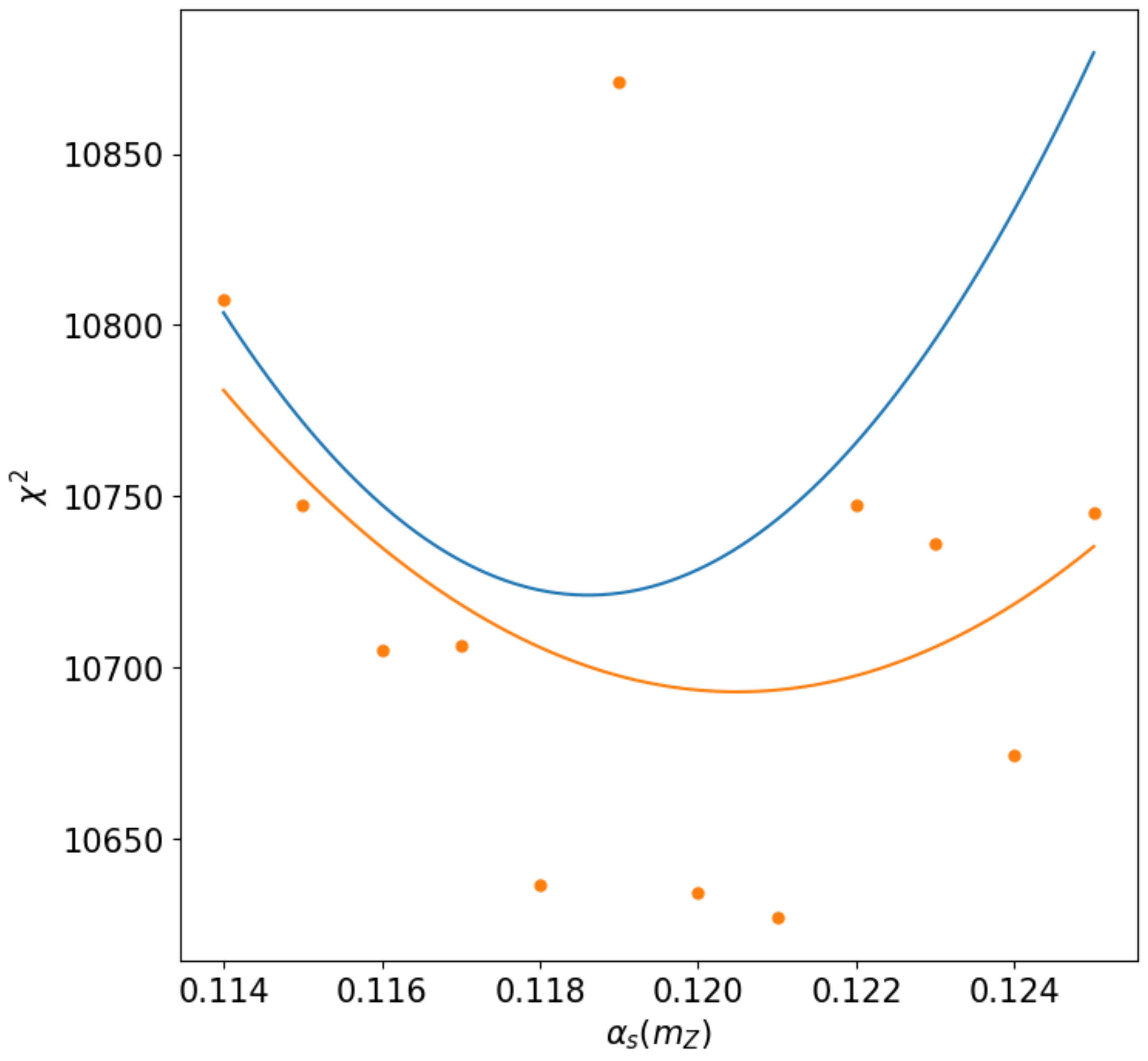

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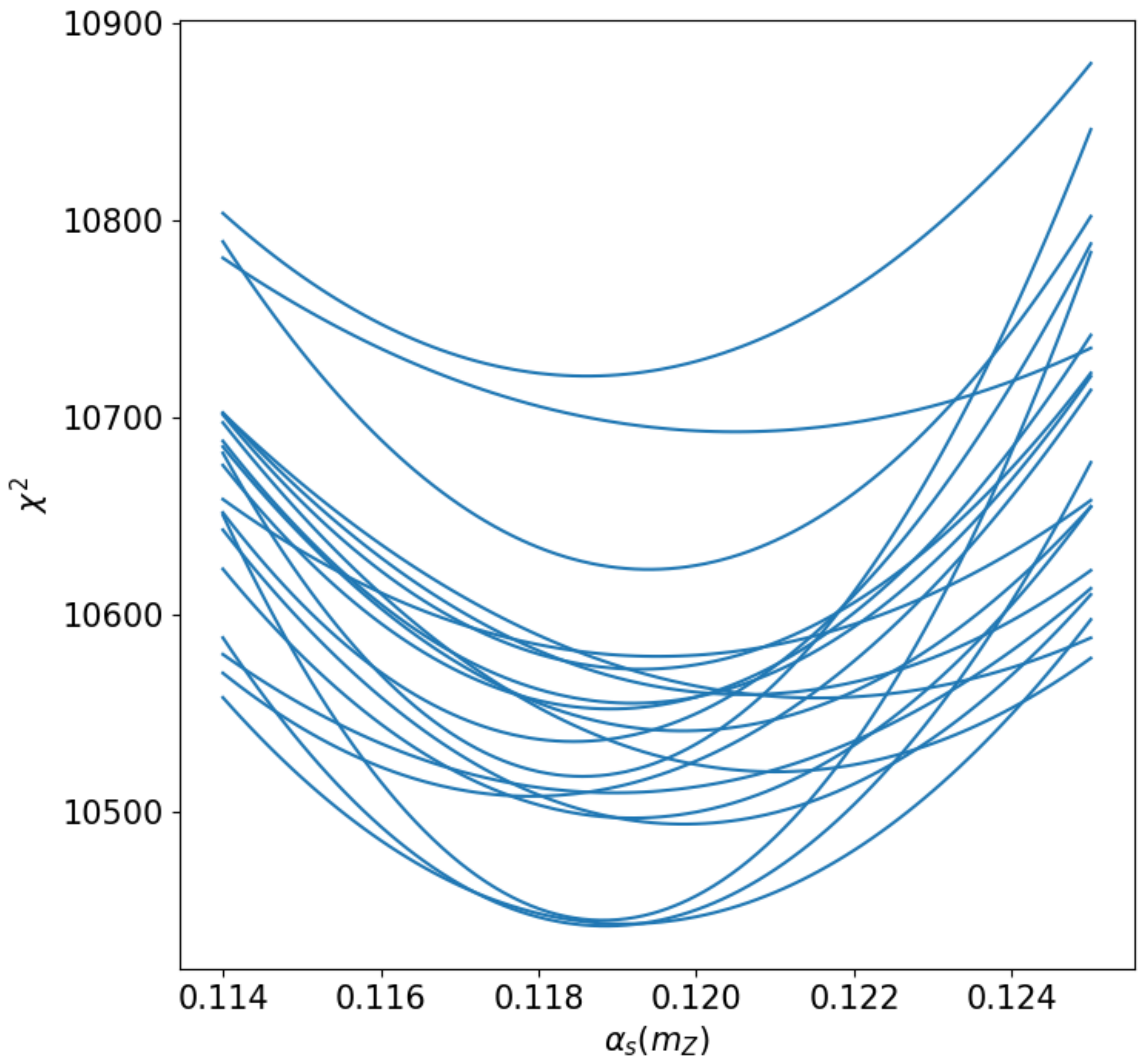
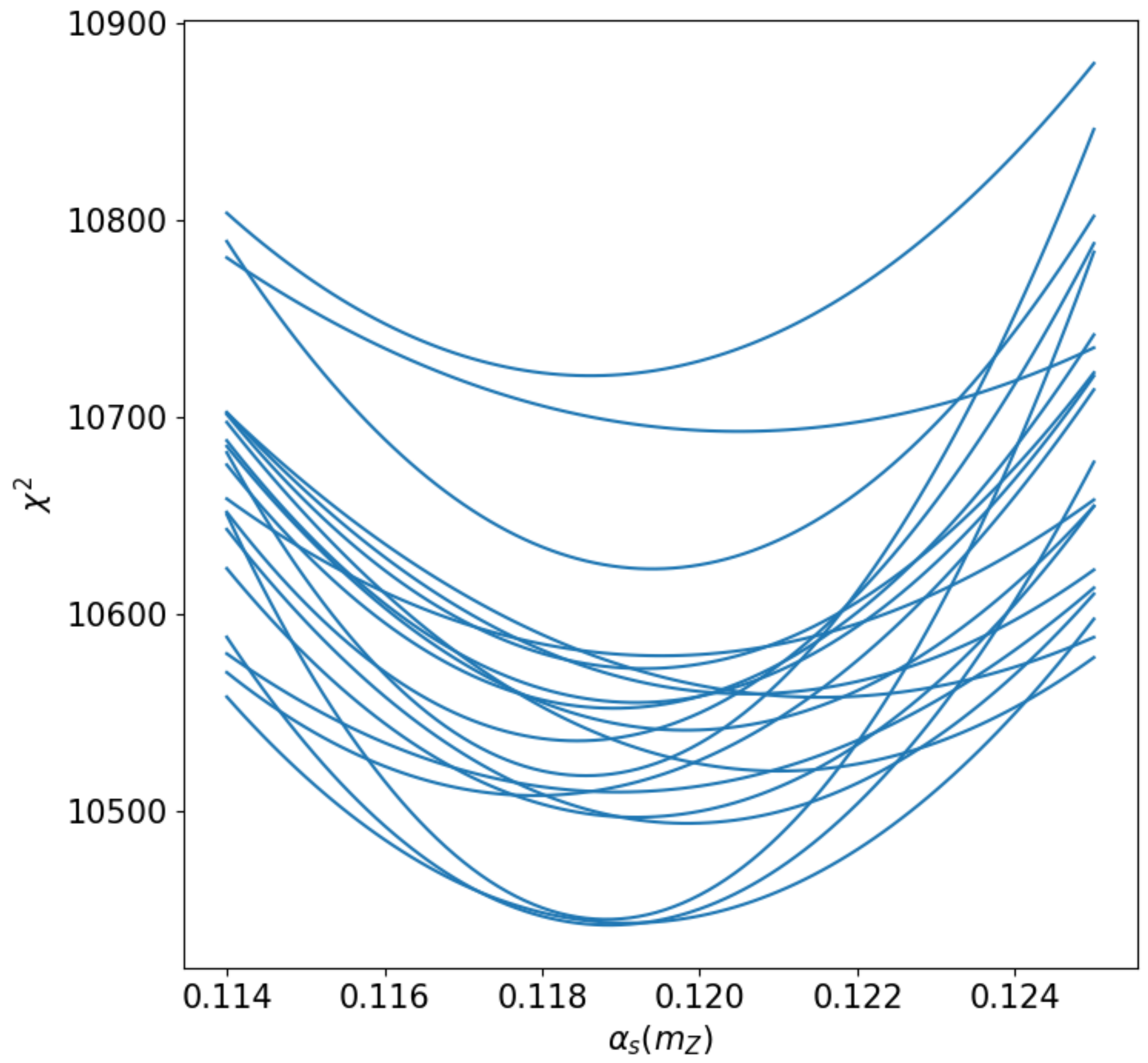


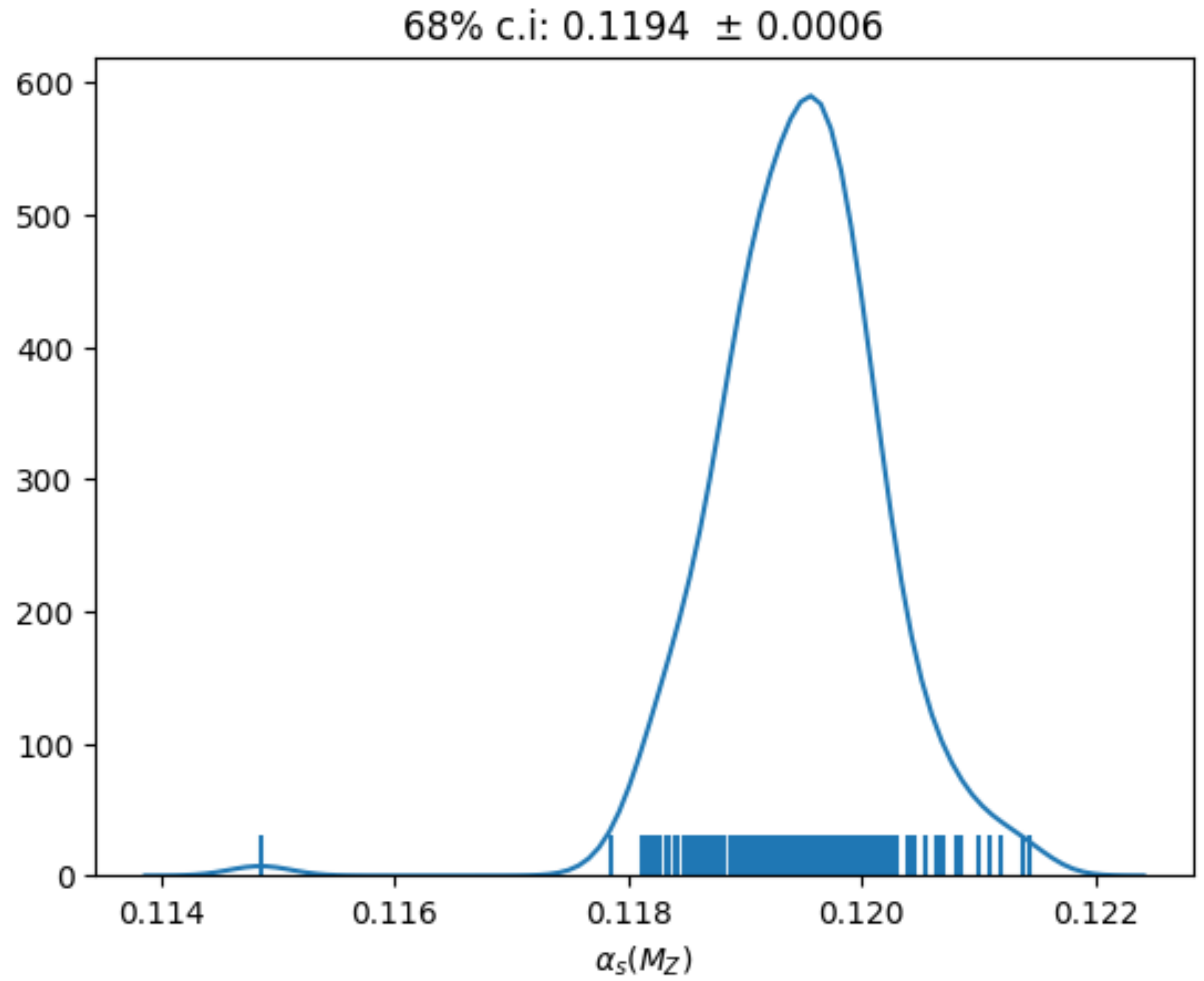
Fit the same data replica at different values of α_s and
fit a parabola for each replica …

Simultaneous minimization of PDF and α_s

Correlated replicas method



Fit the same data replica at different values of α_s and fit a parabola for each replica …



… then look at the distribution of minima of the parabolas

α_s from correlated theory uncertainties

Theory Covariance Method [arXiv:2105.05114]

The "correlated replicas" method is computationally costly because it involves fitting PDFs at many values of α_s

Alternatively, α_s can be determined in a **Bayesian framework** from nuisance parameters:

1. Model the theory uncertainty as a shift correlated for all datapoints
   $T \to T + \lambda \cdot \beta$, for $\beta \equiv T(\alpha_s^+) - T(\alpha_s^-)$
   $$P(T \mid D, \lambda) \propto \exp(\chi^2) = \exp\left(-\frac{1}{2}(T + \lambda \cdot \beta - D)^T C^{-1} (T + \lambda \cdot \beta - D)\right)$$
2. Choose a prior
   $$P(\Delta\alpha_s) \propto \exp\left(-\frac{1}{2}\lambda^2\right)$$
3. Marginalise over λ to get $P\left(T|D\right)$
4. Compute the posterior for λ
   $$P(\lambda \mid T, D) = \frac{P(T \mid D, \lambda) P(\lambda)}{P(T \mid D)} \propto \exp\left[-\frac{1}{2} Z^{-1} (\lambda - \bar{\lambda})\right]$$

$$Z = 1 - \beta^T (C + \beta\beta^T)^{-1} \beta \qquad \bar{\lambda}(T, D) = \beta^T (C + \beta\beta^T)^{-1} (D - T)$$

This idea can be extended to a real PDF fit [arXiv:2105.05114]

1) Perform fit with $C^{exp} \to C^{exp} + C^{\alpha_s}$, $C^{\alpha_s} = \beta\beta^T$
2) Once the fit has completed, compute α_s shift preferred by data as encoded in the fit

Prior dependence in the Theory Covariance Method

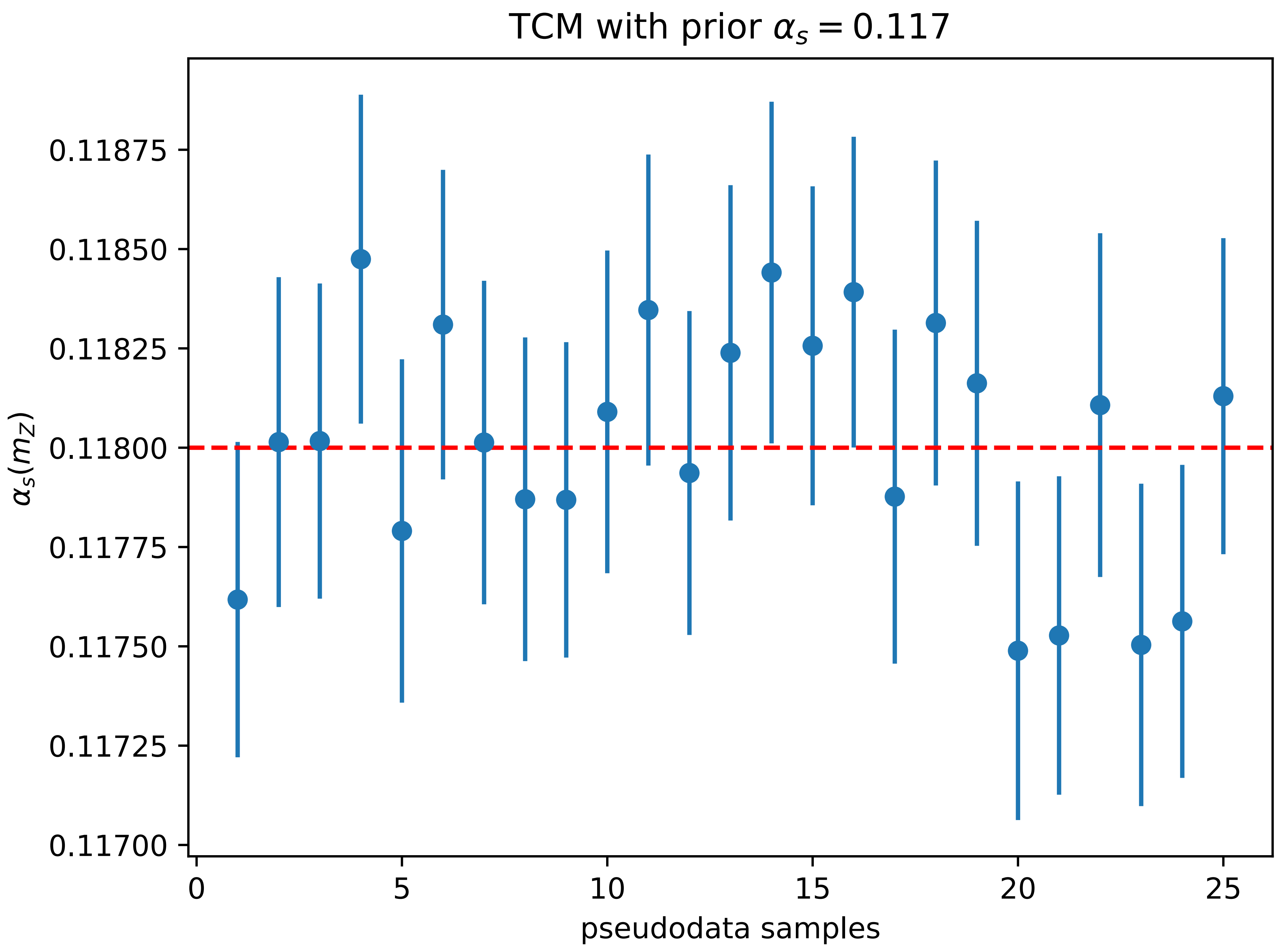
For some aspects of the fit we have to assume a value of $\alpha_s(m_Z)$, in reality we don't know the result so what if we choose "wrong"?

Consider the following

Pseudodata at $\alpha_s(m_Z) = 0.118$

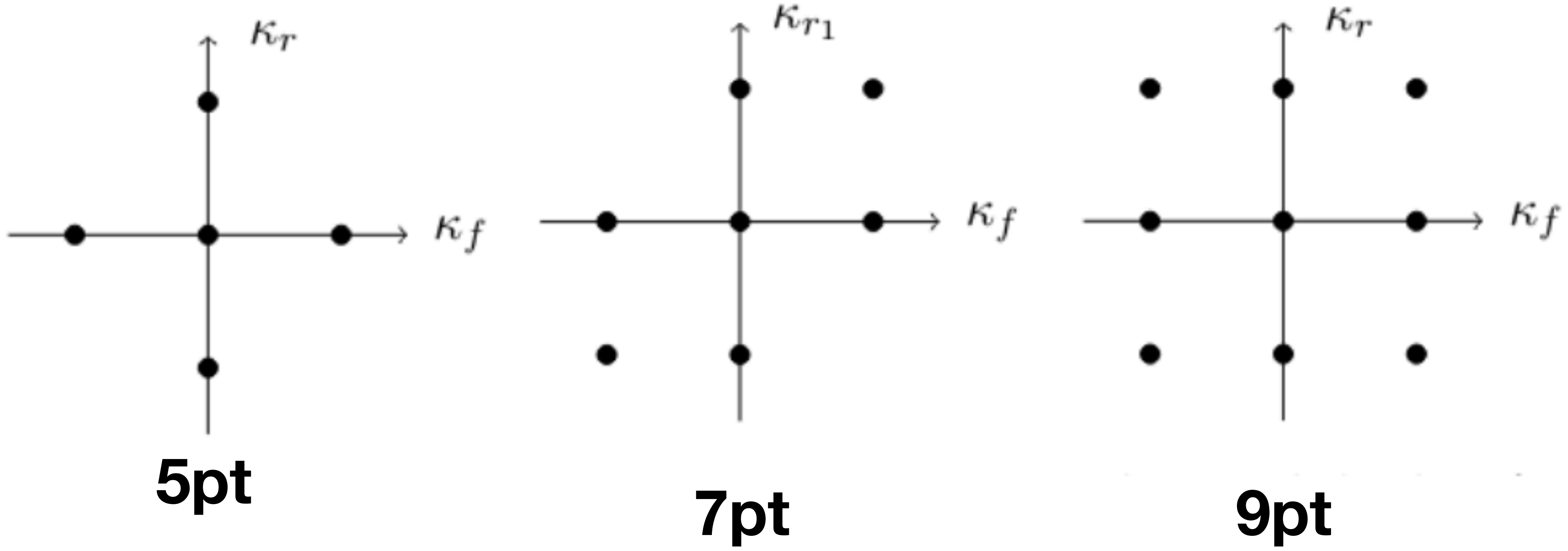
Prior assumption is $\alpha_s(M_Z) = 0.117$

Result moves towards the true result. We update assumption and iterate!



Theory uncertainties in PDFs

Missing higher order uncertainties (MHOUs) are estimated through 7 point scale variations



- In a fit we minimize the χ^2:

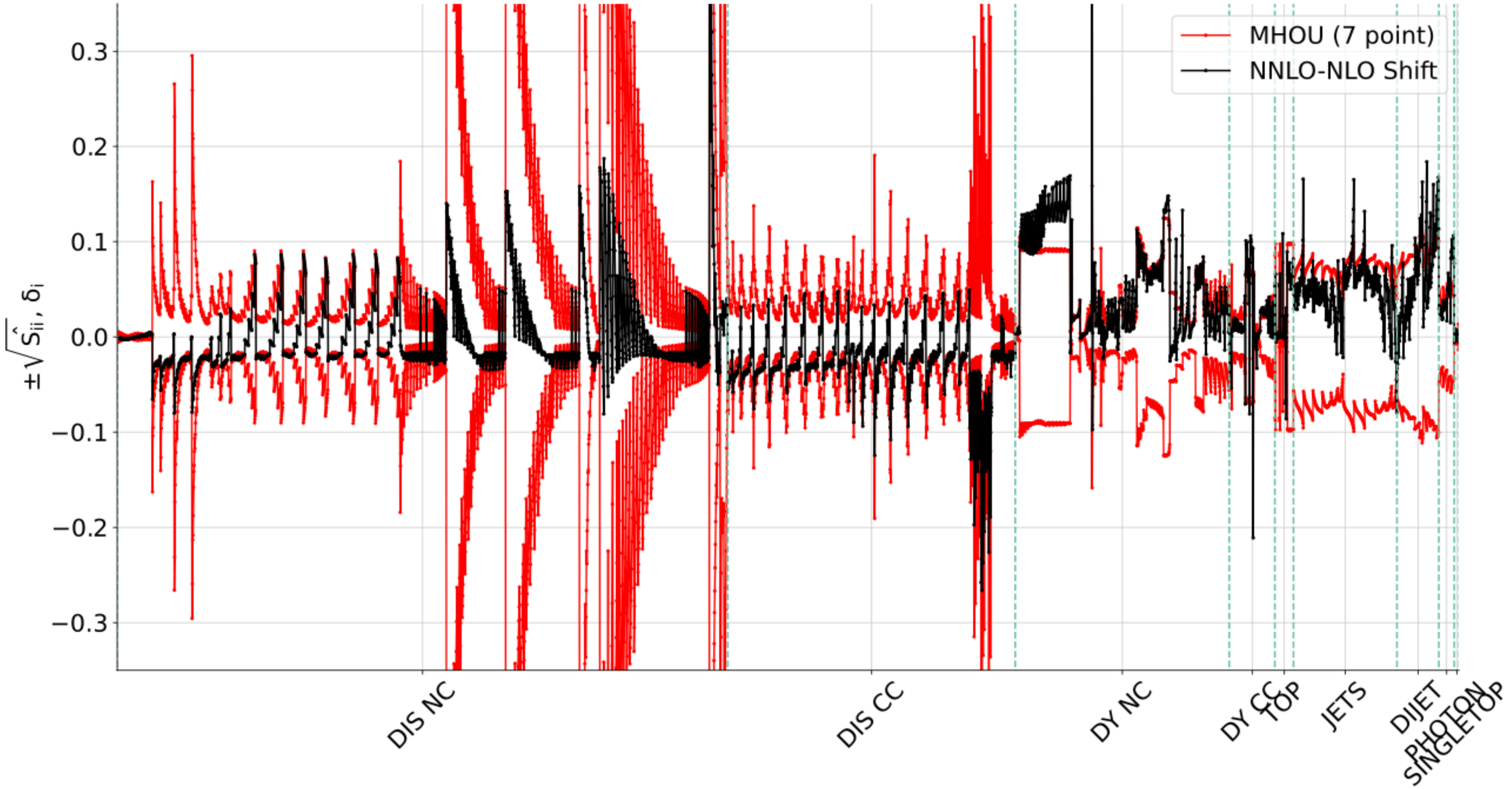
$$P(T \mid D\lambda) \propto \exp\left(-\frac{1}{2}(T-D)^T C^{-1}(T-D)\right) \equiv \exp\left(\chi^2\right)$$

- To account for MHOUs we treat the theory covmat on the same footing as the experimental covmat: $C = C_{\mathrm{exp}} + C_{\mathrm{MHOU}}$

$$C_{\mathrm{MHOU},ij} = n_m \frac{1}{V_m} \sum \left(T_i(\kappa_f, \kappa_r) - T_i(0,0)\right)\left(T_j(\kappa_f, \kappa_r) - T_j(0,0)\right)$$

Validating the MHOU covmat

The MHOU covmat is validated by comparing the **shifts from scale variations at NLO** to the known **NNLO-NLO shifts**



PDFs at approximate N^{3}LO

DGLAP evolution from EKO: github.com/NNPDF/eko

DIS coefficients from Yadism: github.com/NNPDF/yadism

A PDF fit requires several theory inputs:

- **DGLAP splitting functions**
  small-x and large-x limits
  Mellin moments
- **Matching conditions for variable flavor number schemes**
  Now exactly known but original aN3LO publications use approximations
- **DIS coefficient functions**
  Massless known, massive limits known
- **Hadronic cross-section**
  Not much is known

Strategy:

- When N^{3}LO theory is known, it is **used**
- When partial information is available, use it while accounting for **parametrisation uncertainty**
- When it is unknown account for **missing higher order uncertainty**

N^{3}LO QCD corrections in PDF determination

Splitting Functions (information is partial)

Singlet (P_{qq}, P_{gg}, P_{gq}, P_{qg})
- large-n_f limit [NPB 915 (2017) 335; arXiv:2308.07958]
- small-x limit [JHEP 06 (2018) 145]
- large-x limit [NPB 832 (2010) 152; JHEP 04 (2020) 018; JHEP 09 (2022) 155]
- 5 (10) lowest Mellin moments [PLB 825 (2022) 136853; ibid. 842 (2023) 137944; ibid. 846 (2023) 138215]

Non-singlet ($P_{NS,v}$, $P_{NS,+}$, $P_{NS,-}$)
- large-n_f limit [NPB 915 (2017) 335; arXiv:2308.07958]
- small-x limit [JHEP 08 (2022) 135]
- large-x limit [JHEP 10 (2017) 041]
- 8 lowest Mellin moments [JHEP 06 (2018) 073]

DIS structure functions (F_L, F_2, F_3)
- DIS NC (massless) [NPB 492 (1997) 338; PLB 606 (2005) 123; NPB 724 (2005) 3]
- DIS CC (massless) [Nucl.Phys.B 813 (2009) 220]
- massive from parametrisation combining known limits and damping functions [NPB 864 (2012) 399]

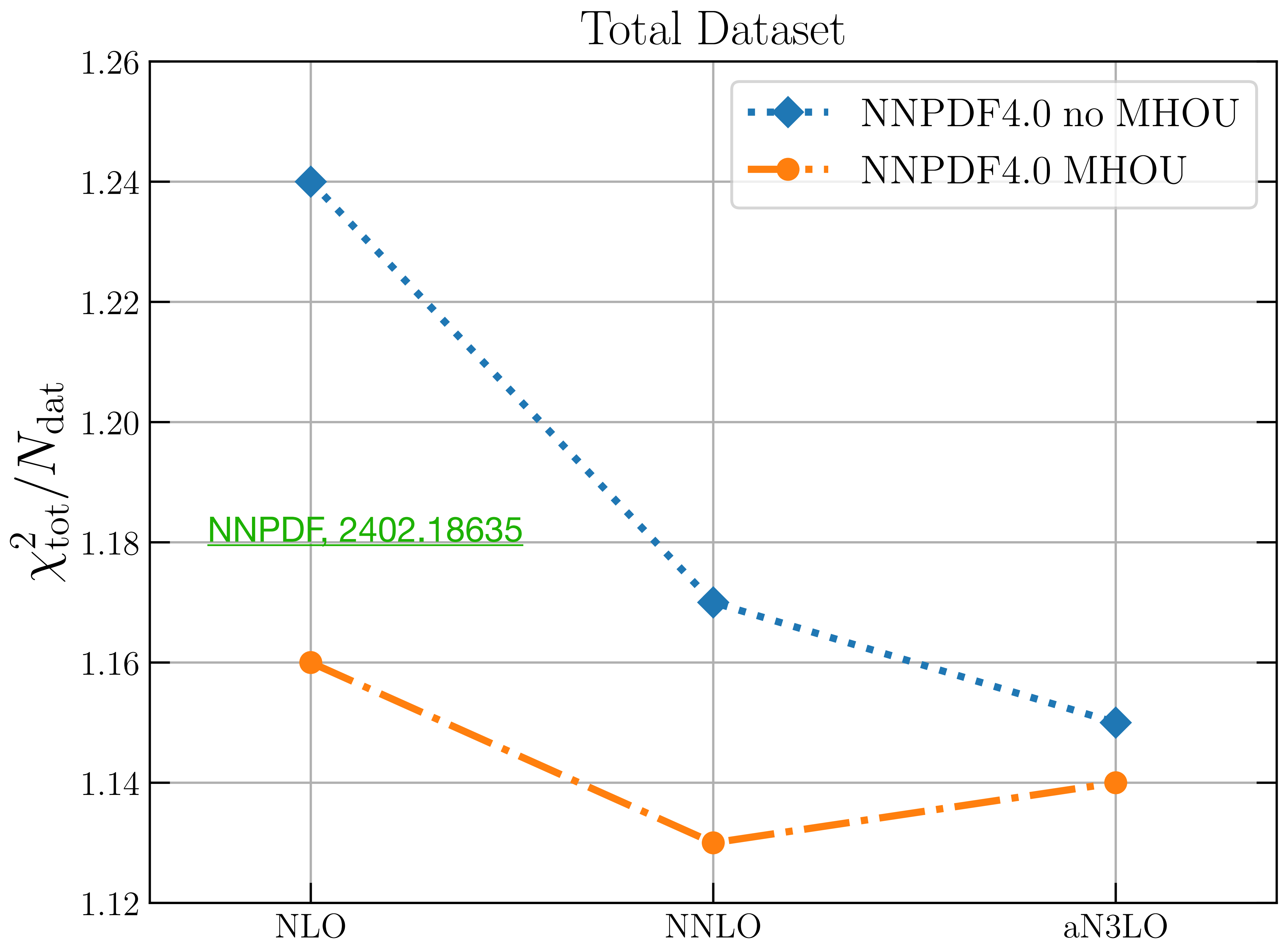
PDF matching conditions
- all known except for $a^3_{H,g}$ [NPB 820 (2009) 417; NPB 886 (2014) 733; JHEP 12 (2022) 134]

Coefficient functions for other processes
- DY (inclusive) [JHEP 11 (2020) 143]; DY (y differential) [PRL 128 (2022) 052001]

Emanuele R. Nocera (UNITO) | Progress from NNPDF | 5 August 2024 | 12 / 25

E. Nocera, Workshop on Hadron Physics and Opportunities Worldwide
Dalian, China, August 2024
(More is known today!)

Fit quality



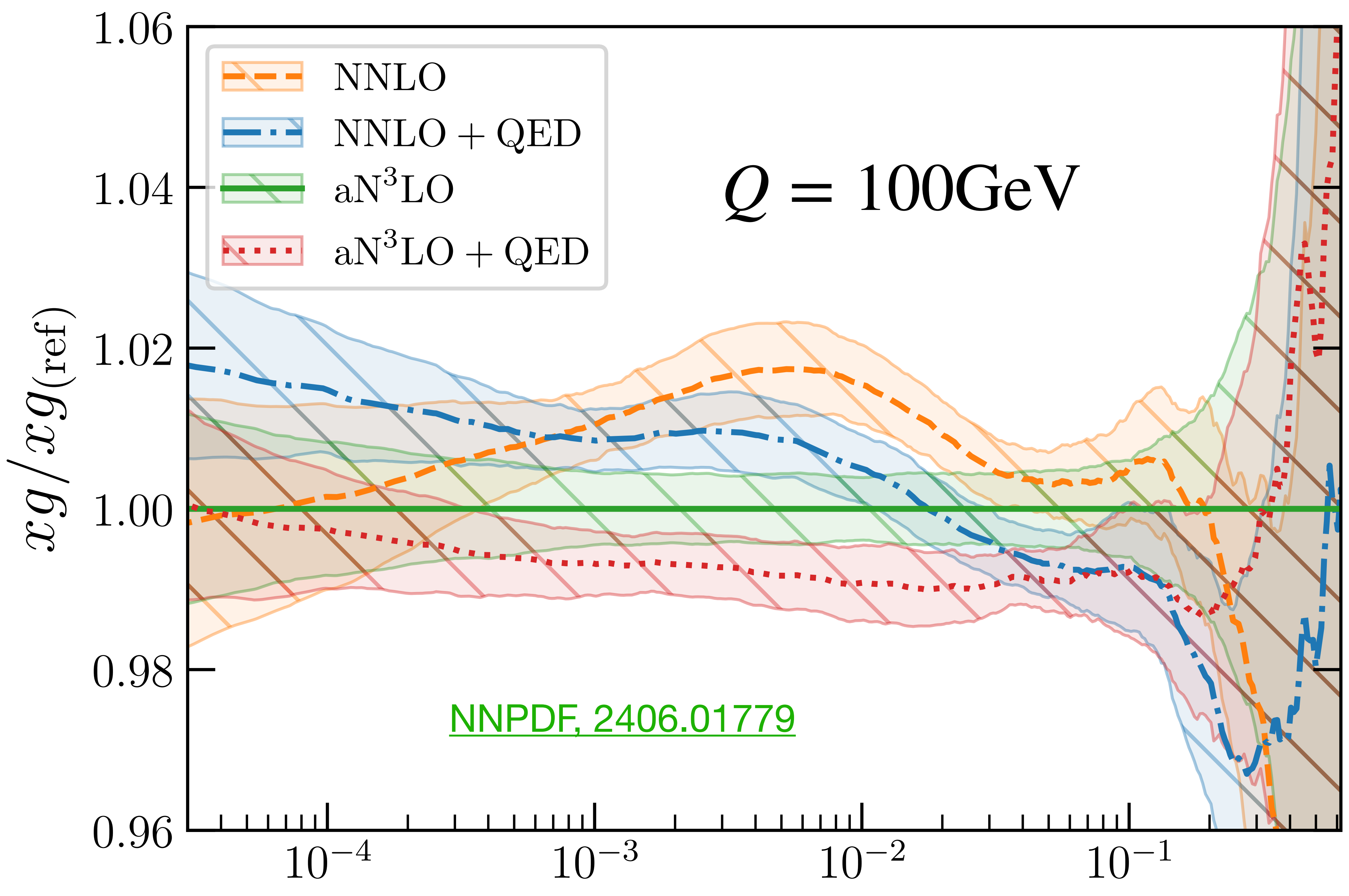
- Without MHOUs the fit improves (lower χ^2) with increasing perturbative order for both NNPDF and MSHT
- With MHOUs the fit depends only weakly on the perturbative order
- At N^{3}LO MHOUs have a small impact on the χ^2

QED corrections and photon PDF

NNPDF4.0QED means:

- NLO QED corrections
  $P = P_{QCD} + P_{QCD \otimes QED}$
  $P_{QCD \otimes QED} = \alpha_{em} P^{(0,1)} + \alpha_{em}\alpha_s P^{(1,1)} + \alpha_{em}^2 P^{(0,2)}$
- Photon PDF

PDFs at $\mathrm{aN^3LO_{QCD}} \otimes \mathrm{NLO_{QED}}$ with photon PDF represents the **most accurate PDFs**



- Photon subtracts momentum from the gluon PDF
- QED effect similar in magnitude to aN3LO corrections