中国科学技术大学
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Capping the cone of positivity bounds: dim-8 Higgs operators in SMEFT

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Joint T06+T08 Session, EPS-HEP 2025, 9 July 2025

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Based: earlier work + 2309.15922 + 2404.04479

Introduction: Positivity bounds

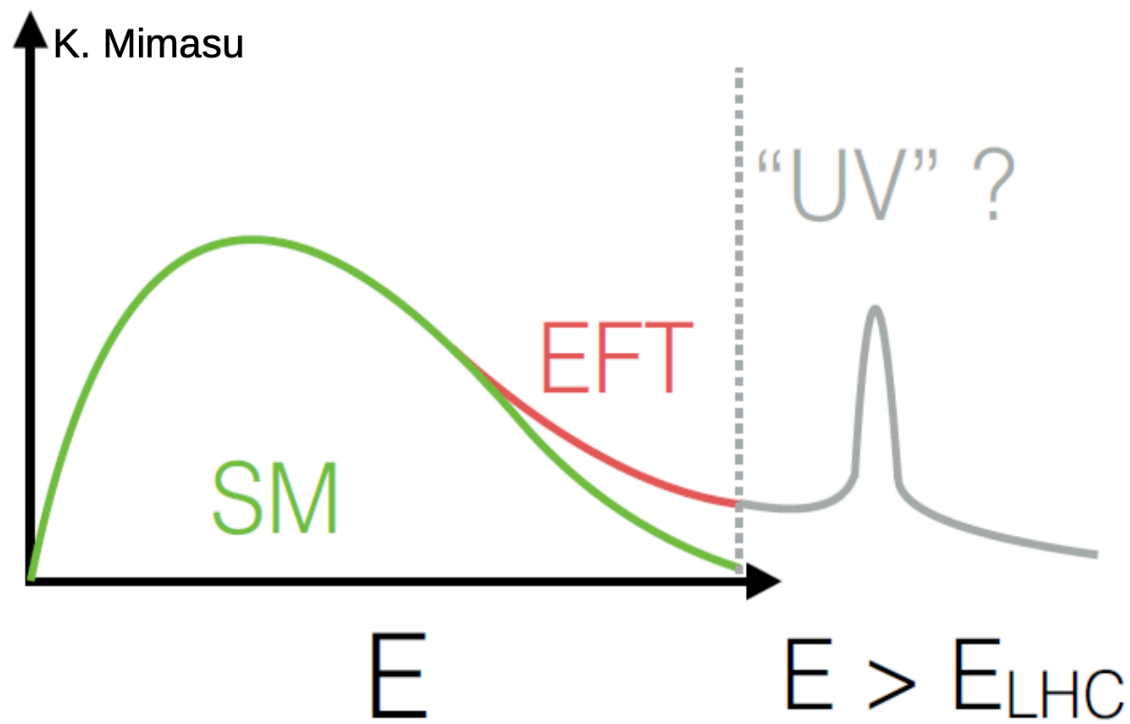
EFT approach to BSM

SM Effective Field Theory

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} + \sum_j \frac{c_j^{(6)} O_j^{(6)}}{\Lambda^2} + \sum_i \frac{c_i^{(8)} O_i^{(8)}}{\Lambda^4} + \cdots$$

$c_i^{(n)}$: Wilson coefficients

- ✦ SM particle contents
- ✦ SM symmetries



- Systematically parametrize new physics
- **Drawback:** Too many Wilson coefficients

- dim-6: $O(10^2)$ terms
- dim-8: $O(10^3)$ terms

Even if consider up to dim-8, or order s^2

Many operators! Huge parameter space!

“Anything goes” for EFT coefficients?

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \sum_j \frac{c_j^{(6)} O_j^{(6)}}{\Lambda^2} + \sum_i \frac{c_i^{(8)} O_i^{(8)}}{\Lambda^4} + \cdots$$

Question: Are Wilson coefficients $c_i^{(n)}$ allowed to take any values, from the purely **theoretical** point of view?

Short answer: **No!**

- Perturbative unitarity bounds: have been widely used
- **Positivity bounds:** new theoretical bounds

Zhiboedov’s talk

Simpest example: $\mathscr{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\lambda}{\Lambda^4}(\partial_\mu\phi\partial^\mu\phi)^2 + \cdots$

“First” positivity bound: $\lambda > 0$

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 2006

Positivity bounds

high energy UV theory

maybe unknown, but assume causality, unitarity, ...

Positivity bounds
(Causality bounds)

a bootstrap approach

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, hep-th/0602178
de Rham, Melville, Tolley & **SYZ**,1702.06134,1706.02712
Arkani-Hamed, Huang & Huang, 2012.15849
Zhang & **SYZ**, 2005.03047
Tolley, Wang & **SYZ**, 2011.02400
Caron-Huot & Duong, 2011.02957
Sinha & Zahed, 2012.04877
Chiang, Huang, Li, Rodina & Weng, 2105.02862
Caron-Huot, Mazac, Rastelli, Simmons-Duffin, 2102.08951
......

low energy EFT
constraints on Wilson coefficients

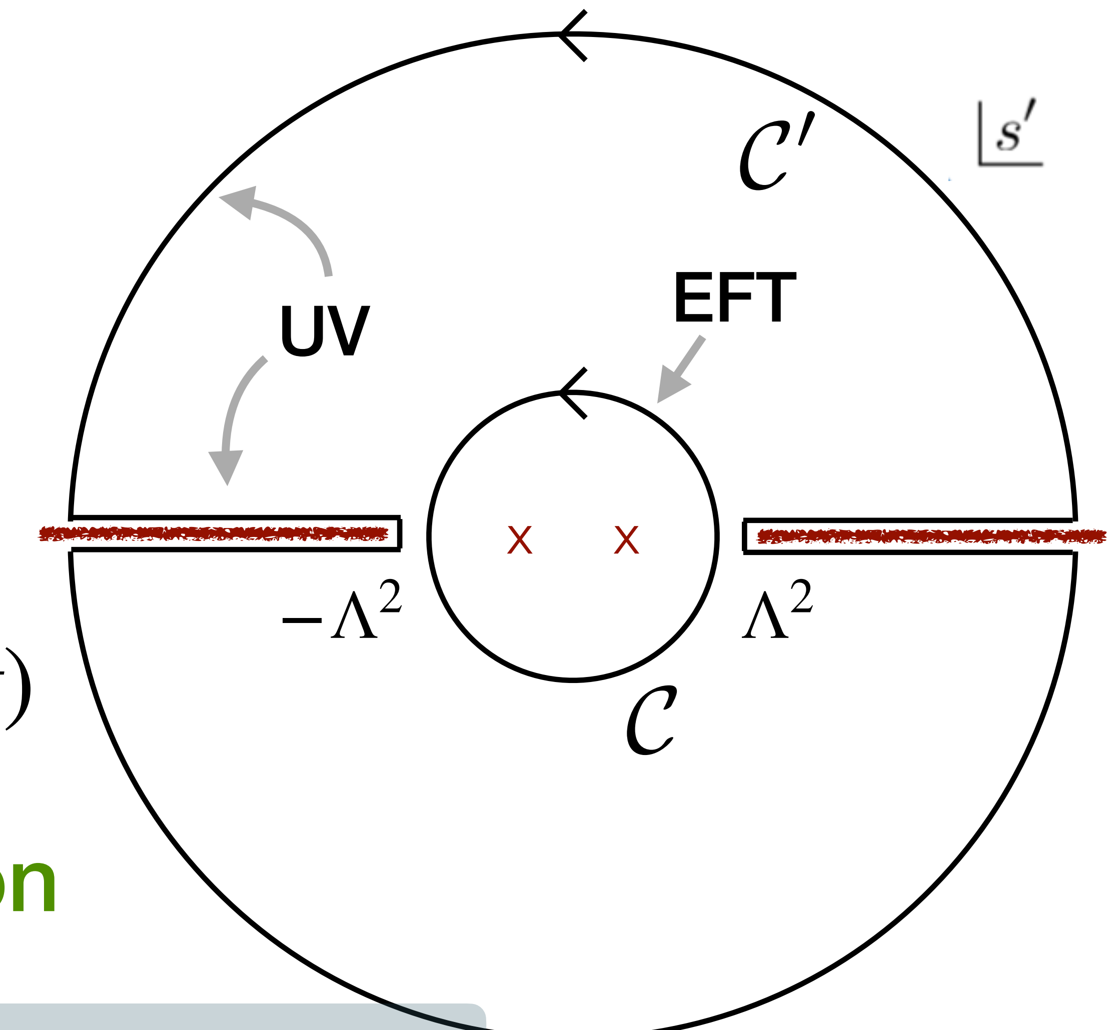
Snowmass White Paper: UV Constraints on IR Physics, de Rham, Kundu, Reece, Tolley & **SYZ**, 2203.06805

Main tool: Dispersion relation

- Analyticity in complex s plane (fixed t)

$$A(s,t) = \frac{1}{2\pi i} \oint_{\mathcal{C}} ds' \, \frac{A(s',t)}{s'-s}$$

- Froissart bound $|A(s' \to \infty, t)| < s'^{2-\epsilon}$
- su crossing symmetry $A(s,t) = A(u,t)$



Twice subtracted dispersion relation

$$A(s,t) \sim \int_{\Lambda^2}^{\infty} \frac{\mathrm{d}\mu}{\pi \mu^2} \left[\frac{s^2}{\mu - s} + \frac{u^2}{\mu - u} \right] \mathrm{Im}\, A(\mu, t)$$

μ: scale of UV particles

EFT amplitude **IR/UV connection** **UV full amplitude**

Sum rules

$$\sum_{i,j} c_{i,j} s^i t^j = A(s,t) \sim \int_{\Lambda^2}^{\infty} \frac{\mathrm{d}\mu}{\pi\mu^2}\left[\frac{s^2}{\mu - s} + \frac{u^2}{\mu - u}\right] \mathrm{Im}\, A(\mu, t)$$

$c_i^{(n)} \sim c_{i,j}$: Wilson coefficients

partial wave expansion:

$$A(s,t) \sim \Sigma_\ell P_\ell(1 + 2t/s) a_\ell(s)$$

- Direct sum rules:

$$c_{i,j} \sim \sum_\ell \int d\tilde{\mu} \frac{D_{i,j}(\eta)}{\mu^{i+j}}$$

$$d\tilde{\mu} \equiv d\mu\, \mathrm{Im} a_\ell(s)$$

$$\eta \equiv \ell(\ell + 1)$$

$D_{i,j}$ is polynomial of η

- Null constraints: $A(u,t) = A(s,t) = A(t,s)$

Tolley, Wang & **SYZ**, 2011.02400
Caron-Huot & Duong, 2011.02957

$$\sum_\ell \int d\tilde{\mu} \frac{\Gamma_{i,j}^{(n)}(\eta)}{\mu^{i+j}} = 0$$

st crossing imposes constraints on $\mathrm{Im} a_\ell$

Dimensional analysis on firm footing

Two-sided bounds:

Constrain wilson coefficients from **both above and below**

$$A(s,t) \sim c_{2,0}s^2 + c_{2,1}s^2 t + c_{2,2}s^2 t^2 + \cdots$$

Wilson coefficients have to be parametrically $O(1)$!

weakly coupled in the IR

Tolley, Wang & **SYZ**, 2011.02400; Caron-Huot & Duong, 2011.02957

Used to be a folklore, called "naturalness/dimensional analysis" but now a rigorous QFT result

Positivity bounds on s^2 coefficients in SMEFT

- lowest order positivity bounds — dim-8 ops
- phenomenologically more relevant

$$A(s,t=0)=\frac{c^{2,0}s^2}{\Lambda^4}+\cdots \quad \Longrightarrow \quad A_{ij\to kl}(s,t=0)=\frac{c^{2,0}_{ij\to kl}s^2}{\Lambda^4}+\cdots$$

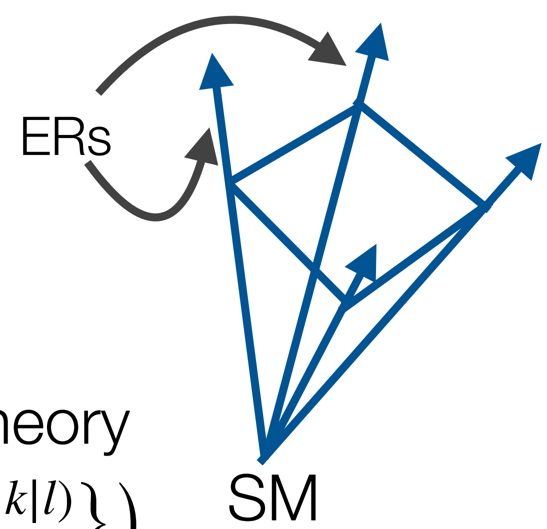
Positive cone for multiple fields

Imposing UV unitarity $\mathrm{Im}A_{ij\to kl}(s>\Lambda^2,0)\geq 0$

$$A_{ij\to kl}(s,0)=\frac{c^{2,0}_{ij\to kl}s^2}{\Lambda^4}+\cdots$$

Positivity bounds: $c^{2,0}_{ij\to kl}$ form (high dimensional) convex cone

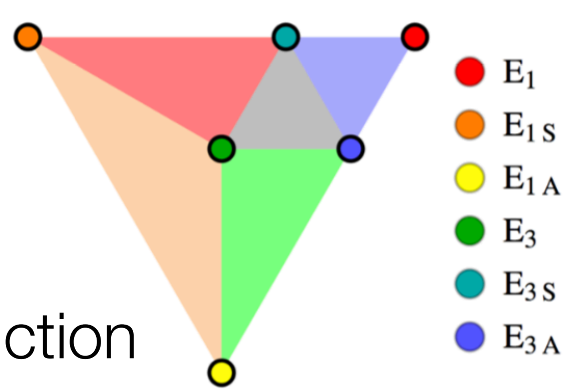
Polyhedral cone



- many symmetries
- finite discrete ERs
- obtained by group theory

$$\mathscr{C}=\mathrm{cone}\left(\left\{P_r^{i(j|k|l)}\right\}\right)$$

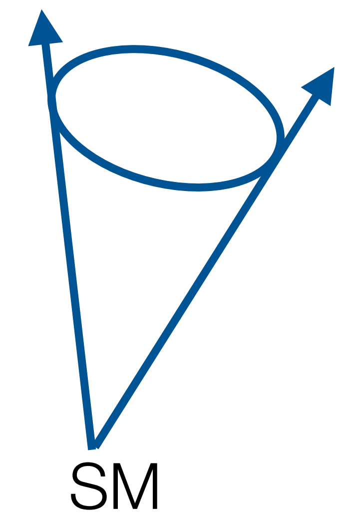
ER ↔ UV states



- importance of dim-8
- useful for UV reconstruction

Zhang & **SYZ**, PRL, 2005.03047

Curly cone



- less symmetries
- infinitely many ERs
- obtained numerically by semi-definite programming

- switch to dual amplitude cone

$$C^*=\{y\,|\,y\cdot x>0,\ x\in C\}$$

- numerically efficient to get optimal bounds

Li, Xu, Yang, Zhang & **SYZ**, PRL, 2101.01191

Most SMEFT space is redundant!

- Transversal Vector Boson Scattering

positivity cone is only
0.681% of 10D sphere

Yamashita, Zhang & **SYZ**, 2009.04490

$$O_{T,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}]$$
$$O_{T,2} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}]$$
$$O_{T,5} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta}$$
$$O_{T,7} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\hat{B}_{\beta\nu}\hat{B}^{\nu\alpha}$$
$$O_{T,8} = \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta}$$

$$O_{T,1} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}]$$
$$O_{T,10} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\tilde{W}^{\alpha\beta}]$$
$$O_{T,6} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\hat{B}_{\mu\beta}\hat{B}^{\alpha\nu}$$
$$O_{T,11} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\hat{B}_{\alpha\beta}\tilde{B}^{\alpha\beta}$$
$$O_{T,9} = \hat{B}_{\alpha\mu}\hat{B}^{\mu\beta}\hat{B}_{\beta\nu}\hat{B}^{\nu\alpha}$$

- 4-gluon SMEFT operators

7D parameter space: 1.6628%

Li, Xu, Yang, Zhang & **SYZ**, 2101.01191

$Q_{G^4}^{(1)}$	$(G^A_{\mu\nu}G^{A\mu\nu})(G^B_{\rho\sigma}G^{B\rho\sigma})$
$Q_{G^4}^{(2)}$	$(G^A_{\mu\nu}\widetilde{G}^{A\mu\nu})(G^B_{\rho\sigma}\widetilde{G}^{B\rho\sigma})$
$Q_{G^4}^{(3)}$	$(G^A_{\mu\nu}G^{B\mu\nu})(G^A_{\rho\sigma}G^{B\rho\sigma})$
$Q_{G^4}^{(4)}$	$(G^A_{\mu\nu}\widetilde{G}^{B\mu\nu})(G^A_{\rho\sigma}\widetilde{G}^{B\rho\sigma})$
$Q_{G^4}^{(7)}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}G^{B\mu\nu})(G^C_{\rho\sigma}G^{D\rho\sigma})$
$Q_{G^4}^{(8)}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}\widetilde{G}^{B\mu\nu})(G^C_{\rho\sigma}\widetilde{G}^{D\rho\sigma})$
Q_G	$f^{ABC}G^{A\nu}_{\mu}G^{B\rho}_{\nu}G^{C\mu}_{\rho}$

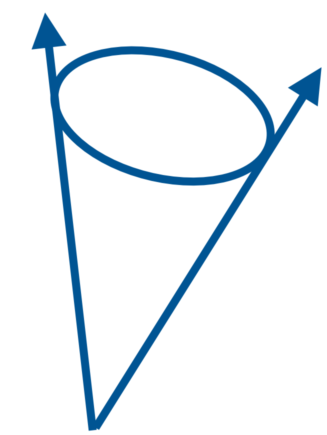
Positivity and beyond

Positivity cone

imposing $\mathrm{Im} A_{ij\to kl}(s > \Lambda^2, 0) \geq 0$ in UV

equivalently, partial waves

$$\mathrm{Im}\, a_\ell^{ijkl} \succeq 0$$



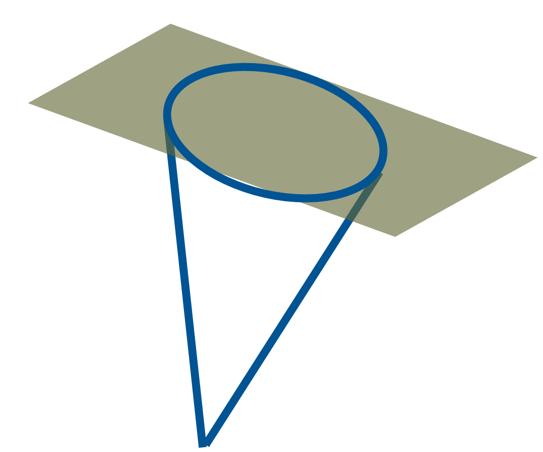
positivity part of unitarity

Capping the positivity cone

- partial wave unitarity

  more info from $\mathrm{Im}\, a_\ell^{ijkl} = \sum_X a_\ell^{ij\to X} \left(a_\ell^{kl\to X}\right)^*$

- null constraints



fuller use of unitarity

Higgs positivity cone

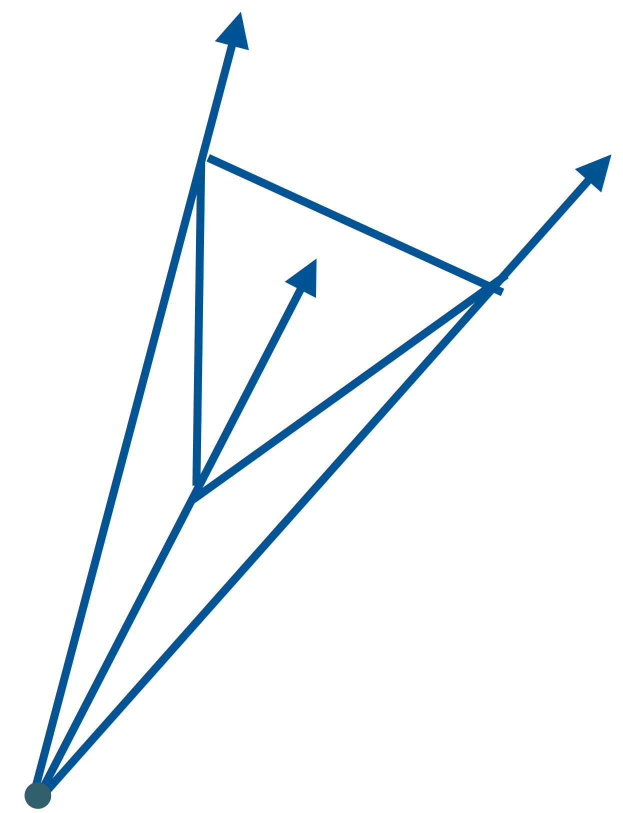
Consider Higgs scattering

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_{\mathrm{SMEFT}} \supset \quad & C_1 \left(D_\mu H^\dagger D_\nu H\right) \left(D^\nu H^\dagger D^\mu H\right) \\ + & C_2 \left(D_\mu H^\dagger D_\nu H\right) \left(D^\mu H^\dagger D^\nu H\right) \\ + & C_3 \left(D^\mu H^\dagger D_\mu H\right) \left(D^\nu H^\dagger D_\nu H\right) \end{aligned}$$

3D trianglar cone

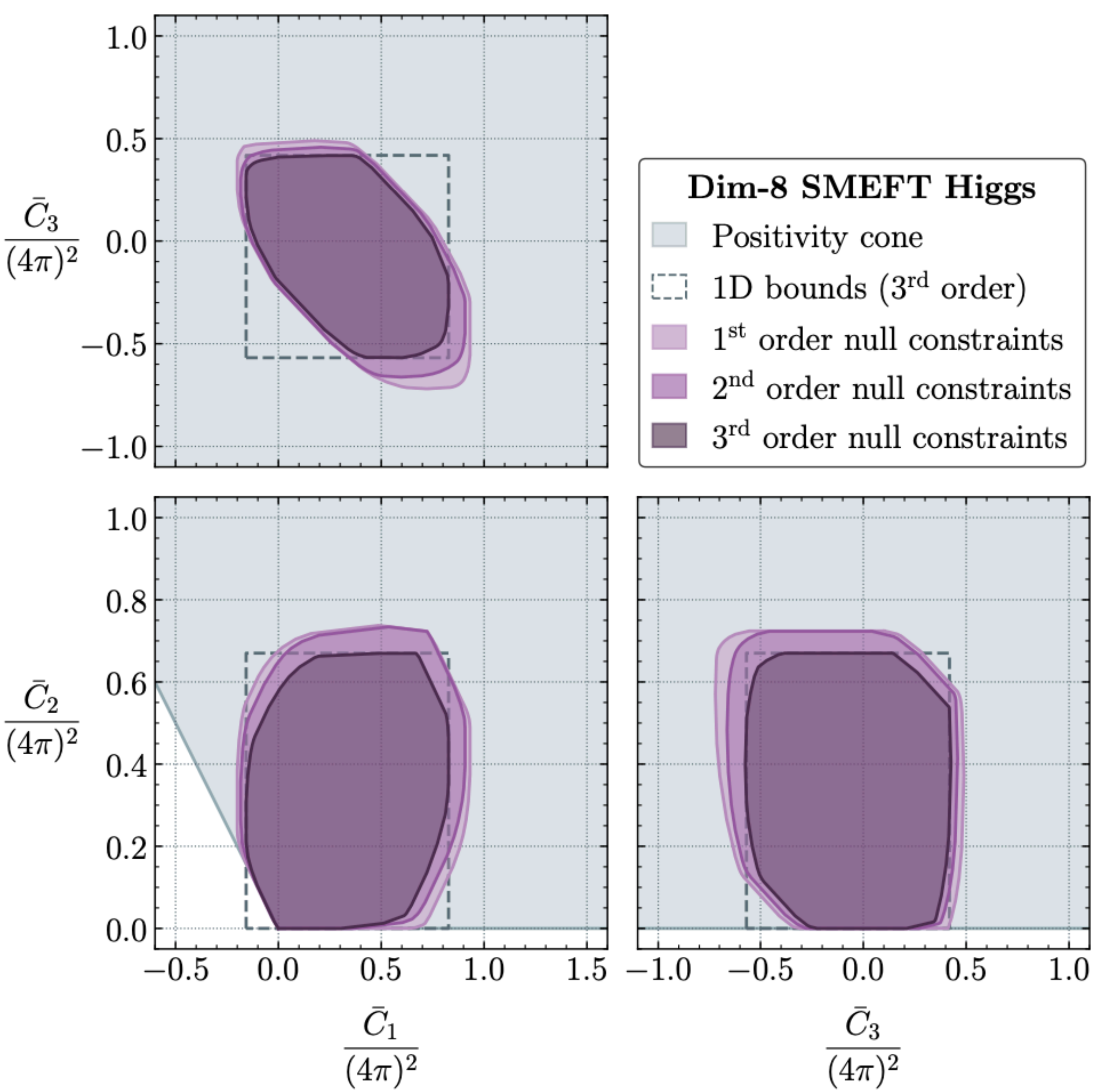
(in space of C_1, C_2, C_3)



Capping Higgs cone: linearized unitarity

Chen, Mimasu, Wu, Zhang & **SYZ**, 2309.15922

2D slices



Discretize UV scales

$$c^{2,0}_{ijkl} = \sum_\ell \int_{\Lambda^2}^{\infty} \frac{d\mu}{\mu^3} \left(\rho^{ijkl}_\ell(\mu) + \rho^{ilkj}_\ell(\mu)\right)$$

$$\Rightarrow c^{2,0}_{ijkl} \approx \frac{1}{\Lambda^4} \sum_{\ell=0}^{\ell_M} \sum_{n=1}^{N} \frac{1}{N} \frac{n}{N} \left(\rho^{ijkl}_{\ell,n} + \rho^{ilkj}_{\ell,n}\right)$$

+ null constraints $\qquad \rho^{ijkl}_\ell = \mathrm{Im}\, a^{ijkl}_\ell$

+ linearized unitarity

$$0 \le \rho^{iiii}_\ell \le 2, \quad \left|\rho^{iijj}_\ell\right| \le 1 - \left|1 - \frac{\rho^{iiii}_\ell + \rho^{jjjj}_\ell}{2}\right|,$$

$$0 \le \rho^{ijij}_\ell \le \frac{1}{2}, \quad \left|\rho^{ijkl}_\ell\right| \le \frac{1}{4} - \left|\frac{1}{4} - \frac{\rho^{ijij}_\ell + \rho^{klkl}_\ell}{2}\right|,$$

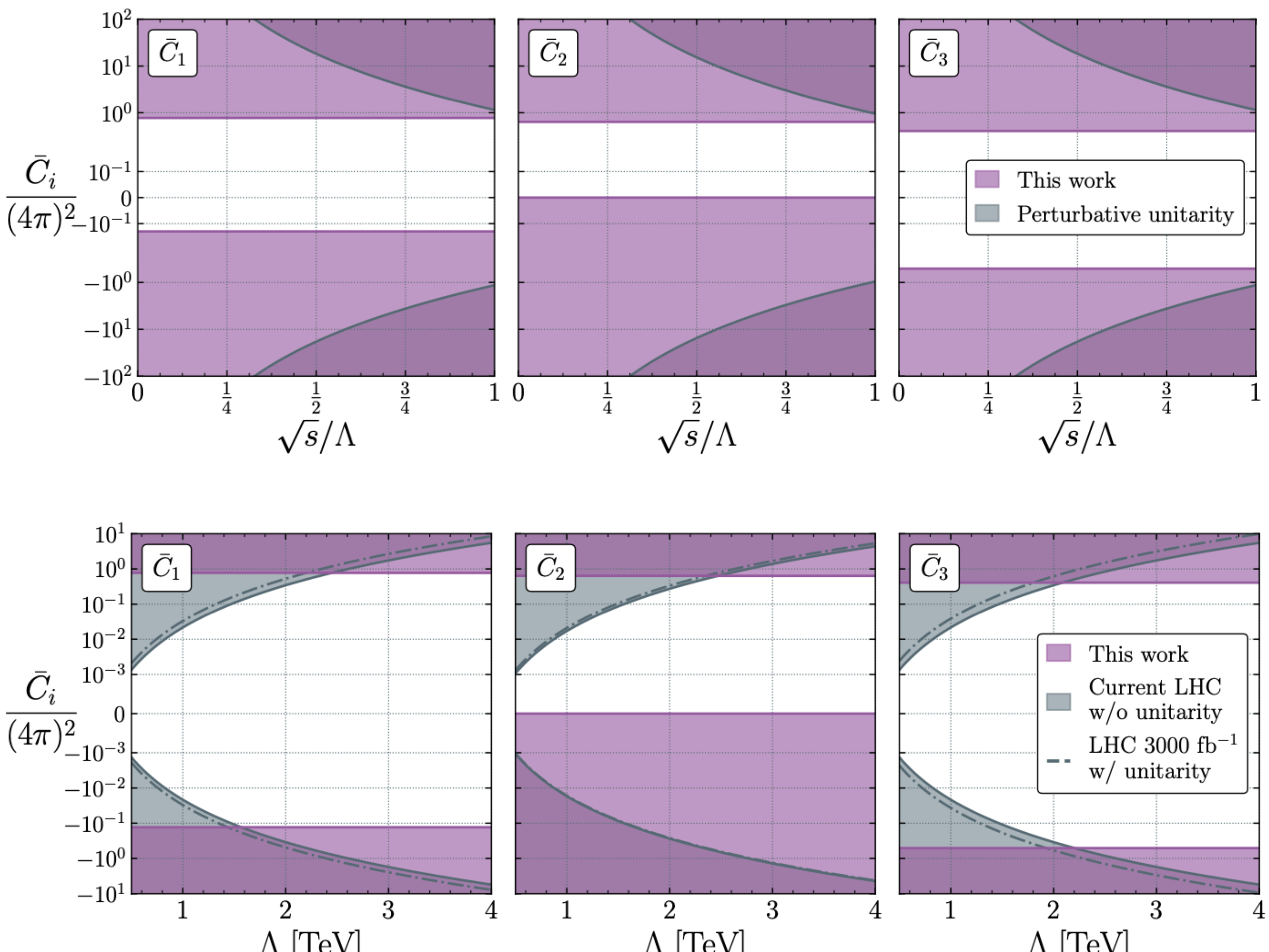
$$\left|(\rho^{iijj}_\ell + \rho^{kkll}_\ell) \pm (\rho^{iikk}_\ell + \rho^{jjll}_\ell)\right| \le 2.$$

⇒ linear programing

Positivity **vs** perturb. unitarity **vs** experi. bounds

Positivity bounds are often stronger

Chen, Mimasu, Wu, Zhang & **SYZ**, 2309.15922



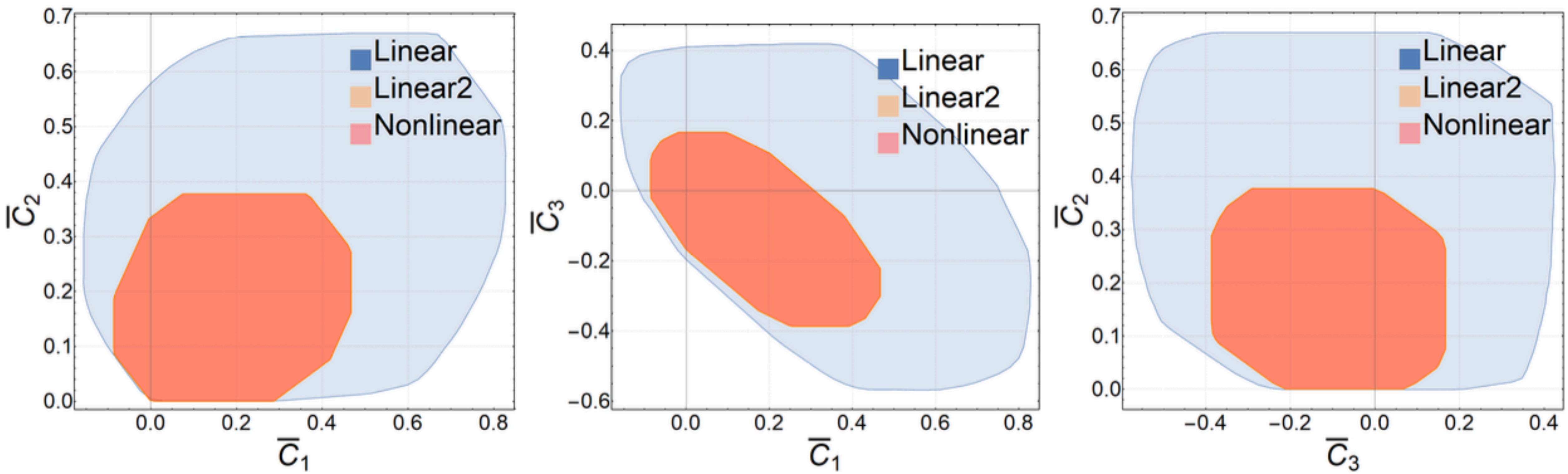
Capping Higgs cone: nonlinear unitarity+full symm.

$$\hat{\mathbb{S}} = \mathbb{I} + i\hat{\mathbb{T}} \qquad \hat{\mathbb{S}}\hat{\mathbb{S}}^\dagger = (\mathbb{I} - \mathbf{Im}\,\mathbb{T})^2 + (\mathbf{Re}\,\mathbb{T})^2 \preceq \mathbb{I}$$

$$\Rightarrow \ \mathbf{Im}\,\mathbb{T}_\ell \succeq 0, \ \ \mathbb{I} - \mathbf{Im}\,\mathbb{T}_\ell \succeq 0$$

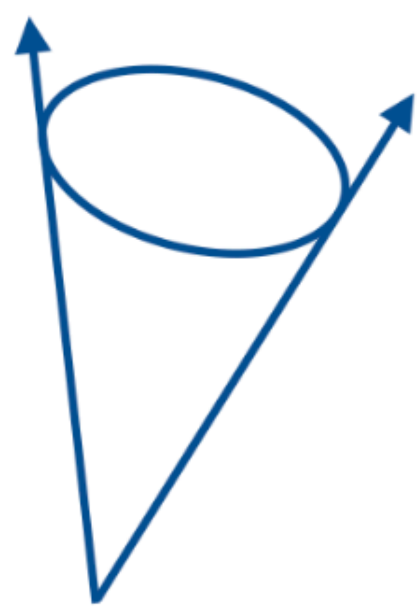
$\Rightarrow$ **semi-definite program**

Hong, Wang & **SYZ**, 2404.04479



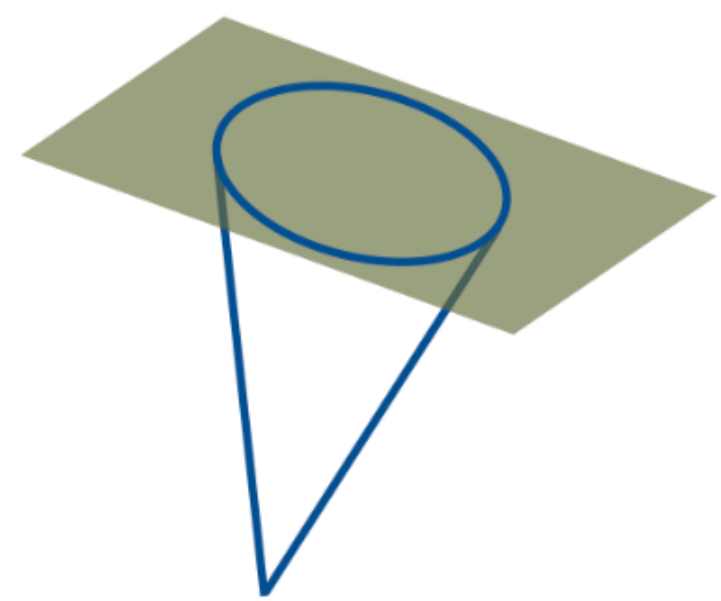
Summary

- Positivity bounds are **robust**—derived from S-matrix axioms
- **dim-8** ops are important to **reverse engineer UV theory**
- Only **small percentage** of naive SMEFT space is consistent theoretically



positivity part of UV unitarity

$(\mathrm{Im}\, a_\ell^{ijkl} \succeq 0)$



fuller use of UV unitarity

+ null constraints

Thank you!

Backup slides

Causality implies analyticity

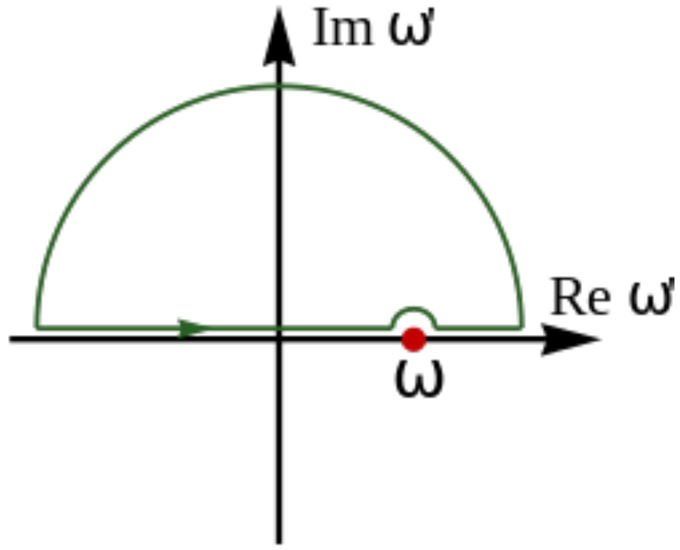
Kramers-Kronig dispersion relation

Titchmarsh's theorem

$$f(t<0)=0$$

$\tilde{f}\,(\omega)$ square-integrable

→ $\tilde{f}\,(\omega)$ analytic in upper ω plane



$$\rightarrow \quad \tilde{f}\,(\omega)=\frac{1}{i\pi}\mathcal{P}\int_{-\infty}^{+\infty}\frac{\mathrm{d}\omega'}{\omega'-\omega}\tilde{f}\,(\omega')$$

eg, complex refractive index n

Relativistic version: response restricted with light-cone

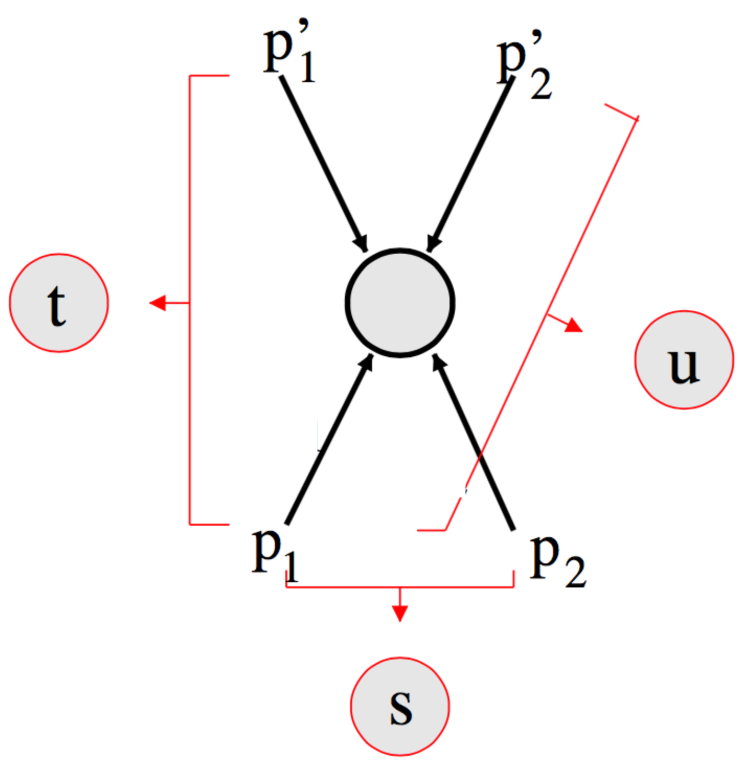
$$f(t,\boldsymbol{x})=\theta(t-\boldsymbol{\xi}\cdot\boldsymbol{x})f(t,\boldsymbol{x})$$
$$\boldsymbol{\xi}^2<1$$

$$\rightarrow \quad \tilde{f}\,(\omega,\boldsymbol{k}_0+\omega\boldsymbol{\xi})=\frac{1}{i\pi}\mathcal{P}\int_{-\infty}^{+\infty}\frac{\mathrm{d}\omega'}{\omega'-\omega}\tilde{f}\,(\omega',\boldsymbol{k}_0+\omega'\boldsymbol{\xi})$$

Analyticity and locality

$A(s,t)$ as **analytic function**

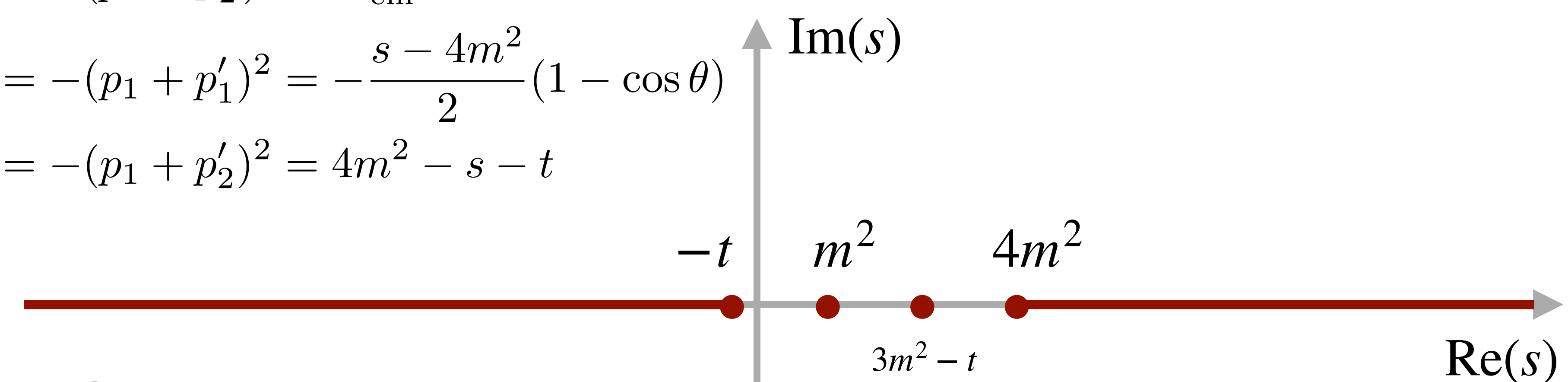
Causality "implies" analyticity



$$s = -(p_1 + p_2)^2 = E^2_{\rm cm}$$

$$t = -(p_1 + p_1')^2 = -\frac{s-4m^2}{2}(1-\cos\theta)$$

$$u = -(p_1 + p_2')^2 = 4m^2 - s - t$$



Crossing symmetry

$$A(s,t) = A(u,t) = A(t,s)$$

some rigorous results in 60'
Martin, …

Locality: $A(s,t)$ is polynomially bounded at high energies

Froissart(-Jin-Martin) bound: Froissart, 1961; Martin, 1963, Jin & Martin, 1964

$$\lim_{s\to\infty} |A(s,t)| < C s^{1+\epsilon(t)}, \quad t < m^2, \quad 0 < \epsilon(t) < 1$$

Unitarity

Unitarity: conservation of probabilities $S^\dagger S = 1 \Rightarrow T - T^\dagger = iT^\dagger T$

Generalized optical theorem

$$A(I \to F) - A^*(F \to I) = i \sum_X \int d\Pi_X (2\pi)^4 \delta^4(p_I - p_X) A(I \to X) A^*(F \to X)$$

optical theorem ($\theta = 0$): $\mathrm{Im}[A(I \to I)] \sim \sum_X \sigma(I \to X) > 0$

Partial wave expansion: $A(s,t) \sim \sum_{\ell=0}^{\infty} (2\ell+1) P_\ell(\cos\theta) a_\ell(s)$

(2-2 scattering, for scalar)

Partial wave unitary bounds:

$$0 \leq |a_\ell(s)|^2 \leq 2\,\mathrm{Im}\, a_\ell(s) \leq 4$$

Forward positivity bounds

Forward limit $t = 0$

$$A(s,0) \sim \int_{\Lambda^2}^{\infty} \frac{\mathrm{d}\mu}{\pi\mu^2}\left[\frac{s^2}{\mu - s} + \frac{s^2}{\mu + s}\right] \mathrm{Im}\, A(\mu, 0)$$

$$c_{2,0}s^2 + c_{4,0}s^4 + \cdots = \left(\int \frac{2\,\mathrm{d}\mu}{\pi\mu^3}\mathrm{Im}\, A(\mu,0)\right)s^2 + \left(\int \frac{2\,\mathrm{d}\mu}{\pi\mu^5}\mathrm{Im}\, A(\mu,0)\right)s^4 + \cdots$$

matching → **Sum rules:**

$$c_{2n,0} = \int \frac{2\,\mathrm{d}\mu}{\pi\mu^{1+2n}}\mathrm{Im}\, A(\mu,0)$$

"First" bounds

Optical theorem
$\mathrm{Im}[A(s,0)] \propto \sigma(s) > 0$ → $c_{2n,0} > 0$

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 2006 + earlier works

Two-sided bounds

Add null constraints to sum rules:

$$\sum_\ell \int d\tilde{\mu} \frac{\Gamma^{(n)}_{i,j}(\eta)}{\mu^{i+j}} = 0$$

$$c_{i,j} \sim \sum_\ell \int d\tilde{\mu} \frac{D_{i,j}(\eta) + \sum_n \alpha_n \Gamma^{(n)}_{i,j}(\eta)}{\mu^{i+j}}$$

can choose α_n to make $D_{i,j} + \Sigma_n \alpha_n \Gamma^{(n)}_{i,j}$ bounded from blow and above

before: $D_{i,j}$ only has **min**

now: $D_{i,j} + \Sigma_n \alpha_n \Gamma^{(n)}_{i,j}$ can have **min and max**

α_n can be positive or negative

➡ **amplitude coeff's $c_{i,j}$ have two-sided bounds**

Tolley, Wang & **SYZ**, 2011.02400

Optimal bounds obtainable by linear programming Caron-Huot & Duong, 2011.02957

Positivity bounds are easy to use

$$M_{S,ij}F_{S,j} > 0$$

$$M_{M,ij}F_{M,j} > 0$$

$$M_{T,ij}F_{T,j} > 0$$

$$M^{abab} = \Sigma_{ijkl} u_i v_j u_k^* v_l^* M^{ijkl} > 0$$

$$F_{S,i} = (F_{S,0}, F_{S,1}, F_{S,2})^T$$

$$F_{M,i} = (F_{M,0}, F_{M,1}, F_{M,2}, F_{M,3}, F_{M,4}, F_{M,5}, F_{M,7})^T$$

$$F_{T,i} = (F_{T,0}, F_{T,1}, F_{T,2}, F_{T,5}, F_{T,6}, F_{T,7}, F_{T,8}, F_{T,9})^T$$

$$M_S = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

$$M_M = \begin{pmatrix} 0 & -2c_W^4 & 0 & -s_W^4 & 0 & s_W^2 c_W^2 & c_W^4 \\ 0 & -2c_W^4 & 0 & -s_W^4 & 0 & -s_W^2 c_W^2 & c_W^4 \\ 0 & -2 & 0 & 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & -1 & 0 & 1 & 1 \\ 0 & -2 & 0 & -1 & 0 & -1 & 1 \end{pmatrix}$$

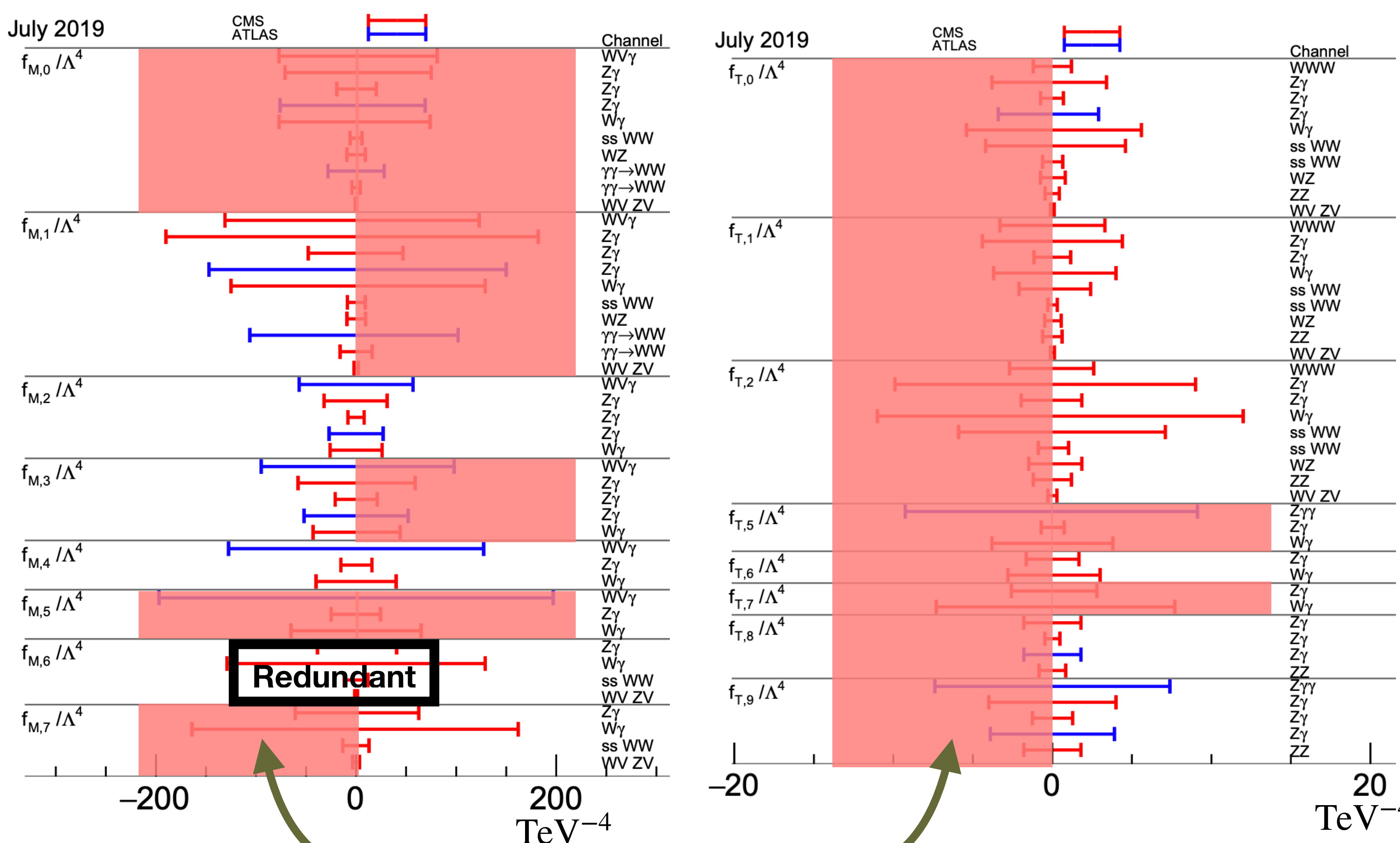
$$M_T = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 8 & 12 & 5 & 0 & 0 & 0 & 0 & 0 \\ 8c_W^8 & 8c_W^8 & 4c_W^8 & 4c_W^4 s_W^4 & 4c_W^4 s_W^4 & 2c_W^4 s_W^4 & 2s_W^8 & s_W^8 \\ 0 & 0 & 4c_W^8 & 0 & 0 & 2c_W^4 s_W^4 & 0 & s_W^8 \\ 0 & 0 & 4c_W^4 & 0 & 0 & s_W^4 & 0 & 0 \\ 0 & 16c_W^4 & 4c_W^4 & 0 & 4s_W^4 & s_W^4 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 1 & 0 & 0 \\ 0 & 16 & 4 & 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 8c_W^4 & 0 & 0 & \left(c_W^2 - s_W^2\right)^2 & 0 & 2s_W^4 \\ 32c_W^4 & 32c_W^4 & 16c_W^4 & -16c_W^2 s_W^2 & 4\left(c_W^2 - s_W^2\right)^2 & 1 - 8s_W^2 c_W^2 & 8s_W^4 & 4s_W^4 \\ 0 & 0 & 4 & 0 & 0 & 2 & 0 & 1 \\ 8 & 8 & 4 & 4 & 4 & 2 & 2 & 1 \end{pmatrix}$$

+ a few nonlinear bounds

Bi, Zhang & **SYZ**,1902.08977

Positivity vs experimental bounds (1)

Constraining aGQC coefficients in VBS

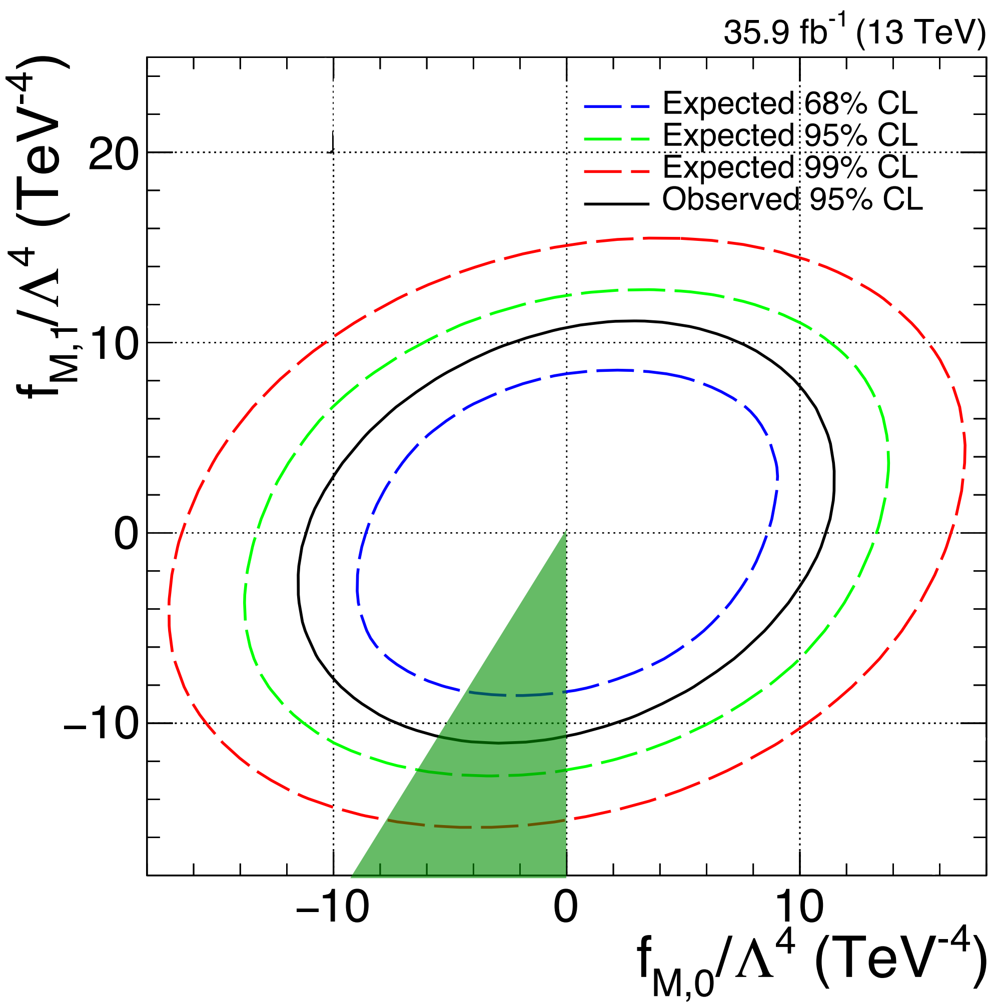


No UV completion/Inconsistent

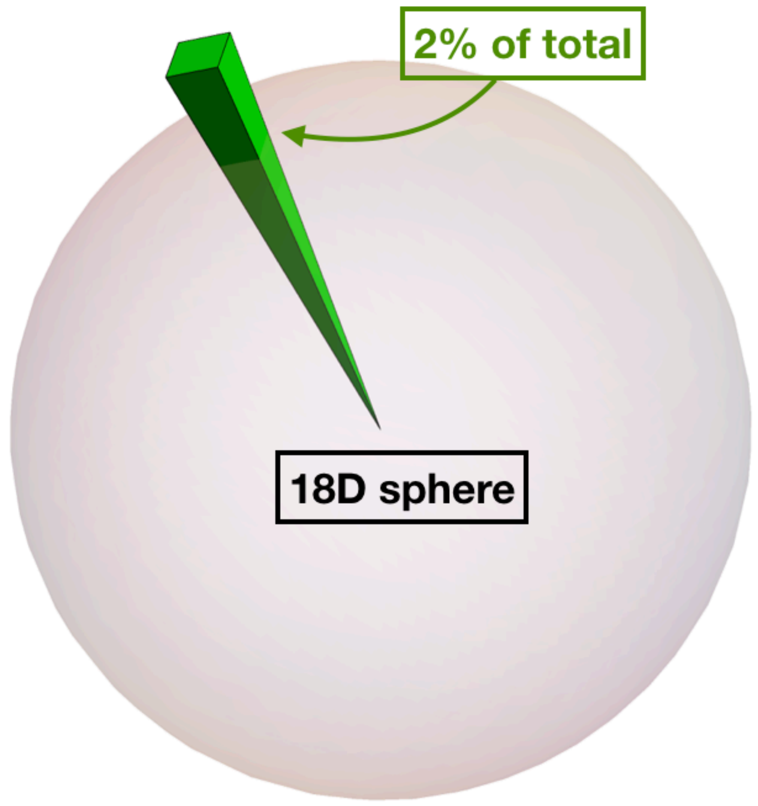
Zhang & **SYZ**,1808.00010

Positivity vs experimental bounds (2)

O_{M0} and O_{M1}



Space of 18 Wilson coeffs for aQGCs



Only <1**%** of the total aQGC parameter space admits an analytic UV completion!

Zhang & **SYZ**,1808.00010
Yamashita, Zhang & **SYZ**, 2009.04490

Stronger positivity bounds?

$$M^{abab} = \sum_{ijkl} u_i v_j u_k^* v_l^* M^{ijkl} = \sum_{ijkl} u_i v_j u_k^* v_l^* c_{2,0}^{ijkl} > 0$$

Is it possible such that

$$M^T = \sum_{ijkl} T_{ijkl} M^{ijkl} > 0, \text{ and } \{T_{ijkl}\} \supset \{u_i v_j u_k^* v_l^*\} ?$$

Yes, T_{ijkl} is more than $u_i v_j u_k^* v_l^*$!

Example: W-boson scatterings in SMEFT

$$F_{T,2} \geq 0, \quad 4F_{T,1} + F_{T,2} \geq 0$$
$$F_{T,2} + 8F_{T,10} \geq 0, \quad 8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0$$
$$12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10} \geq 0$$
$$4F_{T,0} + 4F_{T,1} + 3F_{T,2} + 12F_{T,10} \geq 0$$

Zhang & **SYZ**, 2005.03047

old: $|a\rangle|b\rangle \to |a\rangle|b\rangle$

new: $|U\rangle \to |U\rangle$

scatterings of entangled states

$$T_{ijkl} \sim \Sigma_n \lambda_n U_{ij}^n U_{kl}^n$$

Best bounds from ERs of $\mathscr{T}$ cone

generalized optical theorem

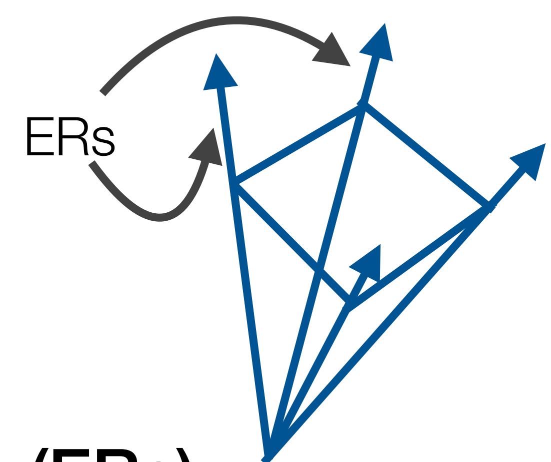
$$T_{ijkl} \in \mathscr{T} \equiv \mathscr{T}^{+} \cap \overrightarrow{\mathbf{S}} \qquad \left\{ \begin{array}{l} \mathscr{T}^{+} \equiv \left\{ T_{ijkl} \mid T_{ij,kl} \succeq 0 \right\} \\ \overrightarrow{\mathbf{S}} \equiv \left\{ T_{ijkl} \mid T_{ijkl} = T_{ilkj} = T_{kjil} = T_{jilk} \right\} \end{array} \right.$$

$\mathscr{T}$ is a spectrahedron

Li, Xu, Yang, Zhang & **SYZ**, 2101.01191

(spectrahedron) = (convex cone of PSD matrices) ∩ (affine-)linear space

To get best bounds, find all ERs of $\mathscr{T}$



all elements of $\mathscr{T}$: $T_{ijkl} = \Sigma_p \alpha_p T^{(p)}_{ijkl}, \ \ \alpha_p > 0$

p enumerates all **Extreme Rays (ERs)**

Best positivity bounds:

$$\sum_{ijkl} T^{(p)}_{ijkl} M^{ijkl} > 0$$

Convex cone $\mathscr{C}$ of amplitudes

$$\mathscr{C} \equiv \{M^{ijkl}\} = \text{cone}\left(\left\{m^{i(j}m^{|k|l)}\right\}\right)$$

$m^{ij} \sim M^{ij\to X}$

X: intermediate state

$\mathscr{C}$ is dual cone of $\mathscr{T}$: $\mathscr{T} \equiv \left\{ T^{ijkl} \middle| T\cdot M \equiv \sum_{ijkl} T_{ijkl}M^{ijkl} > 0 \right\}$

For m^{ij} to be extremal, it can not be split to two amplitudes

$$\Rightarrow \quad m^{ij}_{(\mathrm{ER})} \sim M^{ij\to X_{\mathrm{irrep}}} \sim C^{r,\alpha}_{i,j}$$

CG coefficient

Get $\mathscr{C}$ cone by symmetries of EFT

$$\Rightarrow \quad \mathscr{C} = \text{cone}\left(\left\{P_r^{i(j|k|l)}\right\}\right)$$

$$P_r^{ijkl} \equiv \sum_\alpha C^{r,\alpha}_{i,j}\left(C^{r,\alpha}_{k,l}\right)^*$$

group projector

Zhang & **SYZ**, 2005.03047

The inverse problem

Structure of $\mathscr{C}$ cone implies

Extremal Ray ⟷ **UV Particle**

$P_r^{i(j|k|l)}$

Example: Higgs $\mathscr{C}$ cone in SMEFT

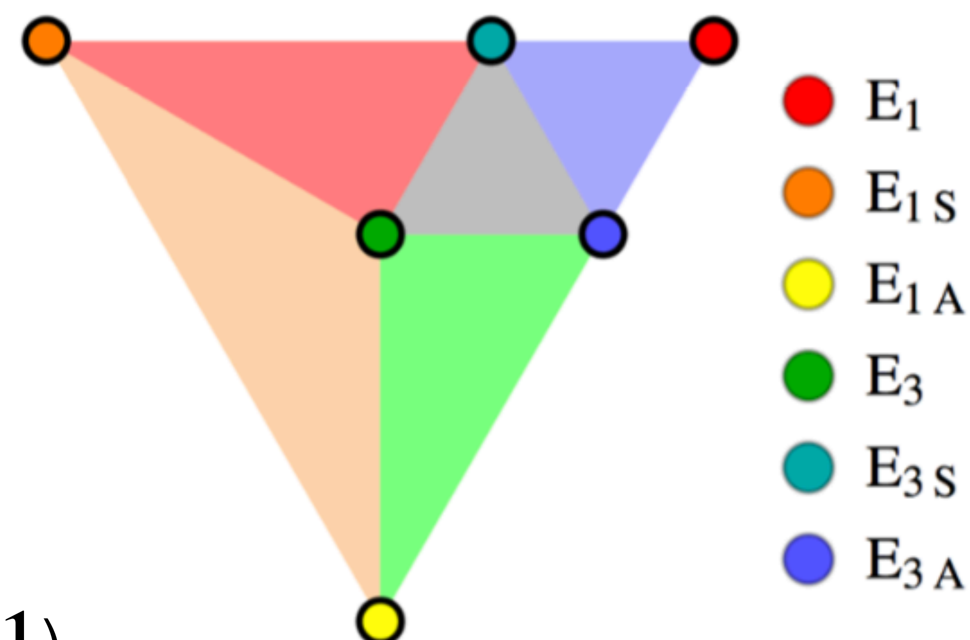
Wilson coeffs fall in blue region

↓

E_1 must exit

↓

new UV state ($SU(2)_L$ singlet, $Y = 1$)



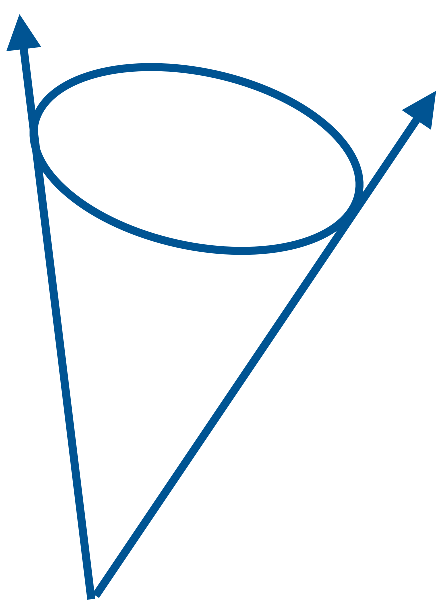
ERs of $\mathscr{C}$ (or dim-8 operators) are important to reverse-engineer the UV model!

Semi-definite program (SDP)

spectrahedron is viable space of a semi-definite program

Use SDP to find best positivity bounds

generally a curly cone



minimize $\sum_{ijkl} T_{ijkl} M^{ijkl}$

subject to $T_{ijkl} \in \mathscr{T} \equiv \mathscr{T}^+ \cap \overrightarrow{\mathbf{S}}$

$\min(T \cdot M) > 0$, then M^{ijkl} is within positivity bounds

Compared to elastic approach ($uvuvM > 0$)

- **stronger bounds**
- **more efficient (polynomial complexity)**

Li, Xu, Yang, Zhang & **SYZ**, 2101.01191