



The impact of NME uncertainties on neutrinoless double beta decay

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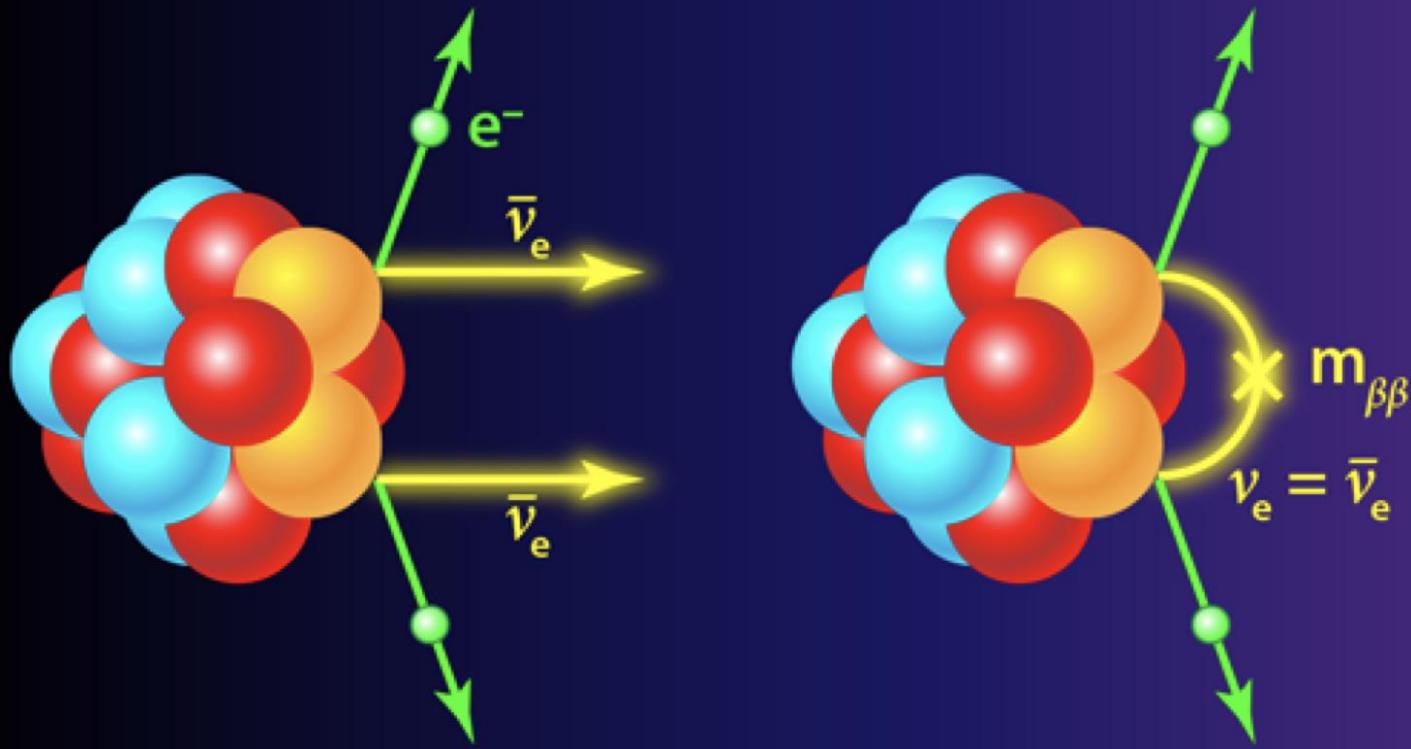
Based on the works JHEP 06 (2023) 104, JHEP 08 (2024) 217

Collaborators: Federica Pompa, Thomas Schwetz; Dong-Liang Fang, Yu-Feng Li, Yi-Yu Zhang

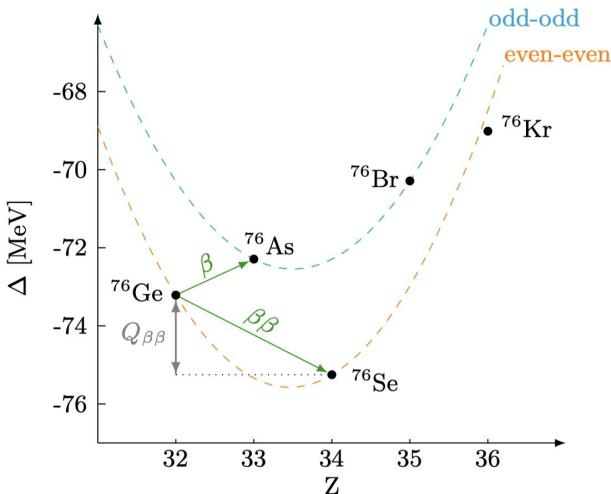
EPSHEP2025, Marseilles

- **Brief introduction**
- **Neutrinoless double beta decay ($0\nu\beta\beta$) in light neutrino exchange mechanism**
Different upper limits of $m_{\beta\beta}$ due to NME uncertainties (short-range contact term)
- **$0\nu\beta\beta$ process in minimal Type-I seesaw**
Constraints of minimal Type-I seesaw from current and future $0\nu\beta\beta$ experiments
- **Summary**

Where
are
you?



Brief background



$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}_e$$

Mayer, 1935; first detected in 1987 by Moe

$$\nu_i^c = \nu_i \quad \text{Majorana, 1937}$$

$$(A, Z) \longrightarrow (A, Z + 2) + 2e^-$$

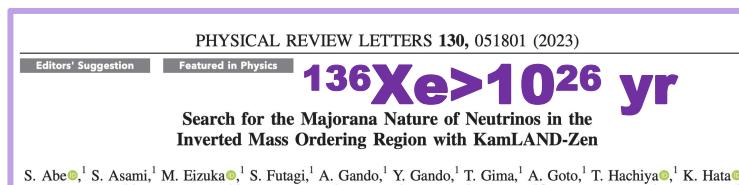
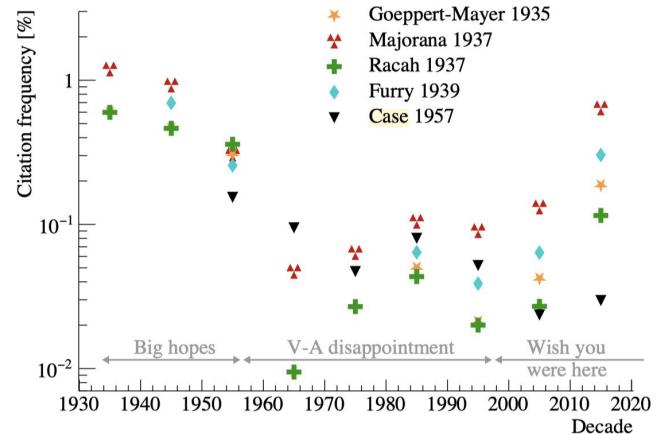
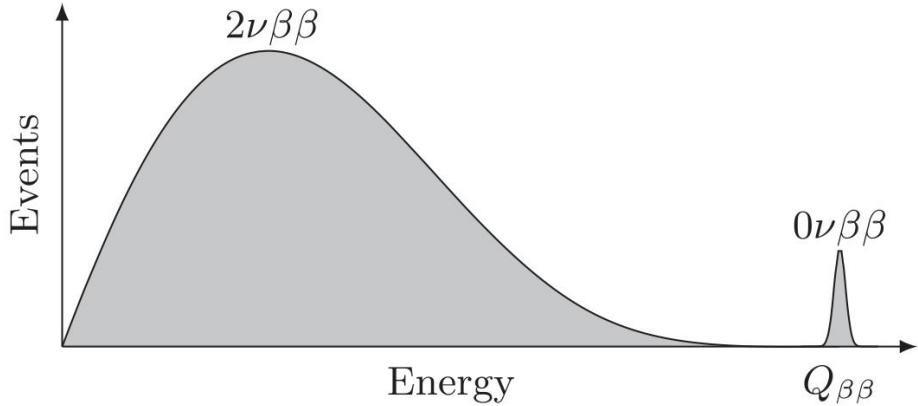
Furry, 1939

Isotope	Daughter	$Q_{\beta\beta}$ (keV) ^a	f_{nat} (%) ^b	f_{enr} (%) ^c	$T_{1/2}^{2\nu\beta\beta}$ (yr) ^d	$T_{1/2}^{0\nu\beta\beta}$ (yr) ^e
⁴⁸ Ca	⁴⁸ Ti	4267.98(32)	30.187(21)	16	$[6.4^{+0.7}_{-0.6}(\text{stat})^{+1.2}_{-0.9}(\text{syst})] \times 10^{19}$	$> 5.8 \times 10^{22}$
⁷⁶ Ge	⁷⁶ Se	2039.061(7)	37.75(12)	92	$(1.926 \pm 94) \times 10^{21}$	$> 1.8 \times 10^{26}$
⁸² Se	⁸² Kr	2997.9(3)	38.82(15)	96.3	$[8.60 \pm 0.03(\text{stat})^{+0.19}_{-0.13}(\text{syst})] \times 10^{19}$	$> 3.5 \times 10^{24}$
⁹⁶ Zr	⁹⁶ Mo	3356.097(86)	32.80(2)	86	$[2.35 \pm 0.14(\text{stat}) \pm 0.16(\text{syst})] \times 10^{19}$	$> 9.2 \times 10^{21}$
¹⁰⁰ Mo	¹⁰⁰ Ru	3034.40(17)	39.744(65)	99.5	$[7.12^{+0.18}_{-0.14}(\text{stat}) \pm 0.10(\text{syst})] \times 10^{18}$	$> 1.5 \times 10^{24}$
¹¹⁶ Cd	¹¹⁶ Sn	2813.50(13)	37.512(54)	82	$2.63^{+0.11}_{-0.12} \times 10^{19}$	$> 2.2 \times 10^{23}$
¹³⁰ Te	¹³⁰ Xe	2527.518(13)	34.08(62)	92	$[7.71^{+0.08}_{-0.06}(\text{stat})^{+0.12}_{-0.15}(\text{syst})] \times 10^{20}$	$> 2.2 \times 10^{25}$
¹³⁶ Xe	¹³⁶ Ba	2457.83(37)	38.857(72)	90	$[2.165 \pm 0.016(\text{stat}) \pm 0.059(\text{syst})] \times 10^{21}$	$> 1.1 \times 10^{26}$
¹⁵⁰ Nd	¹⁵⁰ Sm	3371.38(20)	35.638(28)	91	$[9.34 \pm 0.22(\text{stat})^{+0.62}_{-0.60}(\text{syst})] \times 10^{18}$	$> 2.0 \times 10^{22}$

10¹⁸ yr – 10²¹ yr

Experimental results and future proposals

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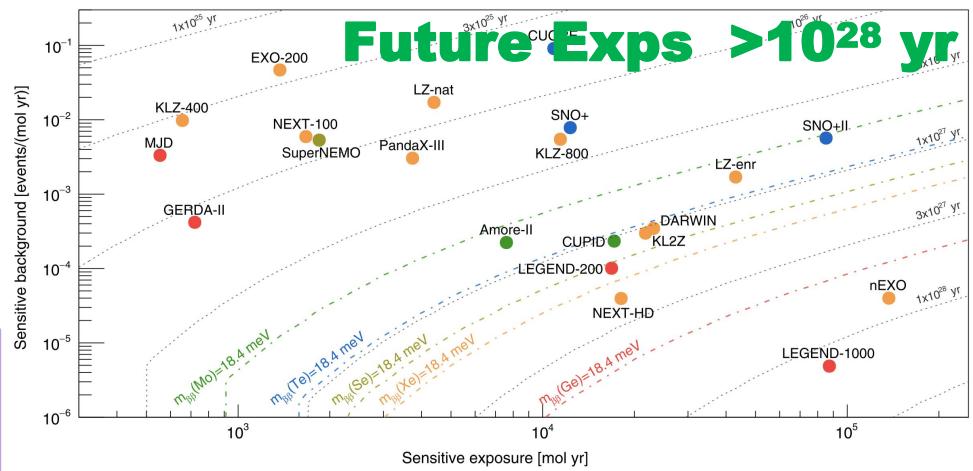
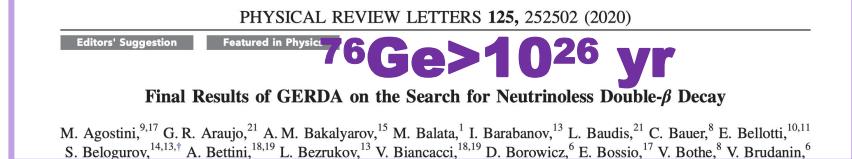


Article
Search for Majorana neutrinos exploiting millikelvin cryogenics with CUORE

130Te>10²⁶ yr

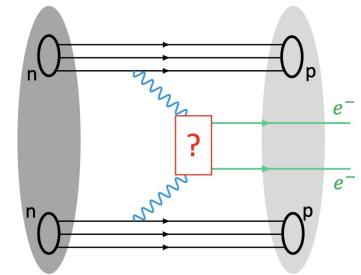
<https://doi.org/10.1088/s41586-022-04497-4> The CUORE Collaboration*

Received: 14 April 2021



Theoretical mechanism → which one dominates?

mechanism	amplitude and particle physics parameter	current limit	test
light neutrino exchange	$\frac{G_F^2}{q^2} U_{ei}^2 m_i $	0.5 eV	oscillations, cosmology, neutrino mass
heavy neutrino exchange	$G_F^2 \left \frac{S_{ei}^2}{M_i} \right $	$2 \times 10^{-8} \text{ GeV}^{-1}$	LFV, collider
heavy neutrino and RHC	$G_F^2 m_W^4 \left \frac{V_{ei}^2}{M_i M_{W_R}^4} \right $	$4 \times 10^{-16} \text{ GeV}^{-5}$	flavor, collider
Higgs triplet and RHC	$G_F^2 m_W^4 \left \frac{(M_R)_{ee}}{m_{\Delta_R}^2 M_{W_R}^4} \right $	$10^{-15} \text{ GeV}^{-1}$	flavor, collider e^- distribution
λ -mechanism with RHC	$G_F^2 \frac{m_W^2}{q} \left \frac{U_{ei} \tilde{S}_{ei}}{M_{W_R}^2} \right $	$1.4 \times 10^{-10} \text{ GeV}^{-2}$	flavor, collider, e^- distribution
η -mechanism with RHC	$G_F^2 \frac{1}{q} \tan \zeta \left U_{ei} \tilde{S}_{ei} \right $	6×10^{-9}	flavor, collider, e^- distribution
short-range \mathcal{R}	$\frac{ \lambda'_{111} }{\Lambda_{\text{SUSY}}^5}$ $\Lambda_{\text{SUSY}} = f(m_{\tilde{g}}, m_{\tilde{u}_L}, m_{\tilde{d}_R}, m_{\chi_i})$	$7 \times 10^{-18} \text{ GeV}^{-5}$	collider, flavor
long-range \mathcal{R}	$\frac{G_F}{q} \left \sin 2\theta^b \lambda'_{131} \lambda'_{113} \left(\frac{1}{m_{b_1}^2} - \frac{1}{m_{b_2}^2} \right) \right $ $\sim \frac{G_F}{q} m_b \frac{ \lambda'_{131} \lambda'_{113} }{\Lambda_{\text{SUSY}}^3}$	$2 \times 10^{-13} \text{ GeV}^{-2}$ $1 \times 10^{-14} \text{ GeV}^{-3}$	flavor, collider
Majorons	$\propto \langle g_\chi \rangle \text{ or } \langle g_\chi \rangle ^2$	$10^{-4} \dots 1$	spectrum, cosmology



Rodejohann, Int.J.Mod.Phys.E 20 (2011)

Phase space factor + nuclear matrix element (NME) ? + new physics parameter ? (effective neutrino mass)

Formula (light neutrino exchange mechanism)

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$$(T_{1/2}^{-1})_\alpha = \tilde{\Gamma}_\alpha(m_{\beta\beta}, M_{\alpha i}) = \frac{\Gamma_\alpha(m_{\beta\beta}, M_{\alpha i})}{\ln 2} = G_\alpha |M_{\alpha i}|^2 m_{\beta\beta}^2$$

$$m_{\beta\beta} = \left| \sum_j U_{ej}^2 m_j \right|$$

Cirigliano et al, Phys.Rev.Lett. 120 (2018) 20, 202001

$$M_{\alpha i} = M_{\alpha i}^{\text{long}} + M_{\alpha i}^{\text{short}} = M_{\alpha i}^{\text{long}}(1 + n_{\alpha i}) \quad n_{\alpha i} = \frac{M_{\alpha i}^{\text{short}}}{M_{\alpha i}^{\text{long}}}$$

$$g_A^{\text{eff}} = q g_A^{\text{free}} \quad g_A^{\text{free}} = 1.27$$

- Quenching effect: correct the NME by q^2 and the decay rate by q^4
(Ab initio many- body theory)
- Short- range NME: Contact operator suggested to contribute to light- neutrino exchange, Cirigliano et al. PRL2018
- We do not know neither the value or the sign of short- range NME well.
- Unknown value of the hadronic coupling g_ν^{NN} , to be determined experimentally or Lattice QCD calculations

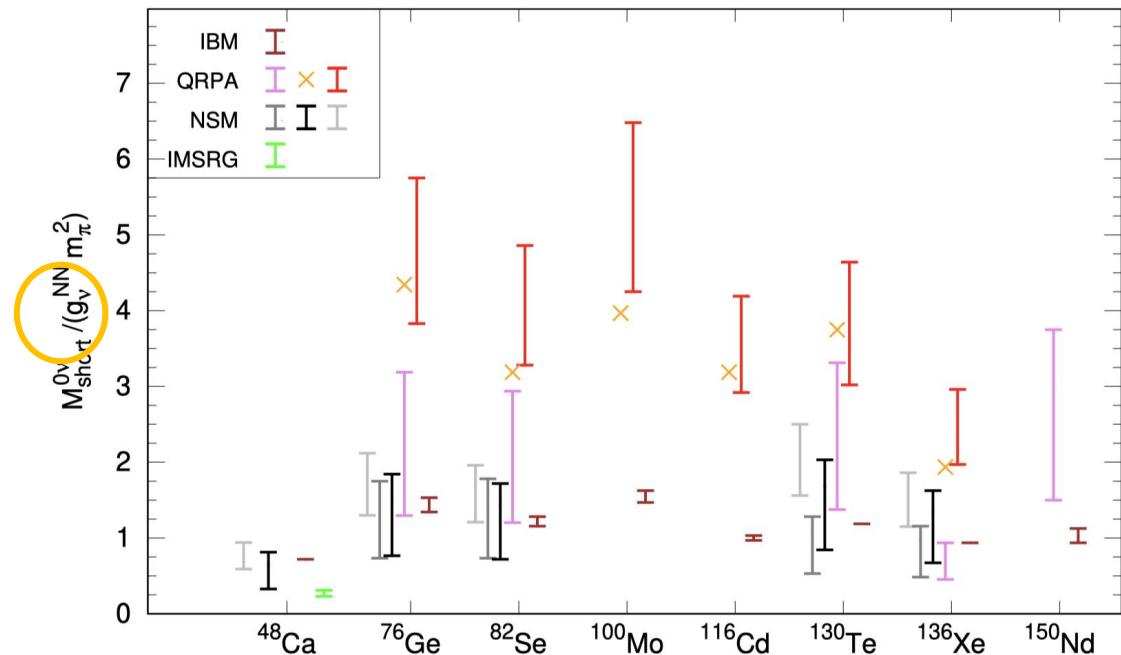
Long-range NME

Nuclear Model	Index [Ref.]	^{76}Ge	^{82}Se	^{100}Mo	^{130}Te	^{136}Xe
NSM	N1 [25]	2.89	2.73	-	2.76	2.28
	N2 [25]	3.07	2.90	-	2.96	2.45
	N3 [26]	3.37	3.19	-	1.79	1.63
	N4 [26]	3.57	3.39	-	1.93	1.76
	N5 [27, 28]	2.66	2.72	2.24	3.16	2.39
QRPA	Q1 [29]	5.09	-	-	1.37	1.55
	Q2 [30]	5.26	3.73	3.90	4.00	2.91
	Q3 [31]	4.85	4.61	5.87	4.67	2.72
	Q4 [32]	3.12	2.86	-	2.90	1.11
	Q5 [32]	3.40	3.13	-	3.22	1.18
	Q6 [33]	-	-	-	4.05	3.38
EDF	E1 [34]	4.60	4.22	5.08	5.13	4.20
	E2 [35]	5.55	4.67	6.59	6.41	4.77
	E3 [36]	6.04	5.30	6.48	4.89	4.24
IBM	I1 [37]	5.14	4.19	3.84	3.96	3.25
	I2 [13]	6.34	5.21	5.08	4.15	3.40

Short-range NME

$$n_{\alpha i} = \frac{M_{\alpha i}^{\text{short}}}{M_{\alpha i}^{\text{long}}}$$

Isotope	NSM %	QRPA %
^{76}Ge	15–42	32–73
^{82}Se	15–41	30–70
^{100}Mo	-	49–108
^{130}Te	17–47	34–77
^{136}Xe	17–47	30–70



Phys. Lett. B 823 (2021) 136720

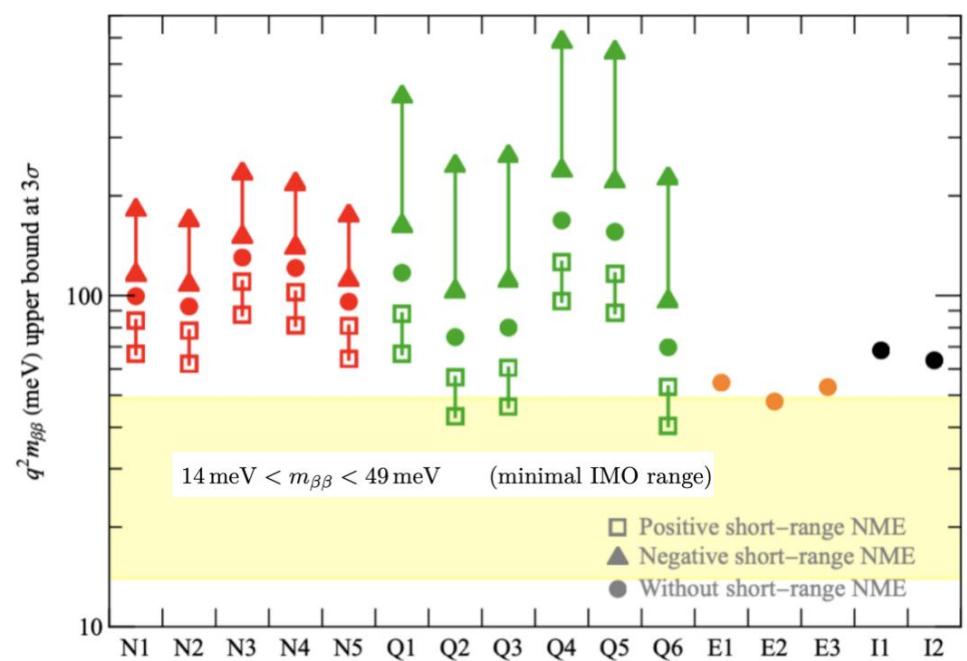
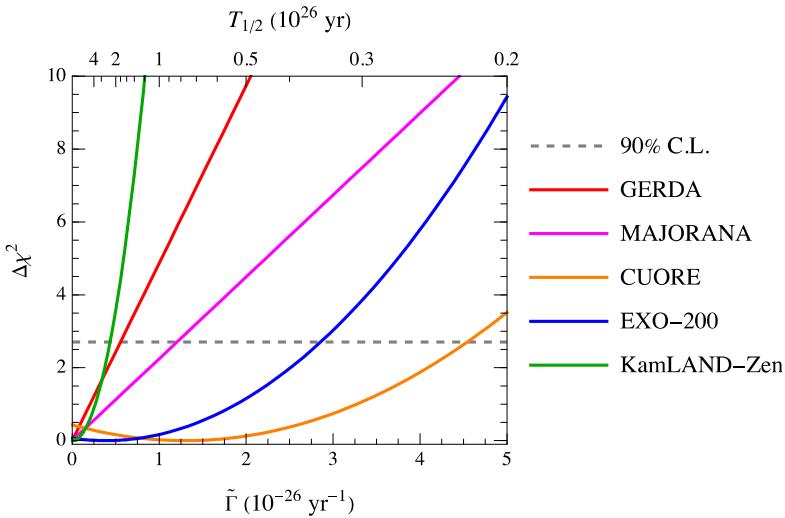
Agostini et al. Rev.Mod.Phys. 95 (2023) 2, 025002

Recent discussion: arXiv:2405.10503

Current constraints

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$$\Delta\chi^2_r(\tilde{\Gamma}_\alpha) = a_r (\tilde{\Gamma}_\alpha)^2 + b_r \tilde{\Gamma}_\alpha + c_r$$



Sensitivities to $(q^2 m_{\beta\beta})^{\text{True}}$ at 3σ

Experiment	Isotope	ε [mol·yr]	b [events/(mol·y)]	PSF [$\text{yr}^{-1} \text{ eV}^{-2}$]
LEGEND-1000	^{76}Ge	8736	$4.9 \cdot 10^{-6}$	$2.36 \cdot 10^{-26}$
SuperNEMO	^{82}Se	185	$5.4 \cdot 10^{-3}$	$10.19 \cdot 10^{-26}$
CUPID	^{100}Mo	1717	$2.3 \cdot 10^{-4}$	$15.91 \cdot 10^{-26}$
SNO+II	^{130}Te	8521	$5.7 \cdot 10^{-3}$	$14.2 \cdot 10^{-26}$
nEXO	^{136}Xe	13700	$4.0 \cdot 10^{-5}$	$14.56 \cdot 10^{-26}$

$$N_{\text{LEGEND-1000}} = \left\{ 0.97 \times \left[\frac{(q^2 m_{\beta\beta})^{\text{True}}}{40 \text{ meV}} \right]^2 \left(\frac{M_{\text{Ge}}^{\text{long}}}{2.66} \right)^2 + 0.04 \right\} \times \frac{T}{1 \text{ yr}}$$

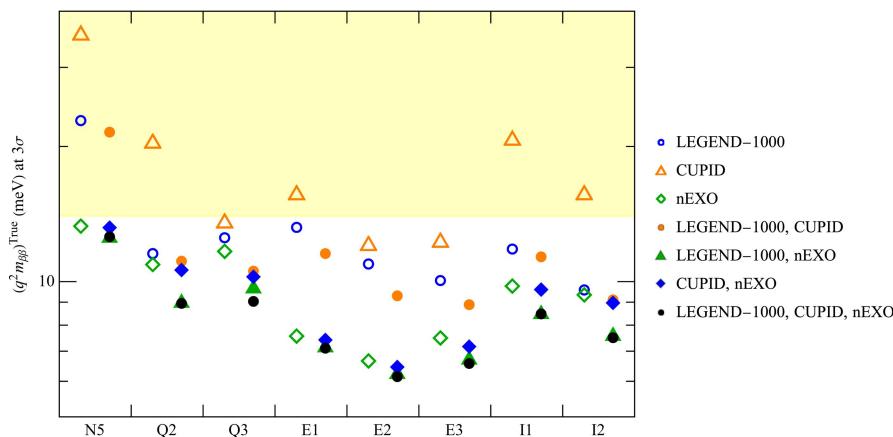
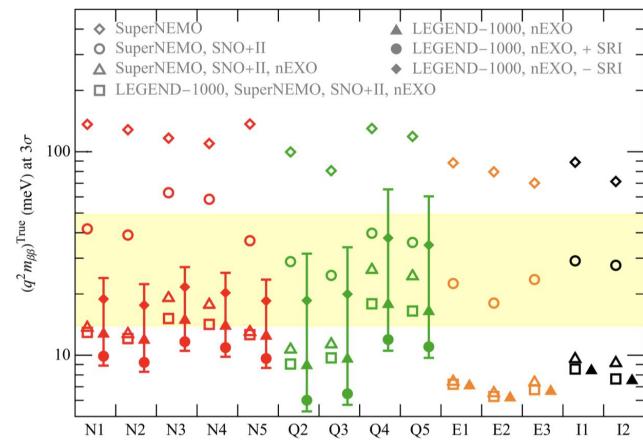
$$N_{\text{SuperNEMO}} = \left\{ 0.09 \times \left[\frac{(q^2 m_{\beta\beta})^{\text{True}}}{40 \text{ meV}} \right]^2 \left(\frac{M_{\text{Se}}^{\text{long}}}{2.72} \right)^2 + 1.0 \right\} \times \frac{T}{1 \text{ yr}}$$

$$N_{\text{nEXO}} = \left\{ 1.64 \times \left[\frac{(q^2 m_{\beta\beta})^{\text{True}}}{40 \text{ meV}} \right]^2 \left(\frac{M_{\text{Xe}}^{\text{long}}}{1.11} \right)^2 + 0.5 \right\} \times \frac{T}{1 \text{ yr}}$$

$$N_{\alpha i} = S_{\alpha i} + B_{\alpha} \quad B_{\alpha} = b_{\alpha} \cdot \varepsilon_{\alpha} \cdot \left(\frac{T}{1 \text{ yr}} \right)$$

$$S_{\alpha i}(m_{\beta\beta}, M_{\alpha i}) = \ln 2 \cdot N_A \cdot \varepsilon_{\alpha} \cdot \left(\frac{T}{1 \text{ yr}} \right) \cdot \tilde{\Gamma}_{\alpha}(m_{\beta\beta}, M_{\alpha i})$$

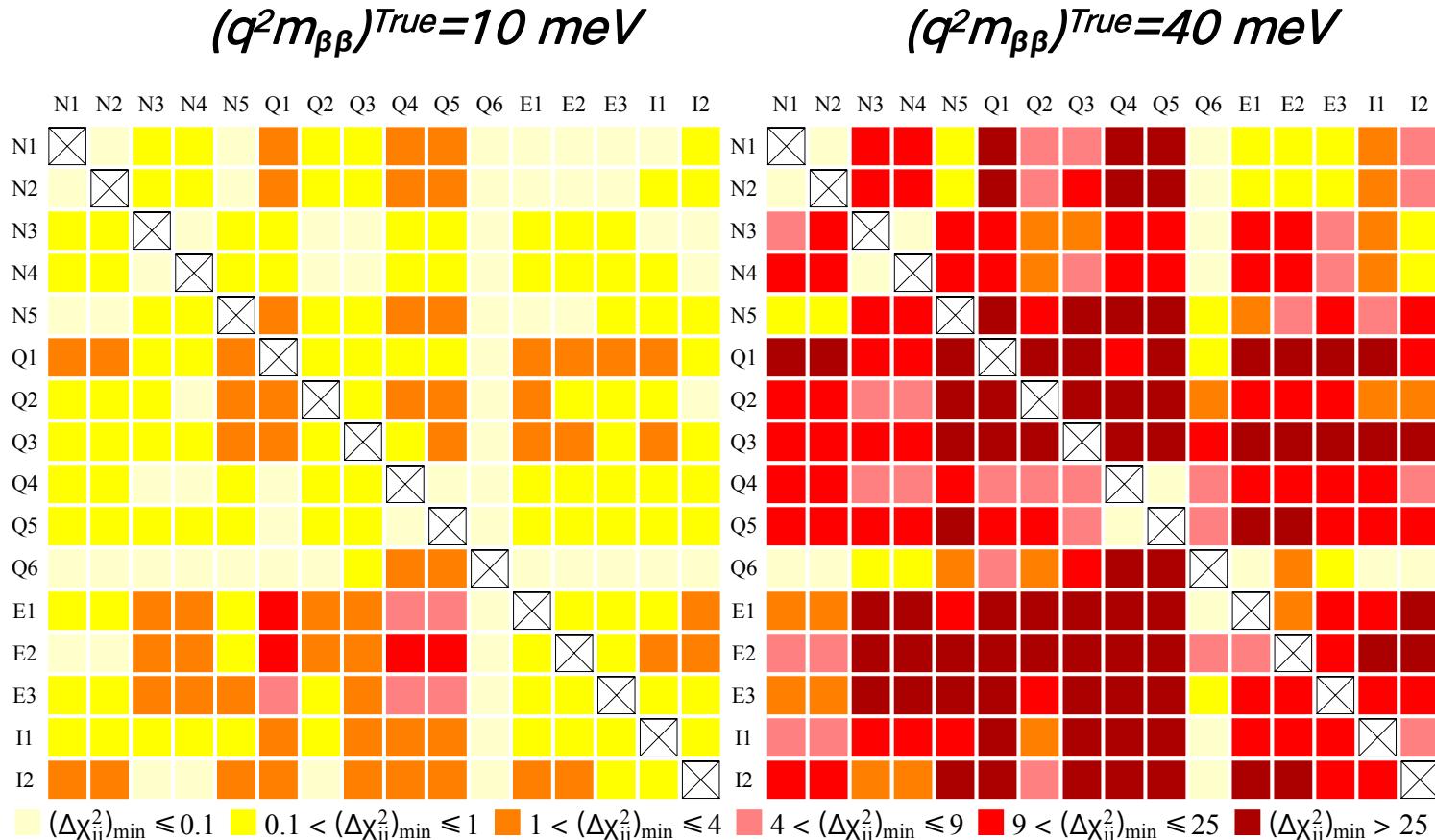
$$\Delta \chi^2_{ij}(m_{\beta\beta}, M_{\alpha j}; m_{\beta\beta}^{\text{True}}, M_{\alpha i}^{\text{True}}) = 2 \sum_{\alpha} \left(N_{\alpha j} - N_{\alpha i}^{\text{True}} + N_{\alpha i}^{\text{True}} \ln \frac{N_{\alpha i}^{\text{True}}}{N_{\alpha j}} \right)$$



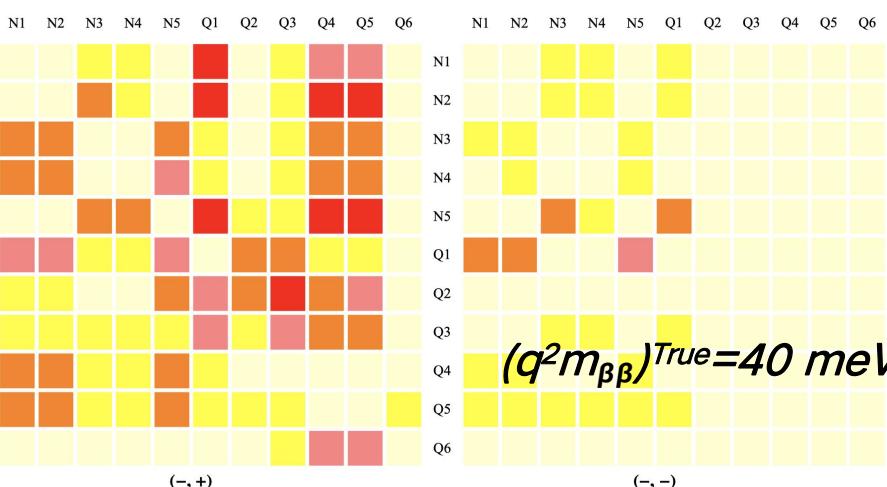
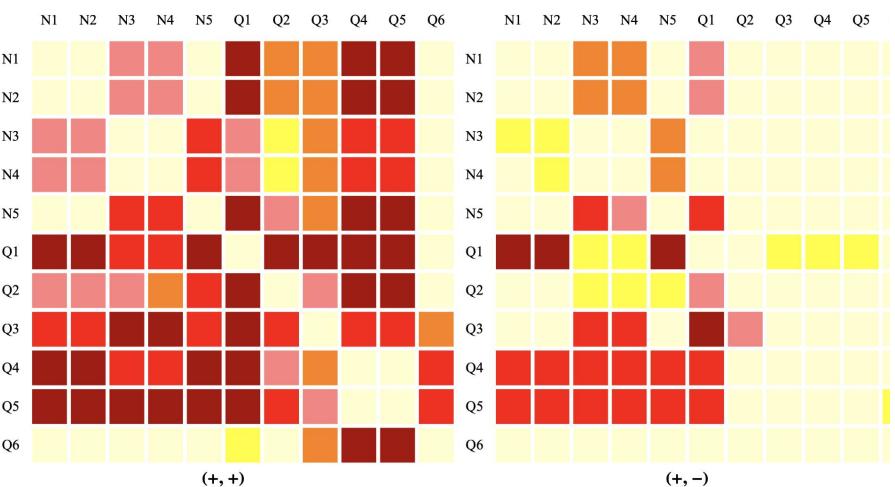
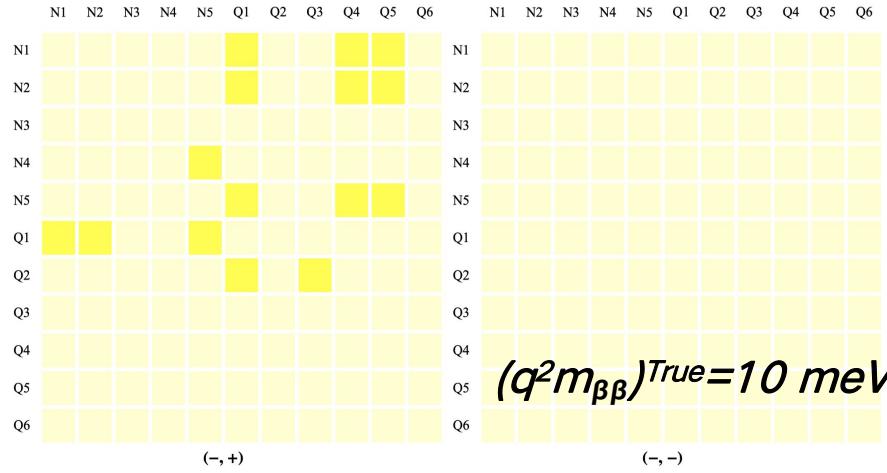
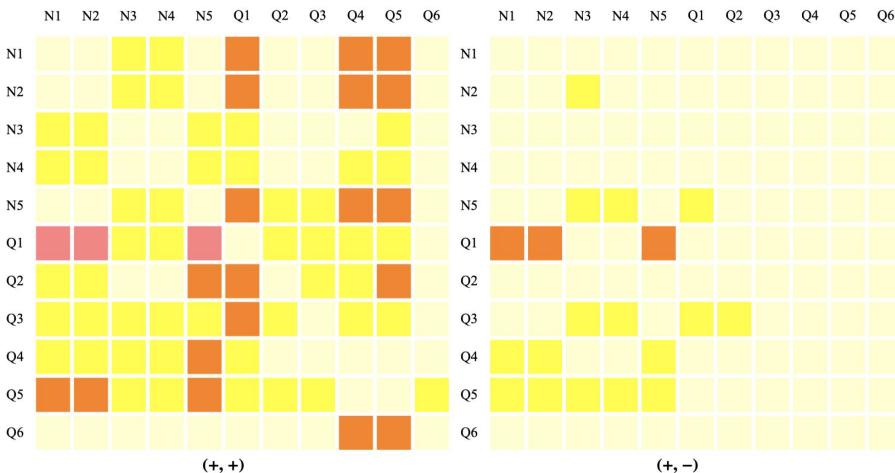
Discrimination without short-range NME

$$(\Delta\chi_{ij}^2)_{\min} = \min_{m_{\beta\beta}} \Delta\chi_{ij}^2(m_{\beta\beta}, M_{\alpha j}; (q^2 m_{\beta\beta})^{\text{True}}, M_{\alpha i}^{\text{True}})$$

Indices i (true model) and j (fitted model) run over the vertical and horizontal axes, respectively



Discrimination with short-range NME, T=10 yr



$$(q^2 m_{\beta\beta})^{True} = 10 \text{ meV}$$

Formulas (minimal type-I seesaw)

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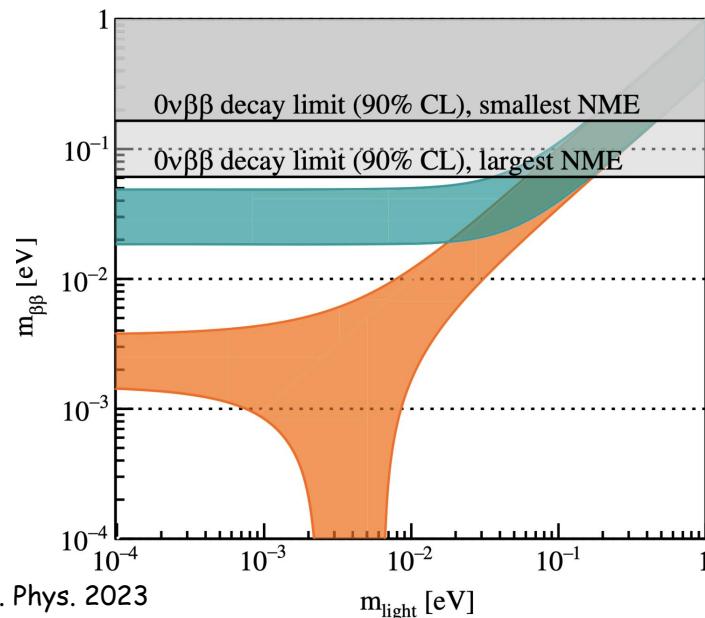
$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} \overline{(\nu_L, N_R^c)} \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + \text{h.c.}$$



$$1/T_{1/2}^{0\nu} = G |M_{0\nu}(0) \cdot m_{\text{eff}}|^2$$

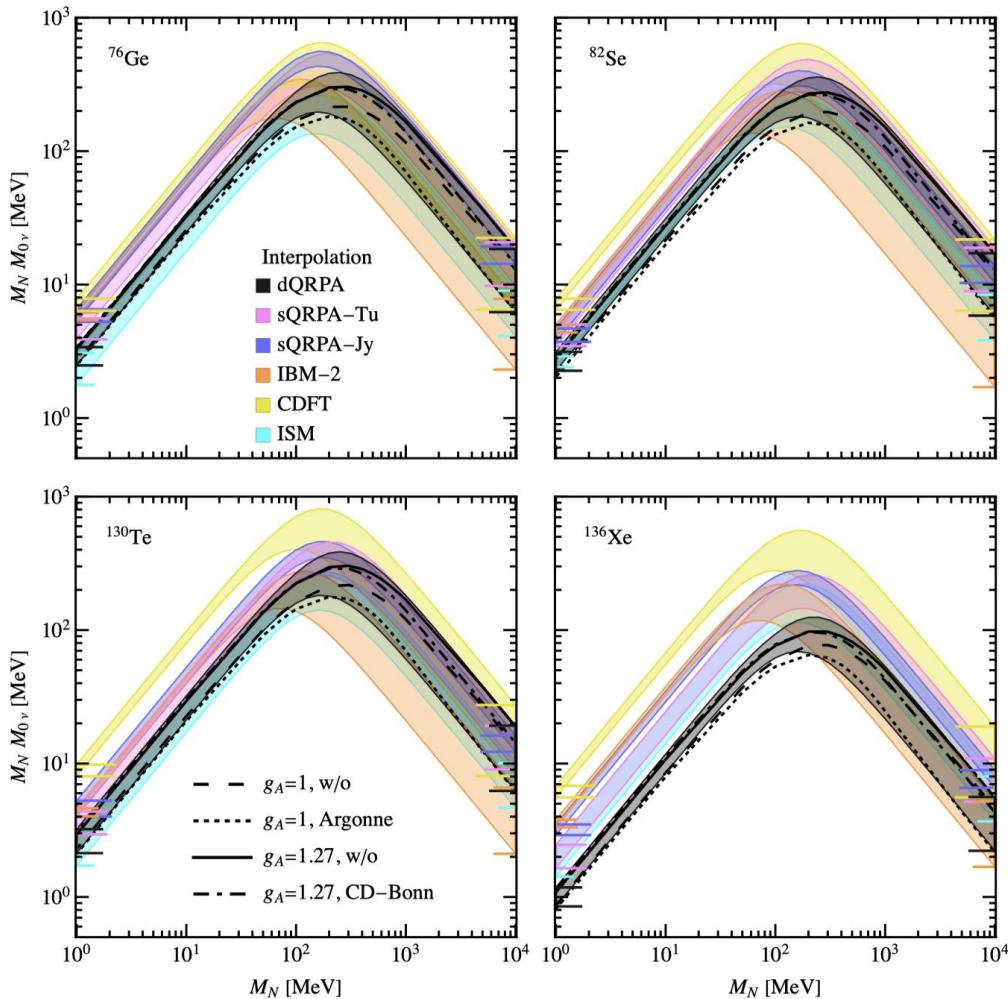
$$|m_{\text{eff}}| = \left| |m_{\text{eff}}^\nu| - |m_{\text{eff}}^\nu| f_\beta(M_2) + [R_{e1}^2] e^{2i\delta_{14}} M_1 [f_\beta(M_1) - f_\beta(M_2)] \right|$$

$$f_\beta(M_N) = M_{0\nu}(M_N)/M_{0\nu}(0) \quad \text{Mass dependent nuclear matrix element (NME)}$$



minimal Type-I seesaw
NMO, [1, 4] meV
IMO, [15, 50] meV

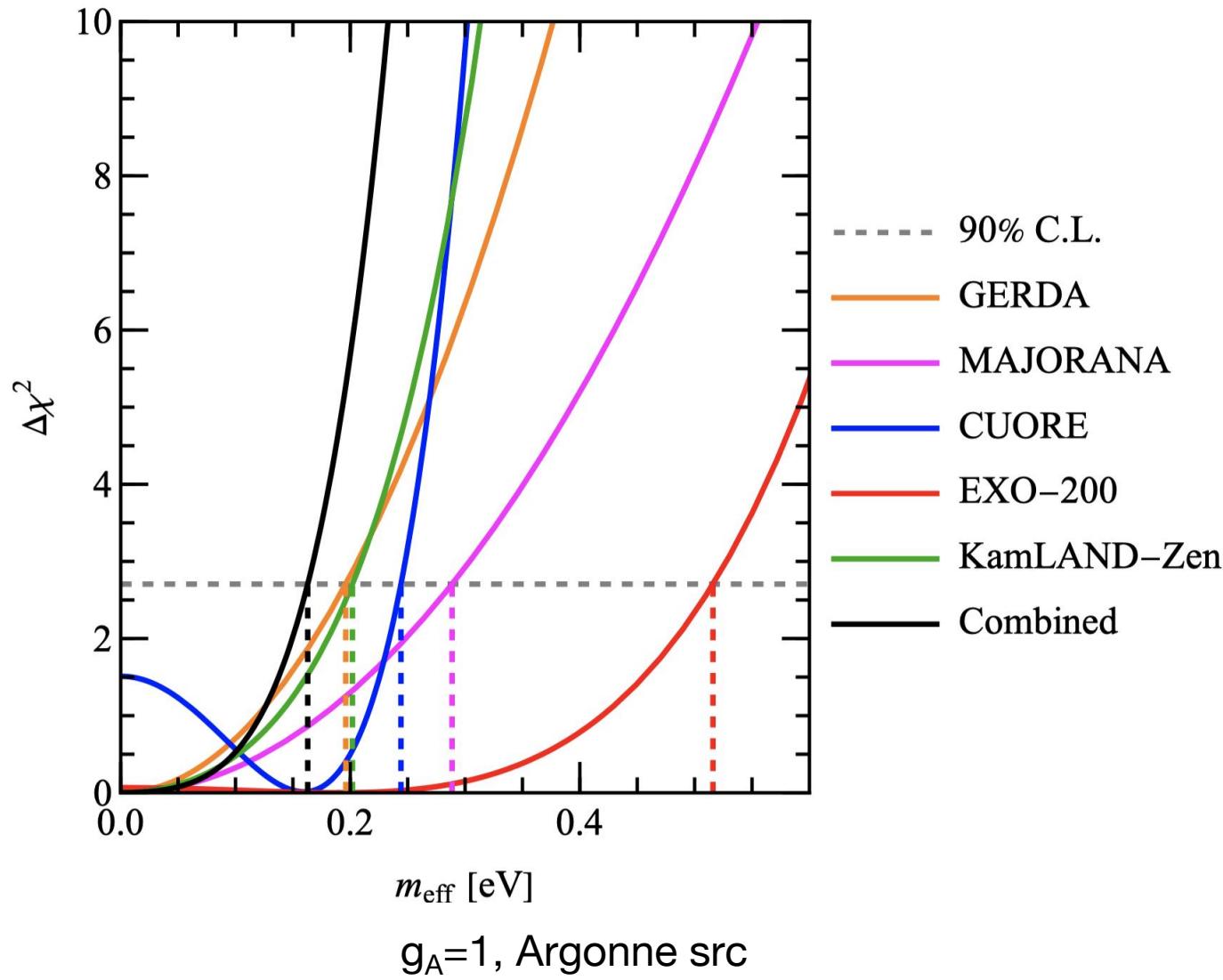
Mass-dependent NME



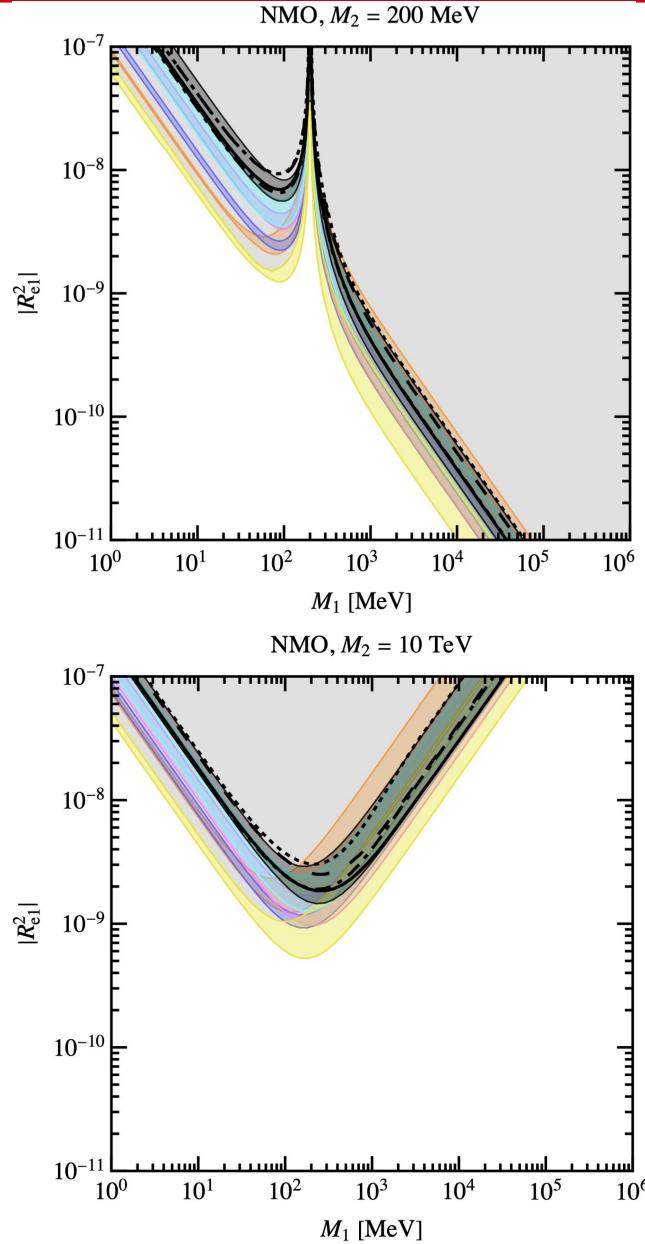
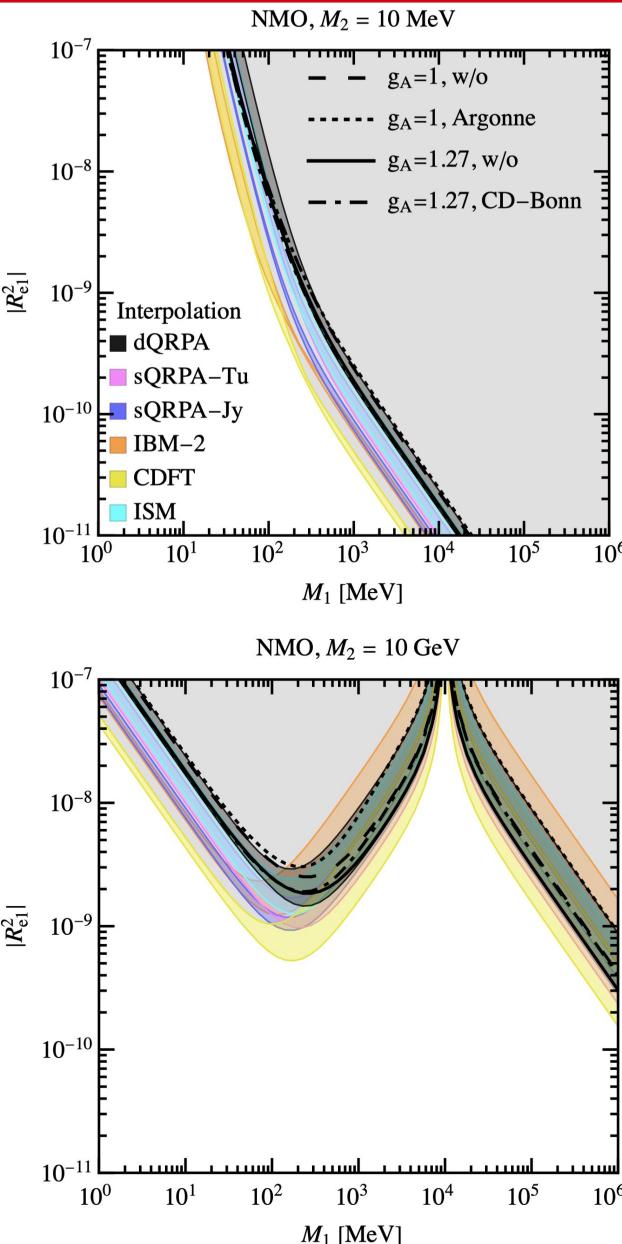
- **dQRPA:** Numerical calculation
 - **Others:** interpolation with two extreme values
- $$M_{0\nu}(m_j) = \frac{m_p m_e}{\langle p^2 \rangle + m_j^2} M_H$$
- **dQRPA:** agrees with ISM for light neutrinos and tends to be consistent with CDFT for heavy neutrinos
 - **In light neutrino mass** the NME from **dQRPA** model is smaller than that of the **IBM-2** model, and in heavy neutrino mass the reverse applies.

$\Delta\chi^2$ functions of m_{eff}

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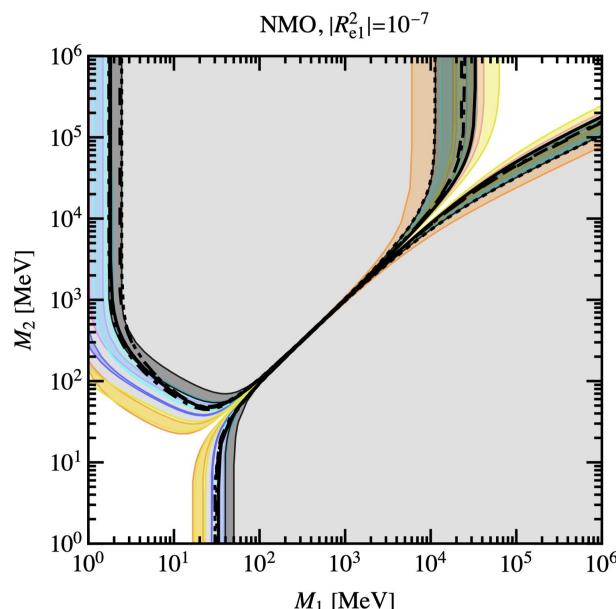
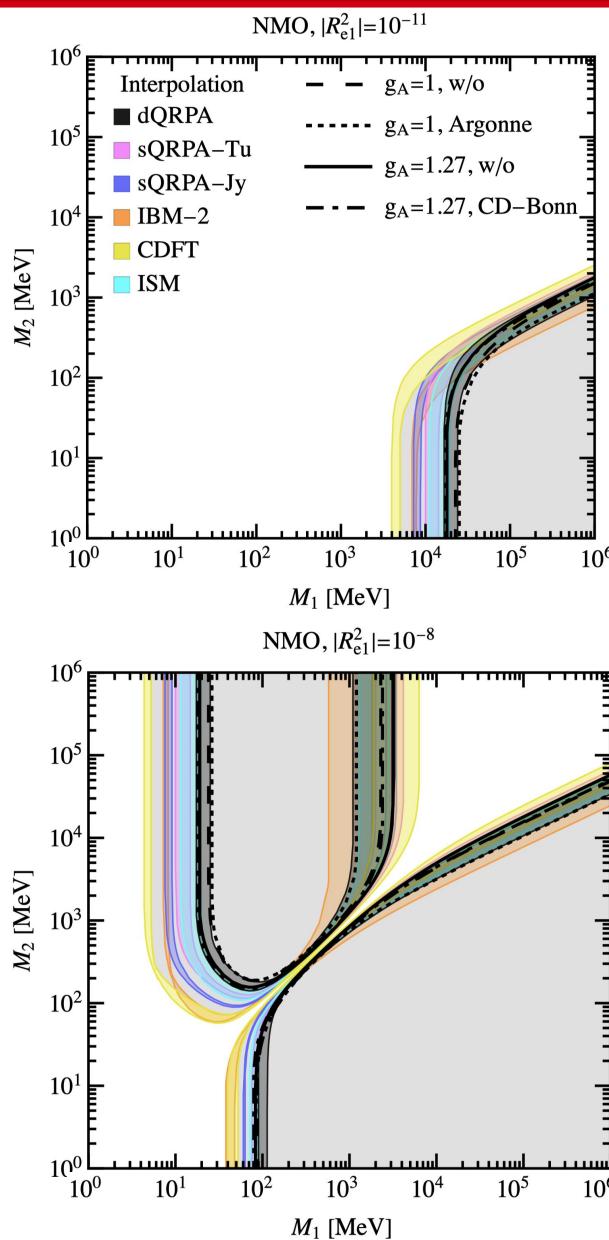
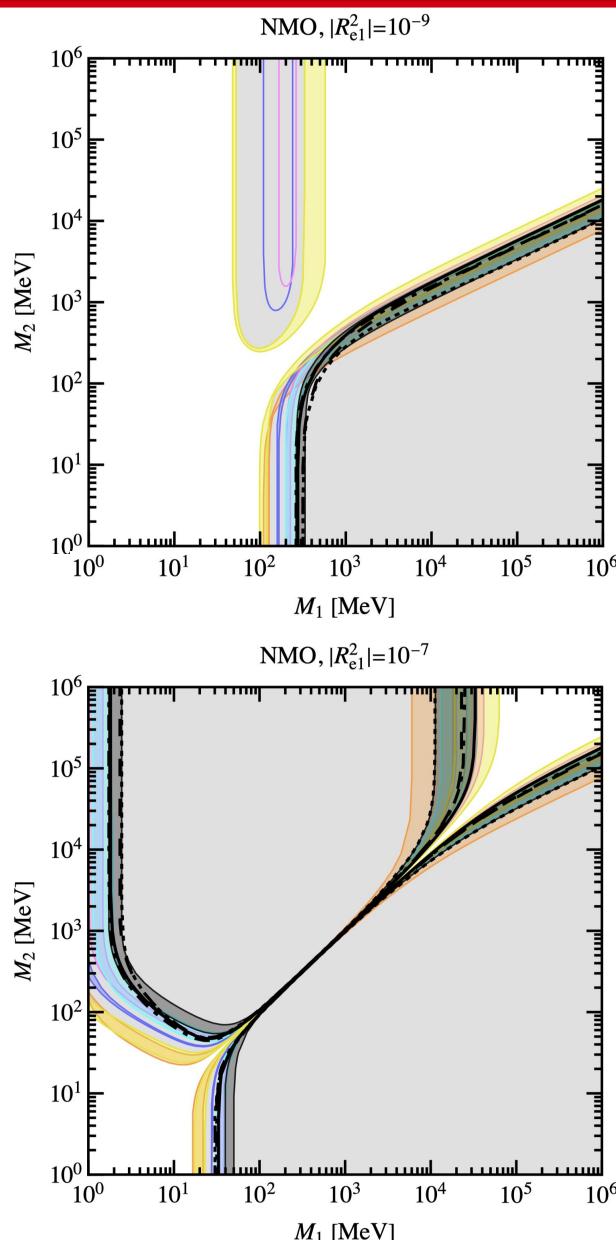
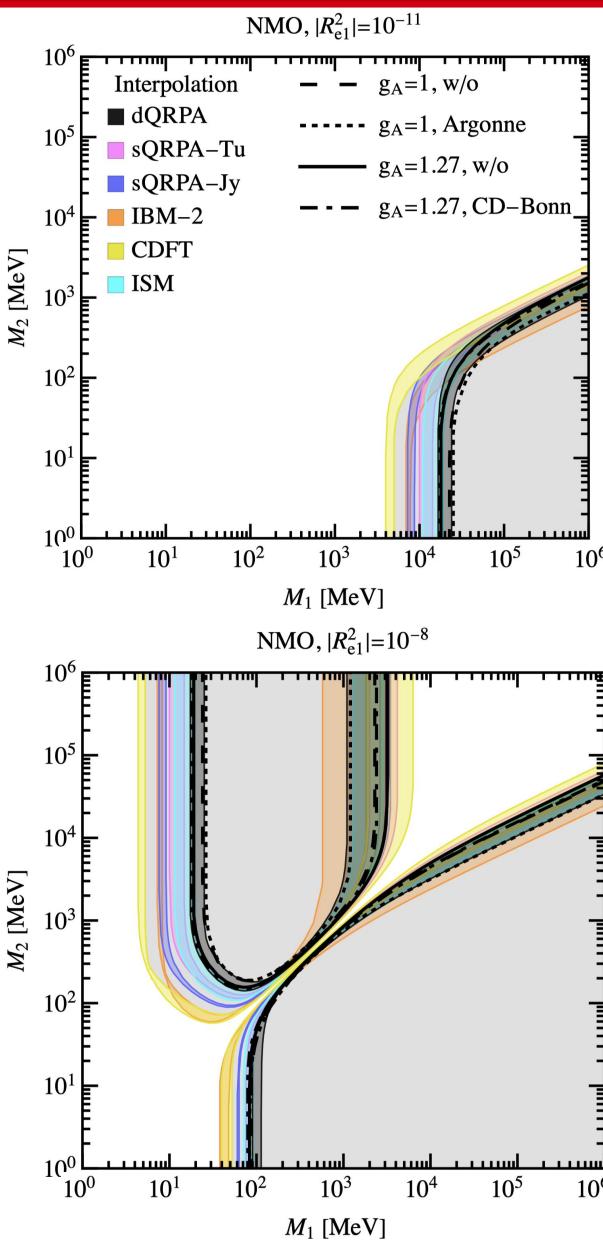


Current limits (M_1 & $|R_{e1}|^2$)



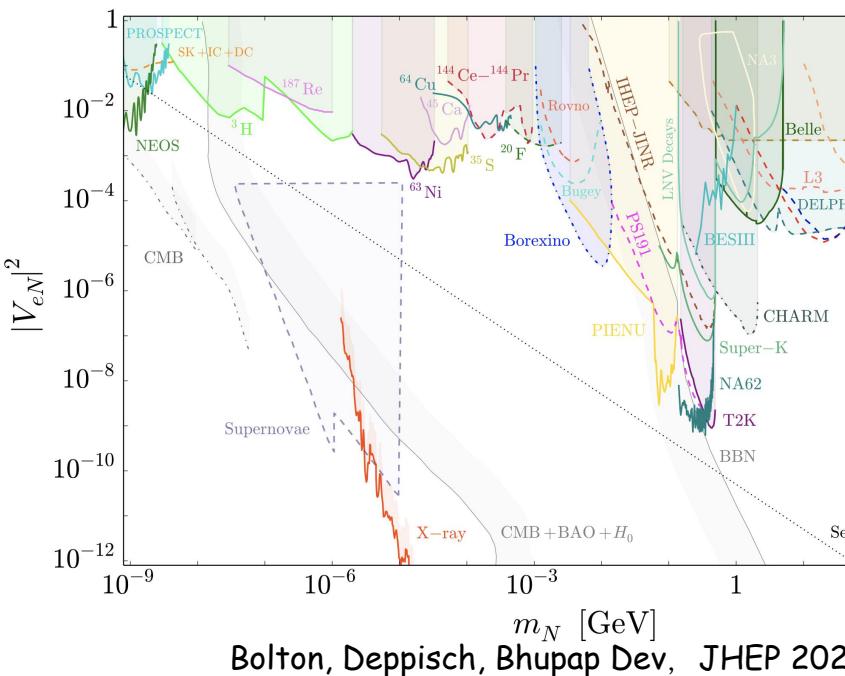
- 3σ C.L.
- Gray regions: excluded regions in the case of CDFT model
- Different choices of parameters and models are scanned (not as Gaussian)
- Both the **Ov $\beta\beta$ -decay** and **oscillation data** are used
- The **IMO** case is similar
- The peak shape

Current limits (M_1 & M_2)

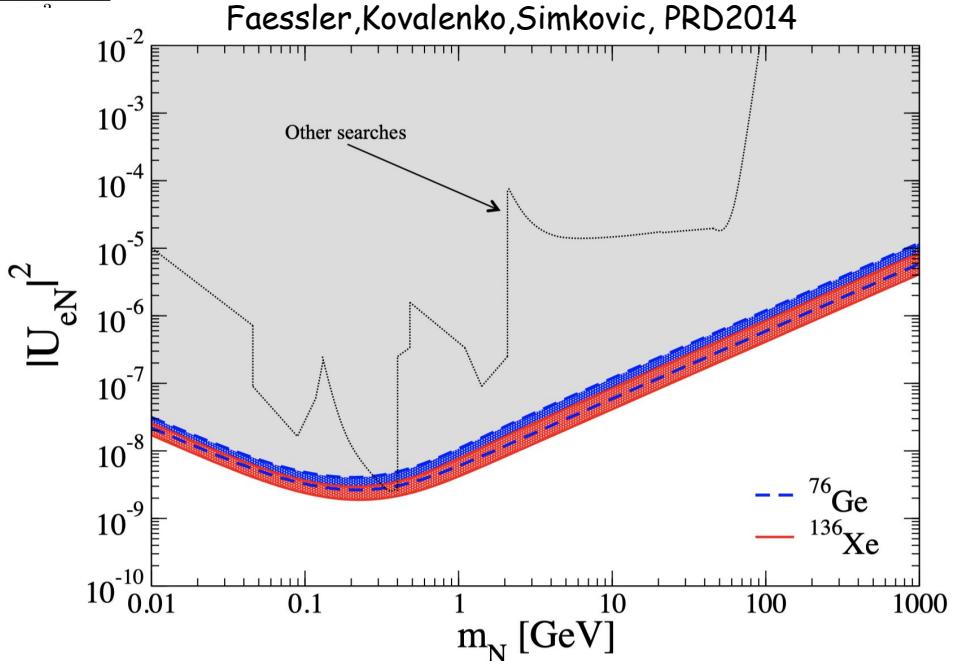


- The IMO case is similar
- The NME hierarchy changes with neutrino mass

Constraints from other probes



- 3+1 case: similar to the case $M_2 \gg M_1$
- $0\nu\beta\beta$ data provide strongest limits in the mass range considered here



Future sensitivities

$$\Delta\chi^2_{ij}(m_{\text{eff}}, (M_{0\nu})_{\alpha j}; m_{\text{eff}}^{\text{True}}, (M_{0\nu})_{\alpha i}^{\text{True}}) = 2 \sum_{\alpha} (N_{\alpha j} - N_{\alpha i}^{\text{True}} + N_{\alpha i}^{\text{True}} \ln \frac{N_{\alpha i}^{\text{True}}}{N_{\alpha j}})$$

Assumed number events

$$N_{\alpha i}^{\text{True}} = B_{\alpha i} + S_{\alpha i}(m_{\text{eff}}^{\text{True}}, (M_{0\nu})_{\alpha i}^{\text{True}})$$

$$N_{\alpha j} = B_{\alpha j} + S_{\alpha j}(m_{\text{eff}}, M_{\alpha j})$$

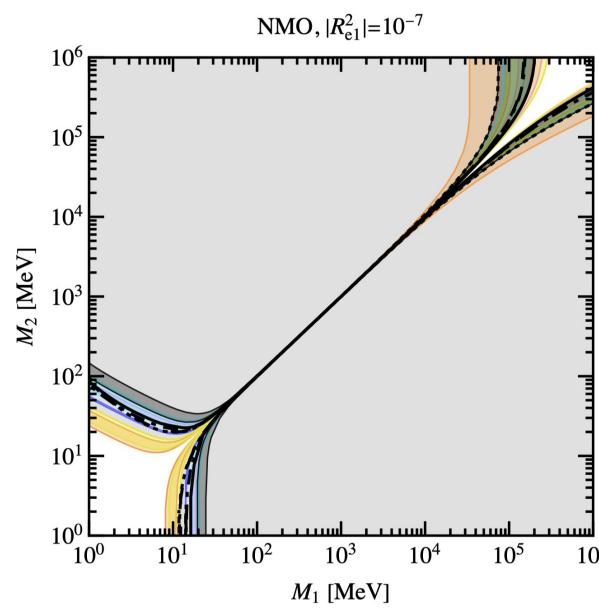
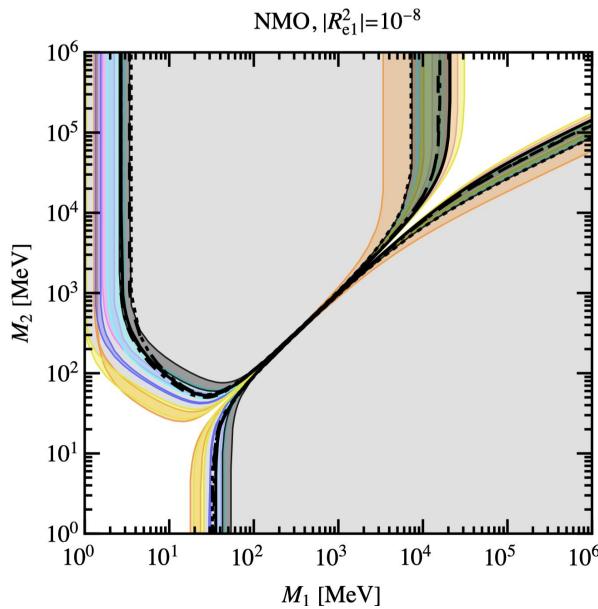
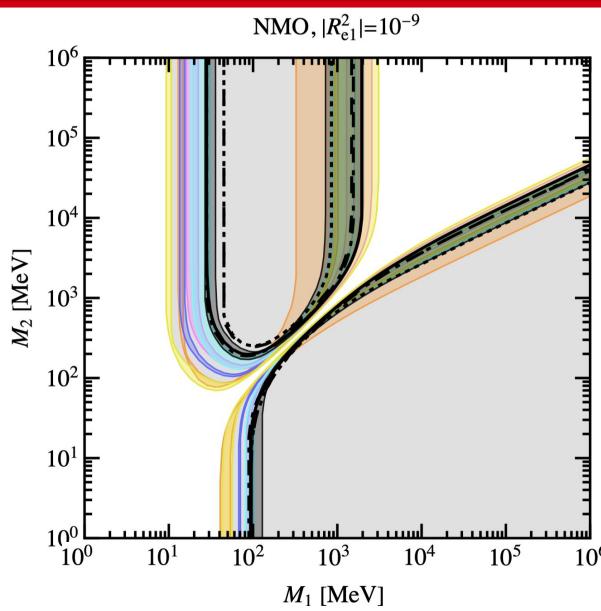
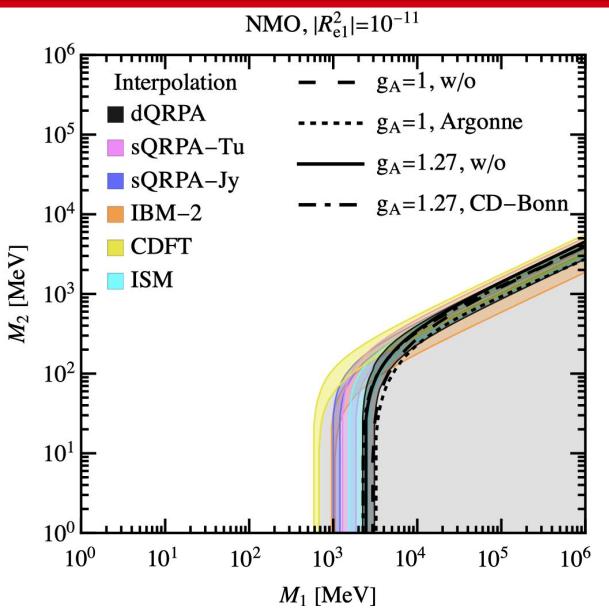
Assuming no positive $0\nu\beta\beta$ signal is observed,
Leading to sensitivities independent of true NME model

$$S_{\alpha i}(m_{\text{eff}}, M_{\alpha i}) = \ln 2 \cdot N_A \cdot \varepsilon_{\alpha} \cdot (T_{1/2}^{0\nu})_{\alpha i}^{-1} \cdot T / (1 \text{ yr})$$

$$B_{\alpha} = b_{\alpha} \cdot \varepsilon_{\alpha} \cdot T / (1 \text{ yr}) \quad \text{T=10 yr}$$

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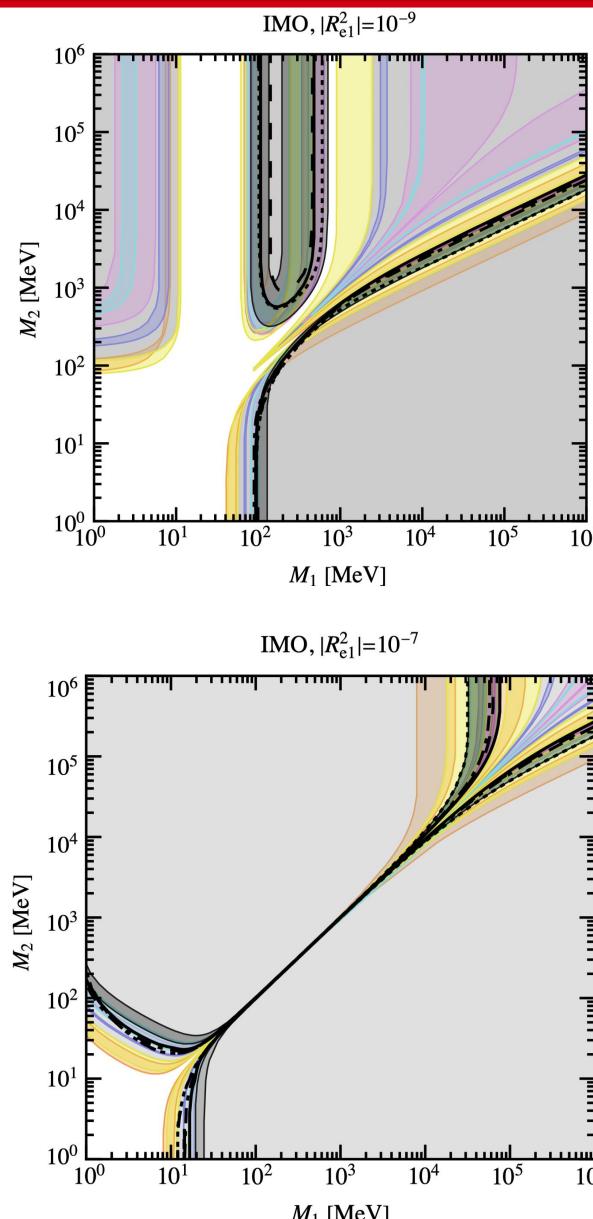
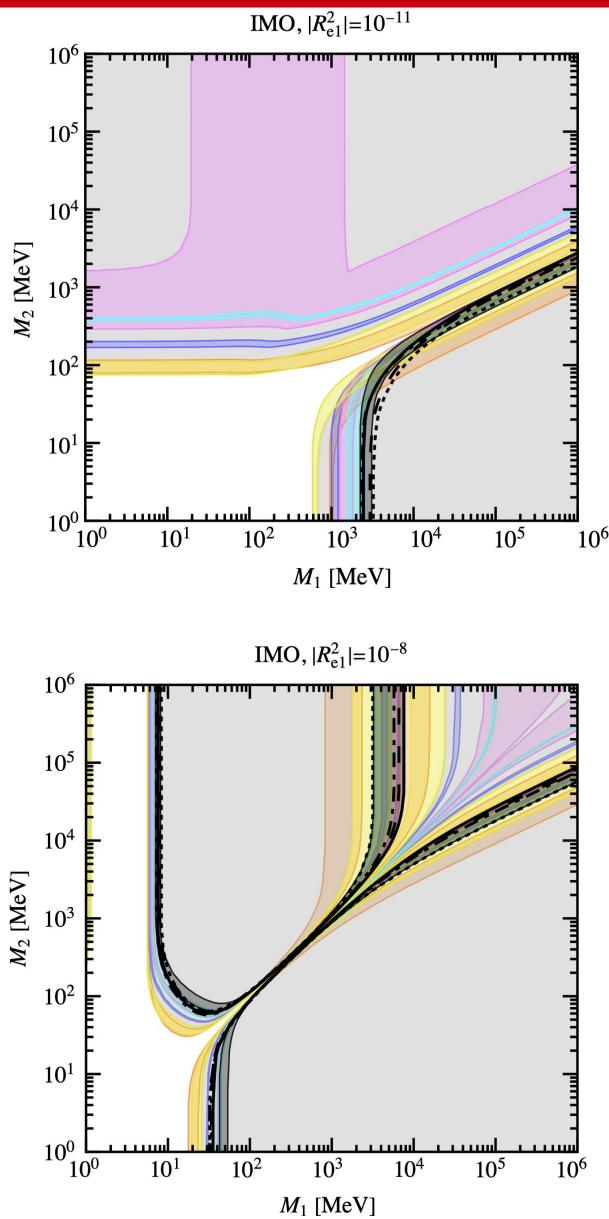
Future sensitivities (M_1 & M_2)



The NMO case

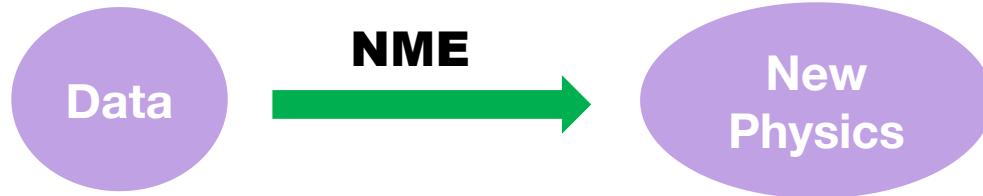
More parameter space can be tested compared to the current experiments

Future sensitivities (M_1 & M_2)

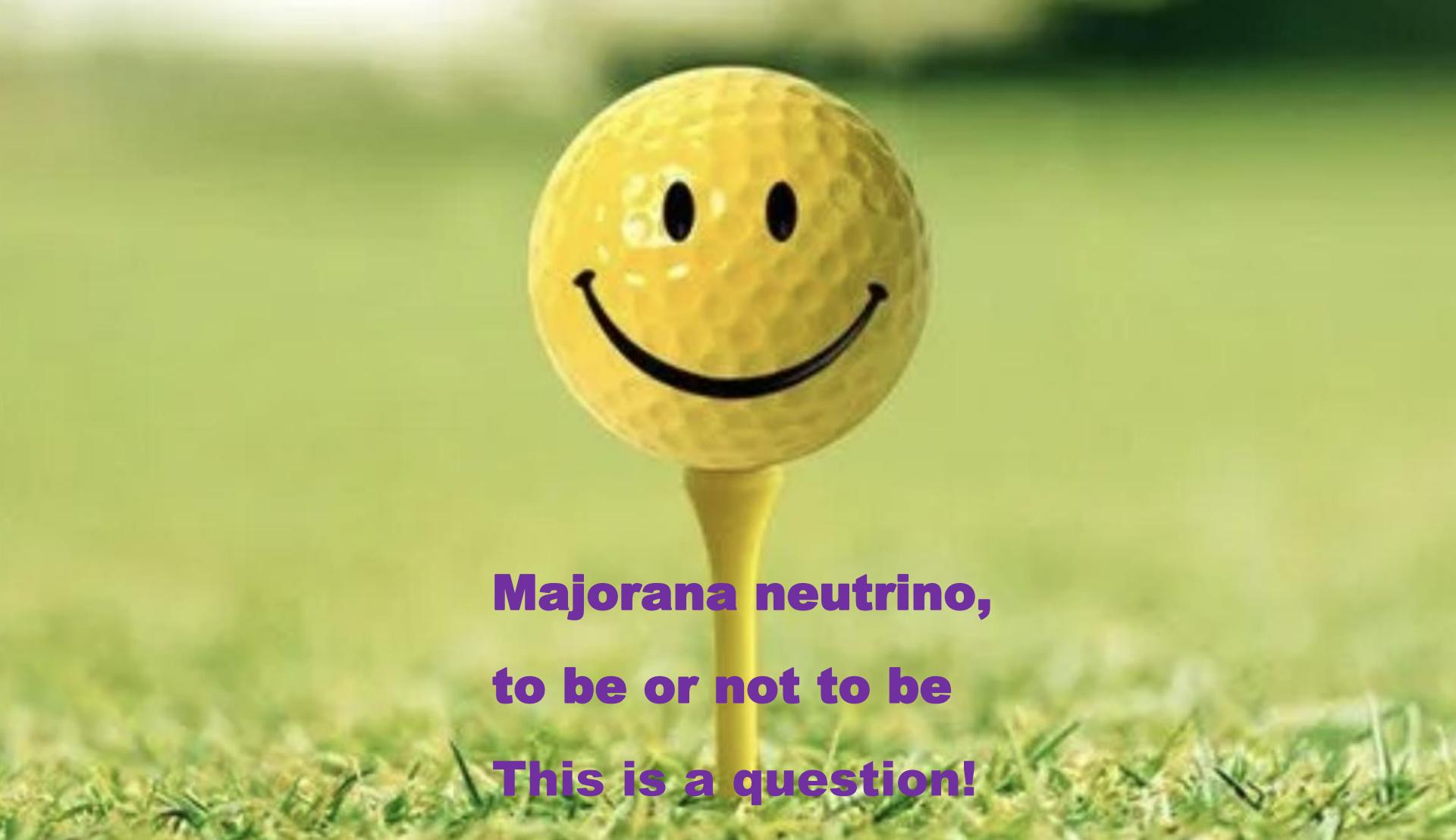


- The IMO case
- The wide pink region in the upper left panel: mainly different δ_{14} values
- Much more parameter space are expected to exclude than NMO case due to zero positive $0\nu\beta\beta$ signal assumed
- By assuming **enough positive $0\nu\beta\beta$ signal**, possible to **discriminate NME calculations** and more parameter space can be **excluded in the NMO case**

- NME uncertainties due to the SRI may lead to the bound on $q^2 m_{\beta\beta}$ varying by a factor of order 5
- Promising discrimination of different NMEs if $(q^2 m_{\beta\beta})_{\text{True}} > 40 \text{ meV}$, positive SRI and 10 year exposure
- Comparison of mass dependent NMEs in different nuclear models
- Derivation of limits and sensitivities on the parameter space of minimal type-I seesaw from current and future $0\nu\beta\beta$ experiments



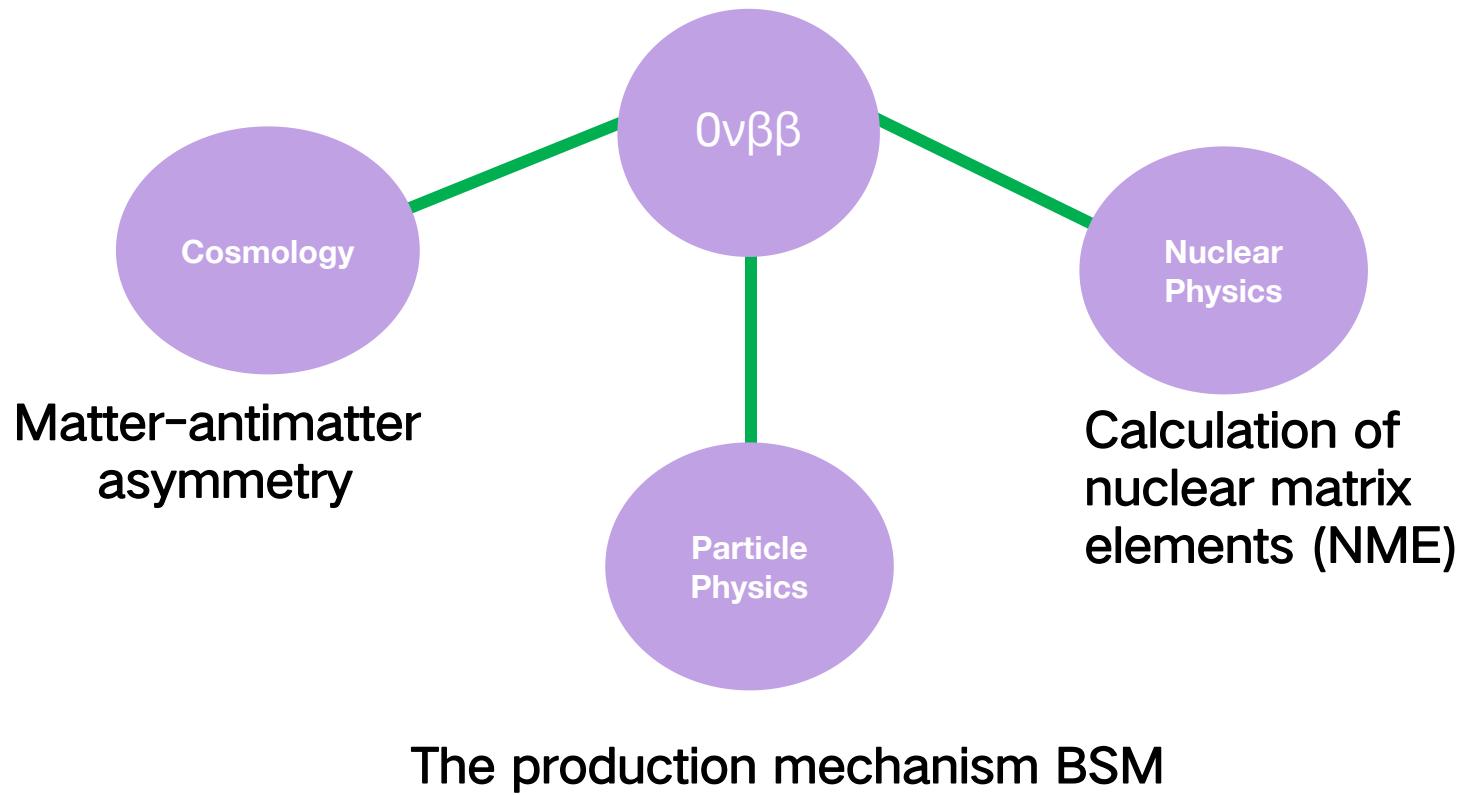
- **Better understanding the short-range NME in $0\nu\beta\beta$**
- **Better understanding the nuclear structure**
- **The quenching problem**
- **NME statistical uncertainties**
- **LEC from lattice calculations**
- **Sensitivities to different particle physics mechanisms and how to distinguish them**
- ...
- **From $0\nu\beta\beta$ to $m_{\beta\beta}$: improving the calculations of NME**
- **From $0\nu\beta\beta$ to discriminating NME models: more information on $m_{\beta\beta}$**



**Majorana neutrino,
to be or not to be
This is a question!**

Thank you!

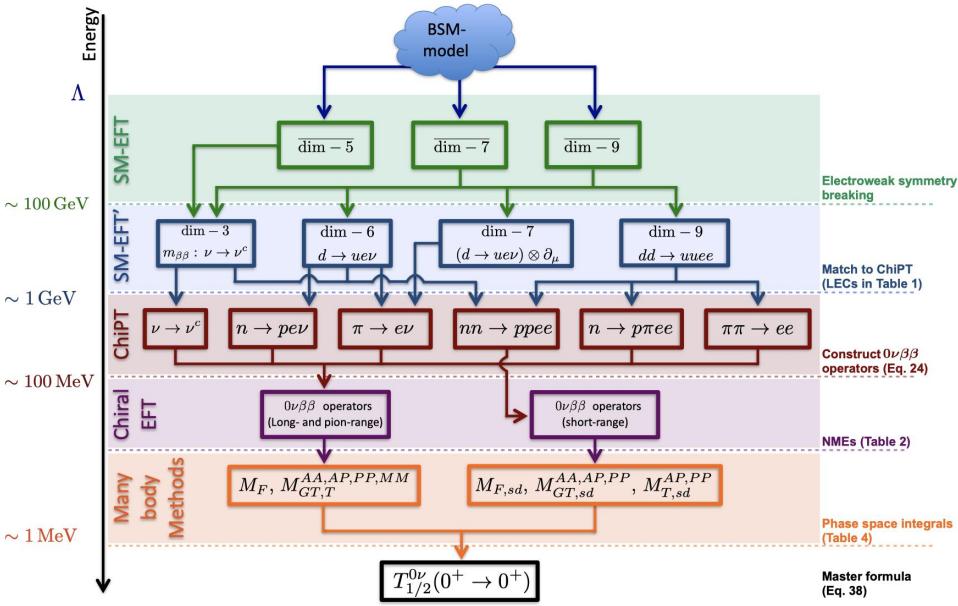
Backups



Experiments:

- Find compromises between nature abundance, Q-value, enrichment and detector techniques
- Key parameter: background ratio, energy resolution, exposure

A general theoretical framework



Cirigliano et al., JHEP 2018

Naive Dimensional Analysis(NDA) problem

- D. B. Kaplan, M. J. Savage, and M. B. Wise, *Nucl. Phys.* **B478**, 629 (1996).
- S. R. Beane, P. F. Bedaque, M. J. Savage, and U. van Kolck, *Nucl. Phys.* **A700**, 377 (2002).
- A. Nogga, R. G. E. Timmermans, and U. van Kolck, *Phys. Rev. C* **72**, 054006 (2005).
- B. Long and C.-J. Yang, *Phys. Rev. C* **86**, 024001 (2012).
- M. Pavón Valderrama and D. R. Phillips, *Phys. Rev. Lett.* **114**, 082502 (2015).

and $F_\pi = 92.2 \text{ MeV}$ is the pion decay constant. However, it is known that Weinberg's power counting leads to inconsistent results in nucleon-nucleon scattering [34–37] and nuclear processes mediated by external currents [38], due to a conflict between naive dimensional analysis and nonperturbative renormalization. We therefore investigate the scaling of g_ν^{NN} by studying the amplitude $\mathcal{A}(nn \rightarrow ppee) \equiv \mathcal{A}_{\Delta L=2}$ with strong interactions H_{strong} included nonperturbatively.

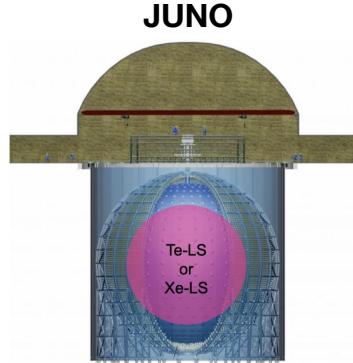
Cirigliano et al., *Phys. Rev. Lett.* **120** (2018) 20, 202001

Experimental techniques

CUPID-Mo



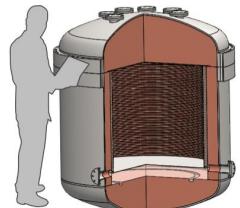
JUNO



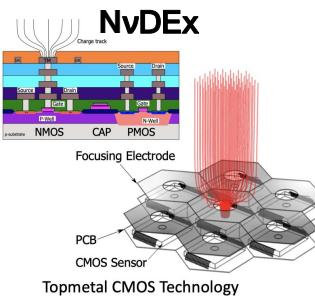
CANDLE CaF scintillating crystal



PandaX-4T LXe TPC



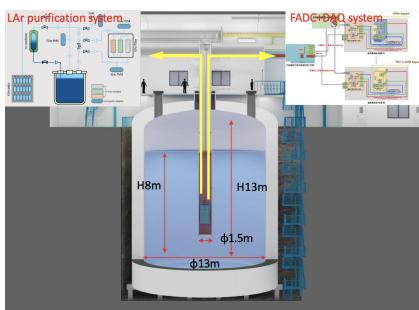
PandaX-III GXe TPC



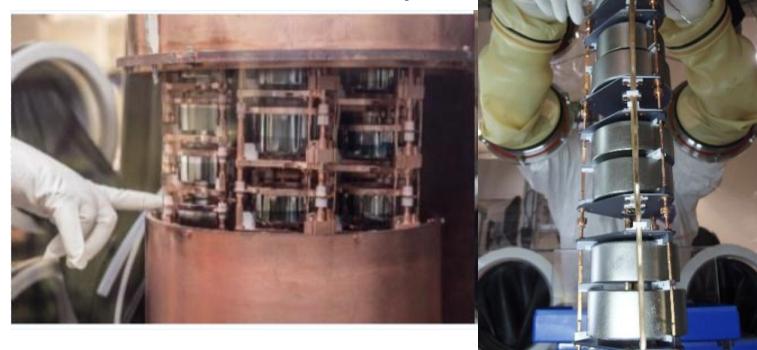
EXO, KamLAND-Zen Liquid Xe



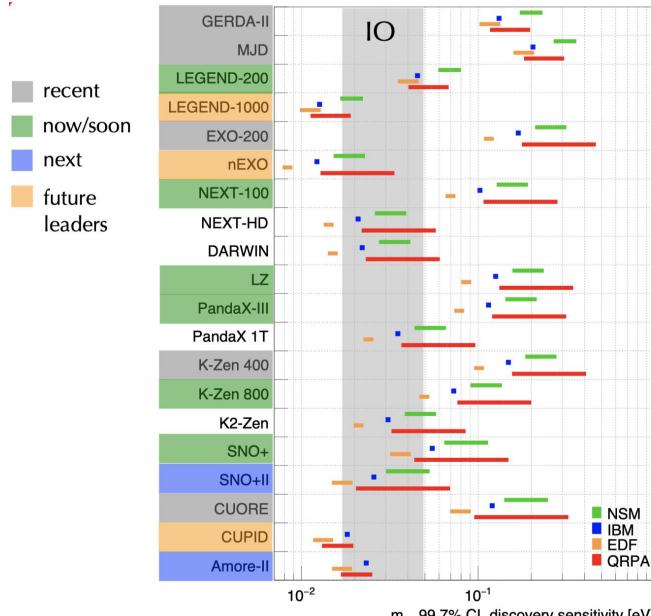
CDEX



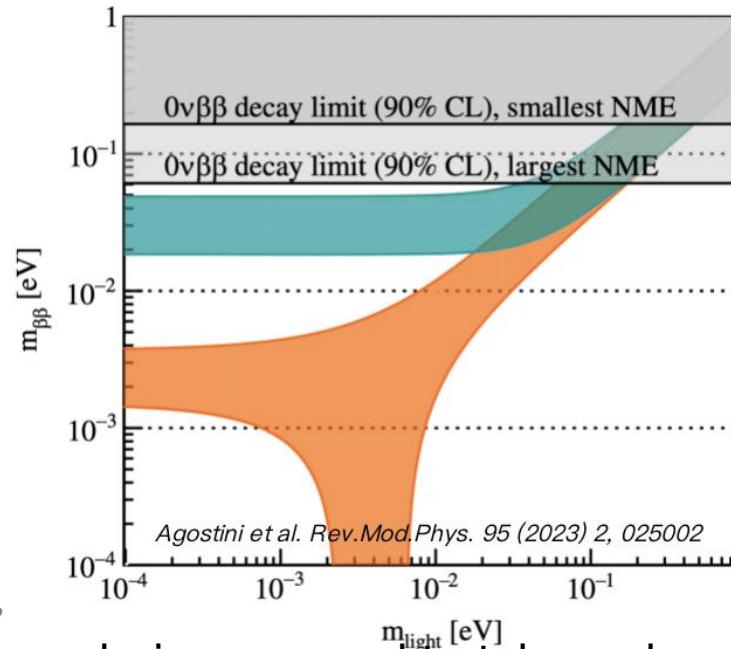
GERDA, MAJORANA Ge crystal



etc.



Agostini *et al.* Rev. Mod. Phys. 95 (2023) 2, 025002

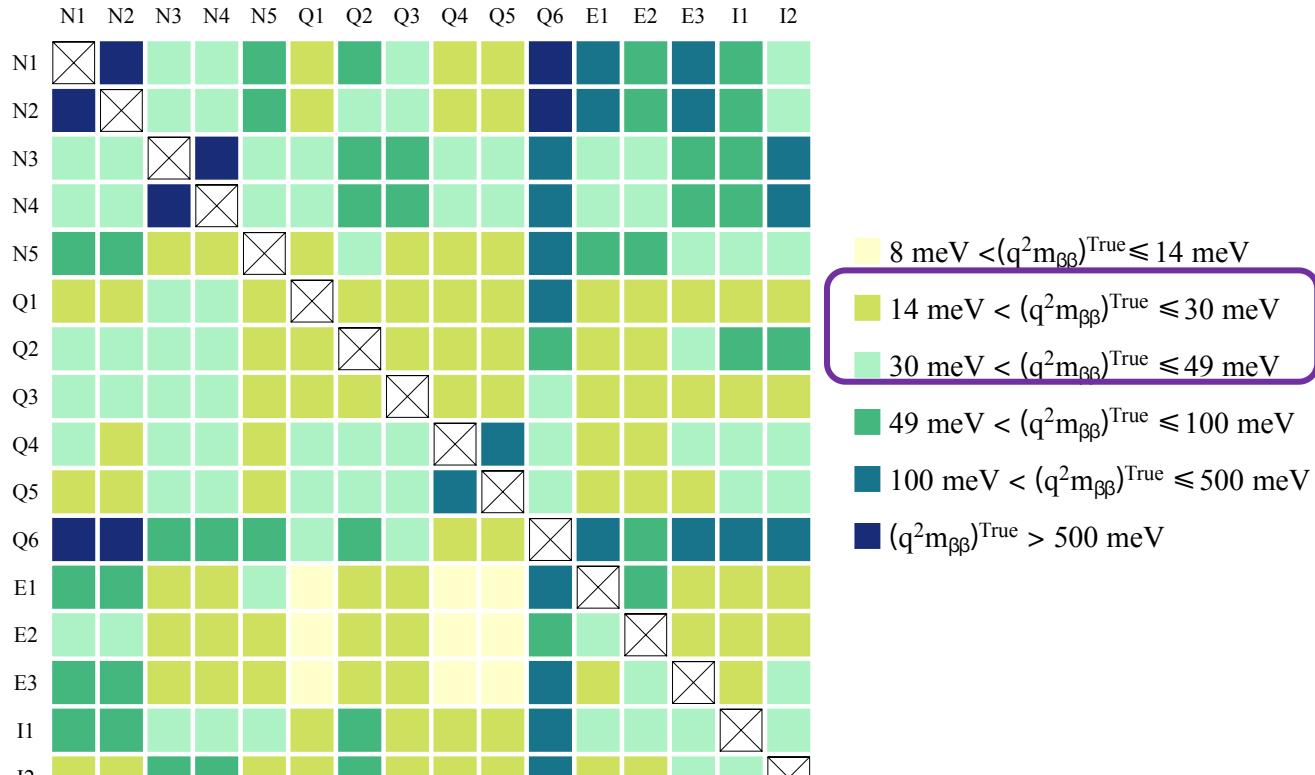


New techniques and more exposure are being pursued to take us beyond the IO.
Discovery could come at any time!

Motivations: Schwetz, Popma, Zhu, JHEP 06 (2023) 104

- Interpreting the constraints/sensitivities on $m_{\beta\beta}$ of current/future 0v $\beta\beta$ experiments
- Checking the possibilities of discriminating NME models in future 0v $\beta\beta$ experiments

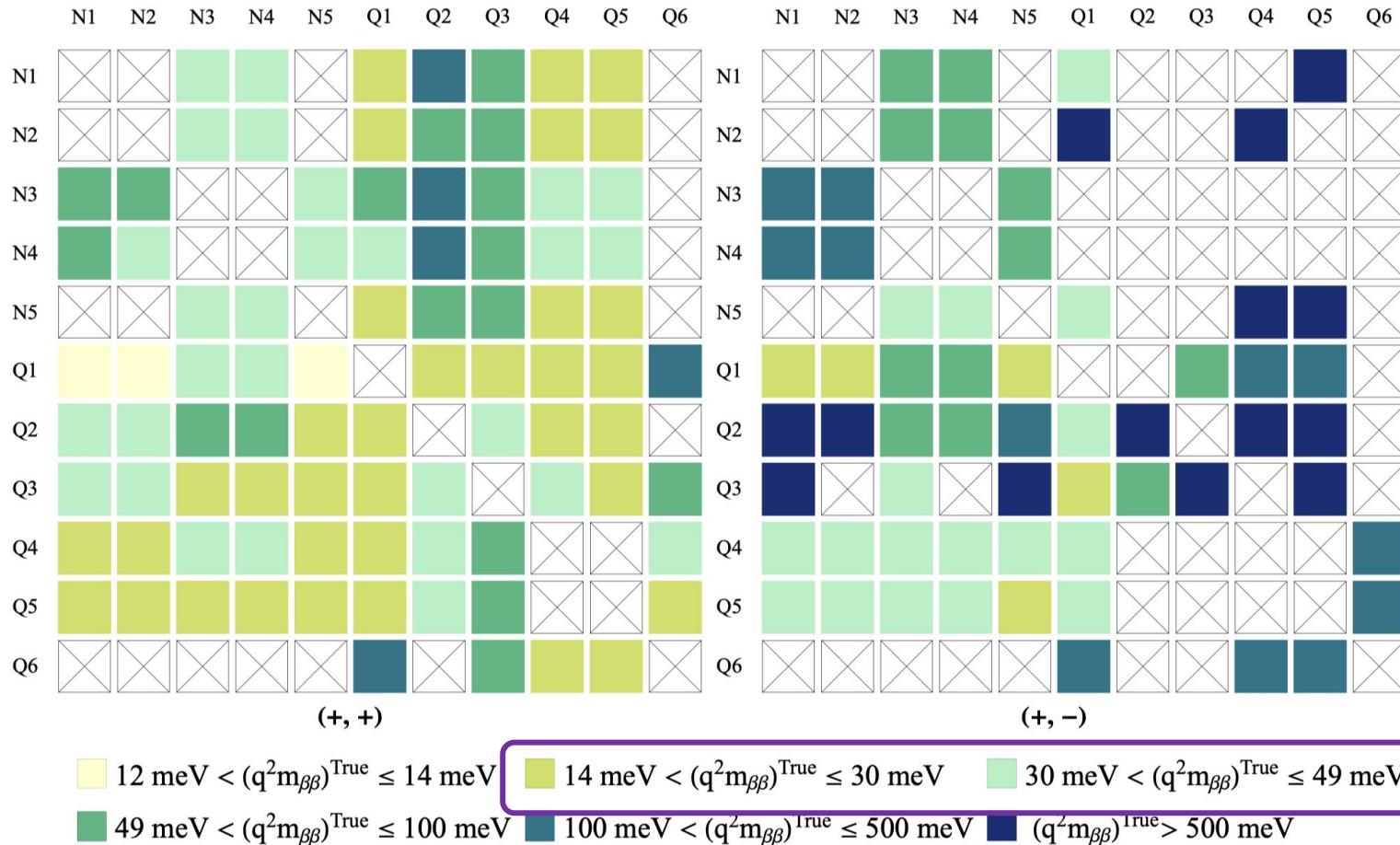
$m_{\beta\beta}^{\text{True}}$ corresponding to discrimination at 3σ



(without short- range NME)

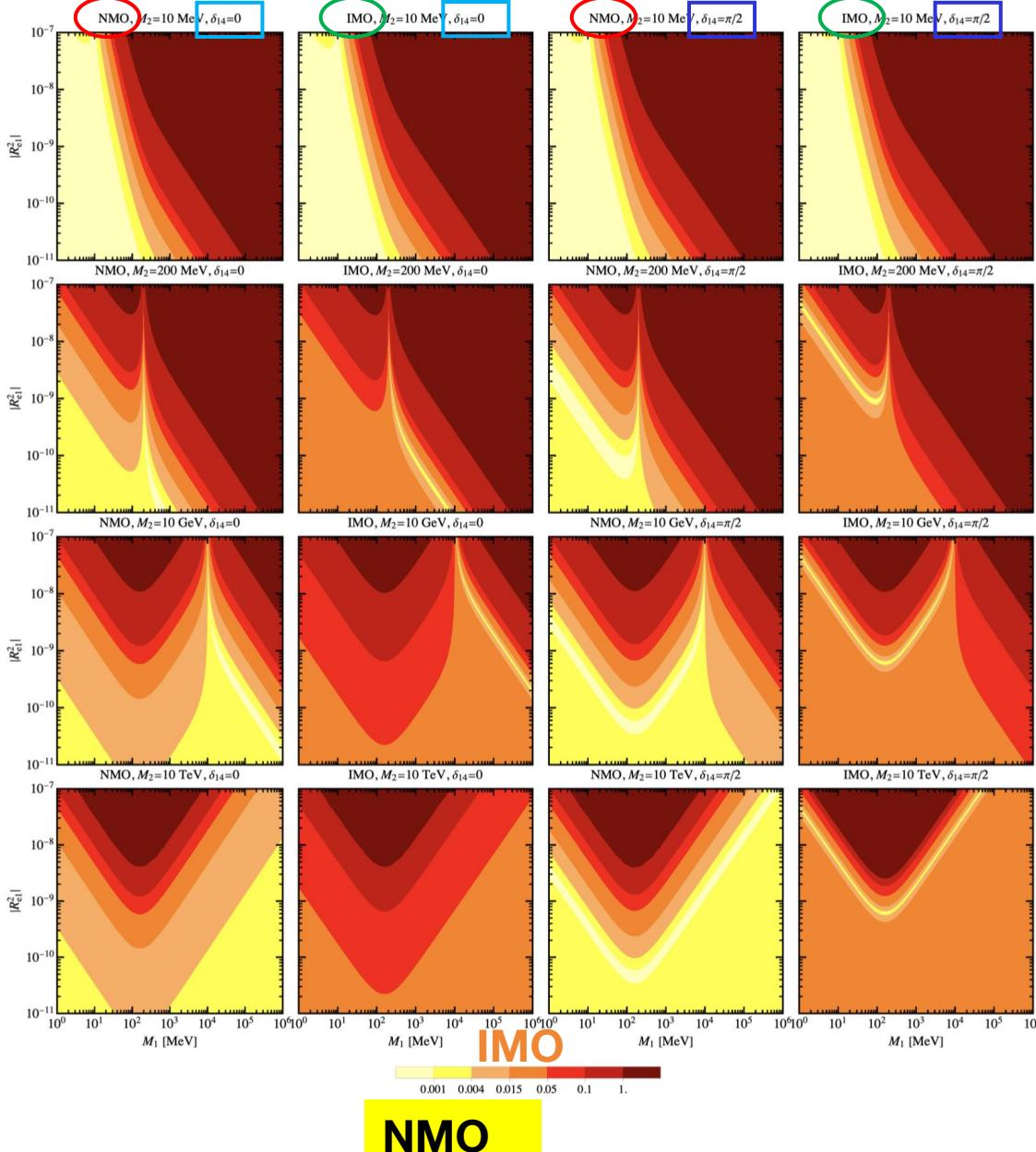
$m_{\beta\beta}^{\text{True}}$ corresponding to discrimination at 3σ

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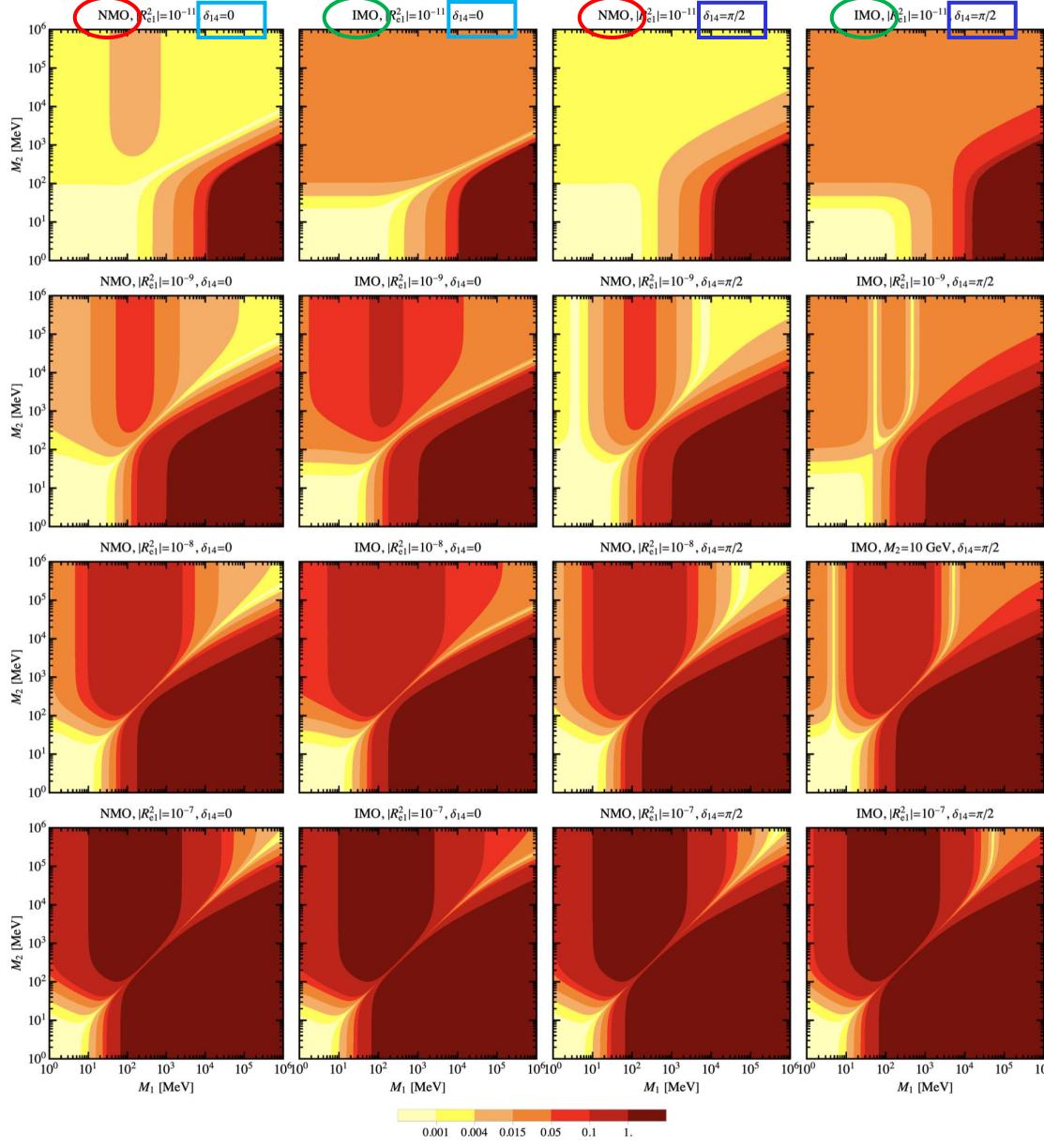
(with short- range NME)

Parameter space of m_{eff}



- $g_A=1$, Argonne src
- Some parameter space can be very easily/hardly excluded by current/future $0\nu\beta\beta$ experiments
- The NMO/IMO can be very different and δ_{14} matters

Parameter space of m_{eff}



NME of light neutrinos

	g_A	src	dQRPA [74]	sQRPA-Tu [75]	sQRPA-Jy [77]	IBM-2 [87]	CDFT [80]	ISM [81]
^{76}Ge	1.27	w/o	3.27				7.61	
		Argonne	3.12	5.157	-	5.98	7.48	2.89
		CD-Bonn	3.40	5.571	6.54	6.16	7.84	3.07
		Miller-Spencer	-	-	-	5.42	6.36	-
	1.00	w/o	2.64	-	-	-	-	-
		Argonne	2.48	3.886	-	-	-	1.77
^{82}Se		CD-Bonn	2.72	4.221	5.26	-	-	1.88
	1.27	w/o	3.01	-	-	-	7.60	-
		Argonne	2.86	4.642	-	4.84	7.48	2.73
		CD-Bonn	3.13	5.018	4.69	4.99	7.83	2.90
		Miller-Spencer	-	-	-	4.37	6.48	-
	1.00	w/o	2.41	-	-	-	-	-
^{130}Te		Argonne	2.26	3.460	-	-	-	2.41
		CD-Bonn	2.49	3.746	3.73	-	-	2.56
	1.27	w/o	3.10				9.55	
		Argonne	2.90	3.888		4.47	9.38	2.76
		CD-Bonn	3.22	4.373	5.27	4.61	9.82	2.96
		Miller-Spencer	-	-	-	4.03	8.03	
^{136}Xe	1.00	w/o	2.29					
		Argonne	2.13	2.945	-	-	-	1.72
		CD-Bonn	2.37	3.297	4.00	-	-	1.84
	1.27	w/o	1.12	-	-	-	6.62	
		Argonne	1.11	2.177		3.67	6.51	2.28
		CD-Bonn	1.18	2.460	3.50	3.79	6.80	2.45
		Miller-Spencer	-	-	-	3.33	5.58	
	1.00	w/o	0.85					
		Argonne	0.86	1.643	-	-	-	1.42
		CD-Bonn	0.89	1.847	2.91	-	-	1.53

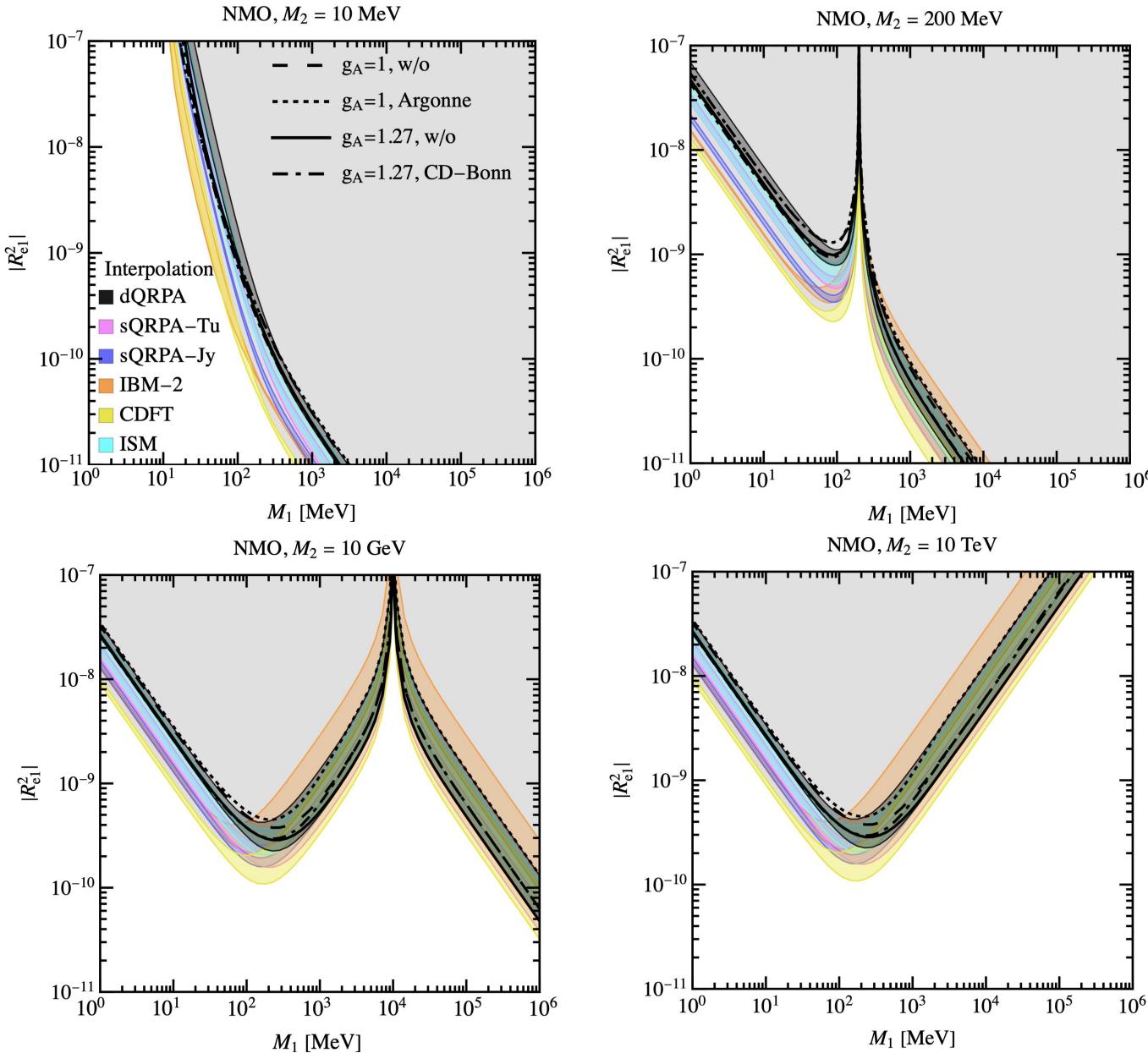
- CDFT biggest
- ISM/dQRPA smallest
- different NME ratios between different isotopes

NME of heavy neutrinos

	g_A	src	dQRPA [74]	sQRPA-Tu [75]	sQRPA-Jy [77]	IBM-2 [87]	CDFT [80]	ISM [81]
^{76}Ge	1.27	w/o	385.4				466.8	
		Argonne	187.3	316		107	267	130
		CD-Bonn	293.7	433	401.3	163	378.1	188
	1.00	Miller-Spencer				48.1	135.7	
		w/o	275.9					
		Argonne	129.7	204				86
^{82}Se	1.27	CD-Bonn	207.2	287	298.3			122
		w/o	358.7				454	
		Argonne	175.9	287		84.4	261.4	121
	1.00	CD-Bonn	273.6	394	287.1	132	369	175
		Miller-Spencer				35.6	132.7	
		w/o	257.4					
^{130}Te	1.27	Argonne	122.1	186	-	-	-	80
		CD-Bonn	193.4	262	214.3	-	-	113
		w/o	401.1				573	
	1.00	Argonne	191.4	292		92	339.2	146
		CD-Bonn	303.5	400	338.3	138	472.8	210
		Miller-Spencer				44	168.5	
^{136}Xe	1.27	w/o	281.2					
		Argonne	130.2	189	-	-	-	97
		CD-Bonn	209.5	264	255.7	-	-	136
	1.00	w/o	117.1				394.5	
		Argonne	66.9	166		72.8	234.3	116
		CD-Bonn	90.5	228	186.3	109	326.2	167
	1.00	Miller-Spencer	-	-	-	35.1	116.3	
		w/o	82.7					
		Argonne	46.3	108	-	-	-	77
		CD-Bonn	62.8	152	137.3	-	-	108

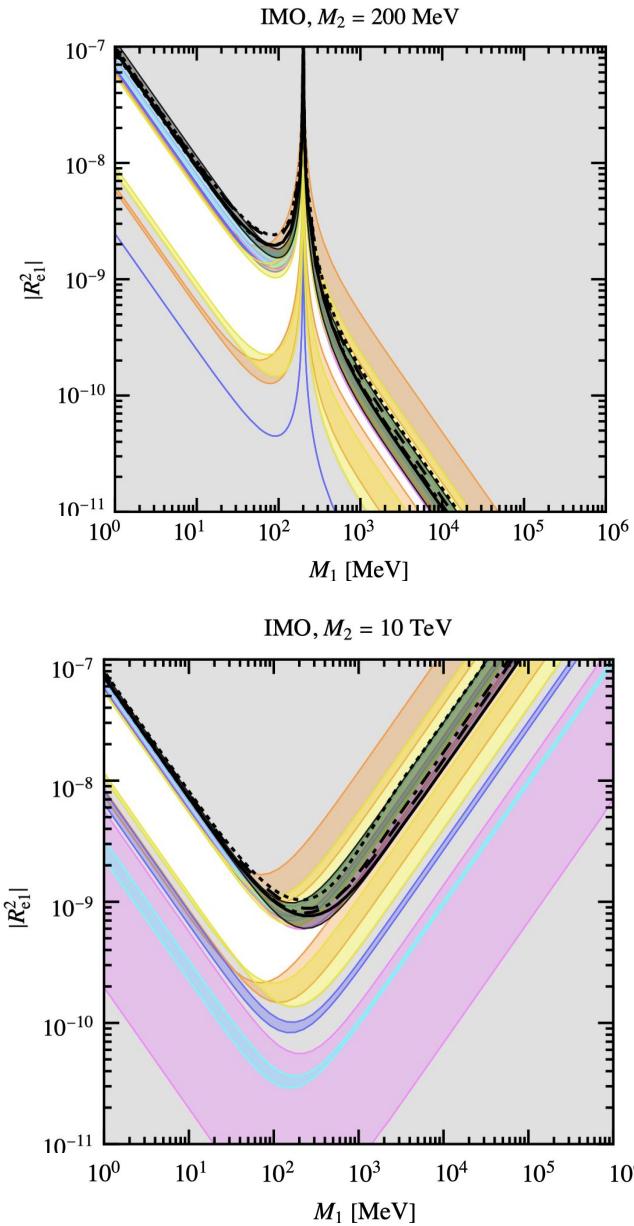
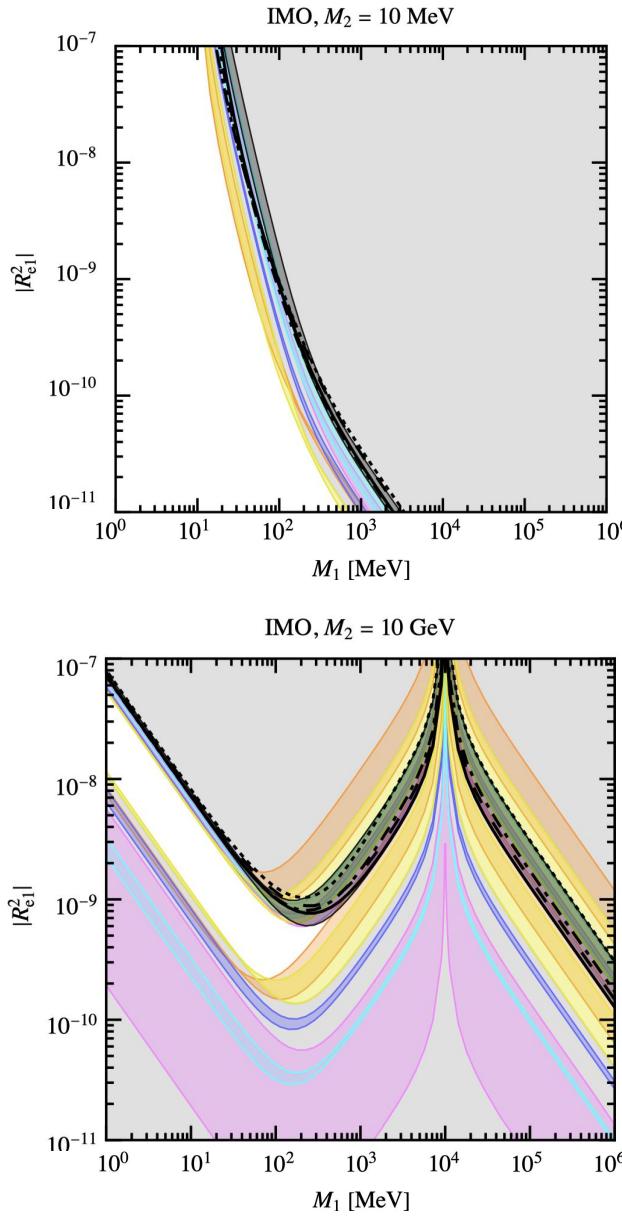
- CDFT biggest
- IBM-2 smallest
- different NME ratios between different isotopes

Future sensitivities (M_1 & $|R_{e1}|^2$)

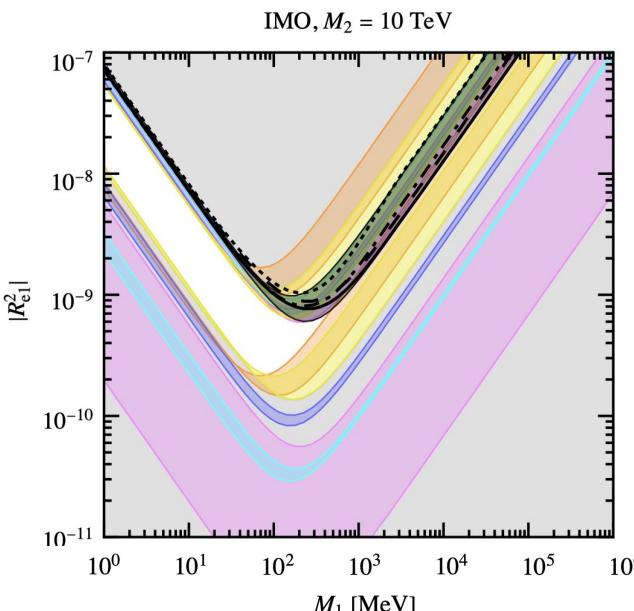
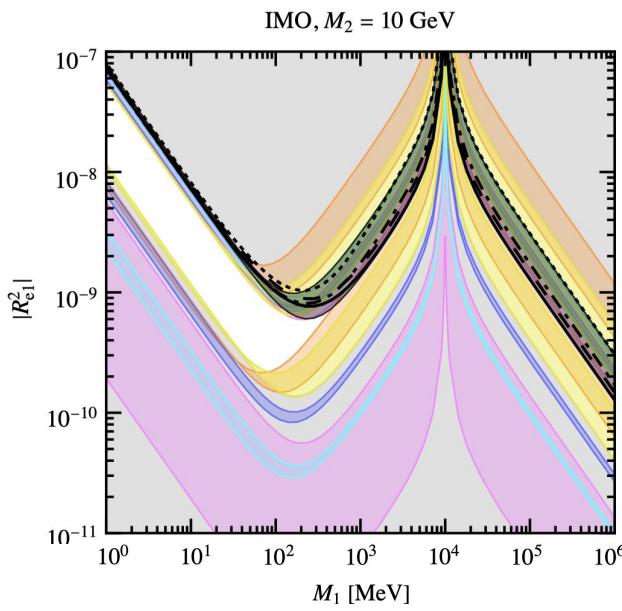


- The NMO case
- More parameter space can be tested compared the current experiments

Future sensitivities (M_1 & $|R_{e1}|^2$)

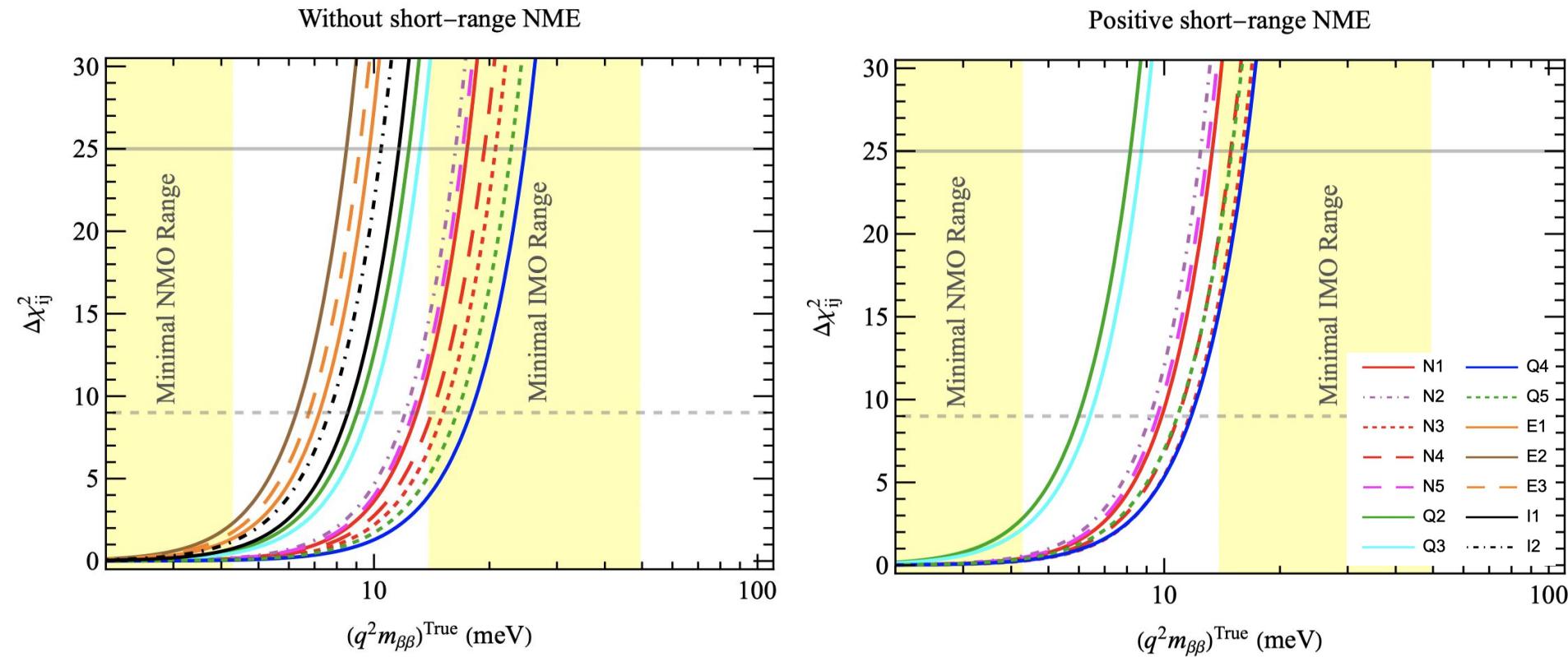


- The IMO case
- Much more parameter space are expected to exclude than NMO case due to zero positive $0\nu\beta\beta$ signal assumed
- By assuming **enough positive $0\nu\beta\beta$ signal**, possible to **discriminate NME calculations** and more parameter space can be **excluded in the NMO case**



The significance of observing one positive signal:

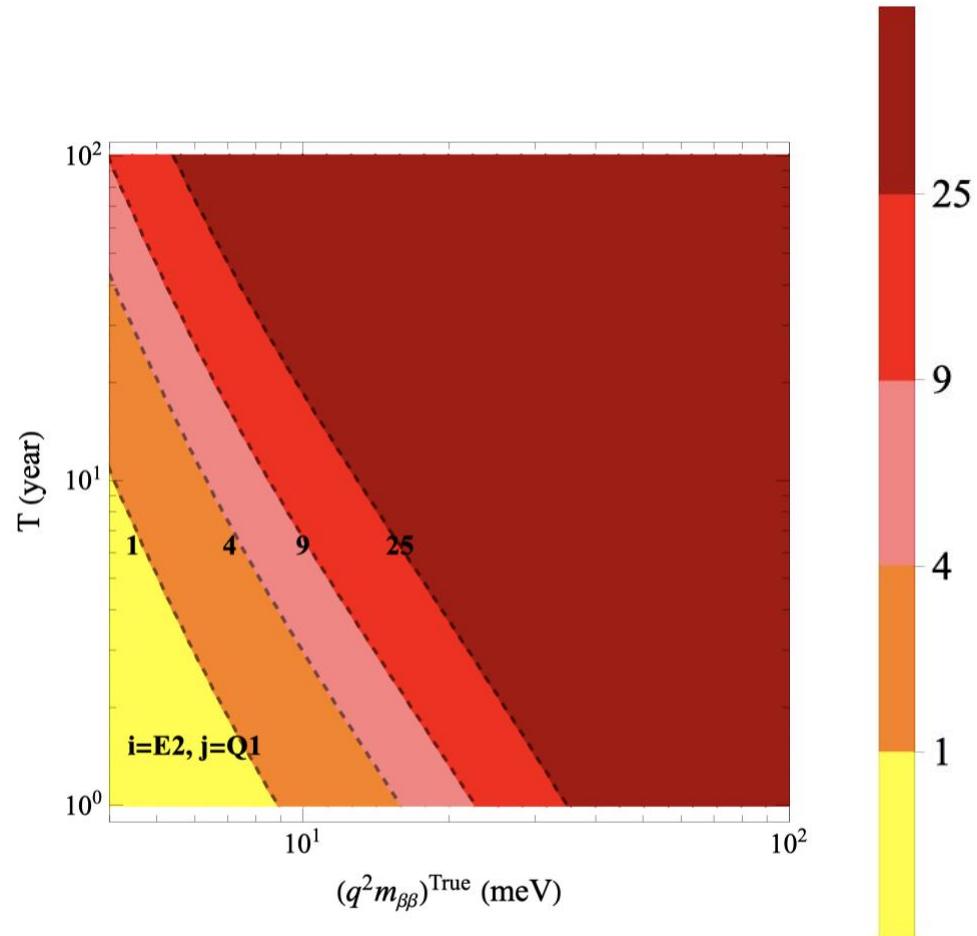
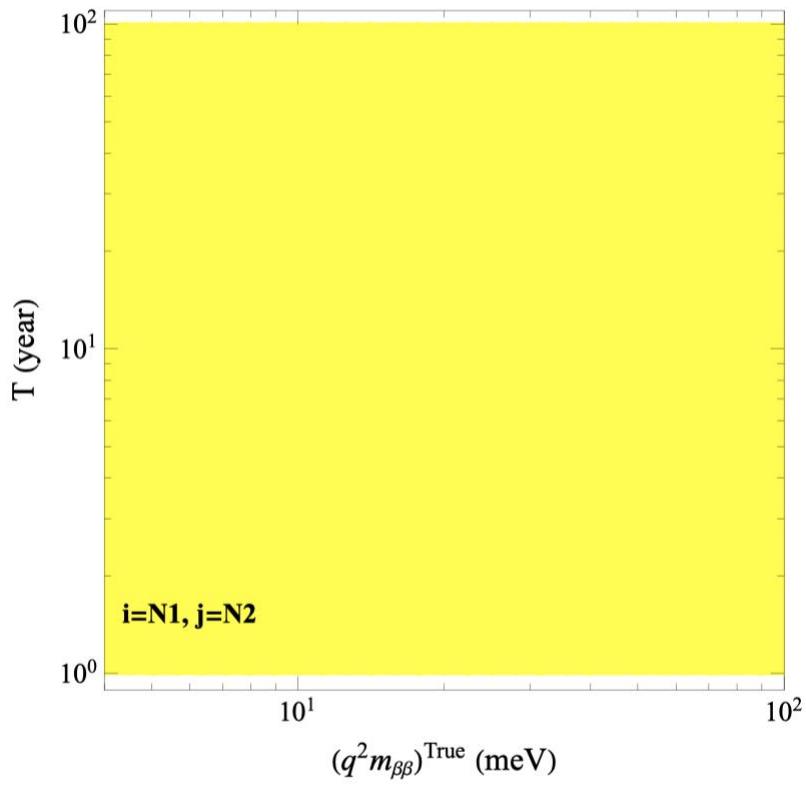
39



$$m_{\beta\beta}=0, T=10 \text{ yr}$$

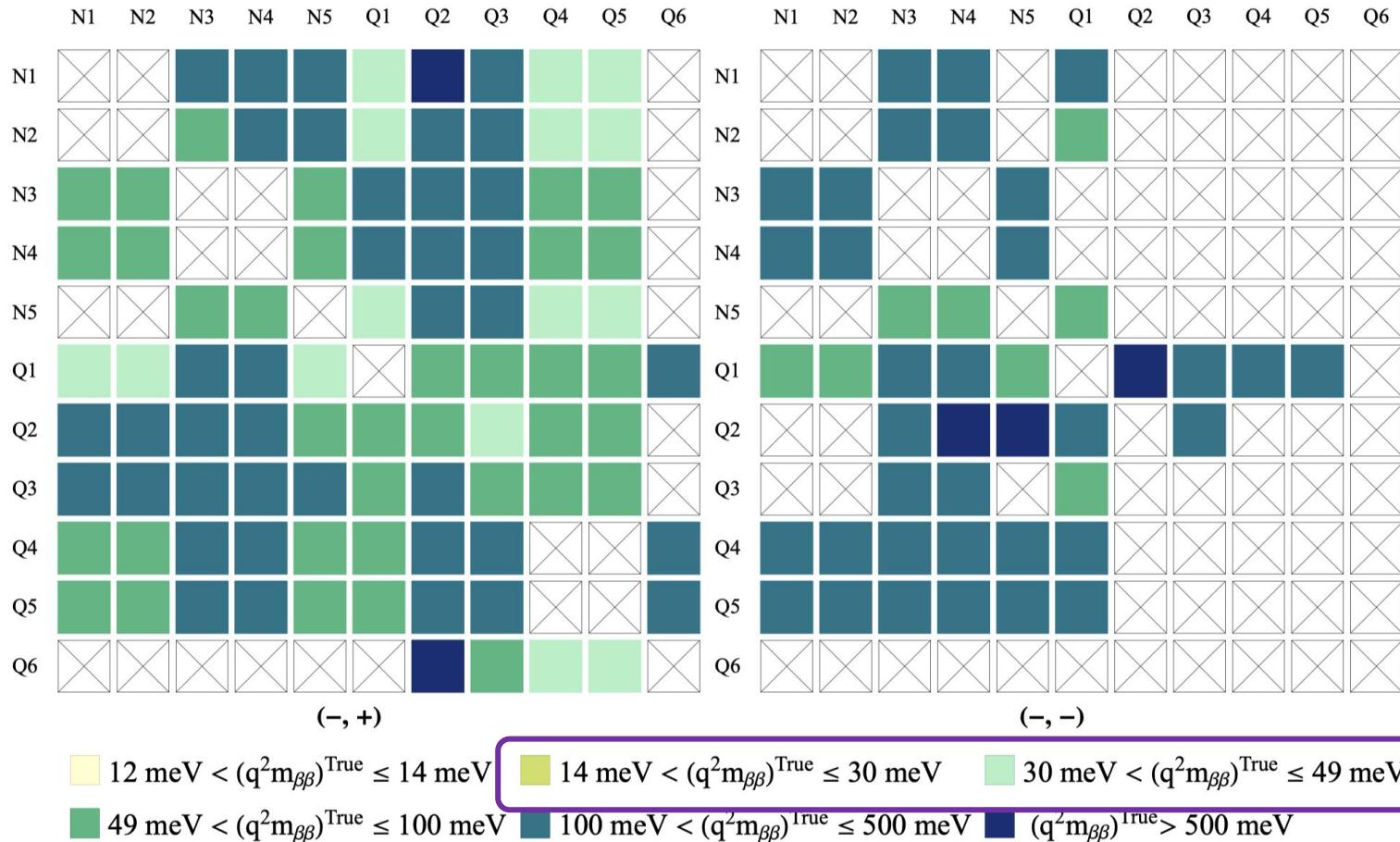
Contours of $(\Delta\chi^2_{ij})_{\min}$ as function of T and $(q^2 m_{\beta\beta})^{\text{True}}$

40



(without short- range NME)

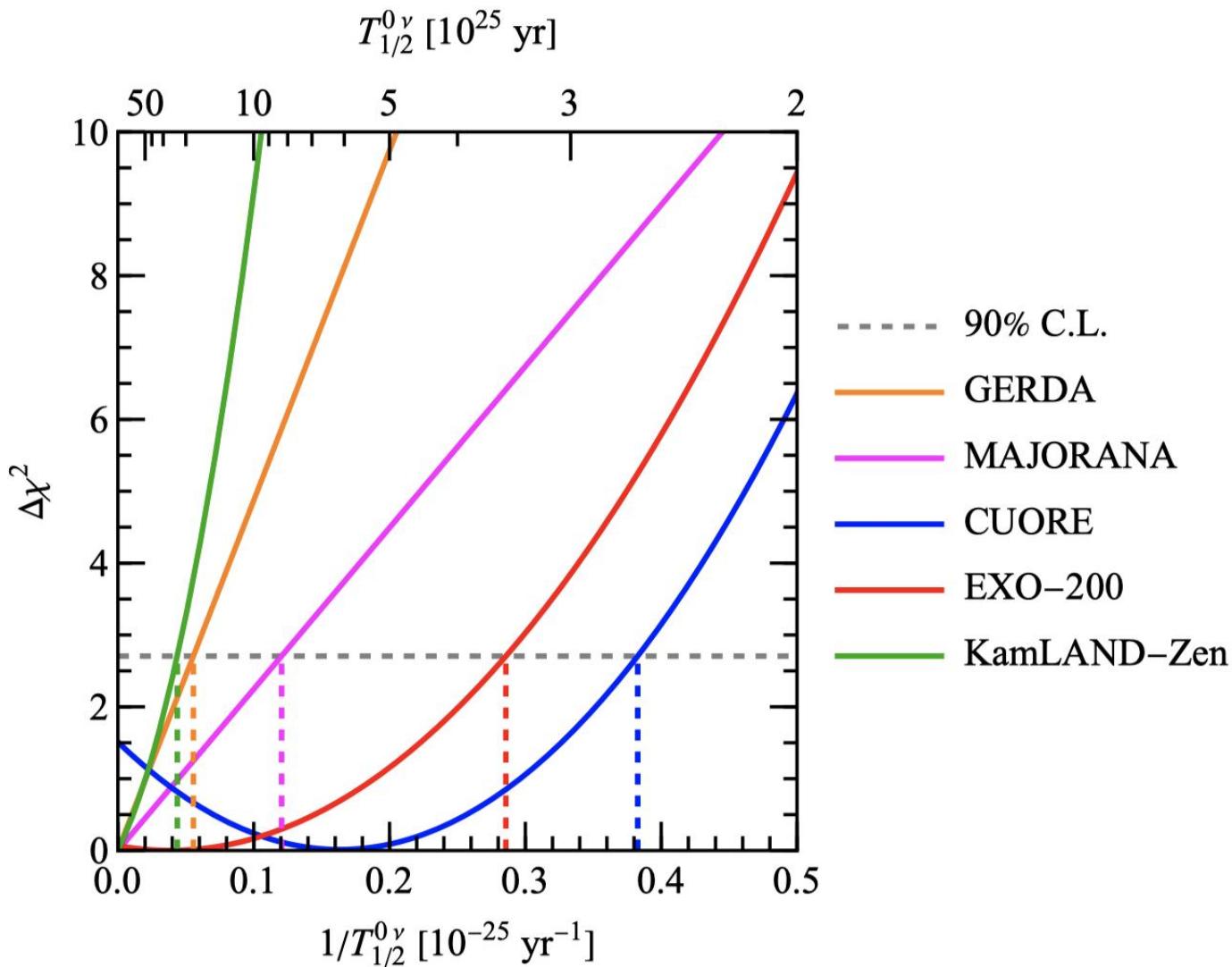
$m_{\beta\beta}^{\text{True}}$ corresponding to discrimination at 3σ



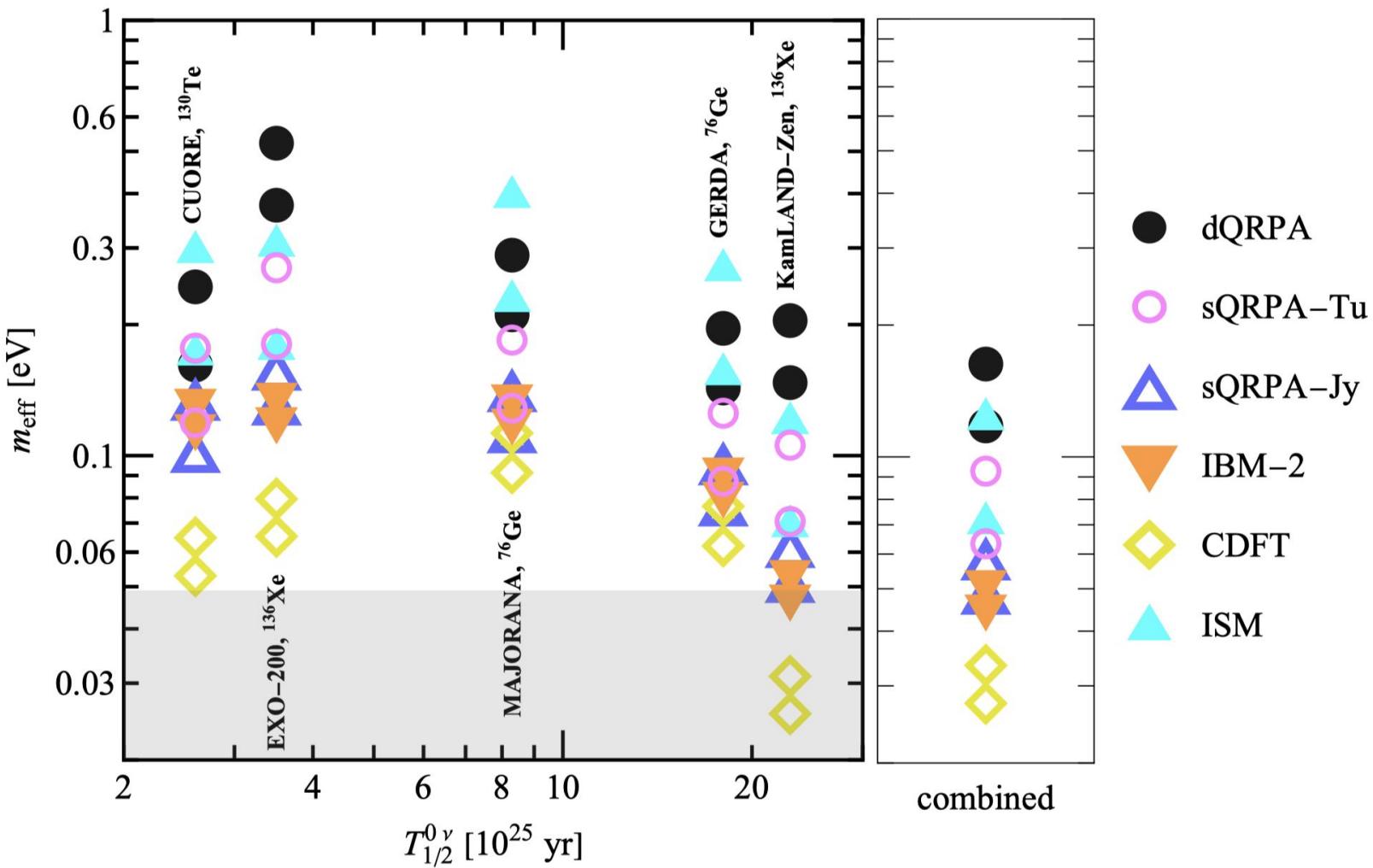
(with short- range NME)

$\Delta\chi^2$ functions of inverse half-life

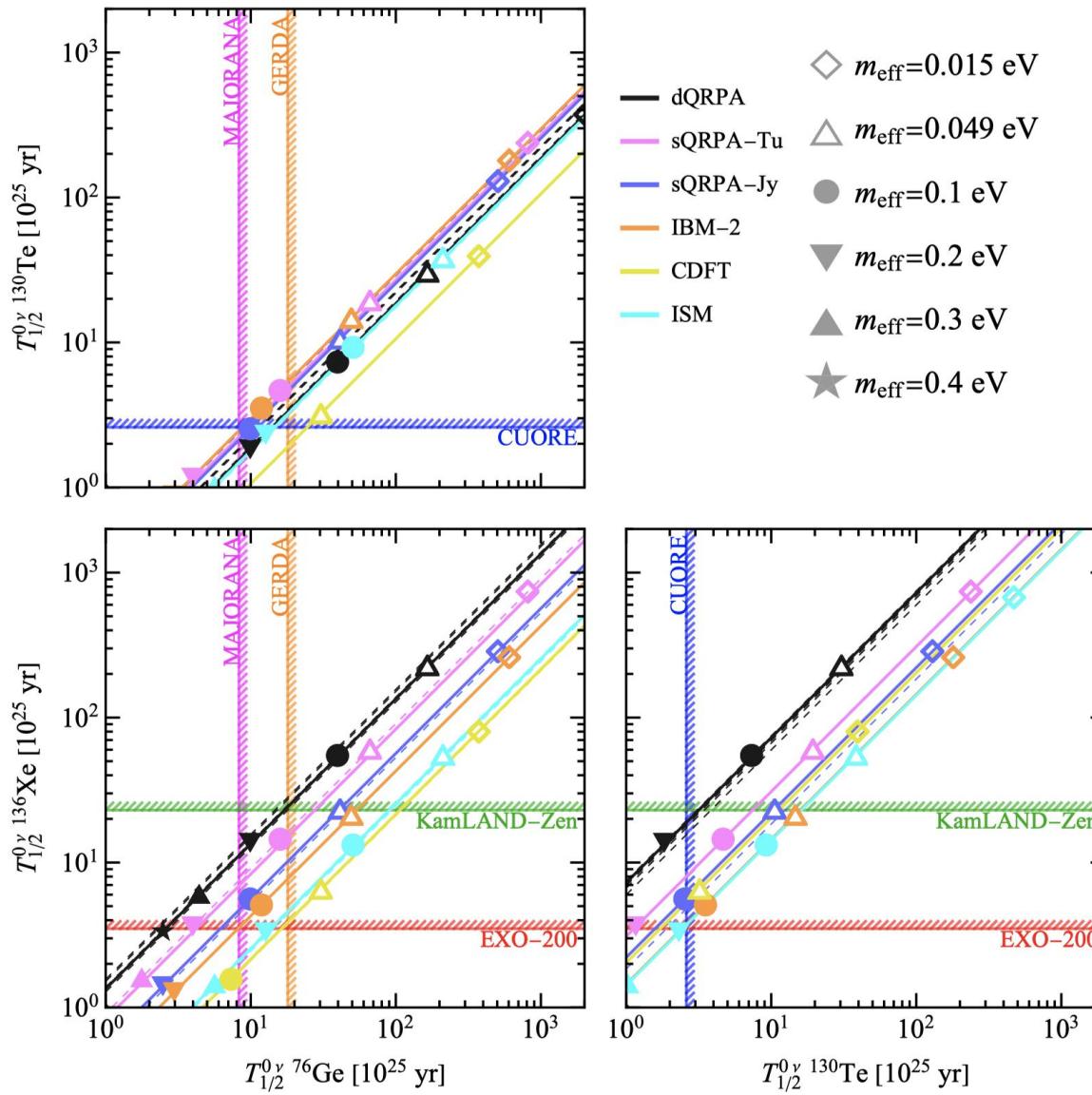
42



The upper limit of m_{eff}



Half-life relations of different isotopes



$g_A=1.27$, CD-bonn src

0v $\beta\beta$ half life

- m_{eff} value
- NME value