Causality Bounds on Gravitational EFTs with Matter Coupling

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Introduction

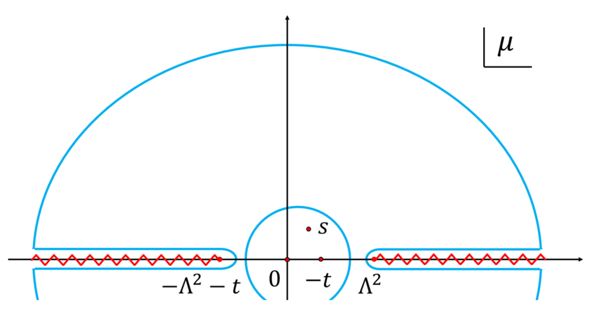
Effective field theories (EFTs) are constrained by causality, unitarity, and locality [1], known collectively as causality/positivity bounds. In this poster, we present these bounds within the context of the EFT of gravity coupled with a scalar field. These theories are popular extensions of general relativity and have been extensively studied in astrophysics and cosmology. They are now being probed by new observational tools, such as the LIGO-Virgo-KAGRA gravitational wave detectors and the Event Horizon Telescope.

We derive the power counting of the Wilson coefficients and discuss how some bounds vary with the coefficient of the leading scalar self-coupling term, $(\partial\phi)^4$. The sharp bounds on the coefficients are derived by formulating an optimization problem to extract information from causality and unitarity. Furthermore, we explore the phenomenological implications of the causality bounds. This poster is primarily based on [2, 3].

Power counting via dispersion relations

Causality, unitarity, and locality impose strong constraints on low-energy effective field theories (EFTs).

The analytical structures of the amplitudes are determined by the causality condition. The analytical structure of $\mathcal{M}(\mu,t)/(\mu-s)$ as a complex function of μ is shown in the figure below.



Λ is the cutoff energy scale of the EFT. In general, $\Lambda \ll M_P$. The theory is assumed to be weakly coupled below Λ, which allows tree level approximation in IR.

The generalized optical theorem, a result of unitarity, gives:

$$\mathrm{Disc}_\mu \mathcal{M}^{A\to B}(\mu,t) \sim \sum_\ell (2\ell+1) c^{A\to X}_{\mu,\ell}\, c^{B\to X,*}_{\mu,\ell}\ , \text{for } \mu \geq \Lambda^2\,,$$

where $c^{A,B\to X}_{\ell,\mu}$ are the UV partial waves to the intermediate state X ($\to X$ is omitted below).

The Regge behavior of the amplitudes is bounded by $\lim_{\mu\to\infty}|\mathcal{M}(\mu, t<0)| < |\mu|^2$. Together with the crossing symmetry of the amplitudes, dispersion relations can be derived using the Cauchy integration formula along the contour in the figure above:

$$\sum_{n=-1}^{2} (\text{Wilson coefficient}) \times t^n = \sum_\ell \int_{\Lambda^2}^{\infty} \mathrm{d}\mu\ (\text{Known functions}) \times c^A_{\mu,\ell}\, c^{B,*}_{\mu,\ell} + \text{crossing}\,.$$

where A, B label the helicities of the incoming and outgoing states. Dispersion relations connect the IR EFT and its UV completion, through which the unitarity condition of the UV completion can impose constraints on the EFT.

The EFT contains all possible couplings allowed by symmetry:

$$S \supset \int \mathrm{d}^4x\sqrt{g}\ \left(\frac{\alpha}{2}(\nabla_\mu\phi\nabla^\mu\phi)^2 + \frac{\beta_1}{2!}\phi\mathcal{G} + \frac{\beta_2}{4}\phi^2\mathcal{G} + \frac{\gamma_0}{3!}\mathcal{R}^{(3)} + \frac{\gamma_1}{3!}\phi\mathcal{R}^{(3)}\right)\,,$$

where $\mathcal{G}$ is the Gauss-Bonnet term, $\mathcal{R}^{(3)} = R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\alpha\beta}R_{\alpha\beta}{}^{\mu\nu}$. Independent in and out states have helicities $A = 00, +0, ++, +-$.

The contributions of some UV partial waves can be estimated from the dispersion relations.

$$\frac{\Lambda}{M_P} \Leftrightarrow c^{++}_{\ell,\mu}\,,\ c^{+-}_{\ell,\mu}\,,\ c^{+0}_{\ell,\mu}\,.$$

Now the only unknown independent partial wave is $c^{00}_{\ell,\mu}$: it is related to the scalar self-coupling constant and, in general, it should survive in the gravity decoupling limit M_P. Thus, it is generally not suppressed by the Planck scale, and its contribution can reach the upper bound given by unitarity.

$$1 \Leftrightarrow c^{00}_{\ell,\mu}\,.$$

This indicates that if all dispersion relations of an IR coefficient involve $c^{00}_{\ell,\mu}$, the scale of this coefficient is sensitive to the scale of the scalar self-coupling α.

In general, the scales of the coupling constants are given by:

$$\widehat{\mathcal{O}}_{\phi R} \sim M_P^2\Lambda^2 \left[\frac{\nabla}{\Lambda}\right]^{N_\nabla} \left[\frac{R}{\Lambda^2}\right]^{N_R} \left[\frac{\phi}{M_P}\right]^{N_\phi} \left[\frac{M_P}{\Lambda}\right]^{\tilde{N}_\phi}.$$

The power of the enhancement factor $\tilde{N}_\phi$ is determined by the number of UV partial wave $c^{00}_{\ell,\mu}$ in the most constraining dispersion relation.

As an illustration, bounds on $\gamma_0, \beta_1, \gamma_1$ are not sensitive to the scale of scalar self coupling α, while bound on β_2 is.

$$\gamma_0 \sim \frac{M_P^2}{\Lambda^4}\,,\quad \beta_1 \sim \frac{M_P}{\Lambda^2}\,.\quad \gamma_1 \sim \frac{M_P}{\Lambda^4}\,,$$
$$\beta_2 \sim \frac{M_P}{\Lambda^3} \quad \text{when} \quad \alpha \sim \frac{1}{\Lambda^4}\,,\quad \beta_2 \sim \frac{1}{\Lambda^2} \quad \text{when} \quad \alpha \sim \frac{1}{M_P^2\Lambda^2}\,.$$

Sharp bounds from optimization

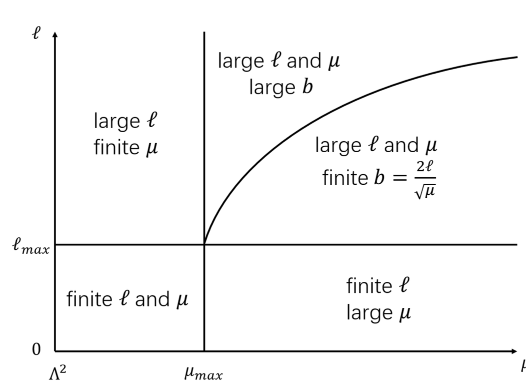
For gravitational theories, t−channel poles appear as a result of the Regge behavior of the amplitude [4]. This prohibits the use of the forward limit of the twice subtracted dispersion relations. Instead, one needs to attach each dispersion relation with a weight function $\phi^{A\to B}_k(p), t = -p^2$ and then integrate over p in the region $0 < p < \Lambda$:

$$\sum_{A,B}\sum_k\sum_{n=-1}^{2} a^{A\to B}_{k,n}\left(\int_0^\Lambda \mathrm{d}p\ \phi^{A\to B}_k(p)(-p^2)^n\right) = \sum_{A,B}\sum_\ell \int \mathrm{d}\mu\ B^{A,B}_{\ell,\mu} c^A_{\ell,\mu} c^{B,*}_{\ell,\mu}\,,$$

where $a^{A\to B}_{k,n}$ are linear combinations of the Wilson coefficients, $B^{A,B}_{\ell,\mu}$ is a known function of ℓ and μ By choosing the weight functions such that $B_{\ell,\mu} \succeq 0$, the LHS gives a constraint

$$\sum_{A,B}\sum_k\sum_{n=-1}^{2} a^{A\to B}_{k,n}\left(\int_0^\Lambda \mathrm{d}p\ \phi^{A\to B}_k(p)(-p^2)^n\right) \geq 0\,.$$

By selecting the strongest bound provided by the weight functions that ensure $B_{\ell,\mu} \succeq 0$, we can derive sharp bounds on the Wilson coefficients. With some approximations, this problem can be formulated as a semi-definite problem (SDP) [5, 6], and can be solved using the SDP solver `SDPB` [7].



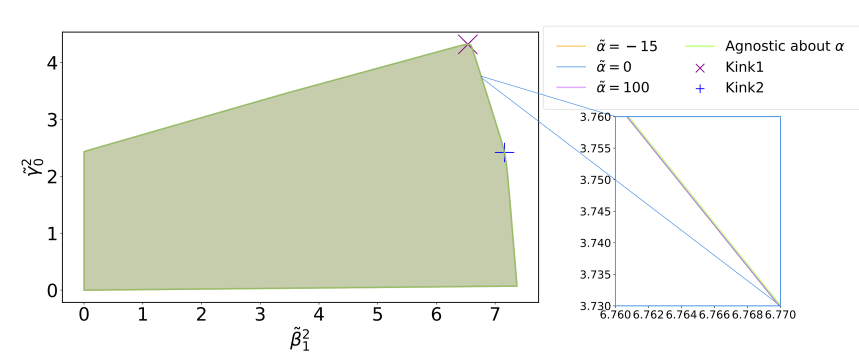
The existence of the t−channel pole indicates the contribution from the region parameterized by the impact parameter $b = 2\ell/\sqrt{\mu}$ is important in the dispersion relations. This region is carefully treated in the optimization scheme [5, 6].

We define dimensionless coupling constants as:

$$\widetilde{\#}_{\widehat{\mathcal{O}}_{\phi R}} = \#_{\widehat{\mathcal{O}}_{\phi R}} \times \frac{\Lambda^{N_\nabla+2N_R-2}M_P^{N_\phi-2}}{\log^{\#}(\Lambda/m_{\mathrm{IR}})}\,.$$

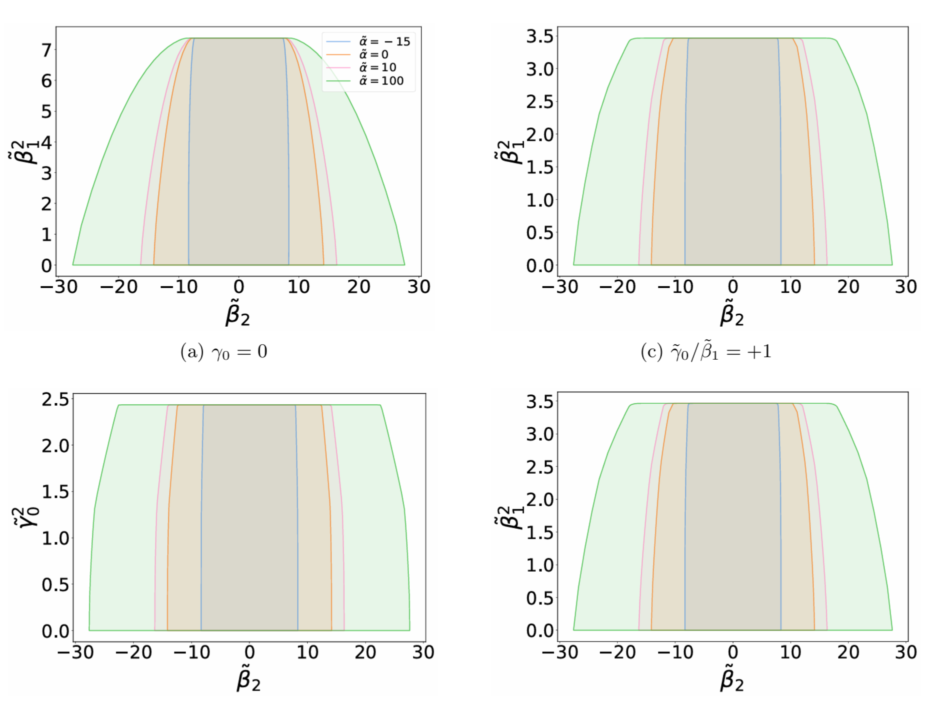
The infrared divergence $\log(\Lambda/m_{\mathrm{IR}})$ arising from the impact parameter space is due to the fact that the gravitational S-matrix is not well-defined in 4 dimensions. In general, the IR cutoff m_{IR} can be taken as the inverse of the Hubble scale of the universe.

The bounds on coefficients γ_0^2 and β_1^2 for various α are shown below



They are insensitive to the scale of the scalar self-coupling α, which is consistent with the power counting via the dispersion relations.

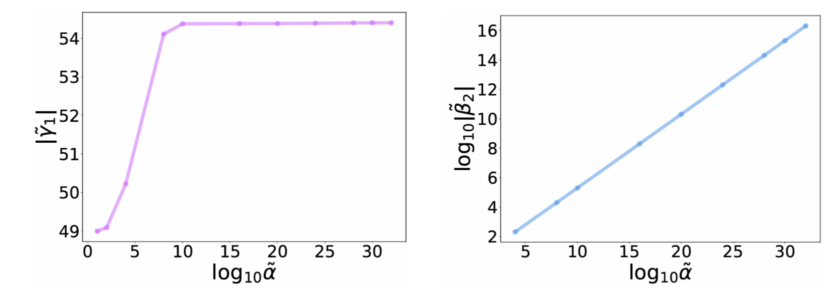
On the other hand, the upper bound on Wilson coefficient $|\beta_2|$ is sensitive to the value of α, as shown in the figure below (The figures are cross sections of the 3D bound on $\tilde{\gamma}_0^2$, $\tilde{\beta}_1^2$ and $\tilde{\beta}_2$.)



(a) $\gamma_0 = 0$ (c) $\tilde{\gamma}_0/\tilde{\beta}_1 = +1$

(b) $\beta_1 = 0$ (d) $\tilde{\gamma}_0/\tilde{\beta}_1 = -1$

We can also see that the upper bound on $|\beta_2|$ is proportional to $\alpha^{1/2}$ as α approaches its upper bound $\alpha \sim 1/\Lambda^4$, as shown in the left figure below. This is consistent with the fact that all the dispersion relations for β_2 contain one UV partial wave $c^{00}_{\ell,\mu}$.

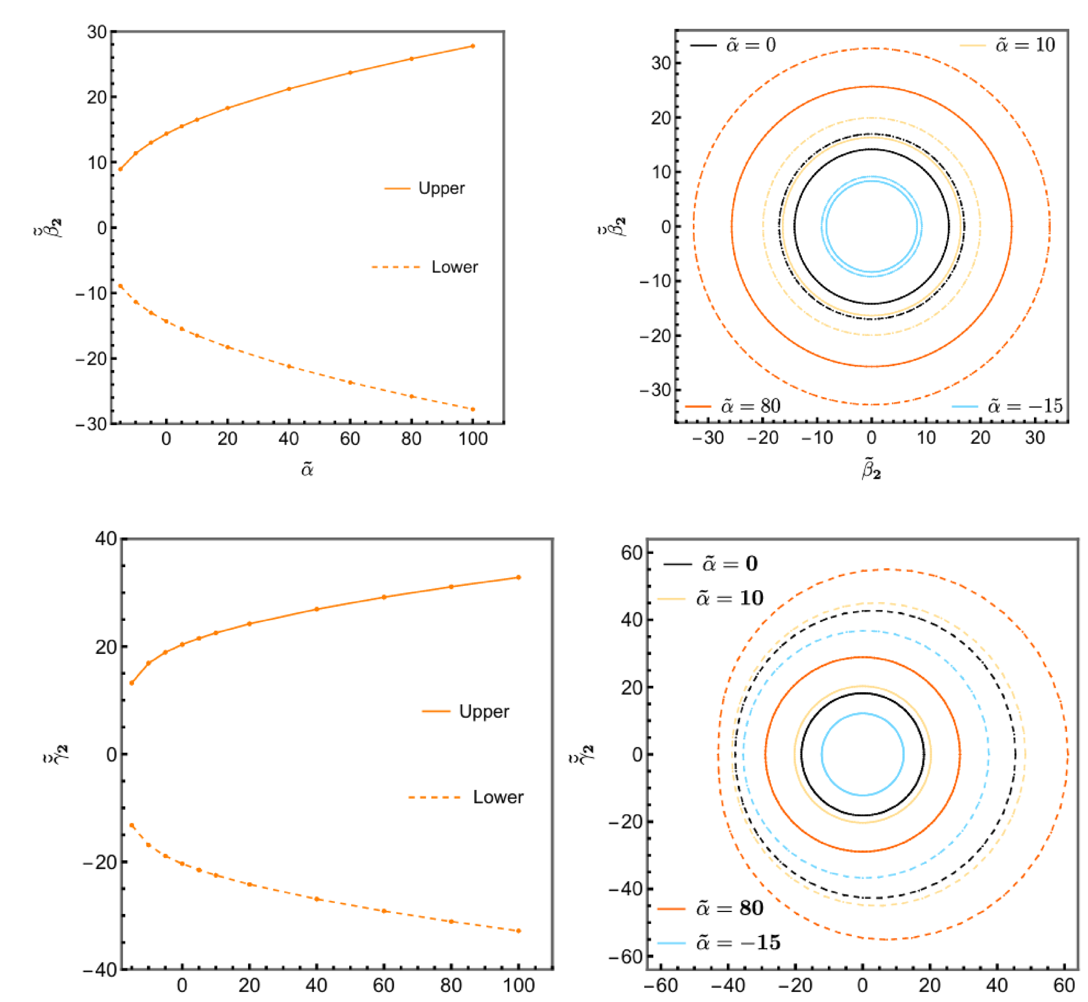


In contrast, the upper bound on $|\gamma_1|$, for which the dispersion relations do not contain $c^{00}_{\ell,\mu}$, is insensitive to the scale of α, as shown in the right figure above.

Bounds on parity violating couplings

Including parity-violating operators:

$$\mathcal{L} \supset \frac{\beta_2}{4}\phi^2\mathcal{G} + \frac{\gamma_2}{2}\nabla_\mu\phi\nabla^\mu\phi\mathcal{R}^{(2)} + \frac{\breve{\beta}_2}{4}\phi^2\tilde{\mathcal{R}}^{(2)} + \frac{\breve{\gamma}_2}{2}\nabla_\mu\phi\nabla^\mu\phi\tilde{\mathcal{R}}^{(2)}\bigg)\,.$$



Generally, the bounds on the parity-violating terms are of the same order as the parity-conserving ones.

Phenomenological implications

- In general, the scales of scalar Gauss-Bonnet couplings are

$$\mathcal{L} \supset M_P^2\sqrt{-g}\left(\frac{\mathcal{O}(1)}{\Lambda^2}\varphi\mathcal{G} + \frac{\mathcal{O}(1)M_P}{\Lambda^3}\varphi^2\mathcal{G}\right)\,,$$

where φ is defined as ϕ/M_P. This means the spontaneous scalarization models are natural where a vanishing $\varphi\mathcal{G}$ term is usually assumed and a sizable $\varphi^2\mathcal{G}$ is required for tachyonic instabilities to take place.

- Combining the positivity bounds and oberveble bounds, the lower bounds on the EFT cuttoff Λ can be derived. The table below shows the lower bounds on Λ from the NS-WD binary J0348+0432 for various equations of state (EoS) of the neutron star

	$\beta_2 > 0$			$\beta_2 < 0$				
EoS	MS1	MPA1	WFF1	MS1	MPA1	ENG	APR4	WFF1
$\Lambda(10^{-10}\mathrm{eV})$	1.4	2.1	3.4	2.9	4.0	4.7	5.2	5.9

- While the observational constraints on parity-violating coefficients are weaker than the parity-conserving counterparts, the causality bounds are of comparable strength and thus may play a more prominent role in constraining strong gravity effects in upcoming observations.

References

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