Gravitational Optomechanics: Photon-Matter Entanglement via Graviton Exchange

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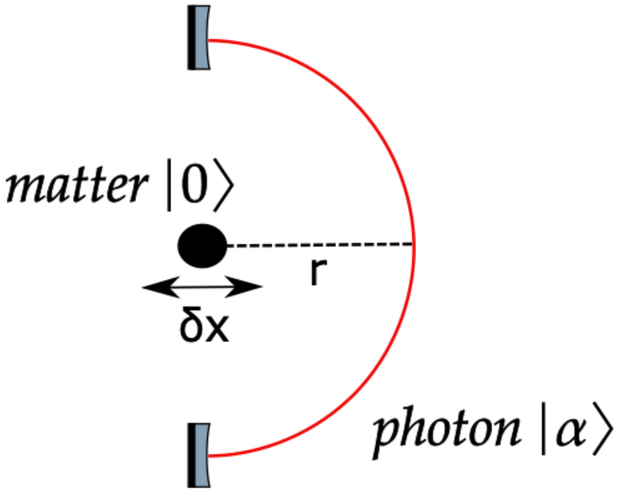
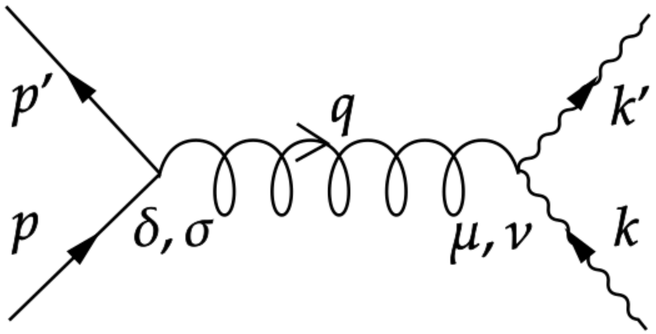
rijksuniversiteit groningen





Motivation: To Test the Quantum Nature of Gravity in a Lab

Biswas, Bose, **Mazumdar**, Toroš, PRD, 108, 064023 (2023)

GORDON AND BETTY MOORE FOUNDATION

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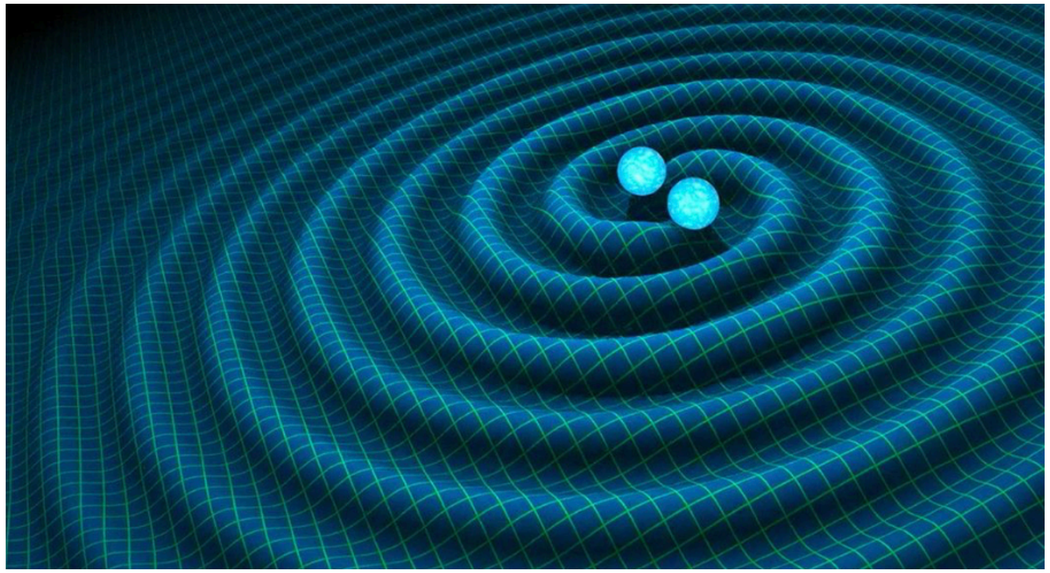
Origin of Spacetime



We don’t know whether spacetime/gravity is classical or quantum?

Aim: To test the nature of spacetime/gravity in a lab

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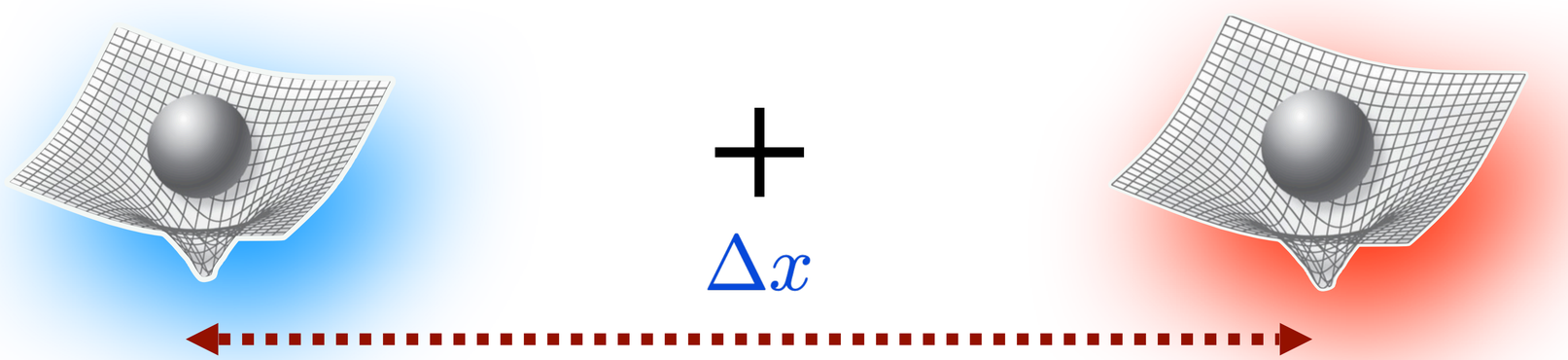
Our quest to understand the fundamental aspects of spacetime pushes frontier technologies to extraordinary heights & brings disruptive quantum technology

Macroscopic Spatial Quantum Superposition

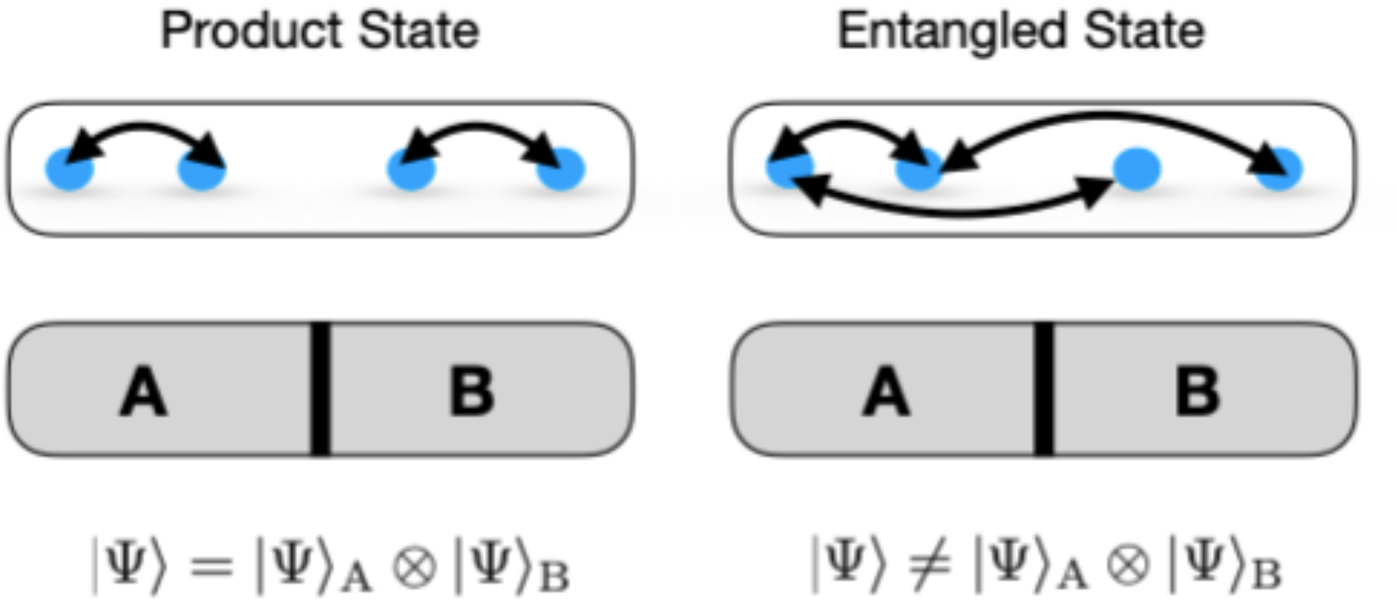
Can space-time with matter be in two places at once?



Bose**, Mazumdar,** Morley, Ulbricht, Toroš, Paternostro, Geraci, Barker, Kim & Milburn
PRL 119, 240401 (2017)

Biswas, Bose, **Mazumdar**, Toroš, PRD, 108, 064023 (2023)

Quantum Interactions, Correlations & Entanglement



Entanglement is a bonafide quantum entity which has no classical analogy

Quantum Interactions —> Entanglement

$$H = \hat{H}_A + \hat{H}_B \quad |\psi_i\rangle = |0\rangle_A |0\rangle_B \qquad H = \hat{H}_A + \hat{H}_B + \lambda \hat{H}_{AB} \quad |\psi_f\rangle \equiv \frac{1}{\sqrt{\mathcal{N}}} \sum_{n,N} C_{nN} |n\rangle |N\rangle$$

$$|\psi_f\rangle \sim \left[|0\rangle + \sum_{n>0} A_n |n\rangle \right] \cdot \left[|0\rangle + \sum_{N>0} B_N |N\rangle \right] + \sum_{n,N>0} \left(C_{nN} - A_n B_N \right) |n\rangle |N\rangle$$

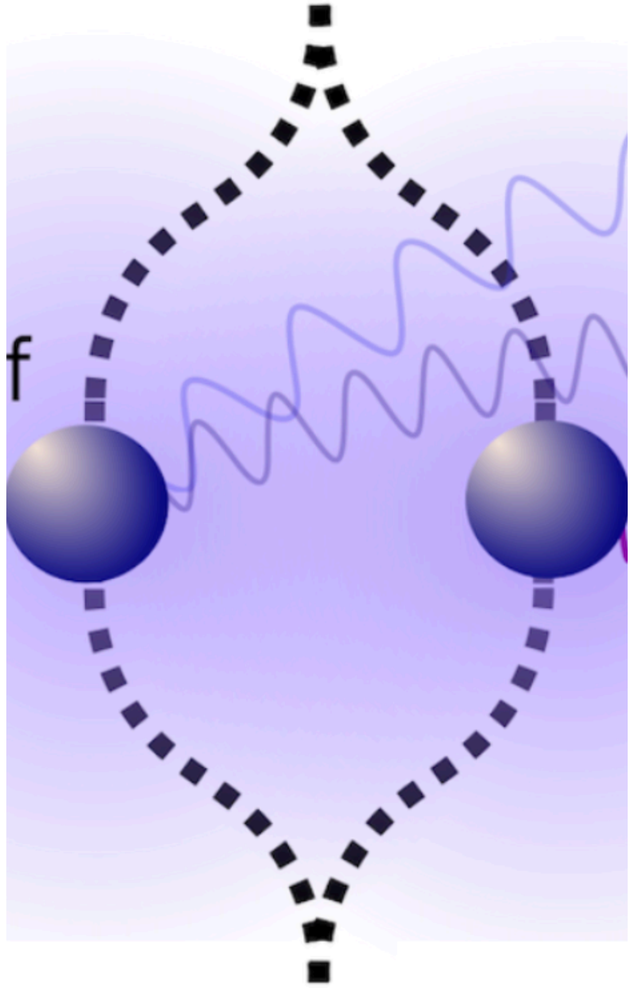
Product State — Non-Product State

$$C_{nN} = \lambda \frac{\langle n | \langle N | \hat{H}_{AB} | 0 \rangle | 0 \rangle}{2E_0 - E_n - E_N} \neq 0$$ Entanglement

$\hat{H}_{AB}$: QED, QCD, Weak, phonon, gravity, or any other quantum Interaction such as Axion/Axion-Like Particle

Bose, Mazumdar, Schut, Toros, 2201.03583

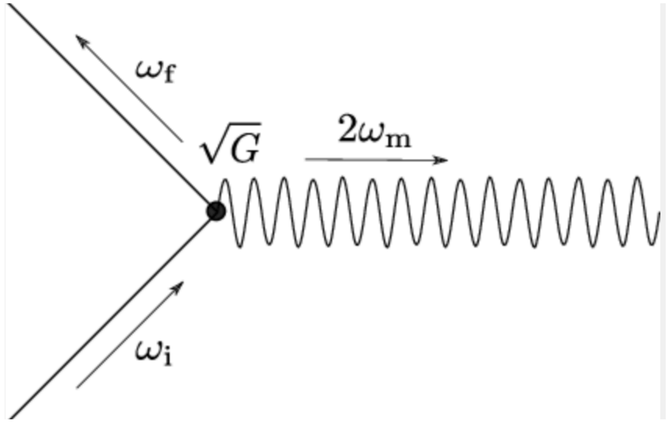
Quantum Interaction Yields Entanglement



$$S = S_G + S_m + S_{GF}$$

$$S_m = \frac{\sqrt{16\pi G}}{2}\int d^4x\ h_{\mu\nu}T^{\mu\nu} \sim \frac{\sqrt{16\pi G}}{2}\sum_{\lambda}\int dk C_k^{\lambda}\hat{g}_{k,\lambda}\hat{X}^2 + H.C.$$

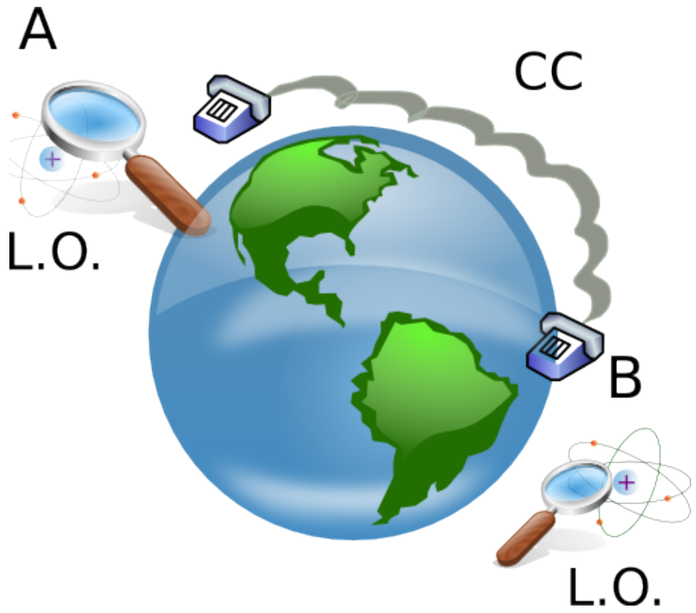
$$\hat{X} \sim \hat{b} + \hat{b}^{\dagger}$$



Genuine Tripartite Entanglement

Carmona Rufo, Mazumdar, Sabin 2411.03293

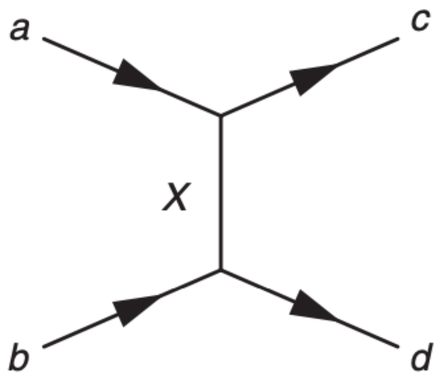
QGEM: No Entanglement via Classical Mediator



If A and B were initially product states, local operations and classical communications (LOCC) would not be able to generate/increase entanglement between them.

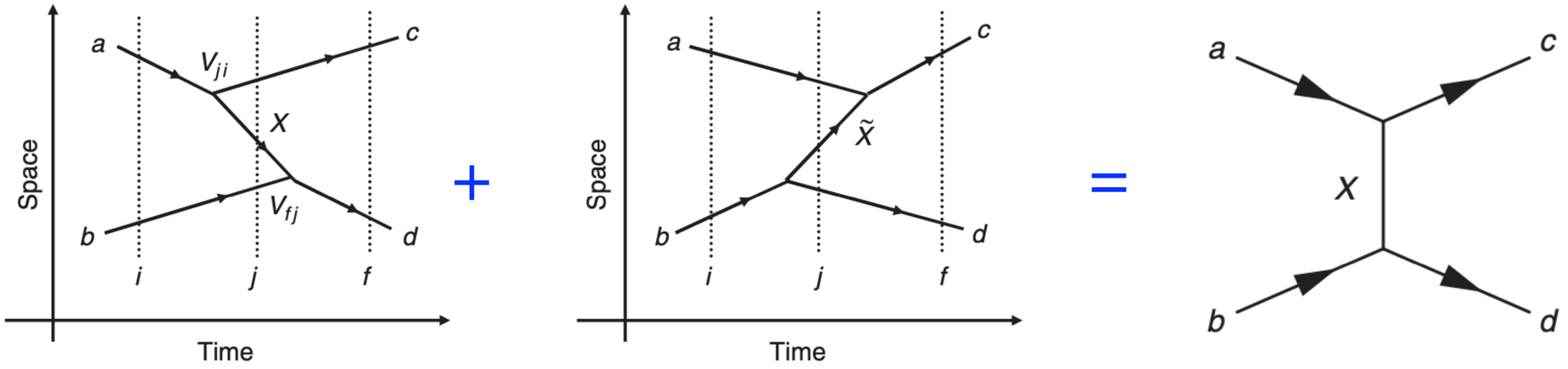
Bennett, et.al, (1996)

LOCC keeps separable states separable & cannot entangle states



LOQC: local operation & quantum communication can entangle

Quantum Field Theory

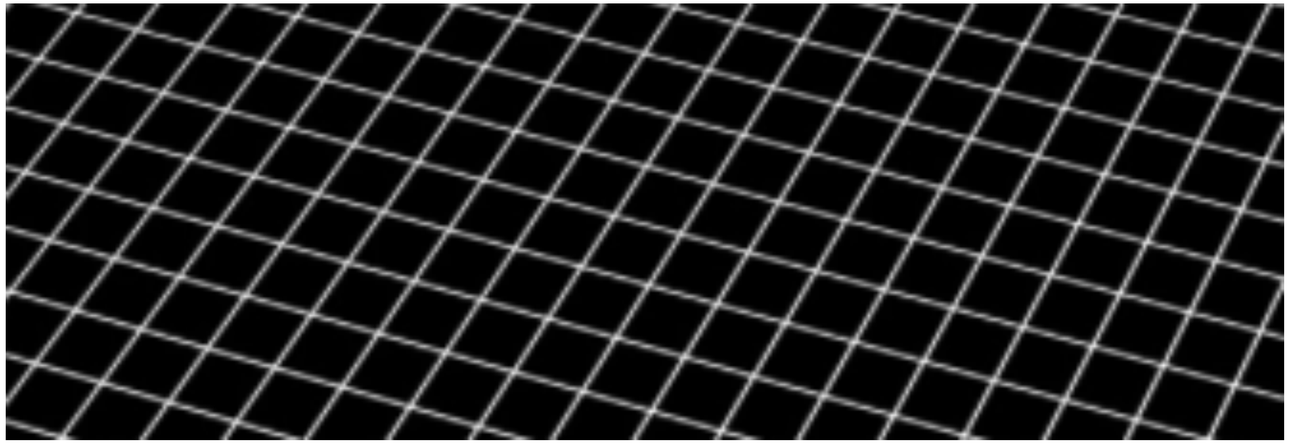


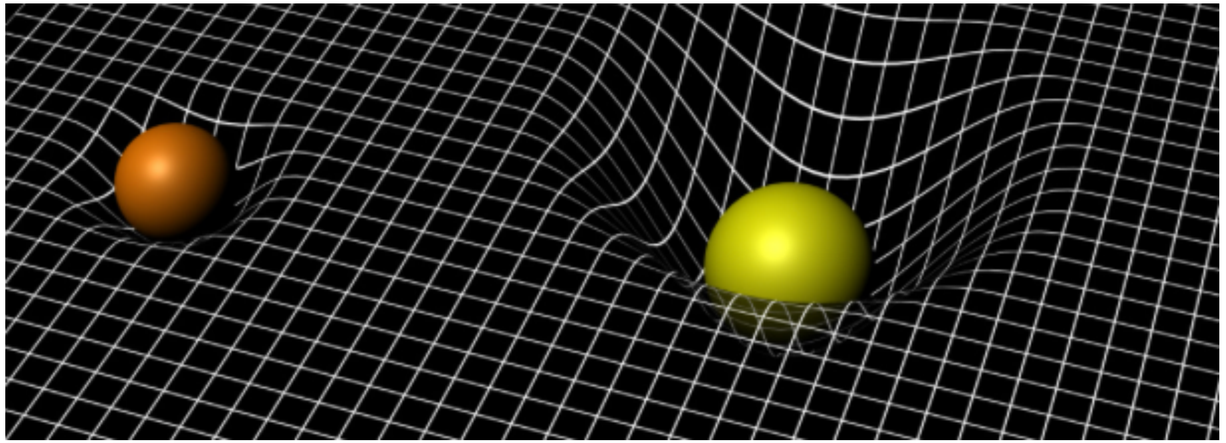
$$\langle f|\, S\, |i\rangle = \langle f|i\rangle - \frac{i\kappa^2}{4}\int \mathrm{d}^4x\mathrm{d}^4x'\,\langle f|\, T^{\mu\nu}(x)G_{\mu\nu\alpha\beta}(x,x')T^{\alpha\beta}(x')\,|i\rangle$$

$$\langle f|\, S-1\,|i\rangle = i\mathcal{A}(2\pi)^4\delta^{(4)}(P_{fi})$$

$$V(\boldsymbol{r}) = -\frac{1}{4E_pE_q}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}}\mathcal{A}(\boldsymbol{k})$$

Quantum nature of Newtonian potential





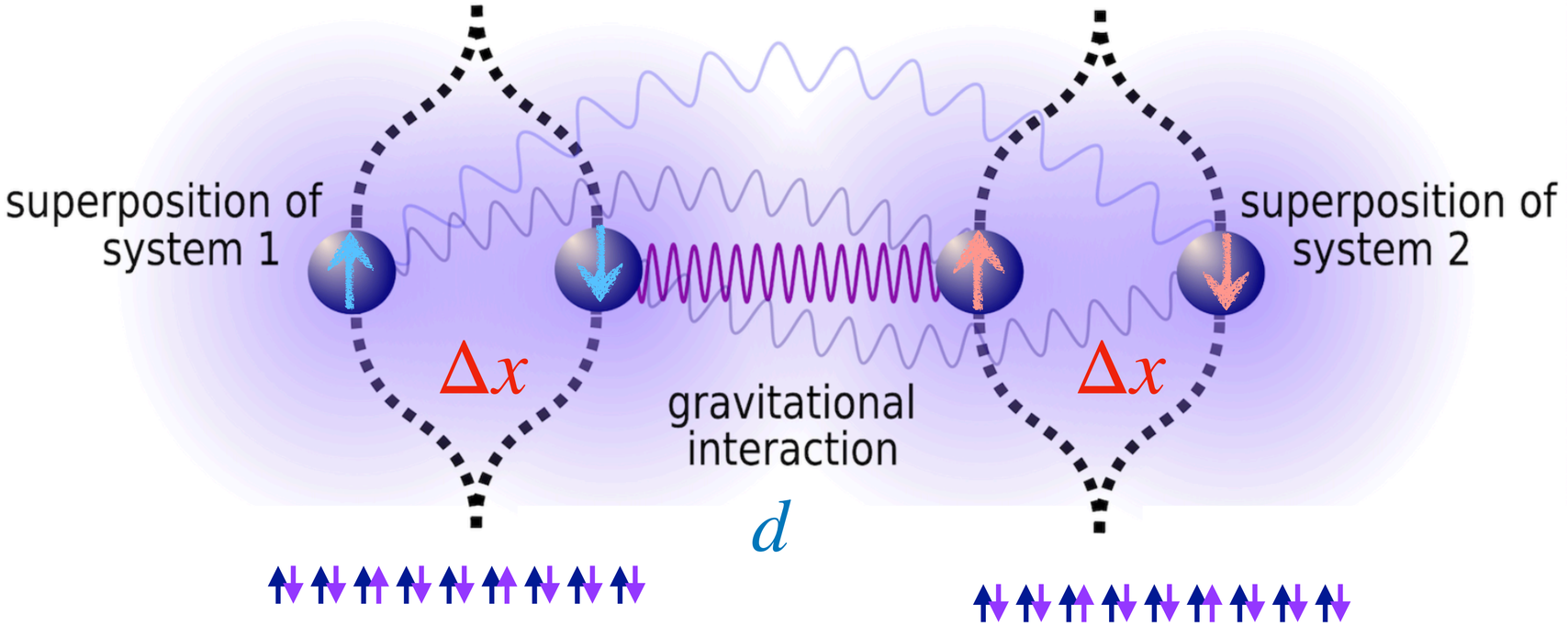
If spacetime is quantum, then the change in the gravitational energy due to the two masses is also a quantum entity

$$\widehat{H}_g \sim -\frac{Gm^2}{\hat{r}} - 7\frac{G\hat{p}^2}{c^2\hat{r}} - \frac{G\hat{p}^4}{c^4m^2\hat{r}} + 2\frac{G^2m^2}{c^2\hat{r}^2} + \frac{62G^2m\hbar}{15\pi c^2\hat{r}^3} + \cdots$$

All the terms have a quantum origin

This interaction between gravity and matter leads to entanglement, and we can witness this entanglement

QGEM: Protocol with Massive Qubits



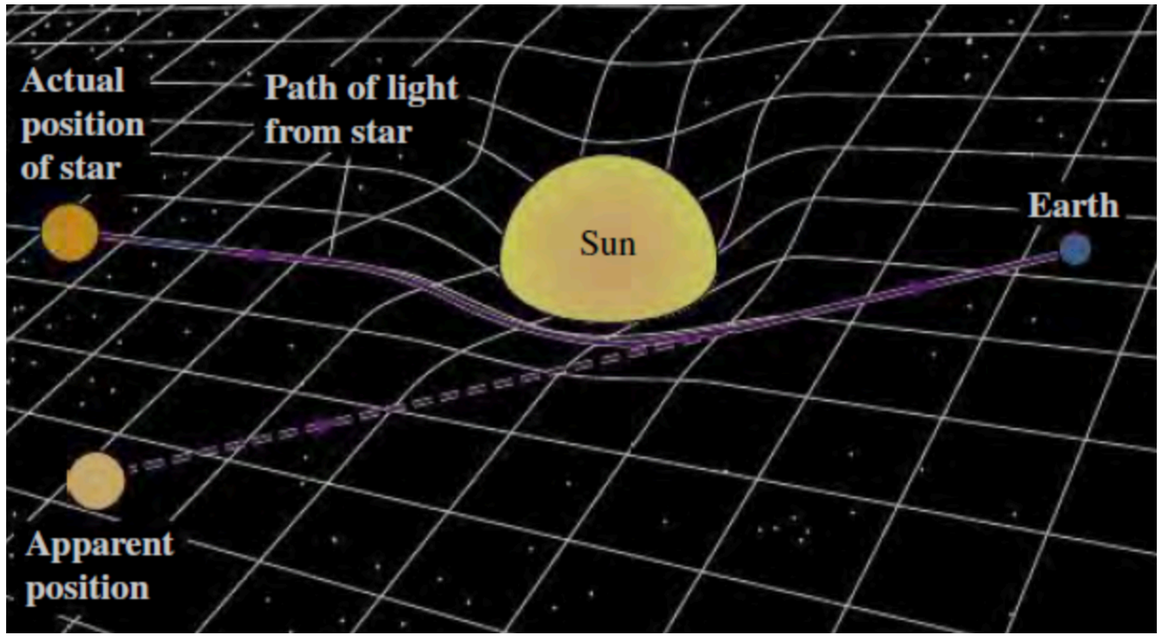
$$\psi(\tau) = \frac{e^{i\phi}}{2}\left[\ |\uparrow\rangle_1|\uparrow\rangle_2 + e^{i\phi_1}|\uparrow\rangle_1|\downarrow\rangle_2 + e^{i\phi_2}|\downarrow\rangle_1|\uparrow\rangle_2 + |\downarrow\rangle_1|\downarrow\rangle_2\ \right]$$

$$\Delta\phi_{ent} = \phi_1 + \phi_2 \sim \frac{Gm^2}{\hbar d}\tau\left(\frac{\Delta x}{d}\right)^2$$

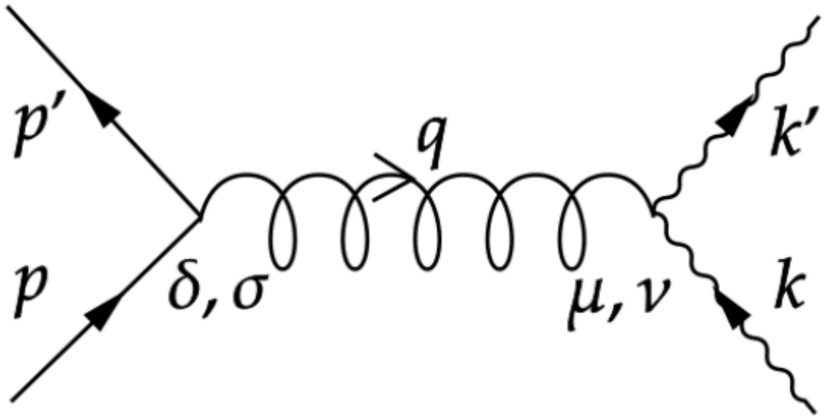
$$m \sim 10^{-14}\text{kg}, \quad \Delta x \sim 100\mu\text{m}, \quad d \sim 500\mu\text{m}, \quad \tau \sim 1s \implies \Delta\phi_{ent} \sim \mathcal{O}(1)$$

Bose**, Mazumdar,** Morley, Ulbricht, Toroš, Paternostro, Geraci, Barker, Kim & Milburn **PRL 119, 240401 (2017)**

Light bending



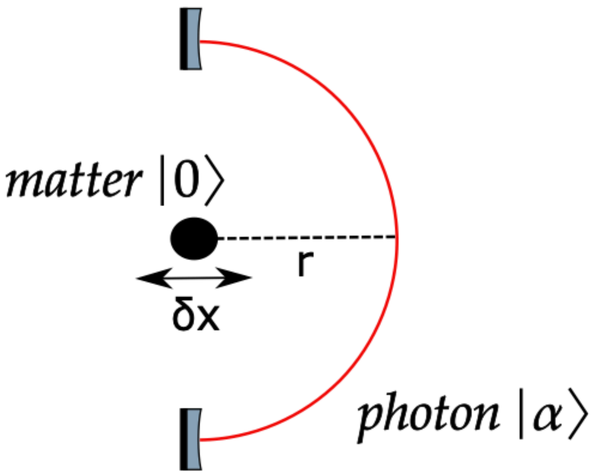
	Newton	Nordström	Einstein
Angle of Light Bending	$\theta_0 = \frac{2m}{r}$	$\theta_0 = 0$	$\theta_0 = \frac{4GM}{R}$

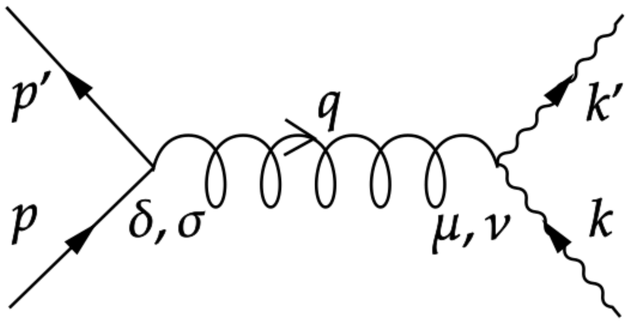


Only Spin-2 will couple to a massless photon

Spn-0 will have to couple to the trace of the EM field, but the trace is zero. Hence, no coupling at all.

Entanglement in light bending





$$V(r) = -\frac{1}{4m\omega}\frac{8\pi G(m\omega)^2}{\pi r}\varepsilon^*(k')\cdot\varepsilon(k) = -\frac{2Gm\omega}{r}\varepsilon^*(k')\cdot\varepsilon(k).$$

$$\hat{V} \approx -2Gm\omega\left[\frac{1}{r} + \frac{2\delta\hat{x}}{r^2} + \mathcal{O}(\delta\hat{x}^2)\right] \otimes \hat{\varepsilon}^*(k')\cdot\hat{\varepsilon}(k)$$

Optomechanical interaction: $\hat{V} = -g_0(\hat{b} + \hat{b}^\dagger)\hat{a}^\dagger\hat{a}$ $\qquad g_0 = \frac{4Gm\omega}{r^2(2m\omega_{\rm m})^{1/2}}$

Biswas, Bose, **Mazumdar**, Toroš, PRD, 108, 064023 (2023)

Linear Entanglement entropy

$$S = 1 - \mathrm{Tr}(\rho_{\mathrm{m}}^2)$$

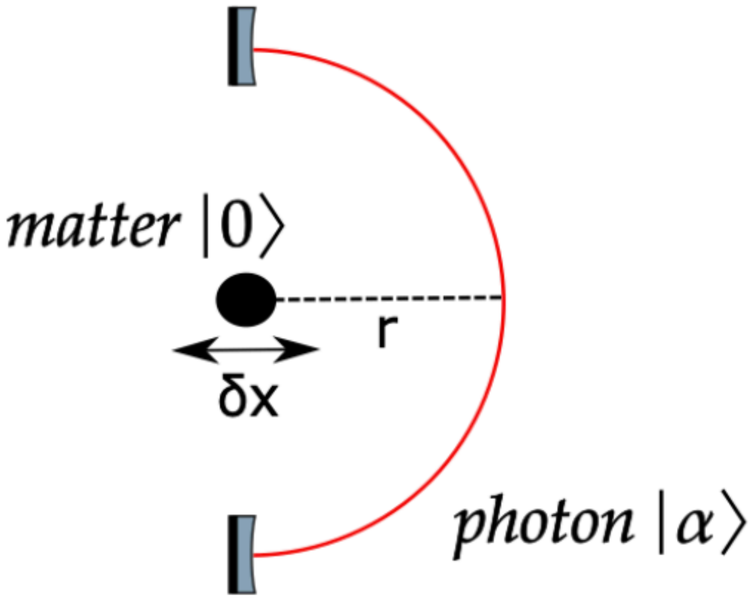
Reduced density matrix by tracing out photon

$$S = 2g^2\tau^2 = \frac{4G^2 m\omega^2 |\alpha|^2 \tau^2}{r^4 \omega_{\mathrm{m}}}$$

$$|\alpha| = \sqrt{\frac{4IV_{\mathrm{c}}}{\omega}}$$

Coherent state is connected to the occupation number of photon, hence the intensity

$$S \propto (\Delta x)^2 m^{5/3}, \quad \Delta x = (\hbar/(2m\omega_m))^{1/2} e^{\xi}$$



Parameter Space

$m \sim 10\mathrm{kg}$, $\omega_m \sim 10^2\mathrm{Hz}$ $\quad I \sim 10^{23}\mathrm{Wcm}^{-2}, \omega \sim 10^{16}\mathrm{Hz}$

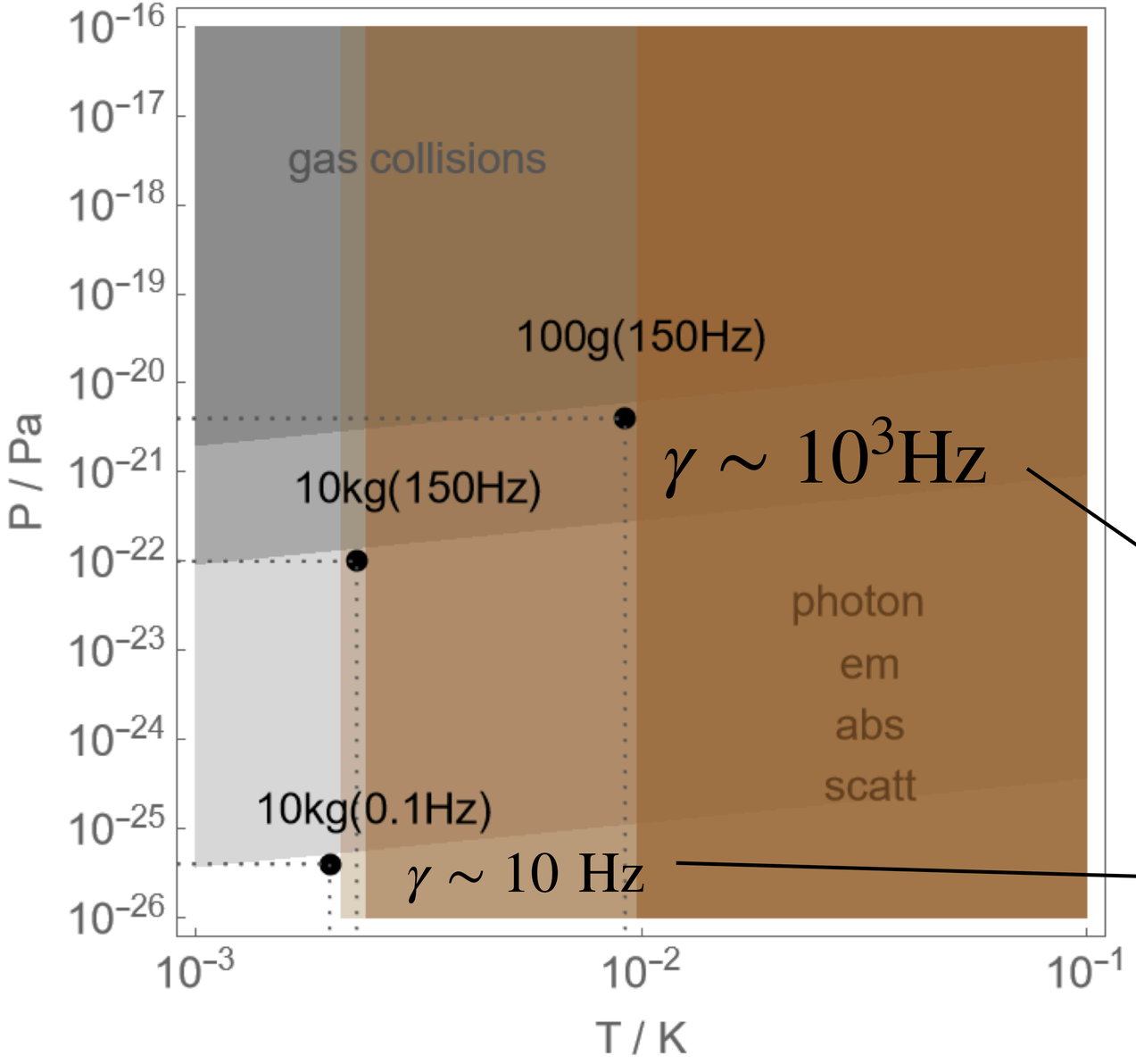
P=1 Peta Watt $\quad r \sim 10^3 - 25\mathrm{cm}$, $b \sim 6\mathrm{cm}$, $\alpha \sim 10^{18}\mathrm{W/cm}^2$

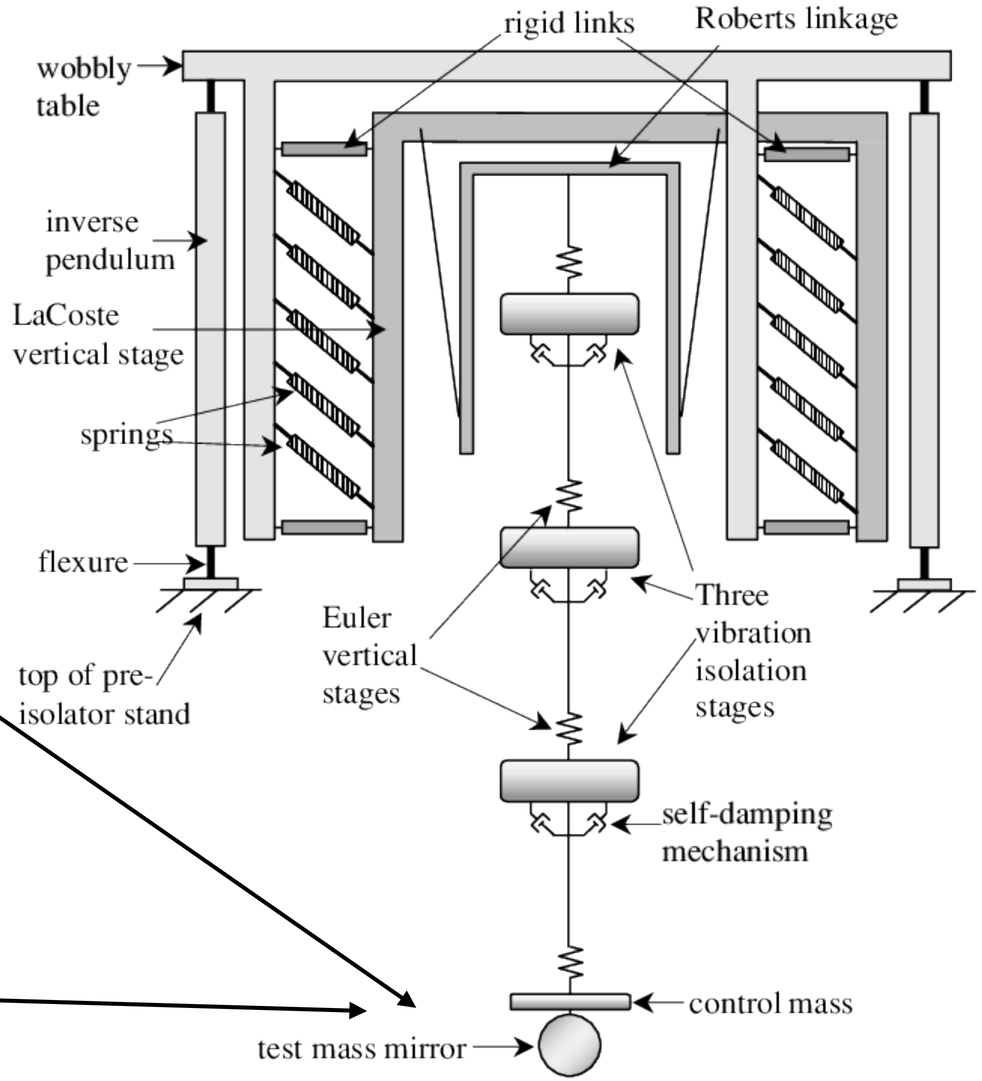
$\tau = 1\mathrm{ms}$, $\quad \xi = 4$, $\quad S \sim \mathcal{O}(1)$

$\omega_m = 0.1\mathrm{Hz}$, $\quad \mathrm{P} = 1\mathrm{GW}, \tau = 2.5\mathrm{s}$, $\xi = 4$, $S \sim \mathcal{O}(1)$

Biggest Challenge: Large Squeezing

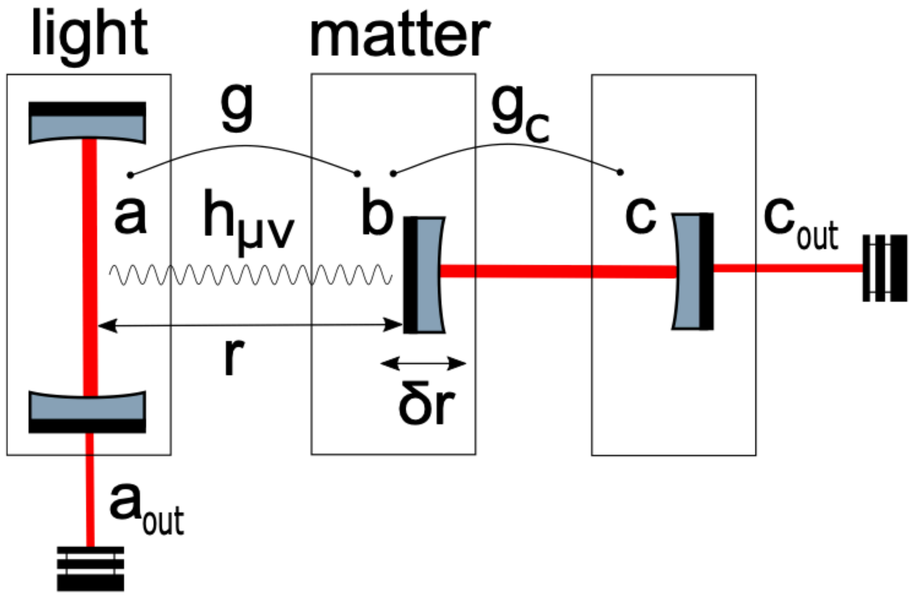
Decoherence & Levitation





LIGO/ ET setup for seismic isolation

Experimental Setup for Entanglement Witness



Duan-Giedke-Cirac-Zoller

Or

Simon's Entanglement

Witness for a continuous variable

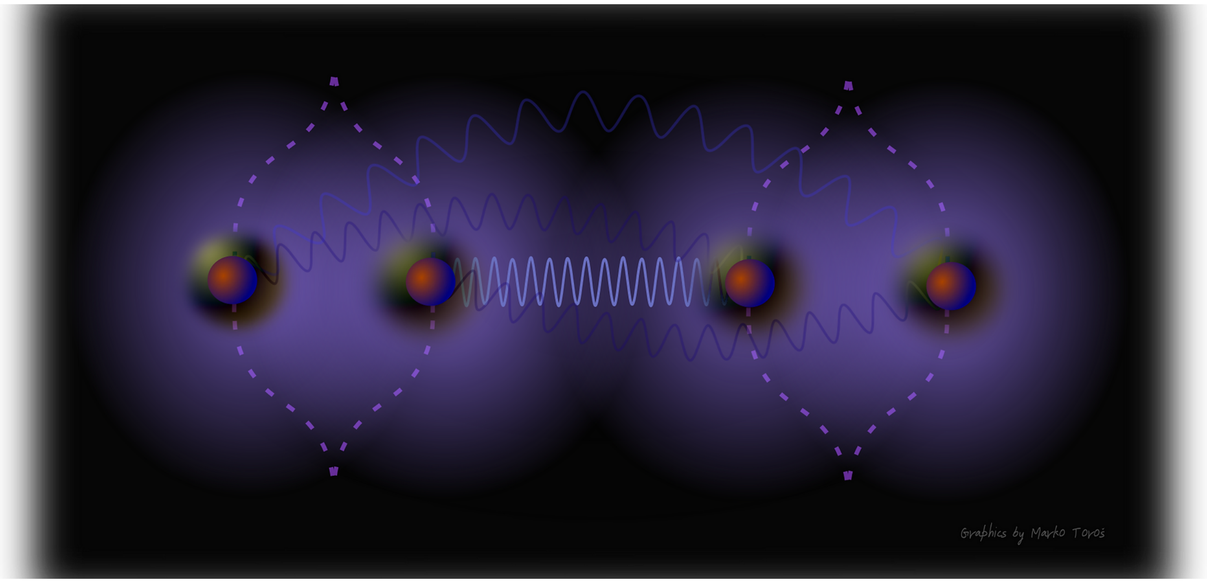
Entanglement witness provides evidence for the spin-2 nature of a massless graviton. Hence, it confirms the quantum nature of gravitational interaction with matter.

nature › research highlights › article

RESEARCH HIGHLIGHT | 11 November 2024

Two places at once: superposed crystal could test whether gravity obeys quantum laws

Method could probe whether a key tenet of quantum mechanics applies to gravity, which has so far resisted quantum theory.



Community effort: join us in this highly disruptive, ambitious technology to understand spacetime

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