Lepton Mixing and charged Lepton Flavour Violation from Inverse Seesaw

Based on "Charged lepton flavour violation from inverse seesaw with flavour and CP symmetries"
(FPDM, C. Hagedorn, '24) ;

"Lepton mixing and cLFV from ISS with non-degenerate heavy states "
(FPDM, C.Hagedorn, '25)

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EPS-HEP 2025, Marseille **08/07/2025**

Flash Introduction to Neutrino Physics

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(https://neutrinos.fnal.gov/history/)

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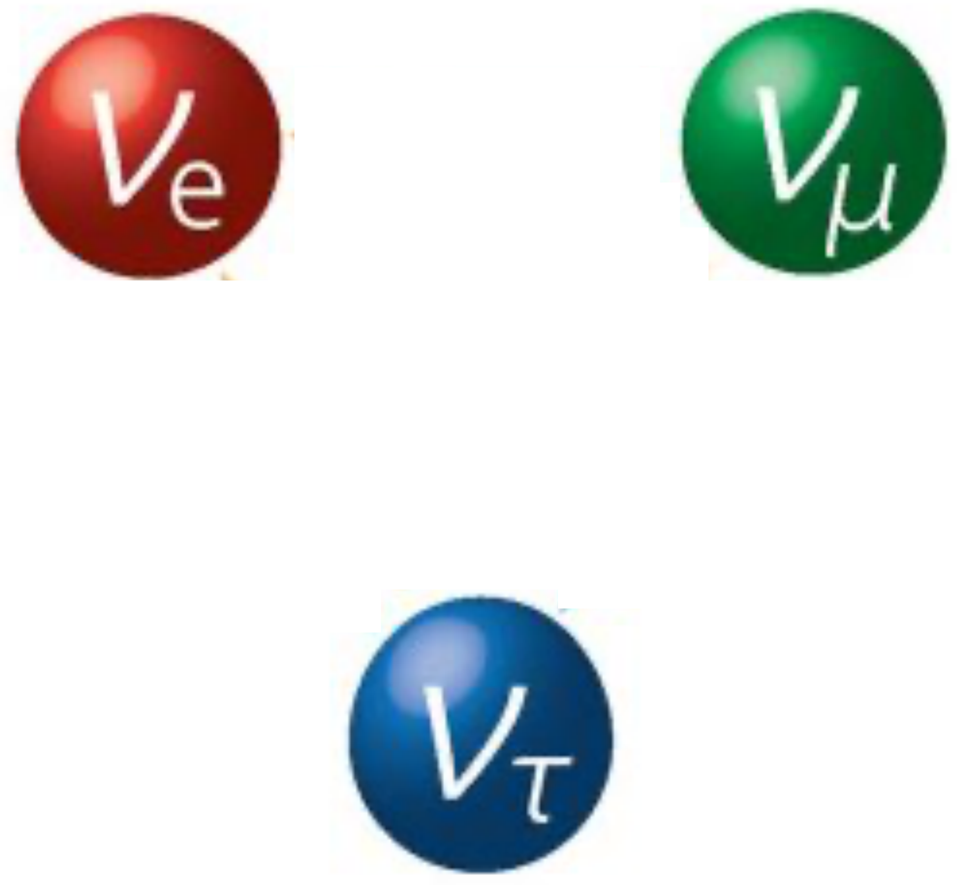

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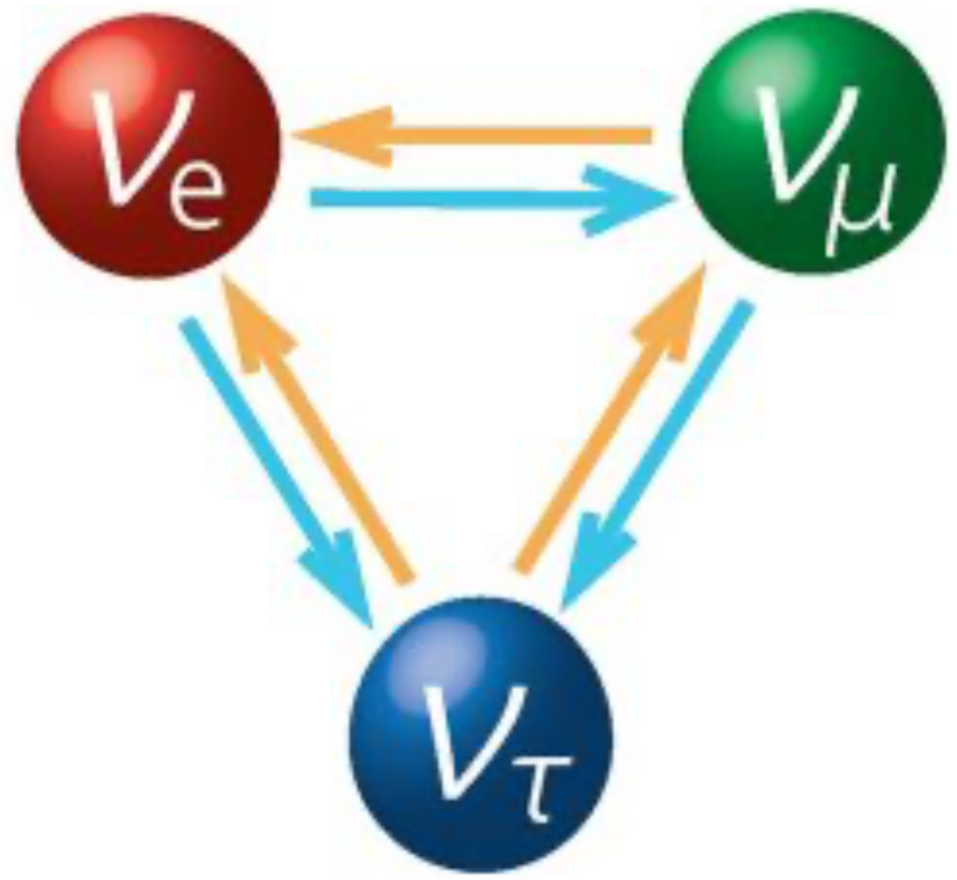

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Formulation of GSW theory of EW interactions (approx 1960-1970): LH and massless neutrinos

Evidences of oscillation in the neutrino sector is incompatible with massless neutrinos!

Flavour dynamics needs explanations!!

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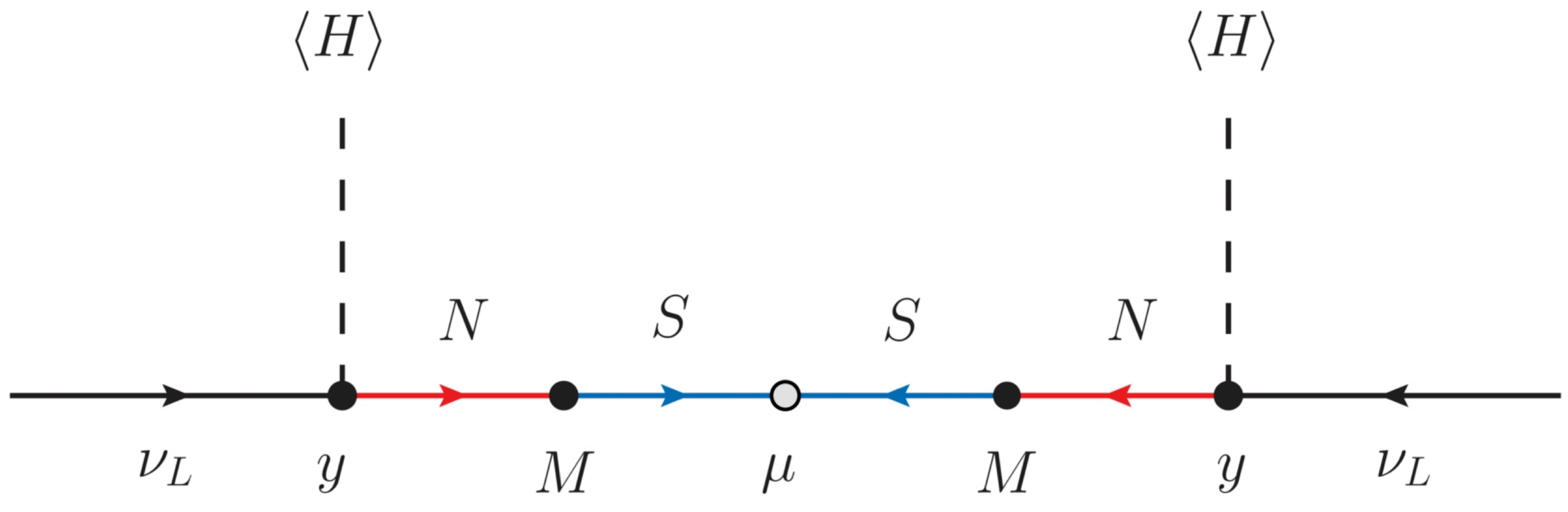

- Majorana Masses generated via Weinberg operators

- In this talk: **Inverse Seesaw**

$$\mathcal{L}_m = -\bar{L}^c Y_D H N^c - \bar{N} M_{NS} S - \frac{1}{2}\bar{S}^c \mu_S S + h.c. = -\begin{pmatrix}\bar{\nu}_L & \bar{N}^c & \bar{S}\end{pmatrix}^c \mathcal{M}\begin{pmatrix}\nu_L \\ N^c \\ S\end{pmatrix}$$

$$\mathcal{M} = \begin{pmatrix} \varnothing & m_D & \varnothing \\ m_D^T & \varnothing & M_{NS} \\ \varnothing & M_{NS}^T & \mu_S \end{pmatrix}$$

$$|\mu_0| \ll |m_D| \ll M_{NS}$$

First Problem: Neutrino Masses

The Inverse Seesaw (ISS)

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(R. N. Mohapatra and J. W. F. Valle, ('86); R. N. Mohapatra ('86); J. Bernabeu, A. Santamaria, J. Vidal, A. Mendez and J. W. F. Valle, ('87))

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$$\mathscr{U}^T \mathscr{M} \mathscr{U} = \mathscr{M}^{(diag)}$$

$$\mathscr{U} = \begin{pmatrix} \tilde{U}_\nu & S \\ T & V \end{pmatrix}$$

$$m_\nu \approx m_D (M_{NS}^{-1})^T \mu_S M_{NS}^{-1} m_D^T \sim y^2 \frac{\mu_0}{M_0^2} v^2$$

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In the basis in which charged lepton mass matrix is diagonal, $\tilde{U}_\nu$ is the (**non-unitary**) leptonic mixing matrix

$$\tilde{U}_\nu = (1 - \eta)\, U_0$$

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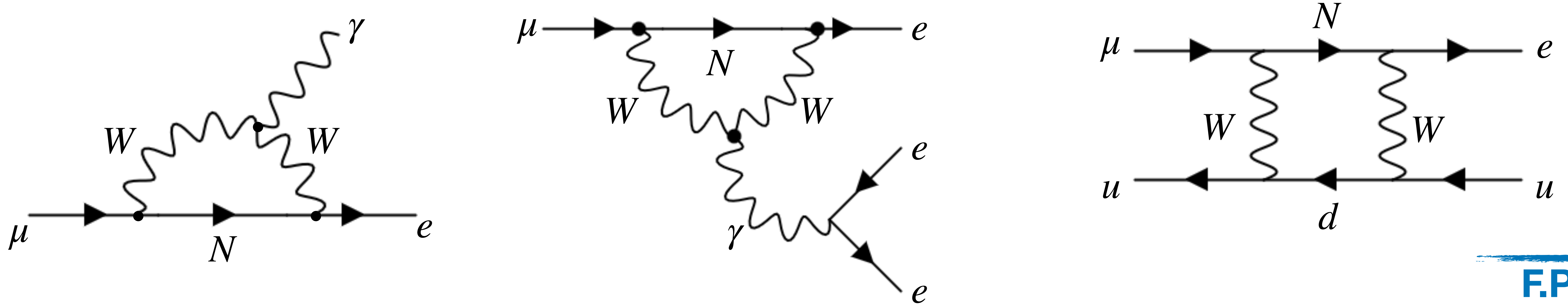
S, T describe the mixing of light and sterile neutrinos

$(S, T \ll \tilde{U}_\nu) \Rightarrow$ **Can induce cLFV processes**

(R. Alonso, M. Dhen, M. B. Gavela, T. Hambye ('12))



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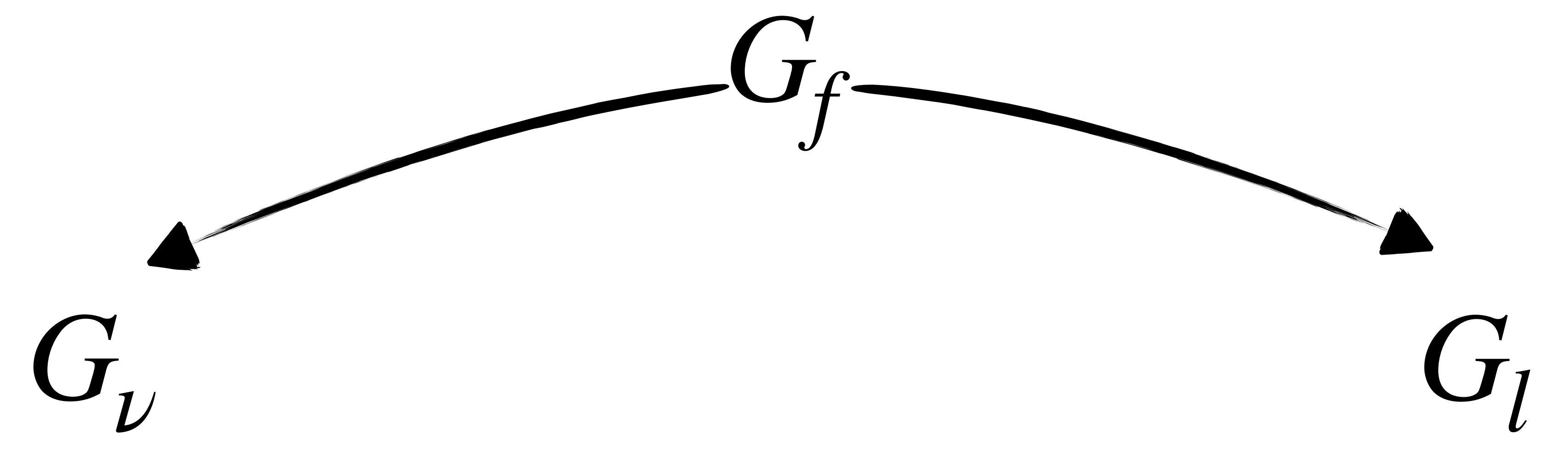
That explains the masses, but…
What about the mixing?



(Residual symmetry of m_ν)

(Residual symmetry of $m_l^\dagger m_l$)

$$\textbf{diag}\{m_e^2, m_\mu^2, m_\tau^2\} = U_l^\dagger m_l^\dagger m_l U_l \qquad m_\nu^{(diag)} = U_\nu^\dagger m_\nu U_\nu^* \qquad U_{PMNS} = U_l^\dagger U_\nu$$

m_ν , $m_l^\dagger m_l$ and the mixing U_{PMNS} is completely fixed by the symmetry!

(R. N. M

bye ('12))

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G_f **and** CP

G_ν (Residual symmetry of m_ν)

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$\Delta(3n^2)$ ($\Delta(6n^2)$) is a non-Abelian finite subgroup of $SU(3)$ of order $3n^2$ ($6n^2$) : $\Delta(6n^2) \sim (Z_n \times Z_n) \rtimes S_3$

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$$cd = dc$$

$$aca^{-1} = c^{-1}d^{-1} \qquad ada^{-1} = c$$

General element of the group is written:

$$g = a^\alpha c^\gamma d^\delta \qquad \alpha, \gamma, \delta \in N$$

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Three possible ways of choosing generators of the residual symmetries:

Case 1) $Z = c^{n/2}$ $\quad X = abc^s d^{2s} X_0$
Case 2) $Z = c^{n/2}$ $\quad X = c^s d^t X_0$
Case 3) $Z = bc^m d^m$ $\quad X = bc^s d^{n-s} X_0$

$$X_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

(F. Feruglio, C. Hagedorn, R. Ziegler, ('13))

Form of the generators depends on the particular representation chosen.

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G_f has a variety of 3-dim. representations 3_l

$3 = 3_1$ Complex, Faithful three-dim. representation

$3' = 3_5$ Real, Unfaithful three-dim. representation

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(C. Hagedorn, J. Kriewald, J. Orloff, A. M. Teixeira, ('21))

$$Y_D = y_0 \frac{\langle H \rangle}{M_0} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad M_{NS} = M_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mu_S = U_S^*(\theta_S) \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix} U_S^T(\theta_S)$$

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(FPDM, C. Hagedorn, ('24))

$$M_{NS} = M_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mu_S = \mu_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$m_D = \langle H \rangle U_L^*(\theta_L) \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} U_R^T(\theta_R)$$

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(FPDM, C. Hagedorn, ('25, To appear soon!))

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In this discussion, we focus on **Options 2** and **3**

Option 2

$$m_\nu \approx \frac{1}{M_0^2} m_D \mu_S m_D^T = \langle H \rangle \frac{\mu_0}{M_0^2} U_L^*(\theta_L) \left[\begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} U_R^T(\theta_R) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_R(\theta_R) \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} \right] U_L^\dagger(\theta_L)$$

Heavy neutral states mass matrix:

$$M_h \approx \begin{pmatrix} \varnothing & M_{NS} \\ M_{NS}^T & \mu_S \end{pmatrix}$$

Heavy neutral states spectrum: **Three almost <u>degenerate</u> Pseudo-Dirac Pairs**

$$m_{(i=4,5,6)} \approx M_0 - \frac{1}{2}\mu_0 \qquad m_{(i=7,8,9)} \approx M_0 + \frac{1}{2}\mu_0$$

Remember the ISS condition:

$$|\mu_0| \ll |m_D| \ll M_{NS}$$

Option 2

$$m_\nu \approx \frac{1}{M_0^2} m_D \mu_S m_D^T = \langle H \rangle \frac{\mu_0}{M_0^2} U_L^*(\theta_L) \left[\begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} U_R^T(\theta_R) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_R(\theta_R) \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} \right] U_L^\dagger(\theta_L)$$

$$U_L(\theta_L) = \Omega(3) R_{ij}(\theta_L)$$

$$U_R(\theta_R) = \Omega(3') R_{kl}(\theta_R) \left(P_{kl}^{ij} \right)^T$$

Option 2

$$m_\nu \approx \frac{1}{M_0^2} m_D \mu_S m_D^T = \langle H \rangle \frac{\mu_0}{M_0^2} U_L^*(\theta_L) \left[\begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} U_R^T(\theta_R) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_R(\theta_R) \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} \right] U_L^\dagger(\theta_L)$$

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$\Omega(3)$, $\Omega(3')$ are unitary
Are determined by residual symmetry (specified by the **CASE** and n, s, t) and its embedding in G_f .

Option 2

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$\Omega(3), \Omega(3')$ are unitary
Are determined by residual symmetry (specified by the **CASE** and n, s, t) and its embedding in G_f .

$R_{ij}(\theta_{L,R})$ is a rotation on the ij plane

Option 2

$$m_\nu \approx \frac{1}{M_0^2} m_D \mu_S m_D^T = \langle H \rangle \frac{\mu_0}{M_0^2} U_L^*(\theta_L) \left[\begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} U_R^T(\theta_R) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_R(\theta_R) \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} \right] U_L^\dagger(\theta_L)$$

Parameters of the theory:

$$M_0 \,, \mu_0 \,, y_i \,, \theta_L \,, \theta_R$$

Option 2

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- In our numerical analysis:

  $M_0 \in [150\ GeV;\ 10\ TeV]$

Option 2

$$m_\nu \approx \frac{1}{M_0^2} m_D \mu_S m_D^T = \langle H \rangle \frac{\mu_0}{M_0^2} U_L^*(\theta_L) \left[\begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} U_R^T(\theta_R) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_R(\theta_R) \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} \right] U_L^\dagger(\theta_L)$$

- In our numerical analysis:

$M_0 \in [150\ GeV;\ 10\ TeV]$ $\quad$ $\mu_0 \in [0.1\ keV;\ 10^2\ keV]$

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- In our numerical analysis:

$M_0 \in [150\ GeV;\ 10\ TeV]$ $\mu_0 \in [0.1\ keV;\ 10^2\ keV]$ $\theta_R \in [0;\ 2\pi]$

Option 2

$$m_\nu \approx \frac{1}{M_0^2} m_D \mu_S m_D^T = \langle H \rangle \frac{\mu_0}{M_0^2} U_L^*(\theta_L) \left[\begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} U_R^T(\theta_R) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_R(\theta_R) \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} \right] U_L^\dagger(\theta_L)$$

- In our numerical analysis:

$$M_0 \in [150\ GeV;\ 10\ TeV] \qquad \mu_0 \in [0.1\ keV;\ 10^2\ keV] \qquad \theta_R \in [0;\ 2\pi]$$

- Fixed by fitting LO predictions of the masses to experimental values

Option 2

$$m_\nu \approx \frac{1}{M_0^2} m_D \mu_S m_D^T = \langle H \rangle \frac{\mu_0}{M_0^2} U_L^*(\theta_L) \left[\begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} U_R^T(\theta_R) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_R(\theta_R) \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} \right] U_L^\dagger(\theta_L)$$

- In our numerical analysis:

$$M_0 \in [150\ GeV;\ 10\ TeV] \qquad \mu_0 \in [0.1\ keV;\ 10^2\ keV] \qquad \theta_R \in [0;\ 2\pi]$$

- Fixed by fitting LO predictions of the masses to experimental values
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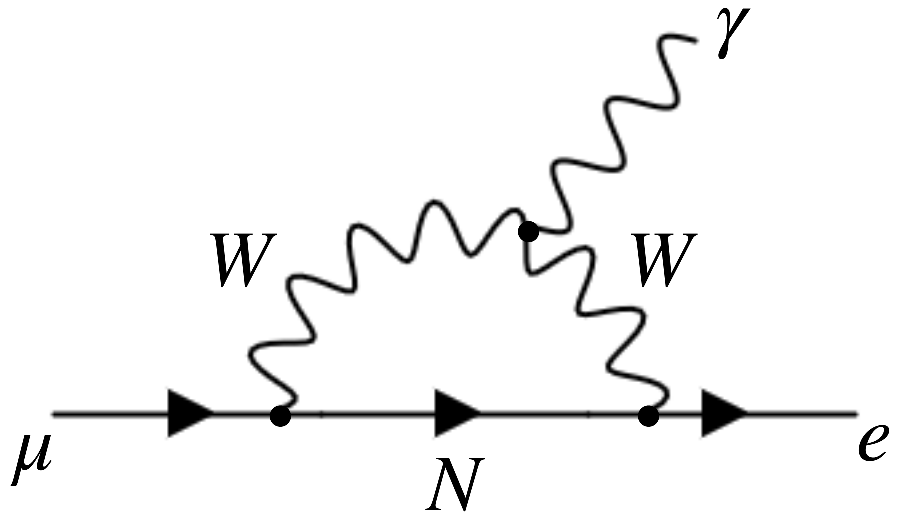
- In our numerical analysis:

$M_0 \in [150\ GeV;\ 10\ TeV]$ $\mu_0 \in [0.1\ keV;\ 10^2\ keV]$ $\theta_R \in [0;\ 2\pi]$

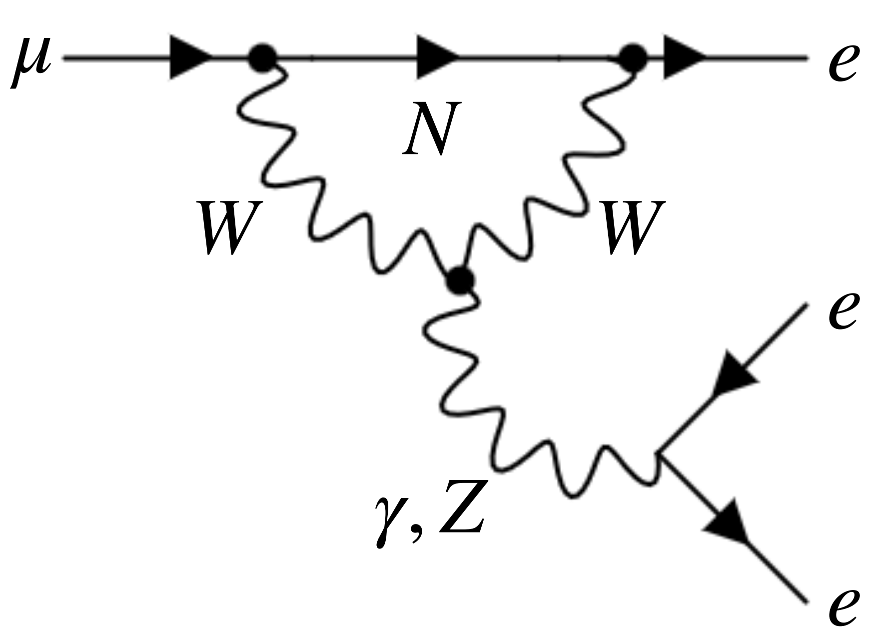
- Fixed by fitting LO predictions of the masses to experimental values
- Fixed by fitting prediction of mixing to lepton mixing data
- We consider $m_0 = 0.03(0.015)\ eV$ for NO(IO)

Option 2

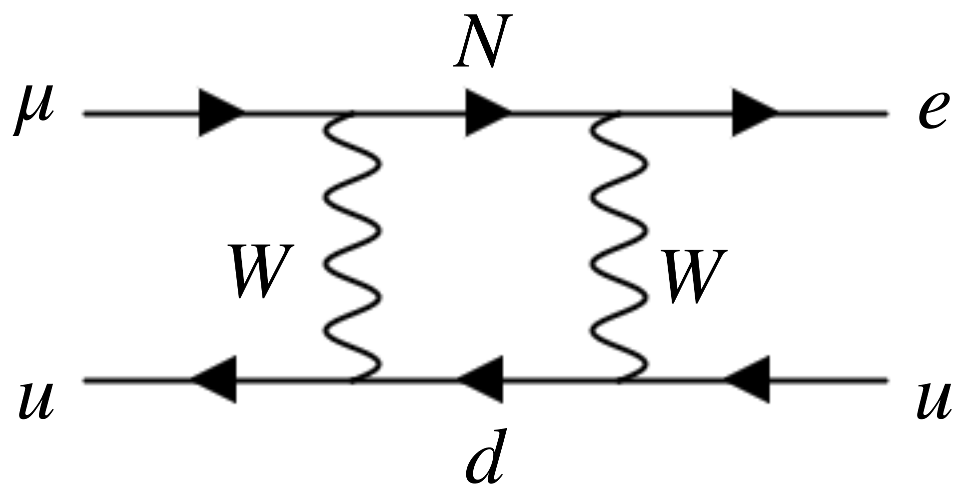
cLFV in the ISS



$\mu \to e\gamma$



$\mu \to 3e$

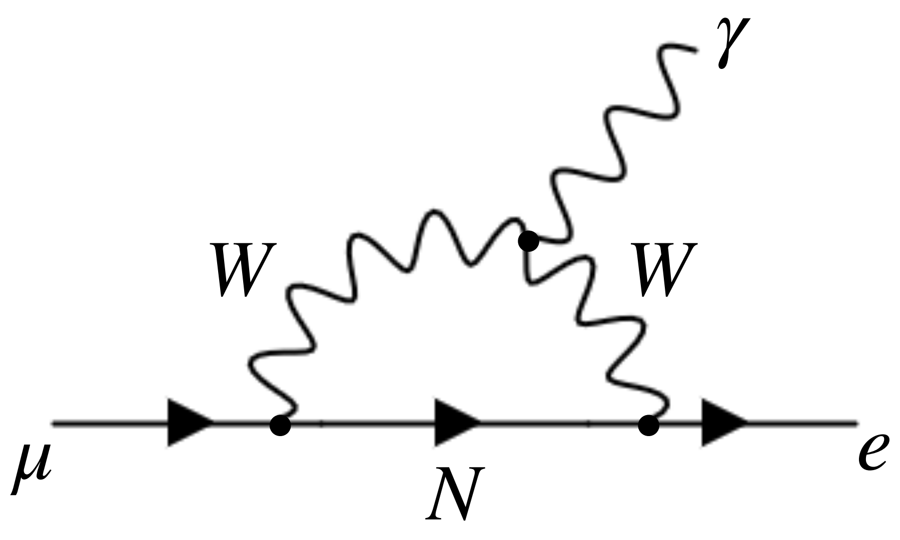


$\mu - e$ **Conversion**

Heavy sterile states and their mixing with light neutrinos can lead to unsuppressed processes that violate flavour in the Charged Lepton Sector!

Option 2

cLFV in the ISS



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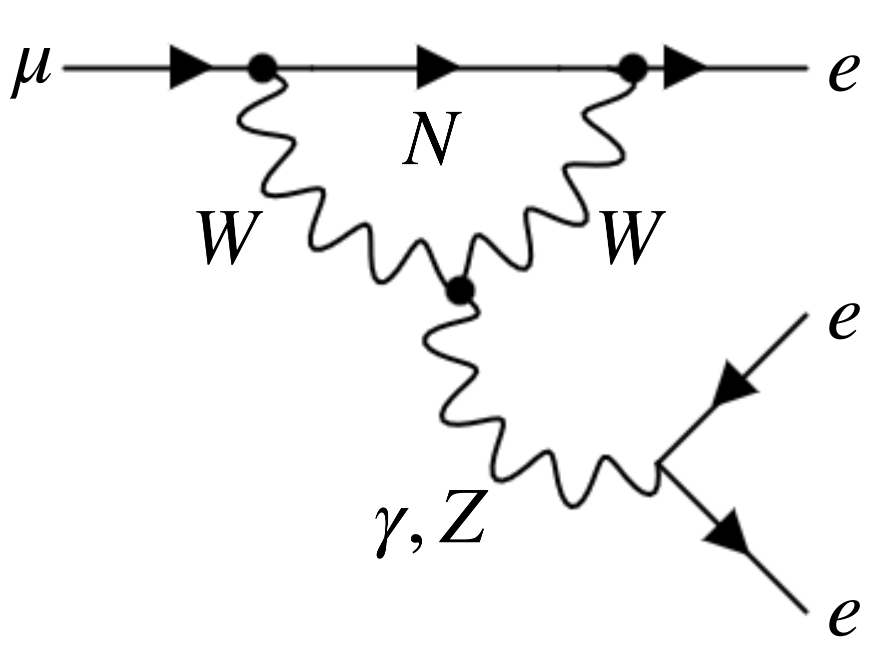
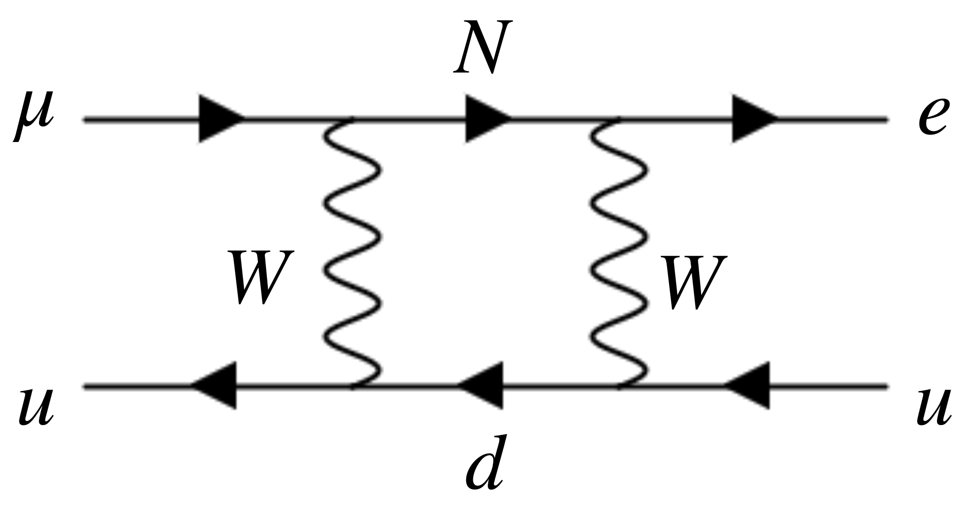


$\mu \to 3e$



$\mu - e$ **Conversion**

Present bounds are non-constraining.

$BR(\mu \to e\gamma) \lesssim 6 \times 10^{-14}$ **Meg-II**

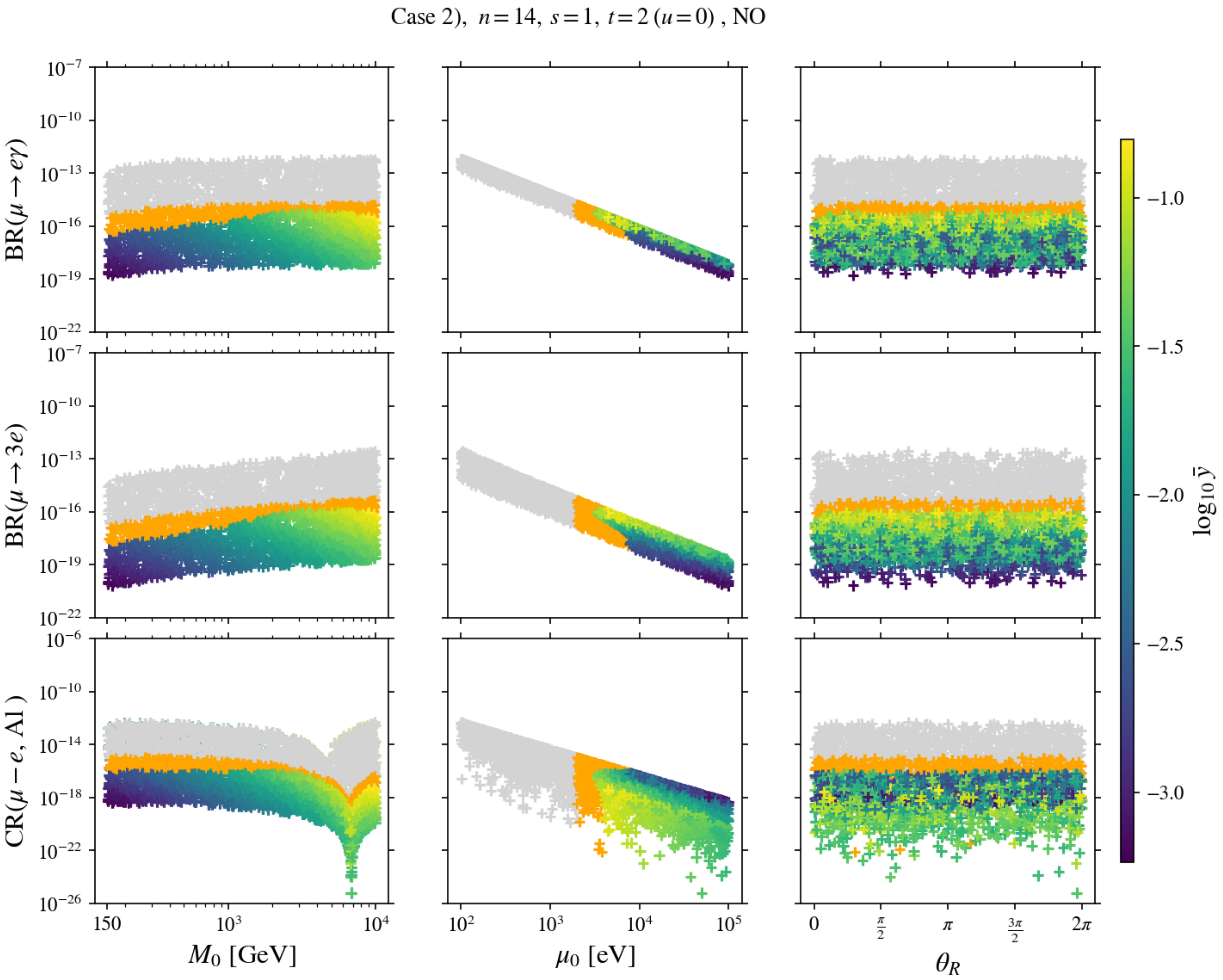
Main constraints come from future bounds:

$BR(\mu \to 3e) \lesssim 20(1) \times 10^{-16}$ **Mu3E Phase-I (II)**

$CR(\mu - e, Al) \lesssim 7.0(8.0) \times 10^{-17}$ **COMET Phase-II (Mu2E)**

Option 2: Case 2

(FPDM, C. Hagedorn, ('24))



Predictions are bounded by both experimental bounds on **cLFV processes** as by bounds on **unitarity violation**

$$\eta \lesssim \begin{pmatrix} 1.3 \times 10^{-3} & & \\ 1.2 \times 10^{-5} & 1.1 \times 10^{-5} & \\ 9.0 \times 10^{-4} & 5.7 \times 10^{-5} & 1.0 \times 10^{-3} \end{pmatrix}$$

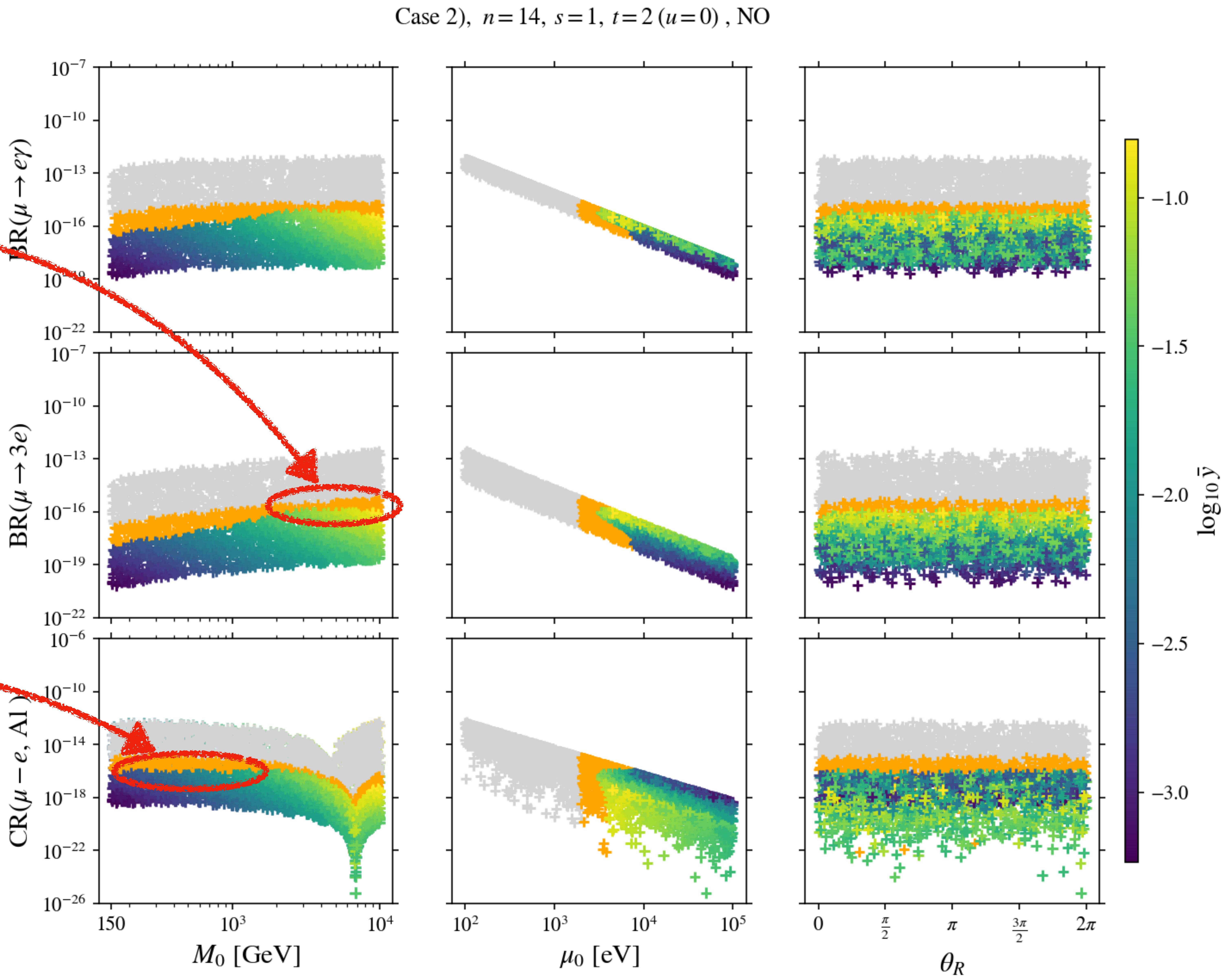
(M. Blennow, E. Fernández-Martínez, J. Hernández-García, J. López-Pavón, X., D. Naredo-Tuero ('23)

Option 2: Case 2

(FPDM, C. Hagedorn, ('24))

Mu3E Bound reached

COMET bound reached



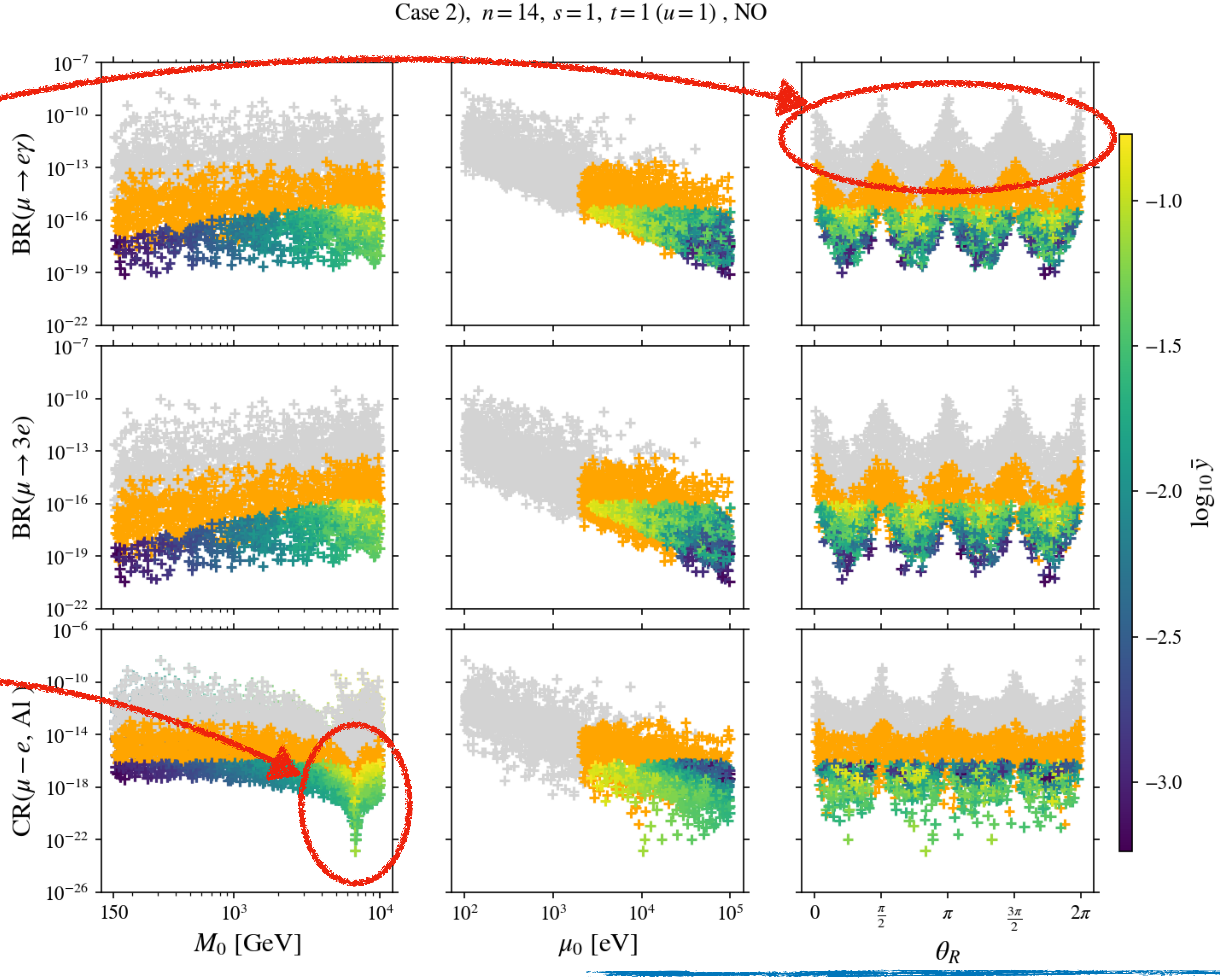
Option 2: Case 2

(FPDM, C. Hagedorn, ('24))

Modulation in function of θ_R

Enhancement of rates for

$$\sin(2\theta_R) \approx 0$$



Cancellation of contributions

$$x_0^{(canc)} \approx 6470 \quad \Rightarrow \quad M_0^{(canc)} \approx 6.5 TeV$$

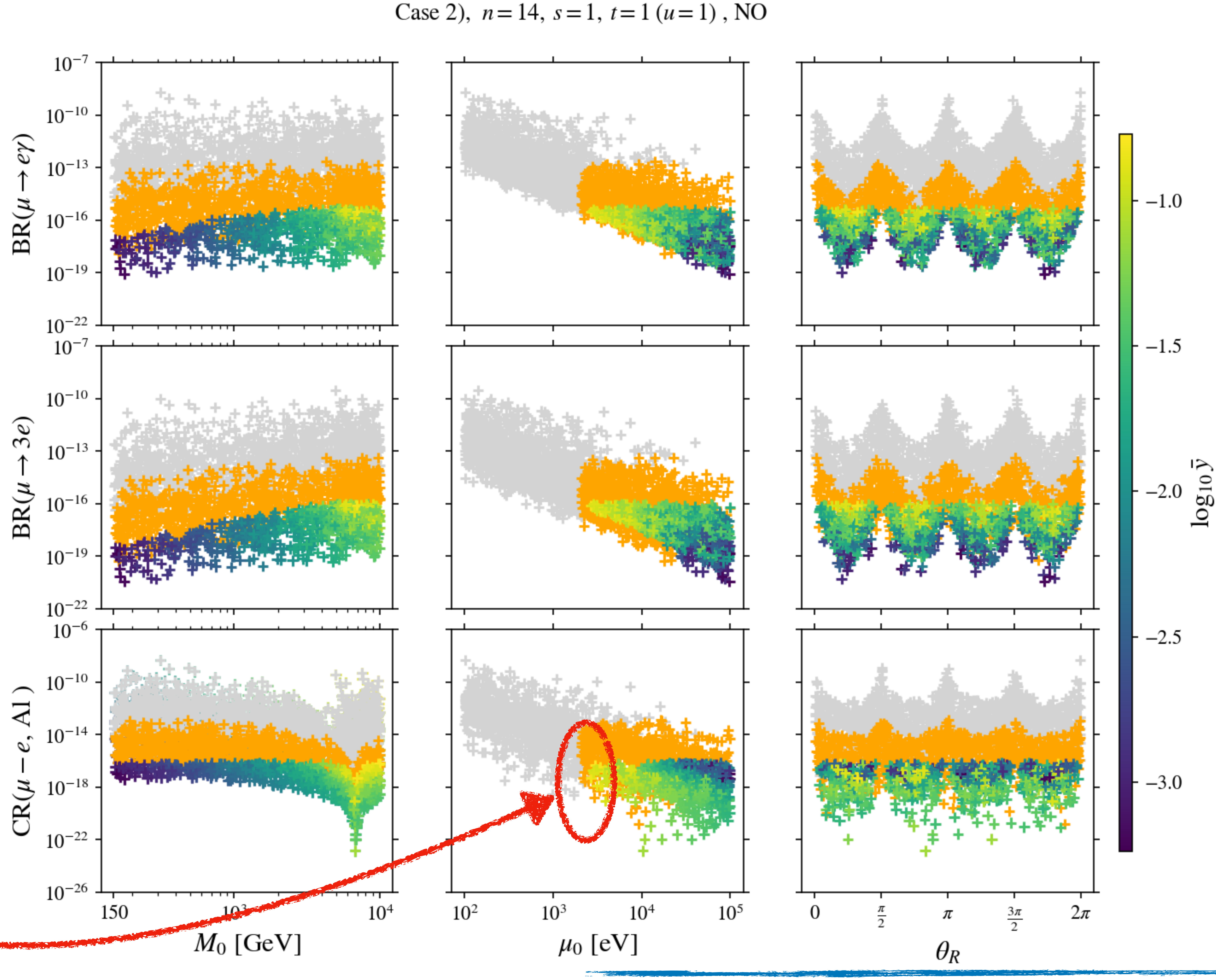
Option 2: Case 2

(FPDM, C. Hagedorn, ('24))

Predictions are compatible with future bounds on $\mu - e$ transitions!

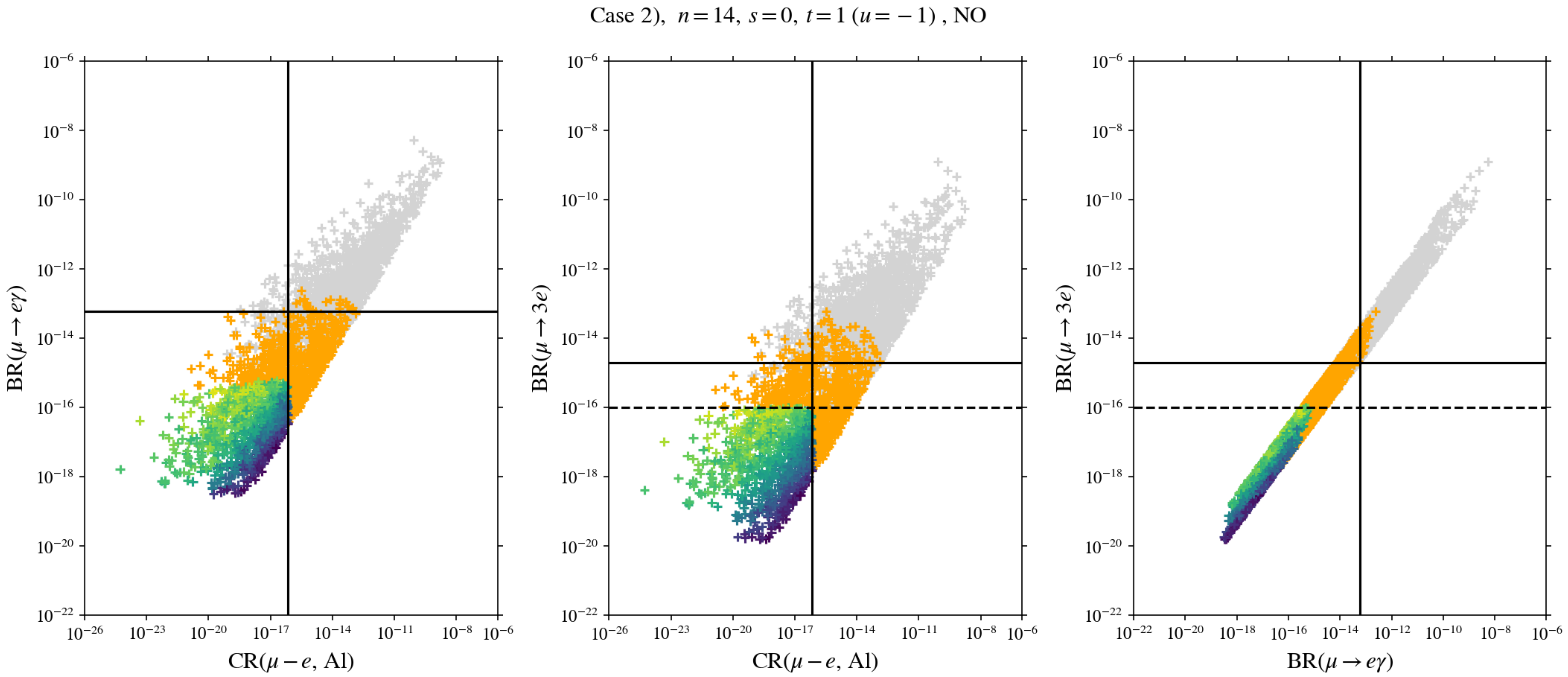
Lower limit for μ_0 can be extrapolated:

$$\mu_0 \gtrsim 2\ keV$$



Option 2: Case 2

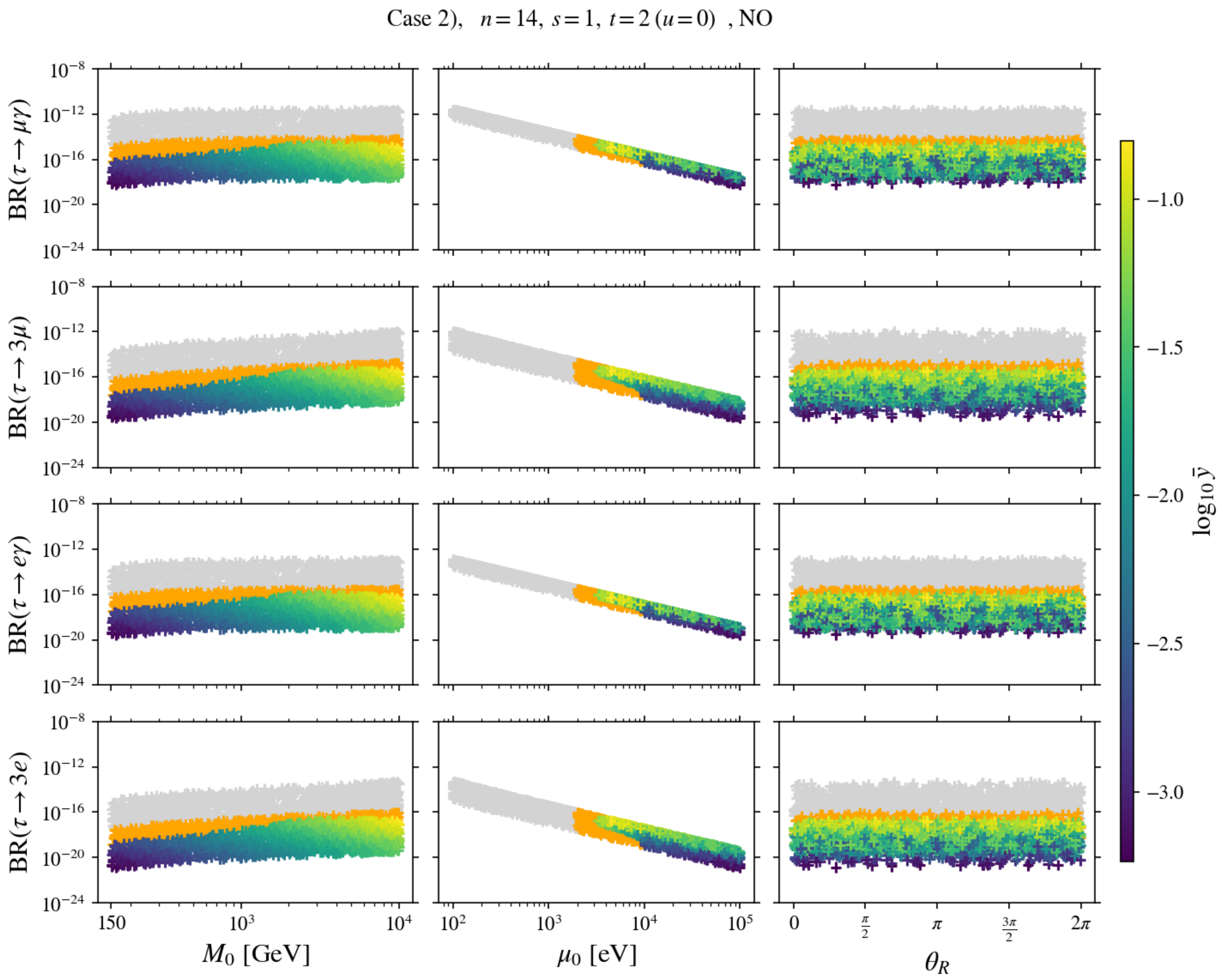
(FPDM, C. Hagedorn, ('24))



Bounds on $BR(\mu \to e\gamma)$ are only mildly constraining

COMET and Mu2E have big potential

Option 2: Case 2



$\tau - l$ transitions are non-constraining!

(FPDM, C. Hagedorn, ('24))

Summary for Option 2:

- For Option 2, all symmetry breaking information is contained in the Yukawa couplings Y_l
- Spectrum of the heavy sterile states is made of three quasi-degenerate pseudo-Dirac couples
- Mixing of heavy sterile states with light neutrinos (described by η) induces testable signals for processes violating Charged Lepton Flavour
- Future experiments could test the parameter space considered in our analysis

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LET'S NOW MOVE ON TO OPTION 3 !

Option 3

(FPDM, C. Hagedorn, ('25, To appear))

$$Y_D = \quad y_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mu_S = \quad \mu_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$M_{NS} = U_N(\theta_N) \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix} U_{S^\dagger}(\theta_S)$$

The flavour symmetry is broken by the M_{NS} mass term.

Option 3

(FPDM, C. Hagedorn, ('25, To appear))

$$Y_D = \quad y_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mu_S = \quad \mu_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

The flavour symmetry is broken by the M_{NS} mass term.

$$M_{NS} = U_N(\theta_N) \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix} U_{S^\dagger}(\theta_S)$$

As for Option 2, the heavy neutral states mass matrix is approximately:

$$M_h \approx \begin{pmatrix} \varnothing & M_{NS} \\ M_{NS}^T & \mu_S \end{pmatrix}$$

The spectrum of heavy sterile states is:

$$m_4 = M_1 - \frac{\mu_0}{2} \qquad m_7 = M_1 + \frac{\mu_0}{2}$$
$$m_5 = M_2 - \frac{\mu_0}{2} \qquad m_8 = M_2 + \frac{\mu_0}{2}$$
$$m_6 = M_3 - \frac{\mu_0}{2} \qquad m_9 = M_3 + \frac{\mu_0}{2}$$

Option 3

(FPDM, C. Hagedorn, ('25, To appear))

T

As per Option 2, spectrum of heavy sterile states is made of three pseudo-Dirac Couples

The main difference respect to Option 2 is the fact that heavy states are now non-degenerate!

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(FPDM, C. Hagedorn, ('25, To appear))

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Mass scales M_i are set by requiring we reproduce light neutrino mass spectrum!

Option 3: Numerical Scan

y_0

- In our numerical analysis:
  $y_0 \in [10^{-4}; 1]$

Option 3: Numerical Scan

μ_0

- In our numerical analysis:

$y_0 \in [10^{-4}; 1]$ $\mu_0 \in [0.1\ keV;\ 10^3\ keV]$

Option 3: Numerical Scan

θ_S θ_S

- In our numerical analysis:

$y_0 \in [10^{-4}; 1]$ $\mu_0 \in [0.1\ keV;\ 10^3\ keV]$ $\theta_S \in [0;\ 2\pi]$

Option 3: Numerical Scan

M_1

M_2

M_3

M_1

M_2

M_3

- In our numerical analysis:

$y_0 \in [10^{-4}; 1] \qquad \mu_0 \in [0.1\ keV;\ 10^3\ keV] \qquad \theta_S \in [0;\ 2\pi]$

- Fixed by fitting LO predictions of the masses to experimental values

Option 3: Numerical Scan

θ_N θ_N

- In our numerical analysis:

$$y_0 \in [10^{-4}; 1] \qquad \mu_0 \in [0.1\ keV;\ 10^3\ keV] \qquad \theta_S \in [0;\ 2\pi]$$

- Fixed by fitting LO predictions of the masses to experimental values
- Fixed by fitting prediction of mixing to lepton mixing data

Option 3: Numerical Scan

- In our numerical analysis:

  $y_0 \in [10^{-4}; 1] \quad \mu_0 \in [0.1\ keV;\ 10^3\ keV] \quad \theta_S \in [0;\ 2\pi]$

- Fixed by fitting LO predictions of the masses to experimental values
- Fixed by fitting prediction of mixing to lepton mixing data
- Points of the scans are generated in a way that lightest sterile state's mass is above $150\ GeV$

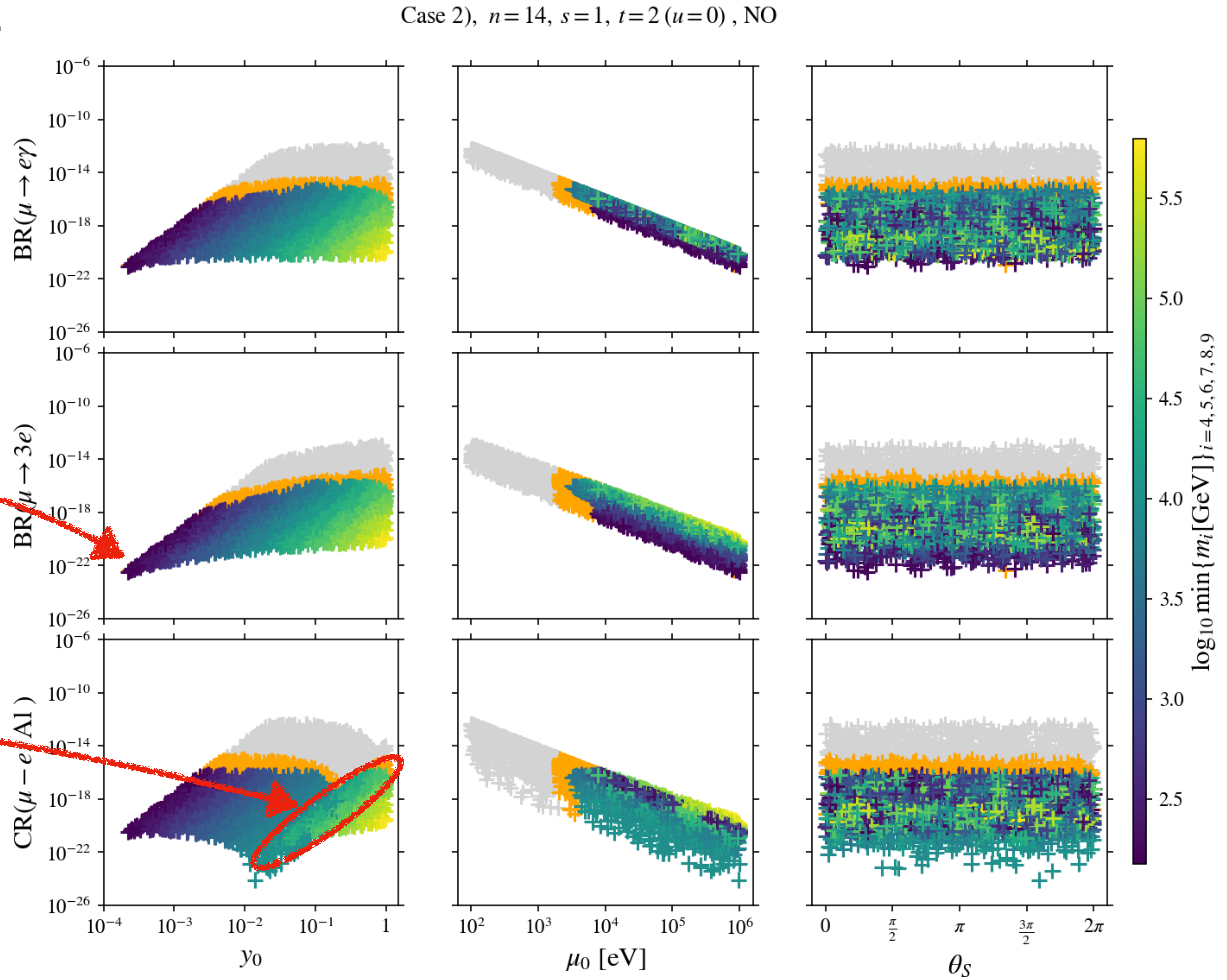
Option 3: Case 2

Predictions are similar to Option 2

Points converge as the mass of the lightest heavy sterile state approaches the threshold of $150\ GeV$

Position of the minima depends on the particular point in parameter space (in particular on μ_0)

Minima is really deep for $m_0 = 0.03(0.015)\ eV$ for NO(IO)

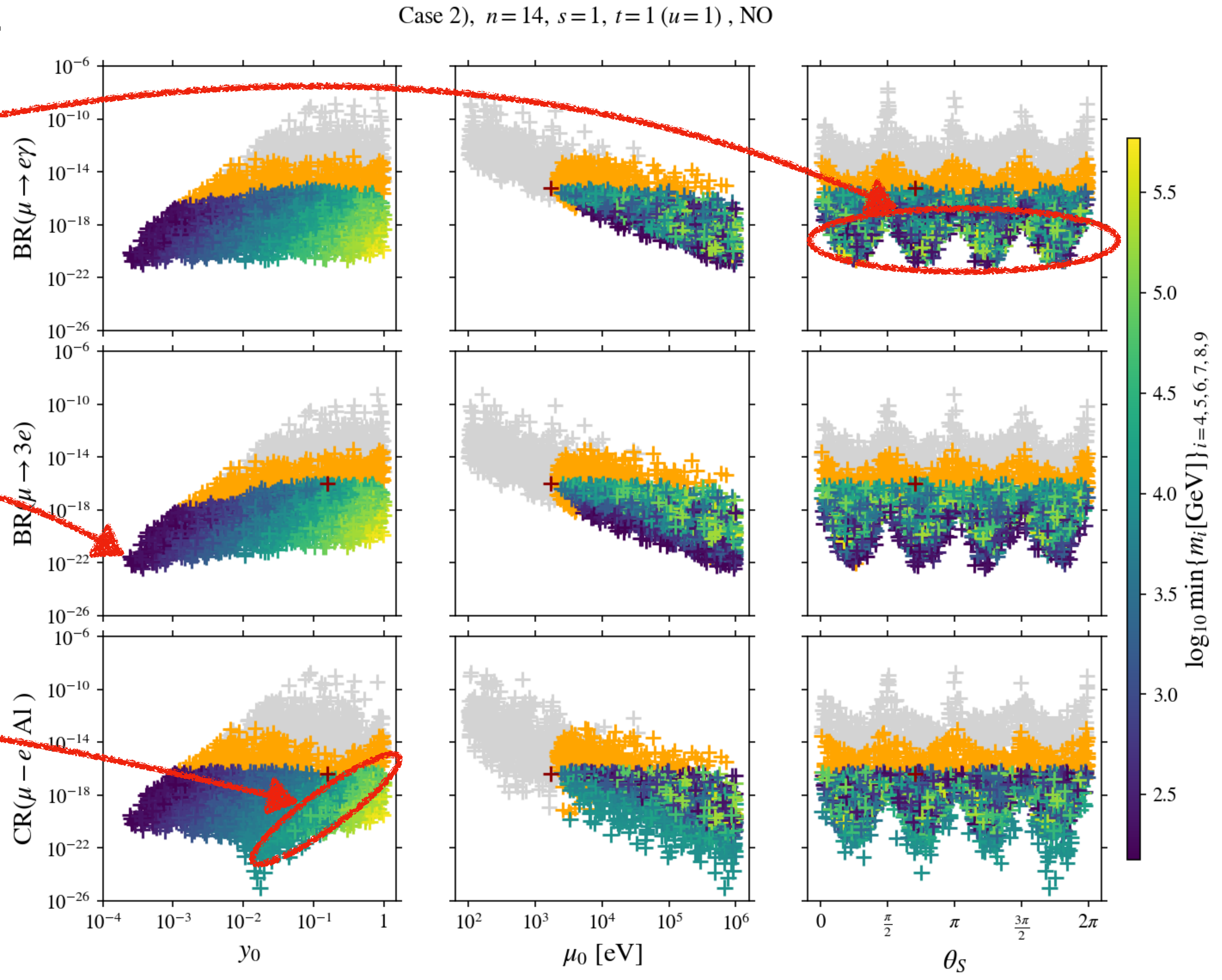


Option 3: Case 2

θ_S dependence is the same as per Option 2

Points converge as the mass of the lightest heavy sterile state approaches the threshold of $150\ GeV$

Position of the minima depends on the particular point in parameter space (in particular on μ_0)



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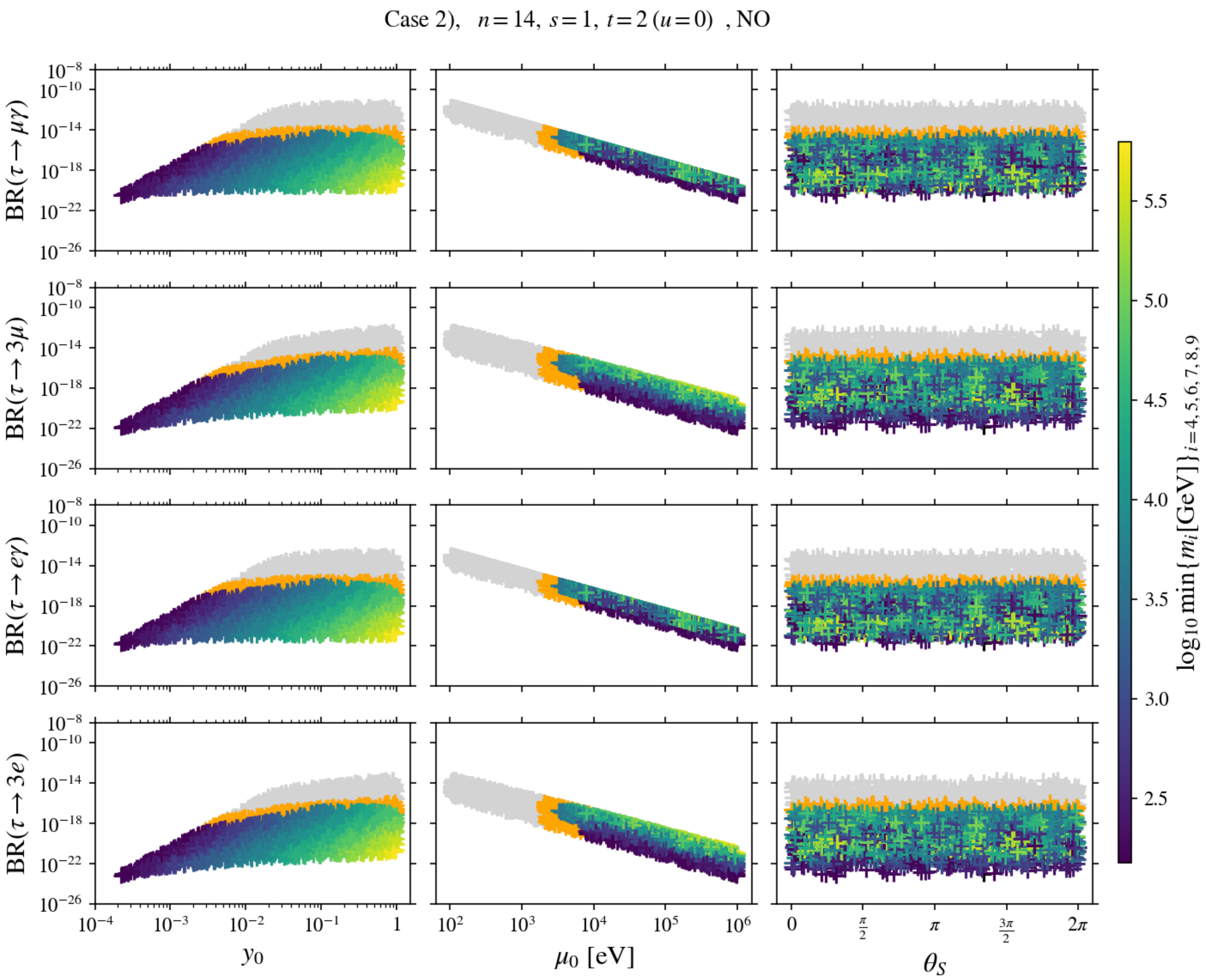


Again, $\tau - l$ transitions are non-constraining!

Option 2 VS. Option 3

We have discussed two Options in an ISS framework to reproduce the light neutrino mass spectrum and Mixing Data

Both solutions are equally successful in this regard

Both predict cLFV signals that could be detectable in the near future

Is there any hope to **distinguish the two options** via these?

Option 2 VS. Option 3

Filter points of the scan corresponding to a range of

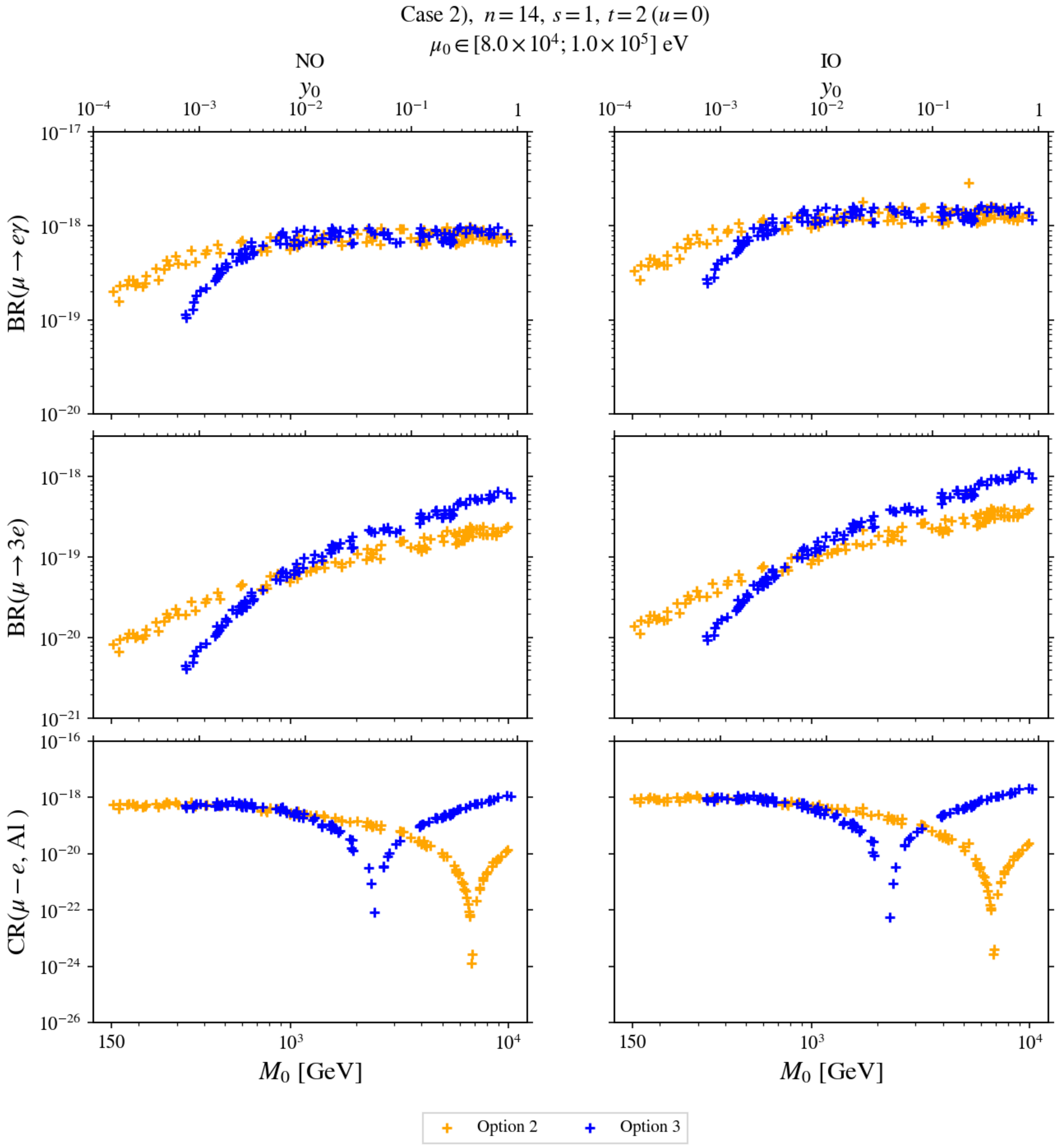
$\mu_0 \in [8 \times 10^4, 10^5]\ GeV$

Upper and Lower limits predicted for the rates are comparable:

Differences are never larger than a factor $\mathcal{O}(3)$ for θ_S -independent cases,

Never larger than a factor $\mathcal{O}(6)$ for θ_S -dependent cases

No sensible difference between the two Options can be found!

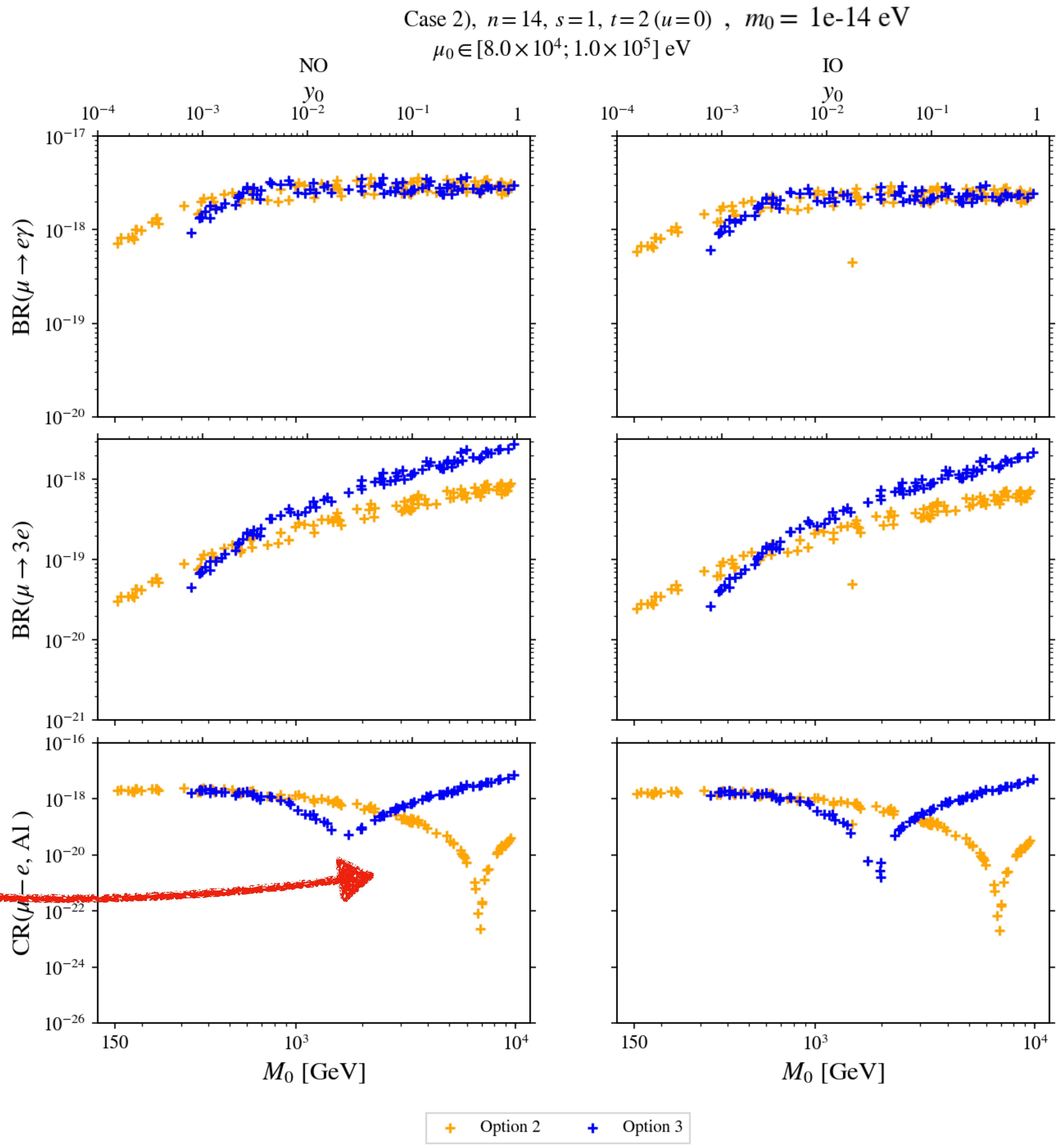


Option 2 VS. Option 3

Filter points of the scan corresponding to a range of $\mu_0 \in [8 \times 10^4, 10^5]\ GeV$

In the decoupling limit, similar considerations hold

Cancellation/Local Minima of the Conversion rate is our best chance at distinguishing the two Options!



SUMMARY AND CONCLUSIONS

- We considered an ISS framework embedded with flavour and CP symmetries
- Two Options of the framework have been considered, each characterised by different representations of the fields and different spectrum of heavy states
    - Option 2 's heavy sterile states spectrum is composed of 3 almost-degenerate pseudo-Dirac couples
    - Option 3 's heavy sterile states spectrum is composed of 3 non-degenerate pseudo-Dirac couples
- Both Options predict cLFV signals which could be tested by future facilities
- Predictions for the two options are really similar, so chances of distinguishing between the two mostly comes from detection of heavy states of the spectrum

THANKS FOR THE ATTENTION…

ANY QUESTIONS?

BACKUP SLIDES

Generalised CP Tranformations

Given a set of fields ϕ, a generalised CP transformation is defined as :

$$\phi(x) \rightarrow \phi' = X\,\phi^*(x_{CP}) \qquad x_{CP} = (x^0, -\vec{x})$$

(W. Grimus, M.N. Rebelo ('95))

(W. Grimus, G. Ecker, H. Neufeld ('84, '87, '88))

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CP is an involution transformation ($CP^2 = 1$), implying:

$X^*X = XX^*$ having taken X unitary, constant and symmetric.

Option 2

$$m_\nu \approx \frac{1}{M_0^2} m_D \mu_S m_D^T = \langle H \rangle \frac{\mu_0}{M_0^2} U_L^*(\theta_L) \left[\begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} U_R^T(\theta_R) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_R(\theta_R) \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} \right] U_L^\dagger(\theta_L)$$

Heavy neutral states mass matrix:

$$M_h \approx \begin{pmatrix} \emptyset & M_{NS} \\ M_{NS}^T & \mu_S \end{pmatrix}$$

Heavy neutral states spectrum: **Three almost <u>degenerate</u> Pseudo-Dirac Pairs**

$$m_{(i=4,5,6)} \approx M_0 - \frac{1}{2}\mu_0 \qquad m_{(i=7,8,9)} \approx M_0 + \frac{1}{2}\mu_0$$

Remember the ISS condition:

$$|\mu_0| \ll |m_D| \ll M_{NS}$$

Option 2

$$m_\nu \approx \frac{1}{M_0^2} m_D \mu_S m_D^T = \langle H \rangle \frac{\mu_0}{M_0^2} U_L^*(\theta_L) \left[\begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} U_R^T(\theta_R) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_R(\theta_R) \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} \right] U_L^\dagger(\theta_L)$$

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$\Omega(3)$, $\Omega(3')$ are unitary
Are determined by residual symmetry (specified by the **CASE** and n, s, t) and its embedding in G_f .

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$R_{ij}(\theta_{L,R})$ is a rotation on the ij plane
Codifies residual freedom in the choice of Ω matrices

Option 2

Example: Case 2

$$Z(\mathbf{r}) = c(\mathbf{r})^{n/2} \qquad X(\mathbf{r}) = c(\mathbf{r})^s d(\mathbf{r})^t X_0(\mathbf{r})$$

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Same structure for all s, t : $\Omega(\mathbf{3})(u,v) = e^{i\frac{v\pi}{n}} U_{TB} R_{13}\left(-\frac{u\pi}{2n}\right)\begin{pmatrix}1 & 0 & 0\\ 0 & e^{-i\frac{v\pi}{n}} & 0 \\ 0 & 0 & -i\end{pmatrix}$

s even, t odd: $\Omega(\mathbf{3'})(u,v) = e^{-i\frac{\pi}{4}} U_{TB} R_{13}\left(\frac{\pi}{4}\right)\begin{pmatrix}i & 0 & 0\\ 0 & e^{-i\frac{\pi}{4}} & 0 \\ 0 & 0 & 1\end{pmatrix}$

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(C. Hagedorn, E. Molinaro, A. Meroni, ('14))

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$$U_{PMNS} = U_L(\theta_L) = \Omega(3)\ R_{ij}(\theta_L)$$

(C. Hagedorn, E. Molinaro, A. Meroni, (‘14))

Option 2

Example: Case 2

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The predictions for mixing angles and CP invariants:

$$\sin^2\theta_{13} = \frac{1}{3}(1 - \cos\phi_u \cos(2\theta_L))$$

$$\sin^2\theta_{12} = \frac{1}{2 + \cos\phi_u \cos(2\theta_L)}$$

$$\sin^2\theta_{23} = \frac{1}{2}\left(1 + \frac{\sqrt{3}\sin\phi_u \cos(2\theta_L)}{2 + \cos\phi_u \cos(2\theta_L)}\right)$$

$$J_{CP} = -\frac{\sin(2\theta_L)}{6\sqrt{3}}$$

$$I_1 = \frac{1}{9}\left(\left[\cos\phi_u + \cos(2\theta_L)\right]\sin\phi_\nu - \sin\phi_u \cos\phi_\nu \sin(2\theta_L)\right)$$

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Option 2

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Predictions of mixing only depend on $\Omega(3)$!!

Generalised CP Tranformations

Given a set of fields ϕ, a generalised CP transformation is defined as :

$$\phi(x) \rightarrow \phi' = X\, \phi^*(x_{CP}) \qquad x_{CP} = (x^0, -\vec{x})$$

(W. Grimus, M.N. Rebelo ('95))

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CP is an involution transformation ($CP^2 = 1$), implying:

$X^*X = XX^*$ having taken X unitary, constant and symmetric.

When combined with Flavour Symmetries, a **consistency condition** is required:

$$X^{-1}\rho(g)X = \rho(g')^* \qquad g, g' \in G_f$$

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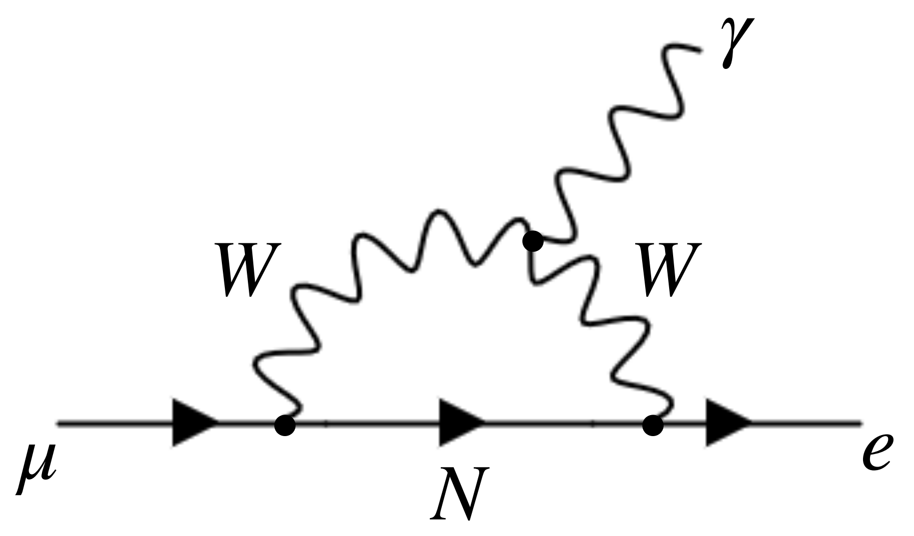
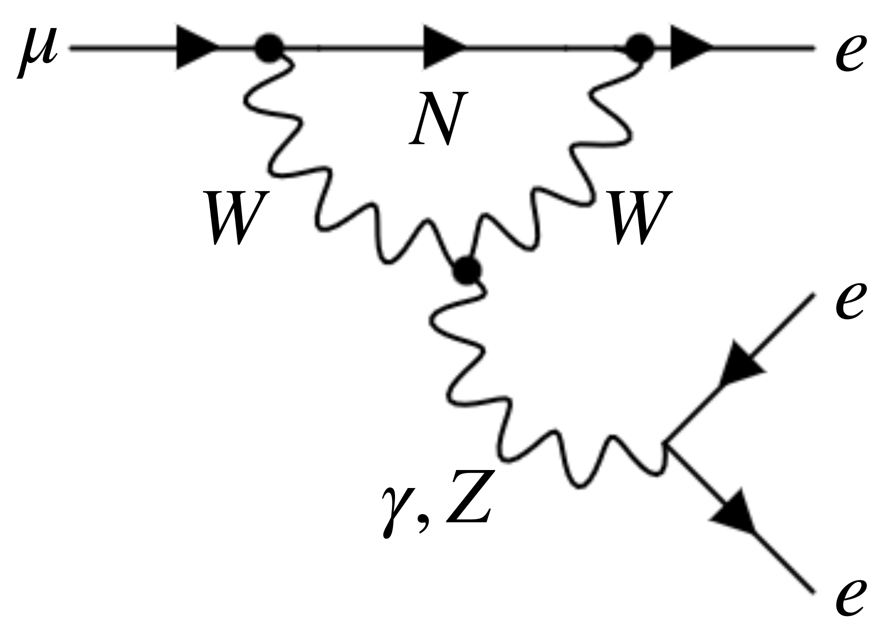
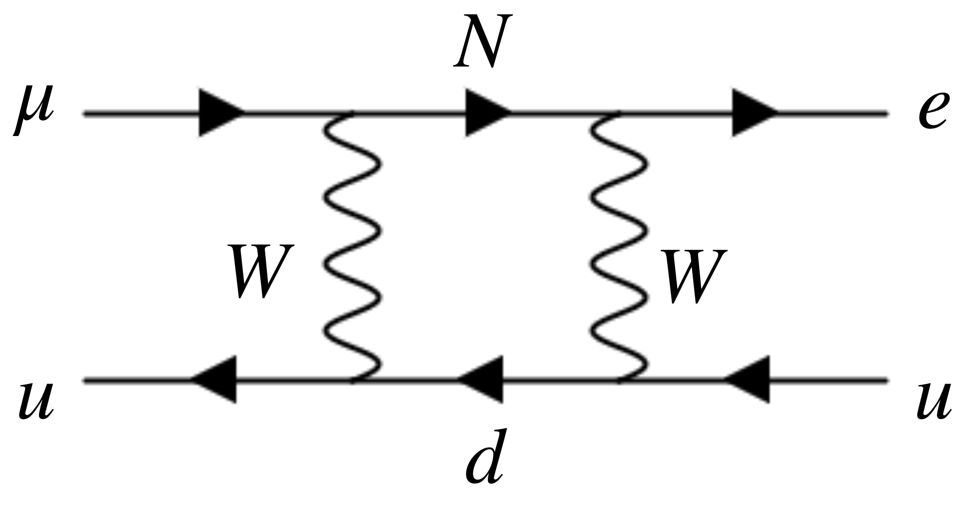
CP has to be an **AUTOMORPHISM** of the flavour group

Option 2

cLFV in the ISS







Relevant (approximated) loop functions for Option 2:

$$G_{\gamma}^{\beta\alpha} \approx \eta_{\alpha\beta} + \mathcal{O}(\frac{Y\mu_0}{M_0}, \frac{\mu_0}{M_0})$$

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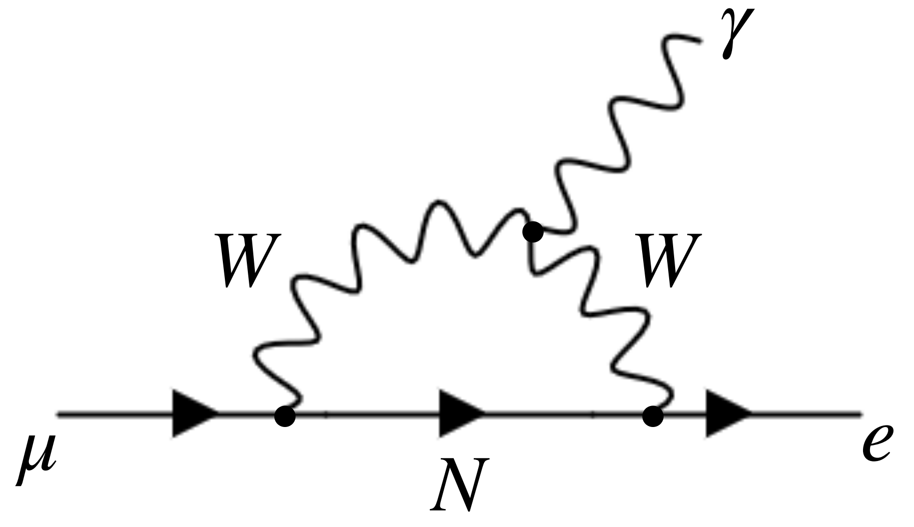
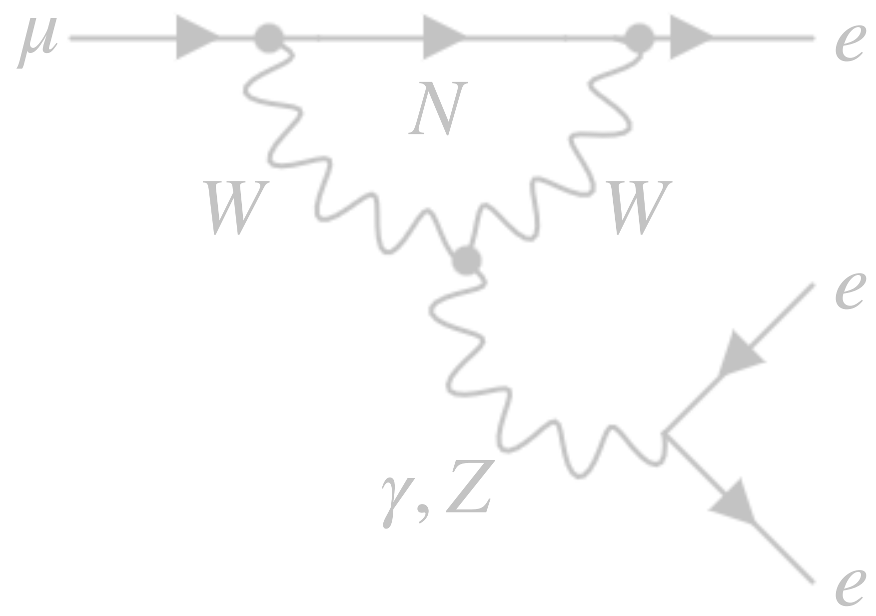
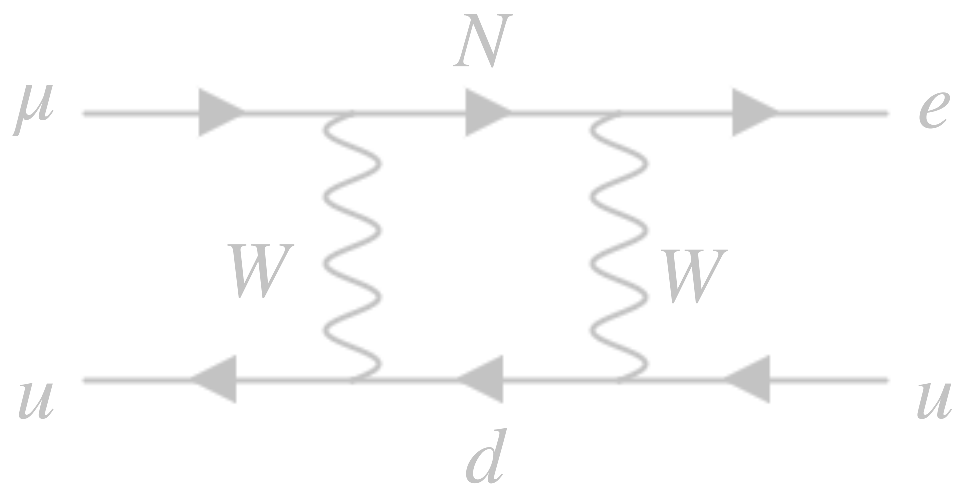
Option 2

cLFV in the ISS







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} **Photon Penguins**

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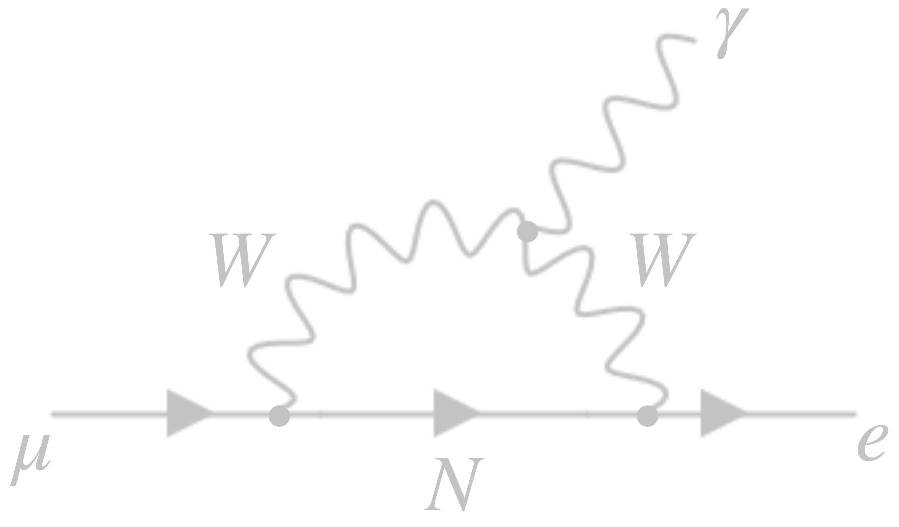
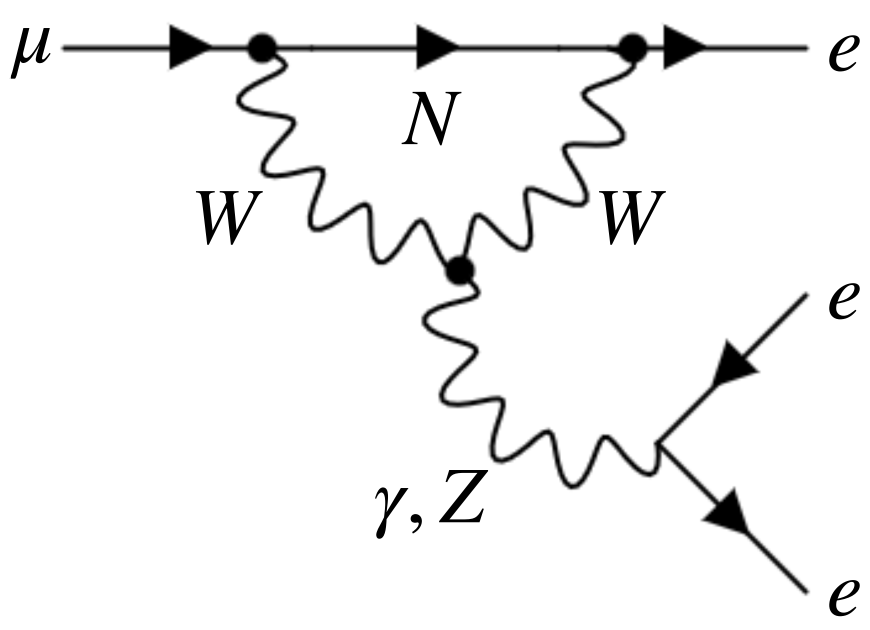
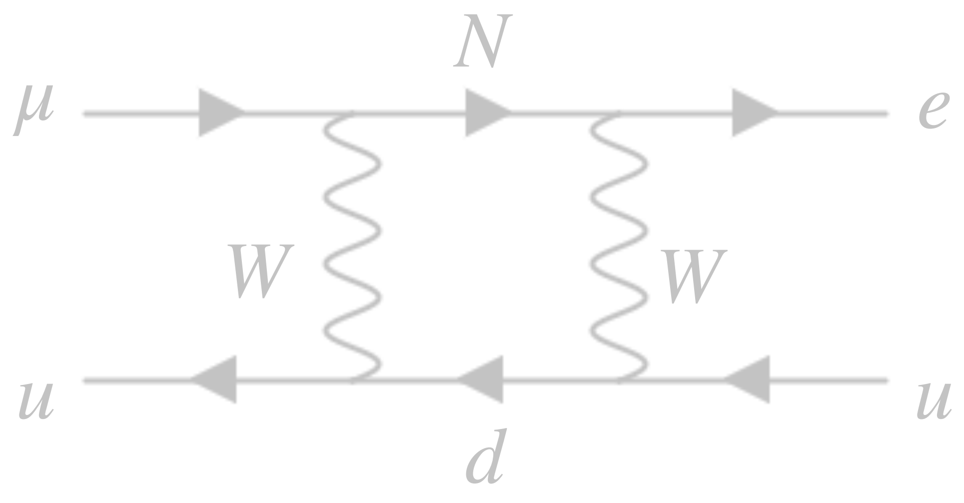
Option 2

cLFV in the ISS







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Option 2

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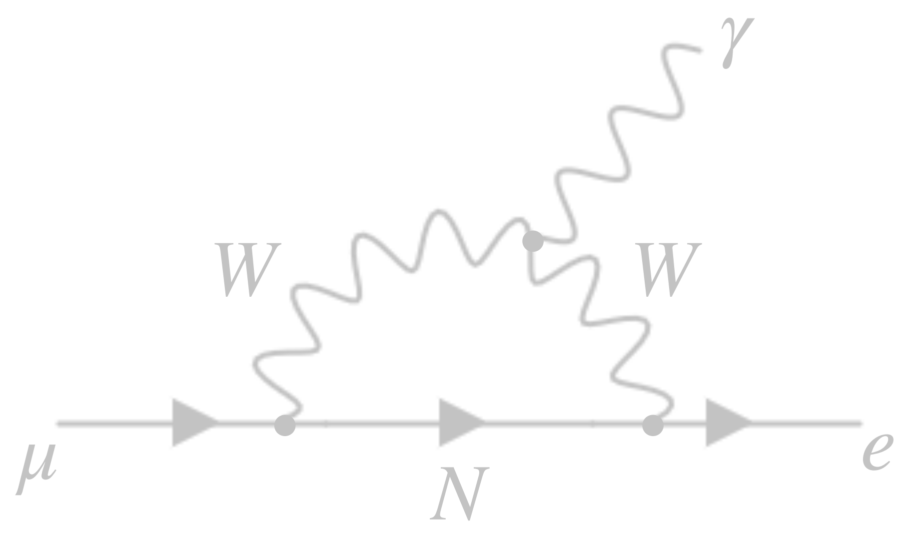

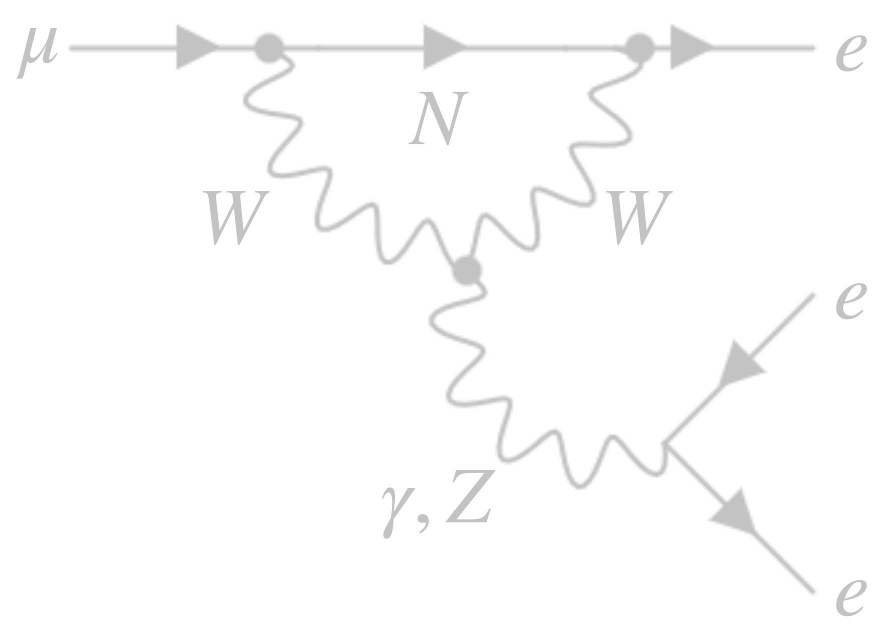

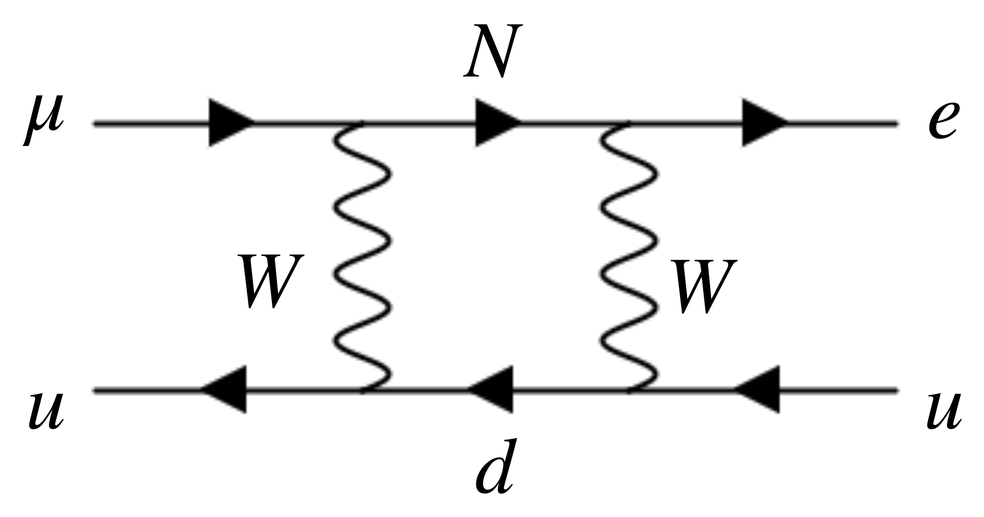

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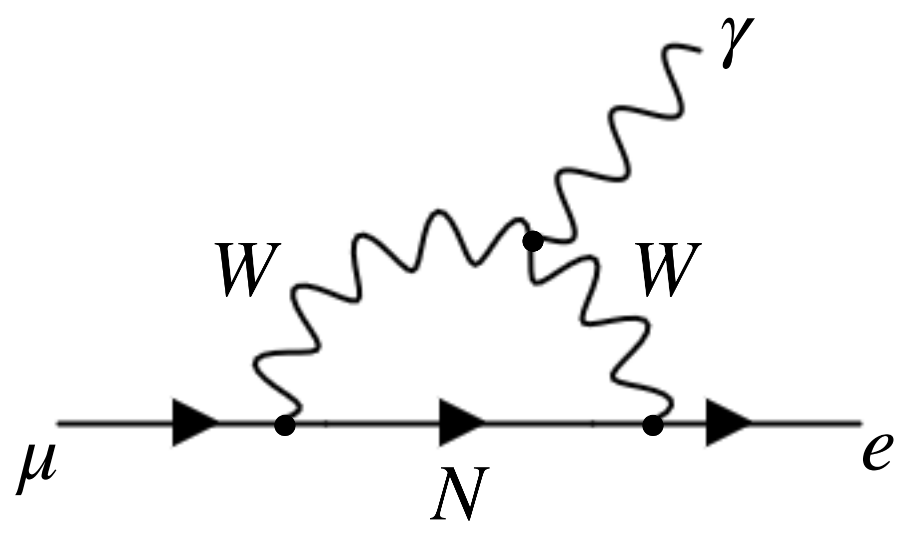
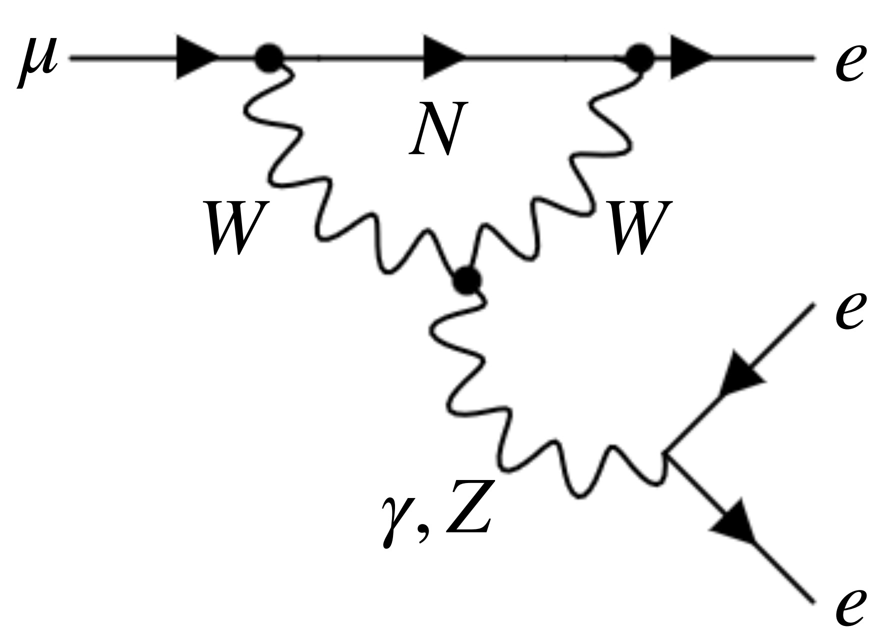
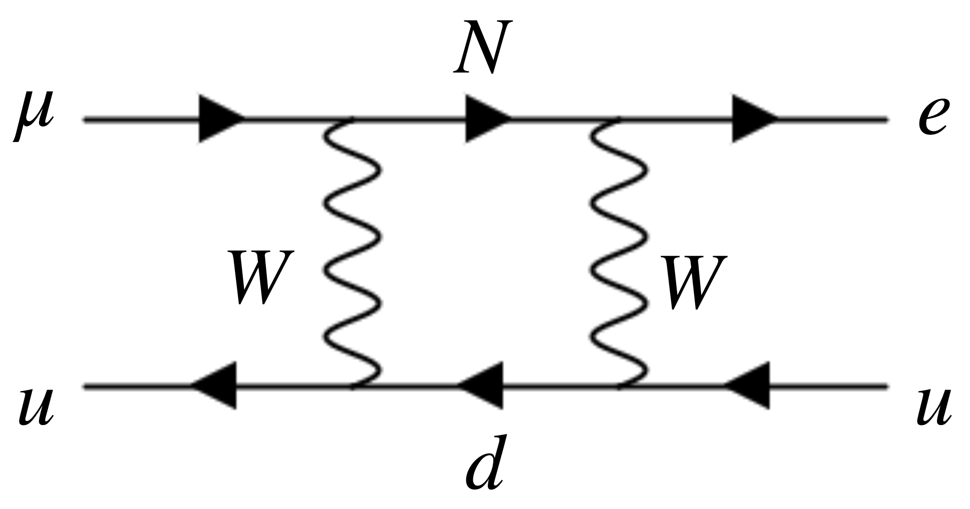
Box Diagrams

Option 2

cLFV in the ISS







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$\eta_{\alpha\beta}$

$\eta_{\alpha\beta}$

$\eta_{\alpha\beta}$

$\eta_{\alpha\beta}$

$\eta_{e\mu}$

$\eta_{e\mu}$

Rates of the processes are proportional to η at LO for Option 2!

$$\eta = \frac{v^2}{4M_0^2} U_L(\theta_L)\, diag(y_1^2, y_2^2, y_3^2\,) U_L(\theta_L)^\dagger$$

Option 2

The form of the η matrix elements depends on the particular case:

Case 1)
$$\eta_{e\mu} = \frac{1}{6}\eta_0' \left[2\Delta y_{21}^2 + \Delta y_{31}^2 \left(-1 + \cos(2\theta_L) - \sqrt{3}\sin(2\theta_L)\right)\right]$$

Case 2)
$$\eta_{\mu e} = \frac{1}{6}\eta_0' \left[2\Delta y_{21}^2 - \Delta y_{31}^2 \left(1 - \cos\left(2\theta_L\right)\cos\phi_u + \sqrt{3}\left(\cos\left(2\theta_L\right)\sin\phi_u - i\sin\left(2\theta_L\right)\right)\right)\right]$$

Case 3)
$$\eta_{e\mu} = \frac{1}{24}\eta_0' \Big\{ 2(1+i\sqrt{3})\left(\Delta y_{21}^2 - 2\Delta y_{31}^2\right)\sin^2\left(\phi_m\right)\xi_2\left(\pi - \phi_m\right) +$$
$$+\Delta y_{21}^2 \left[\cos\left(2\theta_L\right)(1+i\sqrt{3})\xi_1\left(2\phi_m\right) + \sqrt{2}\sin\left(2\theta_L\right)\left(2\xi_2\left(\phi_m - 3\phi_s\right) + i(i+\sqrt{3})\xi_2\left(\phi_m + 3\phi_s\right)\right)\right]\Big\}$$

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$$\left(\phi_u = \frac{2\pi(2s-t)}{n}\right)$$

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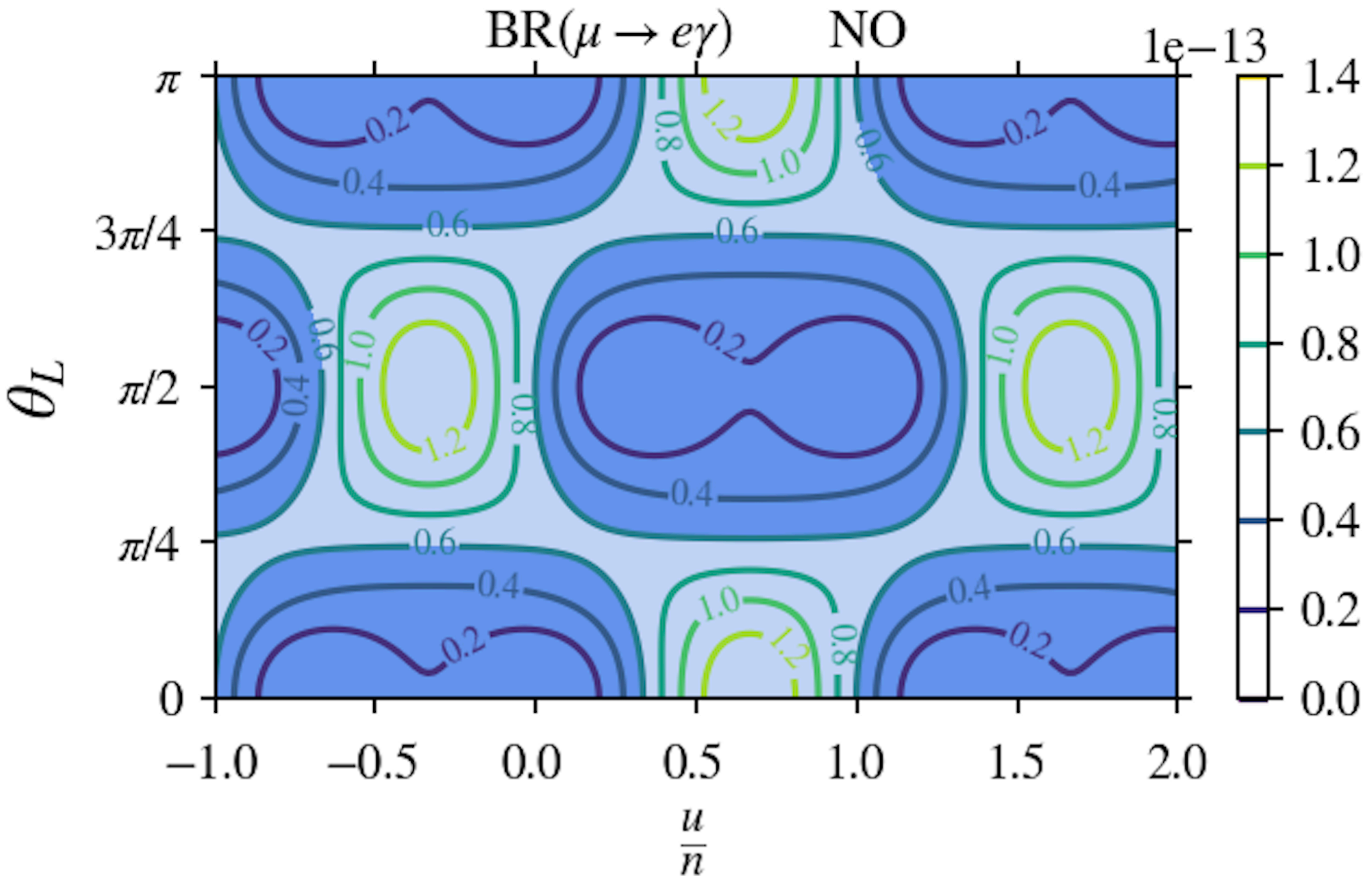
We show some results for Case 2…

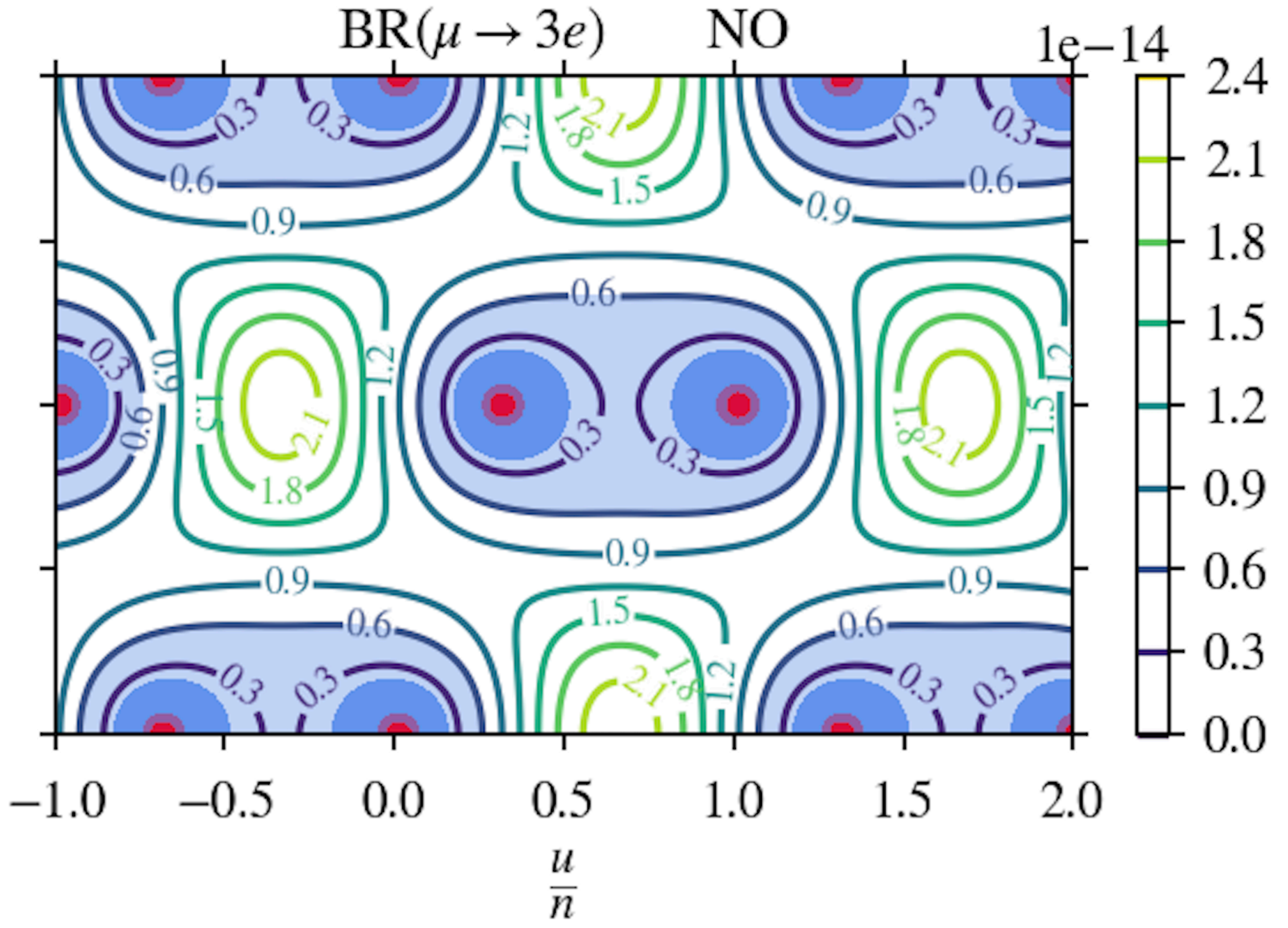
Option 2: Case 2

$M_0 = 3\ TeV$
$\mu_0 = 1\ keV$

t **even**



$BR(\mu \to e\gamma) \lesssim 6 \times 10^{-14}$ **Meg-II**



$BR(\mu \to 3e) \lesssim 20(1) \times 10^{-16}$ **Mu3E Phase-I (II)**

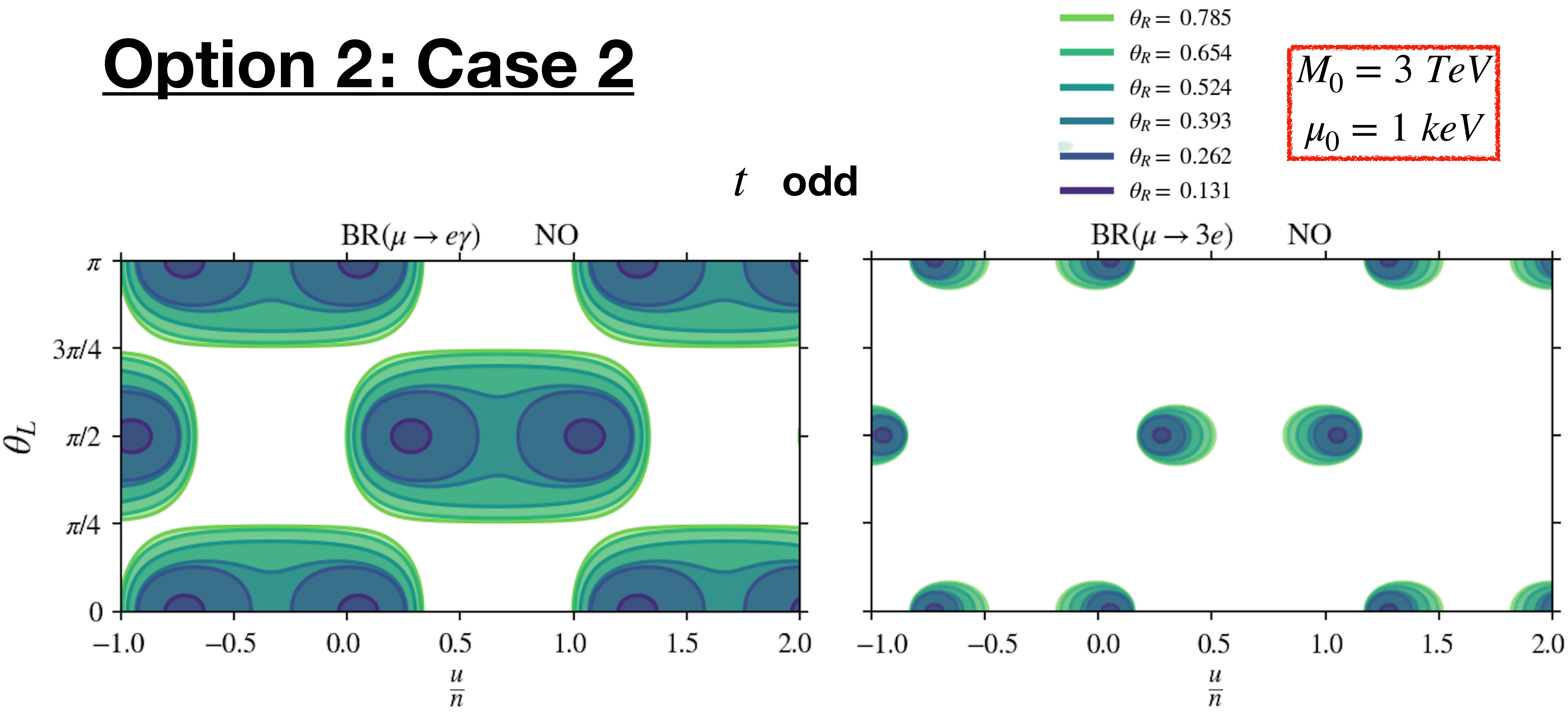
(FPDM, C. Hagedorn, ('24))

Option 2: Case 2

$M_0 = 3\ TeV$

$\mu_0 = 1\ keV$

t **odd**



$BR(\mu \to e\gamma) \lesssim 6 \times 10^{-14}$ **Meg-II**

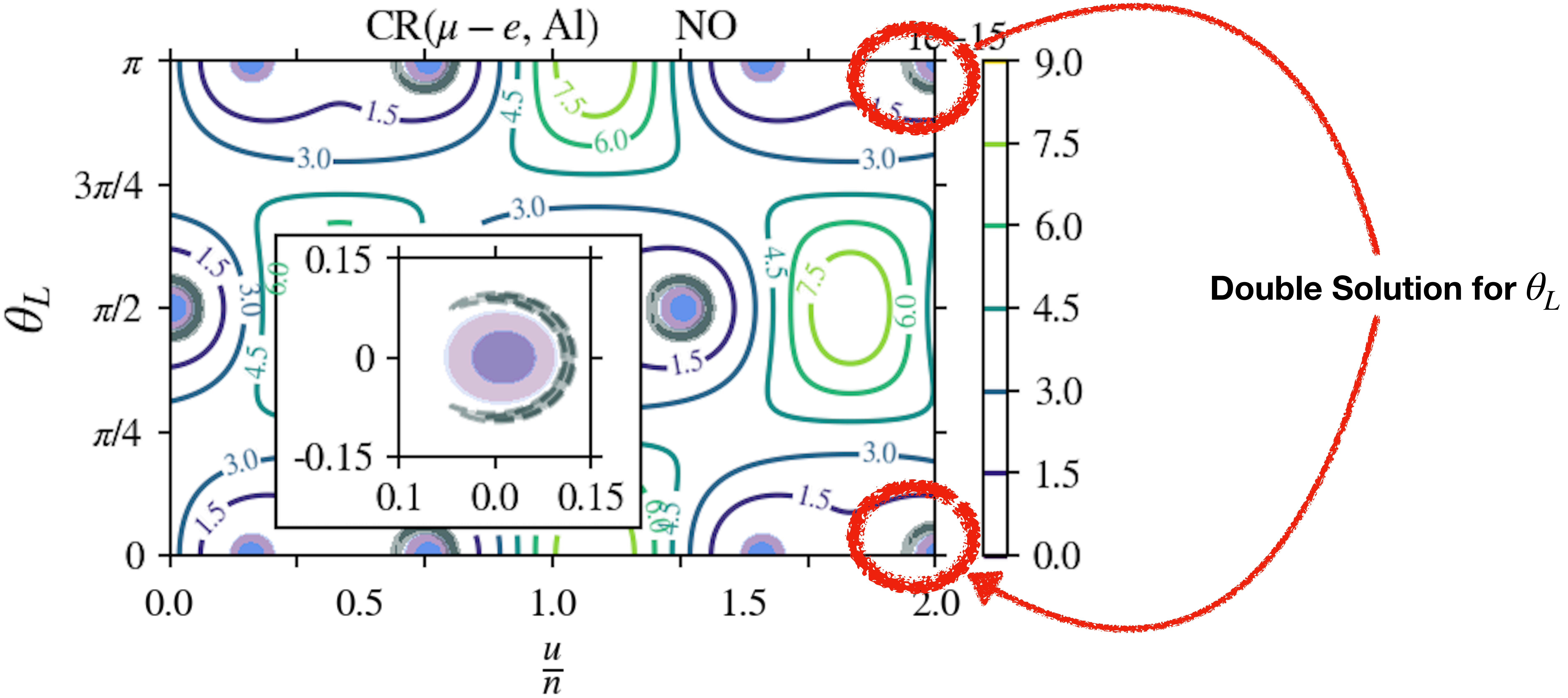
$BR(\mu \to 3e) \lesssim 20 \times 10^{-16}$ **Mu3E Phase-I (II)**

(FPDM, C. Hagedorn, ('24))

Option 2: Case 2

$M_0 = 3\ TeV$
$\mu_0 = 1\ keV$

t **even**



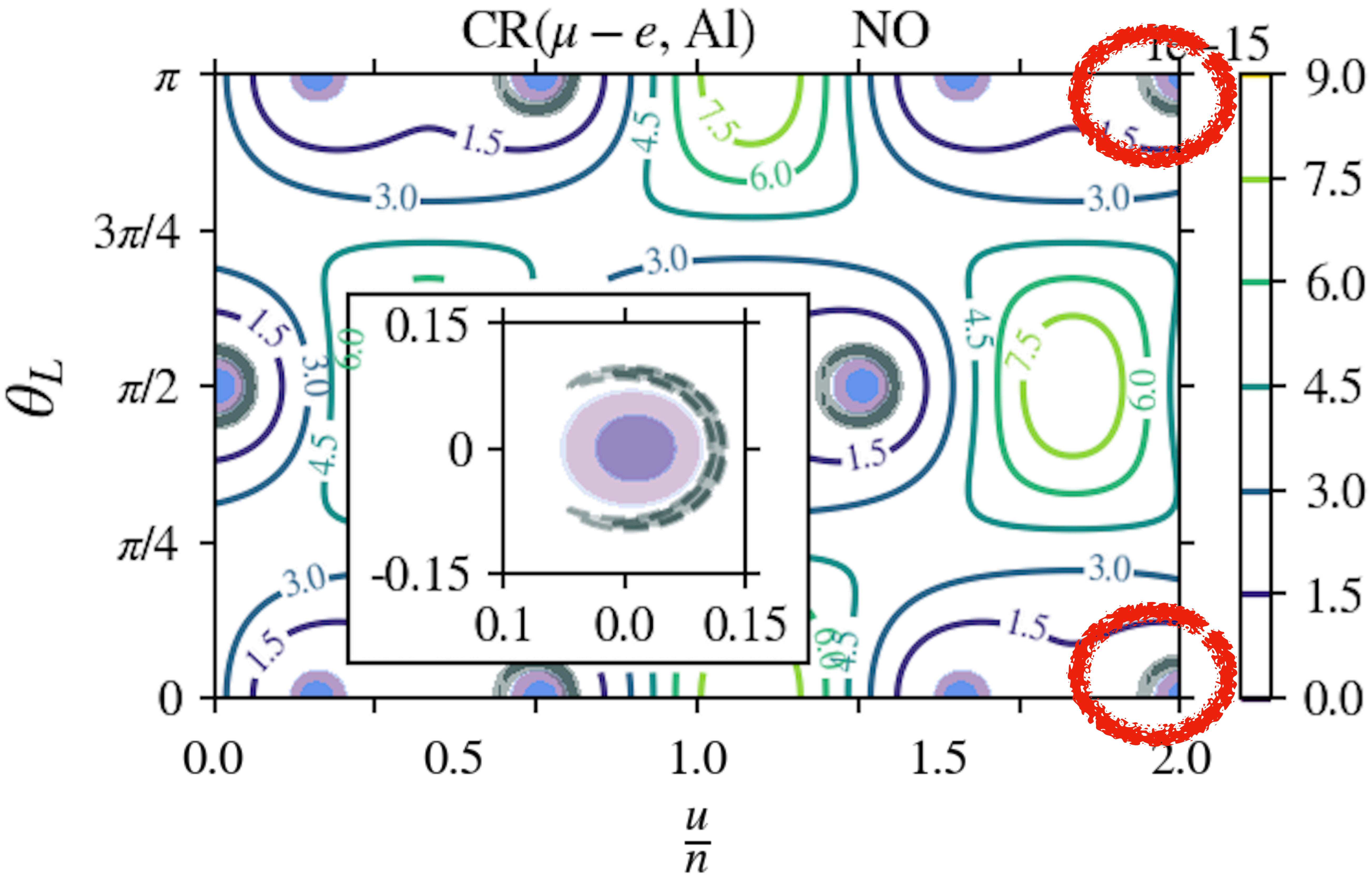
$CR(\mu - e, Al) \lesssim 7.0(8.0) \times 10^{-17}$ **COMET Phase-II (Mu2E)**

(FPDM, C. Hagedorn, ('24))

Option 2: Case 2

$M_0 = 3\ TeV$
$\mu_0 = 1\ keV$

t **even**



	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$
$n=14;\ s=t=1$	0.34	0.021(0.022)	0.559(0.561)
$n=14;\ s=t=0$	0.341	0.022	0.5
$n=14;\ s=0;\ t=1$	0.34	0.022	0.436
$n=14;\ s=1;\ t=2$	0.341	0.022	0.5

$\tilde{\theta}_L \approx 0.183\ (2.959)$

Best fit results: ($\chi^2_{mix} \leq 12$)

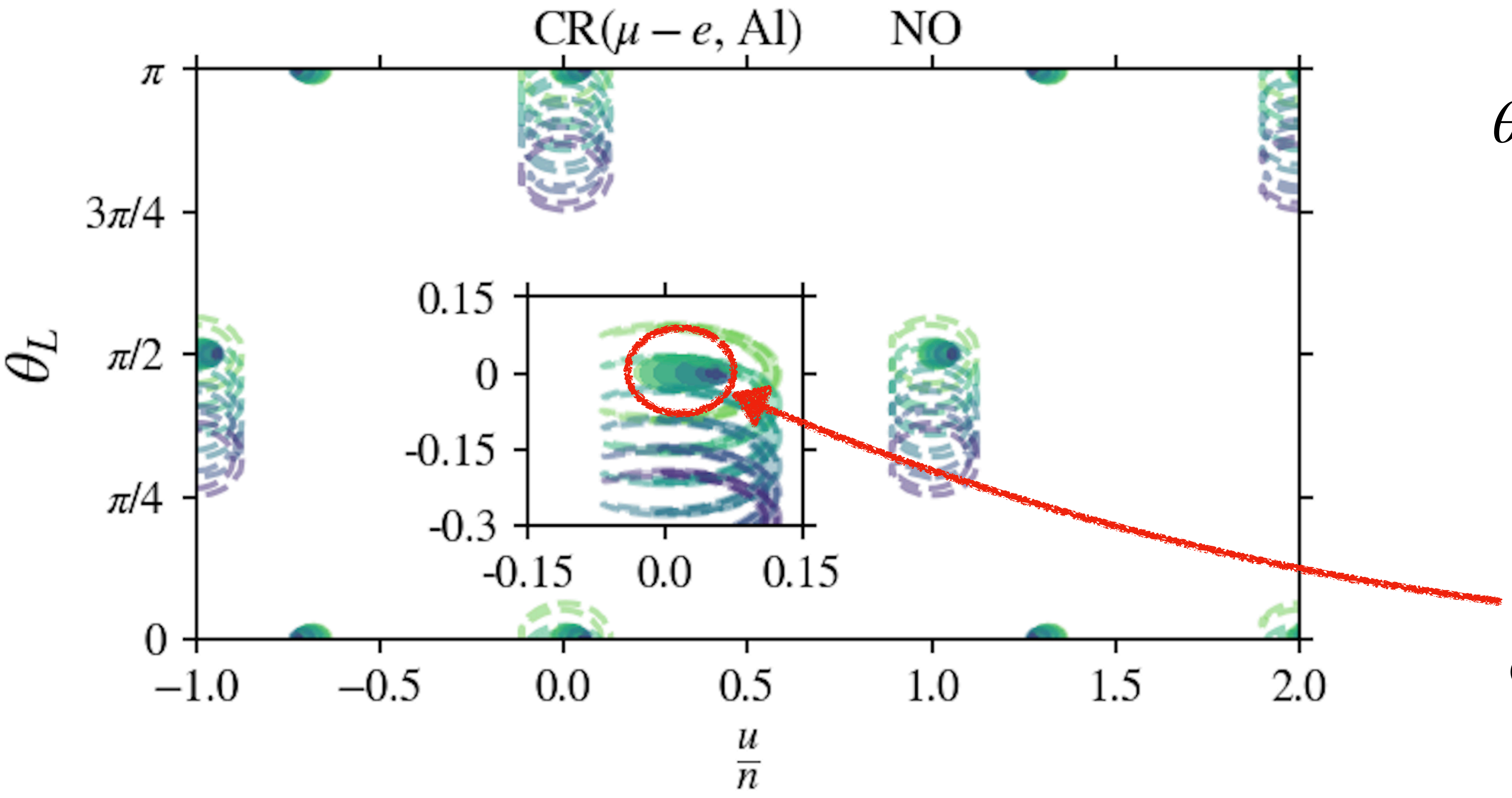
$CR(\mu - e, Al) \lesssim 7.0(8.0) \times 10^{-17}$ **COMET Phase-II (Mu2E)**

(FPDM, C. Hagedorn, ('24))

Option 2: Case 2

$M_0 = 3\ TeV$
$\mu_0 = 1\ keV$

t **odd**



$CR(\mu - e, Al) \lesssim 7.0 \times 10^{-17}$ **COMET Phase-II**

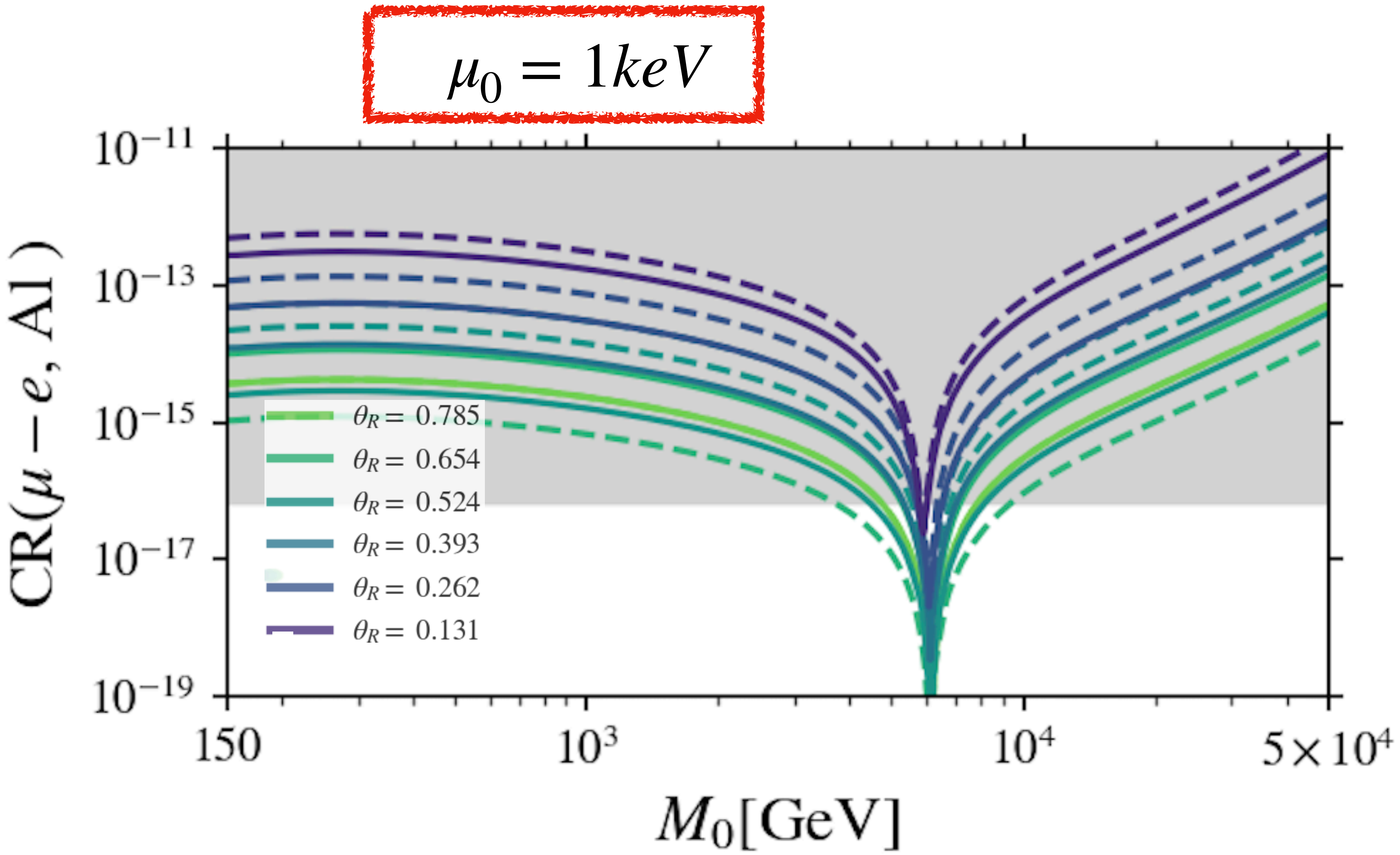
(FPDM, C. Hagedorn, ('24))

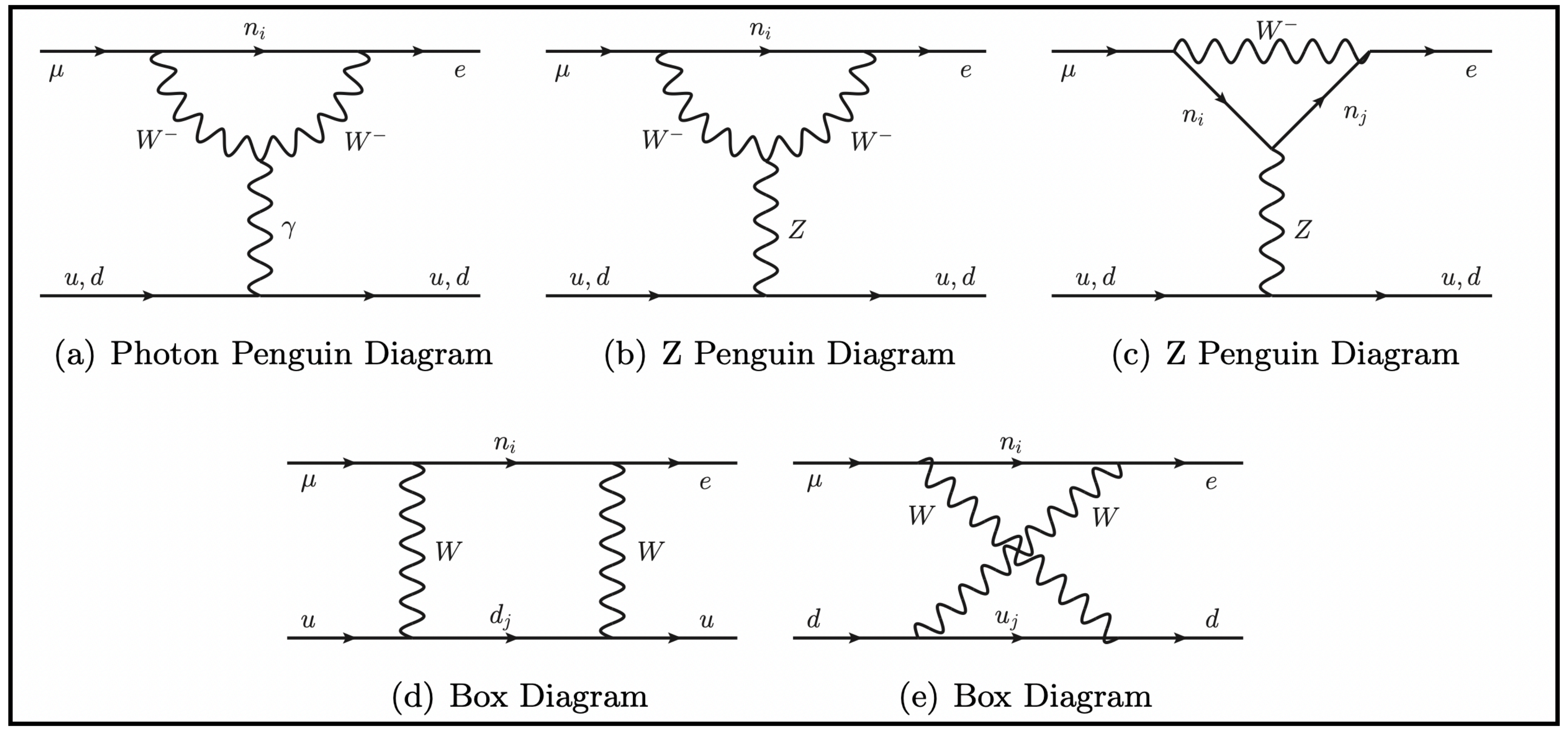
θ_R Shifts best-fit regions upward:

$$\tan(\delta_\theta) = -\frac{y_i y_j \cot(2\theta_R)}{y_i^2 + y_j^2}$$

Different choices of θ_R give different predictions for cLFV

Option 2: Case 2





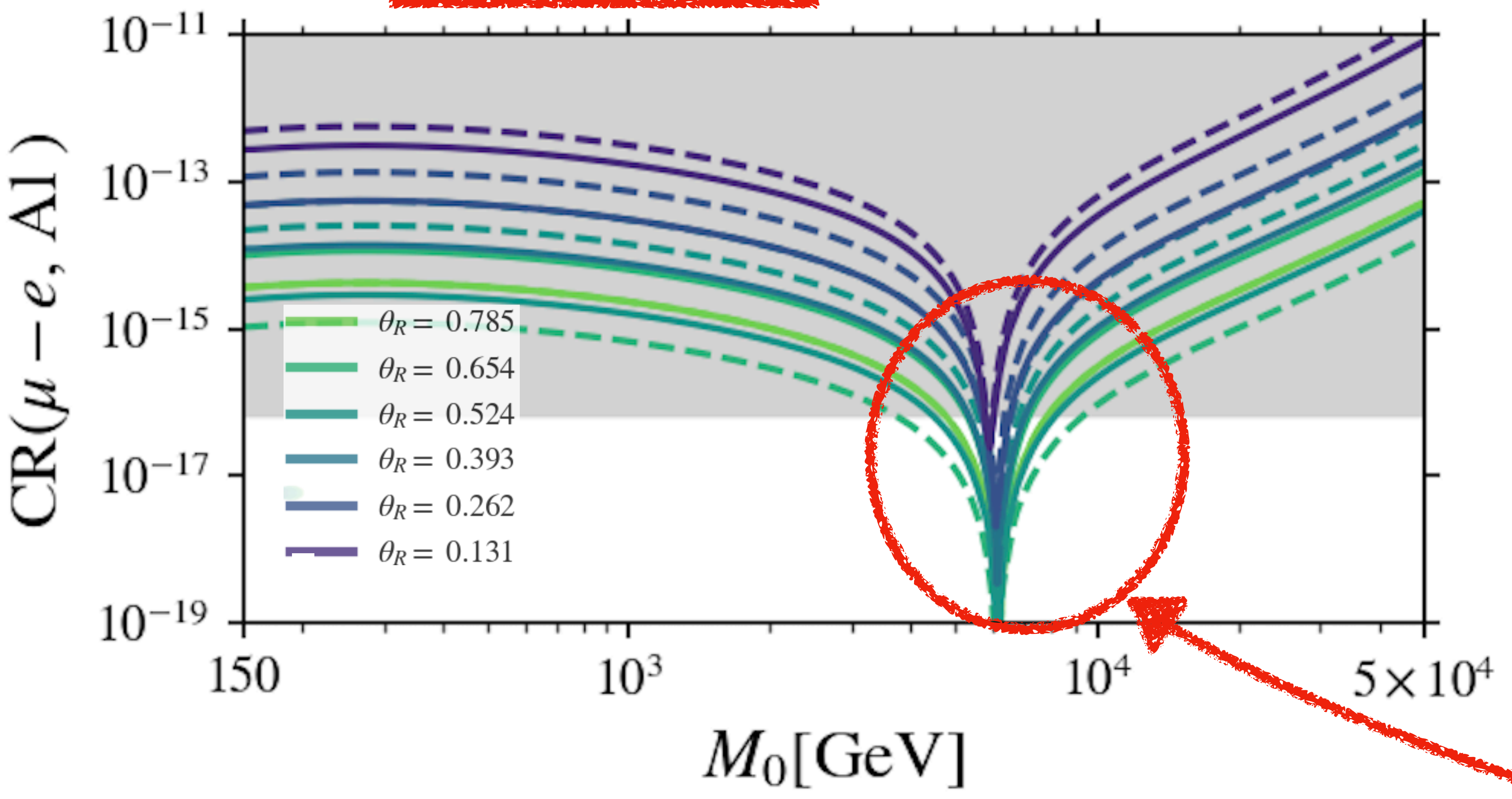
(R. Alonso, M. Dhen, M. B. Gavela, T. Hambye ('12))

Conversion Rate has some interesting properties:

$$CR(\mu - e, Al) = \frac{2G_F^2\alpha_w^2 m_\mu^5}{(4\pi)^2\Gamma_{capt}}\left|4V^{(p)}\left(2\tilde{F}_u^{\mu e} + \tilde{F}_d^{\mu e}\right) + 4V^{(n)}\left(\tilde{F}_u^{\mu e} + 2\tilde{F}_d^{\mu e}\right) + s_w^2\frac{G_\gamma^{\mu e}D}{2e}\right|^2$$

Option 2: Case 2

$\mu_0 = 1keV$

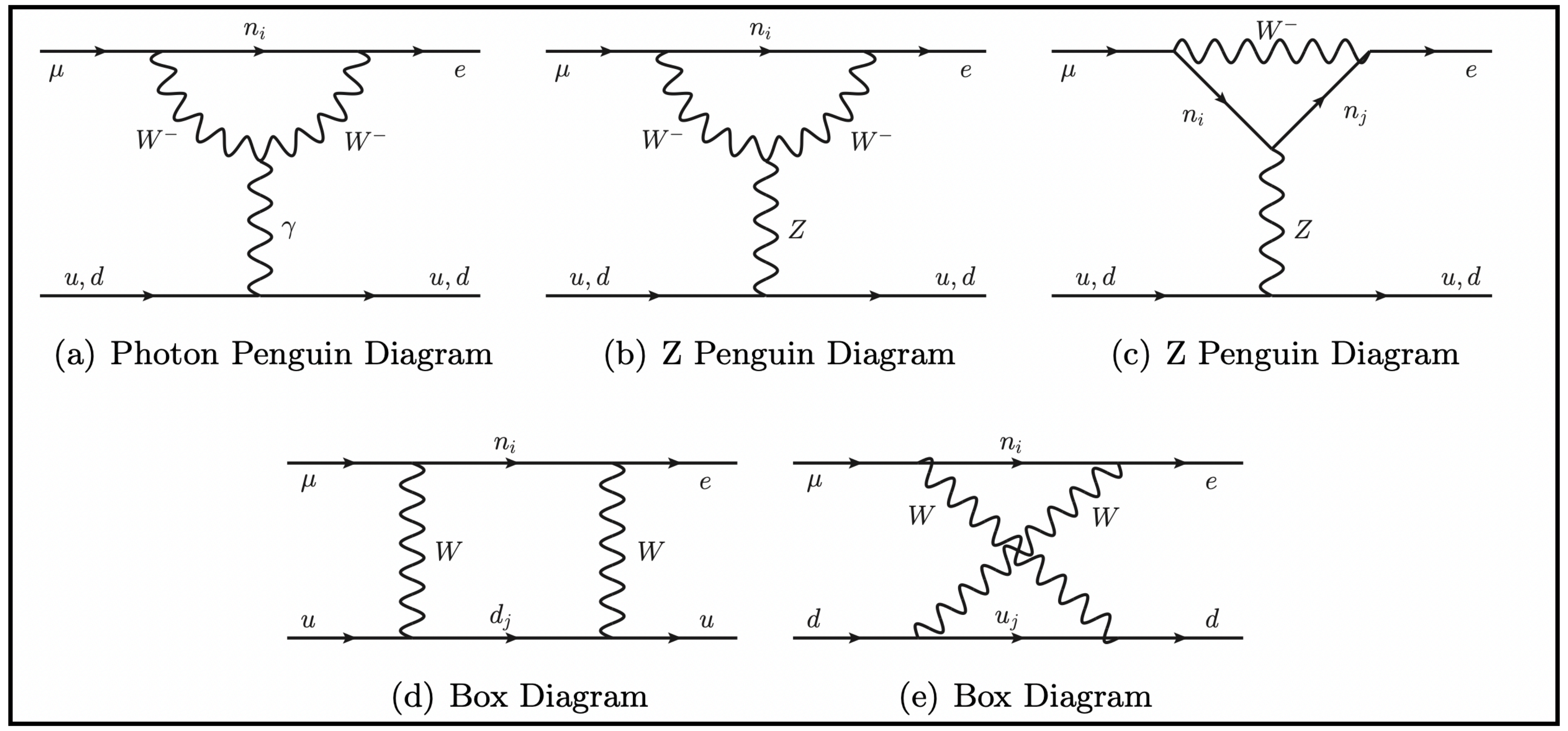


Conversion Rate has some interesting properties:

$$CR(\mu - e, Al) = \frac{2G_F^2\alpha_w^2 m_\mu^5}{(4\pi)^2\Gamma_{capt}}\left| 4V^{(p)}\left(2\tilde{F}_u^{\mu e} + \tilde{F}_d^{\mu e}\right) + 4V^{(n)}\left(\tilde{F}_u^{\mu e} + 2\tilde{F}_d^{\mu e}\right) + s_w^2 \frac{G_\gamma^{\mu e} D}{2e}\right|^2$$

Cancellation of the rate:

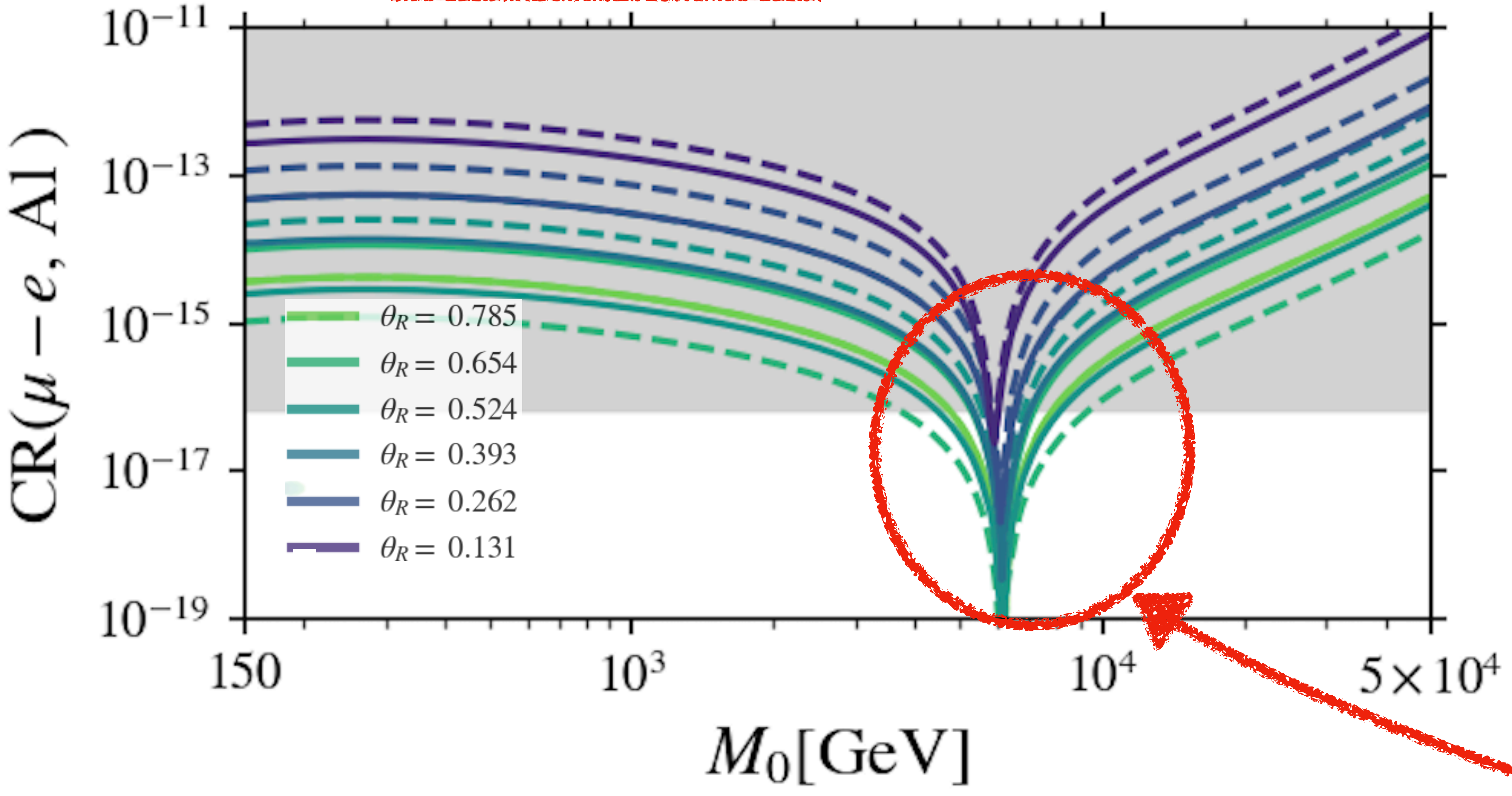
Doesn't depend on θ_R of μ_0 !



(R. Alonso, M. Dhen, M. B. Gavela, T. Hambye ('12))

Option 2: Case 2

$\mu_0 = 1keV$



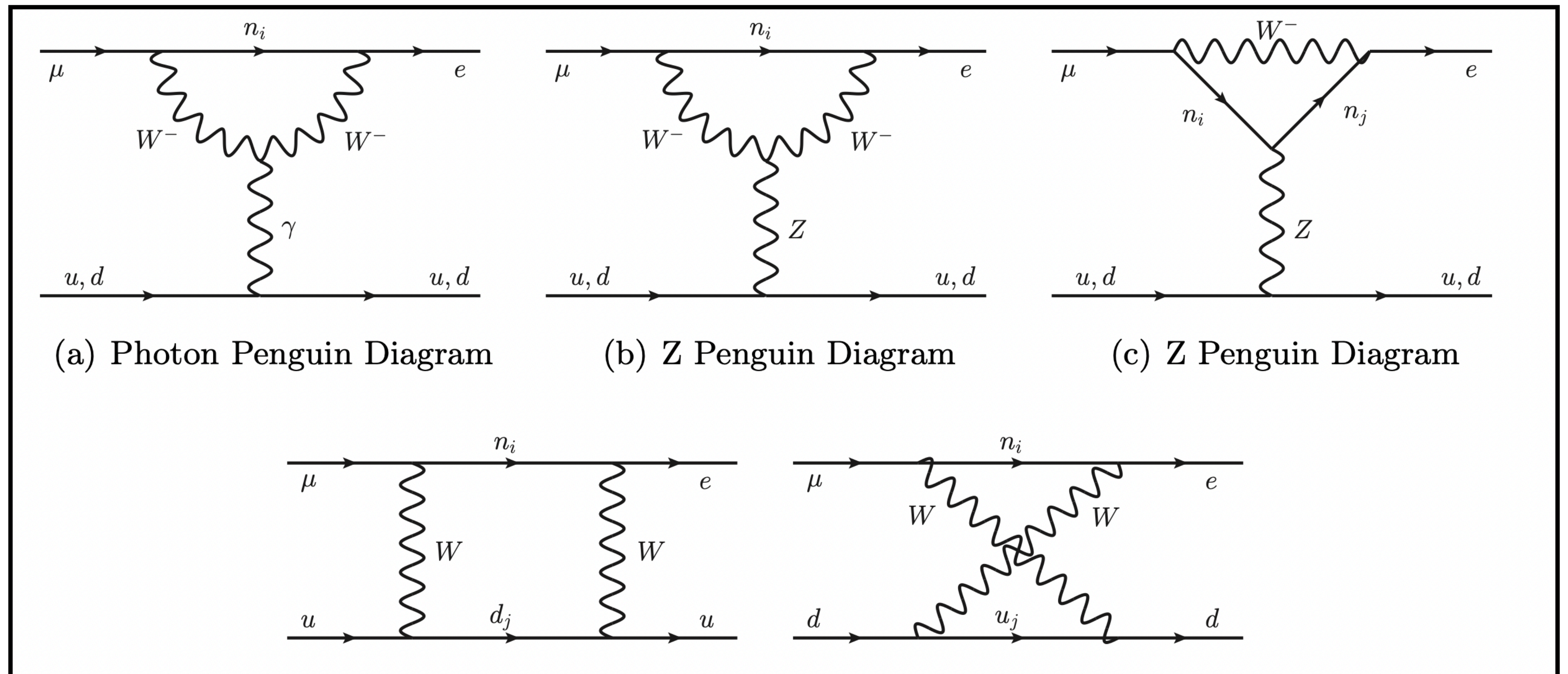
Conversion Rate has some interesting properties:

$$CR(\mu - e, Al) = \frac{2G_F^2\alpha_w^2 m_\mu^5}{(4\pi)^2\Gamma_{capt}}\left|4V^{(p)}\left(2\tilde{F}_u^{\mu e}+\tilde{F}_d^{\mu e}\right)+4V^{(n)}\left(\tilde{F}_u^{\mu e}+2\tilde{F}_d^{\mu e}\right)+s_w^2\frac{G_\gamma^{\mu e}D}{2e}\right|^2$$

Up and down quark contributions have different sign due to different charge and weak isospin

Cancellation of the rate:

Doesn't depend on θ_R of μ_0 !



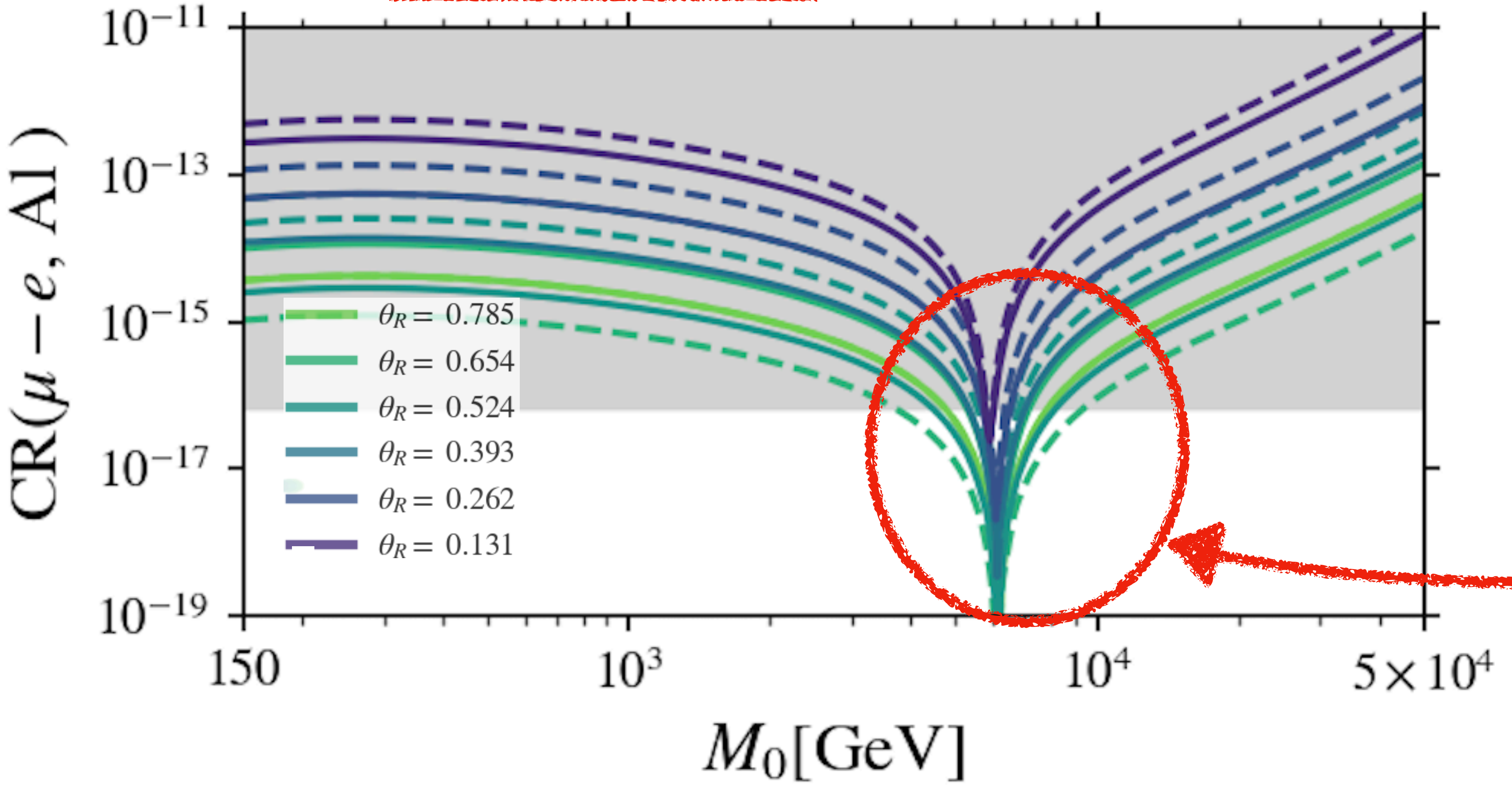
(a) Photon Penguin Diagram (b) Z Penguin Diagram (c) Z Penguin Diagram

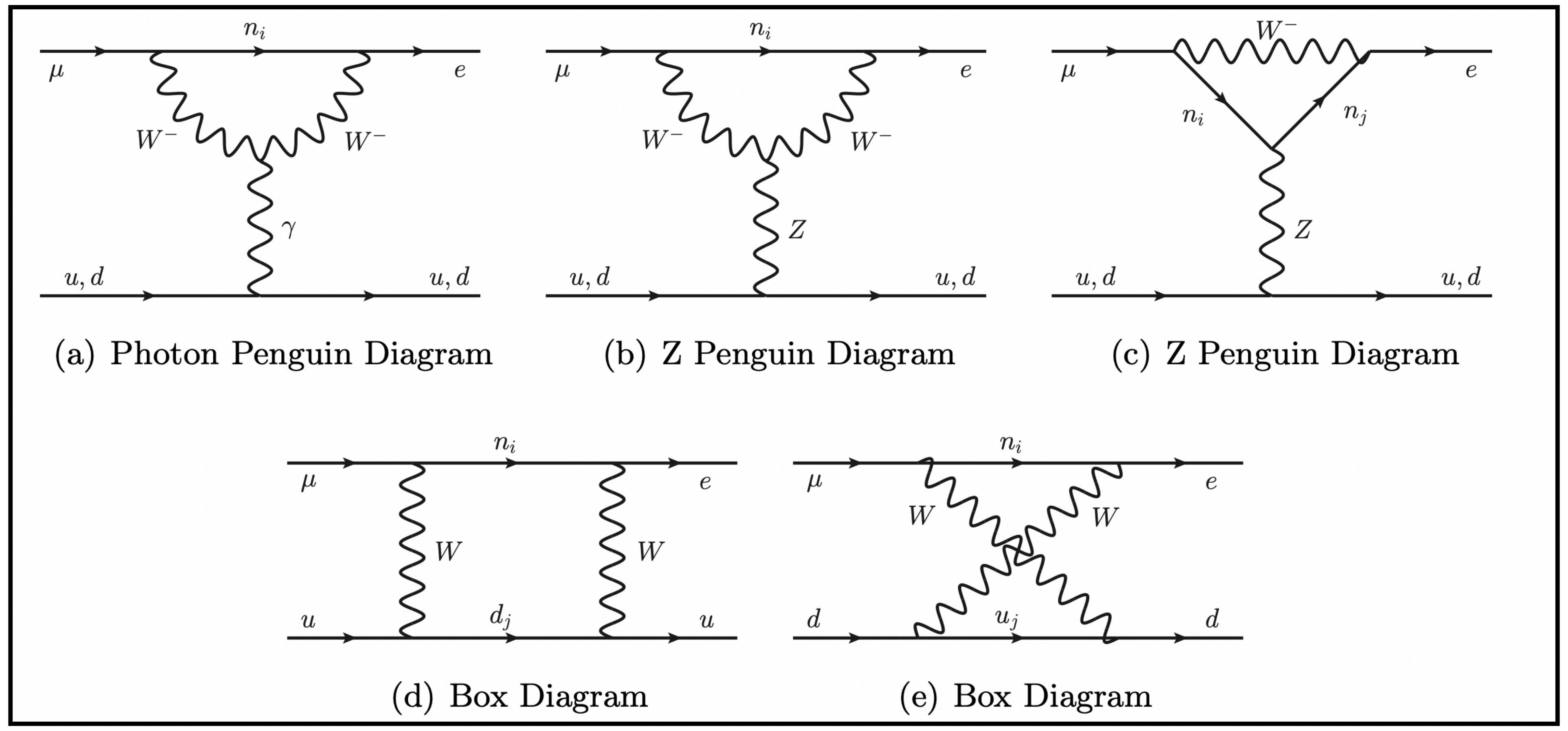
(d) Box Diagram (e) Box Diagram

(R. Alonso, M. Dhen, M. B. Gavela, T. Hambye ('12))

Option 2: Case 2

$\mu_0 = 1keV$





(R. Alonso, M. Dhen, M. B. Gavela, T. Hambye ('12))

Conversion Rate has some interesting properties:

$$CR(\mu - e, Al) = \frac{2G_F^2\alpha_w^2 m_\mu^5}{(4\pi)^2\Gamma_{capt}}\left|4V^{(p)}\left(2\tilde{F}_u^{\mu e} + \tilde{F}_d^{\mu e}\right) + 4V^{(n)}\left(\tilde{F}_u^{\mu e} + 2\tilde{F}_d^{\mu e}\right) + s_w^2\frac{G_\gamma^{\mu e}D}{2e}\right|^2$$

$$M_0^2 = exp\left(\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8} + \frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3} - \frac{3}{8}\right)V^{(p)}}\right)M_W^2$$

Due to factorisation of $\eta_{e\mu}$, cancellation of the rate is also <u>independent of the particular case</u>!!

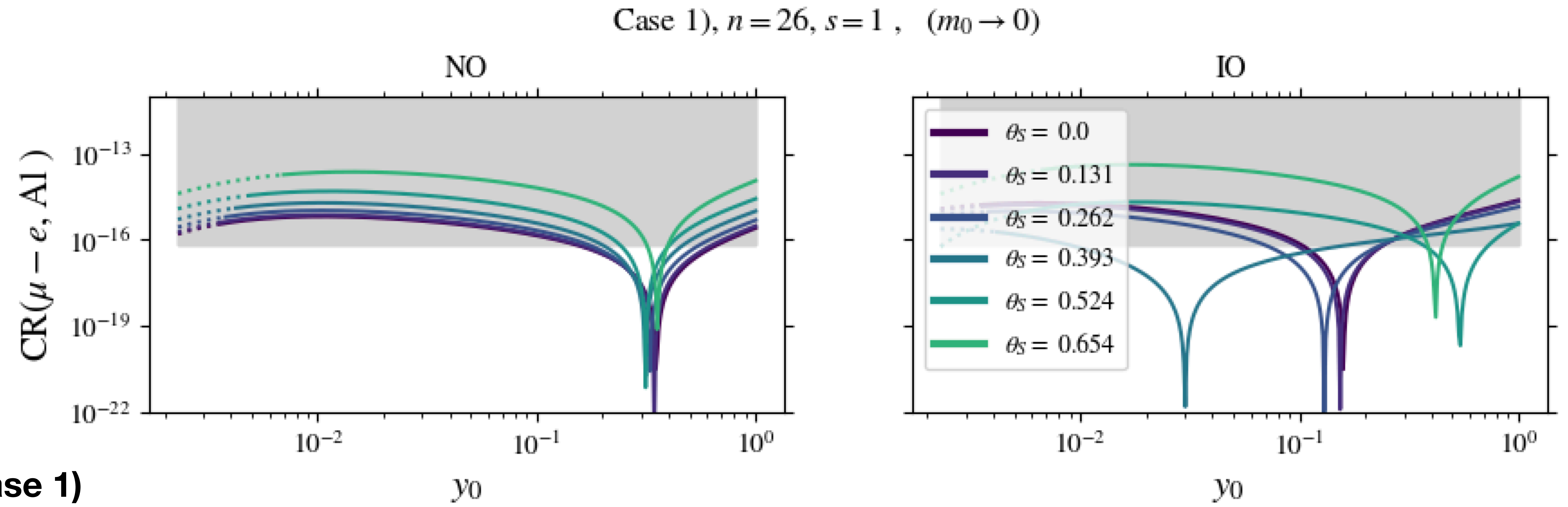
$$x_0^{(canc)} \approx 6470 \quad \Rightarrow \quad M_0^{(canc)} \approx 6.5TeV$$

Option 3: Decoupling Limit Predictions

In the decoupling limit, we wither have $m_1 \to 0$, $m_3 \to 0$ for NO and IO.

$$\bar{x}^{(NO)} = \exp\left(\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8} + \frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3} - \frac{3}{8}\right)V^{(p)}} - \frac{r \log \frac{m_2}{m_3}}{r + \frac{m_2}{m_1^2}} \right)$$

$$\bar{x}^{(IO)} = \exp\left(\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8} + \frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3} - \frac{3}{8}\right)V^{(p)}} + \frac{\frac{m_2}{m_1}\log\frac{m_2}{m_1}}{1 + r - \frac{m_2}{m_1}} \right)$$



$(\mu_0 = 3\ keV)$

Case 1)

- **Stronger dependence on the different values of θ_S than for non-decoupling limit**
- **Single solution for θ_N**

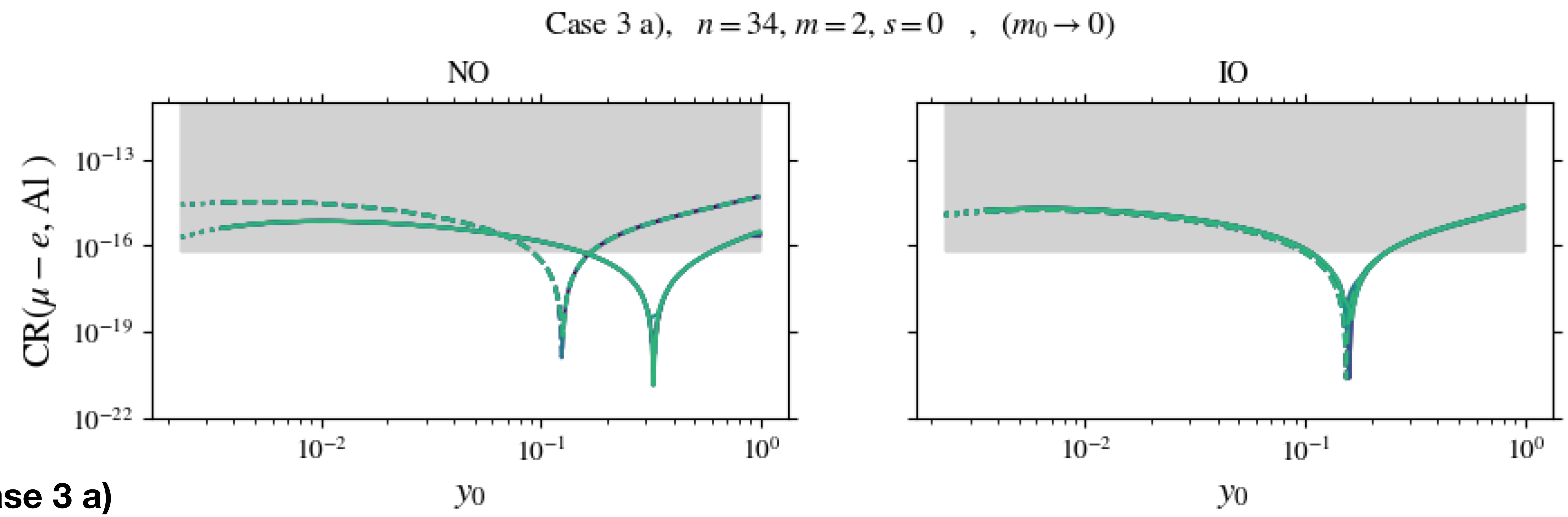
(FPDM, C. Hagedorn, ('25, To appear))

Option 3: Decoupling Limit Predictions

In the decoupling limit, we wither have $m_1 \to 0$, $m_3 \to 0$ for NO and IO.

$$\bar{x}^{(NO)} = \exp\left(\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8} + \frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3} - \frac{3}{8}\right)V^{(p)}} - \frac{r \log \frac{m_2}{m_3}}{r + \frac{m_2}{m_1^2}} \right)$$

$$\bar{x}^{(IO)} = \exp\left(\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8} + \frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3} - \frac{3}{8}\right)V^{(p)}} + \frac{\frac{m_2}{m_1} \log \frac{m_2}{m_1}}{1 + r - \frac{m_2}{m_1}} \right)$$



$(\mu_0 = 3\ keV)$

Case 3 a)

- **Double solution for θ_N results in different values of r**
- **Effect is strongly visible in the decoupling limit, and only for NO.**

(FPDM, C. Hagedorn, ('25, To appear))

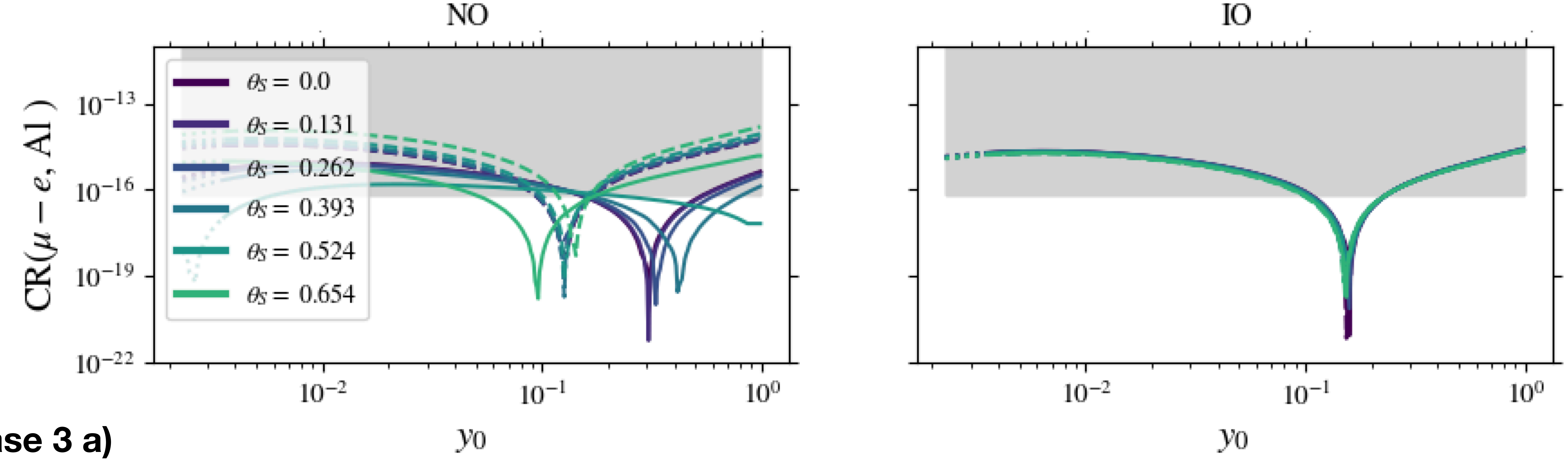
Option 3: Decoupling Limit Predictions

In the decoupling limit, we wither have $m_1 \to 0$, $m_3 \to 0$ for NO and IO.

$$\bar{x}^{(NO)} = \exp\left(\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8} + \frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3} - \frac{3}{8}\right)V^{(p)}} - \frac{r \log \frac{m_2}{m_3}}{r + \frac{m_2}{m_1^2}} \right)$$

$$\bar{x}^{(IO)} = \exp\left(\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8} + \frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3} - \frac{3}{8}\right)V^{(p)}} + \frac{\frac{m_2}{m_1}\log\frac{m_2}{m_1}}{1 + r - \frac{m_2}{m_1}} \right)$$

Case 3 a), $n=16, m=1, s=0$, $(m_0 \to 0)$



$(\mu_0 = 3\ keV)$

Case 3 a)

- **Double solution for θ_N results in different values of r, and dependence on θ_S**
- **Effect is strongly visible in the decoupling limit, and only for NO.**

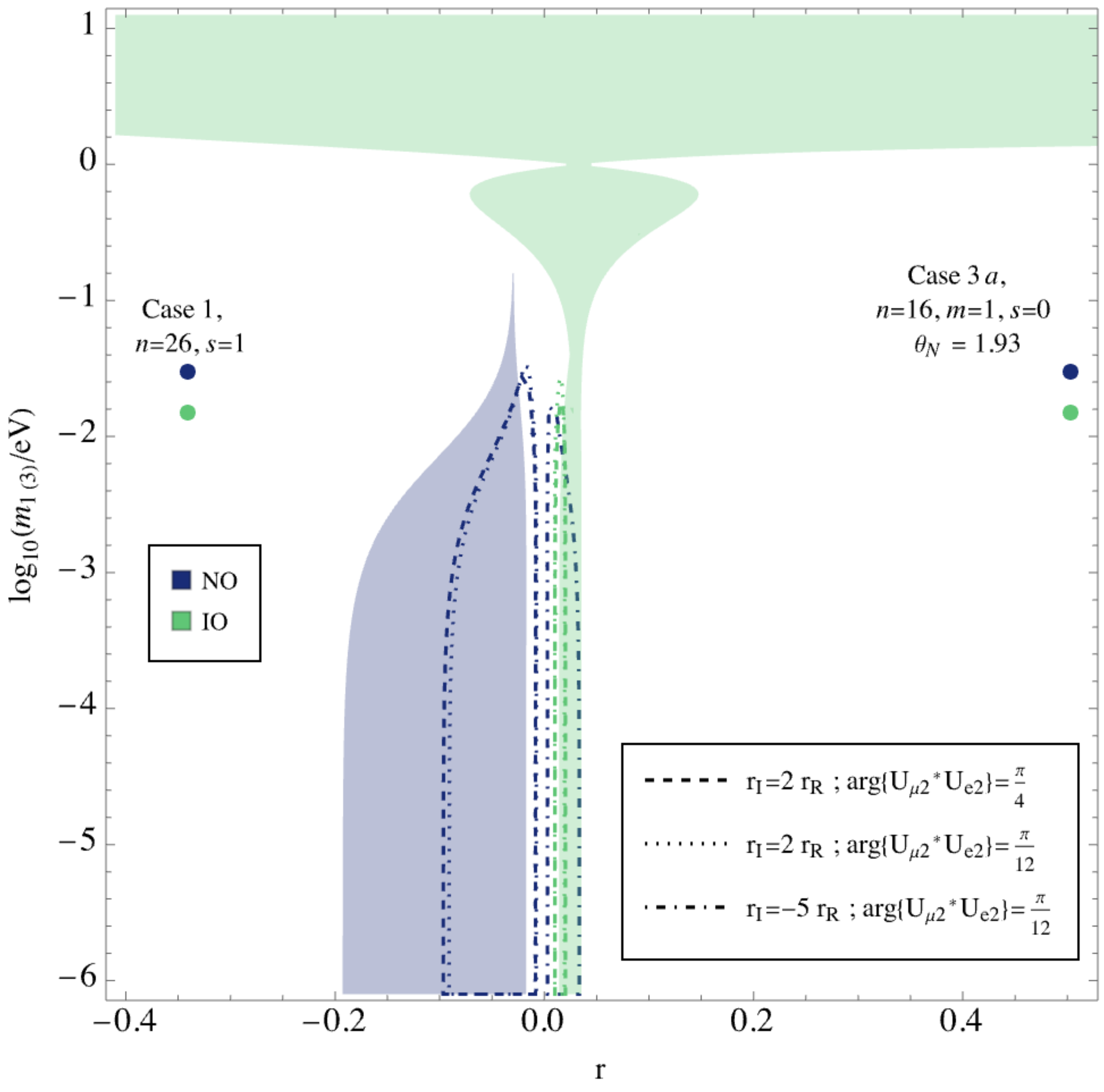
(FPDM, C. Hagedorn, ('25, To appear))

$$\frac{\textbf{Re}\left\{U^*_{\mu 3}U_{e3}\right\}}{\textbf{Re}\left\{U^*_{\mu 2}U_{e2}\right\}} = \frac{\textbf{Im}\left\{U^*_{\mu 3}U_{e3}\right\}}{\textbf{Im}\left\{U^*_{\mu 2}U_{e2}\right\}} \quad , \quad U = U_N(\theta_N)$$

It can be proven that, when a cancellation is not possible, this is replaced by a **local minimum**

(FPDM, C. Hagedorn, ('25, To appear))

Regions in which $\bar{X}$ exists



$$\frac{\mathrm{Re}\left\{U^*_{\mu 3}U_{e3}\right\}}{\mathrm{Re}\left\{U^*_{\mu 2}U_{e2}\right\}} = \frac{\mathrm{Im}\left\{U^*_{\mu 3}U_{e3}\right\}}{\mathrm{Im}\left\{U^*_{\mu 2}U_{e2}\right\}} \quad , \quad U = U_N(\theta_N)$$

It can be proven that, when a cancellation is not possible, this is replaced by a **local minimum**

For $BR(\mu \to e\gamma)$, the cancellation happens for :

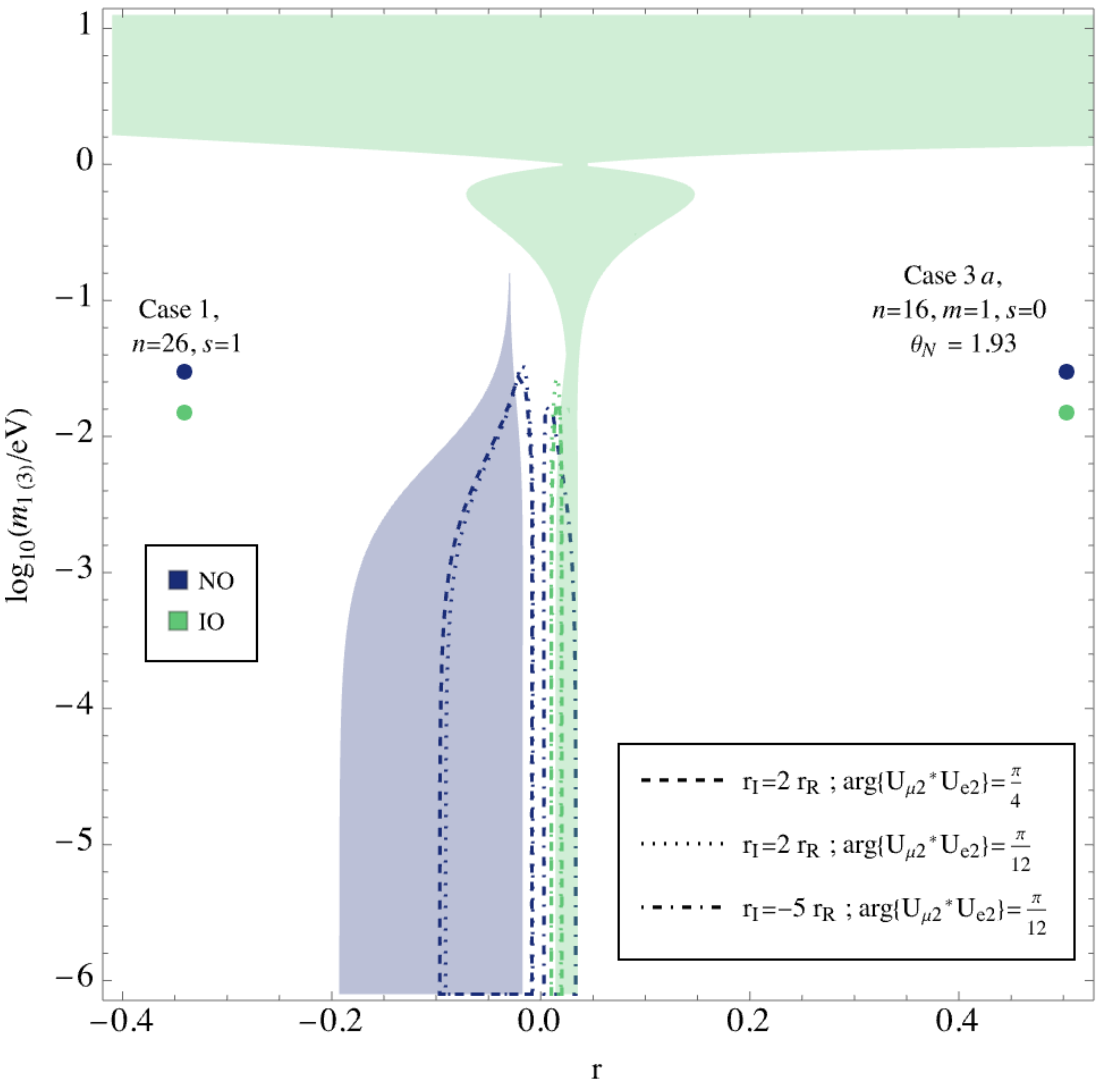
$$\bar{X} = -3\frac{-1+w_2^2+r(-1+w_3^2)}{-1+w_2+r(-1+w_3)} \cdot$$

$$\cdot\, W\left(-e^{\frac{11}{6}}\frac{-1+w_2+r(-1+w_3)}{3(-1+w_2^2+r(-1+w_3^2))} w_2^{-\frac{w_2^2}{-1+w_2^2+r(-1+w_3^2)}} w_3^{-\frac{r w_3^2}{-1+w_2^2+r(-1+w_3^2)}}\right)$$

$$w_2 = \frac{m_2}{m_1} \qquad w_3 = \frac{m_3}{m_1}$$

(FPDM, C. Hagedorn, ('25, To appear))

Regions in which $\bar{X}$ exists



$$\frac{\mathbf{Re}\left\{U^*_{\mu 3}U_{e3}\right\}}{\mathbf{Re}\left\{U^*_{\mu 2}U_{e2}\right\}} = \frac{\mathbf{Im}\left\{U^*_{\mu 3}U_{e3}\right\}}{\mathbf{Im}\left\{U^*_{\mu 2}U_{e2}\right\}} \quad , \quad U = U_N(\theta_N)$$

It can be proven that, when a cancellation is not possible, this is replaced by a **local minimum**

For $BR(\mu \to e\gamma)$, the cancellation happens for :

$$\bar{X} = -3\frac{-1 + w_2^2 + r(-1 + w_3^2)}{-1 + w_2 + r(-1 + w_3)} \cdot$$

$$\cdot\, W\left(-e^{\frac{11}{6}}\frac{-1 + w_2 + r(-1 + w_3)}{3(-1 + w_2^2 + r(-1 + w_3^2))} w_2^{-\frac{w_2^2}{-1 + w_2^2 + r(-1 + w_3^2)}} w_3^{-\frac{r w_3^2}{-1 + w_2^2 + r(-1 + w_3^2)}}\right)$$

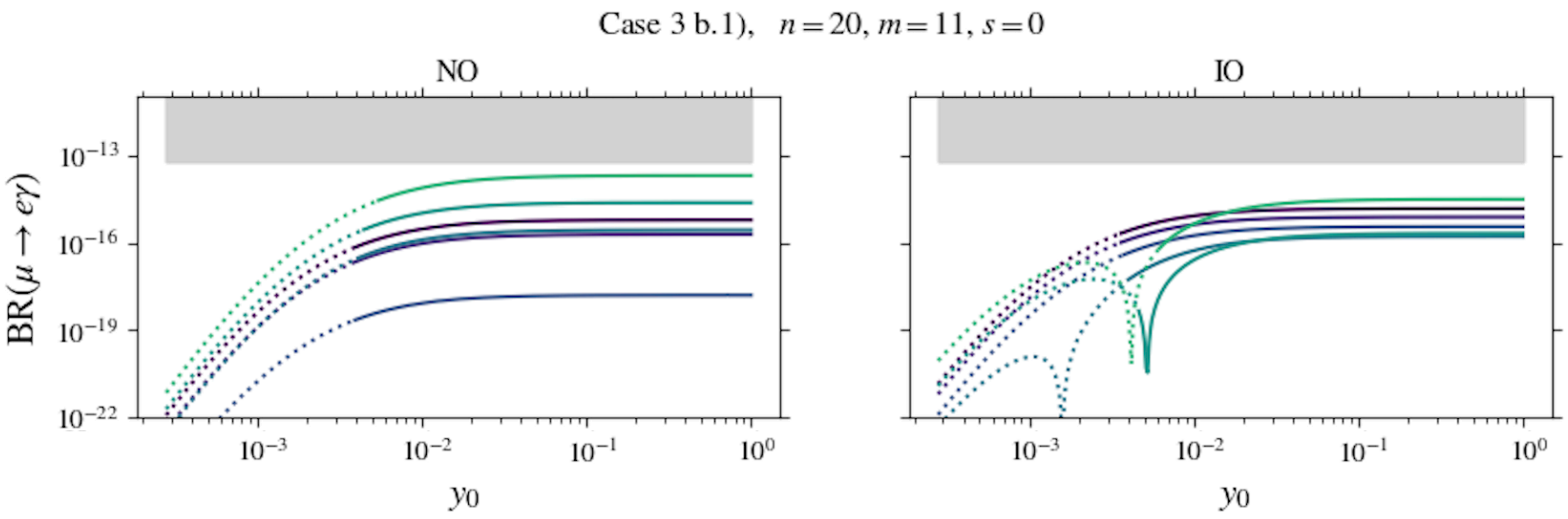
$$w_2 = \frac{m_2}{m_1} \qquad w_3 = \frac{m_3}{m_1}$$

Expression of the Location of the local minima is similar but way more involved…

(FPDM, C. Hagedorn, ('25, To appear))

Regions in which $\bar{X}$ exists

$$\frac{\textbf{Re}\left\{U^*_{\mu 3}U_{e3}\right\}}{\textbf{Re}\left\{U^*_{\mu 2}U_{e2}\right\}} = \frac{\textbf{Im}\left\{U^*_{\mu 3}U_{e3}\right\}}{\textbf{Im}\left\{U^*_{\mu 2}U_{e2}\right\}} \quad , \quad U = U_N(\theta_N)$$



$(\mu_0 = 3\ keV)$

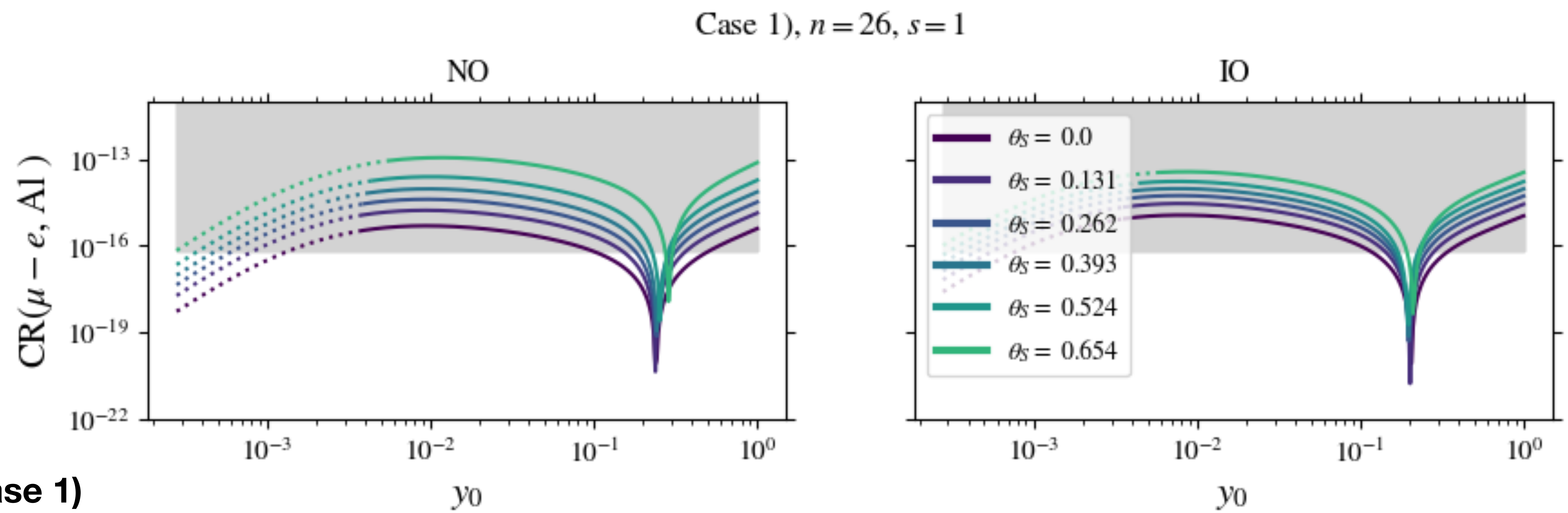
Cancellations of $BR(\mu \to e\gamma)$ can generally happen for values of the lightest sterile state mass close to $150\ GeV$

(FPDM, C. Hagedorn, ('25, To appear))

The Conversion rate also shows a cancellation for:

Differently from Option 2, cancellation depends upon the particular CASE (expression of r and spectrum)

$$\bar{x} = \exp\left(\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8} + \frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3} - \frac{3}{8}\right)V^{(p)}} + \frac{\frac{m_2}{m_1}\log\frac{m_2}{m_1} + r\frac{m_3}{m_1}\log\frac{m_3}{m_1}}{\frac{m_2}{m_1} - 1 + r\left(\frac{m_3}{m_1} - 1\right)} \right)$$



$(\mu_0 = 3\ keV)$

Case 1)

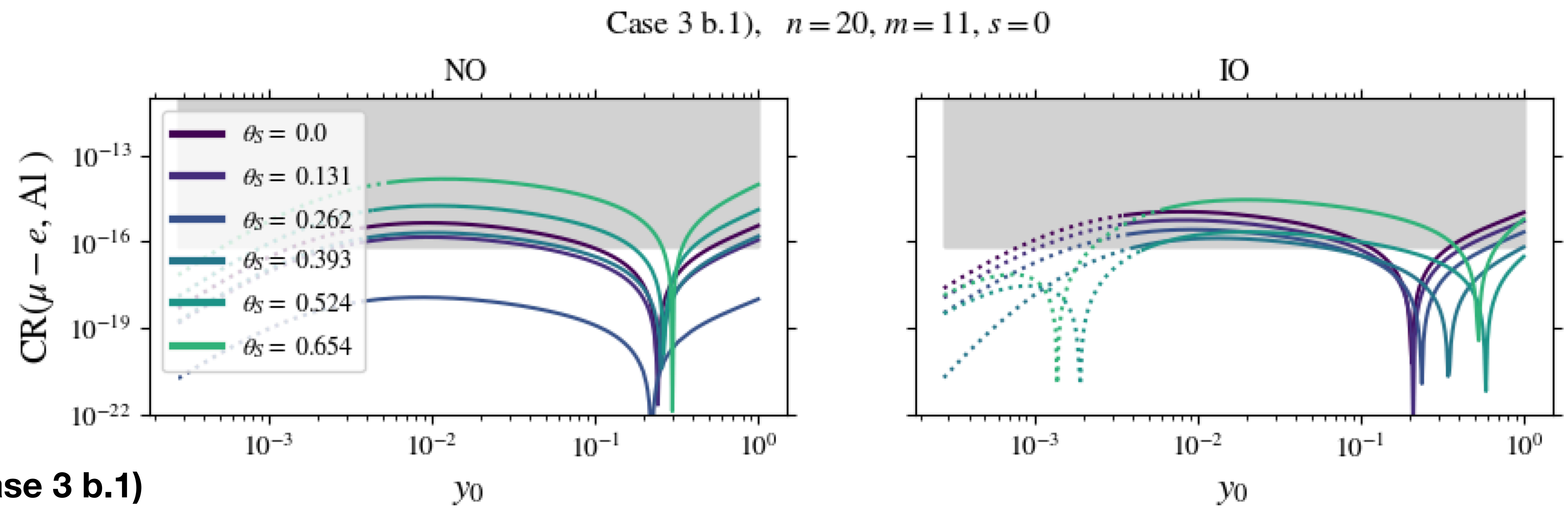
- **Weak dependence on the different values of θ_S**
- **Single solution for θ_N**

(FPDM, C. Hagedorn, ('25, To appear))

Differently from Option 2, cancellation depends upon the particular CASE (expression of r and spectrum)

The Conversion rate also shows a cancellation for:

$$\bar{x} = \exp\left(\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8} + \frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3} - \frac{3}{8}\right)V^{(p)}} + \frac{\frac{m_2}{m_1}\log\frac{m_2}{m_1} + r\frac{m_3}{m_1}\log\frac{m_3}{m_1}}{\frac{m_2}{m_1} - 1 + r\left(\frac{m_3}{m_1} - 1\right)} \right)$$



$(\mu_0 = 3\ keV)$

Case 3 b.1)

- **Stronger dependence on the different values of θ_S**
- **Single solution for θ_N**

(FPDM, C. Hagedorn, ('25, To appear))

When no cancellation happens, we find a local minima:

$$r_R = \frac{\textbf{Re}\left\{U^*_{\mu 3}U_{e3}\right\}}{\textbf{Re}\left\{U^*_{\mu 2}U_{e2}\right\}} \quad ; \quad r_I = \frac{\textbf{Im}\left\{U^*_{\mu 3}U_{e3}\right\}}{\textbf{Im}\left\{U^*_{\mu 2}U_{e2}\right\}} \quad \textbf{with} \quad r_R \neq r_I$$

$$y_0^{(min)} = \sqrt{\frac{2m_{(1,3)}}{\mu_0 v^2}M_W^2}\,\textbf{exp}\left(\frac{1}{2}\frac{-\mathscr{K}_R - \mathscr{K}_I\phi^2}{-\mathscr{K}_R^2 - \mathscr{K}_I^2\phi^2}\right)$$

$$\mathscr{K}_{R,I} = -\left[\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8} + \frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3} - \frac{3}{8}\right)V^{(p)}} + \frac{\frac{m_2}{m_1}\log\frac{m_2}{m_1} + r\frac{m_3}{m_1}\log\frac{m_3}{m_1}}{\frac{m_2}{m_1} - 1 + r\left(\frac{m_3}{m_1} - 1\right)}\right]^{-1}$$

With:

$$\phi = \left|\textbf{arg}\left\{U^*_{\mu 2}U_{e2}\right\}\frac{-54e(V^{(n)} + V^{(p)}) + s_w^2(3D - 148eV^{(p)})\left(-1 + \frac{m_2}{m_1} + r_R\left(-1 + \frac{m_3}{m_1}\right)\right) - 2e(9V^{(n)} + (-9 + 32s_w^2)V^{(p)})\left(\frac{m_2}{m_1}\log\frac{m_2}{m_1} + r_I\frac{m_3}{m_1}\log\frac{m_3}{m_1}\right)}{-54e(V^{(n)} + V^{(p)}) + s_w^2(3D - 148eV^{(p)})\left(-1 + \frac{m_2}{m_1} + r_I\left(-1 + \frac{m_3}{m_1}\right)\right) - 2e(9V^{(n)} + (-9 + 32s_w^2)V^{(p)})\left(\frac{m_2}{m_1}\log\frac{m_2}{m_1} + r_R\frac{m_3}{m_1}\log\frac{m_3}{m_1}\right)}\right|^2$$

(FPDM, C. Hagedorn, ('25, To appear))

When no cancellation happens, we find a local minima:

$$r_R = \frac{\mathbf{Re}\left\{U^*_{\mu 3}U_{e3}\right\}}{\mathbf{Re}\left\{U^*_{\mu 2}U_{e2}\right\}} \quad ; \quad r_I = \frac{\mathbf{Im}\left\{U^*_{\mu 3}U_{e3}\right\}}{\mathbf{Im}\left\{U^*_{\mu 2}U_{e2}\right\}} \quad \textbf{with} \quad r_R \neq r_I$$

$$y_0^{(min)} = \sqrt{\frac{2m_{(1,3)}}{\mu_0 v^2}M_W^2}\,\mathbf{exp}\left(\frac{1}{2}\frac{-\mathcal{K}_R - \mathcal{K}_I\phi^2}{-\mathcal{K}_R^2 - \mathcal{K}_I^2\phi^2}\right)$$

$$\mathcal{K}_{R,I} = -\left[\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8}+\frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3}-\frac{3}{8}\right)V^{(p)}} + \frac{\frac{m_2}{m_1}\log\frac{m_2}{m_1} + r\frac{m_3}{m_1}\log\frac{m_3}{m_1}}{\frac{m_2}{m_1} - 1 + r\left(\frac{m_3}{m_1}-1\right)}\right]^{-1}$$

With:

$$\phi = \left|\mathbf{arg}\left\{U^*_{\mu 2}U_{e2}\right\}\frac{-54e(V^{(n)}+V^{(p)}) + s_w^2(3D - 148eV^{(p)})\left(-1+\frac{m_2}{m_1}+r_R\left(-1+\frac{m_3}{m_1}\right)\right) - 2e(9V^{(n)} + (-9+32s_w^2)V^{(p)})\left(\frac{m_2}{m_1}\log\frac{m_2}{m_1} + r_I\frac{m_3}{m_1}\log\frac{m_3}{m_1}\right)}{-54e(V^{(n)}+V^{(p)}) + s_w^2(3D - 148eV^{(p)})\left(-1+\frac{m_2}{m_1}+r_I\left(-1+\frac{m_3}{m_1}\right)\right) - 2e(9V^{(n)} + (-9+32s_w^2)V^{(p)})\left(\frac{m_2}{m_1}\log\frac{m_2}{m_1} + r_R\frac{m_3}{m_1}\log\frac{m_3}{m_1}\right)}\right|^2$$

Way too complicated... What does this mean???

(FPDM, C. Hagedorn, ('25, To appear))

When no cancellation happens, we find a local minima: $r_R = \frac{\text{Re}\left\{U^*_{\mu 3}U_{e3}\right\}}{\text{Re}\left\{U^*_{\mu 2}U_{e2}\right\}}$; $r_I = \frac{\text{Im}\left\{U^*_{\mu 3}U_{e3}\right\}}{\text{Im}\left\{U^*_{\mu 2}U_{e2}\right\}}$ **with** $r_R \neq r_I$

$$y_0^{(min)} = \sqrt{\frac{2m_{(1,3)}}{\mu_0 v^2}M_W^2}\,\textbf{exp}\left(\frac{1}{2}\frac{-\mathcal{K}_R - \mathcal{K}_I\phi^2}{-\mathcal{K}_R^2 - \mathcal{K}_I^2\phi^2}\right)$$

$$\mathcal{K}_{R,I} = -\left[\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8} + \frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3} - \frac{3}{8}\right)V^{(p)}} + \frac{\frac{m_2}{m_1}\log\frac{m_2}{m_1} + r\frac{m_3}{m_1}\log\frac{m_3}{m_1}}{\frac{m_2}{m_1} - 1 + r\left(\frac{m_3}{m_1} - 1\right)}\right]^{-1}$$

General structure of the rate is $\mathscr{R} \propto$ **Re**$\{\bar{\mathscr{L}}\}^2$+**Im**$\{\bar{\mathscr{L}}\}^2$

(FPDM, C. Hagedorn, ('25, To appear))

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$$y_0^{(min)} = \sqrt{\frac{2m_{(1,3)}}{\mu_0 v^2} M_W^2} \textbf{exp}\left(\frac{1}{2} \frac{-\mathscr{K}_R - \mathscr{K}_I \phi^2}{-\mathscr{K}_R^2 - \mathscr{K}_I^2 \phi^2}\right)$$

$$\mathscr{K}_{R,I} = -\left[\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8} + \frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3} - \frac{3}{8}\right)V^{(p)}} + \frac{\frac{m_2}{m_1}\log\frac{m_2}{m_1} + r\frac{m_3}{m_1}\log\frac{m_3}{m_1}}{\frac{m_2}{m_1} - 1 + r\left(\frac{m_3}{m_1} - 1\right)}\right]^{-1}$$

General structure of the rate is $\mathscr{R} \propto$ **Re**$\{\bar{\mathscr{L}}\}^2$+**Im**$\{\bar{\mathscr{L}}\}^2$

Re$\{\bar{\mathscr{L}}\}^2$ **and Im**$\{\bar{\mathscr{L}}\}^2$ **cancel independently at** $y_0^{(R,I)}$ **respectively**

(FPDM, C. Hagedorn, ('25, To appear))

When no cancellation happens, we find a local minima:

$$r_R = \frac{\mathrm{Re}\left\{U^*_{\mu 3}U_{e3}\right\}}{\mathrm{Re}\left\{U^*_{\mu 2}U_{e2}\right\}} \quad ; \quad r_I = \frac{\mathrm{Im}\left\{U^*_{\mu 3}U_{e3}\right\}}{\mathrm{Im}\left\{U^*_{\mu 2}U_{e2}\right\}} \quad \text{with} \quad r_R \neq r_I$$

$$y_0^{(min)} = \sqrt{\frac{2m_{(1,3)}}{\mu_0 v^2}M_W^2}\,\mathbf{exp}\left(\frac{1}{2}\frac{-\mathcal{K}_R - \mathcal{K}_I\phi^2}{-\mathcal{K}_R^2 - \mathcal{K}_I^2\phi^2}\right)$$

$$\mathcal{K}_{R,I} = -\left[\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8}+\frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3}-\frac{3}{8}\right)V^{(p)}} + \frac{\frac{m_2}{m_1}\log\frac{m_2}{m_1} + r\frac{m_3}{m_1}\log\frac{m_3}{m_1}}{\frac{m_2}{m_1} - 1 + r\left(\frac{m_3}{m_1}-1\right)}\right]^{-1}$$

General structure of the rate is $\mathcal{R} \propto$ **Re**$\{\bar{\mathcal{L}}\}^2$+**Im**$\{\bar{\mathcal{L}}\}^2$

Re$\{\bar{\mathcal{L}}\}^2$ **and Im**$\{\bar{\mathcal{L}}\}^2$ **cancel independently at** $y_0^{(R,I)}$ **respectively**

$$y_0^{(R)} = y_0^{(I)} \Leftrightarrow r_I = r_R$$

(FPDM, C. Hagedorn, ('25, To appear))

When no cancellation happens, we find a local minima: $r_R = \frac{\textbf{Re}\left\{U^*_{\mu 3}U_{e3}\right\}}{\textbf{Re}\left\{U^*_{\mu 2}U_{e2}\right\}}$; $r_I = \frac{\textbf{Im}\left\{U^*_{\mu 3}U_{e3}\right\}}{\textbf{Im}\left\{U^*_{\mu 2}U_{e2}\right\}}$ **with** $r_R \neq r_I$

$$y_0^{(min)} = \sqrt{\frac{2m_{(1,3)}}{\mu_0 v^2} M_W^2}\,\textbf{exp}\left(\frac{1}{2}\frac{-\mathcal{K}_R - \mathcal{K}_I\phi^2}{-\mathcal{K}_R^2 - \mathcal{K}_I^2\phi^2}\right)$$

$$\mathcal{K}_{R,I} = -\left[\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8} + \frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3} - \frac{3}{8}\right)V^{(p)}} + \frac{\frac{m_2}{m_1}\log\frac{m_2}{m_1} + r\frac{m_3}{m_1}\log\frac{m_3}{m_1}}{\frac{m_2}{m_1} - 1 + r\left(\frac{m_3}{m_1} - 1\right)}\right]^{-1}$$

General structure of the rate is $\mathcal{R} \propto \textbf{Re}\{\bar{\mathcal{L}}\}^2 + \textbf{Im}\{\bar{\mathcal{L}}\}^2$

$\textbf{Re}\{\bar{\mathcal{L}}\}^2$ and $\textbf{Im}\{\bar{\mathcal{L}}\}^2$ cancel independently at $y_0^{(R,I)}$ respectively

$$y_0^{(R)} = y_0^{(I)} \Leftrightarrow r_I = r_R$$

The minima is shallower the closer r_R and r_I are

Let's see some examples!!

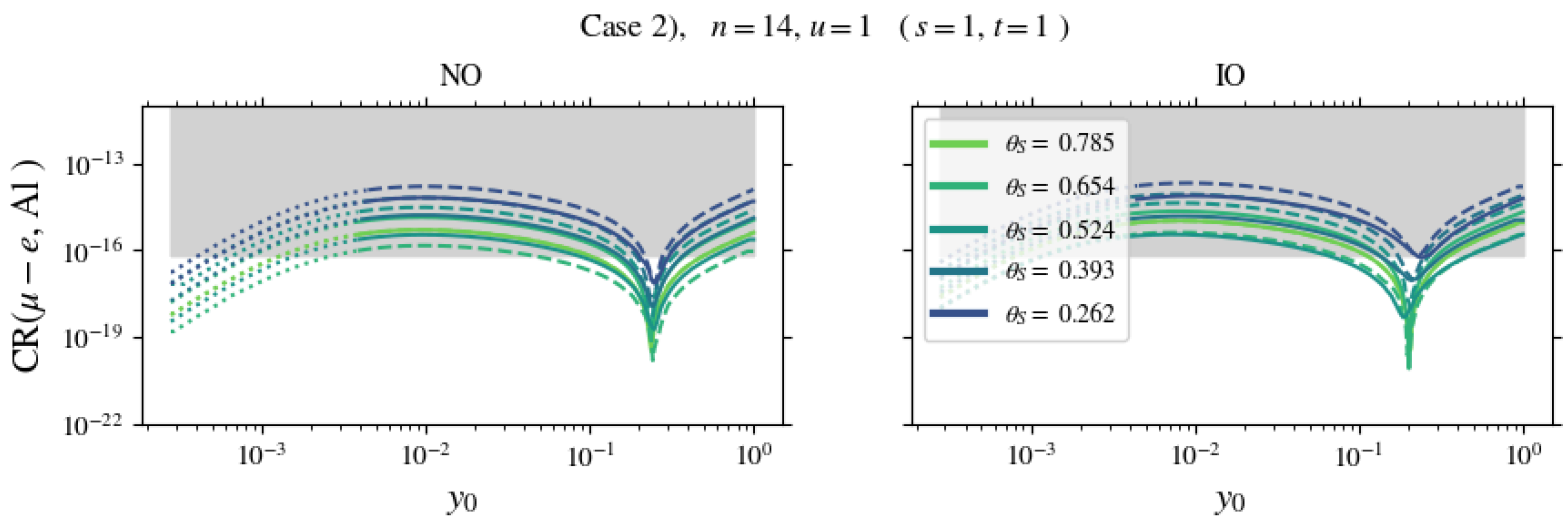
(FPDM, C. Hagedorn, ('25, To appear))

When no cancellation happens, we find a local minima:

$$r_R = \frac{\text{Re}\left\{U^*_{\mu 3}U_{e3}\right\}}{\text{Re}\left\{U^*_{\mu 2}U_{e2}\right\}} \quad ; \quad r_I = \frac{\text{Im}\left\{U^*_{\mu 3}U_{e3}\right\}}{\text{Im}\left\{U^*_{\mu 2}U_{e2}\right\}} \quad \textbf{with} \quad r_R \neq r_I$$

$$y_0^{(min)} = \sqrt{\frac{2m_{(1,3)}}{\mu_0 v^2}M_W^2 \exp\left(\frac{1}{2}\frac{-\mathscr{K}_R - \mathscr{K}_I\phi^2}{-\mathscr{K}_R^2 - \mathscr{K}_I^2\phi^2}\right)}$$

$$\mathscr{K}_{R,I} = -\left[\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8} + \frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3} - \frac{3}{8}\right)V^{(p)}} + \frac{\frac{m_2}{m_1}\log\frac{m_2}{m_1} + r\frac{m_3}{m_1}\log\frac{m_3}{m_1}}{\frac{m_2}{m_1} - 1 + r\left(\frac{m_3}{m_1} - 1\right)}\right]^{-1}$$



Case 2)

- **Dependence on θ_S is rather weak**
- **Minima is really shallow due to $r_R \approx r_I$**
- **Shallower minima for IO**

(FPDM, C. Hagedorn, ('25, To appear))

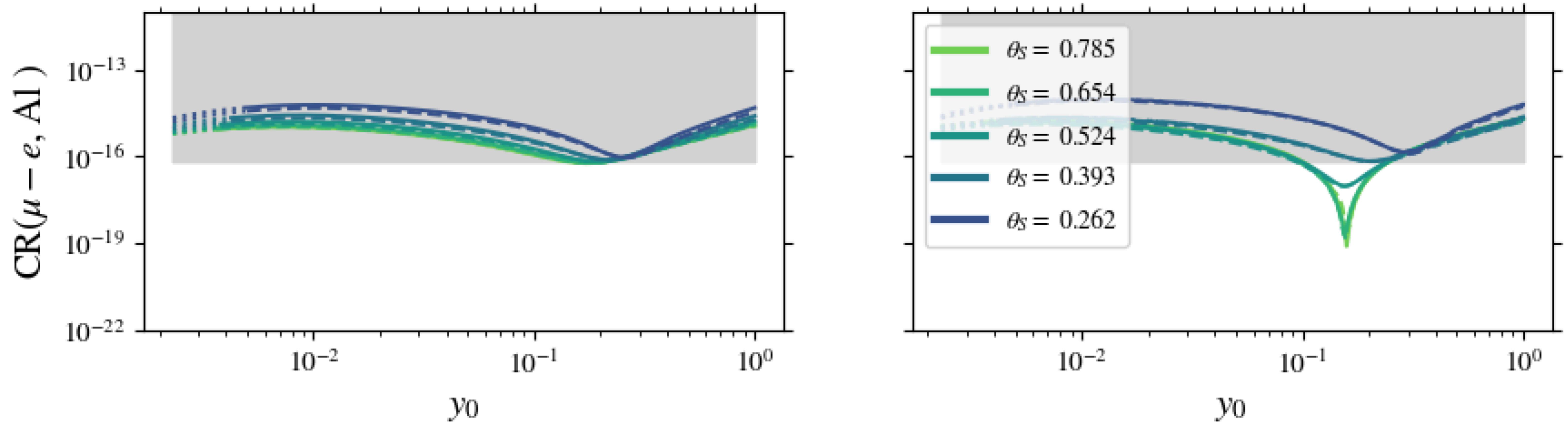
When no cancellation happens, we find a local minima:

$$r_R = \frac{\text{Re}\left\{U^*_{\mu 3} U_{e3}\right\}}{\text{Re}\left\{U^*_{\mu 2} U_{e2}\right\}} \quad ; \quad r_I = \frac{\text{Im}\left\{U^*_{\mu 3} U_{e3}\right\}}{\text{Im}\left\{U^*_{\mu 2} U_{e2}\right\}} \quad \text{with} \quad r_R \neq r_I$$

$$y_0^{(min)} = \sqrt{\frac{2m_{(1,3)}}{\mu_0 v^2} M_W^2 \exp\left(\frac{1}{2}\frac{-\mathcal{K}_R - \mathcal{K}_I \phi^2}{-\mathcal{K}_R^2 - \mathcal{K}_I^2 \phi^2}\right)}$$

$$\mathcal{K}_{R,I} = -\left[\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8} + \frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3} - \frac{3}{8}\right)V^{(p)}} + \frac{\frac{m_2}{m_1}\log\frac{m_2}{m_1} + r\frac{m_3}{m_1}\log\frac{m_3}{m_1}}{\frac{m_2}{m_1} - 1 + r\left(\frac{m_3}{m_1} - 1\right)}\right]^{-1}$$

Case 2), $n=14, u=1 \quad (s=1, t=1) \quad , \quad (m_0 \to 0)$



Case 2)

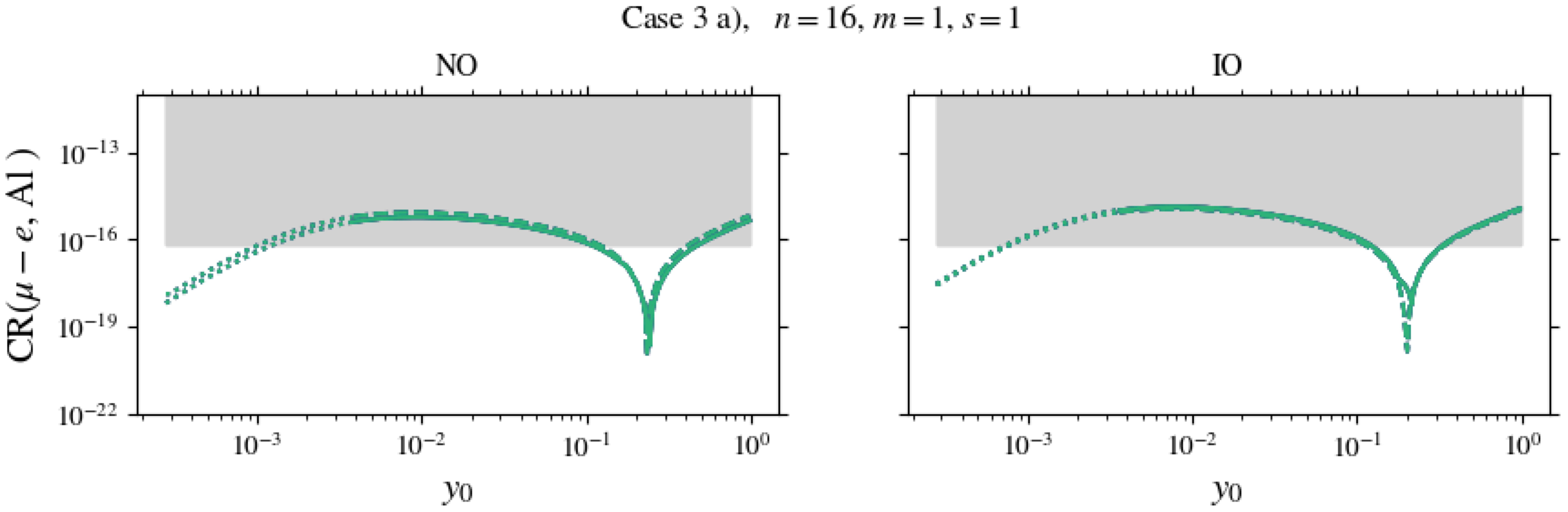
- **In the decoupling limit minima is much shallower**
- **Stronger dependence on θ_S**
- **Shallower minima for NO**

(FPDM, C. Hagedorn, ('25, To appear))

When no cancellation happens, we find a local minima: $r_R = \frac{\textbf{Re}\left\{U^*_{\mu 3}U_{e3}\right\}}{\textbf{Re}\left\{U^*_{\mu 2}U_{e2}\right\}}$; $r_I = \frac{\textbf{Im}\left\{U^*_{\mu 3}U_{e3}\right\}}{\textbf{Im}\left\{U^*_{\mu 2}U_{e2}\right\}}$ **with** $r_R \neq r_I$

$$y_0^{(min)} = \sqrt{\frac{2m_{(1,3)}}{\mu_0 v^2} M_W^2 \textbf{exp}\left(\frac{1}{2}\frac{-\mathcal{K}_R - \mathcal{K}_I\phi^2}{-\mathcal{K}_R^2 - \mathcal{K}_I^2\phi^2}\right)}$$

$$\mathcal{K}_{R,I} = -\left[\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8}+\frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3}-\frac{3}{8}\right)V^{(p)}} + \frac{\frac{m_2}{m_1}\log\frac{m_2}{m_1} + r\frac{m_3}{m_1}\log\frac{m_3}{m_1}}{\frac{m_2}{m_1} - 1 + r\left(\frac{m_3}{m_1}-1\right)}\right]^{-1}$$



Case 3 a)

- **As per Case 2, minima is really shallow due to** $r_R \approx r_I$

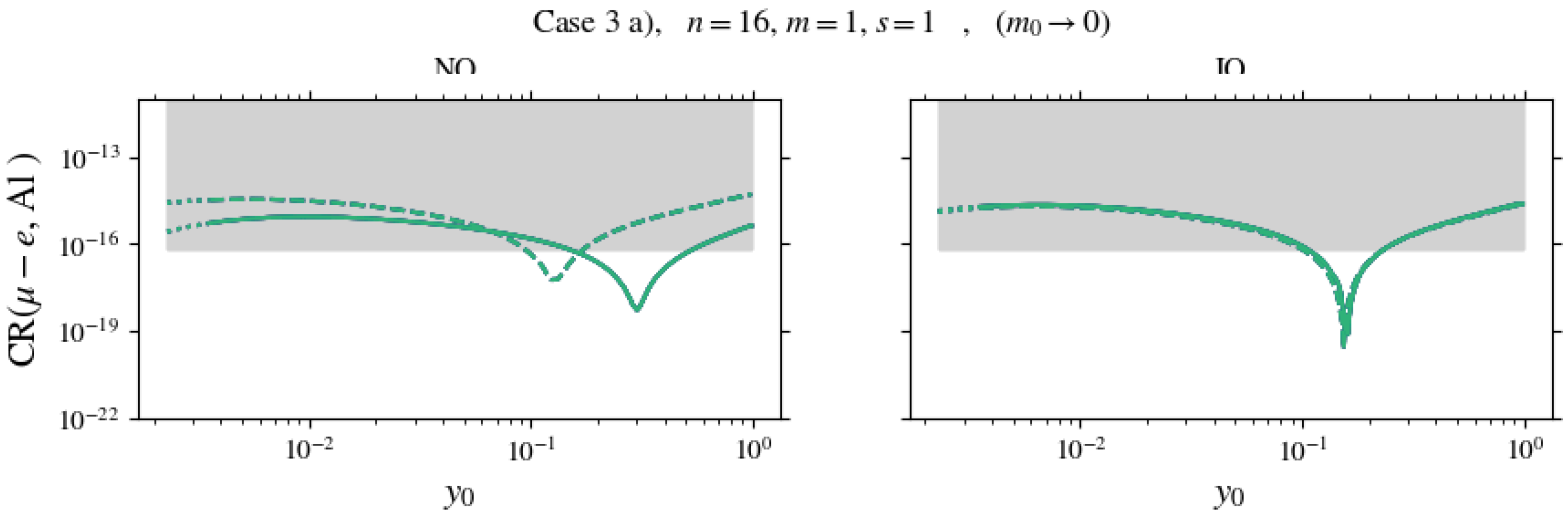
(FPDM, C. Hagedorn, ('25, To appear))

When no cancellation happens, we find a local minima:

$$r_R = \frac{\text{Re}\left\{U^*_{\mu 3}U_{e3}\right\}}{\text{Re}\left\{U^*_{\mu 2}U_{e2}\right\}} \quad ; \quad r_I = \frac{\text{Im}\left\{U^*_{\mu 3}U_{e3}\right\}}{\text{Im}\left\{U^*_{\mu 2}U_{e2}\right\}} \quad \text{with} \quad r_R \neq r_I$$

$$y_0^{(min)} = \sqrt{\frac{2m_{(1,3)}}{\mu_0 v^2}M_W^2 \exp\left(\frac{1}{2}\frac{-\mathcal{K}_R - \mathcal{K}_I\phi^2}{-\mathcal{K}_R^2 - \mathcal{K}_I^2\phi^2}\right)}$$

$$\mathcal{K}_{R,I} = -\left[\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8}+\frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3}-\frac{3}{8}\right)V^{(p)}} + \frac{\frac{m_2}{m_1}\log\frac{m_2}{m_1} + r\frac{m_3}{m_1}\log\frac{m_3}{m_1}}{\frac{m_2}{m_1} - 1 + r\left(\frac{m_3}{m_1}-1\right)}\right]^{-1}$$



Case 3 a)

- **In the decoupling limit, for NO, different predictions for different solutions for θ_N**
- **Shallow minima for NO only**

(FPDM, C. Hagedorn, ('25, To appear))

Option 3: Case 2

In the decoupling limit, predictions are of the same order of magnitude

Local minima is much shallower than in the non-decoupling limit

Lower limit is raised due to the shallower minima