Symmetric Mass Generation

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Two paradigms to generate mass:

Chiral Symmetry Breaking (χSM)

Symmetric Mass Generation (SMG)

- Generate mass without symmetry breaking

- UV limit: N_f massless flavors gauged with SU(N_c): SU$(N_f)_V \times$ SU$(N_f)_A \times$ U$(1)_V \times$ U$(1)_A$

Chiral Symmetry Breaking (χSM)

- IR limit: Spontaneous symmetry breaking
  - → Fermions are confined
  - → Chiral symmetry breaks
  - → Massless Goldstone bosons
  - → Bilinear condensate $\langle\psi\bar{\psi}\rangle \neq 0$
  - → Non-Goldstone states are gapped
- If a system has 't Hooft anomalies, massless Goldstones are necessary to satisfy the anomaly matching condition

Symmetric Mass Generation (SMG)

- IR limit: Symmetric mass generation
  - → Fermions are confined
  - → No symmetry breaking
  - → No massless Goldstones bosons
  - → Bilinear condensate $\langle\psi\bar{\psi}\rangle = 0$
  - → Expect $\langle\psi_1\bar{\psi}_1\psi_2\bar{\psi}_2\rangle \neq 0$ and/or instantons
  - → All bound states are gapped
- If all 't Hooft anomalies are canceled SMG is possible

Two paradigms to generate mass:

Chiral Symmetry Breaking (χSM)

Symmetric Mass Generation (SMG)

- Generate mass without symm try breaking

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  - → A
- If all 't H e canceled SMG is poss

Traditionally:

Confinement cannot happen w/o χSB

Recently:

Consider generalized symmetries

[García-Etxebarria, Montero JHEP 08 (2019) 003]

How to cancel 't Hooft anomalies (qualitative example by Cenke Xu for illustration)

- Vector N_f massless flavors gauged with SU(N_c): $\mathrm{SU}(N_f)_V \times \mathrm{SU}(N_f)_A \times U(1)_V \times \cancel{U(1)}_A$
- $U(1)_A : \psi \to e^{i\alpha\gamma_5}\psi,\ \bar{\psi} \to \bar{\psi}e^{i\alpha\gamma_5}$ is explicitly broken by the anomaly

$$Z \to \exp\{i\alpha N_c N_f Q\}Z, \quad Q = \int d^4x F\bar{F}$$

- Spin-Z_4 : $\psi_L \to i\psi_L, \quad \psi_R \to -i\psi_R,\ (\phi \to -\phi,\ A_\mu \to A_\mu)$ is a symmetry of the action
  - → The measure changes: $Z \to \exp\left\{i\frac{2\pi}{4}N_c N_f Q\right\} Z$
  - → Not anomalous, if $N_f N_c Q \ mod \ 4 = 0$
- If the system breaks $U(1)_A \to$ Spin-Z_4 an SMG phase is possible
  - → Breaking could be explicit (add 4-fermion interaction or lattice discretization) or spontaneous

Rigorous examples [García-Etxebarria, Montero JHEP 08 (2019) 003]

- 2-D: Eight Weyl (massless Majorana) fermions cancel all anomalies (chiral and time reversal)

→ Add 4-fermion interaction ⇒ SMG phase is predicted [Fidkowski, Kitaev PRB 81 (2010) 134509]

[Review by Wang, You Symmetry 14 (2022) 7]

- 4-D: 16 Weyl fermions or 8 massless Dirac fermions to cancel all 't Hooft anomalies

  [You, BenTov, Xu, arXiv:1402.4151] [Butt et al. PRD 104 (2021) 094504]

  → Counting in the UV: $N_f \times N_c \bmod 8 = 0$

  → Possible 4-D candidate models:
    - SU(2) gauge + $N_f = 4$ fundamental flavors
    - SU(3) gauge + $N_f = 8$ fundamental flavors
    - SU(4) gauge + $N_f = 2, 4, 6, \ldots$ fundamental flavors

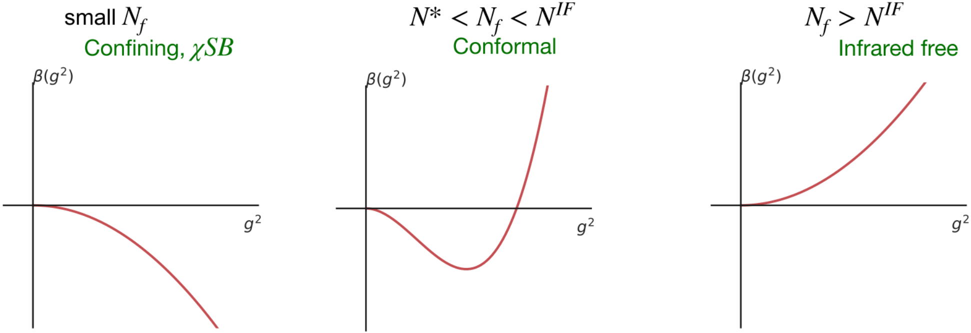
- Anomaly cancellation is necessary but not sufficient condition

  → Does SMG exist in any of these systems? Where?

  → Does it exist in the continuum limit at a strongly coupled fixed point?

Phases of gauge-fermion systems

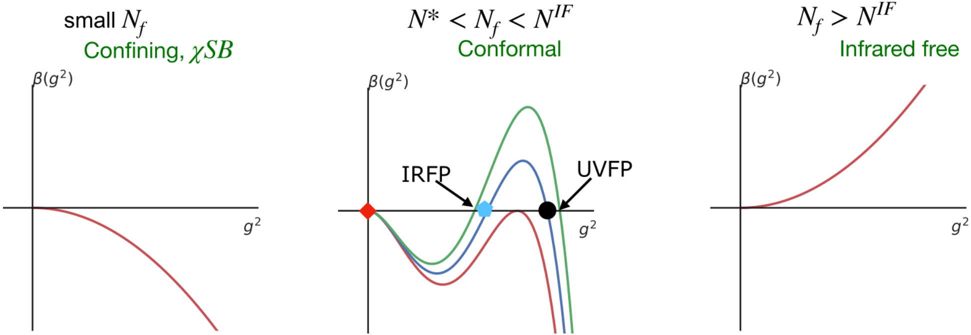
- SU(N_c) gauge + N_f (fundamental) massless flavors



- RG β function $\beta(g^2) = -\mu^2 \frac{dg^2}{d\mu^2}$ characterizes the phases
  → Known up to 5-loop in $\overline{\text{MS}}$ [Baikov, Chetyrkin, Kühn PRL118(2017)082002]
- Perturbatively emerges the IR fixed point at $g_0^2 = \infty$ for $N_f = N^*$ and moves to $g_0^2 = 0$ for $N_f \to N^{IF}$

Phases of gauge-fermion systems

- SU(N_c) gauge + N_f (fundamental) massless flavors



small N_f
Confining, χSB

$N^* < N_f < N^{IF}$
Conformal

$N_f > N^{IF}$
Infrared free

- Nonperturbatively the IR fixed point could emerge at finite g_*^2 if $\beta(g^2) \sim (\delta - \delta_*) - (g - g_*)^2$

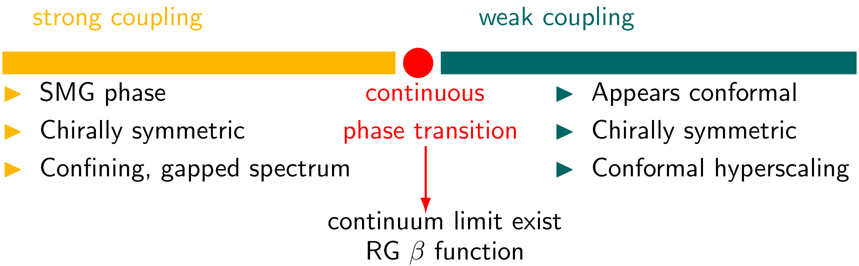
  → "Conformality lost" at IR-UV fixed point merger
  [Kaplan et al. PRD80 (2009) 125005] [Vecchi PRD82 (2010) 045013] [Gorbenko et al. JHEP10 (2018) 108]

  → UVFP: phase transition, new phase, new relevant operator

Numerically investigated examples in 4-D

- Similar results for two systems using $m_f = 0$
  - → SU(3) gauge + $N_f = 8$ [Hasenfratz PRD 106 (2022) 014513] [Hasenfratz, OW (LSD) PoS Lattice 2024 146]
  - → SU(2) gauge + $N_f = 4$ [Butt et al. PRL 134 (2025) 031602]

- Phase structure



strong coupling

weak coupling

- SMG phase
- Chirally symmetric
- Confining, gapped spectrum

continuous phase transition

continuum limit exist
RG β function

- Appears conformal
- Chirally symmetric
- Conformal hyperscaling

Numerical investigations: dynamical LFT simulations

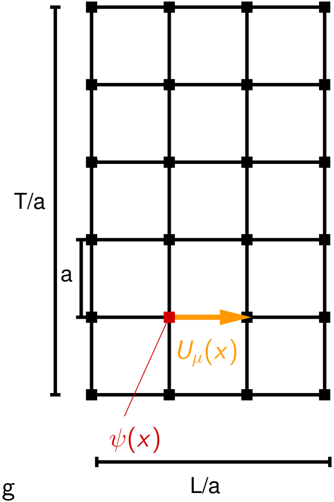
- Wick-rotate to Euclidean time $t \to i\tau$
- Discretize space-time introducing the lattice spacing a
- Restrict volume to a hypercube of finite extent $(L/a)^3 \times T/a$

- Formulate problem in terms of Feynman's path integral

$$\langle \mathcal{O} \rangle_E = \frac{1}{Z} \int \mathcal{D}[\psi, \overline{\psi}]\, \mathcal{D}[U]\, \mathcal{O}[\psi, \overline{\psi}, U]\, e^{-S_E[\psi, \overline{\psi}, U]}$$

- The lattice regularizes the theory
  - → Finite lattice spacing a: UV regulator
  - → Finite volume of length L/a: IR regulator

- Stochastic interpretation of the path integral
  - ⇝ Markov chain Monte Carlo simulations w/ importance sampling
  - → Requires High Performance Computing (HPC)



Simulations

- nHYP-smeared staggered fermions ⇝ Dirac-Kähler fermions
- Additional Pauli-Villars fields
- Plaquette gauge action with adjoint term

- Volume $16^3 \times 32$, $24^3 \times 64$, $32^3 \times 64$, $48^3 \times 96$
- $\beta_b = 8.10, 8.20, 8.30, 8.40, 8.50, 8.60, 8.70, 8.80, 8.90, 9.10, 9.20, 9.30, 9.50$

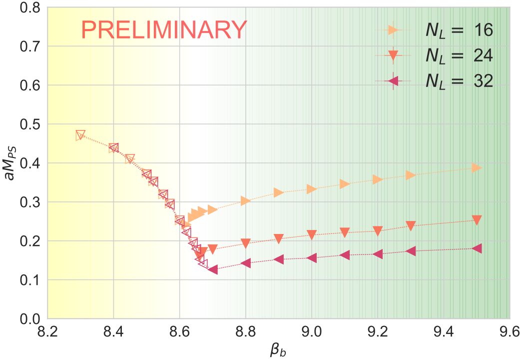
  → Simulations cross from weak coupling to symmetric mass generation (SMG) phase

  [Hasenfratz PRD 106 (2022) 014513]

- HMC: QEX (Osborn, Jin, Peterson)
- Spectrum: MILC (DeTar et al., . . . , Schaich, Hasenfratz)
- Gauge flow: QLUA (Pochinsky et al.)

Pseudoscalar correlator (would-be Goldstone pion)

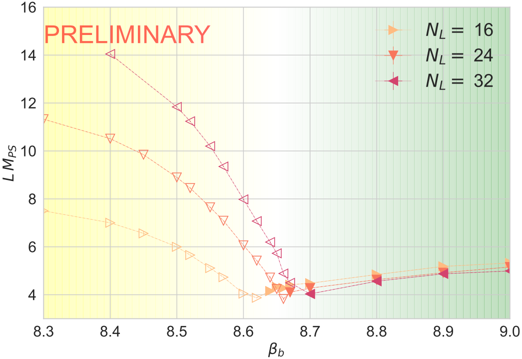
- aM_{PS}



volume independent
"gapped phase"

$1/L$ volume scaling
conformal phase

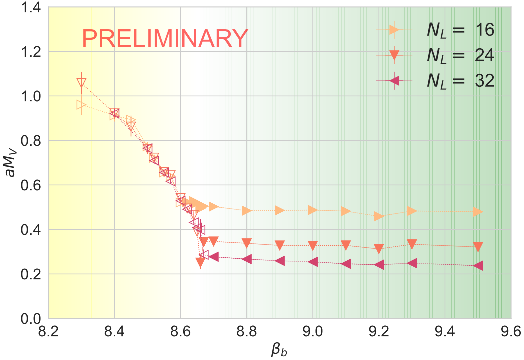
- LM_{PS}



- Conformal scaling for $\beta_b > 8.6$
  $M_{PS}L \approx \text{const}$

Vector correlator

- aM_{VT}



volume independent
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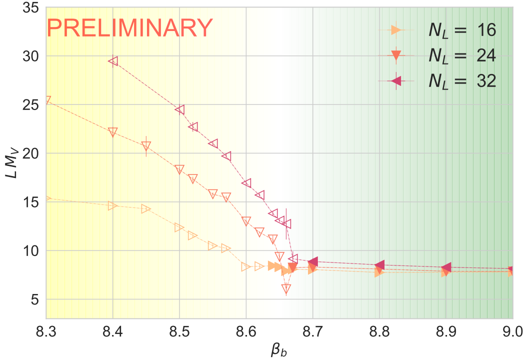

- Conformal scaling for $\beta_b > 8.6$
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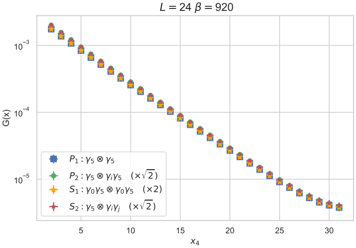
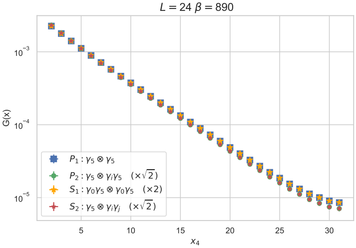
Pseudoscalar correlator (would-be Goldstone pion)

- Weak coupling
  $\beta_b = 9.20$ (conformal)



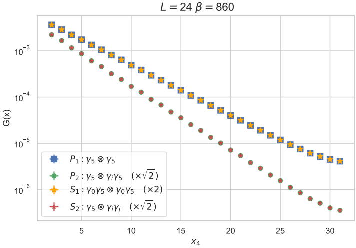
- Degenerate pseudoscalar and scalar (parity doubling)
- No taste splitting

- $\beta_b = 8.90$



- Degenerate pseudoscalar and scalar (parity doubling)

- Strong coupling
  $\beta_b = 8.60$ (SMG)



- Degenerate pseudoscalar and scalar (parity doubling)
- No chiral symmetry breaking

Vector correlator

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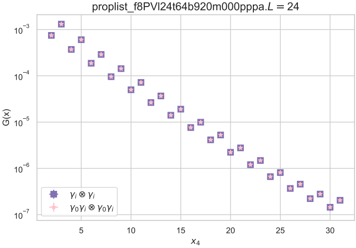
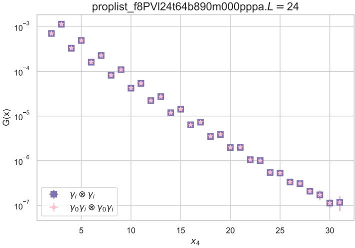


- Degenerate vector and axial (parity doubling)

- $\beta_b = 8.90$



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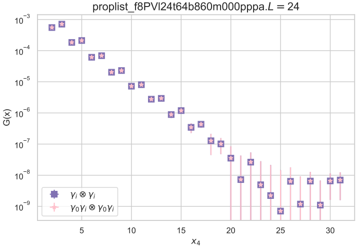

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- No chiral symmetry breaking

Order of the phase transition

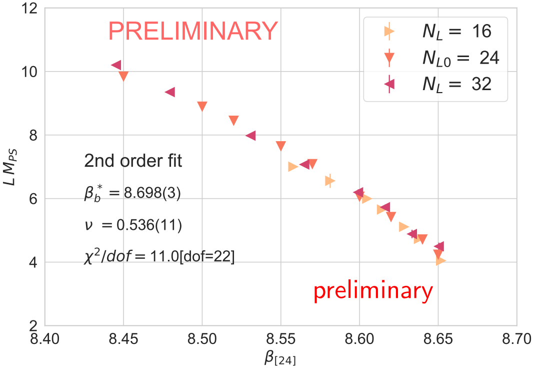
- Finite size scaling/curve collapse analysis near the critical point $g \to g^*$
  [Hasenfratz PRD 106 (2022) 014513]
  → $\mathcal{O}$ is a dimensionless operator: $\mathcal{O}(g, L) = f(L/\xi)$
  → ξ correlation length at g
  → $f(x = L/\xi)$ unique curve, independent of L

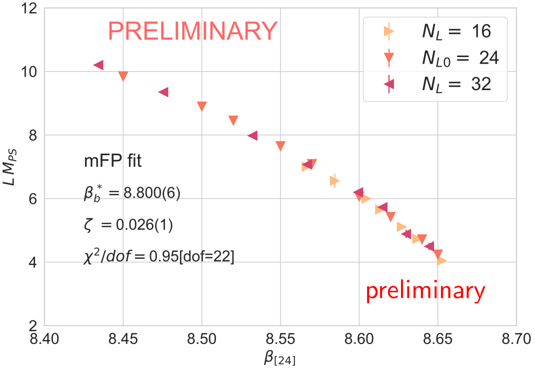
- Second order scaling: $\xi \propto |g - g_*|^{-\nu}$
  → First order scaling is similar and has $\nu = 1/d = 0.25$

- Merged fixed point (BKT) scaling: $\xi \propto \exp\{\zeta/|g - g_*|\}$ if $\beta(g^2) \sim (g^2 - g_*^2)^2$

- Obtain exponents and g^* by standard curve-collapse analysis

Outlook: finite size scaling analysis





- Finite size scaling

  2$^{\text{nd}}$ order: $L \cdot m(\beta; L) = f^{(c)}_{\text{2nd}} \left((\beta/\beta_c - 1)\, L^{1/\nu} \right)$ BKT: $L \cdot m(\beta; L) = f^{(c)}_{\text{BKT}} \left(L \exp\left(-\zeta |\beta/\beta_c - 1|^{-\nu} \right) \right)$

Summary

- SMG is possible in 4-D, if all ’t Hooft anomalies are canceled

- Large scale simulations to scrutinize SU(3) with $N_f = 8$ fundamental flavors at strong coupling using additional Pauli-Villars fields
  - → Preliminary findings support picture of a conformal and an SMG phase
  - → No chiral symmetry breaking
  - → Indications for a new non-trivially interacting UVFP in 4-D
  - → Implies: SU(3) + $N_f = 8$ fundamental flavors is likely the onset of the conformal window
  - → Maybe: merged fixed point with marginal operator like QCD but not asymptotically free
  - → Are staggered (Dirac-Kähler) fermions special?

- Ongoing $48^3 \times 96$ volume simulations

T14, Fri 9:50
Salle Joliette