



# Measurement of the strong-phase differences between $D^0$ and $\bar{D}^0$ decays with quantum-correlated $D\bar{D}$ at BESIII

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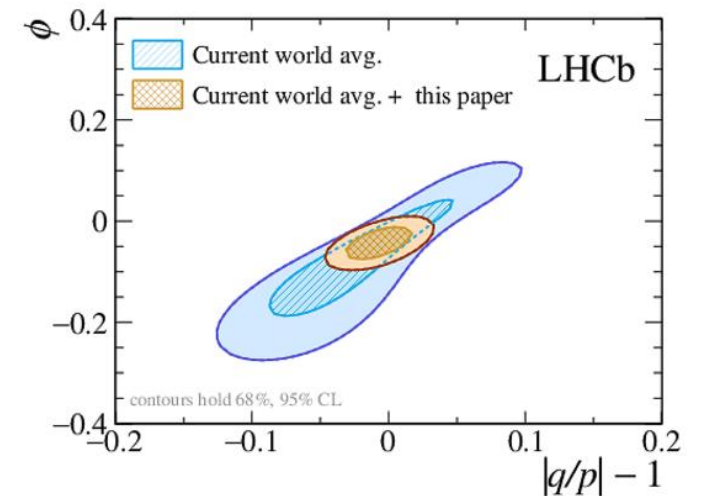
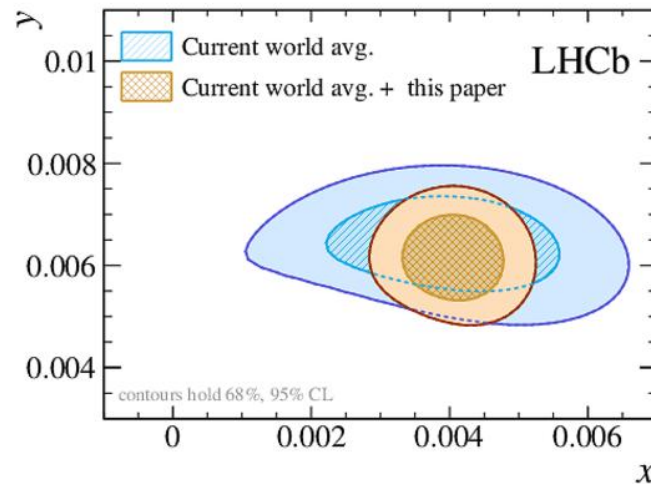
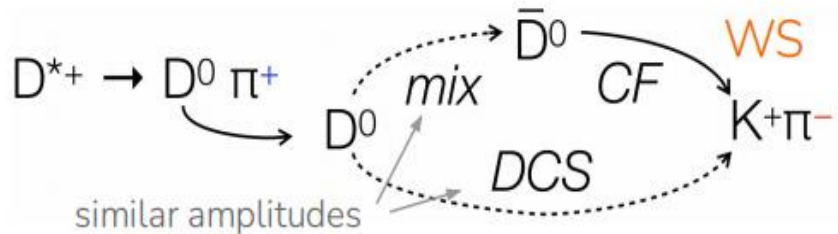


## Outline

- Strong-phase inputs to charm and bottom CPV studies
- Measurement of the strong-phase differences in  $D^0/\bar{D}^0 \rightarrow f$  decays
  - Analysis Method
  - Self-conjugate decays  $f = K_{S,L}^0 \pi^+ \pi^-, h^+ h^- \pi^+ \pi^-$
  - Quasi flavour eigenstates  $f = K^- n \pi$
  - Quasi  $CP$  eigenstates  $f = h^+ h^- \pi^0$
- Prospects and Summary

# Input to CPV studies in the charm sector

Strong-phase parameters provide essential inputs to charm mixing and indirect CPV studies in all the three kinds of decays: self-conjugate decays, quasi-flavour eigenstates, and quasi-CP eigenstates

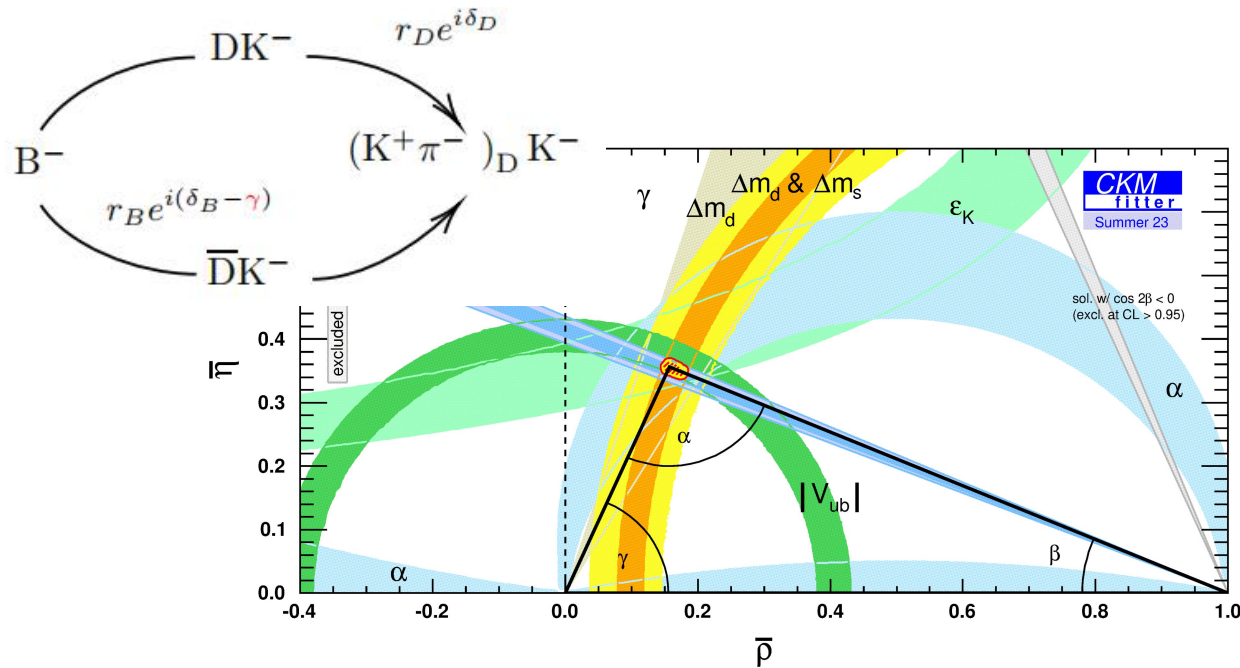


**Phys. Rev. Lett. 127, (2021) 111801**

**Observation of non-zero  $x$  with  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$**

# Input to CPV studies in the bottom sector

- Strong-phase parameters provide essential inputs to precision measurement of CKM angle  $\gamma$  with tree-level  $B \rightarrow DK$  decays
- $D$  decays to the three categories of self-conjugate decays, quasi-flavour eigenstates and (quasi-)CP eigenstates, referred as *BPGGSZ*, *ADS* and *GLW* methods

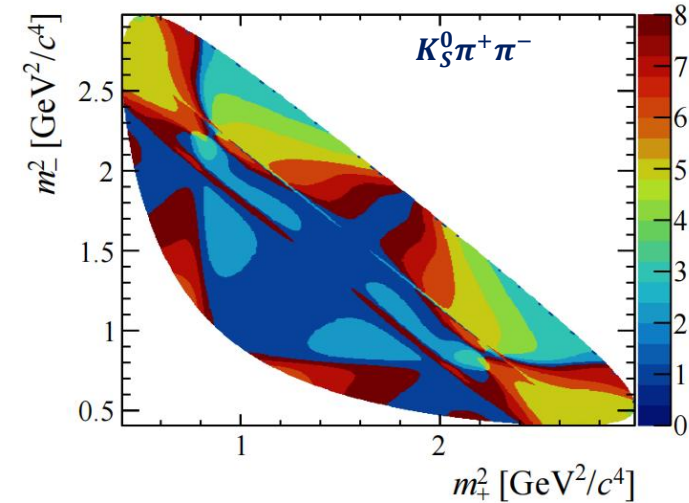


- Direct measurement:  $(64.6 \pm 2.8)^\circ$
- Over-constrained:  $(66.23^{+1.86}_{-0.72})^\circ$
- The uncertainty contributed by the strong-phase inputs should be further reduced to test the CKM unitarity triangle and search for new physics.

# Parameterize the strong-phase parameters

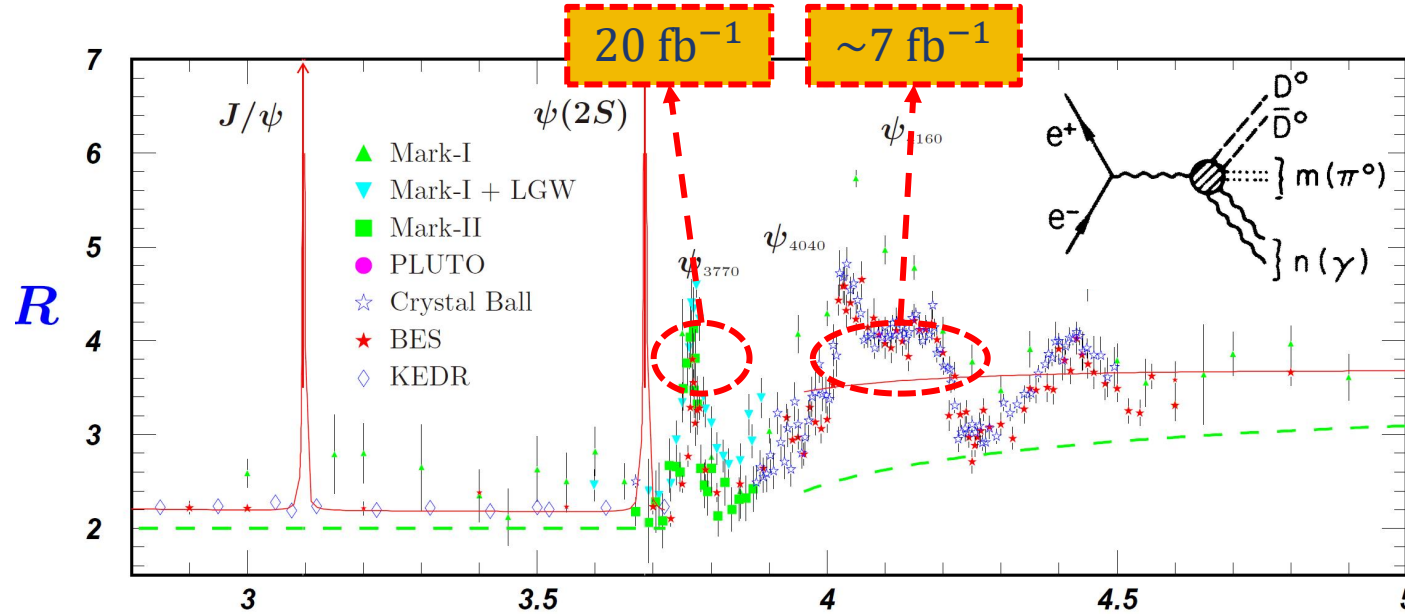
Strong-phase differences can be obtained with amplitudes however with model assumptions. It is more desirable to perform model-independent measurements

- Quasi flavour eigenstates  $D \rightarrow K^- n \pi$ 
  - $r_D^F \exp(-i\delta_D^F) = \left(\frac{\bar{A}_F}{A_F}\right), R_F \exp(-i\delta_D^F) = \frac{\int dX A_F^*(X) A_F(X)}{A_F \bar{A}_F}$
- Quasi  $CP$  eigenstates  $D \rightarrow h^+ h^- \pi^0$ 
  - $F_+ = \frac{\Gamma_+}{\Gamma_+ + \Gamma_-} = \frac{\int dX (|A_F|^2 + |\bar{A}_F|^2 + 2|A_F||\bar{A}_F| \cos \delta_D^F(X))}{\int dX 2(|A_F|^2 + |\bar{A}_F|^2)} = \frac{1 + \int dX \cos \delta_D^F(X)}{2}$
- Self-conjugate decays  $D \rightarrow K_{S,L}^0 h^+ h^-, h^+ h^- \pi^+ \pi^-$ 
  - $c_i = \frac{1}{\sqrt{F_i F_{-i}}} \int_i dX |A(X)| |\bar{A}(X)| \cos [\delta_D(X)]$
  - $s_i = \frac{1}{\sqrt{F_i F_{-i}}} \int_i dX |A(X)| |\bar{A}(X)| \sin [\delta_D(X)]$
  - $F_i = \frac{\int_i dX |A_F|^2}{\int dX |A_F|^2}$



- Exploit the variation of strong-phase differences across the phase-space of multiple-body decays help to optimize the sensitivity to charm CPV and  $\gamma$  measurements
- The binning scheme can be developed with the amplitude model, the strong-phase measurements are still model-independent

# Analysis method



At  $\psi(3770)$

C-odd for  $e^+e^- \rightarrow D^0\bar{D}^0$

$$\Gamma[(e^+e^-)\psi(3770) \rightarrow [S]_D[T]_{\bar{D}}]$$

$$\propto A_S^2 A_T^2 [(r_D^S)^2 + (r_D^T)^2 - 2R_S R_T r_D^S r_D^T \cos(\delta_D^S - \delta_D^T)]$$

Charm mixing can be neglected to  $O(x^4, y^4)$

At higher energy points

C-odd for  $e^+e^- \rightarrow D^{(*)0}\bar{D}^{*0} + c.c. \rightarrow \pi^0(\pi^0)D^0\bar{D}^0$

C-even for  $e^+e^- \rightarrow D^0\bar{D}^{*0} + c.c. \rightarrow \gamma D^0\bar{D}^0$

C-even for  $e^+e^- \rightarrow D^{*0}\bar{D}^{*0} + c.c. \rightarrow \gamma\pi^0 D^0\bar{D}^0$

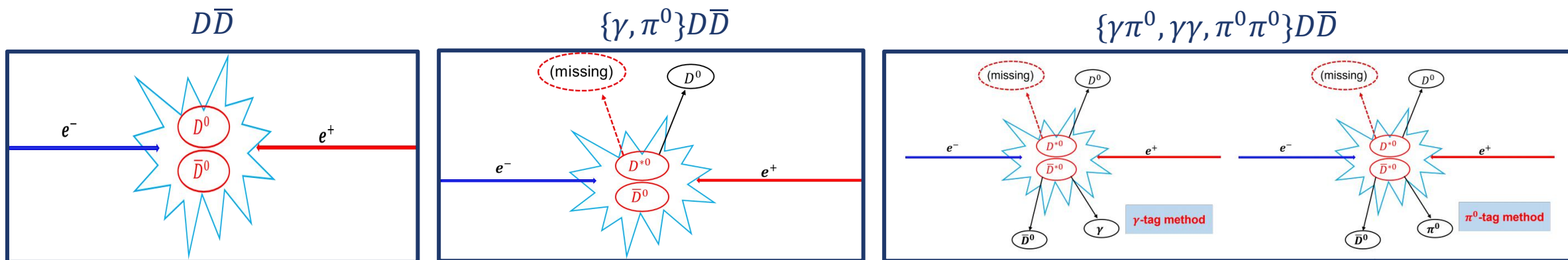
$$\Gamma[(e^+e^-)_{4009-4230} \rightarrow \gamma(\pi^0)[S]_D[T]_{\bar{D}}]$$

$$\propto A_T^2 [(r_D^S)^2 + (r_D^T)^2 + 2R_S R_T r_D^S r_D^T \cos(\delta_D^S - \delta_D^T)]$$

$$- 2y(R_S r_D^S \cos \delta_D^S + R_T r_D^T \cos \delta_D^T) + 4x(R_S r_D^S \sin \delta_D^S + R_T r_D^T \sin \delta_D^T)]$$

Sensitivity to charm mixing could be achieved

# Double-tag method



- Study the joint decay of  $D$  and  $\bar{D}$ , referred as the *double tag* method
- Reconstruct one  $D$  in the signal decay and the other in the tag decay
- Constrain the tag-side strong-phase differences to extract the parameters in the signal side

| Tag side       |                                                                    | Branching fraction level                                                                |
|----------------|--------------------------------------------------------------------|-----------------------------------------------------------------------------------------|
| Flavour        | $K^- e^+ \nu_e, K^- \pi^+, K^- \pi^+ \pi^0, K^- \pi^+ \pi^+ \pi^-$ | $10^{-1} - 10^{-2}$                                                                     |
| CP             | Even                                                               | $K^+ K^-, \pi^+ \pi^-, K_S^0 \pi^0 \pi^0, \pi^+ \pi^- \pi^0, K_L^0 \pi^0, K_L^0 \omega$ |
|                | Odd                                                                | $K_S^0 \pi^0, K_S^0 \omega, K_S^0 \eta, K_S^0 \eta', K_L^0 \pi^0 \pi^0$                 |
| Self-conjugate | $K_S^0 \pi^+ \pi^-, K_L^0 \pi^+ \pi^-$                             | $10^{-2}$                                                                               |



# Study of the $D \rightarrow K_{S,L}^0 \pi^+ \pi^-$ decay

- $CP$  tags

- $M_i \propto K_i + K_{-i} - 2c_i \sqrt{K_i K_{-i}}$
- $M'_i \propto K'_i + K'_{-i} + 2c'_i \sqrt{K'_i K'_{-i}}$

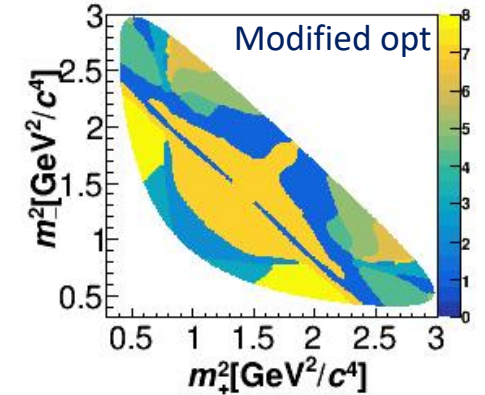
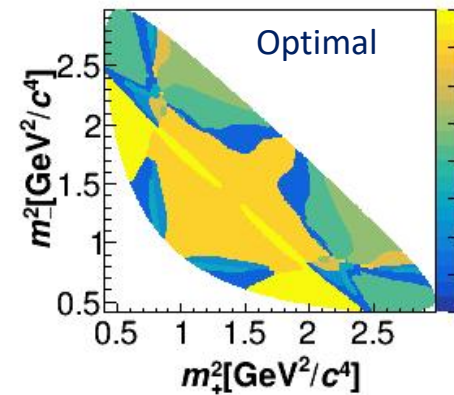
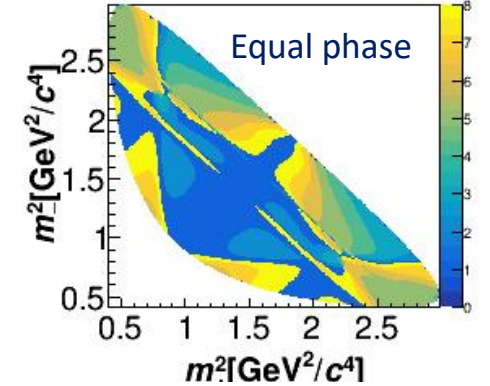
- Self-conjugate tags

- $M_{ij} \propto K_i K_{-j} + K_{-i} K_j - 2\sqrt{K_i K_{-i} K_j K_{-j}}(c_i c_j + s_i s_j)$
- $M_{ij} \propto K_i K'_{-j} + K_{-i} K'_j + 2\sqrt{K_i K_{-i} K'_j K'_{-j}}(c_i c'_j + s_i s'_j)$

- Flavour tags

- $M_i \propto K_i + (r_D^F)^2 K_{-i} - 2R_D r_D^F \sqrt{K_i K_{-i}}(\cos \delta_D^F c_i - \sin \delta_D^F s_i)$
- $M'_i \propto K'_i + (r_D^F)^2 K'_{-i} + 2R_D r_D^F \sqrt{K'_i K'_{-i}}(\cos \delta_D^F c'_i - \sin \delta_D^F s'_i)$
- $K_i$  are the fractional flavour-specific signal decay events in each bin

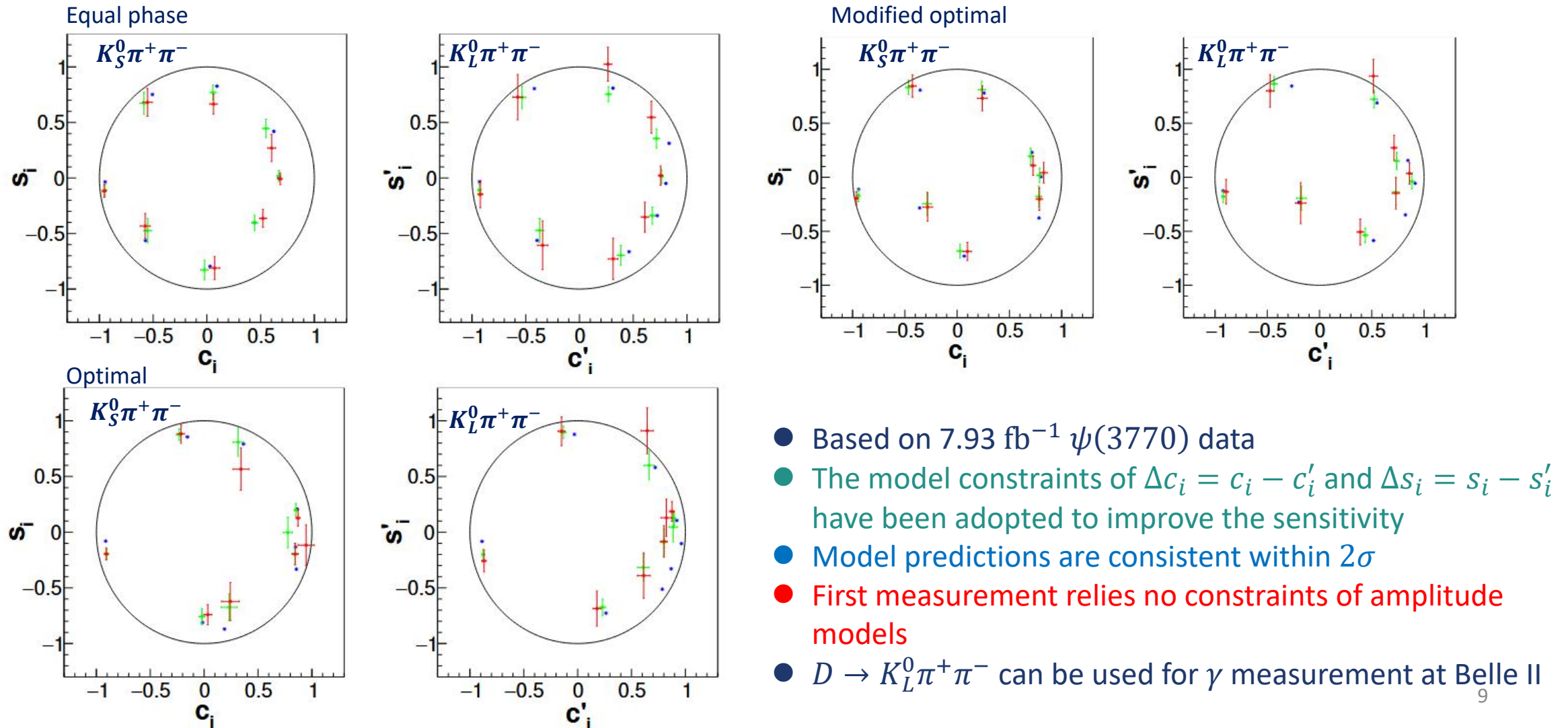
Binning schemes for  $D \rightarrow K_{S,L}^0 \pi^+ \pi^-$   
 Equal phase for charm mixing study  
 Optimal for  $\gamma$  measurement



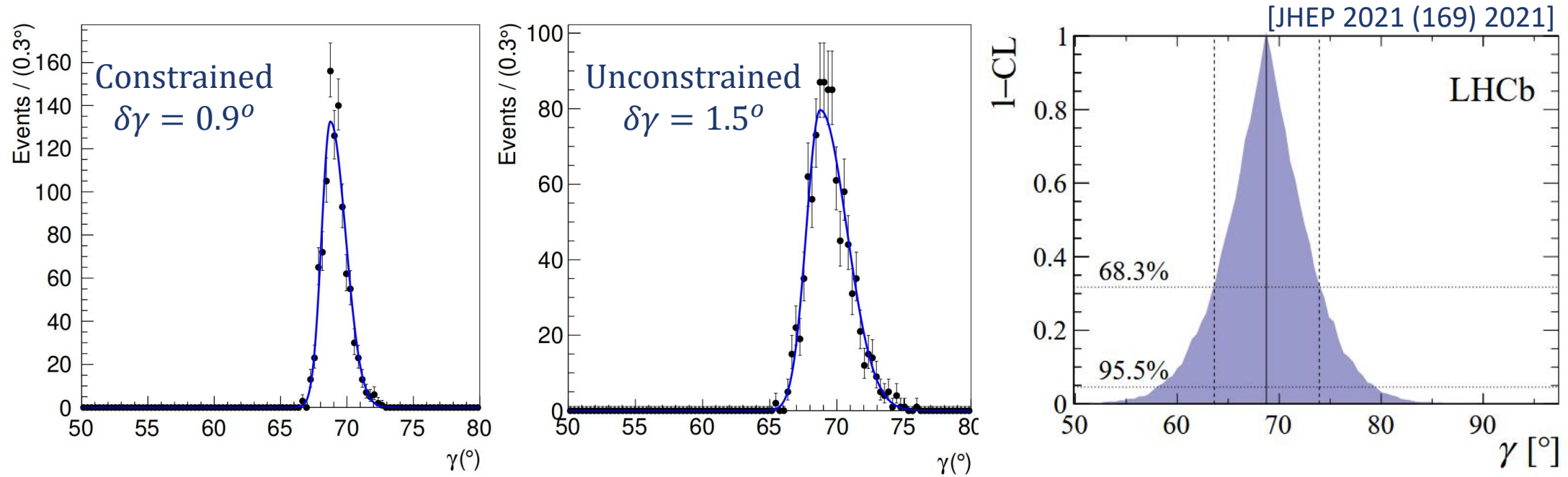
$D \rightarrow K_L^0 \pi^+ \pi^-$  can be studied with the same binning scheme  
 and can help to improve the measurement of  $D \rightarrow K_S^0 \pi^+ \pi^-$



# Results of $c_i^{(')}$ , $s_i^{(')}$ in $D \rightarrow K_{S(L)}^0 \pi^+ \pi^-$



# Results of $c_i^{(')}$ , $s_i^{(')}$ in $D \rightarrow K_{S(L)}^0 \pi^+ \pi^-$

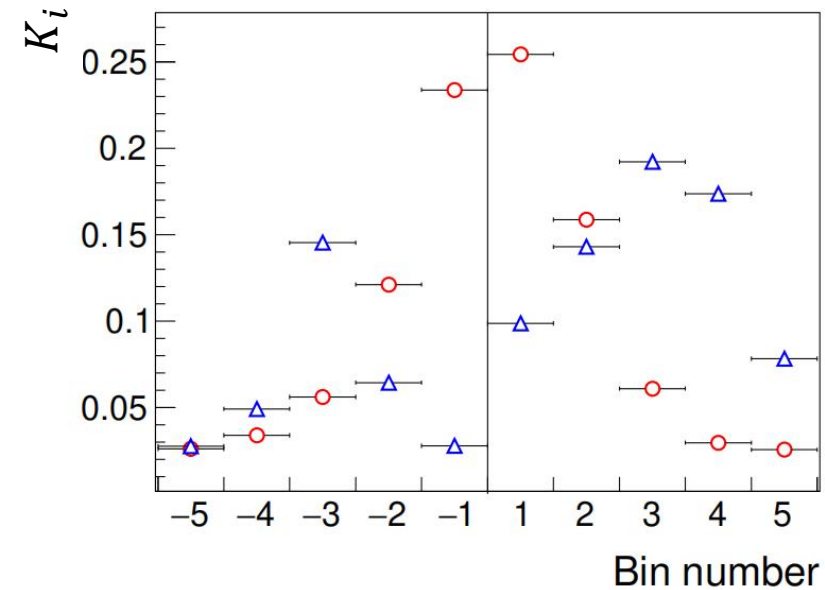
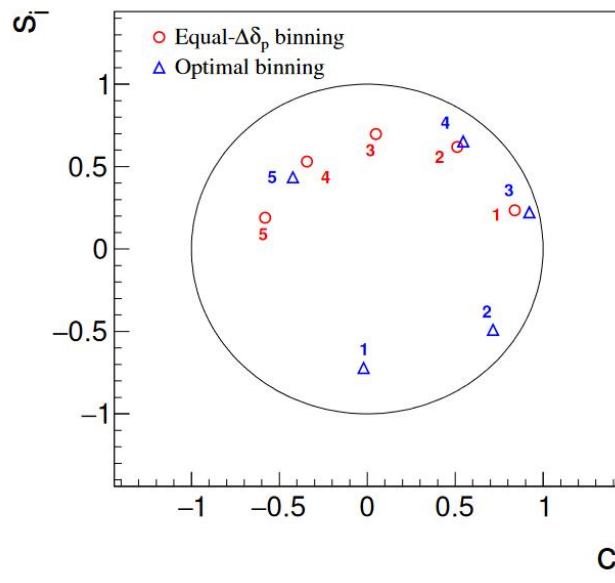


- The uncertainty contribution to  $\gamma$  is improved by  $\sim 1.4$  compared to the previous BESIII measurement
- The constrained strong-phase parameters would contribute an uncertainty to  $\gamma$  sub-leading to the experimental systematic uncertainty
- Mean value of  $\gamma$  shift about  $0.5^\circ$  taking the unconstrained parameters as inputs

$$\begin{aligned}
 x_-^{DK} &= (5.68 \pm 0.96 \pm 0.20 \pm \mathbf{0.23}) \times 10^{-2} \\
 y_-^{DK} &= (6.55 \pm 1.14 \pm 0.25 \pm \mathbf{0.35}) \times 10^{-2} \\
 x_+^{DK} &= (-9.30 \pm 0.98 \pm 0.24 \pm \mathbf{0.18}) \times 10^{-2} \\
 y_+^{DK} &= (-1.25 \pm 1.23 \pm 0.26 \pm \mathbf{0.28}) \times 10^{-2} \\
 \gamma &= (68.7_{-5.1}^{+5.2})^\circ
 \end{aligned}$$

# Study of the decay $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

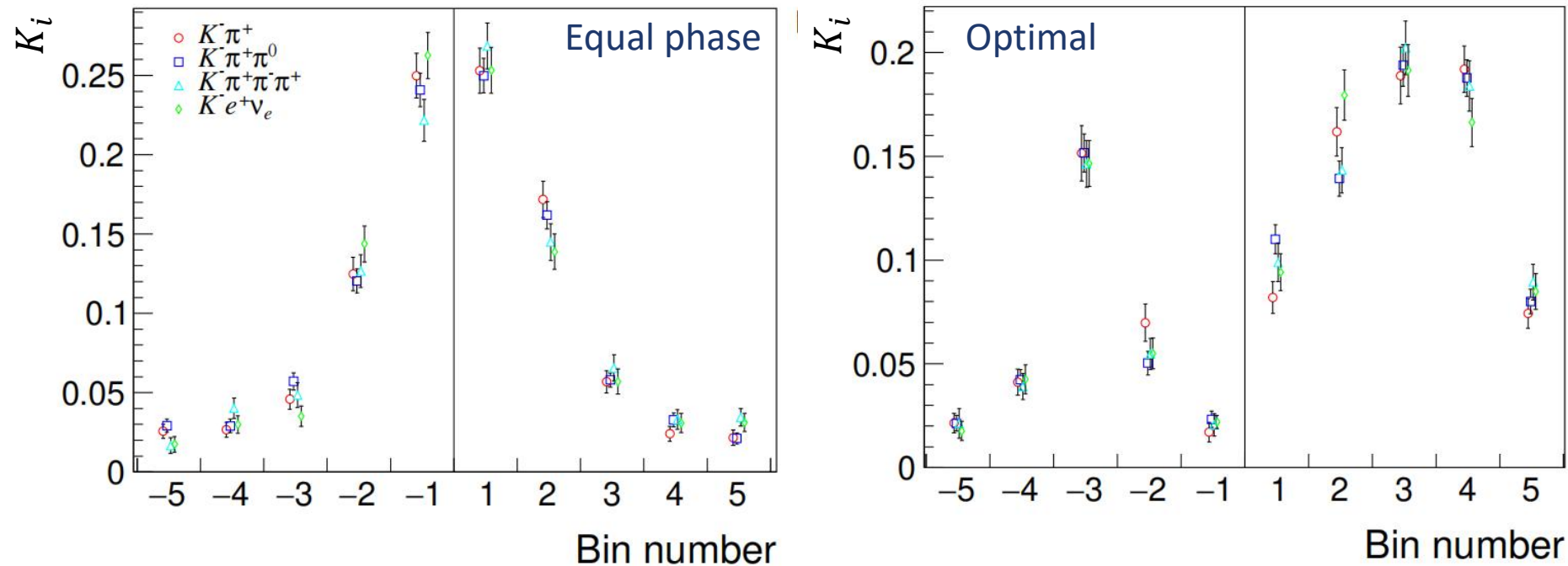
- Binning schemes are developed in the 5D phase space ( $m_{\pi^+\pi^+}$ ,  $m_{\pi^-\pi^-}$ ,  $\cos \theta_{\pi^+\pi^+}$ ,  $\cos \theta_{\pi^-\pi^-}$ ,  $\phi_{+/-}$ ) based on the amplitude model [Chin. Phys. C 48, 083001 (2024)]
- Phase space fold under CP transformation and exchange of identical particles, where the strong-phase differences are invariant
- The optimal binning scheme is constructed where negative (positive) bins are chosen to have  $|A/\bar{A}| < (>)1$  to optimize the sensitivity to  $\gamma$  angle
- The binning method is optimized by maximizing the sensitivity compared to an unbinned analysis, which is 85%



# Study of the decay $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

- Flavour tags are used to measure the fractional binned signal events

- $$M_i \propto K_i + (r_D^F)^2 K_{-i} - 2R_D r_D^F \sqrt{K_i K_{-i}} (\cos \delta_D^F c_i - \sin \delta_D^F s_i)$$
- $$M'_i \propto K'_i + (r_D^F)^2 K'_{-i} + 2R_D r_D^F \sqrt{K'_i K'_{-i}} (\cos \delta_D^F c'_i - \sin \delta_D^F s'_i)$$



# Results of $c_i$ , $s_i$ in $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

$CP$  tags

$$M_i \propto K_i + K_{-i} - 2c_i \sqrt{K_i K_{-i}}$$

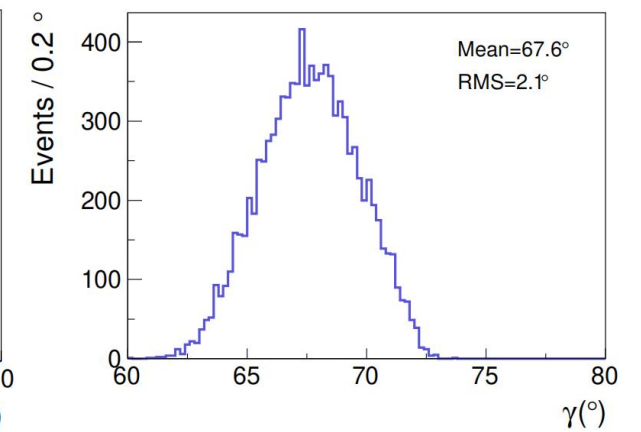
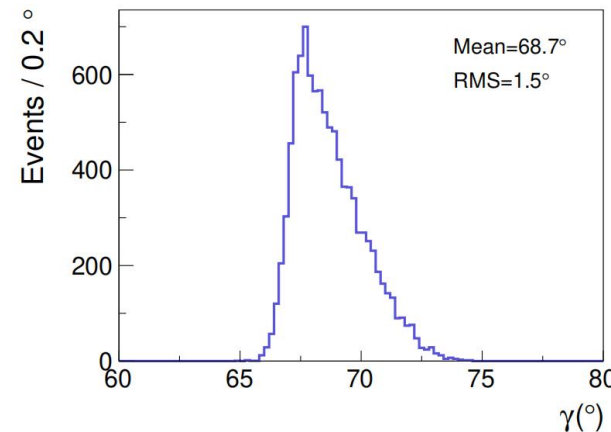
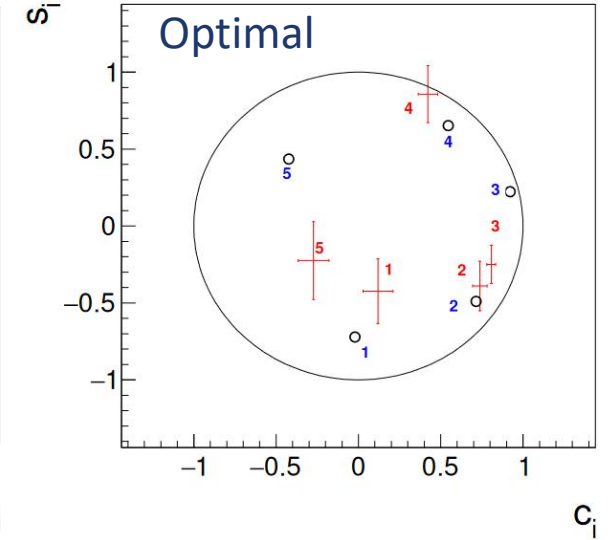
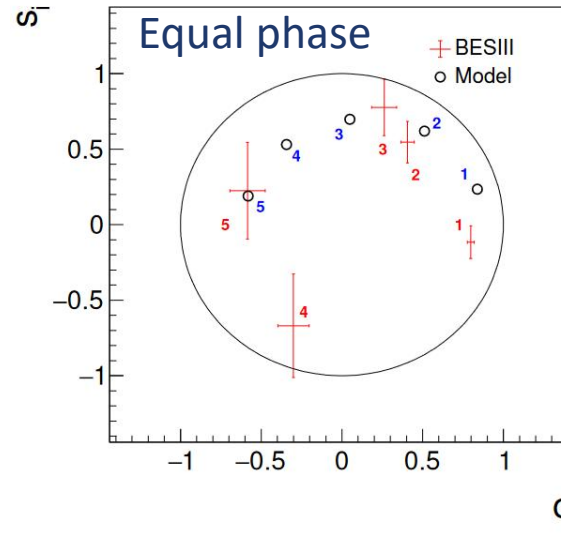
$$M'_i \propto K'_i + K'_{-i} + 2c'_i \sqrt{K'_i K'_{-i}}$$

Self-conjugate tags

$$M_{ij} \propto K_i K_{-j} + K_{-i} K_j - 2\sqrt{K_i K_{-i} K_j K_{-j}}(c_i c_j + s_i s_j)$$

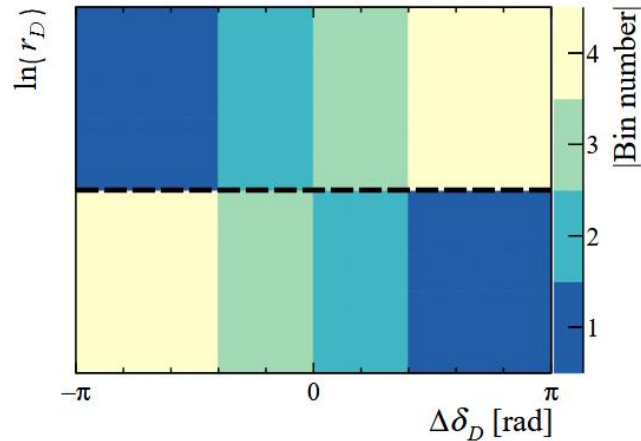
$$M_{ij} \propto K_i K'_{-j} + K_{-i} K'_j + 2\sqrt{K_i K_{-i} K'_j K'_{-j}}(c_i c'_j + s_i s'_j)$$

- The statistical uncertainty of  $\gamma$  measured with binned  $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$  is found to be around  $6^\circ$ , only next to the  $D \rightarrow K_S^0 \pi^+ \pi^-$  decay
- The uncertainty contributed by BESIII inputs is around  $2.1^\circ$  with around  $3 \text{ fb}^{-1} \psi(3370)$  data
- It can be further improved with the total BESIII  $\psi(3770)$  dataset

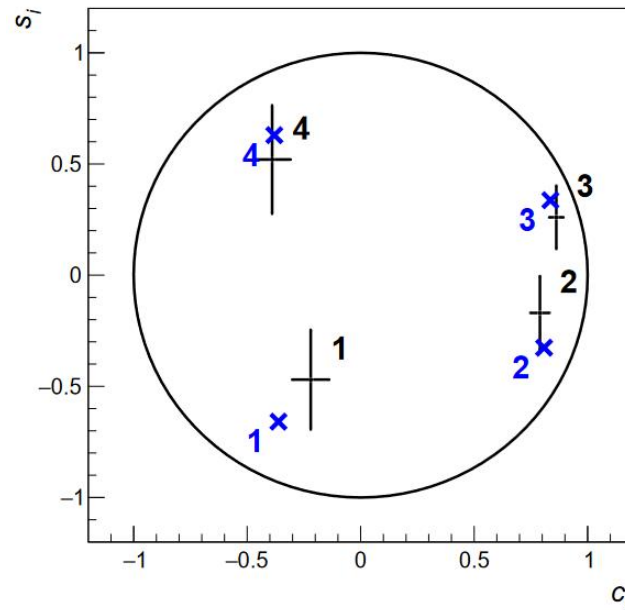


# Study of the decay $D \rightarrow K^+ K^- \pi^+ \pi^-$

- Binning scheme is developed with the decay amplitude model in the 5D phase space EPJC 83 (2023) 547
- The binning method is found to have a sensitivity about 85% that of the model-dependent measurement
- Three categories of tags have been used



Binning scheme available at [10.5281/zenodo.6940031](https://zenodo.org/record/6940031)



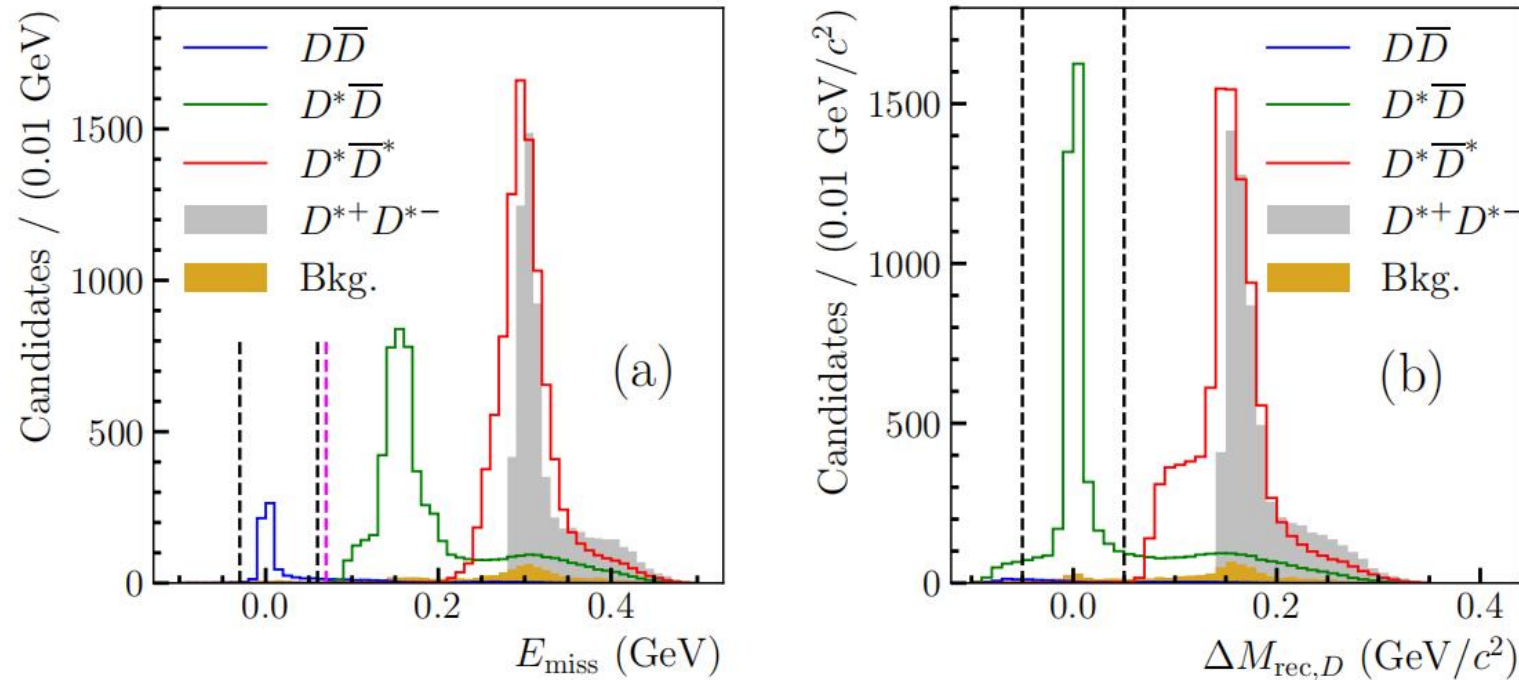
| Variable | Results        | Model |
|----------|----------------|-------|
| $K_{-4}$ | 0.0871(27)(38) | 0.086 |
| $K_{-3}$ | 0.294(5)(6)    | 0.297 |
| $K_{-2}$ | 0.400(7)(3)    | 0.398 |
| $K_{-1}$ | 0.252(7)(12)   | 0.267 |
| $K_1$    | 0.119(6)(6)    | 0.110 |
| $K_2$    | 0.405(12)(2)   | 0.401 |
| $K_3$    | 0.818(10)(10)  | 0.833 |

- The first strong-phase measurement with the total  $20 \text{ fb}^{-1}$  dataset
- Model predictions of strong-phase differences and fractional  $D^0 \rightarrow KK\pi\pi$  signal events are in agreement with the measurements
- The uncertainty contributing to  $\gamma$  is found to be around  $10^\circ$ , the statistical uncertainty of  $\gamma$  is  $12.7^\circ$



# First exploration of the C-even correlation

Based on simulation of  $(K^-\pi^+)_D(K^+\pi^-)_{\bar{D}}$  decays produced in different mechanism

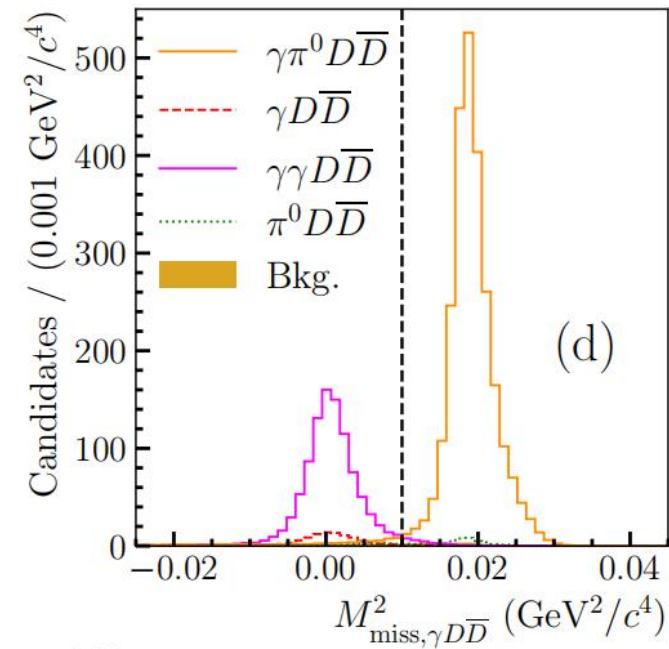
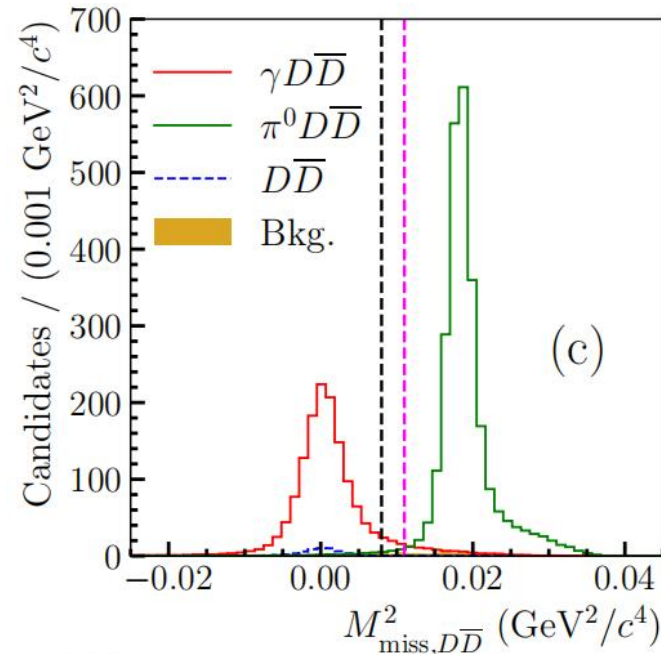


- The  $D$  and  $\bar{D}$  decays are jointly reconstructed
- Distinguish between  $D\bar{D}$ ,  $D^*\bar{D}$  and  $D^*D^*$ 
  - “Missing energy” that the transition  $\gamma/\pi^0$  particle carries
  - Minimum difference between the recoil mass of each  $D$  and the  $D^*$  nominal mass



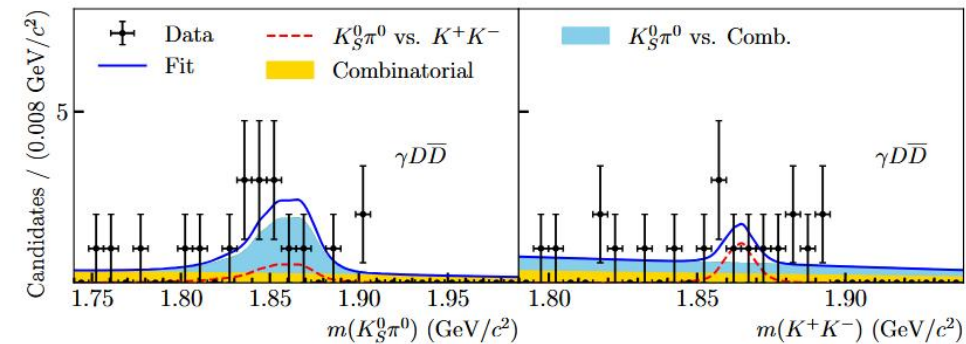
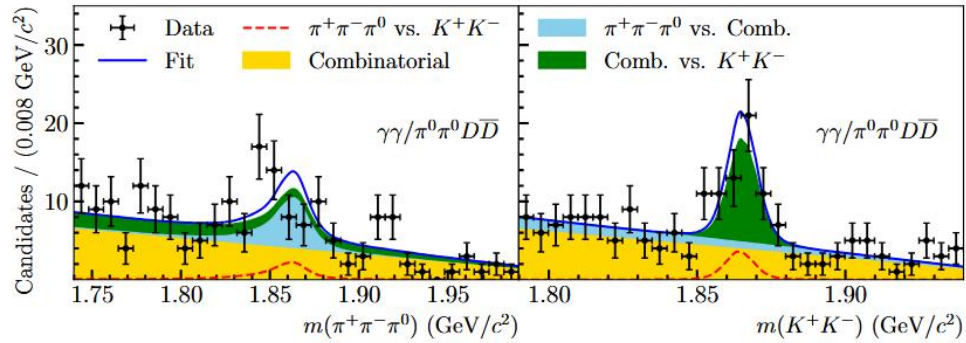
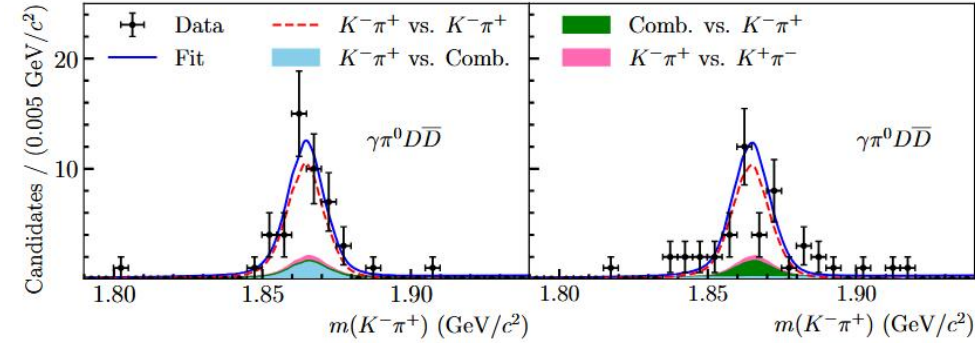
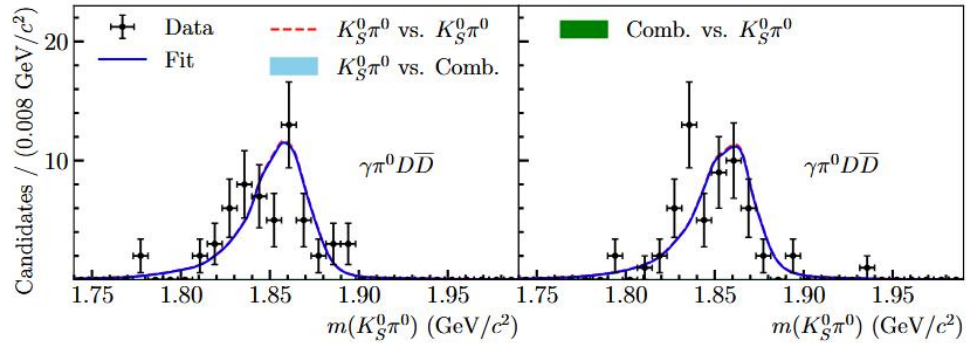
# First exploration of the C-even correlation

Based on simulation of  $(K^-\pi^+)_D(K^+\pi^-)_{\bar{D}}$  decays produced in different mechanism

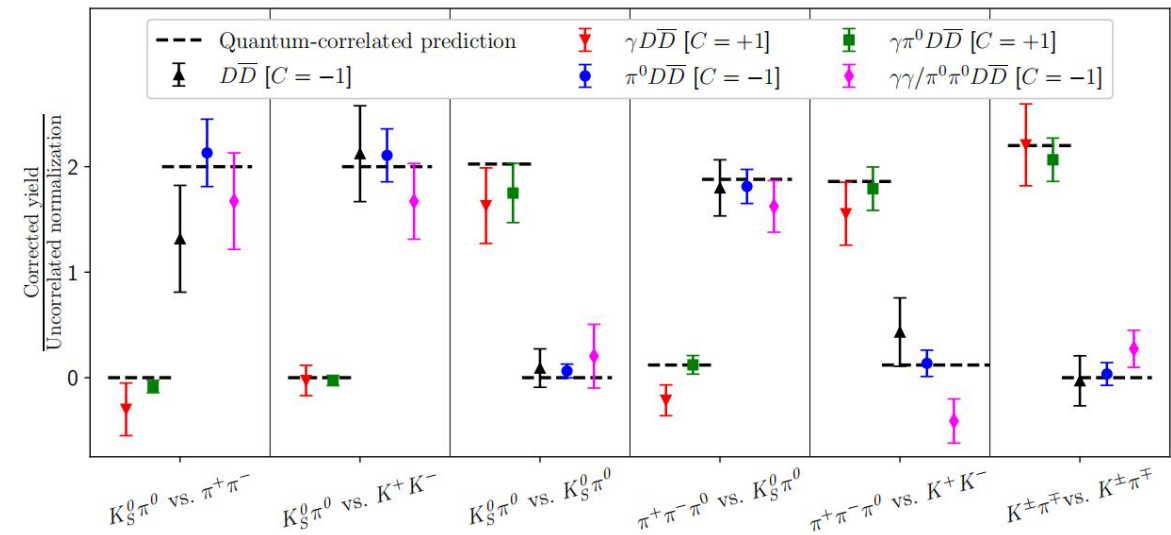


- $D^*\bar{D} \rightarrow \gamma/\pi^0 D\bar{D}$ 
  - Well separated by the “squared missing mass” of the two  $D$  mesons
- $D^*\bar{D}^* \rightarrow \gamma\gamma/\gamma\pi^0 D\bar{D}$ 
  - Reconstruct one of the accompanying photon
  - Well separated by the “squared missing mass” of the two  $D$  mesons and the reconstructed photon

# First exploration of the C-even correlation

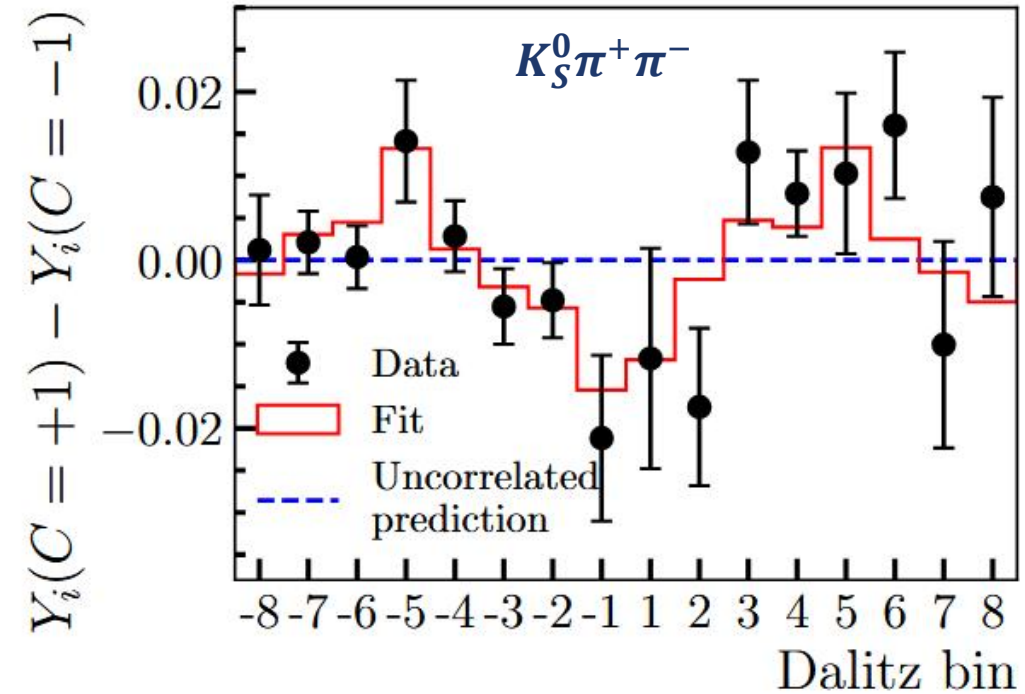
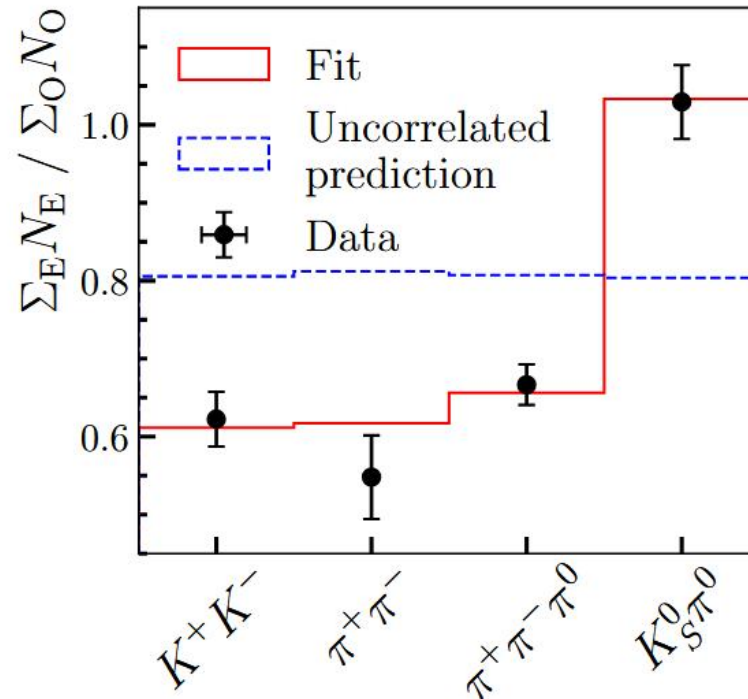


- Clear difference of the observed CP even (odd) vs CP even (odd) signal events in  $\gamma\pi^0 D\bar{D}$  and  $\gamma\gamma/\pi^0\pi^0 D\bar{D}$  samples
- CP even vs CP odd signal decays are suppressed in  $\gamma D\bar{D}$  sample
- Like-sign signal event are observed in C-even sample



7.13 fb<sup>-1</sup> from 4.13-4.23 GeV

# Novel measurement of $\delta_{K\pi}$



Comparing the DT yields in C-even and C-odd correlated samples yields

$$\delta_D^{K\pi} = (192.8_{-12.4}^{+11.0+1.9})^{\circ}$$

The first measurement with C-even correlated samples

Implying these samples can be used for measurement of other decays

Feasibility of performing charm mixing and indirect CPV studies with larger samples produced at future Super Tau-Charm Facility are also discussed

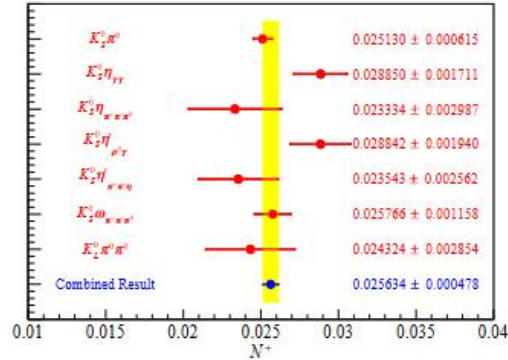
# Measurement of CP-even fraction in $D \rightarrow \pi^+ \pi^- \pi^0, K^+ K^- \pi^0$

CP tags

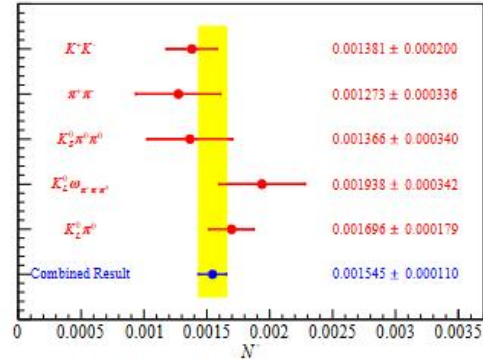
$$\Gamma = A_S^2 A_T^2 [1 - (2F_+ - 1) \cos \delta_D^{CP\pm}]$$

Self-conjugate tags

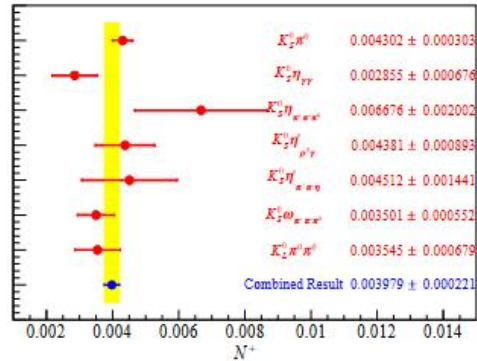
$$\Gamma_i = A_S^2 A_T^2 [K_i + K_{-i} - 2\sqrt{K_i K_{-i}} c_i (2F_+ - 1)]$$



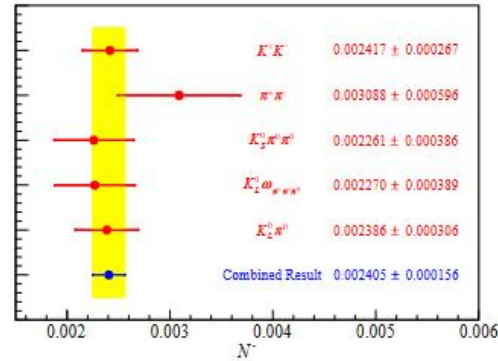
(a)  $N^+$  for  $\pi^+ \pi^- \pi^0$



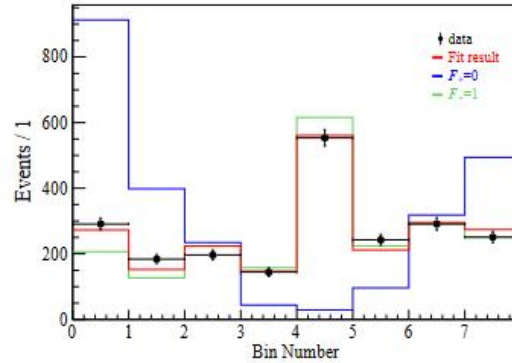
(b)  $N^-$  for  $\pi^+ \pi^- \pi^0$



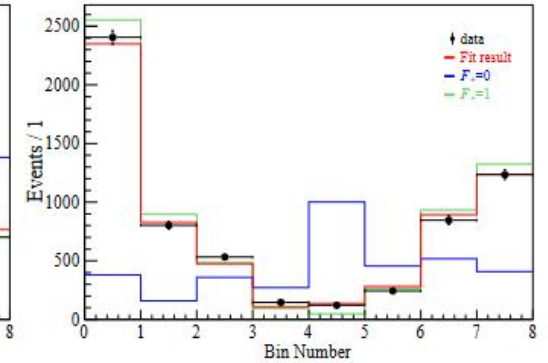
(c)  $N^+$  for  $K^+ K^- \pi^0$



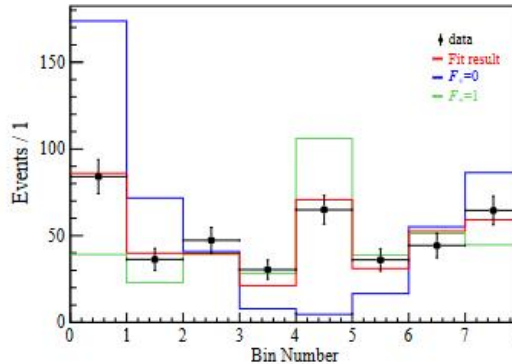
(d)  $N^-$  for  $K^+ K^- \pi^0$



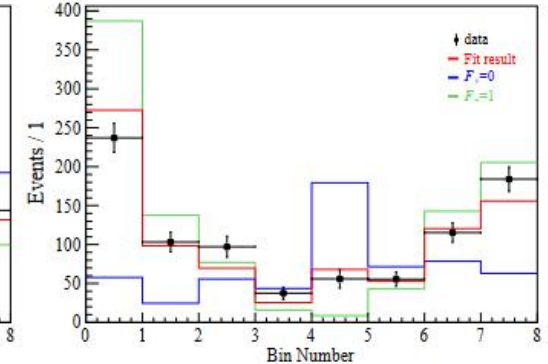
(a)  $K_S^0 \pi^+ \pi^-$  versus  $\pi^+ \pi^- \pi^0$



(b)  $K_L^0 \pi^+ \pi^-$  versus  $\pi^+ \pi^- \pi^0$



(c)  $K_S^0 \pi^+ \pi^-$  versus  $K^+ K^- \pi^0$



(d)  $K_L^0 \pi^+ \pi^-$  versus  $K^+ K^- \pi^0$

|               | $F_{+}^{\pi\pi\pi^0}$          | $F_{+}^{KK\pi^0}$           |
|---------------|--------------------------------|-----------------------------|
| This analysis | $0.9406 \pm 0.0055 \pm 0.0033$ | $0.631 \pm 0.014 \pm 0.011$ |
| CLEO-c        | $0.973 \pm 0.017$              | $0.732 \pm 0.055$           |



# Summary

| BESIII         | Decay                              | Parameters       | 2.93 fb <sup>-1</sup> $\psi(3770)$             | Prospects                                                               |
|----------------|------------------------------------|------------------|------------------------------------------------|-------------------------------------------------------------------------|
| Self-conjugate | $K_{S,L}^0 \pi^+ \pi^-$            | $c_i, s_i$       | PRL 124 (2020) 241802<br>PRD 101 (2020) 112002 | 8 fb <sup>-1</sup> JHEP 06 (2025) 086<br>20 fb <sup>-1</sup> ongoing    |
|                | $K_{S,L}^0 K^+ K^-$                | $c_i, s_i$       | PRD 102 (2020) 052008                          | 20 fb <sup>-1</sup> ongoing                                             |
|                | $\pi^+ \pi^- \pi^+ \pi^-$          | $F^+ / c_i, s_i$ | PRD 106 (2022) 092004<br>PRD 110 (2024) 112008 | 20 fb <sup>-1</sup> ongoing                                             |
|                | $K^+ K^- \pi^+ \pi^-$              | $F^+ / c_i, s_i$ | PRD 107 (2023) 032009                          | 20 fb <sup>-1</sup> arXiv: 2502.12873                                   |
|                | $K_{S,L}^0 \pi^+ \pi^- \pi^0$      | $F^+ / c_i, s_i$ | PRD 108 (2023) 032003                          | $c_i, s_i$ : ongoing                                                    |
| “Flavour”      | $K^- \pi^+ \pi^+ \pi^-$            | $\delta_D, R_D$  | JHEP 05 (2021) 164                             | 8 and 20 fb <sup>-1</sup> ongoing<br>$D^* \bar{D}^{(*)}$ ongoing        |
|                | $K^- \pi^+ \pi^0$                  | $\delta_D, R_D$  | JHEP 05 (2021) 164                             | 8 and 20 fb <sup>-1</sup> ongoing<br>$D^* \bar{D}^{(*)}$ ongoing        |
|                | $K^- \pi^+$                        | $\delta_D$       | EPJC 82 (2022) 1009                            | 20 fb <sup>-1</sup> ongoing<br>$D^* \bar{D}^{(*)}$ : 2506.07906         |
|                | $K_S^0 K^\pm \pi^\mp$              | $\delta_D, R_D$  | —                                              | 20 fb <sup>-1</sup> ongoing                                             |
| Quasi CP       | $\pi^+ \pi^- \pi^0, K^+ K^- \pi^0$ | $F^+ / c_i, s_i$ | —                                              | 8 fb <sup>-1</sup> PRD 111 (2025) 012007<br>20 fb <sup>-1</sup> ongoing |

- Progresses have been made at BESIII to provide strong-phase inputs to charm and bottom CPV studies
- Many analyses are ongoing and novel method have been investigated
- Larger data samples at future Super Tau-Charm Facility [Front.Phys.(Beijing) 19 (2024) 1, 14701]
  - Improved strong-phase differences are of great importance in the era of LHCb upgrade and Belle II
  - Time-integrated charm mixing and CPV studies complementary to LHCb and Belle II [arXiv:2502.08907]