Evolution of the chiral condensate in AdS/QCD with time-dependent temperature

Floriana Giannuzzi

INFN Bari, Italy

EPS-HEP 2025

Marseille, 8 July 2025

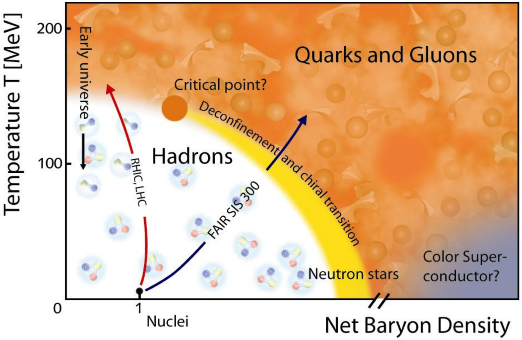
based on 2503.20726 (to appear on PRD)

Funded by the European Union
NextGenerationEU

Chiral phase transition

Phase transition restoring chiral symmetry at higher temperatures

$$\langle \bar{q} q \rangle \neq 0 \quad \rightarrow \quad \langle \bar{q} q \rangle = 0$$



- Study chiral PT at $\mu = 0$ in AdS/QCD for $n_f = 2$

Chiral condensate at finite temperature in AdS/QCD

Bottom-up approach: study QCD through an effective field theory in $5d$ AdS space

QCD operators described by fields

Ingredients:

- Metric of $5d$ AdS space with black hole
$$ds^2 = -A(z,v)dv^2 + \Sigma(z,v)^2 e^{B(z,v)} dx_\perp^2 + \Sigma(z,v)^2 e^{-2B(z,v)} dy^2 - \frac{2}{z^2} dv dz$$
Black-hole horizon at $z = z_h$
- Dilaton $\phi(z) = c^2 z^2$ breaks conformal invariance
- Field $X = e^{i\pi}(X_0 + S)e^{i\pi} \quad \Leftrightarrow \quad \bar{q}q$ operator

$\Rightarrow$ study the vev $X_0(z,v) = \dfrac{1}{2} X_q(z,v) \mathrm{diag}(1,1) \qquad$ for $n_f = 2$

The action is: [Fang *et al.*, PLB 762 (2016), 86]

$$S \propto \int d^5x\, \sqrt{-g}\, e^{-\phi(z)} \left[(\partial_M X_q)(\partial^M X_q) + m_5^2 X_q^2 + \frac{\lambda}{4} X_q^4 \right]$$

$m_5^2(z) = -3 - \mu^2 z^2$

Parameters: $c = 440$ MeV, $\mu = 1450$ MeV, $\lambda = 80$

Equation of motion:

$$\partial^M (\sqrt{-g}\, e^{-\phi(z)}\, \partial_M X_q(z,v)) - m_5^2 \sqrt{-g}\, e^{-\phi(z)}\, X_q(z,v) - \frac{\lambda}{2} \sqrt{-g}\, e^{-\phi(z)}\, X_q(z,v)^3 = 0$$

The chiral condensate $\langle \bar{q} q \rangle = \gamma\sigma$ from the expansion

$$X_q \xrightarrow[z \to 0]{} m_q\, \gamma\, z + \sigma\, z^3 + \left(c^2 m_q \gamma + (m_q \gamma)^3 \lambda - \frac{1}{2} m_q \gamma \mu_c^2 \right) z^3 \log z + \mathcal{O}(z^5)$$

At equilibrium

Metric with static black hole:

[Cao *et al.*, PRD 107 (2023), 086001]

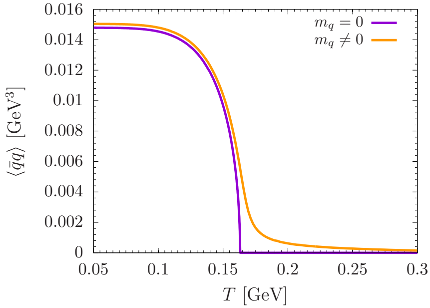
$$\Sigma(z,v) = 1/z$$
$$B(z,v) = 0$$
$$A(z) = \frac{1}{z^2}(1 - z^4/z_h^4) \qquad z_h = \frac{1}{\pi T}$$

- $m_q = 0$: 2nd order PT at $T_c \sim 163$ MeV

  $\langle \bar{q}q \rangle = (245\ \text{MeV})^3$ at $T = 0$

- $m_q = 3.22$ MeV: crossover

  $\langle \bar{q}q \rangle = (247\ \text{MeV})^3$ at $T = 0$



Out of equilibrium

Matter produced in HIC undergoes rapid expansion and cooling $\Rightarrow$ include out-of-equilibrium effects

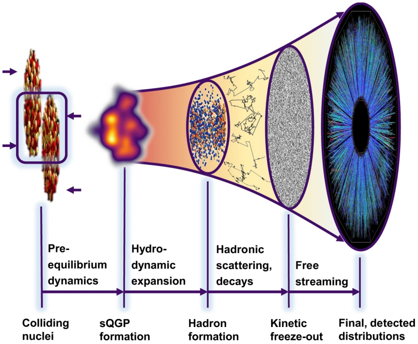


Fig. from Commun Phys 8, 55 (2025)

How does χ_{PT} change?

$\Rightarrow$ assume a time-dependent metric in the bulk and either on-shell or perturbed initial conditions

Metric functions and scalar field $X_q(z,v)$ depend on time, quark mass m_q and dilaton $\phi(z)$ are constant

Equation of motion $u = z/z_h(v)$:

$$X_q''\left(2u\dot{z}_h + Az_h^2u^2\right) + z_h\dot{X}_q\left(-3\frac{\Sigma'}{\Sigma} + \phi'\right) - 2z_h\dot{X}_q' +$$
$$+X_q'\left(6\dot{z}_h\frac{\Sigma'}{\Sigma}u - u\dot{z}_h\phi' + 2\dot{z}_h + 3u^2z_h^2A\frac{\Sigma'}{\Sigma} - u^2z_h^2A\phi' + u^2z_h^2A' + 2uz_h^2A - 3z_h\frac{\dot{\Sigma}}{\Sigma}\right) +$$
$$-\frac{m_5^2}{u^2}X_q - \frac{\lambda}{2u^2}X_q^3 = 0$$

We solve it with Chebyshev pseudospectral method

We assume two different backgrounds characterised by different $z_h(v)$

Toy model

We assume the metric functions are:

$$\Sigma(z,v) = 1/z$$

$$B(z,v) = 0$$

$$A(z,v) = 1/z^2(1 - z^4/z_h(v)^4)$$

with

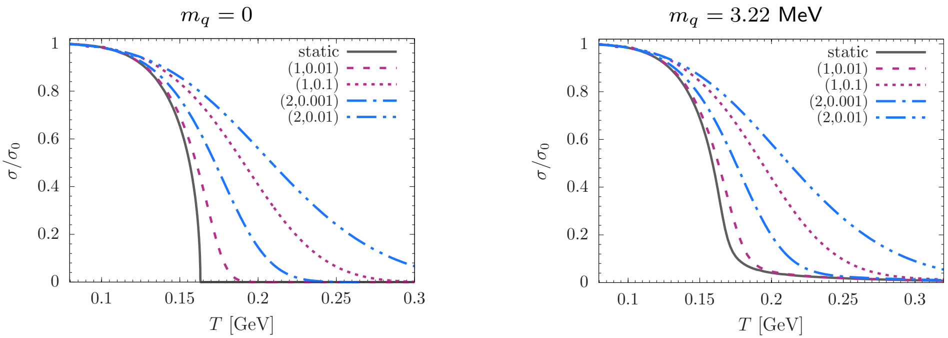
$$z_h(v)/z_h(0) = 1 \pm \Lambda_\ell \,(v/z_h(0))^a \qquad a = 1, 2$$

Define effective temperature $T(v) = \dfrac{1}{\pi\, z_h(v)}$

$$T(v)/T(0) = (1 \pm \Lambda_\ell \,(v/z_h(0))^a)^{-1}$$

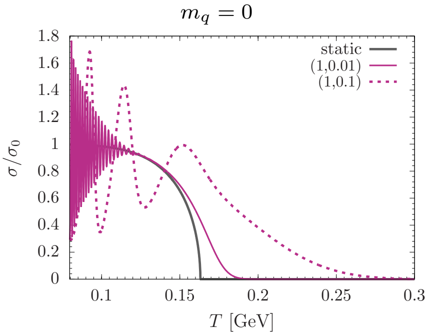
$\Rightarrow$ Rapid variation for large Λ_ℓ

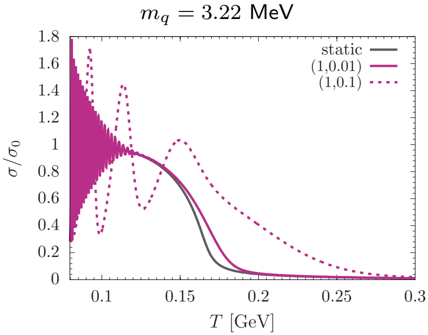
Increasing temperature (—), starting from equilibrium solution at $T(0) \sim 80$ MeV



- In both cases chiral condensates smoothly goes to zero
- The curves deviate more from the equilibrium one as a and Λ_ℓ increase

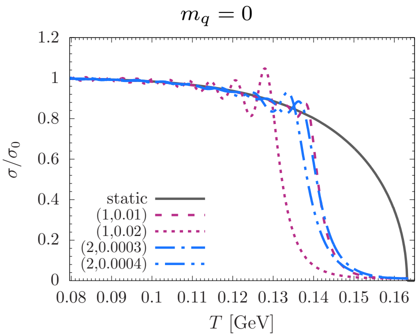
Increasing temperature (−), perturbed initial condition at $T(0) \sim 80$ MeV with $\sigma = 0.016$ GeV^3

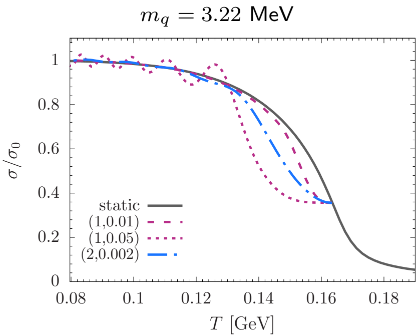




- Oscillations disappear when the system approaches the equibrium curve
- For smaller Λ_ℓ the oscillations are faster and the system reaches equilibrium at lower temperatures

Decreasing temperature (+), starting from equilibrium solution at $T(0) \sim T_c$

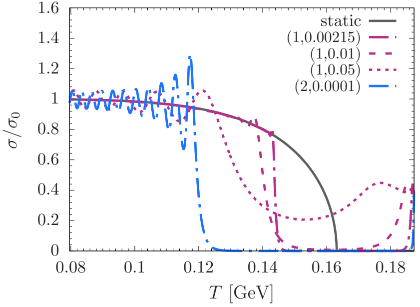




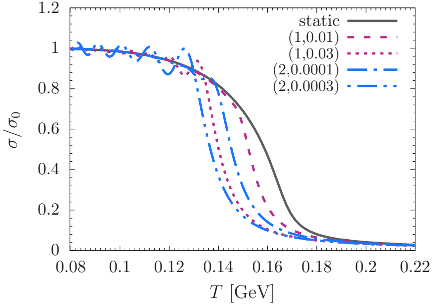
- χ_{SB} at $T < T_c$, which increases with decreasing Λ_ℓ and a
- At the beginning the system tends to stay close to the initial condition, then σ starts growing, running after the static curve
- Eventually, when the dynamical solution approaches the static one, oscillations can occur

Decreasing temperature $(+)$, perturbed initial condition

$m_q = 0$
IC at $T(0) \sim 187$ MeV with $\sigma = 0.02$ GeV^3



$m_q = 3.22$ MeV
IC at $T(0) \sim 320$ MeV with $\sigma = 0$



- Bigger differences in the chiral limit
- In the chiral limit, if the initial condition is $X_q = 0$, the solution vanishes for any subsequent time

Viscous hydrodynamics background

Choose a $5d$ metric that gives the $4d$ EMT obtained in viscous hydrodynamics through Bjorken expansion:

$T^{\mu}_{\nu} = \frac{N_c^2}{2\pi^2}\mathrm{diag}(-\epsilon, p_\perp, p_\perp, p_{||})$ with

$$
\begin{aligned}
\epsilon(v) &= \frac{3\pi^4\Lambda^4}{4(\Lambda v)^{4/3}}\left[1 - \frac{2c_1}{(\Lambda v)^{2/3}} + \frac{c_2}{(\Lambda v)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda v)^2}\right)\right] \\
p_{\|}(v) &= \frac{\pi^4\Lambda^4}{4(\Lambda v)^{4/3}}\left[1 - \frac{6c_1}{(\Lambda v)^{2/3}} + \frac{5c_2}{(\Lambda v)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda v)^2}\right)\right] \\
p_{\perp}(v) &= \frac{\pi^4\Lambda^4}{4(\Lambda v)^{4/3}}\left[1 - \frac{c_2}{(\Lambda v)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda v)^2}\right)\right]
\end{aligned}
$$

$c_1 = \frac{1}{3\pi}$, $c_2 = \frac{1 + 2\log 2}{18\pi^2}$

[Janik and Peschanski, PRD 73 (2006) 045013]

Effective temperature from $\epsilon(v) = \frac{3}{4}\pi^4 T(v)^4$

$$
T(v) = \frac{\Lambda}{(\Lambda v)^{1/3}}\left[1 - \frac{1}{6\pi(\Lambda v)^{2/3}} + \frac{-1 + \log 2}{36\pi^2(\Lambda v)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda v)^2}\right)\right]
$$

The metric that produces this EMT is:

$$\begin{aligned} A(z,v) &= \frac{1}{z^2}\left(1-\frac{4z^4}{3}\epsilon(v)\right) \\ \Sigma(z,v) &= \frac{1}{z}\left(v+z\right)^{1/3} \\ B(z,v) &= \frac{z^4}{3}(p_\perp(v)-p_{||}(v))-\frac{2}{3}\log(v+z) \end{aligned}$$

[de Haro *et al.*, Commun. Math. Phys. 217 (2001) 595-622,
Bellantuono *et al.*, PRD 94 (2016), 025005]

BH horizon at $A(z,v)=0$:

$$z_h(v)=\frac{(\Lambda v)^{1/3}}{\pi\Lambda}\left[1+\frac{c_1}{2(\Lambda v)^{2/3}}+\frac{5c_1^2-2c_2}{8(\Lambda v)^{4/3}}+\mathcal{O}\left(\frac{1}{(\Lambda v)^2}\right)\right]$$

Gauge/gravity duality used to study the thermalization of the system driven far from equilibrium by boundary sourcing

[Chesler and Yaffe, PRL 102 (2009) 211601]

Considering different quenches, found:

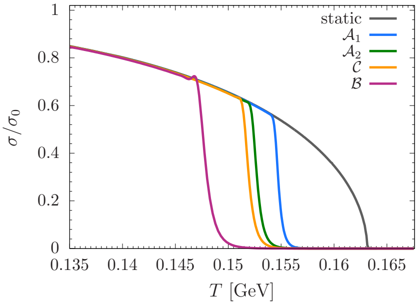
- $\Lambda = 2.25$ GeV (model $\mathcal{A}_1$)
- $\Lambda = 1.73$ GeV (model $\mathcal{A}_2$)
- $\Lambda = 1.12$ GeV (model $\mathcal{B}$)
- $\Lambda = 1.59$ GeV (model $\mathcal{C}$)

[Bellantuono *et al.*, JHEP 07 (2015) 053]

$T(v)$ varies more slowly with time when Λ is larger

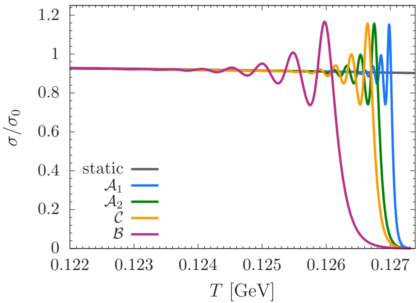
$m_q = 0$

Initial condition: $X_q = 10^{-5}u^3$ at $T \sim 168$ MeV



$m_q = 0$

Initial condition: $X_q = 0.001u^3$ at $T = 127.3$ MeV

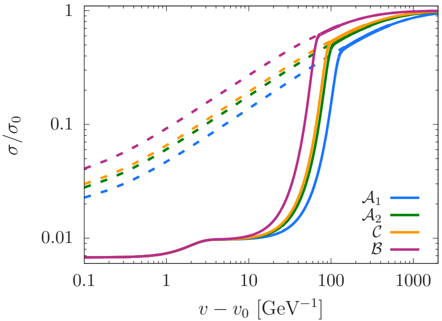


- If a fluctuation of X_q occurs at $T \gtrsim T_c$, thermalization and transition to a chirally broken phase at $T < T_c$
- If a fluctuation of X_q occurs at a much lower temperature, short oscillations are observed before equilibrium
- In model $\mathcal{B}$, which has the smallest Λ, equilibrium reached at lower temperatures

Prethermalization

Transient quasistationary state different from the true thermal equilibrium

- We observe prethermalization in chiral limit if $T(0) \sim T_c$ and if it slowly changes with time
- Initial condition: $X_q(u, v_0) = 10^{-4} u^3$
- Model $\mathcal{B}$ (smallest Λ) has the shortest prethermalization stage
- Equilibrium is eventually reached



Duration of prethermalization depends on how fast temperature changes with time and how much the initial condition deviates from the equilibrium one

Conclusions

We have studied the chiral condensate at finite temperature in AdS/QCD in a time-dependent background

- Model:
  - Two different scenarios: power-law time dependence of $z_h(v)$ and the one foreseen by viscous hydrodynamics
  - Two different initial conditions: equilibrium solution and a perturbed profile
- Results:
  - Deviation from equilibrium increases as the rate of temperature change over time becomes more pronounced
  - In the chiral limit, if the chiral condensate initially vanishes, the transition does not occur
  - At low temperatures the chiral condensate oscillates around the equilibrium value when the solution approaches the static one with an energy excess
  - A prethermalization stage found in the chiral limit if initially $T \sim T_c$