On-shell amplitude approach to spinning binaries in GR and beyond

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In collaboration with Adam Falkowski:
[2407.16457], [2411.12909],
[2507.XXXXX] (also with Edoardo Alviani)

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Image credit: Mark Garlick/Science Photo Library/Getty Images

Plan for this talk:

1. Motivation
2. Scattering Amplitudes and Observables
3. Scalar-tensor theories
4. Waveforms for scattering of compact objects with scalar-hair **[2411.12909]**
5. Waveforms for scattering of compact objects in shift-symmetric scalar-tensor theories **[2407.16457]**
6. Outlook

1. Motivation

Image Credit: EGO*

→ LIGO/VIRGO collaboration: First detection of **Gravitational Waves** (GWs) in **2015**

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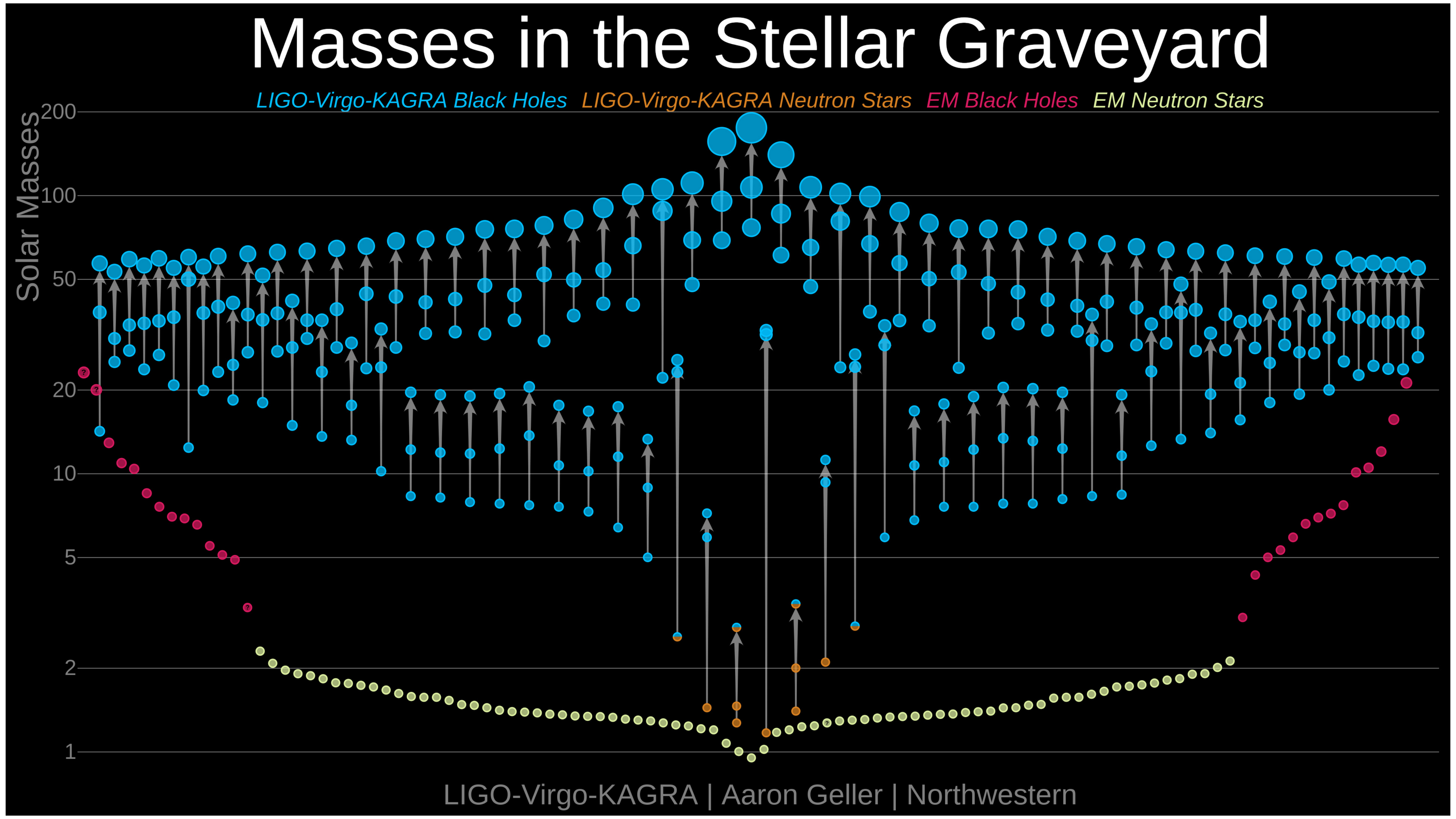
Has since then inspired an unprecedented interest in GW detection, especially with the upcoming **new generation of GW interferometers** (ET, Cosmic Explorer, LISA)



Credit:LIGO-Virgo/Aaron Geller/Northwestern University*

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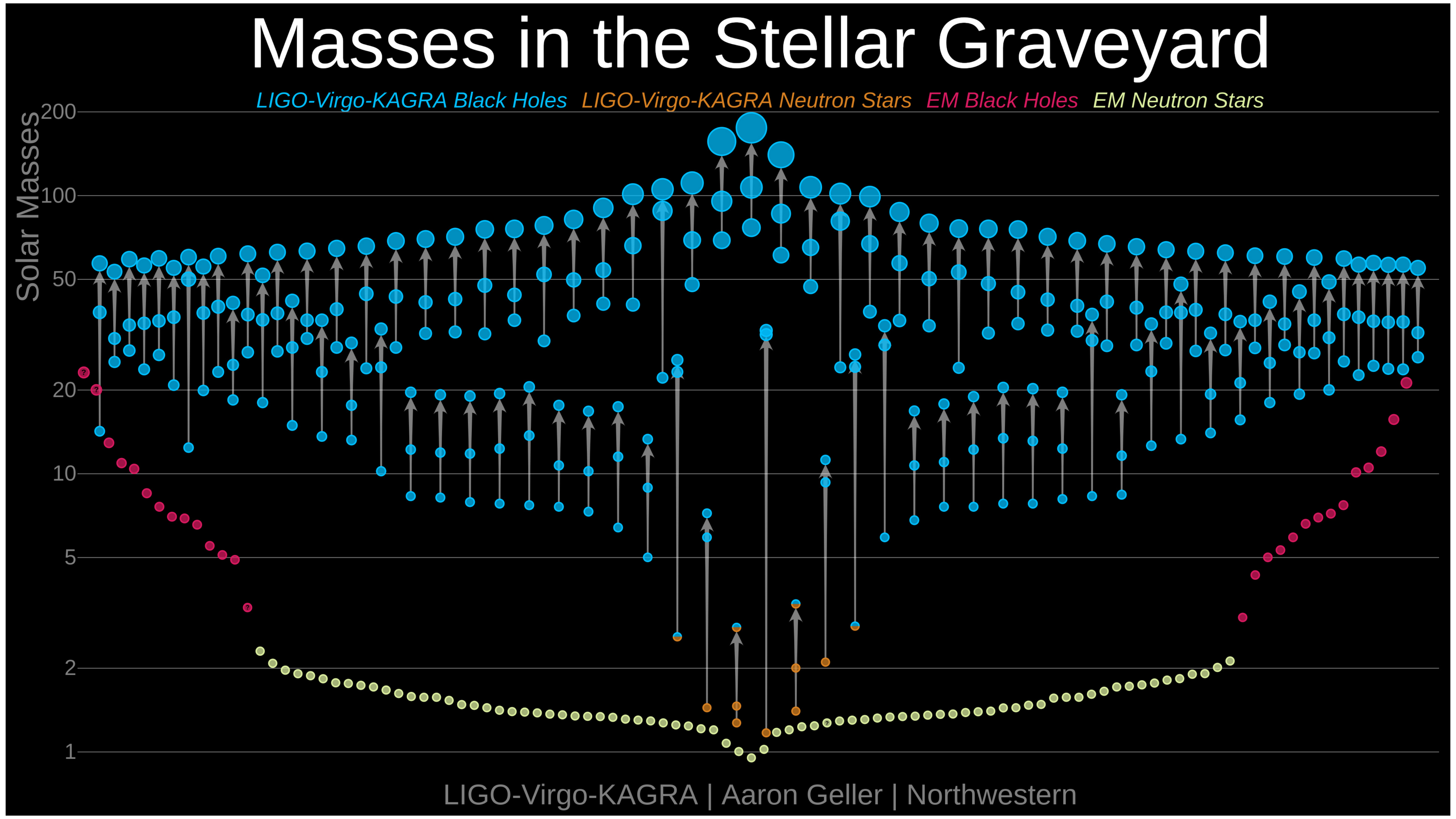
New era of **high precision** measurements of GWs:

Highly accurate GW templates

New window to test General Relativity (GR)



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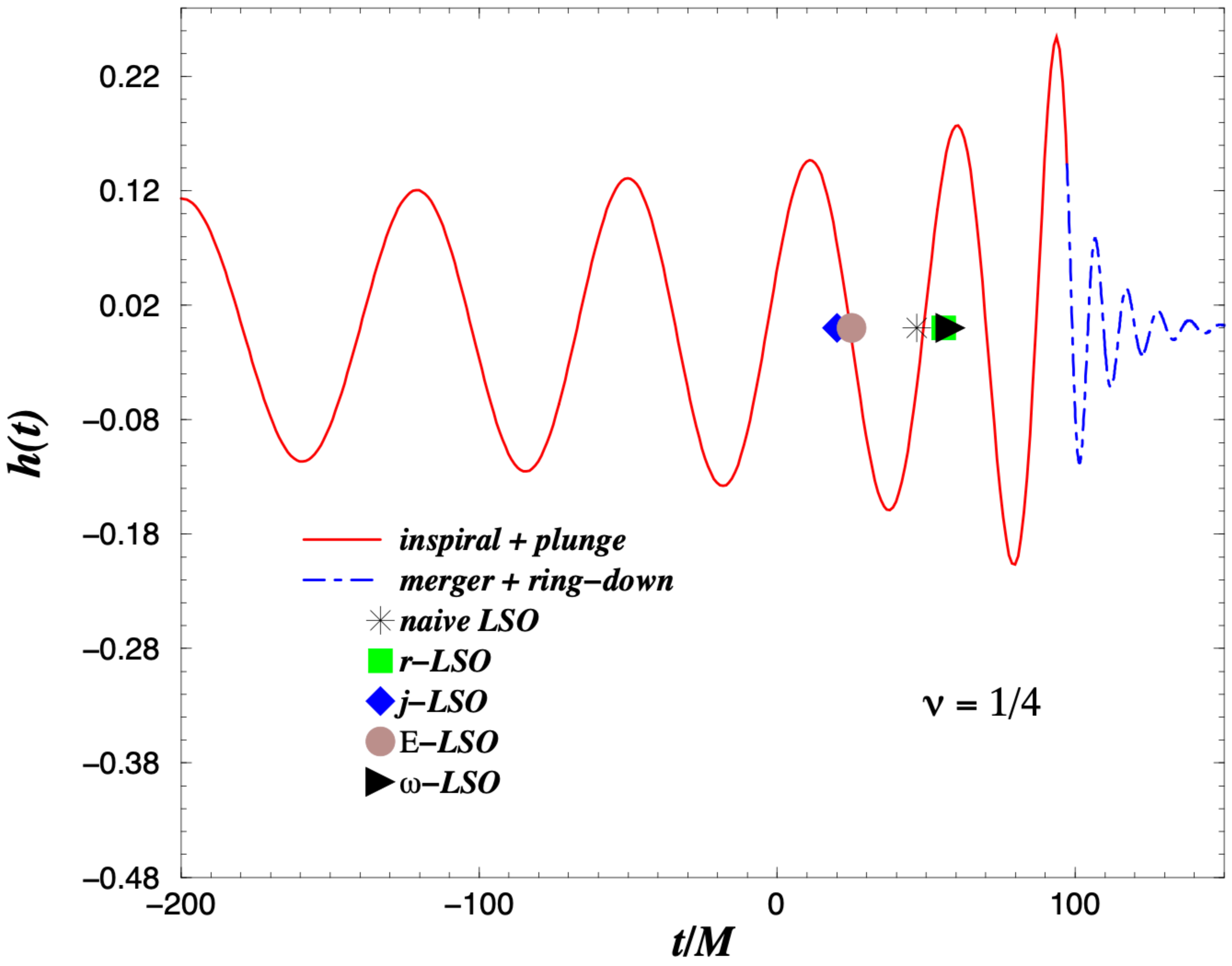
The phases of the binary problem:

Phys.Rev.D 62 (2000) 064015 [Buonanno, Damour]*



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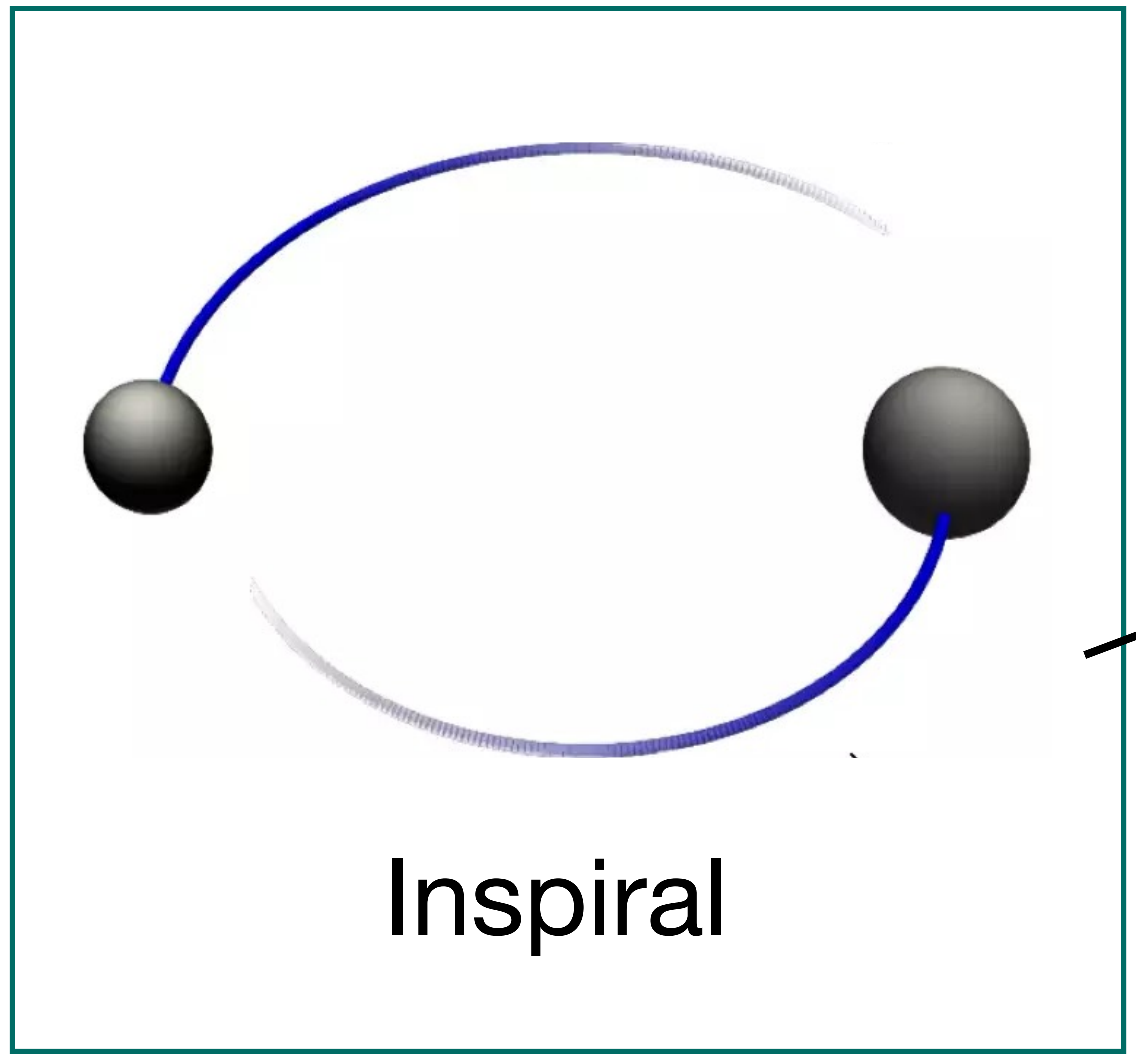
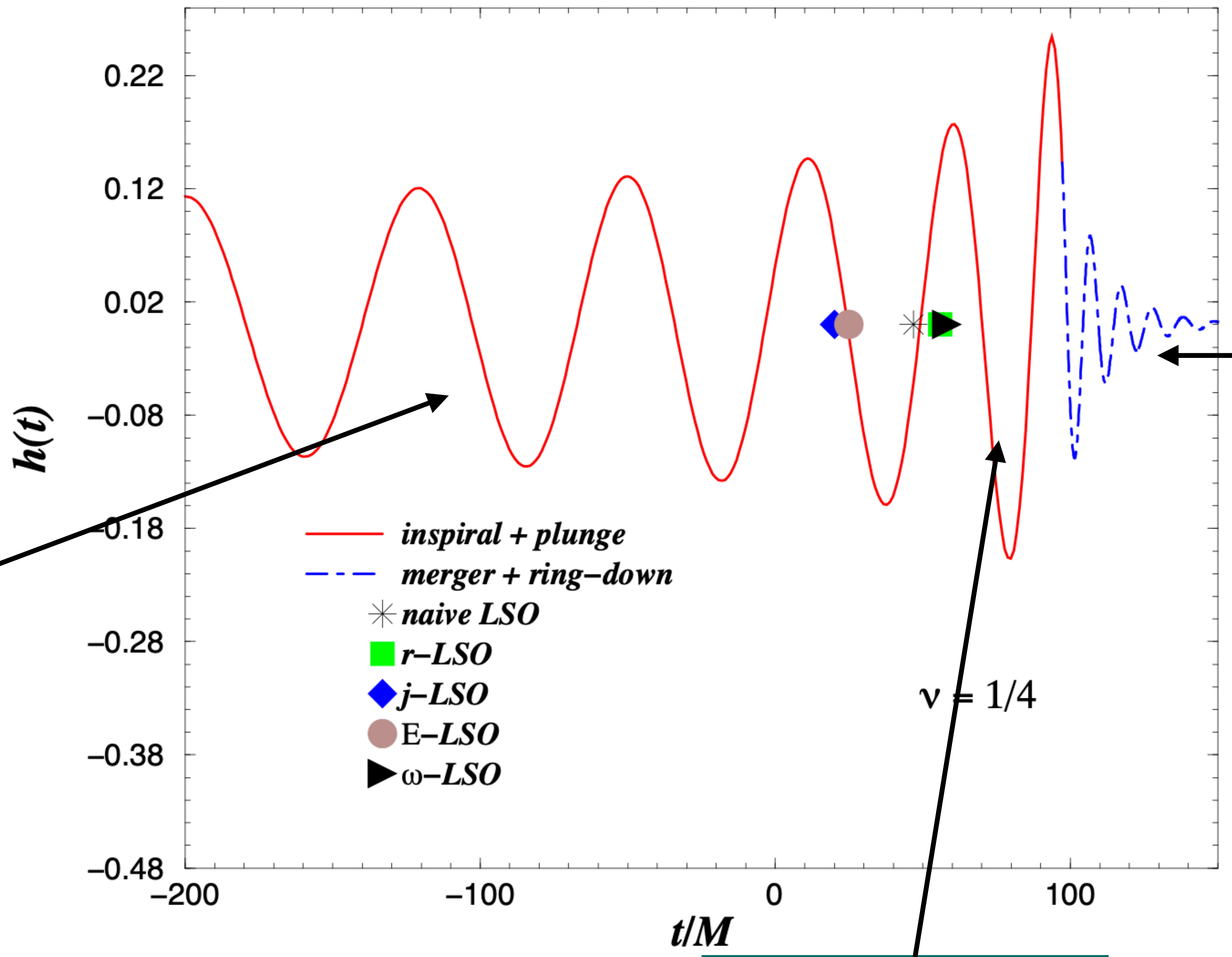
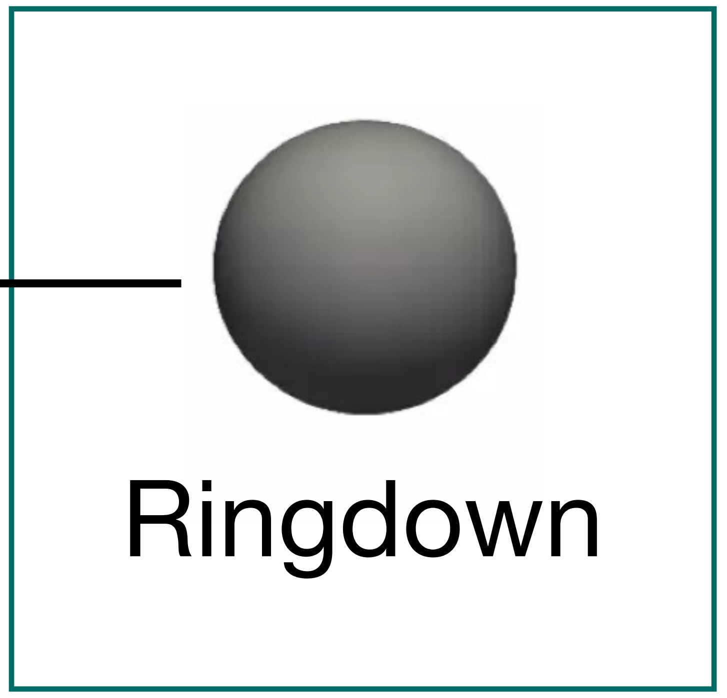
Analytical approaches



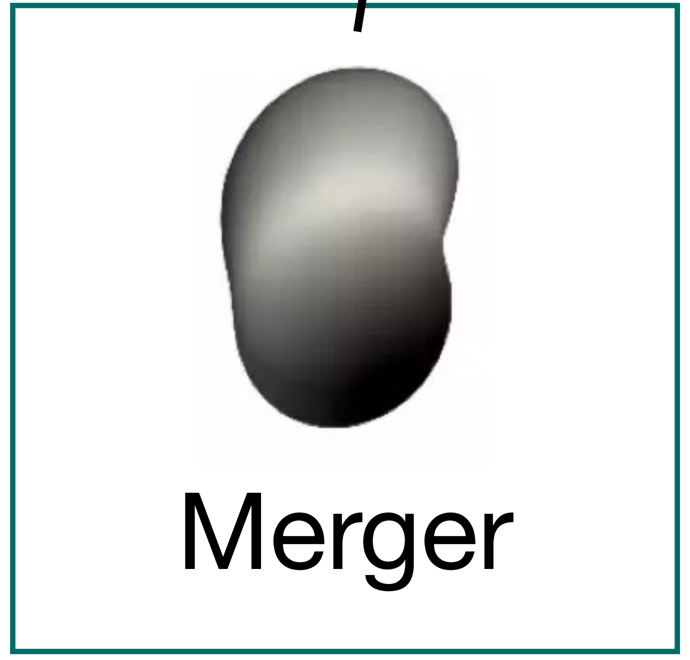
Focus specifically on PM expansion; Expansion around weak gravitational fields, $G \ll 1$

Powerful approach, containing information at all $\frac{v}{c}$ orders, compared to PN expansion.





Black Hole Perturbation Theory (BHPT)

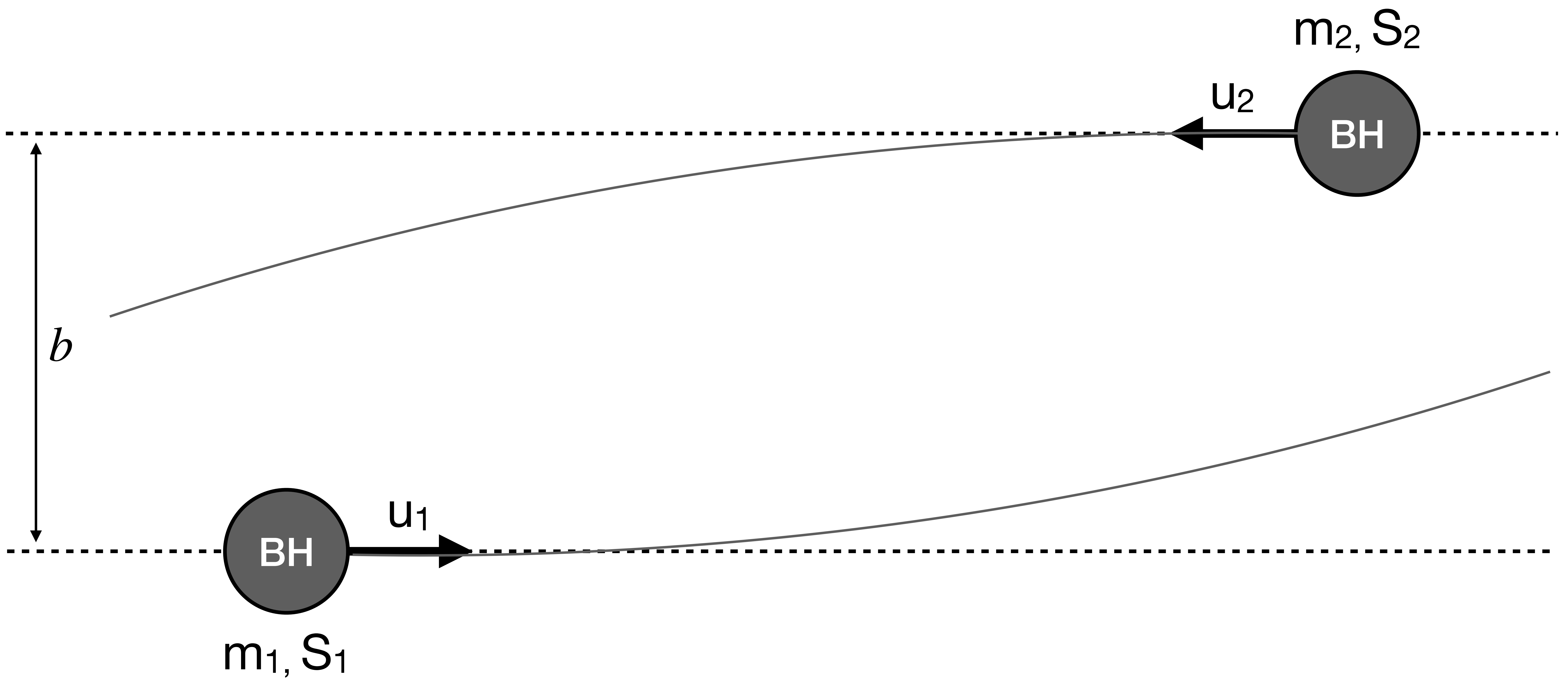


Numerical Relativity

Scattering information can be used for the **bound** problem!

Illustration Credit: Max-Planck-Gesellschaft*

2. Scattering Amplitudes and Observables



Weak field expansion:

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_{Pl}} h_{\mu\nu}$$

PM expansion:

$$R_s \ll b$$

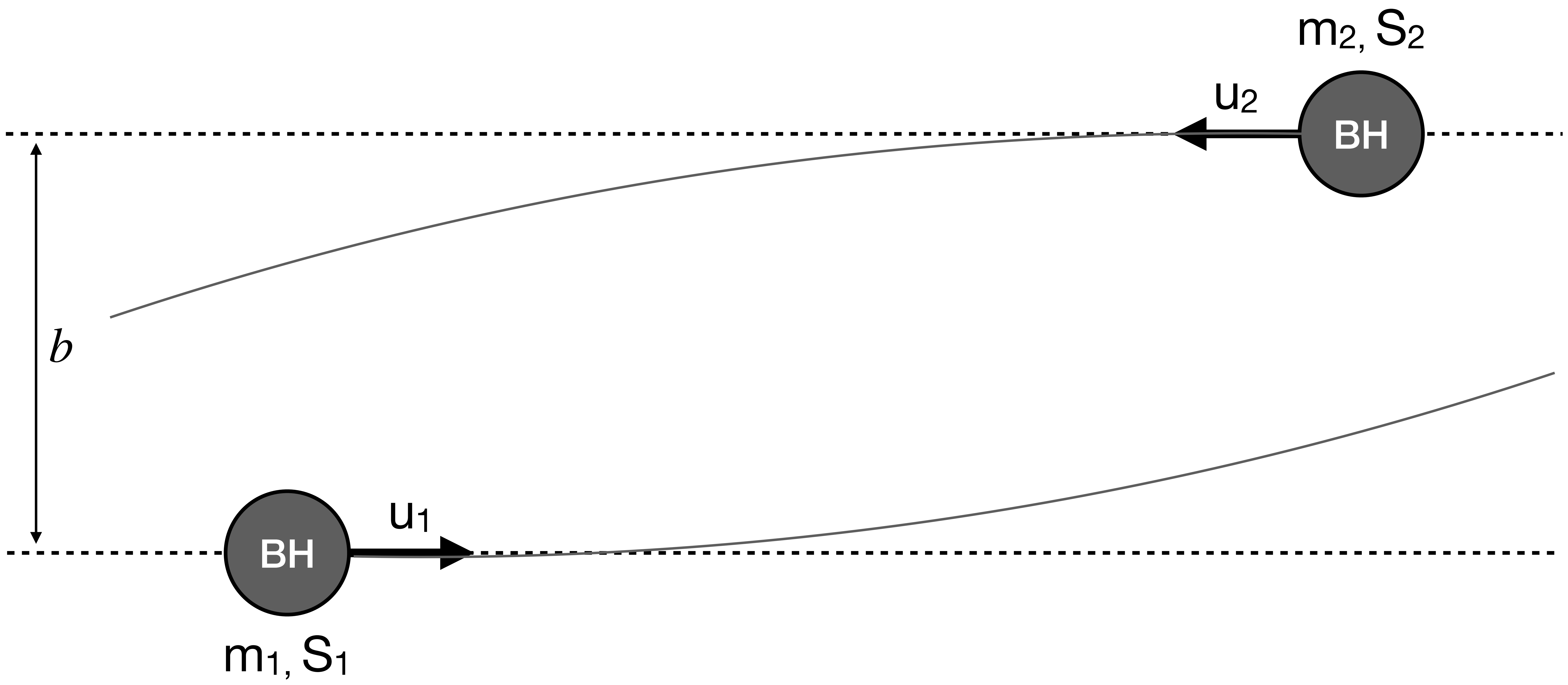
Spin expansion:

$$\frac{S}{m} \ll b$$

- **Focus**: **Classical scattering problem in GR**

→ Can we describe this problem using the Scattering Amplitudes used in QFT?

2. Scattering Amplitudes and Observables



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- **Focus:** **Classical scattering problem in GR**

⟶ Can we describe this problem using the Scattering Amplitudes used in QFT?

Yes! ⟶ Use of the **KMOC formalism**

Waveforms at leading order:

JHEP 02 (2019) 137 [Kosower, Maybee, O'Connell]
Phys.Rev.D 106 (2022) 5, 0567007 [Cristofoli, Gonzo, Kosower, O'Connell]

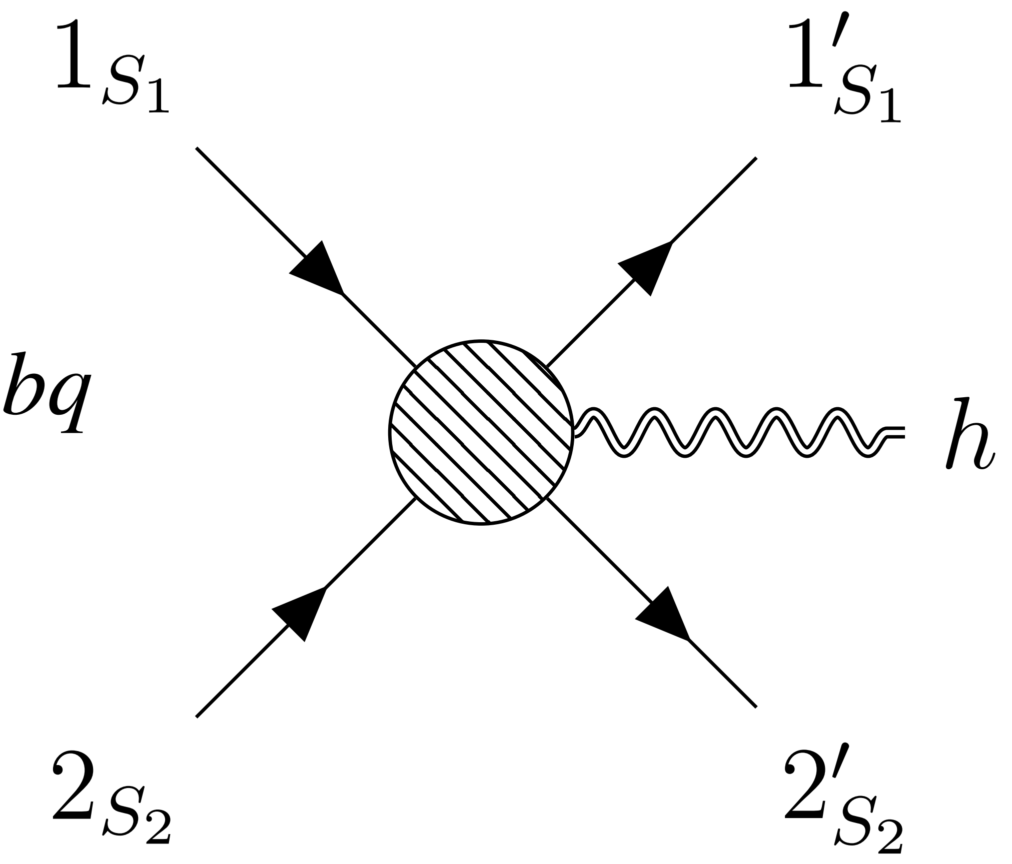
retarded time

$$R_{GR}(t) \sim \frac{h_{GR}(t)}{|\boldsymbol{x}|}, \quad |\boldsymbol{x}| \to \infty, \quad t \equiv x^0 - |\boldsymbol{x}|, \qquad R_{GR} \equiv {}_{\text{out}}\langle \psi | h^{\mu\nu}(x) | \psi \rangle_{\text{out}} \epsilon^{-}_{\mu\nu}$$

strain

on-shell measure

$$h_{GR}(t) \equiv h_+ \pm i h_\times \sim \int d\omega e^{-i\omega t} \int d\Phi(q) e^{-ibq}$$



$$\underbrace{\hphantom{1_{S_1} \quad\quad\quad 2'_{S_2}}}_{\mathcal{M}_5^{\text{cl}}(q,k)\,|_{k^\mu=\omega n^\mu}}$$

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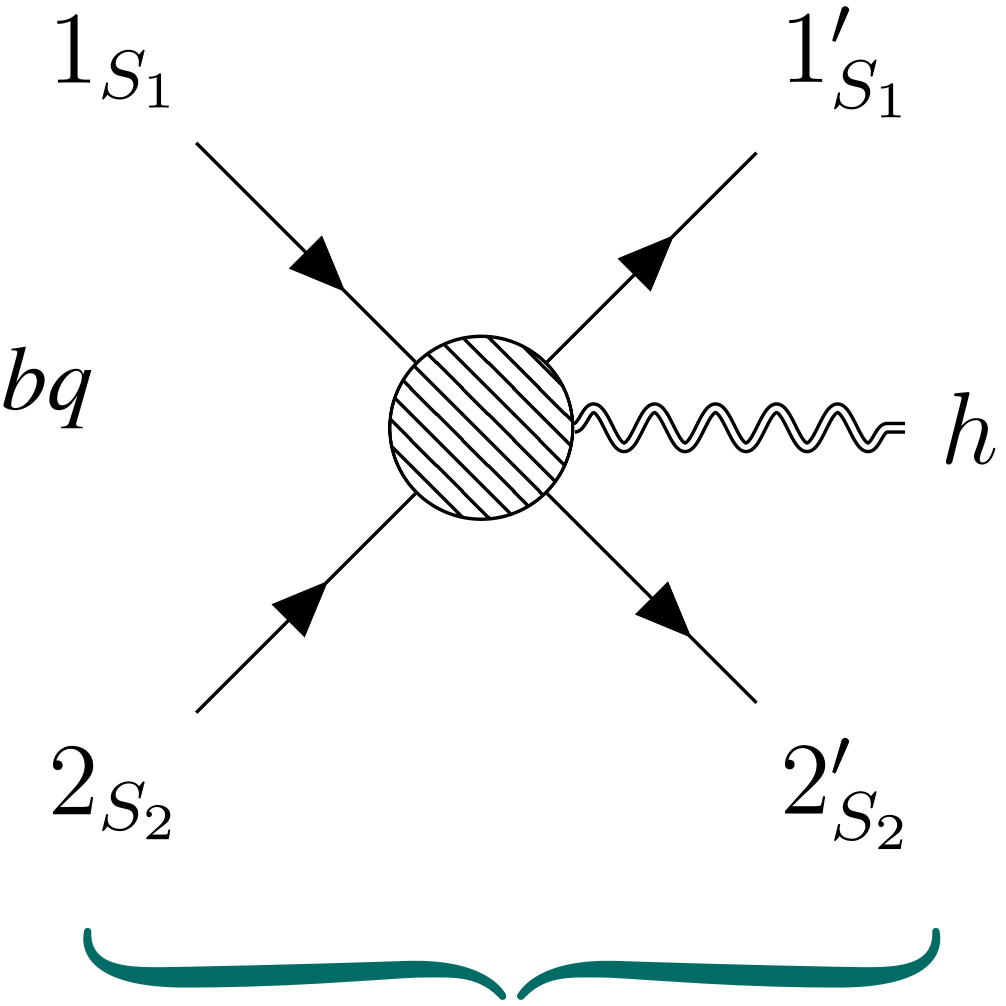

$$\mathcal{M}_5^{\text{cl}}(q,k)\,|_{k^\mu = \omega n^\mu}$$

But, **why?**

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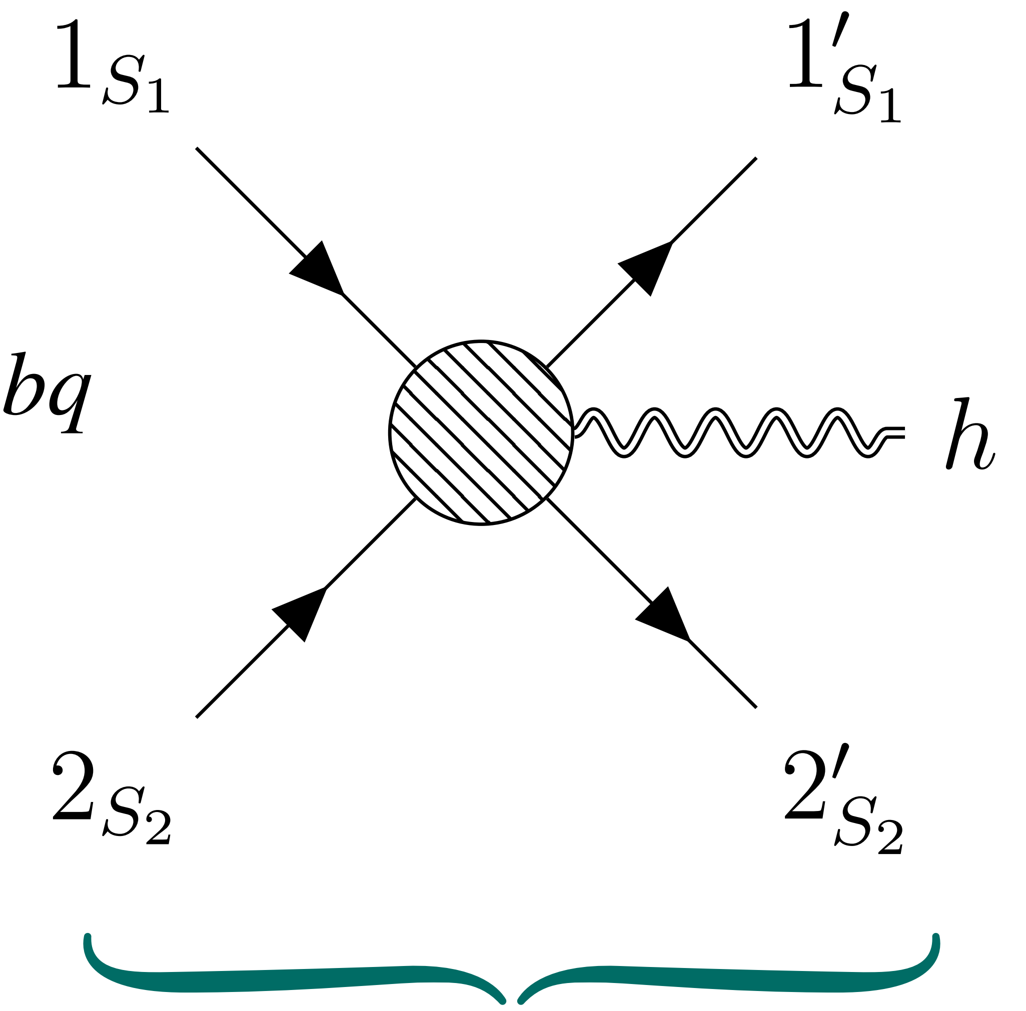

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But, **why?**

- Computations organized in a **perturbative expansion** using a **simple algorithm**.
- **Analytic results**, in places where either PN approximations or NR was used before.
- Can exploit many **modern techniques used in particle physics** to simplify the problem.
- Can straightforwardly **extend to beyond GR predictions**

On-shell techniques:

Basic QFT Principles: Poincaré invariance, locality, unitarity of the Scattering Matrix

QFT assumptions: Introduction of auxiliary fields, Lagrangians, gauge invariance, Feynman rules…

Scattering Amplitude:

$$\mathscr{M}^{h_1,\ldots,h_m}(p_1,\ldots,p_n)$$

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On-shell techniques

Scattering Amplitude:
$\mathscr{M}^{h_1,\ldots,h_m}(p_1,\ldots,p_n)$

Use spinors as variables instead of momenta

$$\left.\begin{array}{l} \text{Massless spinors: } p_\mu \sigma^\mu_{\alpha\dot\alpha} = \lambda_\alpha \tilde\lambda_{\dot\alpha} \\ \\ \text{Massive spinors: } p_\mu \sigma^\mu_{\alpha\dot\alpha} = \varepsilon_{IJ}\chi^I_\alpha \tilde\chi^J_{\dot\alpha} \end{array}\right\} \text{Obey little group transformation rules}$$

$\langle ij\rangle = \varepsilon_{\alpha\beta}\lambda_i^\alpha \lambda_j^\beta$, $[ij] = \varepsilon_{\dot\alpha\dot\beta}\tilde\lambda_i^{\dot\alpha}\tilde\lambda_j^{\dot\beta}$

$\langle \mathbf{ij}\rangle = \langle i^I j^J\rangle$, $[\mathbf{ij}] = [i^I j^J]$

Example: Leading order GR waveform

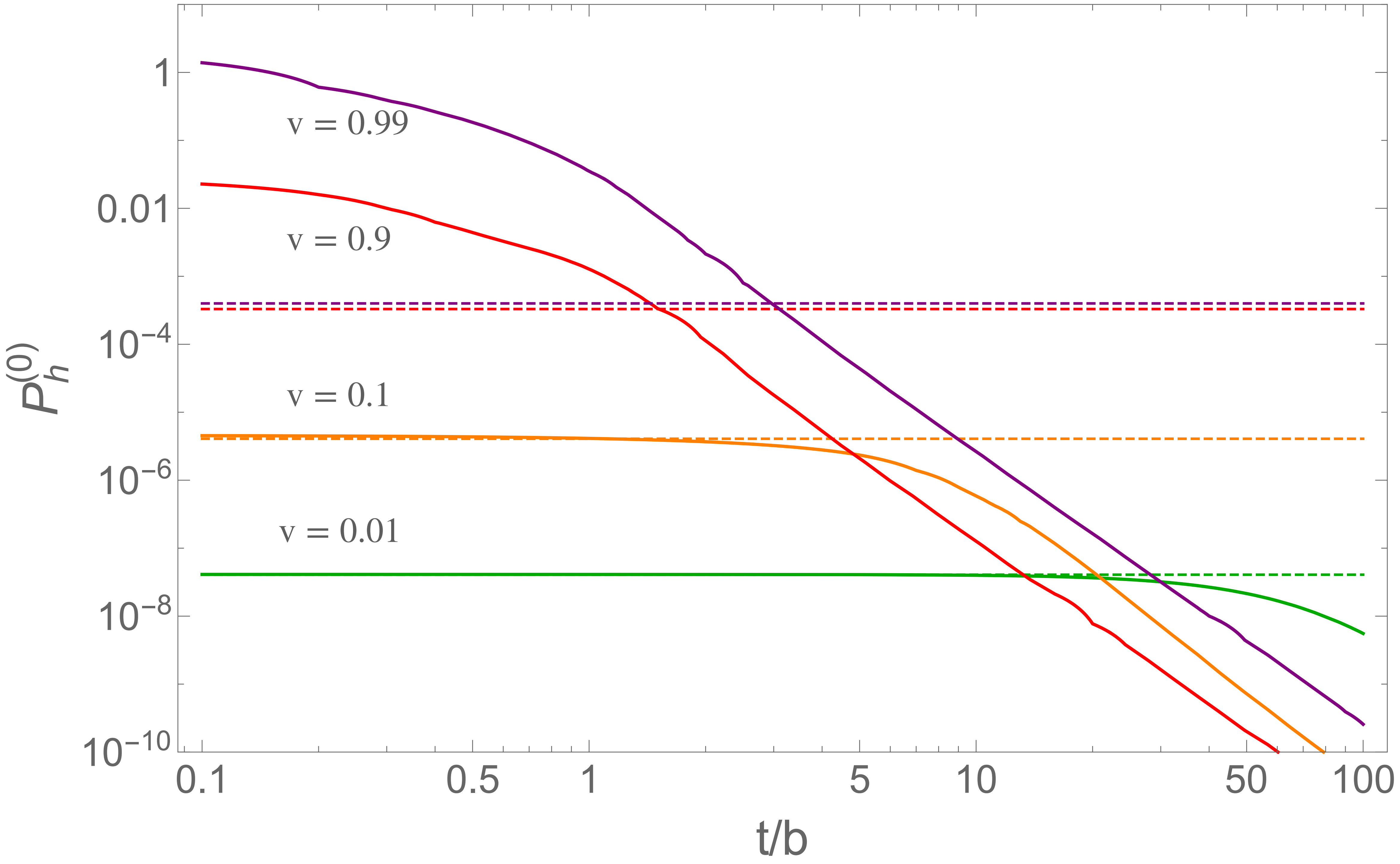
See works by [De Angelis, Gonzo, Novichkov+ Jacobsen Mogull, Plefka, Steinhoff+Aoude, Haddad, Heissenberg, Helset+Brandhuber, Brown, Chen, Gowdy, Travaglini]

Eur.Phys.J.C 85 (2025) 1, 74 [Falkowski, **PM**]

Waveform in GR calculated as expansion in spin: $W_h = W_h^{(0)} + W_h^{(1)} + \dots$

<u>At leading order</u> : $$W_h^{(0)} = -\frac{m_1 m_2}{512\pi^2 M_{\text{Pl}}^3 b(\hat{u}_1 n)^2\sqrt{\gamma^2-1}}\frac{1}{\sqrt{z^2+1}}\mathcal{R}\left\{\frac{\left(\mathcal{F}_1^-(z)\right)^2+\left(\mathcal{F}_2^-(z)\right)^2}{\gamma(\hat{u}_2 n)-(\hat{u}_1 n)+z(\tilde{b}n)+i\sqrt{z^2+1}(\tilde{v}n)}\right\}\Bigg|_{z=T_1} + (1\leftrightarrow 2)$$

Emitted power in gravitational waves: $\frac{dP_h}{d\Omega} = 2\,|\,\partial_t W_h|^2$



$$\mathcal{F}_1^-(z) = \langle n\,|\left(\hat{u}_1\hat{u}_2 + 2\gamma z\hat{u}_1\tilde{b} + z\tilde{b}\hat{u}_2 + 2i\gamma\sqrt{z^2+1}\,\hat{u}_1\tilde{v} + i\sqrt{z^2+1}\,\tilde{v}\hat{u}_2\right)|\,n\rangle$$

$$\mathcal{F}_2^-(z) = \langle n\,|\left(\hat{u}_1 - z\tilde{b} - i\sqrt{z^2+1}\,\tilde{v}\right)\hat{u}_2\,|\,n\rangle$$

$$\partial_t W_h^{(0)} = -\,e^{-2i\phi}\frac{m_1 m_2}{16\pi^2 M_{\text{Pl}}^3 b^2}\left\{(\hat{\boldsymbol{v}}\hat{\boldsymbol{n}})(\hat{\boldsymbol{b}}\hat{\boldsymbol{n}}) + i(\hat{\boldsymbol{v}}\times\hat{\boldsymbol{b}})\cdot\hat{\boldsymbol{n}}\right\}v + O(v^2)$$

velocity suppression

$$T_i \equiv \frac{t - b_i n}{(\hat{u}_i n)b} \qquad \hat{u}_i = \frac{u_i}{\sqrt{\gamma^2-1}} \qquad \gamma = u_1 u_2 = \frac{1}{\sqrt{1-v^2}}$$

$$v^\mu \equiv \varepsilon^{\mu\alpha\beta\rho}u_{1\alpha}u_{2\beta}\tilde{b}_\rho \qquad \tilde{v}^\mu = \frac{v^\mu}{\sqrt{-v^2}}$$

$$b \equiv \sqrt{-(b_1-b_2)^2} \qquad \tilde{b}^\mu = \frac{b_1^\mu - b_2^\mu}{b}$$

3. Scalar-tensor theories

- **Scalar-tensor theories** have long stood as a promising avenue to study **extensions of GR**
- They consist of gravity theories with the introduction of an additional **massless scalar** degree of freedom

$$\boxed{S_{GR}[g_{\mu\nu}] \rightarrow S_{ST}[g_{\mu\nu}, \phi]}$$

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Example: Scalar Gauss-Bonnet and Dynamical Chern Simons gravity

$$S = S_{EH}[R] + S_{SGB,DCS}[\phi, g_{\mu\nu}] + S_m[\Psi_m, \mathscr{A}(\phi)g_{\mu\nu}] \quad ,$$

- $S_{EH} = \int d^4x \frac{M_{Pl}^2}{2}\sqrt{-g}R$
- $S_{SGB,DCS} = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{\Lambda^2}\left(f(\phi)\mathscr{G} + \tilde{f}(\phi)R\tilde{R}\right) + \frac{1}{2}\left(\partial^\mu\phi\partial_\mu\phi\right)\right]$

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$$\boxed{R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2}$$

$$\boxed{R^{\mu\nu\rho\sigma}\tilde{R}_{\mu\nu\rho\sigma} \quad , \quad \tilde{R}^{\mu}{}_{\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\rho\sigma}{}^{\alpha\beta}R^{\mu}{}_{\nu\alpha\beta}}$$

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Broad window of resolvability! *Eur.Phys.J.C* 85 (2025) 1, 74 [Falkowski, **PM**]

Phys.Rev.D 107 (2023) 4, 044030 [Silva, Ghosh, Buonanno]
arXiv: 2406.13654 [Julié, Pompili, Buonanno]
Phys.Rev.Lett. 126 (2021) 18, 181101 [Silva, Holgado, Cárdenas-Avendaño, Yunes]

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Already GW observations are used to constrain them!!!

$$\boxed{R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2}$$

$$\boxed{R^{\mu\nu\rho\sigma}\tilde{R}_{\mu\nu\rho\sigma} \quad , \quad \tilde{R}^{\mu}{}_{\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\rho\sigma}{}^{\alpha\beta}R^{\mu}{}_{\nu\alpha\beta}}$$

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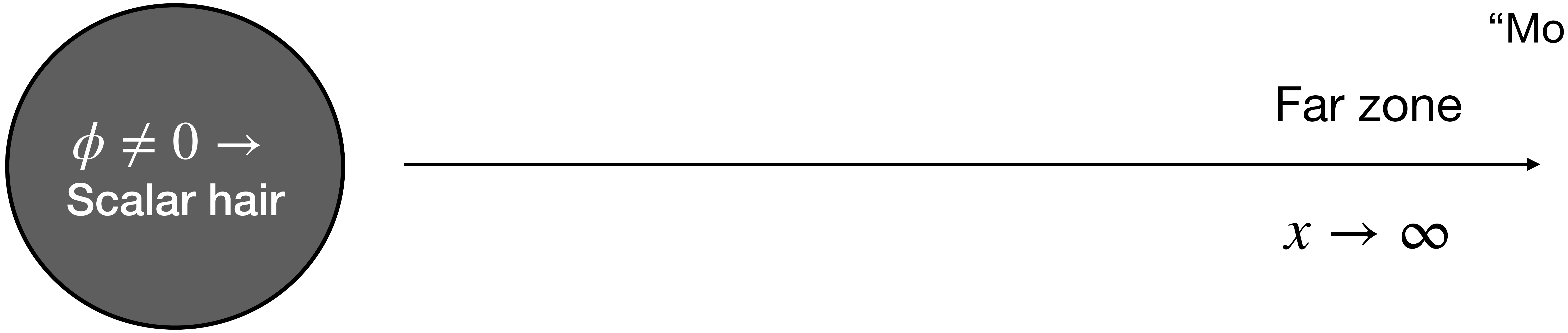
Experimental window:

$$\frac{\sqrt{\alpha}}{\Lambda} \lesssim 0.22km \quad , \quad \frac{\sqrt{\tilde{\alpha}}}{\Lambda} \lesssim 9.5km$$

Scalar hair in scalar-tensor theories:

Compact objects can acquire scalar hair in ST theories ⟶ Exactly the case for SGB and DCS!

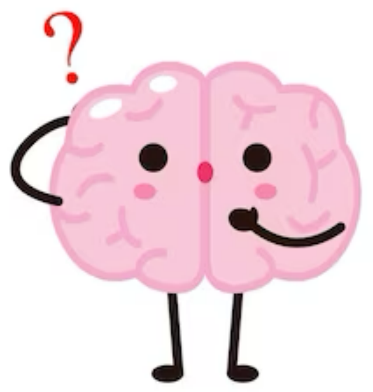


BH solution in ST theory

“Monopole hair” “Dipole hair”

$$\phi = \frac{\boxed{c_i}}{r} + \frac{\boxed{d_i}}{r^2} + \dots$$

How can we model this behaviour with amplitudes?



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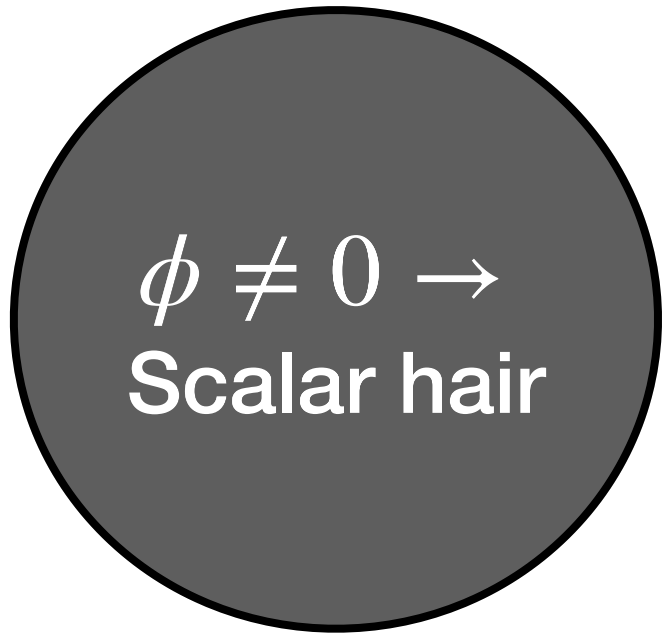

BH solution in ST theory

$$\xrightarrow[x \to \infty]{\text{Far zone}} \qquad \phi = \frac{\overset{\text{“Monopole hair”}}{c_i}}{r} + \frac{\overset{\text{“Dipole hair”}}{d_i}}{r^2} + \dots$$

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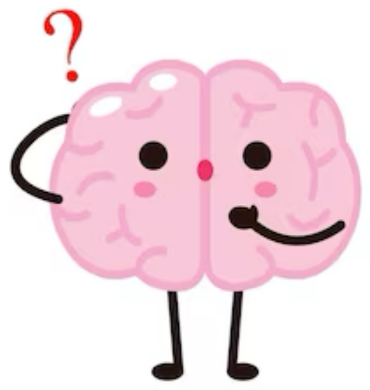


We model the BH as a point-particle interacting with the scalar field in a ST fashion

Neglect it, heavily suppressed

Most general effective metric that respects causality is:

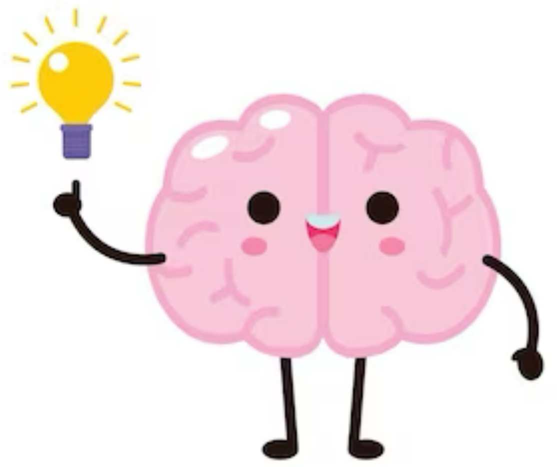
$$\tilde{g}_{\mu\nu} = \underbrace{\exp\Big[C\big(\frac{\phi}{M_{Pl}}\big)\Big] g_{\mu\nu}}_{\text{Conformal coupling}} + \underbrace{\cancel{D\big(\frac{\phi}{M_{Pl}}\big)\frac{D_\mu\phi D_\nu\phi}{M_{Pl}^2\Lambda^2}}}_{\text{Disformal coupling}},$$

Generate 3-point interaction for arbitrary spinning BH:

$$\boxed{\exp\Big[C\big(\frac{\phi}{M_{Pl}}\big)\Big] \approx 1 + c\frac{\phi}{M_{Pl}}}$$

Simple mass redefinition
at all spin orders:

$$m \rightarrow e^{C/2} m$$



4. Waveforms for scattering of compact objects with scalar-hair

LO Scalar Waveforms-Spinless part:

$$W_\phi^{(0)} = -\frac{m_1 m_2}{32\pi^2 M_{Pl}^3\sqrt{\gamma^2-1}(\hat{u}_1 n)^2 b}\frac{1}{\sqrt{1+T_1^2}}\left\{\frac{(\gamma^2-1)[c_1(\hat{u}_2 n)^2 + c_2(\hat{u}_1 n)^2][\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n)T_1]}{[-(\hat{u}_1 n) + \gamma(\hat{u}_2 n) + (\tilde{b} n)T_1]^2 + (\tilde{v} n)^2(1+T_1^2)}\right.$$

$$\left. -\frac{c_1}{2}\frac{(\hat{u}_1 n) + (2\gamma^2-3)y(\hat{u}_2 n) - (2\gamma^2-1)(\tilde{b} n)T_1}{\gamma^2-1} + \frac{C_1^{(0)}}{2}c_2(\hat{u}_1 n)\right\} + (1 \leftrightarrow 2)\,.$$

$v \ll 1$ limit

$$\left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \frac{1}{b^4} \xrightarrow{\boxed{\text{Kepler's law}}} \left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim v^8 \longrightarrow$$

Agreement with existing PN results for SGB!

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

Class.Quant.Grav. 39 (2022) 3, 035002 [Shiralilou, Hinderer, Nissanke, Ortiz, Witek]

Connect to observables: Power emitted in scalar radiation

$$\frac{dP_\phi}{d\Omega} = (\partial_t W_\phi)^2$$

JHEP 05 (2025) 045 [Falkowski, **PM**]

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scalar "monopole" charges $c_i = c_i(m_i, a_i, \frac{\alpha}{\Lambda^2})$

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Zeroth spin order contact term

$v \ll 1$ limit

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4. Waveforms for scattering of compact objects with scalar-hair

$$\boxed{c_i \sim \alpha + \mathcal{O}(a^2)}$$

Phys.Rev.D 100 (2019) 10, 104061 [Julié, Berti]

LO Scalar Waveforms-Spinless part:

scalar “monopole” charges $c_i = c_i(m_i, a_i, \frac{\alpha}{\Lambda^2})$ ⟶ Can be matched to existing results if it’s secondary hair!

$$W_\phi^{(0)} = -\frac{m_1 m_2}{32\pi^2 M_{Pl}^3\sqrt{\gamma^2-1}(\hat{u}_1 n)^2 b}\frac{1}{\sqrt{1+T_1^2}}\Bigg\{\frac{(\gamma^2-1)[c_1(\hat{u}_2 n)^2 + c_2(\hat{u}_1 n)^2][\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)T_1]}{[-(\hat{u}_1 n) + \gamma(\hat{u}_2 n) + (\tilde{b}n)T_1]^2 + (\tilde{v}n)^2(1+T_1^2)}$$

$$-\frac{c_1}{2}\frac{(\hat{u}_1 n) + (2\gamma^2-3)y(\hat{u}_2 n) - (2\gamma^2-1)(\tilde{b}n)T_1}{\gamma^2-1} + \frac{C_1^{(0)}}{2}c_2(\hat{u}_1 n)\Bigg\} + (1\leftrightarrow 2)\,.$$

Zeroth spin order contact term

$v \ll 1$ limit

$$\left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \frac{1}{b^4} \xrightarrow{\boxed{\text{Kepler's law}}} \left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim v^8 \longrightarrow$$

Agreement with existing PN results for SGB!

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

Class.Quant.Grav. 39 (2022) 3, 035002 [Shiralilou, Hinderer, Nissanke, Ortiz, Witek]

Connect to observables: Power emitted in scalar radiation

$$\frac{dP_\phi}{d\Omega} = (\partial_t W_\phi)^2$$

JHEP 05 (2025) 045 [Falkowski, **PM**]

More results:

LO Gravitational Waveforms-Spinless part:

$$W_h^{(0)} = -\frac{c_1c_2m_1m_2}{512\pi^2 M_{Pl}^3\sqrt{\gamma^2-1}(\hat{u}_1n)^2b}\frac{1}{\sqrt{1+T_1^2}}\mathrm{Re}\left\{\frac{(\lambda_n[\gamma\hat{u}_2\sigma + T_1\tilde{b}\sigma + i\sqrt{T_1^2+1}\tilde{v}\sigma]\hat{u}_1\bar{\sigma}\lambda_n)^2}{\gamma(\hat{u}_2n) - (\hat{u}_1n) + T_1(\tilde{b}n) + i\sqrt{T_1^2+1}(\tilde{v}n)}\right\} + (1 \leftrightarrow 2)\,.$$

$v \ll 1$ limit

$$\left.\frac{dP_h}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \frac{v^2}{b^4} \xrightarrow{\boxed{\text{Kepler's law}}} \left.\frac{dP_h}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim v^{10} \longrightarrow$$

Agreement and expected suppression compared to scalar radiation!

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

Class.Quant.Grav. 39 (2022) 3, 035002 [Shiralilou, Hinderer, Nissanke, Ortiz, Witek]

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$v \ll 1$ limit

$$\left.\frac{dP_h}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \frac{v^2}{b^4} \xrightarrow{\boxed{\text{Kepler's law}}} \left.\frac{dP_h}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim v^{10} \longrightarrow$$

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Class.Quant.Grav. 39 (2022) 3, 035002 [Shiralilou, Hinderer, Nissanke, Ortiz, Witek]

Comments:

- Can show that there are no linear-in-spin corrections to $W_h^{(1)}$ at this order!
- Hairy objects (only) lead to **non-vanishing memory effect** for both scalar and gravitational waveforms.
- In fact, we also derived that the NR **power** emitted is **exactly the same as in GR** up to setting $c_1 c_2 \to 1$
- For reference, $\left.\frac{dP_{h,GR}}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim v^{10} \longrightarrow$ Interactions impact severely the GW signal!

5. Waveforms for scattering of compact objects in shift-symmetric scalar-tensor theories

LO scalar waveform for scalar Gauss-Bonnet:

$$
\begin{aligned}
W_\phi = \frac{m_1 m_2}{8\pi^2 b^3 M_{Pl}^3 \Lambda^2 (\hat{u}_1 n)^2 \sqrt{\gamma^2-1}} \Bigg(\alpha \Bigg\{ -\frac{d^2}{dz^2} \Bigg[\frac{1}{\sqrt{z^2+1}} \frac{2(\gamma^2-1)^2 (\hat{u}_2 n)^2 \big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z\big]}{\big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z\big]^2 + (\tilde{v} n)^2 (z^2+1)} \Bigg] + \big[(\hat{u}_1 n) + \gamma\big(2\gamma^2-3\big)(\hat{u}_2 n)\big] \frac{2z^2-1}{(z^2+1)^{5/2}} + \big(2\gamma^2-1\big)(\tilde{b} n) \frac{3z}{(z^2+1)^{5/2}} \Bigg\} \\
+ \frac{2\alpha}{b} \frac{d^3}{dz^3} \mathrm{Re} \Bigg\{ \frac{1}{\sqrt{z^2+1}} \frac{(\hat{u}_2 n) - \gamma(\hat{u}_1 n) + \gamma(\tilde{b} n) z + i\gamma(\tilde{v} n)\sqrt{z^2+1}}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z + i(\tilde{v} n)\sqrt{z^2+1}} \Big[z(\tilde{v} n) - i\sqrt{z^2+1}(\tilde{b} n) \Big] \times \Bigg[\frac{(a_1 n)}{(\hat{u}_1 n)} - \gamma(a_1 \hat{u}_2) - (a_2 \hat{u}_1) - (a_1^A - a_2^A)(z\tilde{b}^A + i\sqrt{z^2+1}\tilde{v}^A) \Bigg] \Bigg\} \\
- \sqrt{\gamma^2-1} \frac{C_1}{b} \frac{d^3}{dz^3} \Bigg\{ \frac{1}{\sqrt{z^2+1}} \mathrm{Re} \Bigg[\Bigg(z\big[(\tilde{v} n) a_1^A + (a_1 \tilde{v}) n^A\big] - i\sqrt{z^2+1}\big[(\tilde{b} n) a_1^A + (a_1 \tilde{b}) n^A\big] \Bigg) \times \Bigg(\hat{u}_2^A - \gamma \hat{u}_1^A + \gamma\big[z\tilde{b}^A + i\sqrt{z^2+1}\tilde{v}^A\big] \Bigg) \Bigg] \Bigg\} \Bigg) \Bigg|_{z=T_1} + (1 \leftrightarrow 2) + \mathcal{O}(a^2) \,.
\end{aligned}
$$

$v \ll 1$ limit

$$\left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \frac{v^6}{b^8} \xrightarrow{\text{Kepler's law}} \left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim v^{22}$$

$$\left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^1)} \sim \frac{v^4}{b^{10}} \xrightarrow{\text{Kepler's law}} \left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^1)} \sim v^{24}$$

5. Waveforms for scattering of compact objects in shift-symmetric scalar-tensor theories

LO scalar waveform for scalar Gauss-Bonnet:

$$W_\phi = \frac{m_1 m_2}{8\pi^2 b^3 M_{Pl}^3 \Lambda^2 (\hat{u}_1 n)^2 \sqrt{\gamma^2-1}} \left(\alpha \left\{ -\frac{d^2}{dz^2} \left[\frac{1}{\sqrt{z^2+1}} \frac{2(\gamma^2-1)^2 (\hat{u}_2 n)^2 \big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z\big]}{\big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z\big]^2 + (\tilde{v} n)^2 (z^2+1)} \right] + \big[(\hat{u}_1 n) + \gamma\big(2\gamma^2 - 3\big)(\hat{u}_2 n)\big] \frac{2z^2-1}{(z^2+1)^{5/2}} + \big(2\gamma^2-1\big)(\tilde{b} n) \frac{3z}{(z^2+1)^{5/2}} \right\} \right.$$

$$+ \frac{2\alpha}{b} \frac{d^3}{dz^3} \mathrm{Re} \left\{ \frac{1}{\sqrt{z^2+1}} \frac{(\hat{u}_2 n) - \gamma(\hat{u}_1 n) + \gamma(\tilde{b} n) z + i\gamma(\tilde{v} n)\sqrt{z^2+1}}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z + i(\tilde{v} n)\sqrt{z^2+1}} \left[z(\tilde{v} n) - i\sqrt{z^2+1}(\tilde{b} n) \right] \times \left[\frac{(a_1 n)}{(\hat{u}_1 n)} - \gamma(a_1 \hat{u}_2) - (a_2 \hat{u}_1) - (a_1^A - a_2^A)(z\tilde{b}^A + i\sqrt{z^2+1}\tilde{v}^A) \right] \right\}$$

$$\left. - \sqrt{\gamma^2-1} \frac{C_1}{b} \frac{d^3}{dz^3} \left\{ \frac{1}{\sqrt{z^2+1}} \mathrm{Re} \left[\left(z\big[(\tilde{v} n) a_1^A + (a_1 \tilde{v}) n^A\big] - i\sqrt{z^2+1}\big[(\tilde{b} n) a_1^A + (a_1 \tilde{b}) n^A\big] \right) \times \left(\hat{u}_2^A - \gamma \hat{u}_1^A + \gamma\big[z\tilde{b}^A + i\sqrt{z^2+1}\tilde{v}^A\big] \right) \right] \right\} \right) \Bigg|_{z=T_1} + (1 \leftrightarrow 2) + \mathcal{O}(a^2)\,.$$

$v \ll 1$ limit

$$\left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \frac{v^6}{b^8} \xrightarrow{\text{Kepler's law}} \left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim v^{22} \longrightarrow$$

Bigger suppression compared to v^8 previously computed for hairy objects

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

Class.Quant.Grav. 39 (2022) 3, 035002 [Shiralilou, Hinderer, Nissanke, Ortiz, Witek]

$$\left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^1)} \sim \frac{v^4}{b^{10}} \xrightarrow{\text{Kepler's law}} \left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^1)} \sim v^{24}$$

6. Outlook

- **Scattering Amplitudes techniques** can be proven to be extremely **useful** in the quest for precision measurements **in the GW era**, probing results to all orders in velocity. Combined with the **on-shell** language, computations for **spinning binaries** remarkably **simplify**.

- Recasting already known problems in the amplitudes' language makes the **search for beyond GR effects easier to handle** and essentially the **usual QFT methods can be used**.

6. Outlook

- **Scattering Amplitudes techniques** can be proven to be extremely **useful** in the quest for precision measurements **in the GW era**, probing results to all orders in velocity. Combined with the **on-shell** language, computations for **spinning binaries** remarkably **simplify**.

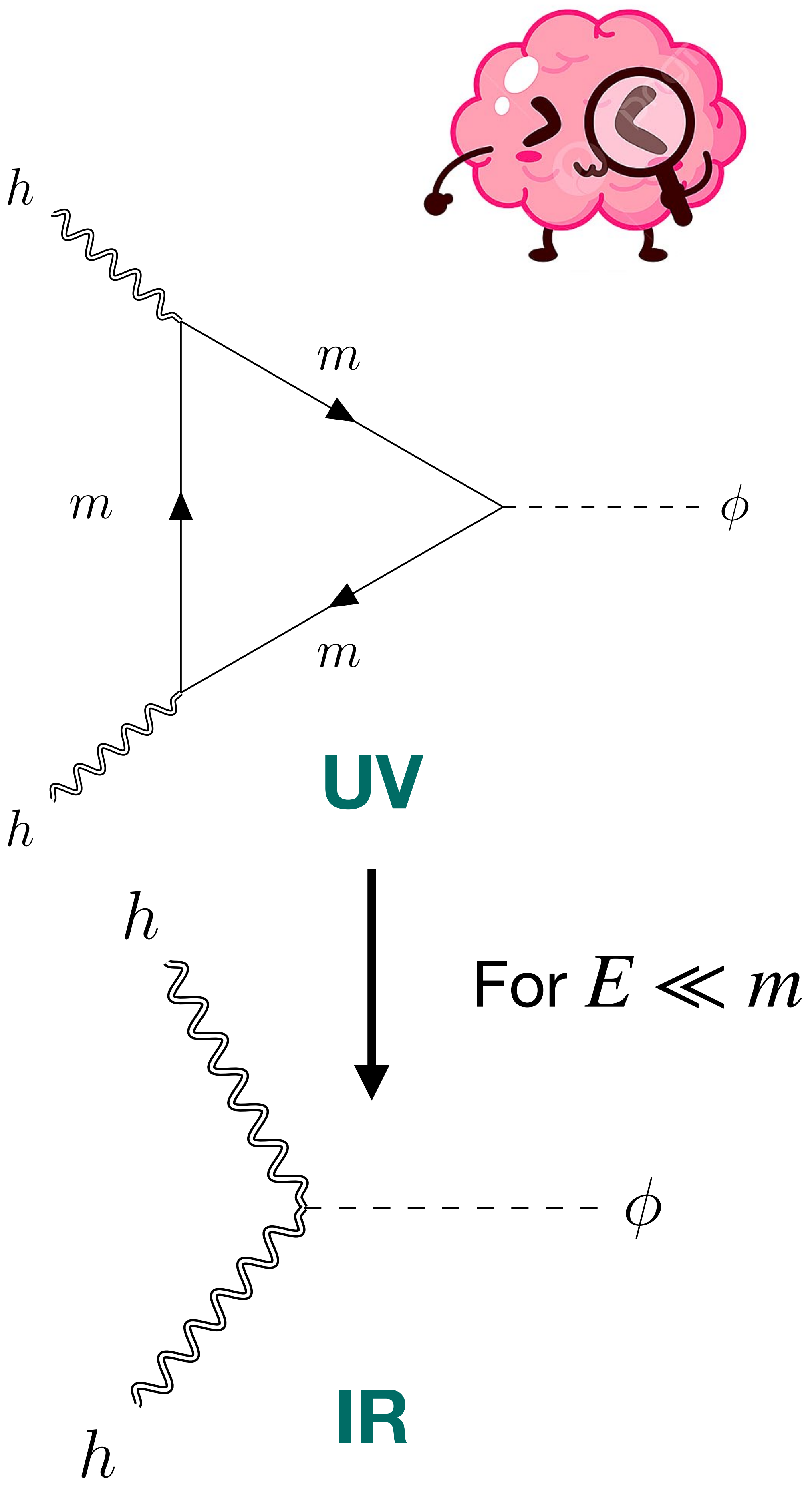
- Recasting already known problems in the amplitudes' language makes the **search for beyond GR effects easier to handle** and essentially the **usual QFT methods can be used**.

What's next?

- Employing similar methods to **study GR (and beyond) effects**, where current PN/NR results are poor, e.g. Description of **dipole hair in DCS from amplitudes** looks promising, but remains still incomplete.

- Could we employ our formalism to **effectively describe new Physics** related to **scalar particles** from GWs (boson clouds around BHs, soliton and boson stars, light axion-like particles, …) ?

- What about **UV completions** of such theories from an **on-shell perspective**? Can exploit their properties to make an efficient **EFT matching**, learning **generic lessons!** (see upcoming work **[2507.XXXXX]** with E.Alviani, A.Falkowski)



1. **SGB** can be generated, but with **shift-symmetry breaking** operators

2. **DCS** can also be generated, with a **unique shift-symmetric** choice, corresponding to a theory of **spin 1/2 fermions** and **a complex scalar** with a **Peccei-Quinn global symmetry** $U(1)_{PQ}$.

The universe seems to be extremely loud!

Thank you for your attention!:)

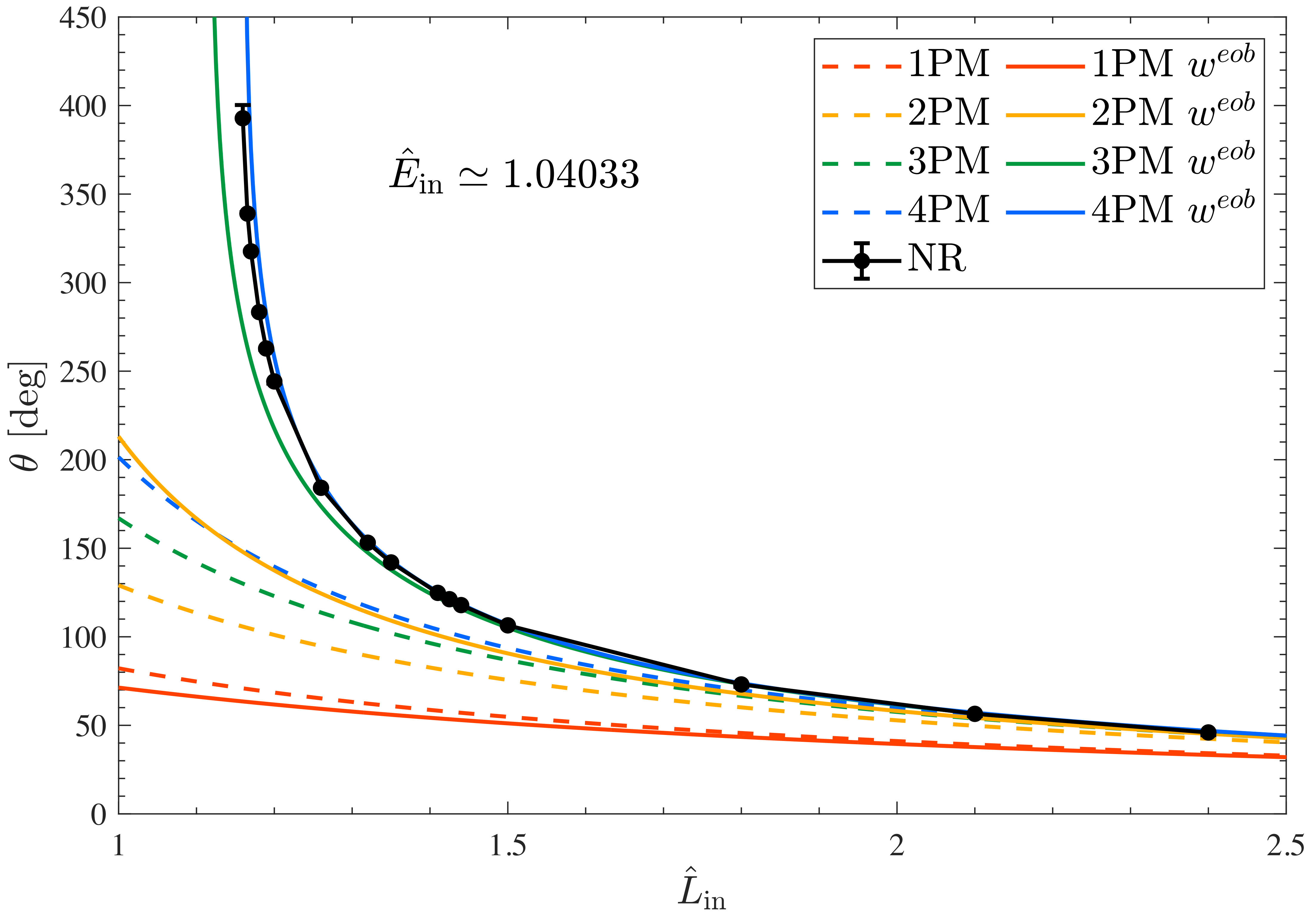
Merci beaucoup!

Artwork by Penelope Cowley*

Backup slides

PM expansion to the test and bound orbits:

Phys.Rev.D 108 (2023) 12, 124016 [Rettegno, Pratten, Thomas, Schmidt, Damour]



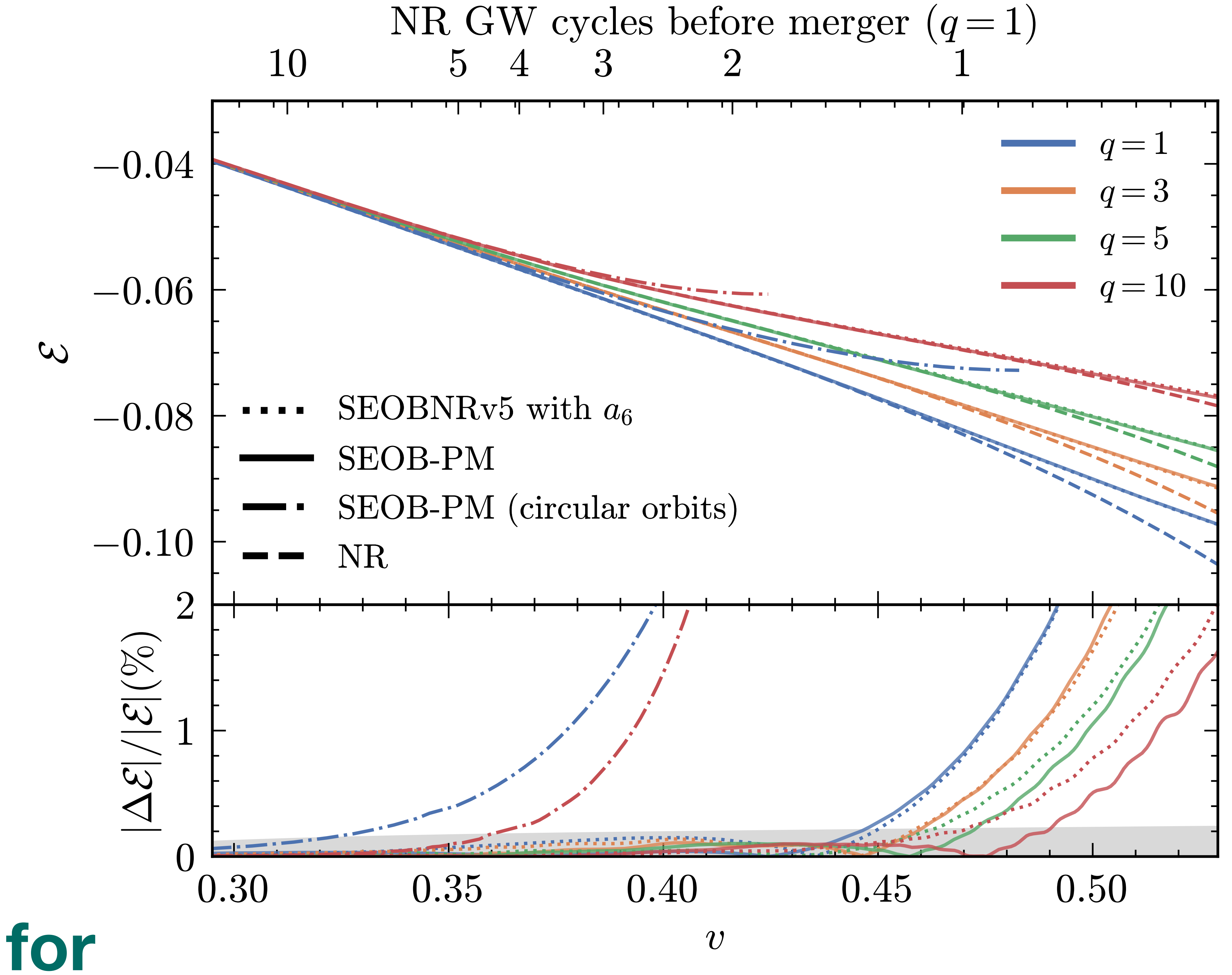
Already PM is doing very well for black hole (BH) scattering!

Bound to Boundary map for binary dynamics:

JHEP 01 (2020) 072 [Kälin, Porto]
JHEP 02 (2020) 120 [Kälin, Porto]
JHEP 04 (2022) 154, JHEP 07 (2022) 002 (erratum) [Cho, Kälin, Porto]

Recent work on a waveform map:

JHEP 05 (2024) 034 [Adamo, Gonzo, Ilderton]

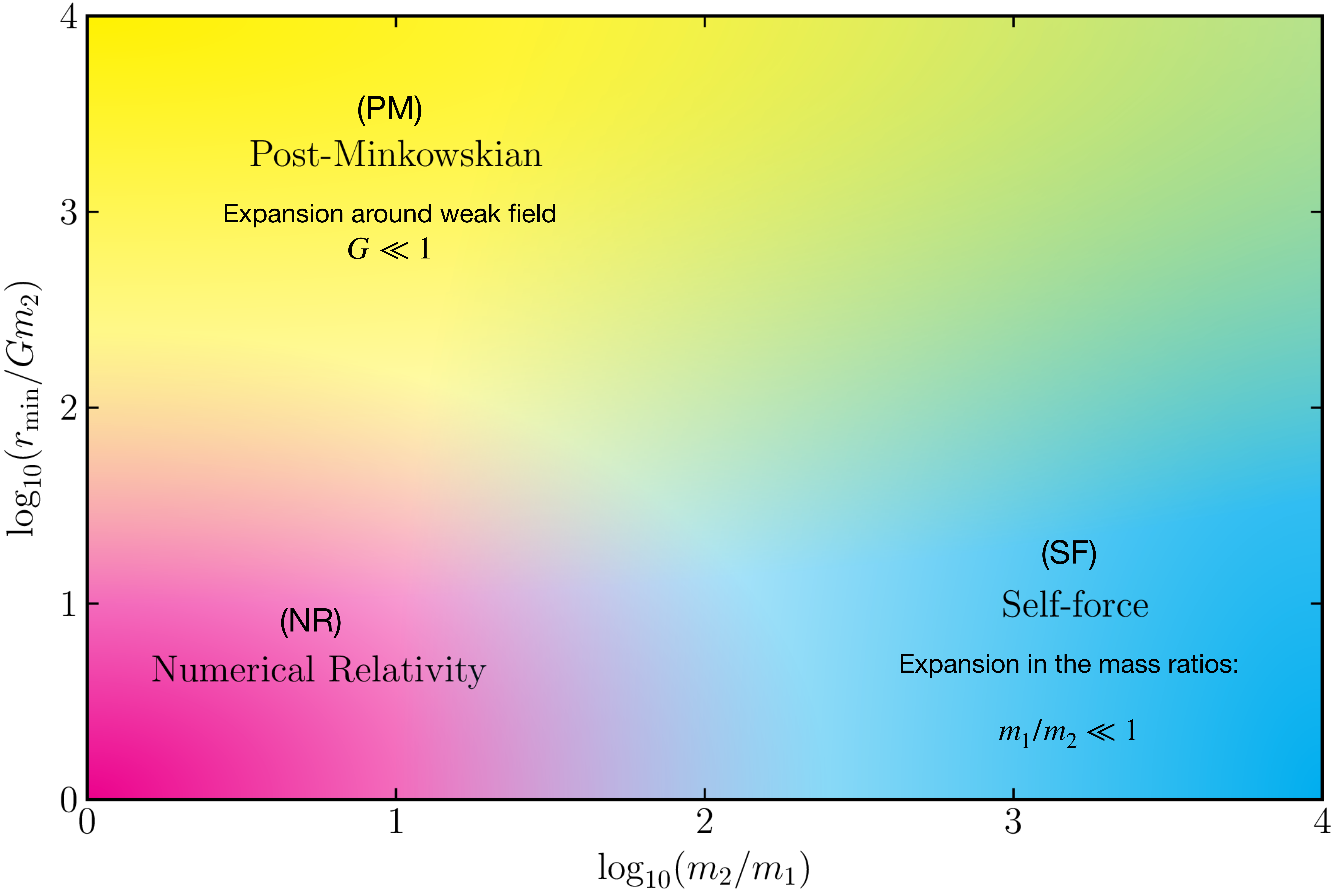


arxiv: 2405.19181 [Buonanno, Mogull, Patil, Pompili]

Good agreement for the bound case as well!

Analytical approaches:

Phys.Rev.D 108 (2023) 2, 024025 [Barack, Bern, Herrmann, Long, Parra-Martinez, Roiban, Ruf, Shen, Solon, Teng, Zeng]

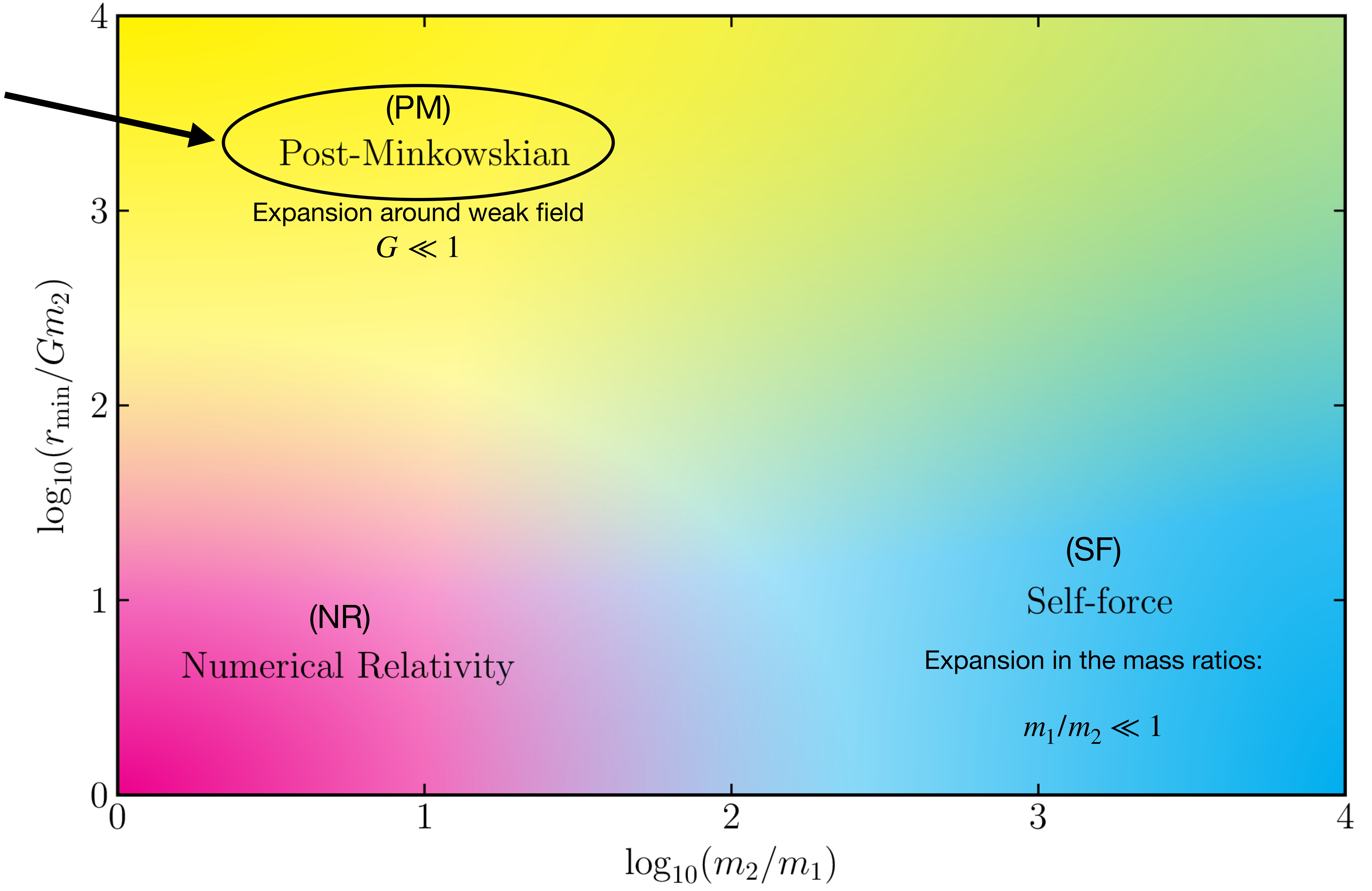


Analytical approaches:

- Enormous program for accurately determining the binary dynamics and computing waveforms **at all orders in** $\frac{v}{c}$, in contrast with Post-Newtonian (PN) approaches.

- Inspiration from **techniques** used in **Scattering Amplitudes and Effective Field Theory**

[Adamo, Alaverdian, Aoude, Bautista, Ben-Shahar, Bern, Bini, Brandhuber, Brown, Buonanno, Cachazo, Cangemi, Chiodaroli, Chen, Cordero, Cristofoli, de la Cruz, Damour, Damgaard, De Angelis, Driesse, Elkhidir, Gatica, Georgoudis, Goldberger, Gowdy, Gonzo, Guevara, Haddad, Heissenberg, Helset, Herrmann, Holstein, Huang, Huang, Ilderton, Jakobsen, Johansson, Kim, Kraus, Kosmopoulos, Kosower, Lee, Levi, Lin, Liu, Luna, Matasan, Maybee, Menezes, Mogull, Mougiakakos, Moynihan, Novichkov, O'Connell, Ochirov, Parra-Martinez, Pichini, Plefka, Porto, Riva, Roiban, Ross, Rothstein, Ruf, Russo, Saketh, Sauer, Scheopner, Sergola, Shen, Siemonsen, Smirnov, Smirnov, Steinhoff, Teng, Travaglini, Vanhove, Vazquez-Holm, di Vecchia, Veneziano, Vernizzi, Vines, Wong, Xu, Yang, Yin, Zeng, et al…]

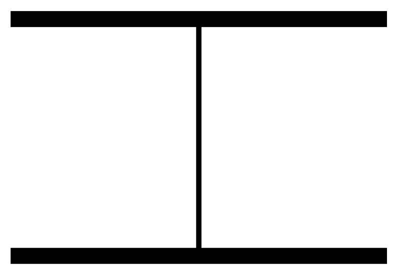


Phys.Rev.D 108 (2023) 2, 024025 [Barack, Bern, Herrmann, Long, Parra-Martinez, Roiban, Ruf, Shen, Solon, Teng, Zeng]

Results obtained in standard GR:

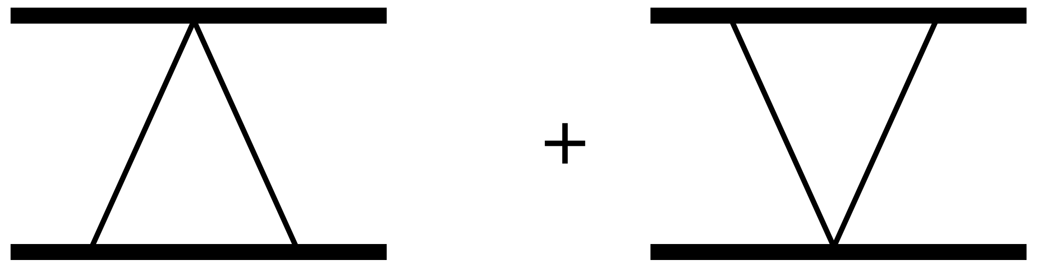
JHEP 10 (2021) 148 [Herrmann, Parra-Martinez, Ruf, Zeng]

tree:



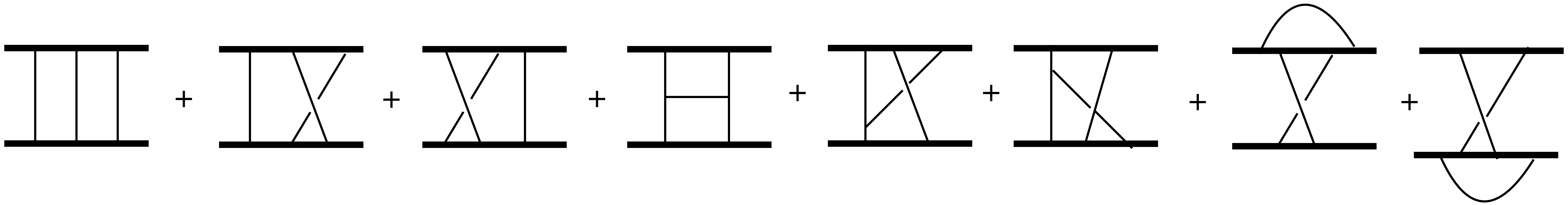
$$\Delta p^{\mu,(0)}_{1,\mathrm{GR}} = \frac{G M^2 \nu}{|b|} \frac{2(2\sigma^2-1)}{\sqrt{\sigma^2-1}} \frac{b^\mu}{|b|} \,.$$

1-loop:



$$\Delta p^{\mu,(1)}_{1,\perp} = \frac{G^2 M^3 \nu}{|b|^2} \frac{3\pi}{4} \frac{(5\sigma^2-1)}{\sqrt{\sigma^2-1}} \frac{b^\mu}{|b|}$$

2-loop:



$$\Delta p^{\mu,(2)}_{1,\perp,\mathrm{cons}} = \frac{G^3 M^4 \nu}{|b|^3} \frac{2}{\sqrt{\sigma^2-1}} \frac{b^\mu}{|b|} \Bigg[h^2(\sigma,\nu) \left(16\sigma^2 - \frac{1}{(\sigma^2-1)^2} \right) - \frac{4}{3}\nu\,\sigma\left(14\sigma^2+25\right) - 8\nu\left(4\sigma^4-12\sigma^2-3\right) \frac{\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} \Bigg]$$

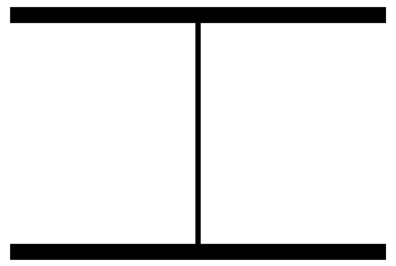
$$\Delta p^{\mu,(2)}_{1,\mathrm{rad}} = \frac{G^3 M^4 \nu^2}{|b|^3} \Bigg\{ \frac{4}{\sqrt{\sigma^2-1}} \frac{b^\mu}{|b|} \Bigg[f_1^{\mathrm{LS}}(\sigma) + f_3^{\mathrm{LS}}(\sigma) \frac{\sigma \operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} \Bigg] + \pi\, \check{u}_2^\mu \Bigg[f_1(\sigma) + f_2(\sigma) \log\left(\frac{\sigma+1}{2}\right) + f_3(\sigma) \frac{\sigma \operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} \Bigg] \Bigg\}$$

…

Results obtained in standard GR:

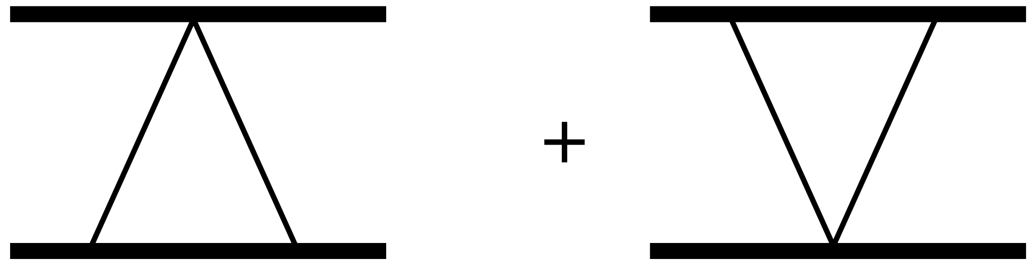
JHEP 10 (2021) 148 [Herrmann, Parra-Martinez, Ruf, Zeng]

tree:



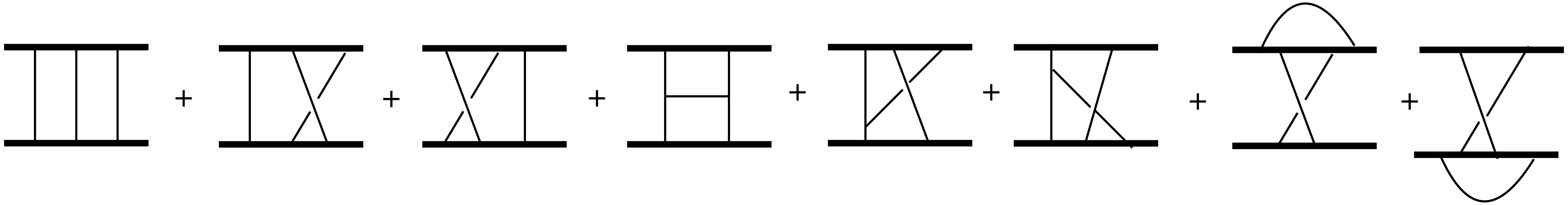
$$\Delta p^{\mu,(0)}_{1,\mathrm{GR}} = \frac{GM^2\nu}{|b|}\frac{2(2\sigma^2-1)}{\sqrt{\sigma^2-1}}\frac{b^\mu}{|b|}\,.$$

1-loop:



$$\Delta p^{\mu,(1)}_{1,\perp} = \frac{G^2M^3\nu}{|b|^2}\frac{3\pi}{4}\frac{(5\sigma^2-1)}{\sqrt{\sigma^2-1}}\frac{b^\mu}{|b|}$$

2-loop:



$$\Delta p^{\mu,(2)}_{1,\perp,\mathrm{cons}} = \frac{G^3M^4\nu}{|b|^3}\frac{2}{\sqrt{\sigma^2-1}}\frac{b^\mu}{|b|}\left[h^2(\sigma,\nu)\left(16\sigma^2-\frac{1}{(\sigma^2-1)^2}\right)\right.$$
$$\left.-\frac{4}{3}\nu\,\sigma\,(14\sigma^2+25)-8\nu\,(4\sigma^4-12\sigma^2-3)\,\frac{\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\right]$$

$$\Delta p^{\mu,(2)}_{1,\mathrm{rad}} = \frac{G^3M^4\nu^2}{|b|^3}\left\{\frac{4}{\sqrt{\sigma^2-1}}\frac{b^\mu}{|b|}\left[f_1^{\mathrm{LS}}(\sigma)+f_3^{\mathrm{LS}}(\sigma)\frac{\sigma\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\right]\right.$$
$$\left.+\pi\,\check{u}_2^\mu\left[f_1(\sigma)+f_2(\sigma)\log\left(\frac{\sigma+1}{2}\right)+f_3(\sigma)\frac{\sigma\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\right]\right\}$$

… →

State of the art is pushing towards 4-loops (5PM) binary dynamics

Phys.Rev.Lett. 132 (2024) 24, 241402 [Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovitsch]
arxiv: 2406.01554 [Bern, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng]

Results obtained in standard GR:

LO order spinning waveform obtained from different approaches (consensus up to $\mathcal{O}(a^4)$):

Phys.Rev.D 110 (2024) 4, L041502 [De Angelis, Novichkov, Gonzo]
Phys.Rev.D 109 (2024) 3, 036007 [Aoude, Haddad, Heissenberg, Helset]
JHEP 02 (2024) 026 [Brandhuber, Brown, Chen, Gowdy, Travaglini]

e.g.: LO spinless waveform:

$$h_f(x)|_{\mathfrak{a}_i=0} = \sum_{i=1}^{2} \frac{\tilde{r}^{-,\mu\nu}_{(i),0} + \tilde{r}^{+,\mu\nu}_{(i),0}}{(p_i \cdot \rho)^2} \mathcal{I}_{(i),\mu\nu}(b_0)$$

$$\mathcal{I}^{\mu\nu}_{(1)}(b) = \frac{K^{\mu\nu}_{(1)}(v_1 \cdot K_{(1)} \cdot \rho) - 2(v_1 \cdot K_{(1)})^{(\mu}(\rho \cdot K_{(1)})^{\nu)}}{4\pi(\gamma^2-1)(\rho \cdot v_2)^2 |b|^2 |\boldsymbol{b}|_{(1)} |b|^2_{2\mathrm{d}}}$$

$$\begin{aligned}
r^{+,\mu\nu}_{(1),0} &= \langle k|p_1 p_2 \gamma^\mu p_1|k\rangle\langle k|p_1 p_2 \gamma^\nu p_1|k\rangle \\
r^{+,\mu\nu}_{(1),1} &= \langle k|p_1 p_2 \gamma^\mu p_1|k\rangle\langle k|p_1 p_2 \gamma^\nu \mathfrak{a}_1|k\rangle \\
r^{+,\mu\nu}_{(1),2} &= \langle k|p_1 p_2 \gamma^\mu \mathfrak{a}_1|k\rangle\langle k|p_1 p_2 \gamma^\nu \mathfrak{a}_1|k\rangle \\
r^{+,\mu\nu}_{(1),3} &= \langle k|p_1 p_2 \gamma^\mu \mathfrak{a}_1|k\rangle\langle k|\mathfrak{a}_1 p_2 \gamma^\nu \mathfrak{a}_1|k\rangle \\
r^{+,\mu\nu}_{(1),4} &= \langle k|\mathfrak{a}_1 p_2 \gamma^\mu \mathfrak{a}_1|k\rangle\langle k|\mathfrak{a}_1 p_2 \gamma^\nu \mathfrak{a}_1|k\rangle
\end{aligned}$$

$$\begin{aligned}
r^{-,\mu\nu}_{(1),0} &= m_1^4 \langle k|p_2 \gamma^\mu|k\rangle\langle k|p_2 \gamma^\nu|k\rangle, \\
r^{-,\mu\nu}_{(1),1} &= m_1^2 \langle k|p_2 \gamma^\mu|k\rangle\langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\nu|k\rangle \\
r^{-,\mu\nu}_{(1),2} &= \langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\mu|k\rangle\langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\nu|k\rangle, \\
r^{-,\mu\nu}_{(1),3} &= \frac{1}{m_1^2}\langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\mu|k\rangle\langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\nu p_1 \mathfrak{a}_1|k\rangle \\
r^{-,\mu\nu}_{(1),4} &= \frac{1}{m_1^4}\langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\mu p_1 \mathfrak{a}_1|k\rangle\langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\nu p_1 \mathfrak{a}_1|k\rangle
\end{aligned}$$

Results obtained in standard GR:

LO order spinning waveform obtained from different approaches (consensus up to $\mathscr{O}(a^4)$):

Phys.Rev.D 110 (2024) 4, L041502 [De Angelis, Novichkov, Gonzo]
Phys.Rev.D 109 (2024) 3, 036007 [Aoude, Haddad, Heissenberg, Helset]
JHEP 02 (2024) 026 [Brandhuber, Brown, Chen, Gowdy, Travaglini]

e.g.: LO spinless waveform:

$$h_f(x)|_{\mathfrak{a}_i=0} = \sum_{i=1}^{2} \frac{\tilde{r}^{-,\mu\nu}_{(i),0} + \tilde{r}^{+,\mu\nu}_{(i),0}}{(p_i \cdot \rho)^2} \mathcal{I}_{(i),\mu\nu}(b_0)$$

$$\mathcal{I}^{\mu\nu}_{(1)}(b) = \frac{K^{\mu\nu}_{(1)}(v_1 \cdot K_{(1)} \cdot \rho) - 2(v_1 \cdot K_{(1)})^{(\mu}(\rho \cdot K_{(1)})^{\nu)}}{4\pi(\gamma^2-1)(\rho \cdot v_2)^2 |b|^2 |\boldsymbol{b}|_{(1)} |b|^2_{2\mathrm{d}}}$$

$$\begin{aligned}
r^{+,\mu\nu}_{(1),0} &= \langle k|p_1 p_2 \gamma^\mu p_1|k\rangle\langle k|p_1 p_2 \gamma^\nu p_1|k\rangle \\
r^{+,\mu\nu}_{(1),1} &= \langle k|p_1 p_2 \gamma^\mu p_1|k\rangle\langle k|p_1 p_2 \gamma^\nu \mathfrak{a}_1|k\rangle \\
r^{+,\mu\nu}_{(1),2} &= \langle k|p_1 p_2 \gamma^\mu \mathfrak{a}_1|k\rangle\langle k|p_1 p_2 \gamma^\nu \mathfrak{a}_1|k\rangle \\
r^{+,\mu\nu}_{(1),3} &= \langle k|p_1 p_2 \gamma^\mu \mathfrak{a}_1|k\rangle\langle k|\mathfrak{a}_1 p_2 \gamma^\nu \mathfrak{a}_1|k\rangle \\
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\end{aligned}$$

$$\begin{aligned}
r^{-,\mu\nu}_{(1),0} &= m_1^4 \langle k|p_2 \gamma^\mu|k\rangle\langle k|p_2 \gamma^\nu|k\rangle, \\
r^{-,\mu\nu}_{(1),1} &= m_1^2 \langle k|p_2 \gamma^\mu|k\rangle\langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\nu|k\rangle \\
r^{-,\mu\nu}_{(1),2} &= \langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\mu|k\rangle\langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\nu|k\rangle, \\
r^{-,\mu\nu}_{(1),3} &= \frac{1}{m_1^2} \langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\mu|k\rangle\langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\nu p_1 \mathfrak{a}_1|k\rangle \\
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\end{aligned}$$

NLO order waveform also looks like a closed case:

JHEP 06 (2023) 048 [Brandhuber, Brown, Chen, De Angelis, Gowdy, Travaglini]
JHEP 07 (2024) 272 [Elkhidir, O'Connell, Sergola, Vazquez-Holm]
JHEP 06 (2023) 004 [Herderschee, Roiban, Teng]
JHEP 2023 (2023) 06, 126 [Georgoudis, Heissenberg, Vazquez-Holm]
JHEP 01 (2024) 139 [Caron-Huot, Giroux, Hannesdottir, Mizera]

Recent result for NLO linear-in-spin effects:

arxiv: 2312.14859 [Bohnenblust, Ita, Kraus, Schlenk]

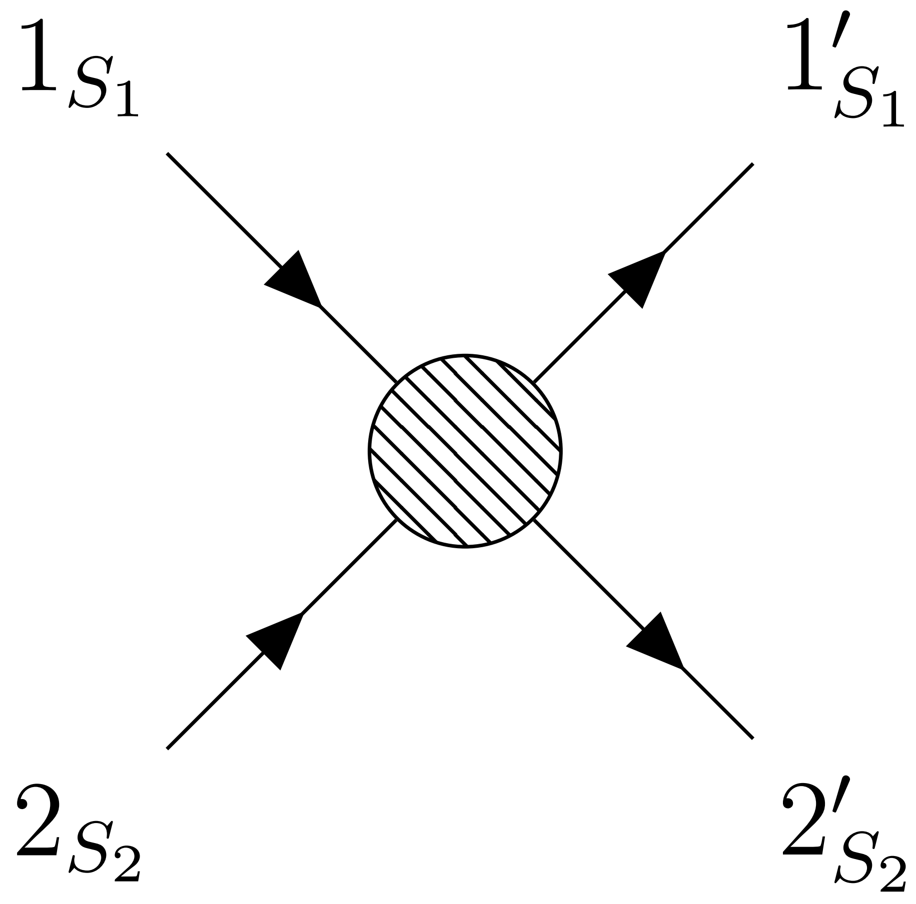
KMOC (Kosower Maybee O’Connell) formalism

JHEP 02 (2019) 137 [Kosower, Maybee, O’Connell]
Phys.Rev.D 106 (2022) 5, 0567007 [Cristofoli, Gonzo, Kosower, O’Connell]

Idea: Relate **Scattering Amplitudes** directly to **classical observables**

⟶ Extract the classical piece of the amplitude through an “$\hbar$” counting prescription

e.g.:



two-particle out state

Quantum momentum operator

two-particle in state

$$\Delta p_1^\mu = {}_{\text{out}}\langle\psi|\hat{P}_1|\psi\rangle_{\text{out}} - {}_{\text{in}}\langle\psi|\hat{P}_1|\psi\rangle_{\text{in}}$$

$$|\psi\rangle_{\text{out}} = S|\psi\rangle_{\text{in}}$$

⟶

Classical Impulse:

$$\Delta p_1^\mu = p_{1,fin.}^\mu - p_{1,in.}^\mu$$

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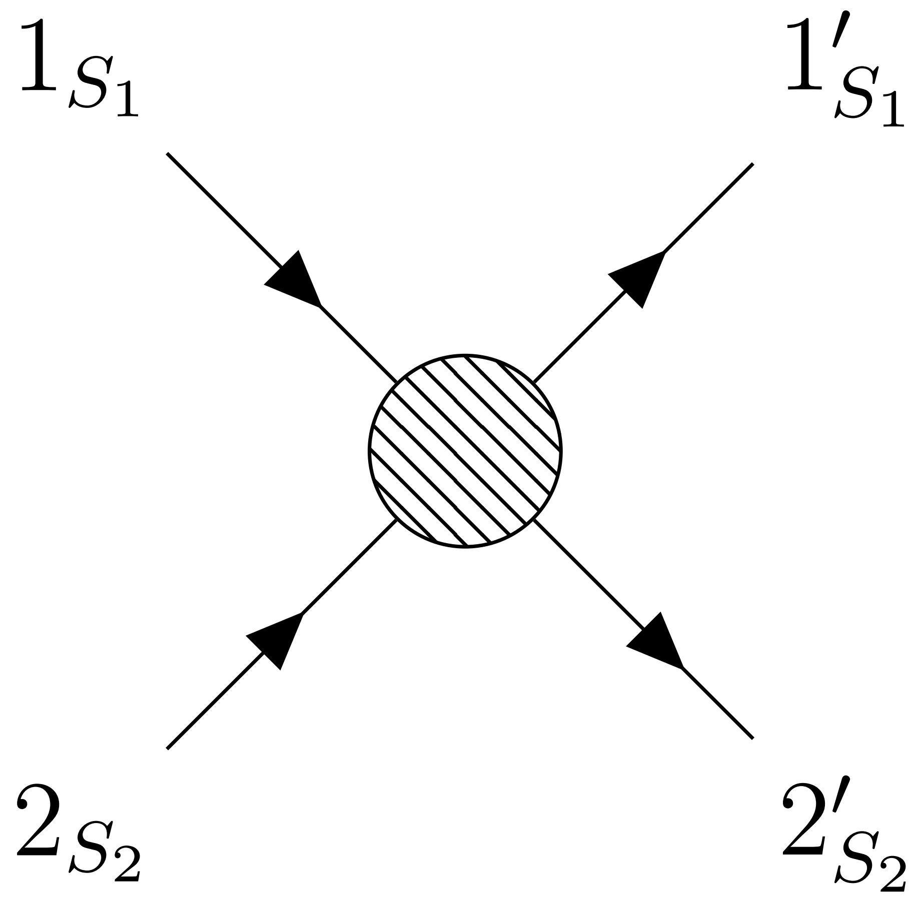

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$$|\psi\rangle_{\text{out}} = S|\psi\rangle_{\text{in}}$$

Classical Impulse:

$$\Delta p_1^\mu = p_{1,fin.}^\mu - p_{1,in.}^\mu$$

At leading order: $\Delta p_1^{\mu,LO} = \frac{i}{4}\left\langle\!\!\left\langle \hbar^2 \int \hat{d}^4 q\, \hat{\delta}(q\cdot p_1)\hat{\delta}(q\cdot p_2) e^{-ib\cdot q} q^\mu \mathcal{M}^{LO}(p_1, p_2 \to p_1 + \hbar q, p_2 - \hbar q)\right\rangle\!\!\right\rangle$

Waveforms from amplitudes

Similar approach can be applied to radiation observables

Examples:

Massless scalar theory

$$R_\phi \equiv {}_{\text{out}}\langle \psi | \phi(x) | \psi \rangle_{\text{out}}$$

Electromagnetism

$$R_\mu \equiv {}_{\text{out}}\langle \psi | A_\mu(x) | \psi \rangle_{\text{out}}$$

or gauge invariant

$$R_{\mu\nu} \equiv {}_{\text{out}}\langle \psi | F_{\mu\nu}(x) | \psi \rangle_{\text{out}} \qquad R_A^{\pm} \equiv {}_{\text{out}}\langle \psi | A^\mu(x) | \psi \rangle_{\text{out}} \epsilon_\mu^{\pm}$$

GR

$$R_{\mu\nu} \equiv {}_{\text{out}}\langle \psi | h_{\mu\nu}(x) | \psi \rangle_{\text{out}}$$

or gauge invariant

$$R_{\mu\nu\alpha\beta} \equiv {}_{\text{out}}\langle \psi | R_{\mu\nu\alpha\beta}(x) | \psi \rangle_{\text{out}} \qquad R_h \equiv {}_{\text{out}}\langle \psi | h^{\mu\nu}(x) | \psi \rangle_{\text{out}} \epsilon_{\mu\nu}^{-}$$

Waveforms from amplitudes

Given radiation observable R_X one defines waveform W_X as

$$R_X(x) = \frac{W_X(t)}{|\boldsymbol{x}|} \qquad |\boldsymbol{x}| \to \infty \qquad t \equiv x^0 - |\boldsymbol{x}|$$

retarded time

Furthermore one defines spectral waveform f_X as Fourier transform:

$$f_X(\omega) = \int dt\; e^{i\omega t} W_X(t)$$

KMOC formalism relates spectral waveform at leading order to integral of 5-point amplitude.

At leading order:

$$f_X(\omega) = \frac{1}{64\pi^3 m_1 m_2} \int d\mu \mathscr{M}^{\text{cl}}[p_1 + w_1, p_2 + w_2 \to p_1, p_2, k]\Big|_{k=\omega n}$$

Integration measure

Matter representing classical bodies

Quantum of particle X

$$d\mu \equiv \delta^4(w_1 + w_2 - k)\Pi_{i=1,2}[e^{ib_i w_i} d^4 w_i \delta(u_i w_i)]$$

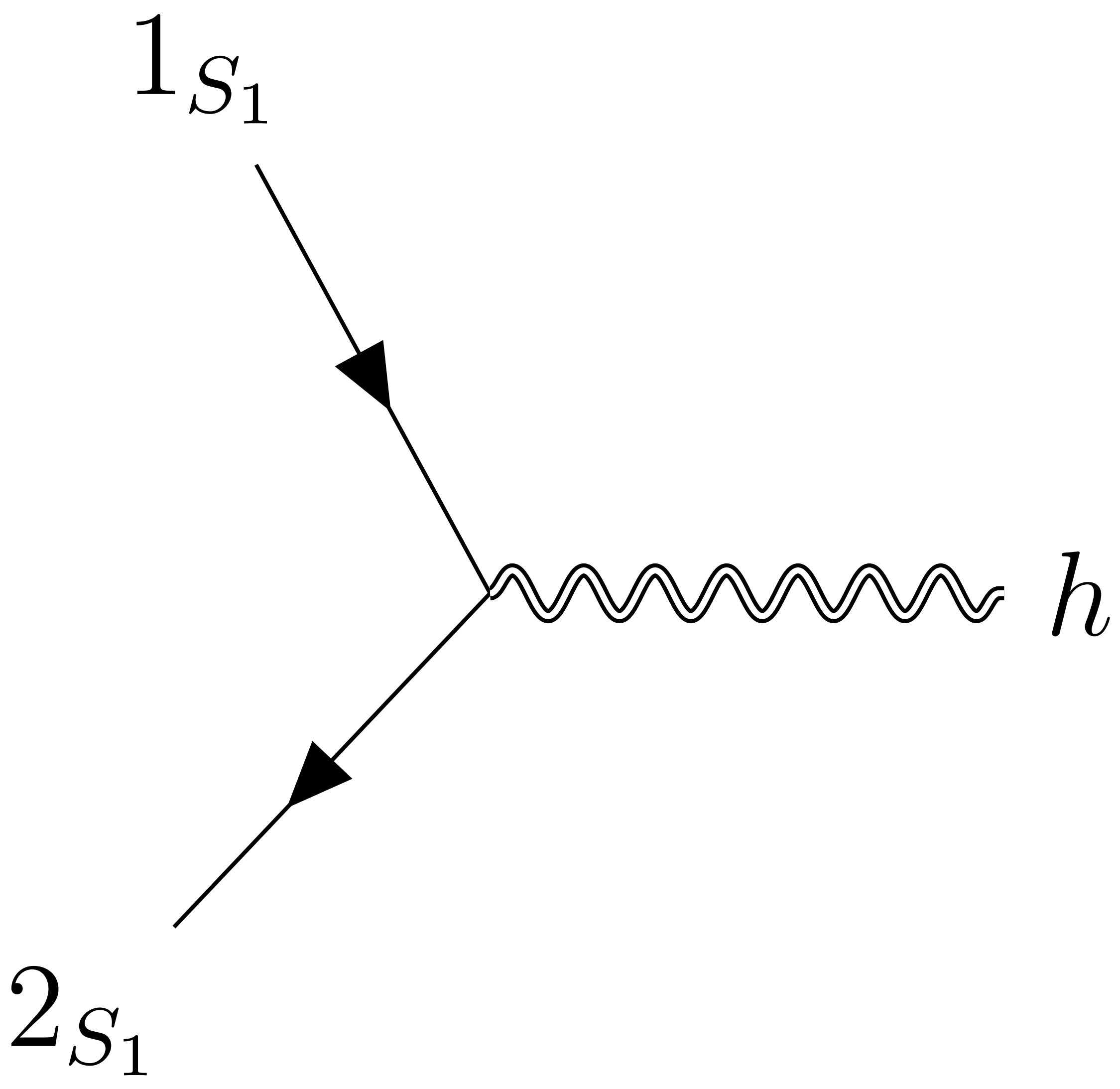
On-shell techniques:

Idea: Build the **on-shell 3-point amplitudes** of the theory

e.g.: **Spinning matter in GR**

$$\mathcal{M}\left[1_{\Phi}2_{\bar{\Phi}}3_h^-\right] = -\frac{\langle 3|p_1|\tilde{\zeta}]^2}{M_{Pl}[3\tilde{\zeta}]^2}\frac{[\mathbf{21}]^{2S}}{m^{2S}},$$

$$\mathcal{M}\left[1_{\Phi}2_{\bar{\Phi}}3_h^+\right] = -\frac{\langle \zeta|p_1|3]^2}{M_{Pl}\langle 3\zeta\rangle^2}\frac{\langle\mathbf{21}\rangle^{2S}}{m^{2S}},$$



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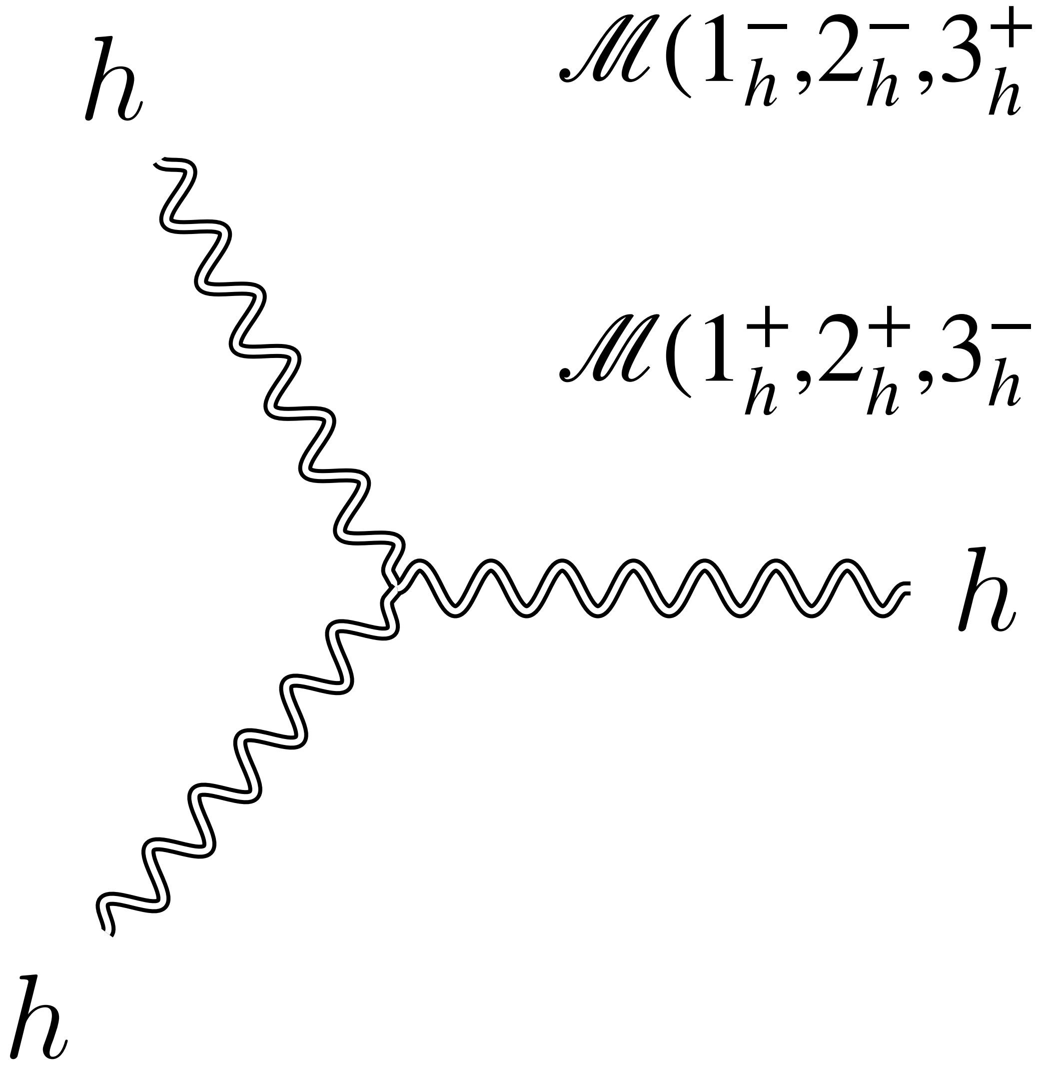

$$\mathcal{M}(1_h^-,2_h^-,3_h^+) = -\frac{1}{M_{Pl}}\frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 23\rangle^2}$$

$$\mathcal{M}(1_h^+,2_h^+,3_h^-) = -\frac{1}{M_{Pl}}\frac{[12]^6}{[13]^2[23]^2}\,.$$

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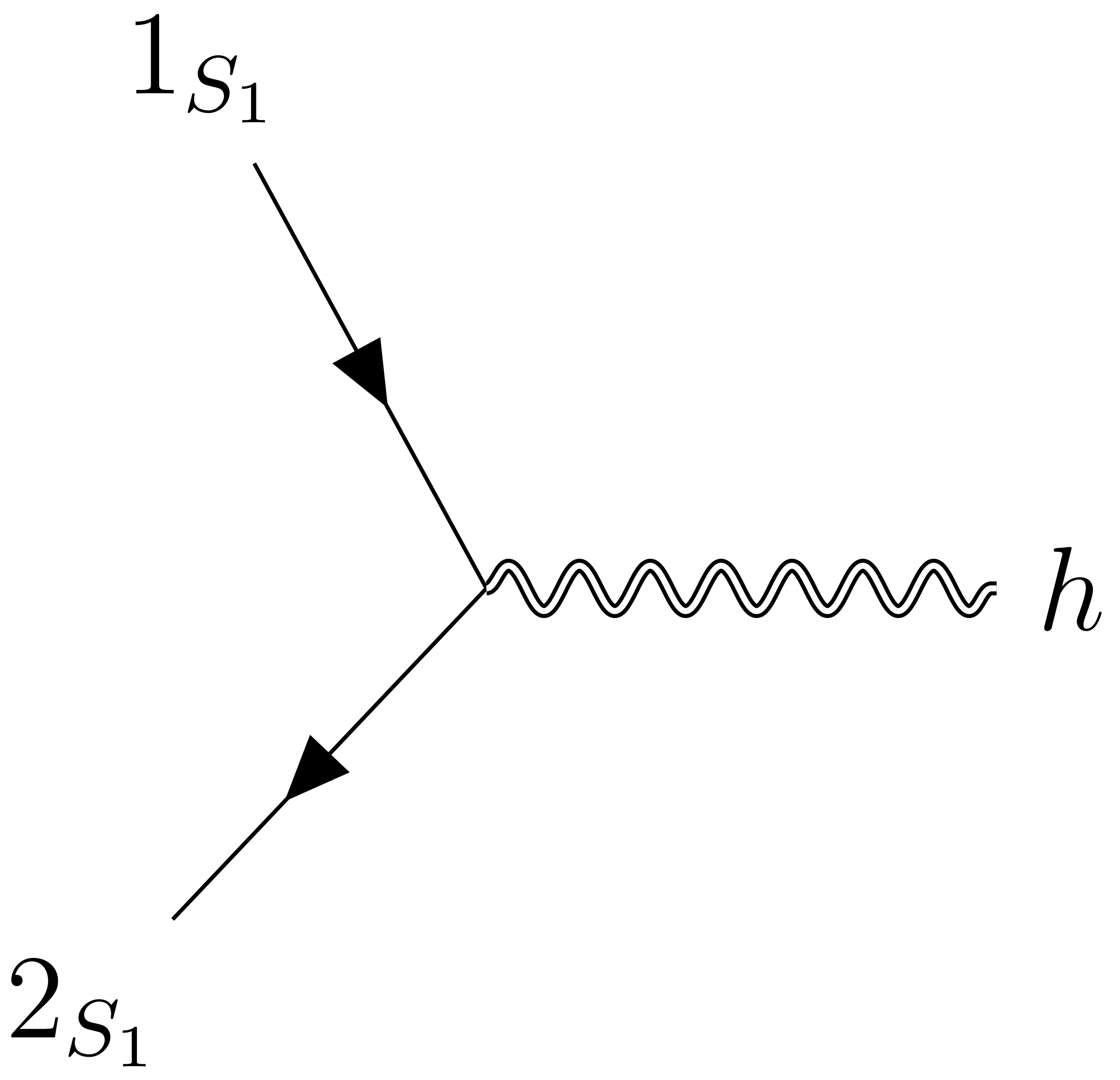

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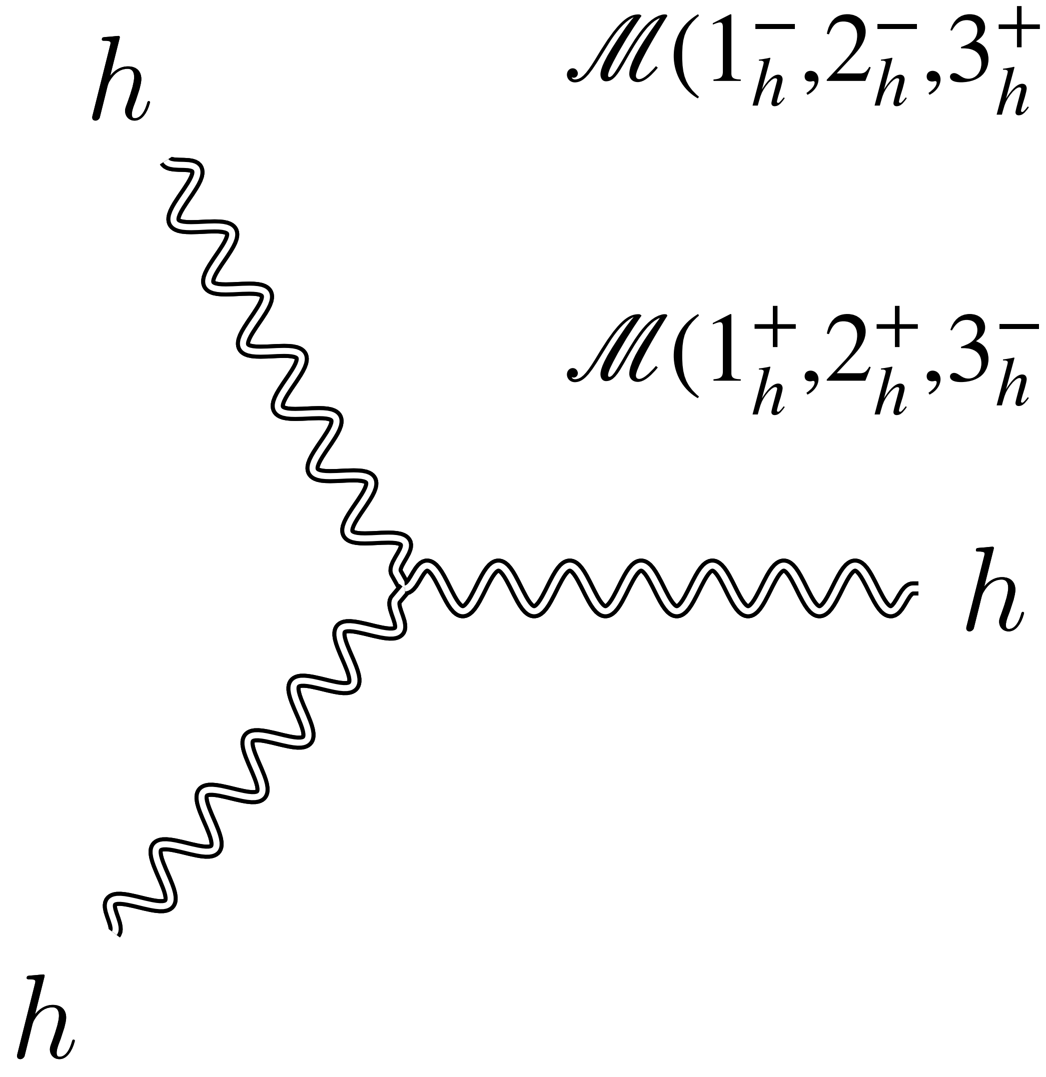

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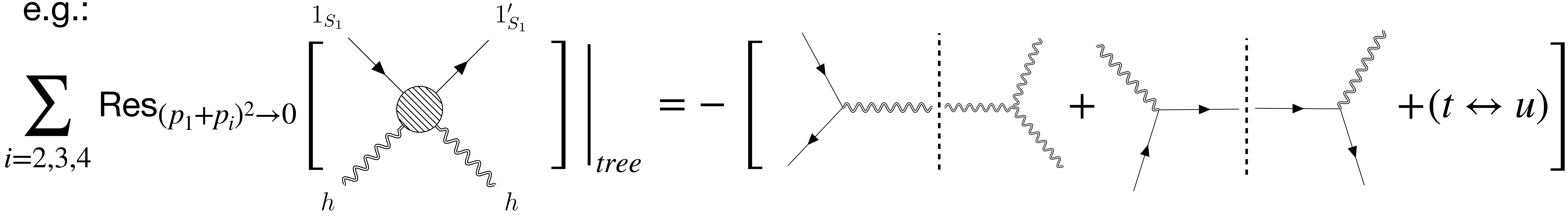
→ Build higher-point amplitudes from their **residues** at kinematic poles in the **complex** plane (up to contact terms)

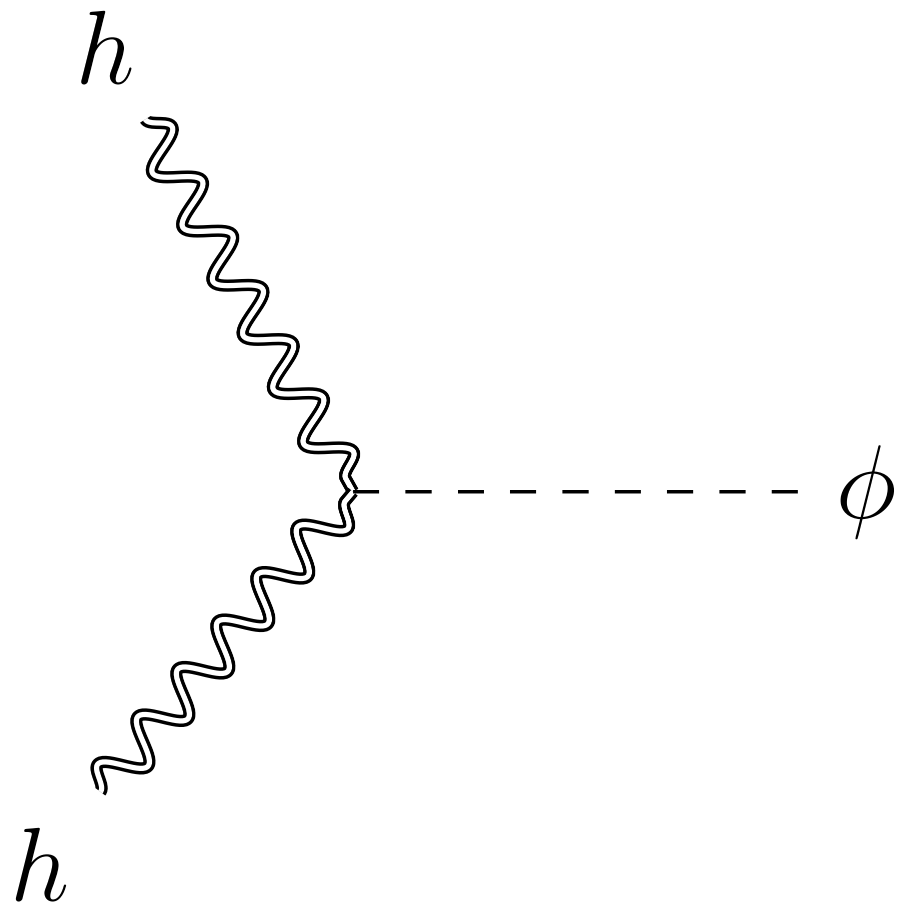
e.g.:



On-shell amplitudes:

Let's work by expanding $f(\phi) \approx const + \frac{\alpha}{M_{Pl}}\phi + \mathcal{O}(\phi^2)\,, \tilde{f}(\phi) \approx const + \frac{\tilde{\alpha}}{M_{Pl}}\phi + \mathcal{O}(\phi^2)$

This action produces an extra 3-point **shift-symmetric** on-shell amplitude which we should consider:



$$\sim \frac{\hat{\alpha}}{\Lambda^2} = \frac{\alpha + i\tilde{\alpha}}{\Lambda^2}$$

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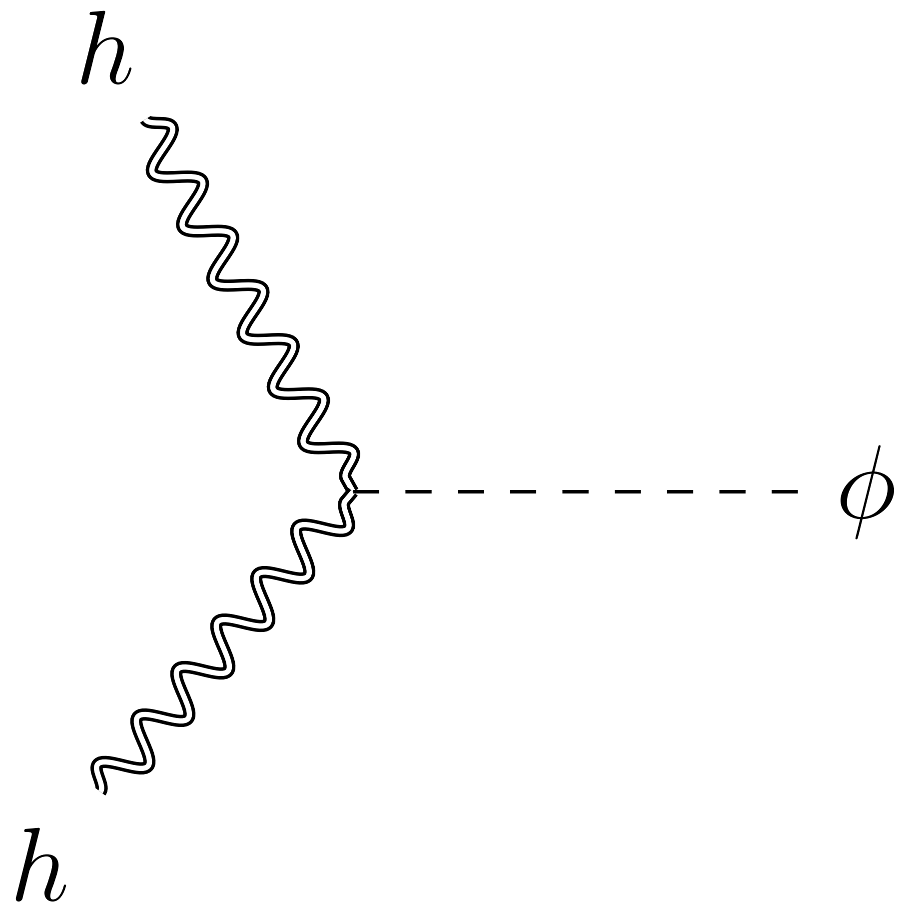

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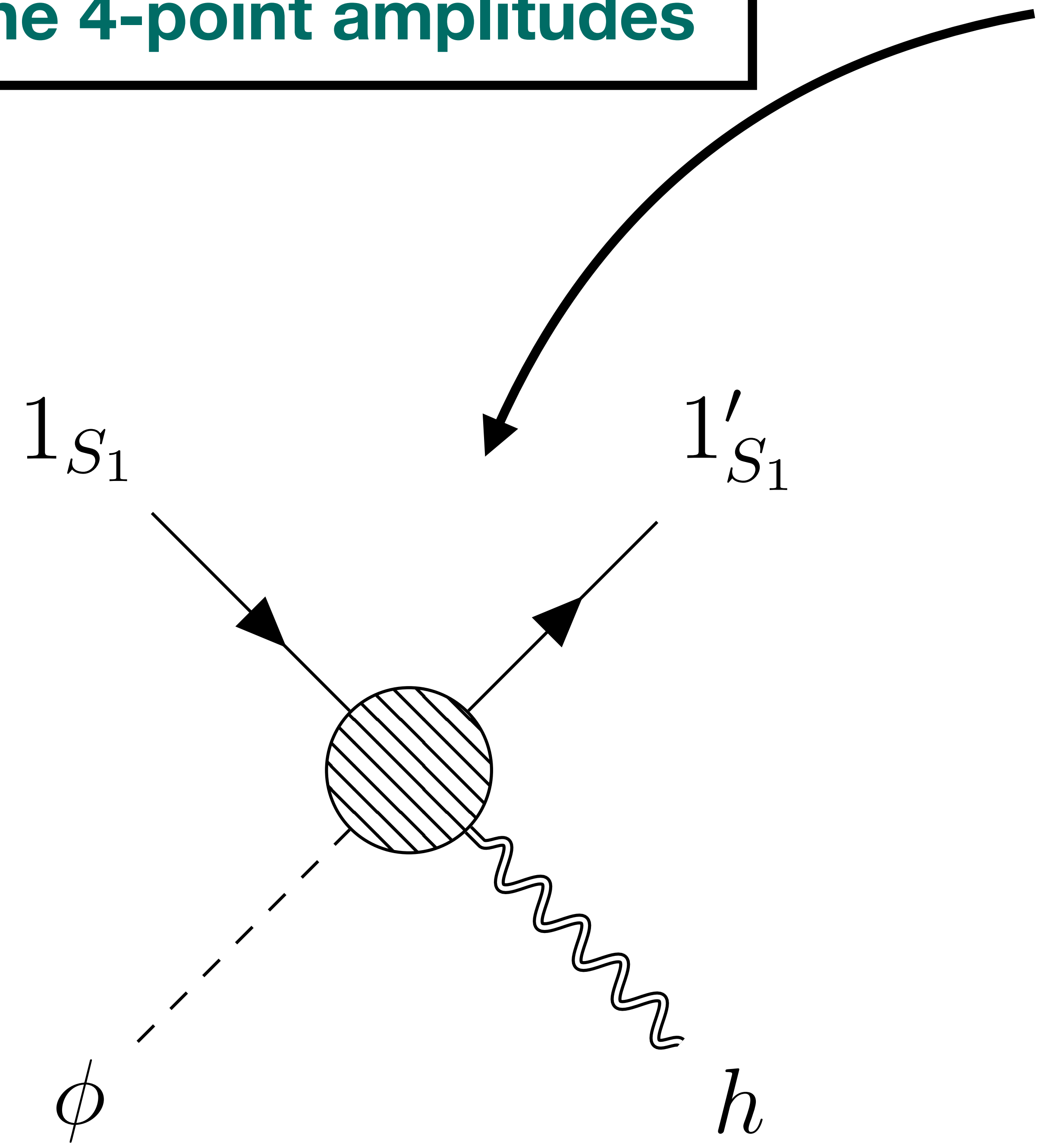
Gives an extra contribution to the 4-point amplitudes



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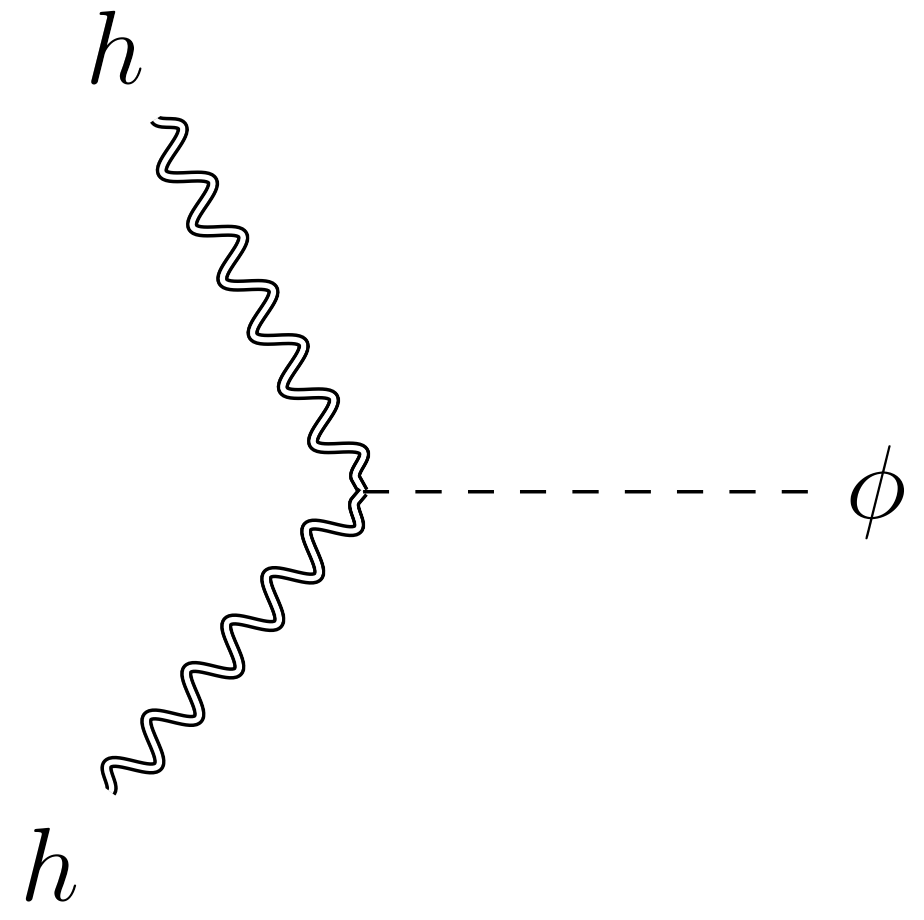

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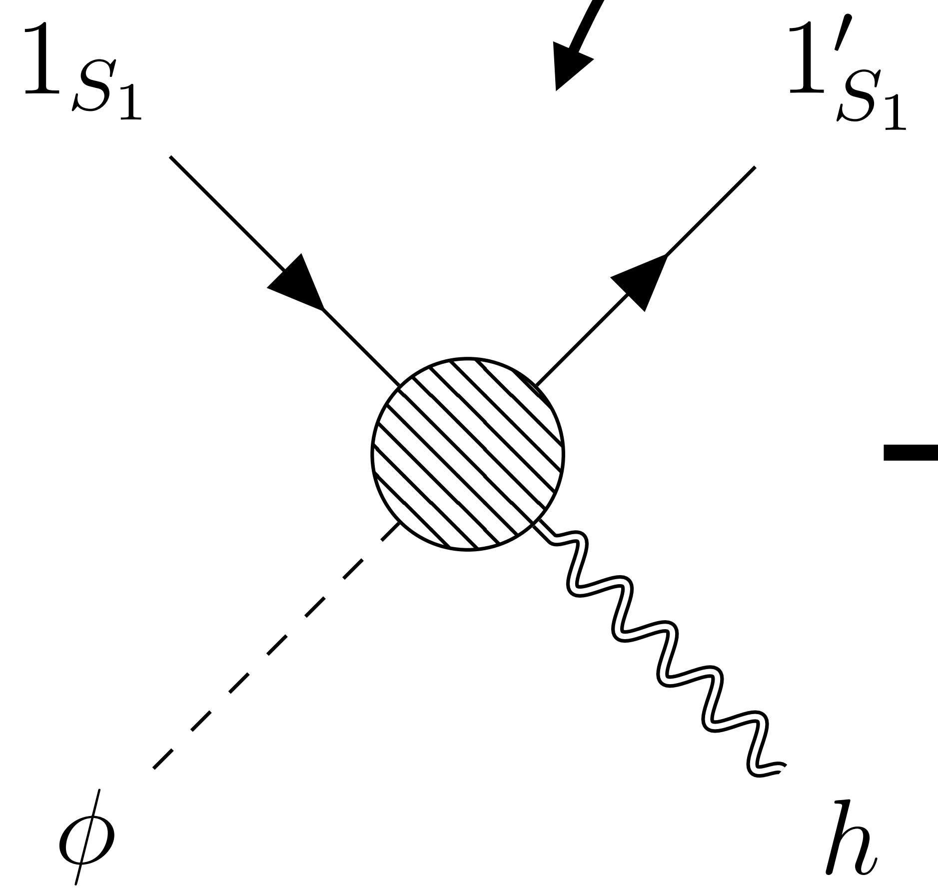
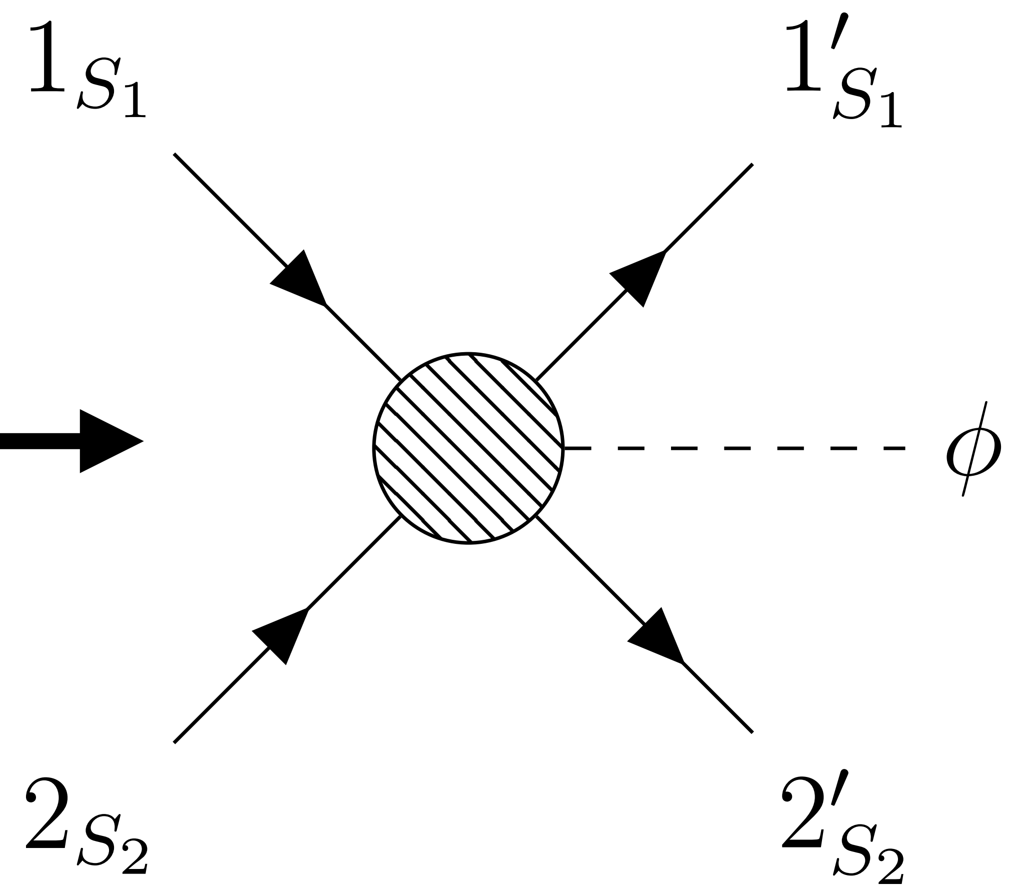
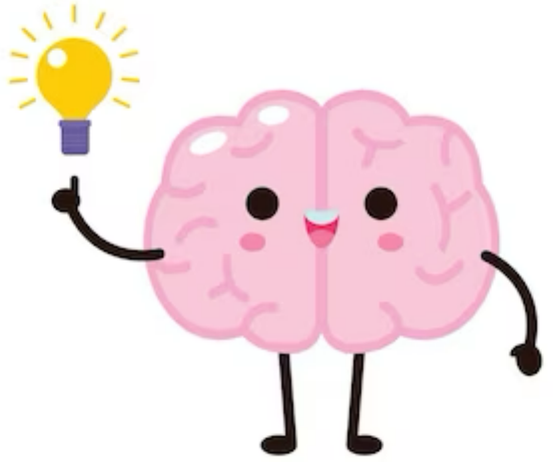


Contributes to the leading order 5-point amplitude



Can compute the Scalar Waveform in the KMOC formalism!

The on-shell way again:



Phys.Rev.D 48 (1993) 3641-3647 [Bekenstein]

We model the BH as a point-particle interacting with the scalar field in a ST fashion

Most general effective metric that respects causality is:

$$\tilde{g}_{\mu\nu} = \underbrace{\exp\left[C\left(\frac{\phi}{M_{Pl}}\right)\right] g_{\mu\nu}}_{\text{Conformal coupling}} + \underbrace{D\left(\frac{\phi}{M_{Pl}}\right)\frac{D_\mu\phi D_\nu\phi}{M_{Pl}^2\Lambda^2}}_{\text{Disformal coupling}},$$

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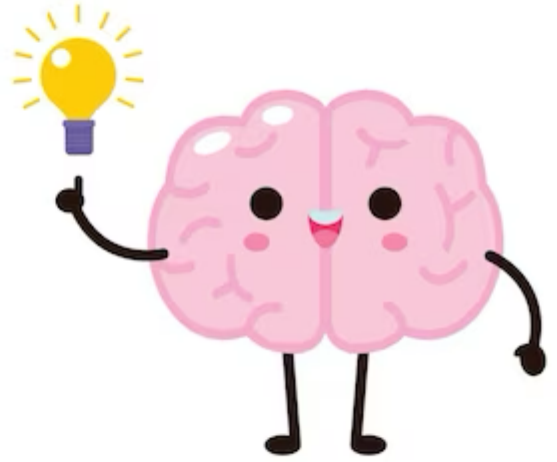

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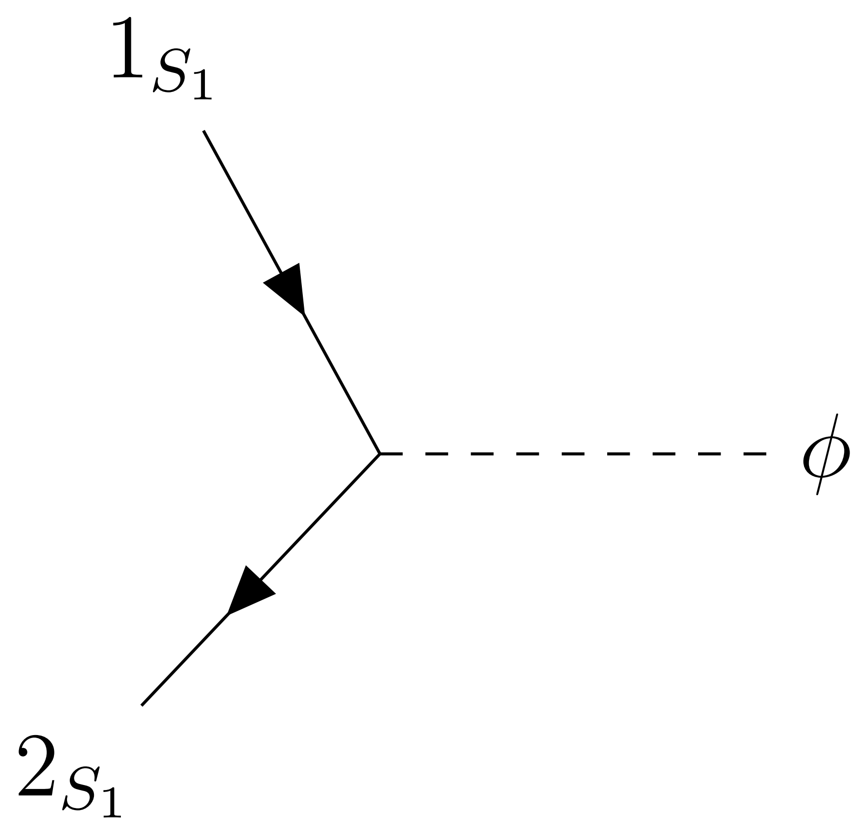
Generate 3-point amplitudes for arbitrary spinning BH:

$$\boxed{\exp\left[C\left(\frac{\phi}{M_{Pl}}\right)\right] \approx 1 + c\frac{\phi}{M_{Pl}}}$$



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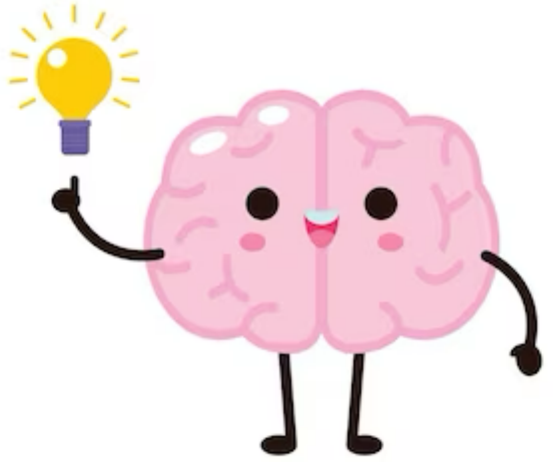

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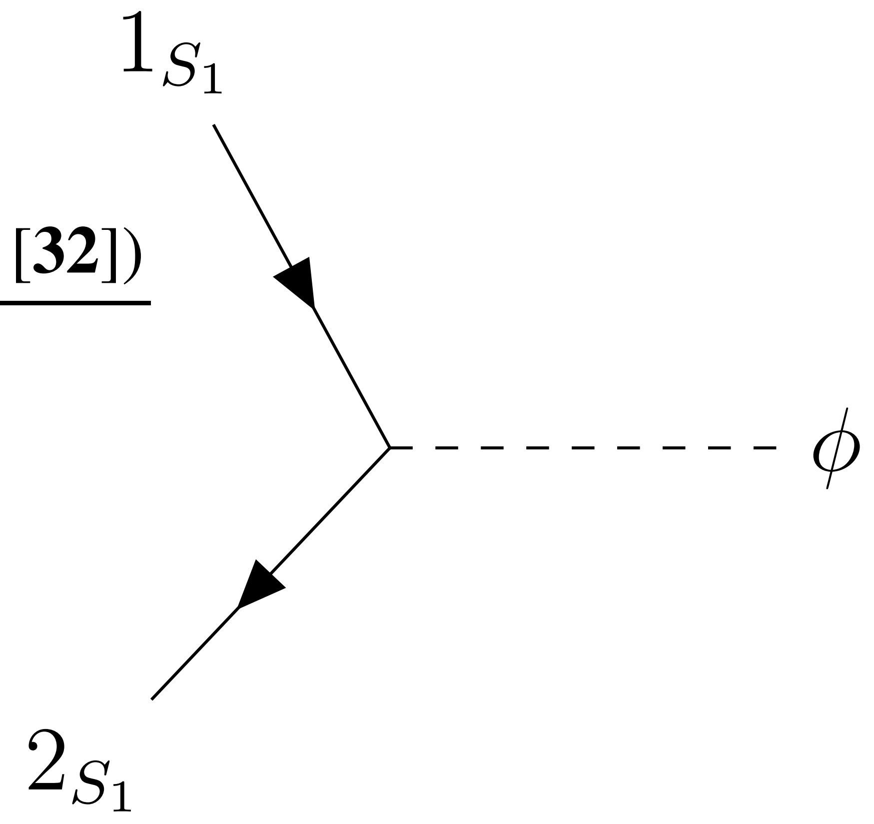

Simple mass redefinition at all spin orders:

$$m \to e^{C/2}m$$

Resemblance to skeletonized action used in GR literature:

Astrophysical Journal, vol. 196, Mar. 1, 1975, pt. 2, p. L59-L62. [Eardley]

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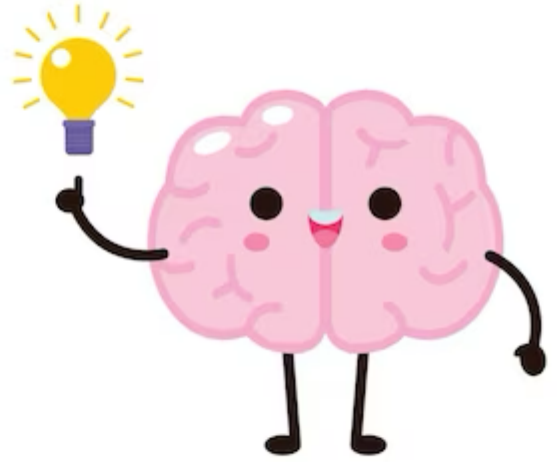

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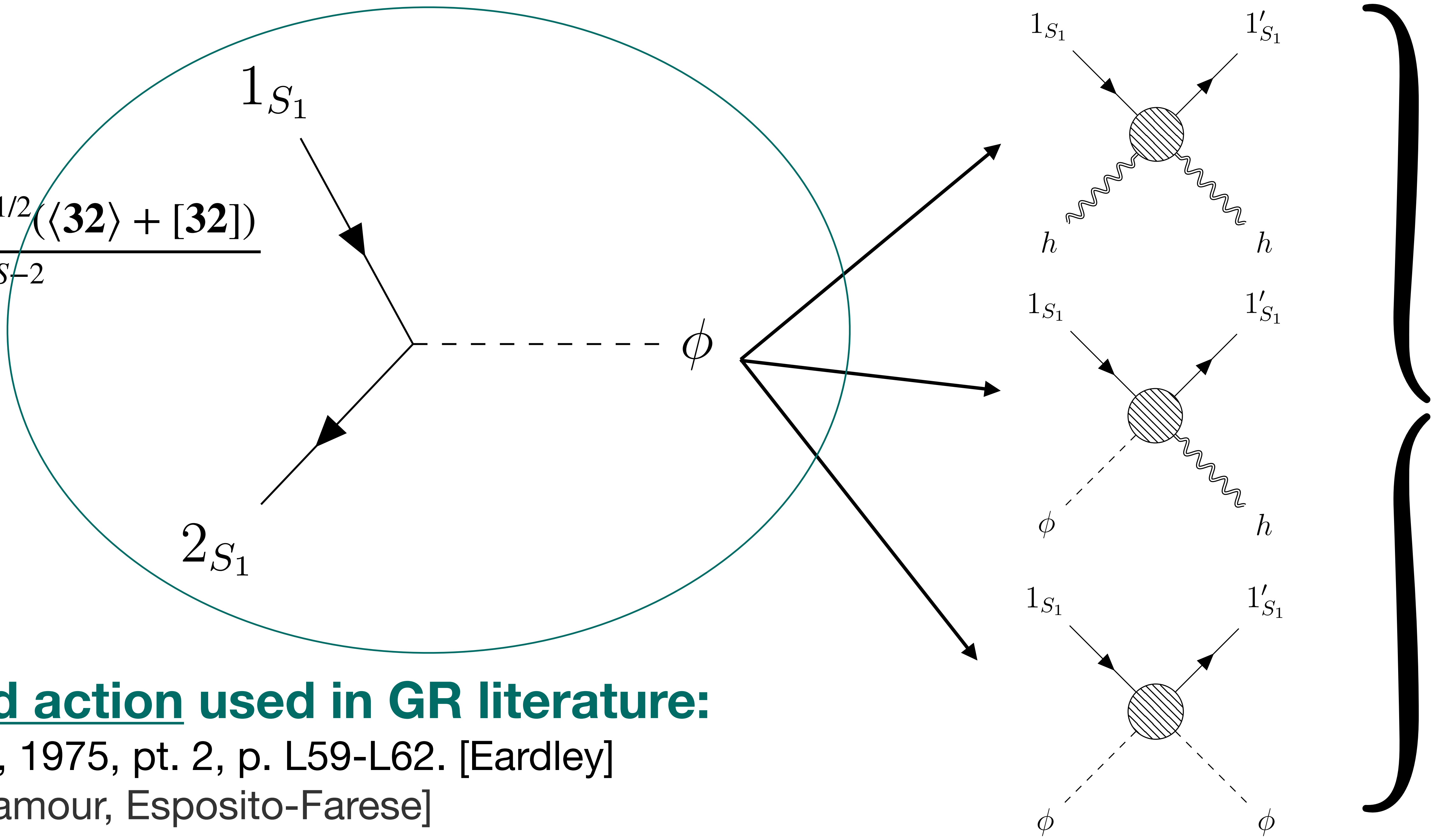

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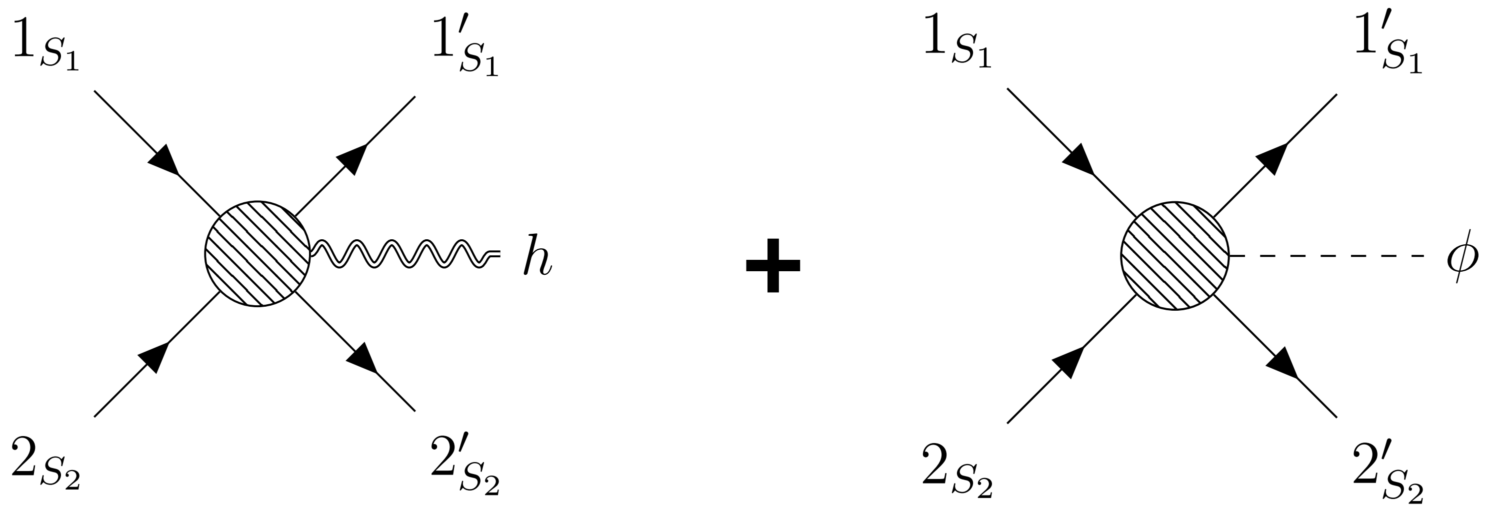

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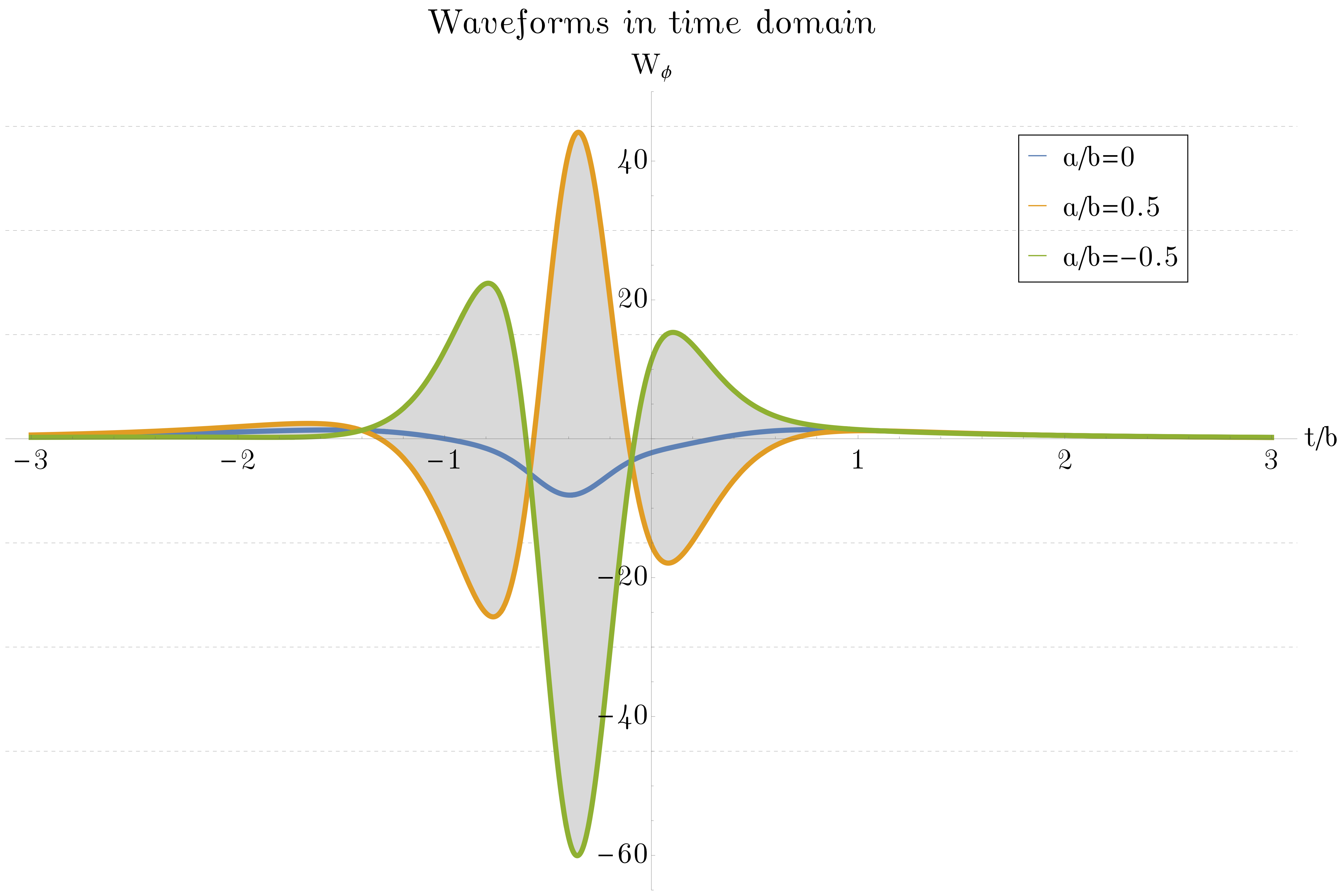
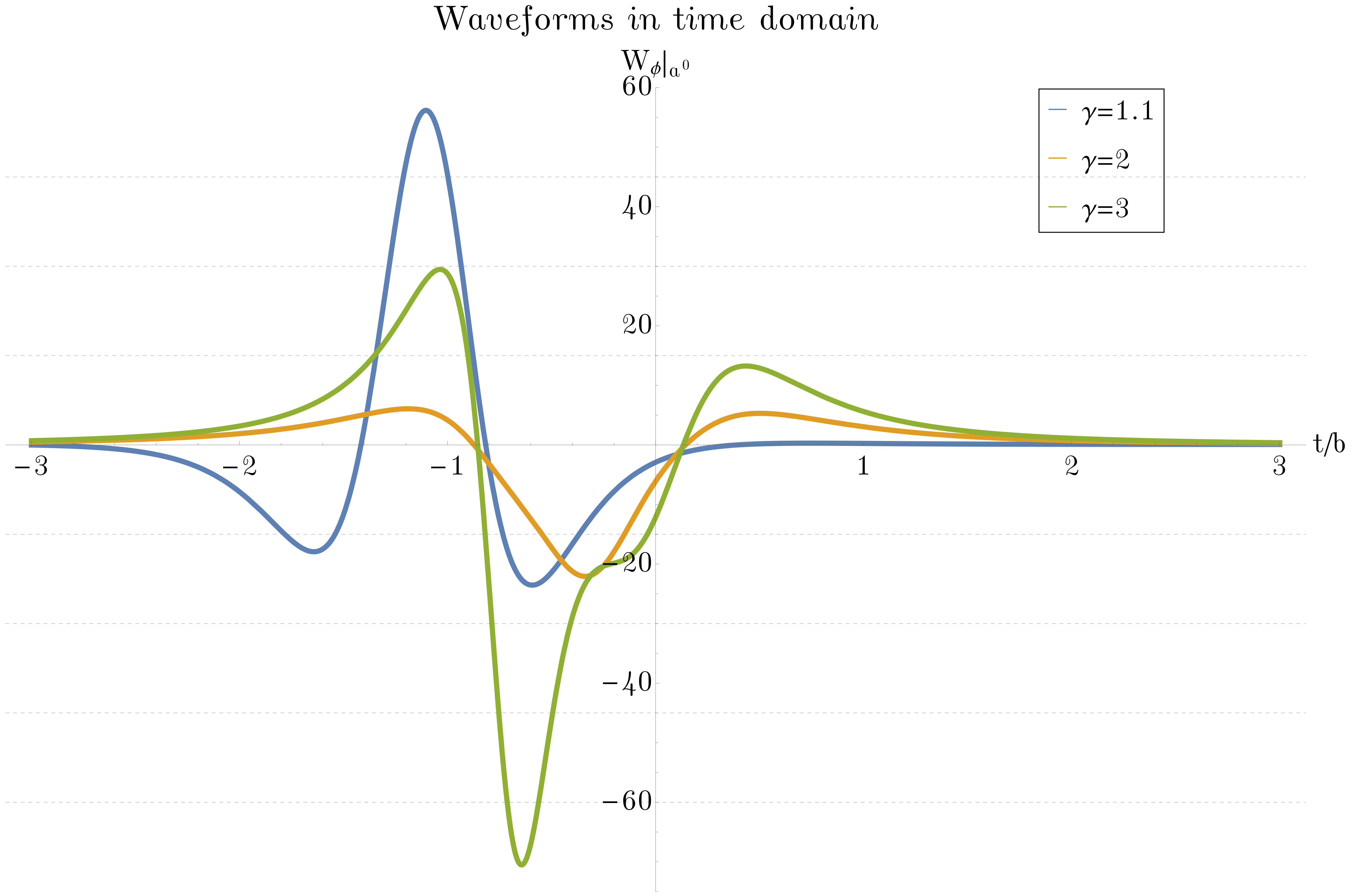
5. Waveforms for scattering of compact objects in shift-symmetric scalar-tensor theories…

LO scalar waveform for scalar Gauss-Bonnet:



1. Scalar waveform for the SGB case up to linear order in spin for different values of the spin magnitude $a \in [-b/2,b/2]$.



2. Scalar piece of the SGB waveform for different values of γ.

6. … and their UV completions from on-shell amplitudes

To appear: 2507.XXXXX [Alviani, Falkowski, **PM**]

$$S_{SGB,DCS} = M_{Pl}\int d^4x\sqrt{-g}\left[\underbrace{\frac{1}{\Lambda^2}\left(\alpha\phi\mathscr{G} + \tilde{\alpha}\phi R\tilde{R}\right)} + \frac{1}{2}\left(\partial^\mu\phi\partial_\mu\phi\right)\right]$$

Nothing more than effective theories!

Can they be mediated by some heavy massive particle in the UV?

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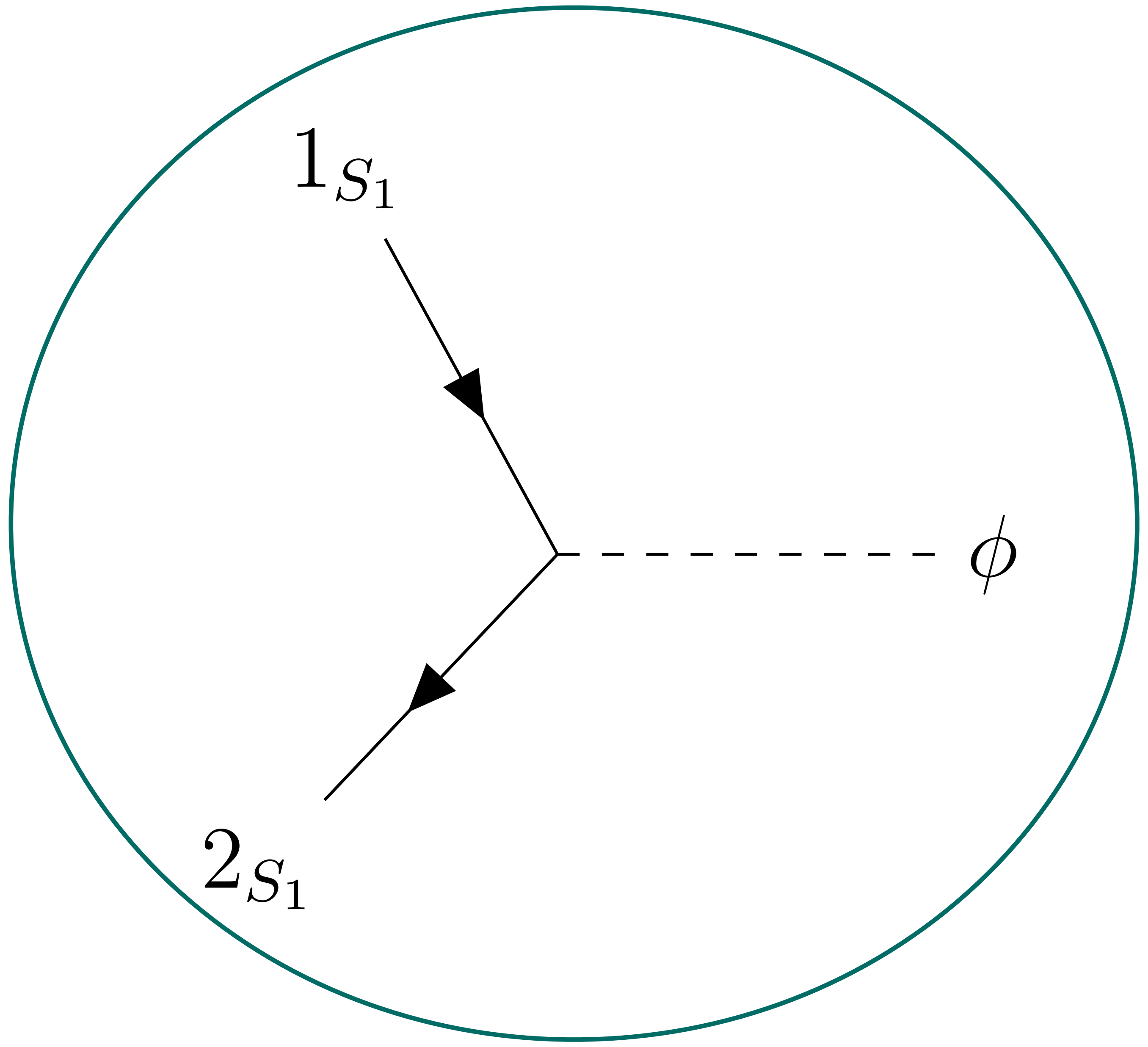

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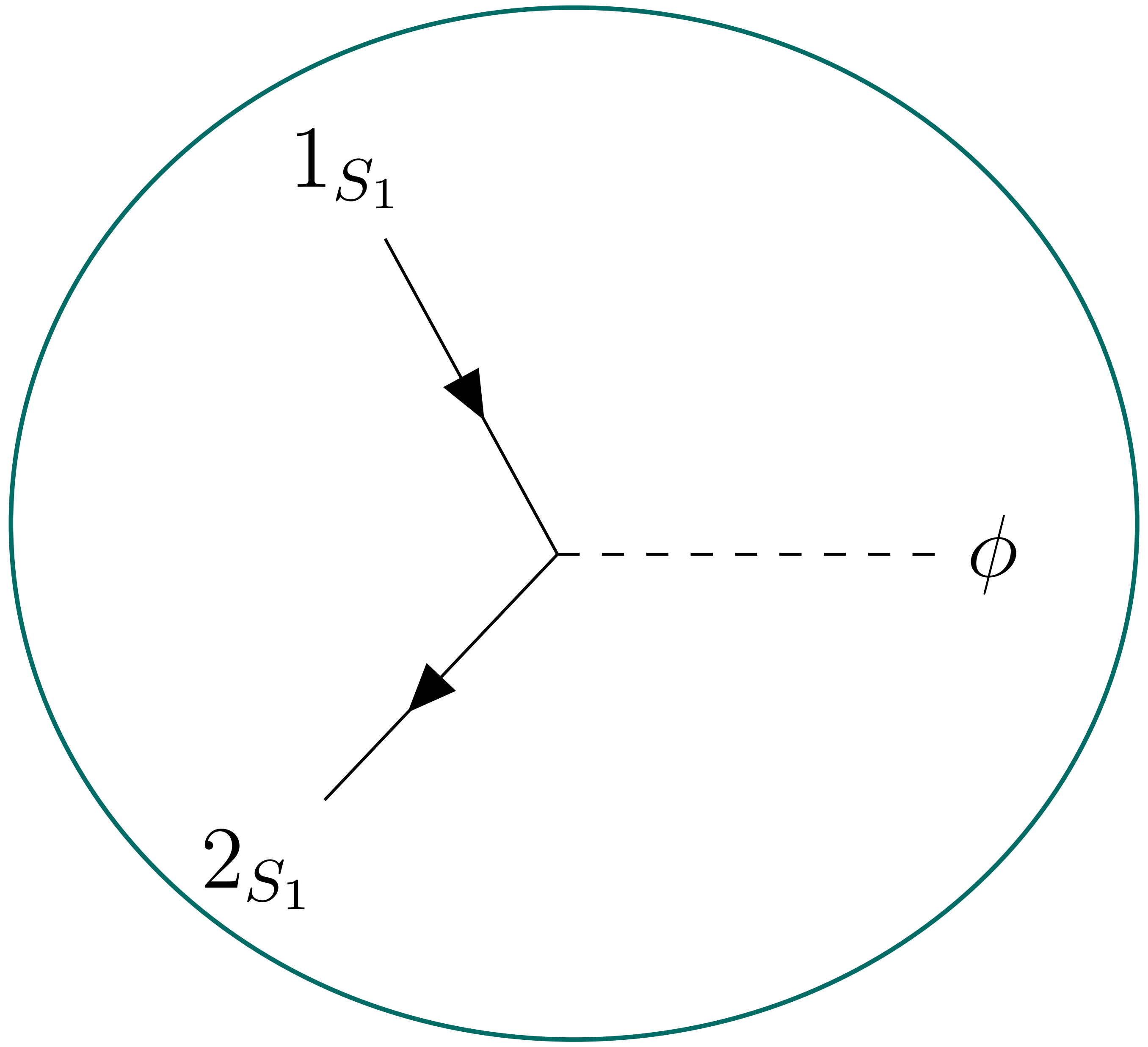
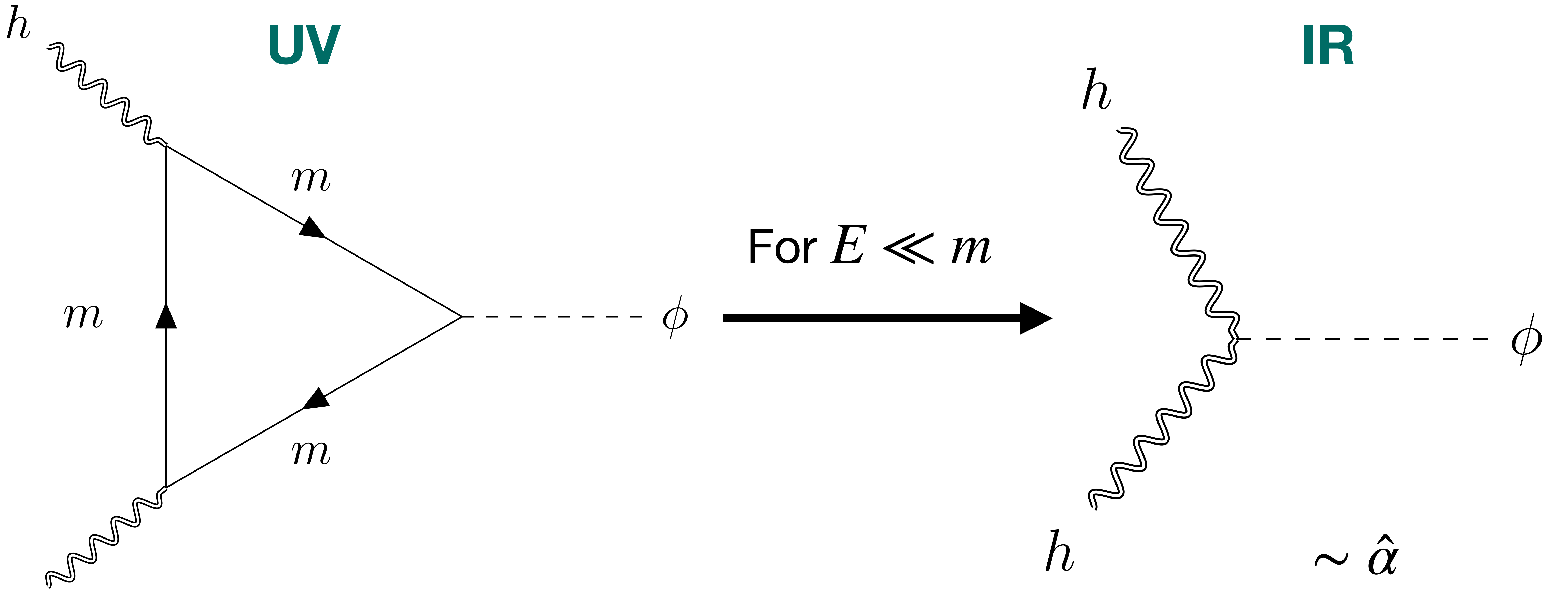
Then, interaction is mediated through this particle in the UV, i.e.





For $E \ll m$

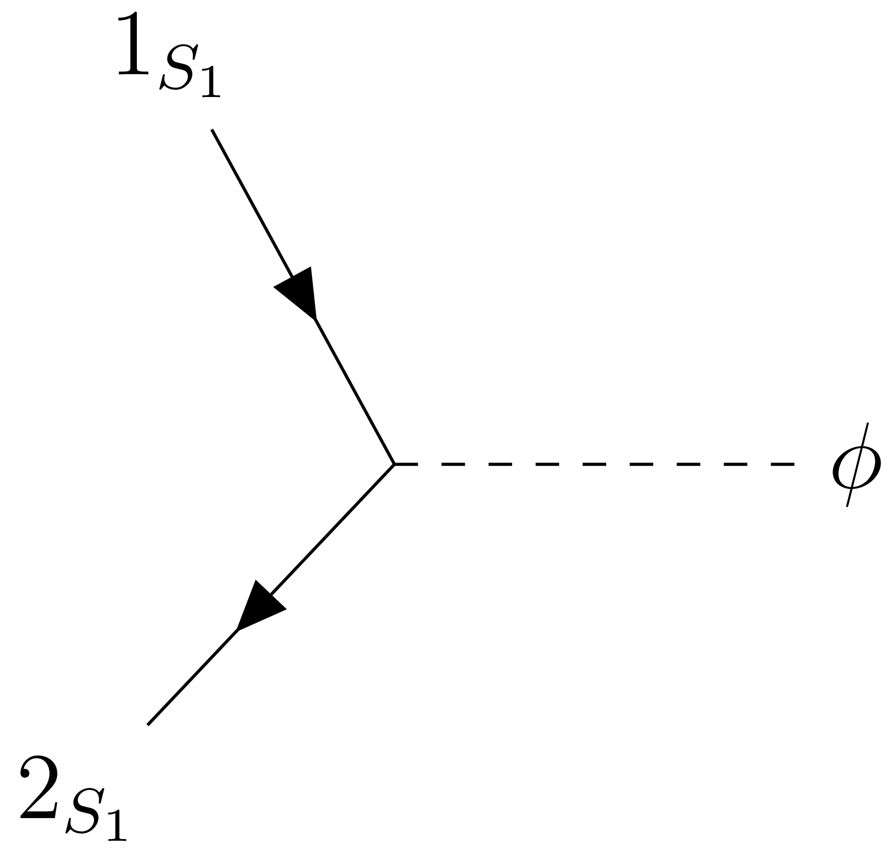
$\sim \hat{\alpha}$

SGB/DCS interaction

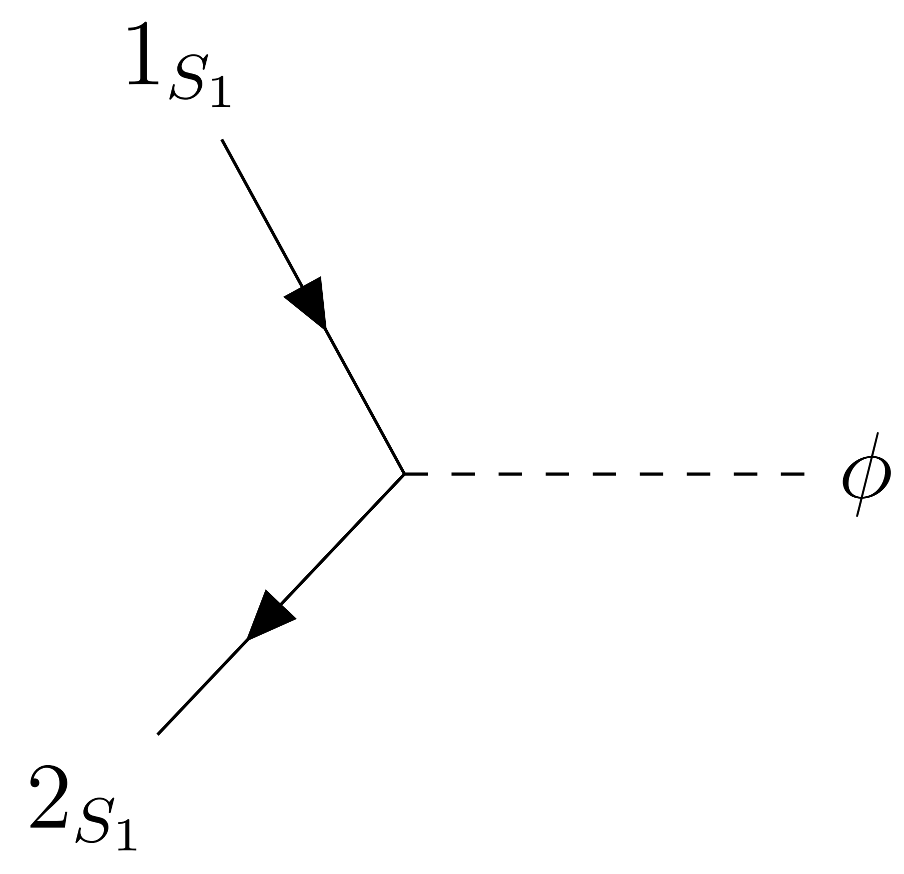
Which amplitude?

Main question:

How should the matter couple in the UV to produce this interaction?



Which amplitude?



Main question:

How should the matter couple in the UV to produce this interaction?

Our guiding principle: Make the IR **shift-symmetry** manifest in the UV amplitudes:

E.g.: In the **on-shell** language: $\mathscr{M}_3^{\rm SS}[1_\phi,2_\Phi,3_{\bar{\Phi}}] = -\frac{\beta}{m^{2S-1}}\left\{\langle\mathbf{32}\rangle+[\mathbf{32}]\right\}^{2S-1}\underbrace{\left\{\langle\mathbf{32}\rangle-[\mathbf{32}]\right\}}_{\sim p_1^\mu}.$

Expectation:

Shift-symmetry encoded by the **vanishing of the amplitudes in the** $p_1^\mu \to 0$ **limit (?)**

Can such UV theories generate shift-symmetric interactions?

Yes and No!

Consequences for EFT matching

General approach:

- Massive scalars: $\mathscr{M}_3[1_\phi, 2_\Phi, 3_{\bar{\Phi}}] = -y_0 m_\Phi$
- Massive fermions: $\mathscr{M}_3[1_\phi, 2_\Phi, 3_{\bar{\Phi}}] = -y_s\left[\langle\mathbf{23}\rangle + [\mathbf{23}]\right] - iy_p\left[\langle\mathbf{23}\rangle - [\mathbf{23}]\right]$
- Massive vectors: $\mathscr{M}_3[1_\phi, 2_V, 3_V] = -\frac{1}{M_*}\left\{y_d\left[\langle\mathbf{23}\rangle^2 + [\mathbf{23}]^2\right] + iy_a\left[\langle\mathbf{23}\rangle^2 - [\mathbf{23}]^2\right] + y_m\langle\mathbf{23}\rangle[\mathbf{23}]\right\}$

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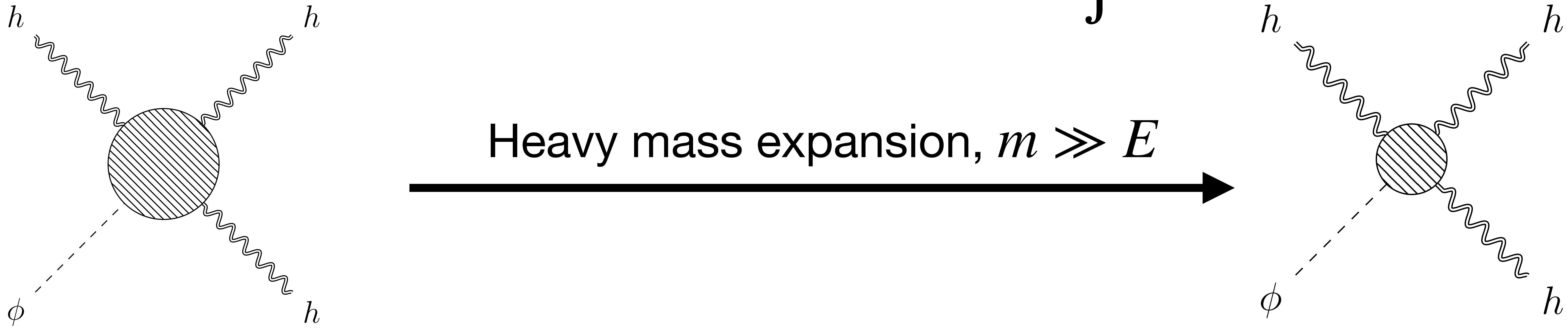
Compute **1-loop discontinuities**, e.g. $\text{Disc}^s\mathscr{M}^{(1)}[1_h^- 2_h^- 3_h^s 4_\phi] = i\int d\Pi_{XY}\mathscr{M}(1_h^- 2_h^- (-Y)_\Phi (-X)_{\bar{\Phi}})\mathscr{M}(3_h^s 4_\phi X_\phi Y_{\bar{\Phi}})$

matter running inside the loops



tree-level diagrams through the SGB interaction

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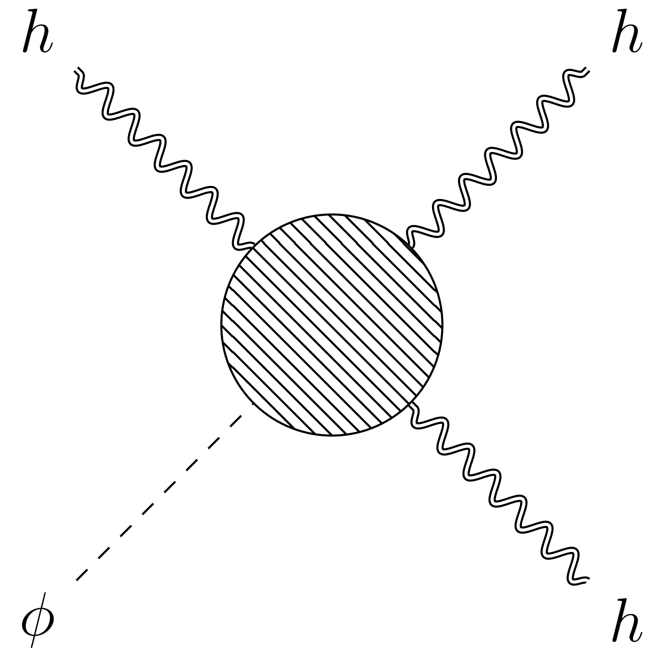
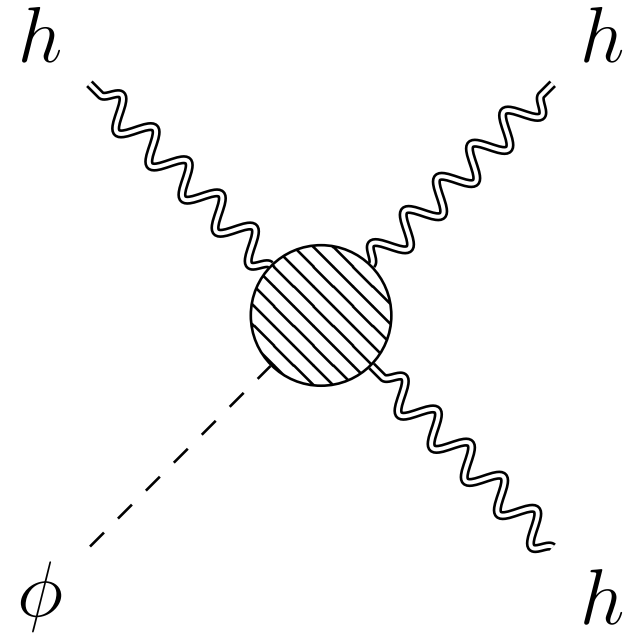


Heavy mass expansion, $m \gg E$ →



tree-level diagrams through the SGB interaction

Results:

1. SGB can be generated, but with **shift-symmetry breaking** operators (may be cancelled but only by fine tuning?)	2. DCS can also be generated, with **unique shift-symmetric** choice, corresponding to a theory of **spin 1/2 fermions** and **a complex scalar** with a **Peccei-Quinn global symmetry** $U(1)_{PQ}$.	3. UV completions with **massive matter** with spin $S \geq 1$ seem to be **non-informative**

Integrals and variables in waveform parametrization:

Single-pole integrals

$$I_2^{\mu_1\ldots\mu_n} = \int \hat{d}w_1 \hat{d}w_2 \hat{\delta}(u_1 w_1)\hat{\delta}(u_2 w_2)\hat{\delta}^{(4)}(w_1 + w_2 - k)\frac{e^{i(b_1 w_1 + b_2 w_2)}}{w_2^2} w_2^{\mu_1} \ldots w_2^{\mu_n}$$

$$= e^{ib_1 k}\int \hat{d}^4 q \hat{\delta}(u_1 \cdot q - u_1 \cdot k)\hat{\delta}(u_2 \cdot q)\frac{e^{-iq\cdot(b_1 - b_2)}}{q^2} q^{\mu_1} \ldots q^{\mu_n}$$

Double-pole integrals

$$J_2^{\mu_1\ldots\mu_n} = \int \hat{d}w_1 \hat{d}w_2 \hat{\delta}(u_1 w_1)\hat{\delta}(u_2 w_2)\hat{\delta}^{(4)}(w_1 + w_2 - k)\frac{e^{i(b_1 w_1 + b_2 w_2)}}{w_2^2 w_1^2} w_2^{\mu_1} \ldots w_2^{\mu_n}$$

$$= e^{ib_1 k}\int \hat{d}^4 q \hat{\delta}(u_1 \cdot q - u_1 \cdot k)\hat{\delta}(u_2 \cdot q)\frac{e^{-iq\cdot(b_1 - b_2)}}{q^2 (k-q)^2} q^{\mu_1} \ldots q^{\mu_n}$$

Parametrization: $q^\mu = z_1 u_1^\mu + z_2 u_2^\mu + z_v \tilde{v}^\mu + z_b \tilde{b}^\mu$

$$v^\mu \equiv \varepsilon^{\mu\alpha\beta\rho} u_{1\alpha} u_{2\beta} \tilde{b}_\rho \quad , \quad \tilde{v}^\mu = \frac{v^\mu}{\sqrt{-v^2}}$$

$$b \equiv \sqrt{-(b_1 - b_2)^2} \quad , \quad \tilde{b}^\mu = \frac{b_1^\mu - b_2^\mu}{b}$$

$$\hat{u}_i = \frac{u_i}{\sqrt{\gamma^2 - 1}}$$

Then follow a residue approach following the logic of

Phys.Rev.D 110 (2024) 4, L041502 [De Angelis, Novichkov, Gonzo]

Integrals and variables in waveform parametrization:

$$I_2^{\mu_1\cdots\mu_n} = -\frac{\pi e^{ib_1 k}(\hat{u}_1 k)^n}{\sqrt{\gamma^2-1}} \int_{-\infty}^{\infty} \frac{dz e^{i(\hat{u}_1 k)bz}}{\sqrt{z^2+1}} \text{Re}\left\{ \left[\gamma \hat{u}_2^{\mu_1} - \hat{u}_1^{\mu_1} + z\tilde{b}^{\mu_1} + i\sqrt{z^2+1}\,\tilde{v}^{\mu_1}\right]\left[\mu_1 \to \mu_2\right]\ldots\left[\mu_1 \to \mu_n\right] \right\}.$$

$$\begin{aligned} J_2^{\mu_1\cdots\mu_n} = \frac{\pi}{2\sqrt{\gamma^2-1}} \int_{-\infty}^{\infty} \frac{dz}{\sqrt{z^2+1}} \Bigg\{ & (\hat{u}_1 k)^{n-1} e^{ib_1 k + i(\hat{u}_1 k)bz} \text{Re}\left[\frac{\left[\gamma \hat{u}_2^{\mu_1} - \hat{u}_1^{\mu_1} + z\tilde{b}^{\mu_1} + i\sqrt{z^2+1}\,\tilde{v}^{\mu_1}\right]\ldots\left[\mu_1 \to \mu_n\right]}{\gamma(\hat{u}_2 k) - (\hat{u}_1 k) + (\tilde{b}k)z + i(\tilde{v}k)\sqrt{z^2+1}} \right] \\ & + (\hat{u}_2 k)^{n-1} e^{ib_2 k + i(\hat{u}_2 k)bz} \text{Re}\left[\frac{\left[\frac{k^{\mu_1}}{\hat{u}_2 k} + \hat{u}_2^{\mu_1} - \gamma \hat{u}_1^{\mu_1} + z\tilde{b}^{\mu_1} - i\sqrt{z^2+1}\,\tilde{v}^{\mu_1}\right]\ldots\left[\mu_1 \to \mu_n\right]}{\gamma(\hat{u}_1 k) - (\hat{u}_2 k) - (\tilde{b}k)z + i(\tilde{v}k)\sqrt{z^2+1}} \right] \Bigg\}. \end{aligned}$$

Resolvability (no-hair compact objects):

$$\alpha_g \sim \frac{m^2}{M_{Pl}^2} = mR_s$$

Scalar radiation for no-hair objects is resolvable if: $\alpha_g \left(\frac{R_s}{b}\right)^2 \frac{1}{\Lambda^4 b^4} \gg 1,$

For example, for $\Lambda \simeq 2 \times 10^{-18} GeV$ and $m \simeq M_{\odot} \simeq 10^{57} \sim GeV$ (corresponding to $R_s \simeq 1.5 \times 10^{19} GeV^{-1}$) we have $b_{\max} \sim 10^{11} R_s$

Linear in spin order effects for the same observable: $\alpha_g \left(\frac{R_s}{b}\right)^2 \frac{1}{\Lambda^4 b^4} \left(\frac{S}{bm}\right) \gg 1$

For our benchmark point, $m \sim M_{\odot}, \Lambda \sim 0.1 \mathrm{km}^{-1}$, this implies $S \gtrsim 10^6 (b/R_s)^7$, with $S_{\max} \sim 10^{76}$ for Kerr black holes in GR.

More results:

The scalar waveform for dynamical Chern-Simons:

$$
\begin{aligned}
W_{\tilde{\phi}} = {} & \frac{m_1 m_2}{8\pi^2 M_{Pl}^3 \Lambda^2 (\hat{u}_1 n)^2 b^3} \left(2\tilde{\alpha}(\tilde{v}n) \frac{d^2}{dz^2} \left\{ \frac{1}{\sqrt{z^2+1}} \left[\gamma z - (\gamma^2-1)(\hat{u}_2 n) \frac{z\big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n)\big] - (\tilde{b}n)}{\big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)z\big]^2 + (\tilde{v}n)^2(z^2+1)} \right] \right\} \right. \\
& + \frac{\tilde{\alpha}}{b\sqrt{\gamma^2-1}} \mathrm{Re} \frac{d^3}{dz^3} \left\{ \frac{1}{\sqrt{z^2+1}} \left(\frac{(a_1 n)}{(\hat{u}_1 n)} - \gamma(a_1 \hat{u}_2) - (a_2 \hat{u}_1) + (a_2^A - a_1^A)\big[z\tilde{b}^A + i\sqrt{z^2+1}\,\tilde{v}^A\big] \right) \left(\frac{2(\gamma^2-1)^2(\hat{u}_2 n)^2}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)z + i(\tilde{v}n)\sqrt{z^2+1}} - (\hat{u}_1 n) - \gamma\big(2\gamma^2-3\big)(\hat{u}_2 n) + \big(2\gamma^2-1\big)\big[z(\tilde{b}n) + i\sqrt{z^2+1}(\tilde{v}n)\big] \right) \right\} \\
& \left. - \frac{1}{\sqrt{\gamma^2-1}} \frac{\tilde{C}_1 a_1^A}{b} \frac{d^3}{dz^3} \left\{ \frac{1}{\sqrt{z^2+1}} \left[(2\gamma^2-1)\big[z^2(\tilde{b}n)\tilde{b}^A - (z^2+1)(\tilde{v}n)\tilde{v}^A\big] - (\gamma^2-1)n^A + \gamma(\gamma^2-2)(\hat{u}_1 n)\hat{u}_2^A - (\gamma^2-2)(\hat{u}_2 n)\hat{u}_2^A + z\gamma(\tilde{b}n)\hat{u}_2^A - z\gamma^2(\hat{u}_1 n)\tilde{b}^A + z\gamma(\hat{u}_2 n)\tilde{b}^A \right] \right\} \right) \Bigg|_{z=T_1} + (1 \leftrightarrow 2) + \mathcal{O}(a^2) \,.
\end{aligned}
$$

LO Scalar Waveforms for CC coupling-Spinning part:

$$
\begin{aligned}
W_{\phi}^{(1)} = {} & \frac{m_1 m_2}{32\pi^2 M_{Pl}^3 (\hat{u}_1 n)^2 b^2} \frac{\partial}{\partial z} \left[\frac{1}{\sqrt{z^2+1}} \mathrm{Re} \left\{ c_1 \big[(\tilde{v}n)z - is_1(\tilde{b}n)\sqrt{z^2+1}\big] \big[-(\hat{u}_1 a_2) + z(\tilde{b}a_2) + is_1\sqrt{z^2+1}(\tilde{v}a_2)\big] \left(\frac{\gamma}{\gamma^2-1} - \frac{(\hat{u}_2 n)}{-\hat{u}_1 n + \gamma(\hat{u}_2 n) + z(\tilde{b}n) + is_1\sqrt{z^2+1}(\tilde{v}n)} \right) \right. \right. \\
& \left. \left. - \frac{c_2(\hat{u}_1 n)}{-\hat{u}_1 n + \gamma(\hat{u}_2 n) + z(\tilde{b}n) + is_1\sqrt{z^2+1}(\tilde{v}n)} \left\{ [is_1(\tilde{b}n)\sqrt{z^2+1} - (\tilde{v}n)z](\hat{u}_2 a_1) + \big[\gamma(\tilde{v}n) + is_1[\gamma(\hat{u}_1 n) - (\hat{u}_2 n)]\sqrt{z^2+1}\big](\tilde{b}a_1) + \big[[(\hat{u}_2 n) - \gamma(\hat{u}_1 n)]z - \gamma(\tilde{b}n)\big](\tilde{v}a_1) \right\} \right\} \right] + (1 \leftrightarrow 2) \,.
\end{aligned}
$$

More results:

LO Scalar Waveforms from non-conformal couplings:

$$W^{(1)}_{non-conf,\phi} = \frac{m_1 m_2}{32\pi^2 M_{Pl}^3 (\hat{u}_1 n)^2 b^2} \frac{\partial}{\partial z} \Bigg[\frac{1}{\sqrt{z^2+1}} \text{Re} \Bigg\{ c_1 \Big[(\tilde{v} n) z - i s_1 (\tilde{b} n) \sqrt{z^2+1} \Big] \Big[-(\hat{u}_1 a_2) + z(\tilde{b} a_2) + i s_1 \sqrt{z^2+1} (\tilde{v} a_2) \Big] \Big(\frac{\gamma}{\gamma^2-1} - \frac{(\hat{u}_2 n)}{-\hat{u}_1 n + \gamma(\hat{u}_2 n) + z(\tilde{b} n) + i s_1 \sqrt{z^2+1} (\tilde{v} n)} \Big)$$

$$- \frac{c_2 (\hat{u}_1 n)}{-\hat{u}_1 n + \gamma(\hat{u}_2 n) + z(\tilde{b} n) + i s_1 \sqrt{z^2+1} (\tilde{v} n)} \Big\{ [i s_1 (\tilde{b} n) \sqrt{z^2+1} - (\tilde{v} n) z] (\hat{u}_2 a_1) + \Big[\gamma (\tilde{v} n) + i s_1 [\gamma(\hat{u}_1 n) - (\hat{u}_2 n)] \sqrt{z^2+1} \Big] (\tilde{b} a_1) + \Big[[(\hat{u}_2 n) - \gamma(\hat{u}_1 n)] z - \gamma(\tilde{b} n) \Big] (\tilde{v} a_1) \Big\} \Bigg\} \Bigg] + (1 \leftrightarrow 2)\,.$$

LO Gravitational Waveforms from non-conformal couplings:

$$W^{(1)}_{non-conf,h} = -\frac{C_d c_2 m_1 m_2 \varepsilon^{\mu\nu\rho\alpha}}{1024\pi^2 M_{Pl}^3 \sqrt{\gamma^2-1}} \frac{\partial}{\partial z} \Bigg\{ \frac{1}{\sqrt{z^2+1}} \text{"Re"} \Bigg\{ \frac{(\lambda_n [\gamma \hat{u}_{2\mu} + z \tilde{b}_\mu + i\sqrt{z^2+1}\, \tilde{v}_\mu] \hat{u}_{1\nu} \sigma_\alpha \bar{\sigma}_\beta \lambda_n)}{-(\hat{u}_1 n) + \gamma(\hat{u}_2 n) + z(\tilde{b} n) + i\sqrt{z^2+1} (\tilde{v} n)}$$

$$\times (\lambda_n [\gamma \hat{u}_2 \sigma + z \tilde{b} \sigma + i\sqrt{z^2+1}\, \tilde{v} \sigma] (\hat{u}_1 \bar{\sigma}) \lambda_n) \Big[\frac{(n a_1)}{(\hat{u}_1 n)} - \gamma(\hat{u}_2 a_1) - z(\tilde{b} a_1) - i\sqrt{z^2+1} (\tilde{v} a_1) \Big] \Bigg\} \Bigg|_{z=T_1} + + (1 \leftrightarrow 2)\,.$$

Contact terms' contributions: An inevitable problem

Huge list of authors who have contributed in solving this problem in QED and Gravity

→ The four -point amplitudes we construct are **healthy at all spin orders**, but **still have to include contact terms' deformations** which can contribute **classically** (very similar to what has already been observed in QED and GR): **Can be done order by order** in the spin expansion, but need a matching procedure to fix their coefficients

$$\mathscr{M}[1_{\Phi_i}2_{\bar{\Phi}_i}3^s_h4_\phi] = \frac{4\hat{\alpha}}{M_{Pl}^2\Lambda^2}t(p_1\epsilon_s)^2\left\{e^{q\cdot a} + P_{\xi,s}(p_3a_1, p_4a_1, wa_1, |a_1|')\right\} \quad ,$$

$$q^\mu = p_3^\mu + p_4^\mu \quad , \quad w^\mu = \frac{(t-m^2)}{2(p_1\epsilon_{3,-})}\epsilon^\mu_{3,-} \quad , \quad |a_1|' = \frac{(t-m^2)}{2m}|a_1|$$

Extending the treatment of
JHEP 05 (2023) 211[Bautista, Guevara, Kavanagh, Vines]

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“Unitary” piece
No spurious poles!

$$q^\mu = p_3^\mu + p_4^\mu \quad , \quad w^\mu = \frac{(t-m^2)}{2(p_1\epsilon_{3,-})}\epsilon^\mu_{3,-} \quad , \quad |a_1|^{'} = \frac{(t-m^2)}{2m}|a_1|$$

e.g. up to $\mathscr{O}(a_1^2)$:

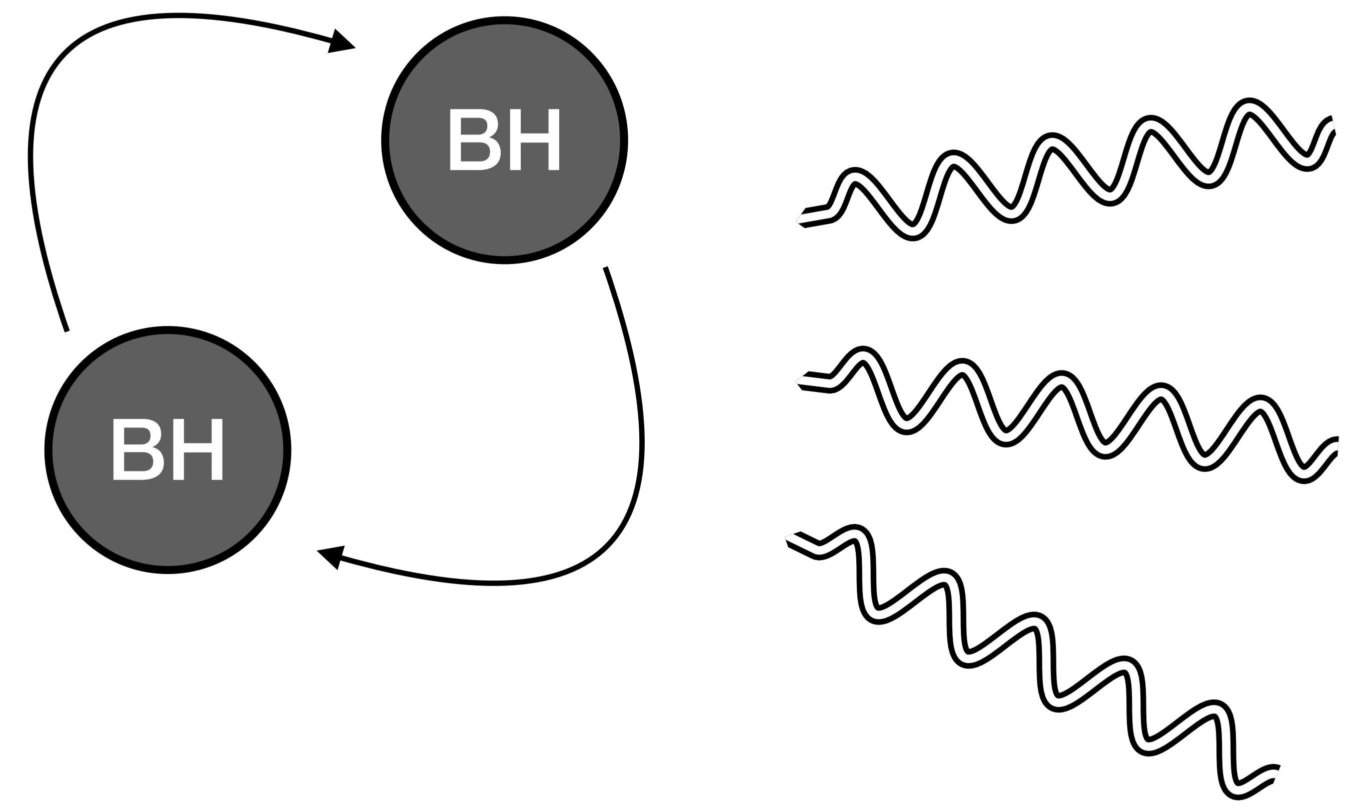
$$P_{\xi,-}\left((p_3a_1),(p_4a_1),(wa_1),|a_1|^{'}\right)\Big|_{\mathscr{O}(a_1^2)} = \xi^{-1}\left(C^{-1}_{000}(wa_1)^2 + D^{-1}_{000}(wa_1)|a_1|^{'}\right)$$

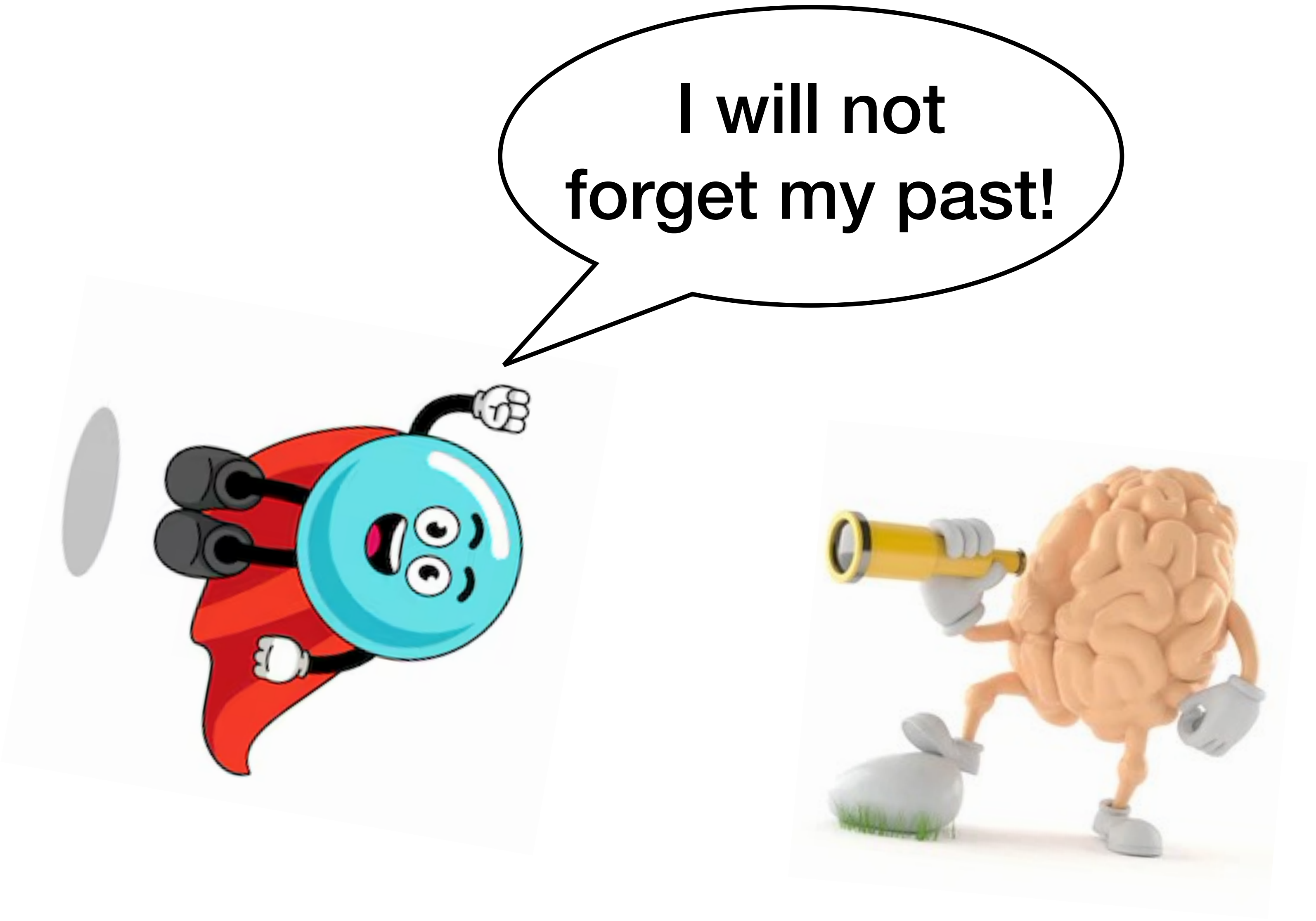
$$+\xi^0\Bigg(C^0_{000}((wa_1)-(p_3a_1)) + C^0_{100}((wa_1)-(p_3a_1))(p_3a_1) + C^0_{010}((wa_1)-(p_3a_1))(p_4a_1)$$

$$+C^0_{001}((wa_1)-(p_3a_1))(wa_1) + C^0_{000}C^0((wa_1)-(p_3a_1))|a_1|^{'}\Bigg).$$

Extending the treatment of
JHEP 05 (2023) 211[Bautista, Guevara, Kavanagh, Vines]

Memory effect:





- **Scalar memory effect:** $\Delta\phi = W_\phi(t=\infty) - W_\phi(t=-\infty)$
- **Gravitational memory effect:** $\Delta h = h(t=\infty) - h(t=-\infty)$

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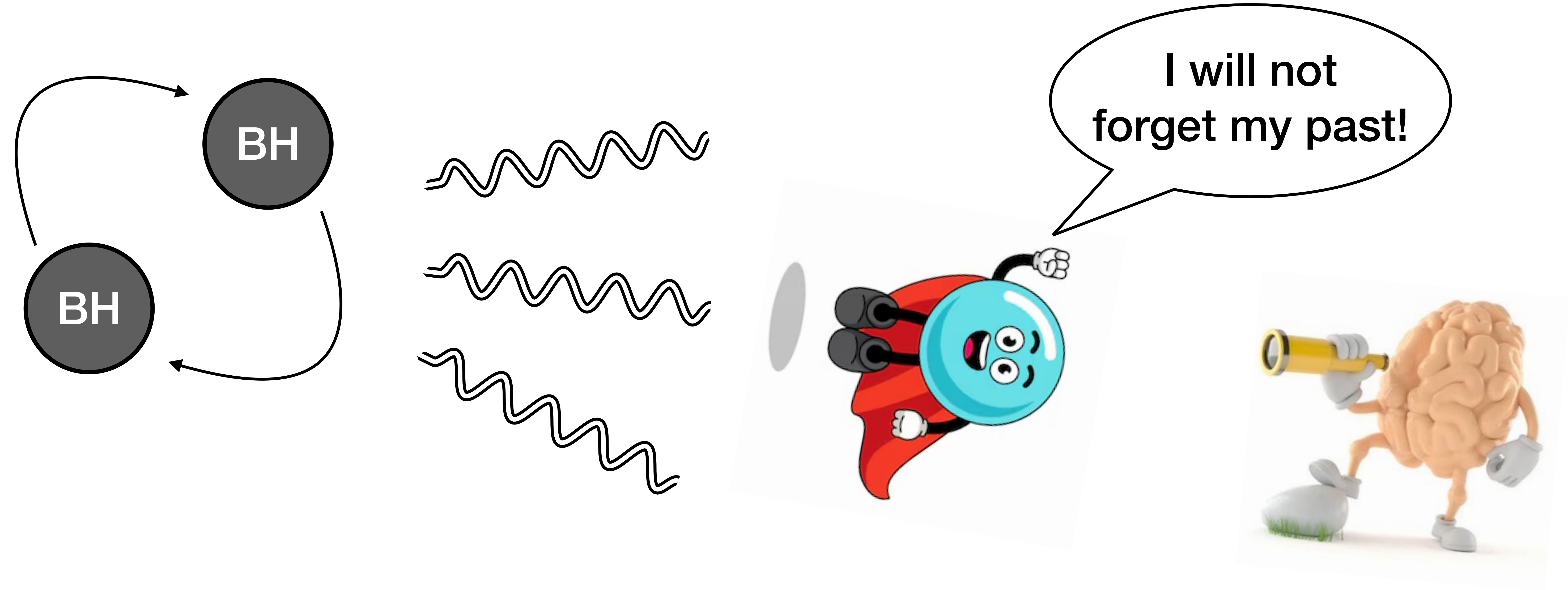

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At LO, can be related to the soft behaviour of the 5-point amplitude

$$\mathscr{M}_5 \to Soft \times \mathscr{M}_4$$

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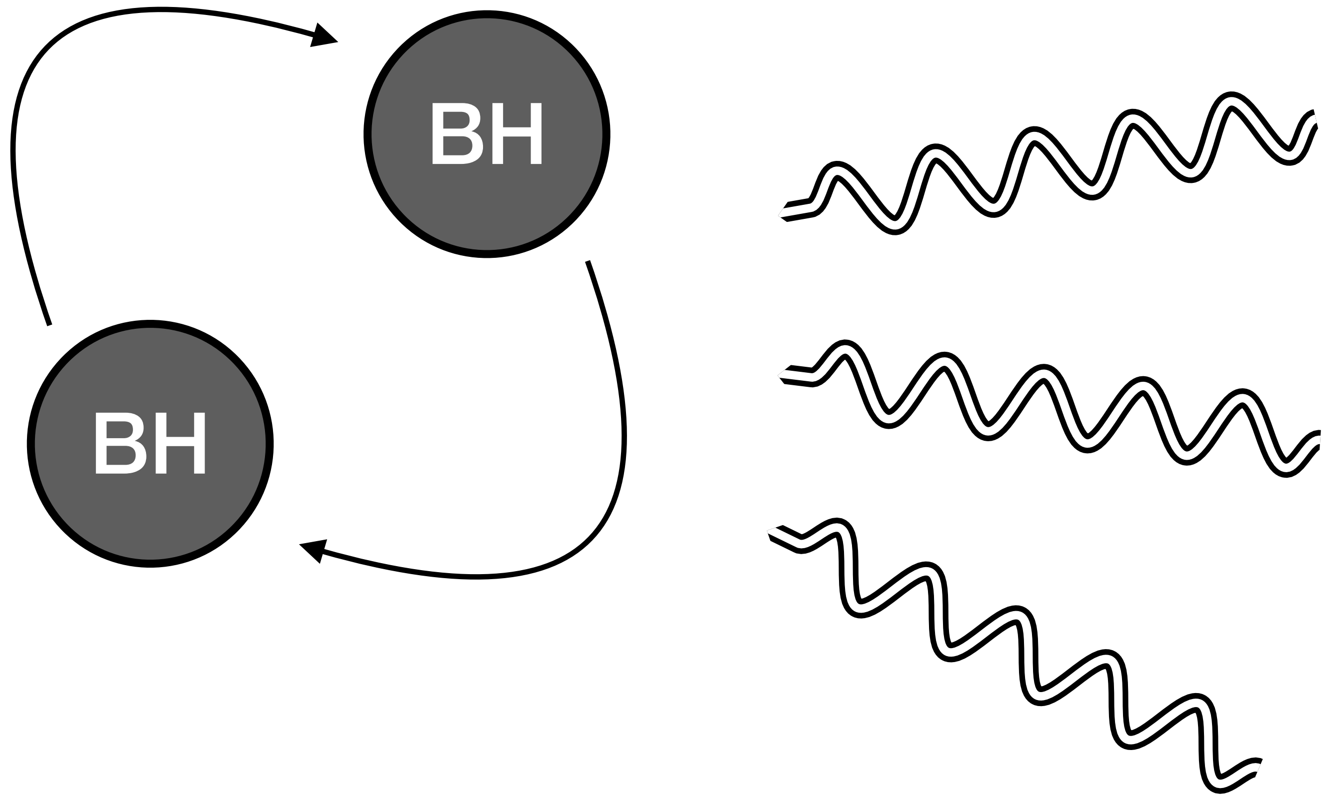

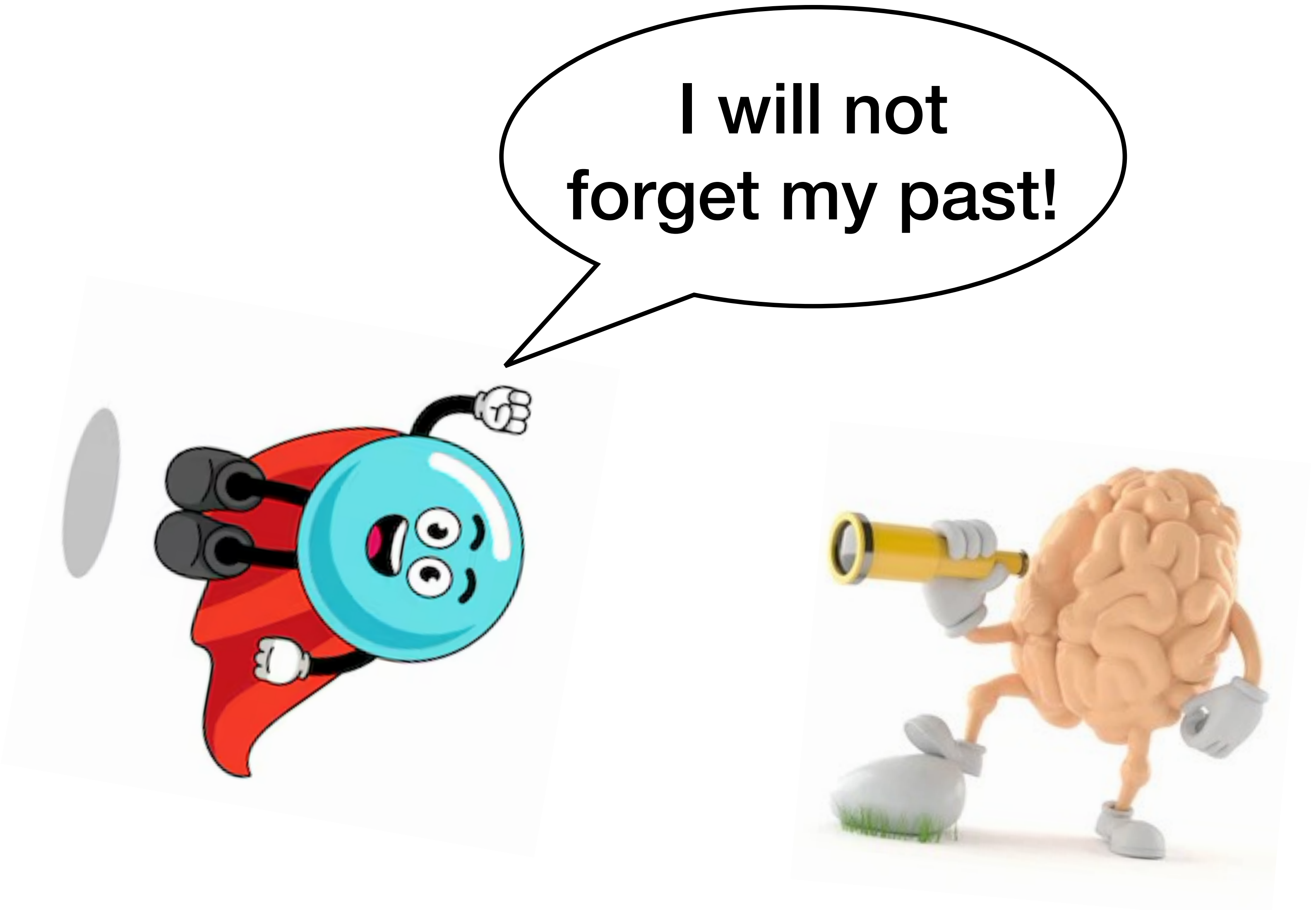

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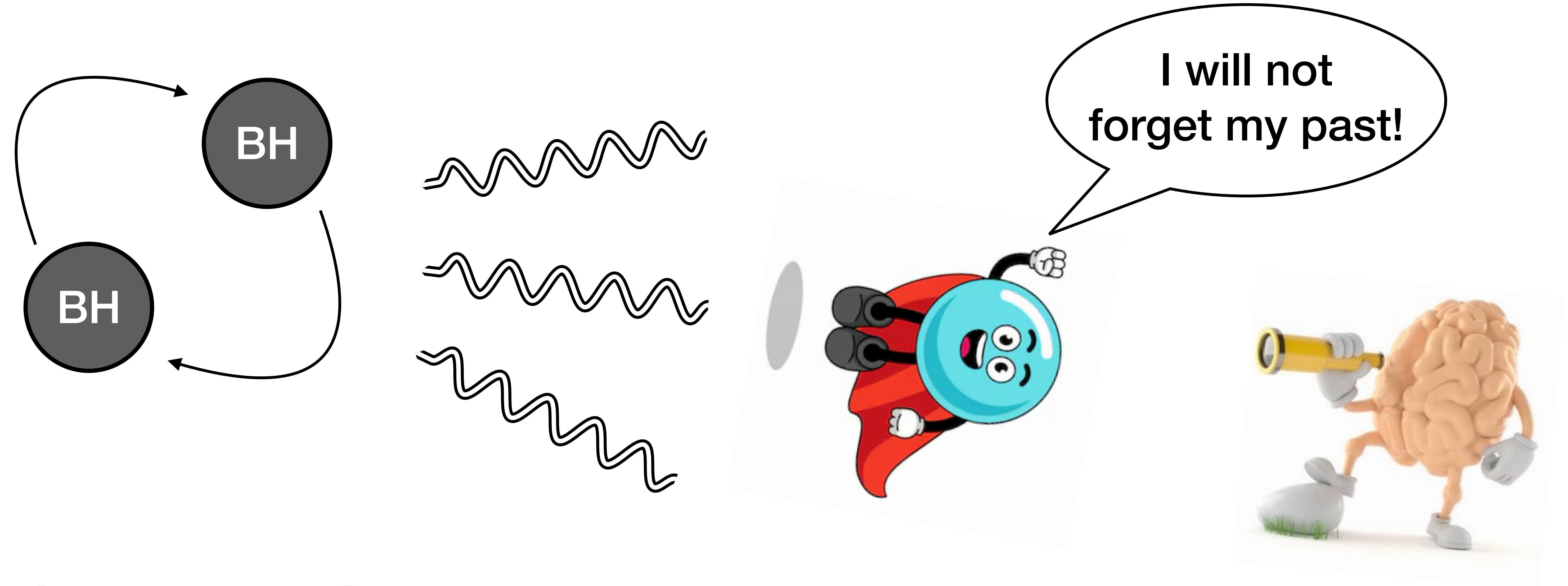
→ **No-hair objects** have **no leading order memory!**
Reason: There are no modifications to the 4-point amplitudes at tree-level.

A more general understanding: Higher order interactions essentially contribute factors of $\dfrac{\partial^n}{\Lambda^n}$ in the action. At the level of the 5-point Amplitude, this translates to:

$$\frac{\partial^n}{\Lambda^n} \to \frac{k^n}{\Lambda^n} \sim \frac{\omega^n}{\Lambda^n} \quad , \text{ for } \quad k^\mu = \omega n^\mu$$ → kills the amplitude in the soft limit

Memory effect:



→ **Hairy objects do have leading order memory!**

→ The gravitational memory effect can be elegantly expressed in terms of the classical soft factor:

$$\Delta h = -\frac{c_1 c_2 m_1 m_2}{4\pi M_{Pl}^3 \sqrt{\gamma^2 - 1}} \frac{1}{|b|} S^{\mathrm{cl}}_{W,s}(n, \tilde{b})$$

$$\Cap$$

$$\boxed{\mathcal{M}^{(0)}_{ST}[1^{S_1}_{\Phi_1} 2^{S_2}_{\Phi_2} 3^{S_1}_{\bar{\Phi}_1} 4^{S_2}_{\bar{\Phi}_2}]}$$

$$S^{\mathrm{cl}}_{W,s}(k,q) = \epsilon_{\mu\nu,s}\left[\frac{p_1^\mu q^\nu + p_1^\nu q^\mu}{p_1 k} - \frac{p_1^\mu p_1^\nu (qk)}{(p_1 k)^2}\right] + (1 \leftrightarrow 2)$$

→ Scalar memory trivially computed: $\Delta\phi = \dfrac{m_1 m_2 (2\gamma^2 - 1)(\tilde{b}n)\left[c_1(\hat{u}_2 n)^2 - c_2(\hat{u}_1 n)^2\right]}{32\pi^2 M_{Pl}^3 (\gamma^2 - 1)^{3/2} (\hat{u}_1 n)^2 (\hat{u}_2 n)^2 b}$

Another perspective: The scalar-charge toy model

Recent work has been using a scalar-charge toy model in order to compare the PM and SF approaches.

$$\mathscr{L} \supset \sqrt{-g}\Big[-2\sqrt{\pi}m_1 Q\psi\phi_1^2\Big]$$

Phys.Rev.D 108 (2023) 2, 024025 [Barack, Bern, Hermann, Long, Parra-Martinez, Roiban, Ruf, Shen, Solon, Teng, Zeng]

arXiv:2304.09200v2 [hep-th] 12 Jul 2023

Comparison of post-Minkowskian and self-force expansions: Scattering in a scalar charge toy model

Leor Barack,[1] Zvi Bern,[2] Enrico Herrmann,[2] Oliver Long,[3,1]
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Abstract

We compare numerical self-force results and analytical fourth-order post-Minkowskian (PM) calculations for hyperbolic-type scattering of a point-like particle carrying a scalar charge Q off a Schwarzschild black hole, showing a remarkably good agreement. Specifically, we numerically compute the scattering angle including the full $\mathcal{O}(Q^2)$ scalar-field self-force term (but ignoring the gravitational self-force), and compare with analytical expressions obtained in a PM framework using scattering-amplitude methods. This example provides a nontrivial, high-precision test of both calculation methods, and illustrates the complementarity of the two approaches in the context of the program to provide high-precision models of gravitational two-body dynamics. Our PM calculation is carried out through 4PM order, i.e., including all terms through $\mathcal{O}(Q^2G^3)$. At the fourth post-Minkowskian order the point-particle description involves two a-priori undetermined coefficients, due to contributions from tidal effects in the model under consideration. These coefficients are chosen to align the post-Minkowskian results with the self-force ones.

Another perspective: The scalar-charge toy model

Recent work has been using a scalar-charge toy model in order to compare the PM and SF approaches.

Phys.Rev.D 108 (2023) 2, 024025 [Barack, Bern, Hermann, Long, Parra-Martinez, Roiban, Ruf, Shen, Solon, Teng, Zeng]

$$\mathscr{L} \supset \sqrt{-g}\Big[\underbrace{-2\sqrt{\pi}m_1 Q}_{\text{Related to our scalar charges } c_i}\underset{\text{scalar}}{\psi}\,\underset{\text{Matter fields}}{\phi_1^2}\Big]$$

⟶ The same type of coupling we have!

Recall: $m \to e^{C/2}m$ for any spin!

Our framework can be **mapped** to this model (in the limit of $c_2 \to 0$) and potentially **extend the results** in a consistent manner to incorporate effects that have not been compared before, e.g. **spin effects, gravitational waveform corrections, …**

Comparison of post-Minkowskian and self-force expansions: Scattering in a scalar charge toy model

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