Six-point ChPT Amplitude for Pions and Kaons

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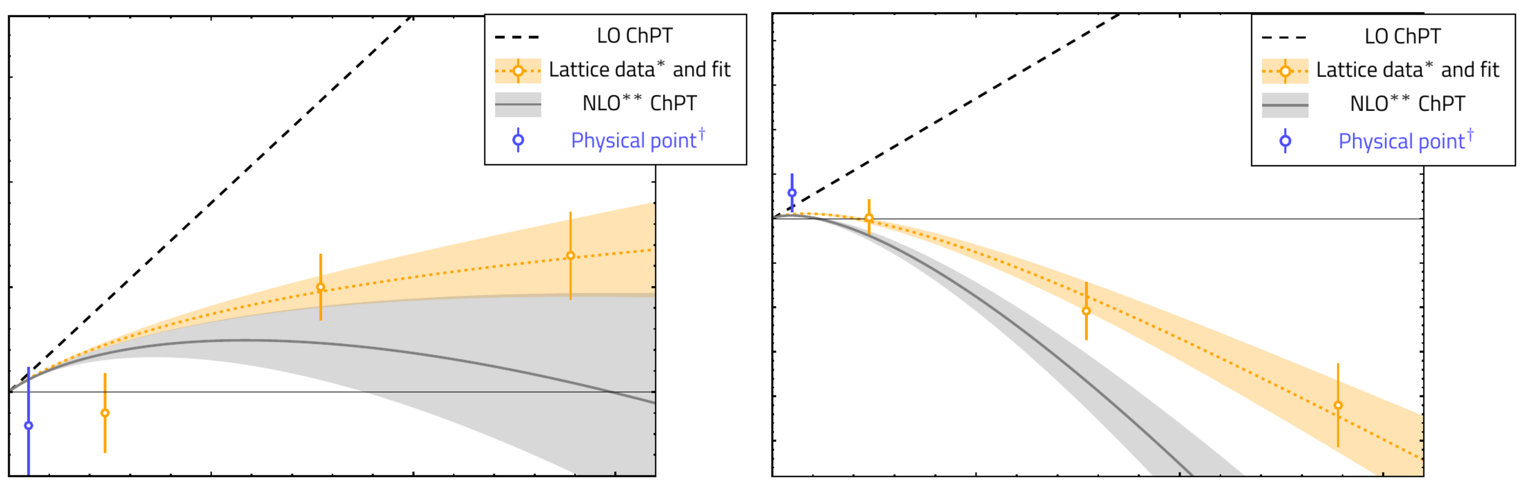
Abstract

We compute all one-loop six-point scattering amplitudes with external pions and kaons within the framework of three-flavor Chiral Perturbation Theory (ChPT). We use distinct pion, kaon and η meson masses, but maintain the isospin limit, thus neglecting mass differences between charged and neutral particles (as this is a QED-dominated effect). Rather than separate amplitudes for each channel, we write generic amplitudes using the flavor Lie algebra, generalizing the concept of stripped amplitudes.

The goal: generalize this



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** **JB**, **MS** et al., *The isospin-3 three-particle K-matrix at NLO in ChPT*, JHEP **05** (2023) 187
† Dawid, S. et al., *Two- and three-meson scattering amplitudes with physical quark masses from lattice QCD*, arXiv:2502.17976

2+1-flavor ChPT

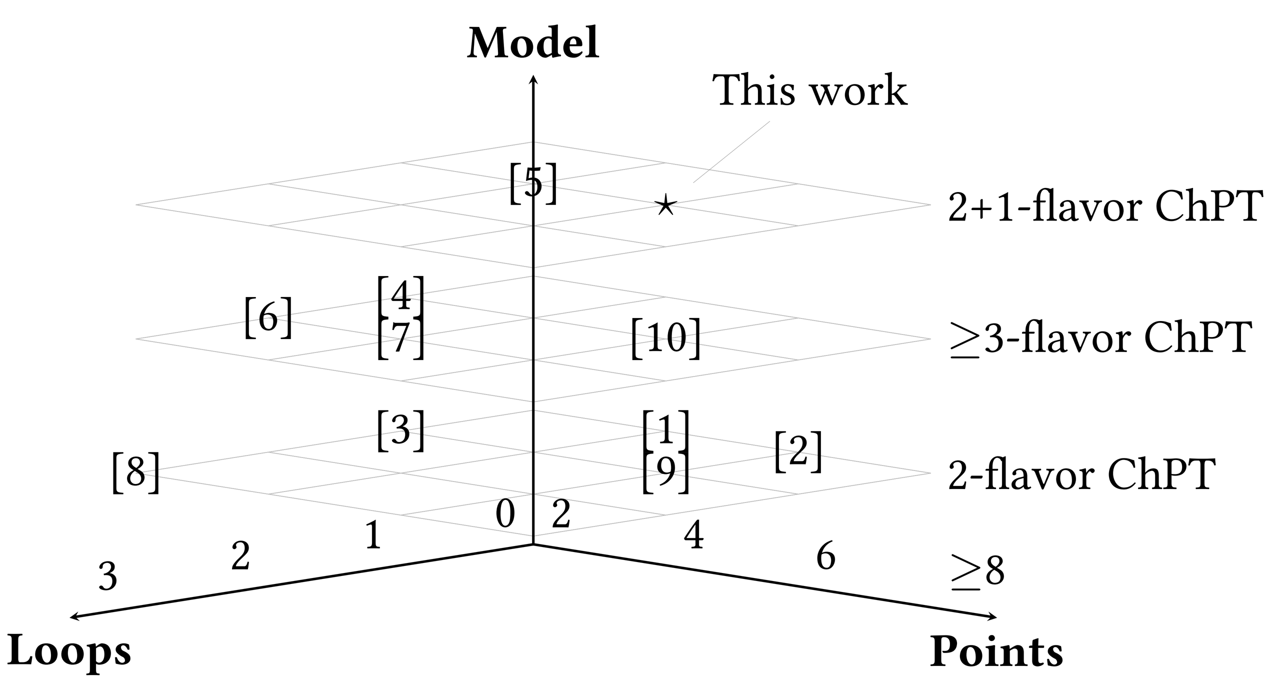
The low-energy effective field theory of QCD with $m_u = m_d < m_s$ and no heavier quarks

$\mathrm{SU}(3)_R \times \mathrm{SU}(3)_L$ chiral symmetry of massless QCD $\longrightarrow$
- *spontaneously* broken by quark condensate $\Rightarrow$ non-linear sigma model
- *explicitly* broken by quark masses $\Rightarrow$ ChPT with meson mass terms

$$\mathcal{L}_{\text{ChPT}} = \left\langle \partial_\mu U \partial^\mu U^\dagger + (U + U^\dagger)\chi \right\rangle + (\text{counterterms})$$

$$U \sim \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -2\frac{\eta}{\sqrt{6}} \end{pmatrix}, \quad \chi \sim \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

(incomplete) ChPT amplitude history



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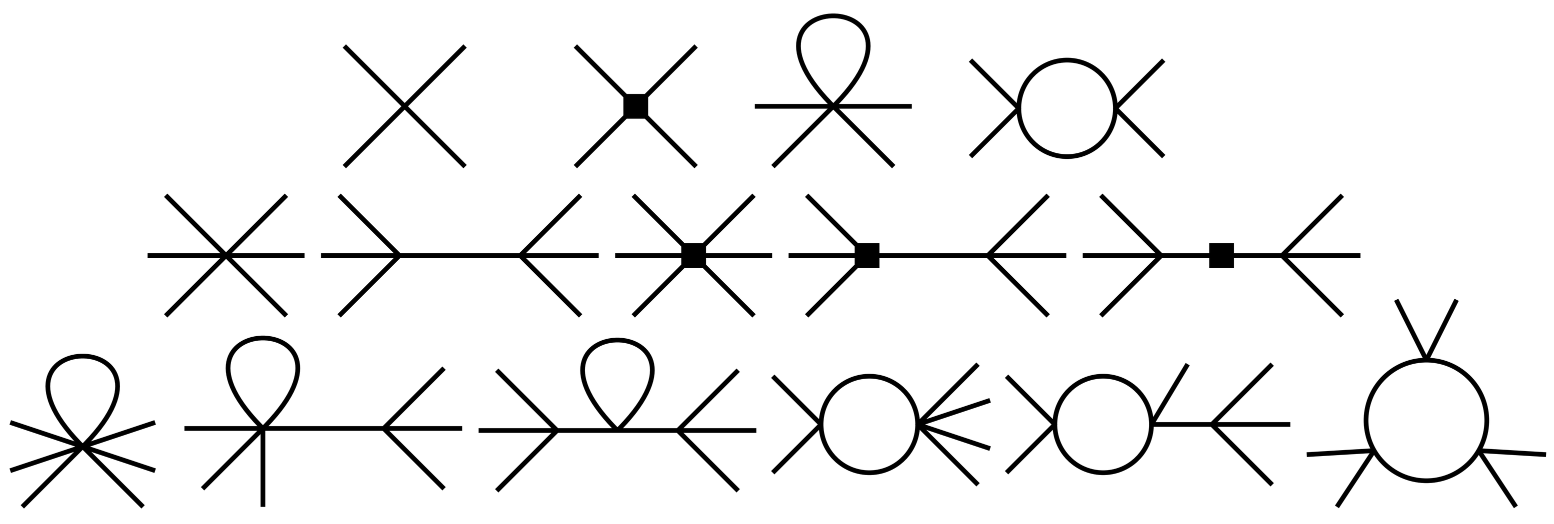
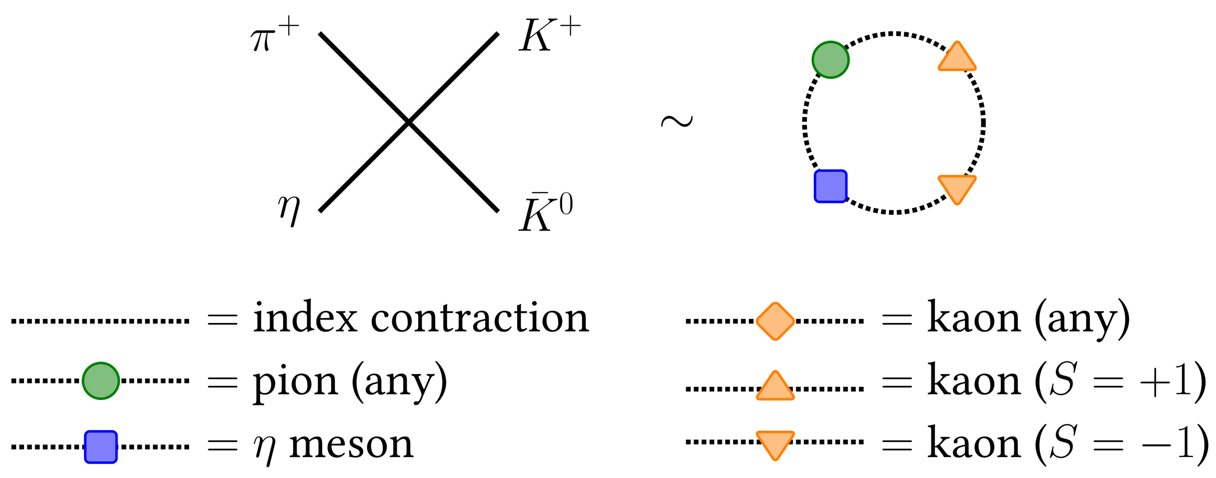
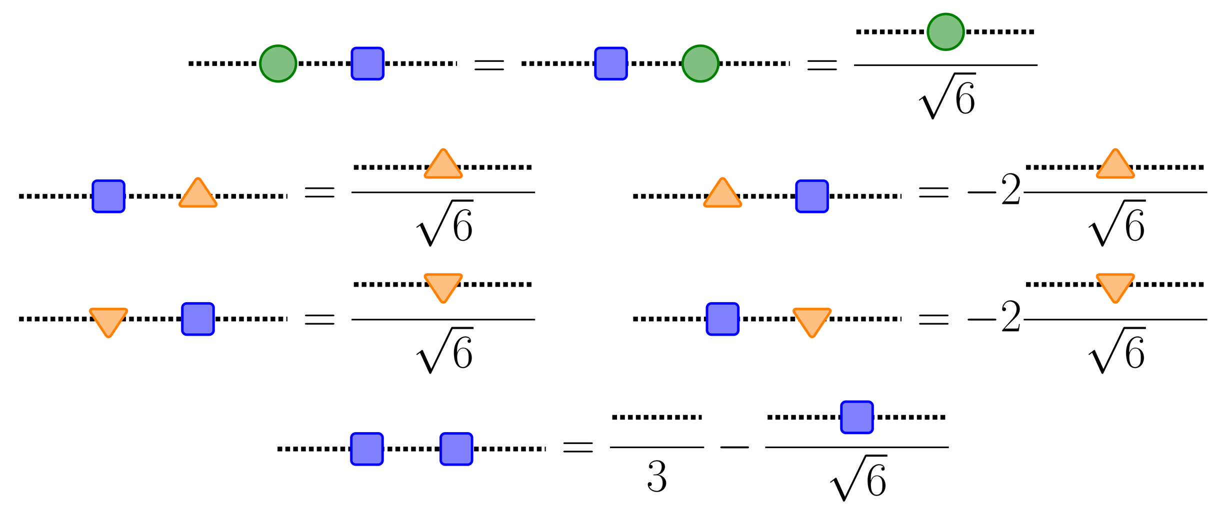
Feynman diagrams



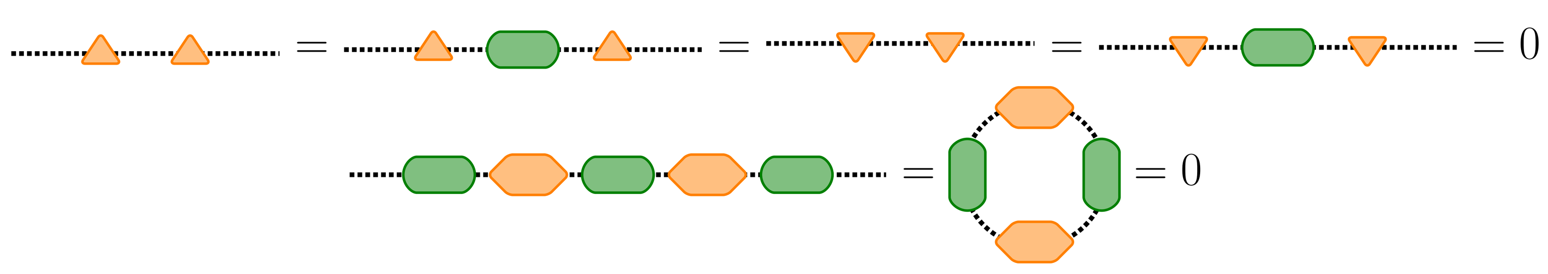
Strangeness-conscious Lie algebra



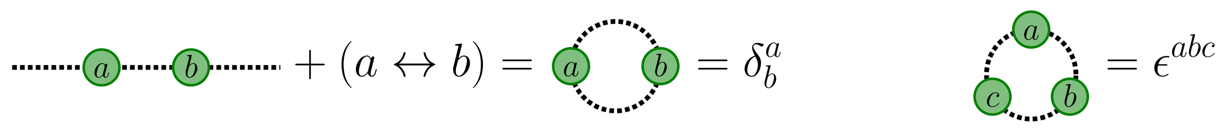
Removal of η's



Strangeness rules

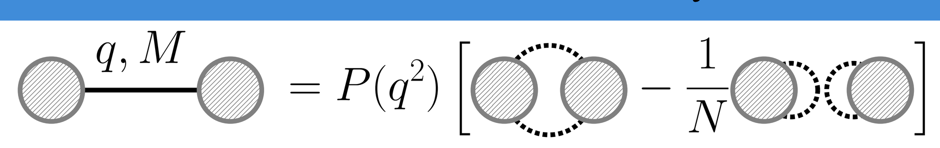


Cayley–Hamilton theorem



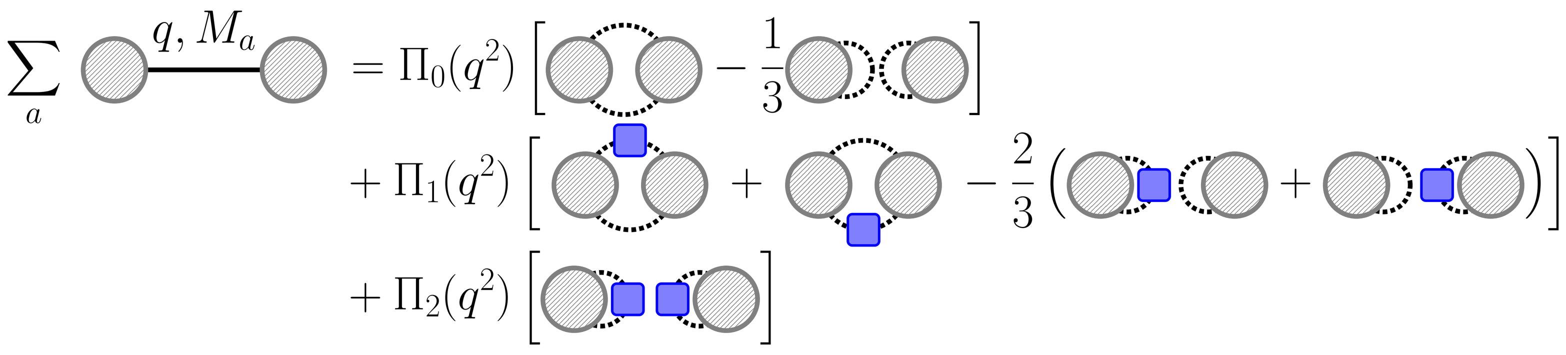
The multi-mass propagator

N-flavor Fierz identity



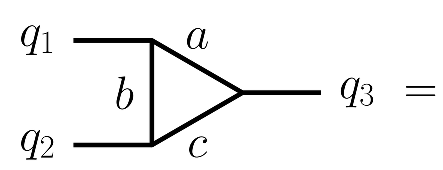
$$P(q^2) = \frac{1}{q^2 - M^2}$$

2+1-flavor Fierz identity



$$\Pi_0 = \frac{P_\pi + 2P_K}{3} \qquad \Pi_1 = \frac{2(P_\pi - P_K)}{\sqrt{6}} \qquad \Pi_2 = P_\eta + \frac{P_\pi - 4P_K}{3}$$

The triangle integral



$$= \int \frac{\mathrm{d}^d l^4}{(2\pi)^d} \frac{(\text{numerator})}{\left[(l-q_1)^2 - M_a^2\right]\left[(l+q_2)^2 - M_b^2\right]\left[l^2 - M_c^2\right]},$$

Numerator up to rank 3, master integral reduction not practical
Large number of symmetries under simultaneous permutation of q_1, q_2, q_3 and M_a, M_b, M_c

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