AsyInt for multi-loop Feynman integrals in asymptotic limits

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Based on [JHEP 09 (2024) 069]

https://gitlab.com/asyint/asyint-public

PSI

EPS-HEP - Marseille - 2025

Swiss National Science Foundation

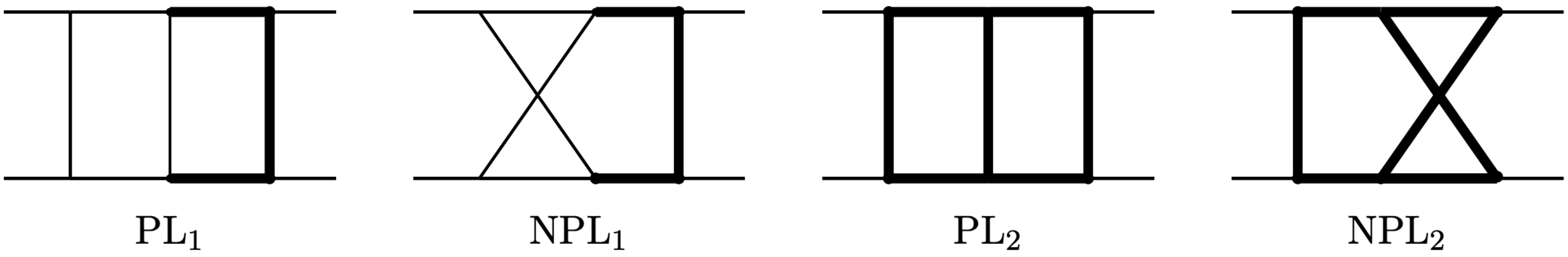
AsyInt

Hantian Zhang — JHEP 09 (2024) 069

Tailored for analytic calculations of massive four-point integrals at high energies

Download at: https://gitlab.com/asyint/asyint-public

Sample Feynman diagrams calculated by AsyInt

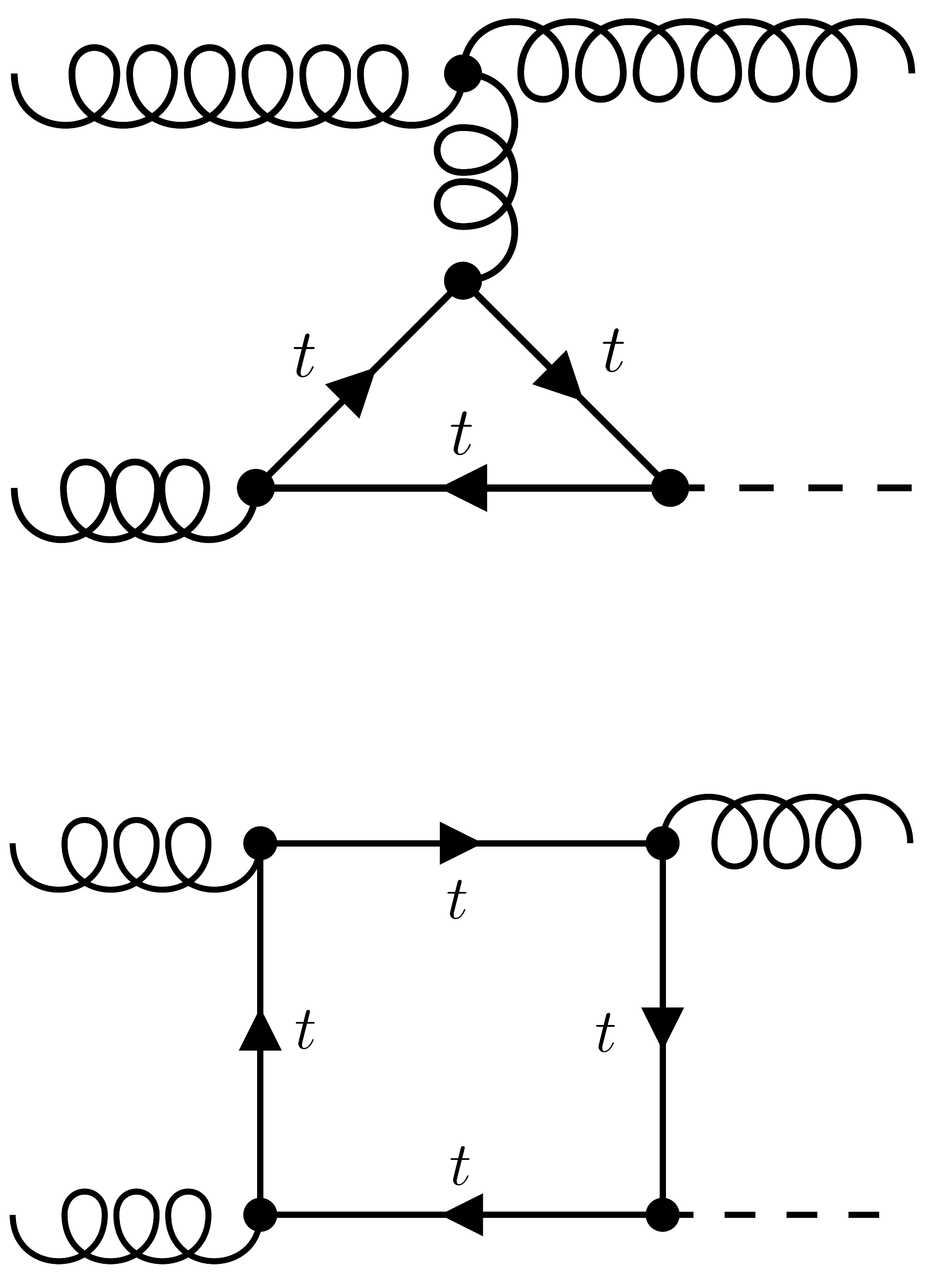


Sample planar and non-planar diagrams. Thick lines denote massive propagators

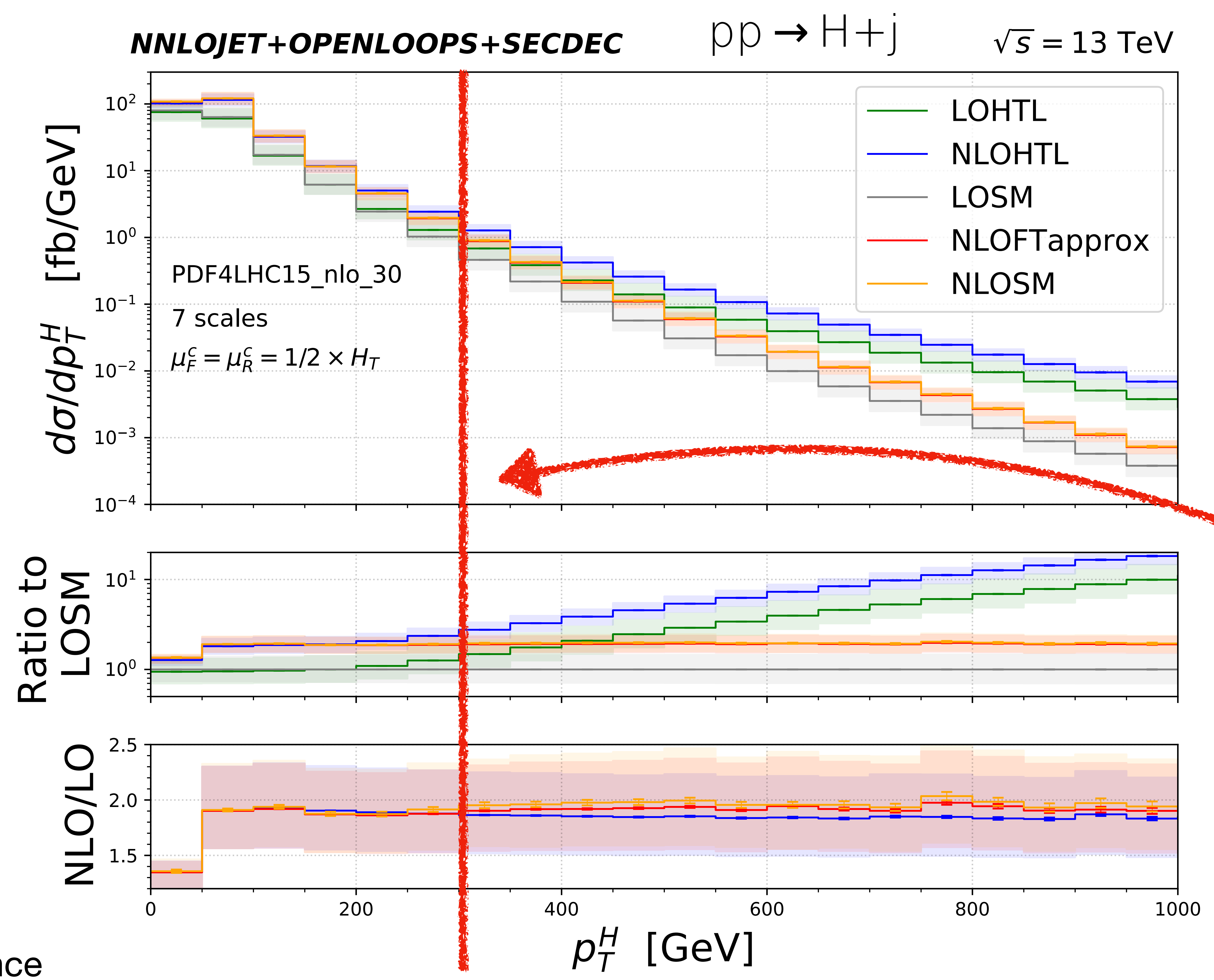
Electroweak and QCD (top-quark) Master Integrals for boosted 2 -> 2 LHC processes

Motivation: probe boosted Higgs boson

- Higgs boson plus jet production with large transversal momentum p_T^H at LHC



NLOSM ⇒ NLO QCD with top-mass dependence



High-energy region

Sensitive to new physics

Large QCD & EW corrections

Massive Feynman integrals

High-energy expansion aims to cover $p_T^H > 300$ GeV region

[Chen, Huss, Jones, Kerner, Lang, Lindert, **Zhang**, **JHEP 03 (2022) 096**]

Overview of analytic $2 \to 2$ high-energy calculations

High-energy expansion

- QCD corrections for $gg \to HH$ [Davies, Mishima, Steinhauser, Wellmann, 18']
- QCD corrections for $gg \to ZH$ [Davies, Mishima, Steinhauser, 21']
- Yukawa and Higgs-selfcoupling corrections for $gg \to HH$ [Davies, Mishima, Schönwald, Steinhauser, **Zhang**, 22', 25']
- QCD master integrals in high-energy expansion [Mishima, 18']
- **AsyInt** and EW master integrals in high-energy expansion [**Zhang**, 24']

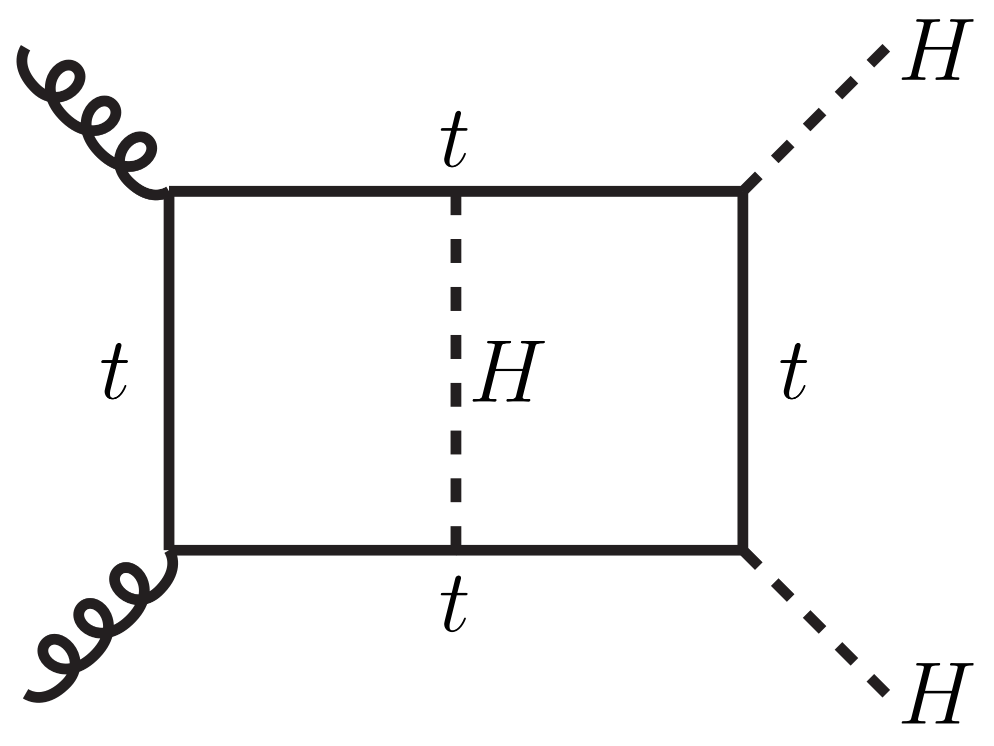
High-energy limit & QCD factorisation

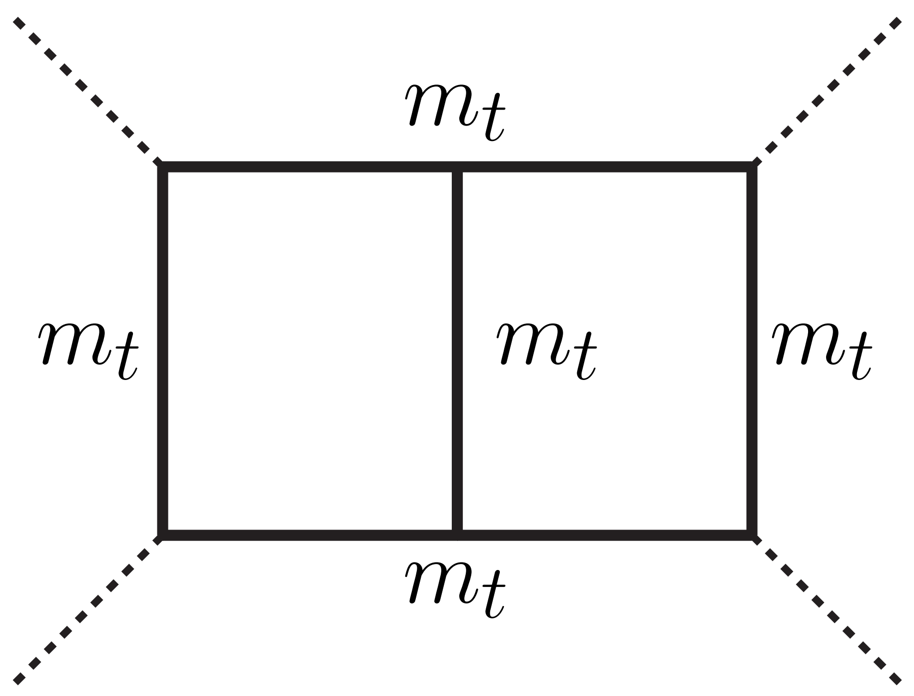
- QCD corrections for $gg \to Hg$ with small bottom mass (no top quark) [Melnikov, Tancredi, Wever, 16']
- QED corrections for massive Bhabha scattering [Penin, 06', Becher, Melnikov, 07']
- Massive QCD factorisation [Mitov, Moch, 06', Wang, Xia, Yang, Ye, 24']
- QCD corrections to $gg \to HH$ with resummation [Jaskiewicz, Jones, Szafron, Ulrich, 24']

Expansion strategies at high energies

- At high energies, SM masses are of a similar order: $m_t \approx m_W, m_Z, m_H \ll \sqrt{s}$
- Two **fast convergent Taylor expansions**: equal-internal-mass and external-mass expansions

  e.g. convergent rates controlled by $m_H^2/s < 0.06$ and $(m_t^2 - m_H^2)/s < 0.01$ for $\sqrt{s} > 500$ GeV





$s, t \gg m_t^2$

High-energy expansion

High energy expansion of master integrals

1. **Asymptotic expansion:** $s, t \gg m_t^2$

2. **System of differential equations for Master Integrals** from IBP reduction [**Kira**]

$$\frac{\partial}{\partial\left(m_t^2\right)}\mathbf{I} = M(s, t, m_t^2, \epsilon)\,\mathbf{I} \quad \text{with} \quad \mathbf{I} = \left(\mathcal{I}_1, \ldots, \mathcal{I}_N\right)^T$$

3. Plug in **power-log ansatz** for each master integral

$$\mathcal{I}_n = \sum C_{(n)}^{ijk}(s,t)\,\epsilon^i\left[m_t^2\right]^j\left[\log(m_t^2)\right]^k$$

4. Solve **boundary master integrals in $m_t^2/s \to 0$ to higher orders in m_t^2 and ϵ using AsyInt**

AsyInt toolkit I: generate MB-integral representations

- Two-loop Feynman integral with n propagators and k numerators

$$\mathcal{I}_{n,k} = \int \prod_{j=1}^{2} \mathrm{d}l_j \, \frac{N_1^{\lambda_1} \cdots N_k^{\lambda_k}}{D_1^{1+\delta_1} \cdots D_n^{1+\delta_n}}$$

δ_i: additional regulators and shifts for dotted propagators (e.g. $\delta_i \to \delta_i + 1$)

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δ_i: additional regulators and shifts for dotted propagators (e.g. $\delta_i \to \delta_i + 1$)

↓ Apply method-of-region ($s, t \gg m_t^2$) with **asy2.1.m** [Smirnov]

`GenerateInput / GenerateInputNum`

↙ **Hard region**

↓ **Asymptotic regions**

AsyInt toolkit I: generate MB-integral representations

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↓

`GenerateInput / GenerateInputNum`

Hard region → **Massless integrals**

Known master integrals

Independent calculation to weight-6 provided in **AsyInt** repository

Asymptotic regions → `AlphaRepForTempInt` → **Template integrals**

Perform parametric integration and Mellin transformations in complex plane

$$(x+y)^{\lambda} = \int_{-i\infty}^{i\infty} \frac{dz}{2\pi i} \frac{\Gamma(-\lambda+z)\Gamma(-z)}{\Gamma(-\lambda)} x^z y^{\lambda-z} \quad \text{with} \quad z \in \mathbb{C}$$

AsyInt toolkit I: generate MB-integral representations

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δ_i: additional regulators and shifts for dotted propagators (e.g. $\delta_i \to \delta_i + 1$)

`GenerateInput / GenerateInputNum`

Hard region

Massless integrals

Asymptotic regions

`AlphaRepForTempInt`

Template integrals

`AsyExp2MB / AsyExpNum2MB`

Generate MB representations for small-m_t expansions
Perform analytic continuations with **MB.m** [Czakon]

Asymptotic Mellin-Barnes integrals to higher orders in m_t and ϵ

δ_i singularities cancel in sum of all asymptotic regions

AsyInt toolkit II: solve MB integrals

↓ MB dimensions reduction

Irreducible MB integrals

Numerical Reconstruction

`AIRecNum1DMB`

Apply PSLQ algorithm given a basis of constants

New constant found in the fully-massive non-planar integral NPL_2

Analytic Summation

`AISum1DMB & AISum2DMB`

Apply Cauchy theorem and extract residues

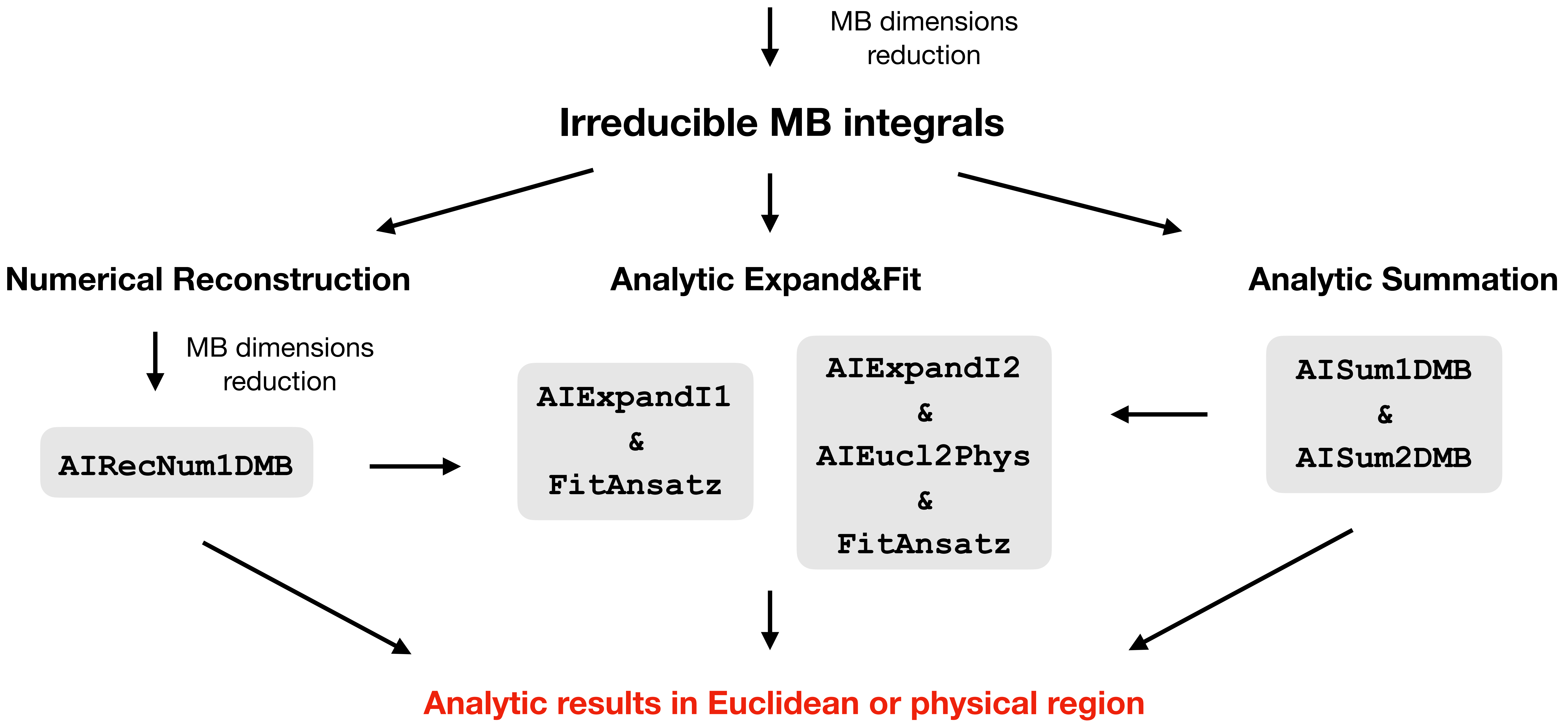
If residue series converge and no arc contributions, sum with **HarmonicSums.m** and **Sigma.m** [Ablinger, Schneider]

$$c_Z = \int_0^\infty \frac{\mathrm{d}\alpha_1\,\mathrm{d}\alpha_2}{\sqrt{\alpha_1\,\alpha_2\,(\alpha_1+\alpha_2+1)\,(\alpha_2\alpha_1+\alpha_1+\alpha_2)}} = 4\sqrt{3}K\left(\frac{1}{2}-\frac{\sqrt{3}}{4}\right)^2$$

[Davies, Schönwald, Steinhauser, **Zhang**, JHEP 04 (2025) 193]

Transcendental weight 1/2 elliptic K and E constants: $\pi = -2K\left(\frac{1}{2}-\frac{\sqrt{3}}{4}\right)\left[\left(\sqrt{3}+1\right)K\left(\frac{1}{2}-\frac{\sqrt{3}}{4}\right)-2\sqrt{3}E\left(\frac{1}{2}-\frac{\sqrt{3}}{4}\right)\right]$

AsyInt toolkit II: solve MB integrals



Expand&Fit for complicated irreducible MB integrals

(type-1): 2-dim 1-scale MB integrals with non-vanishing arc contributions

(type-2): 2-dim 2-scale MB integrals

Analytic Expand&Fit method

Mellin-Barnes (MB) integrals with non-vanishing arc

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, JHEP 08 (2022) 259]

- Close the contour to left or right semi-circle, and sum up residues, e.g.

$$\int_{-i\infty}^{+i\infty} \frac{dz_1}{2\pi i} f(z_1) = -\sum_{k=0}^{\infty} \mathrm{Res}_{z_1=k}\left[f(z_1)\right] - \int_{\mathrm{arc}} \frac{dz_1}{2\pi i} f(z_1) \quad \text{with} \quad f(z_1) = \frac{z_1^8 \Gamma(-z_1)^2 \Gamma(z_1)^2}{(z_1+1)^3 (z_1+2)^3}$$

$$-18\zeta(3) - \frac{3\pi^2}{2} - \frac{21\pi^4}{10} + 240$$

Analytic Expand&Fit method

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$$-18\zeta(3) - \frac{3\pi^2}{2} - \frac{21\pi^4}{10} + 240$$

arc integral non-zero

solve arc contribution by adding auxiliary scale:

$$\int_{\mathrm{arc}} \frac{dz_1}{2\pi i} \xi^{z_1} f(z_1) = -\sum_{k=0}^{\infty} \frac{k^6}{(1+k)^3(2+k)^3} \xi^k \log(\xi) \overset{\xi \to 1}{=} -1$$

Analytic Expand&Fit method

Mellin-Barnes (MB) integrals with non-vanishing arc

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, JHEP 08 (2022) 259]

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Expand&Fit method

[**Zhang**, JHEP 09 (2024) 069]

- for 2-dim 1-scale MB integral with nested non-vanishing arc contributions

$$\int \frac{dz_1}{2\pi i} \frac{dz_2}{2\pi i} \left(\frac{-t}{-s}\right)^{z_1} f\left(\Gamma, \psi^{(i)}; z_1, z_2\right) \Rightarrow \text{HPLs}$$

- for 2-dim 2-scale MB integral in non-planar diagrams

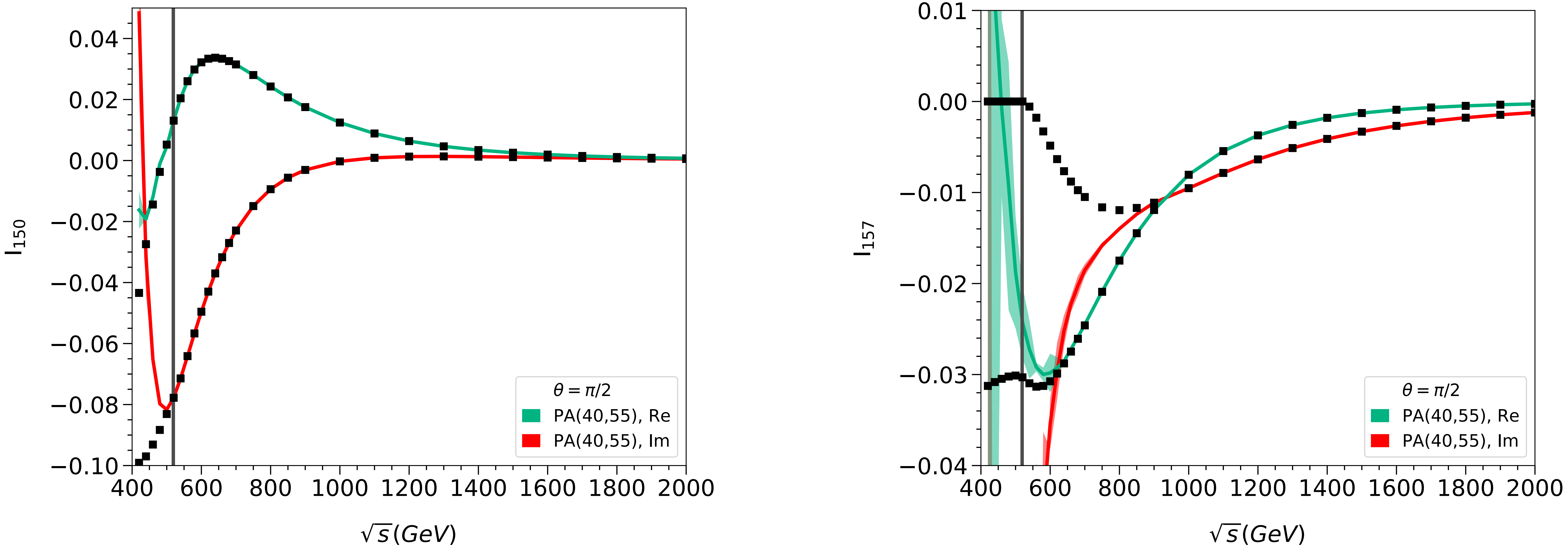
$$\int \frac{dz_1}{2\pi i} \frac{dz_2}{2\pi i} \left(\frac{-t}{-s}\right)^{z_1} \left(\frac{-u}{-s}\right)^{z_2} f\left(\Gamma, \psi^{(i)}; z_1, z_2\right) \Rightarrow \text{HPLs}$$

(1). Expand in $(-t) \to 0$ limit to more than a hundred terms
(2). Solve expanded MB integrals exactly
(3). Reconstruct analytic results with ansat in Euclidean region (for planar integrals) or in physical region (with analytic continuation for non-planar integrals)

Non-planar fully massive (EW) integrals

Analytic solution in terms of Harmonic PolyLogarithms (HPLs) [Davies, Schönwald, Steinhauser, **Zhang**, **JHEP 04 (2025) 193**]

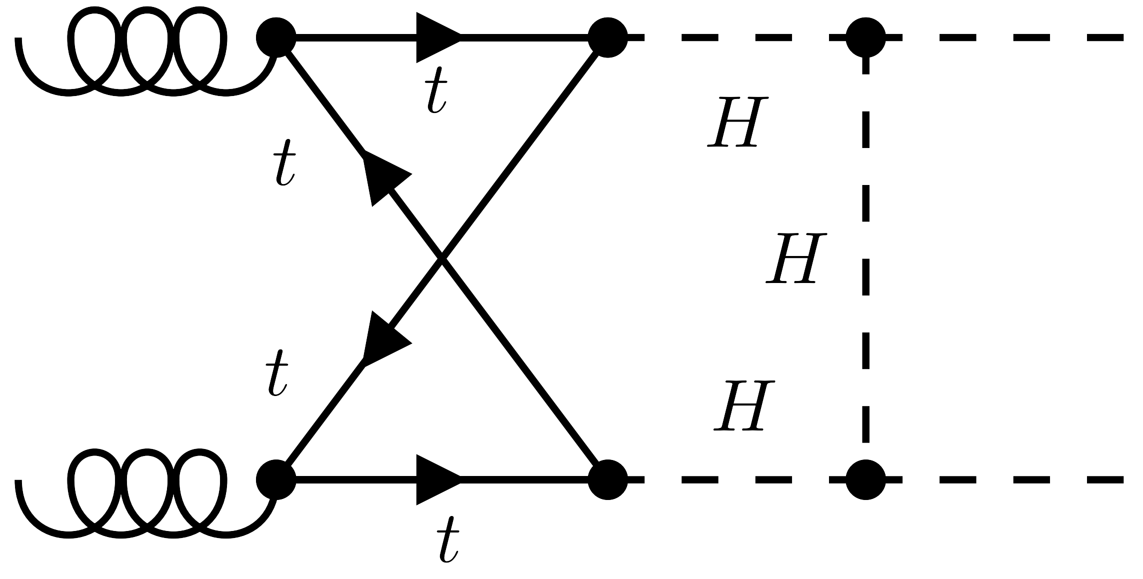
Fixed scattering angle plots



Red Green curves (analytic), Black (numerics from AMFlow)

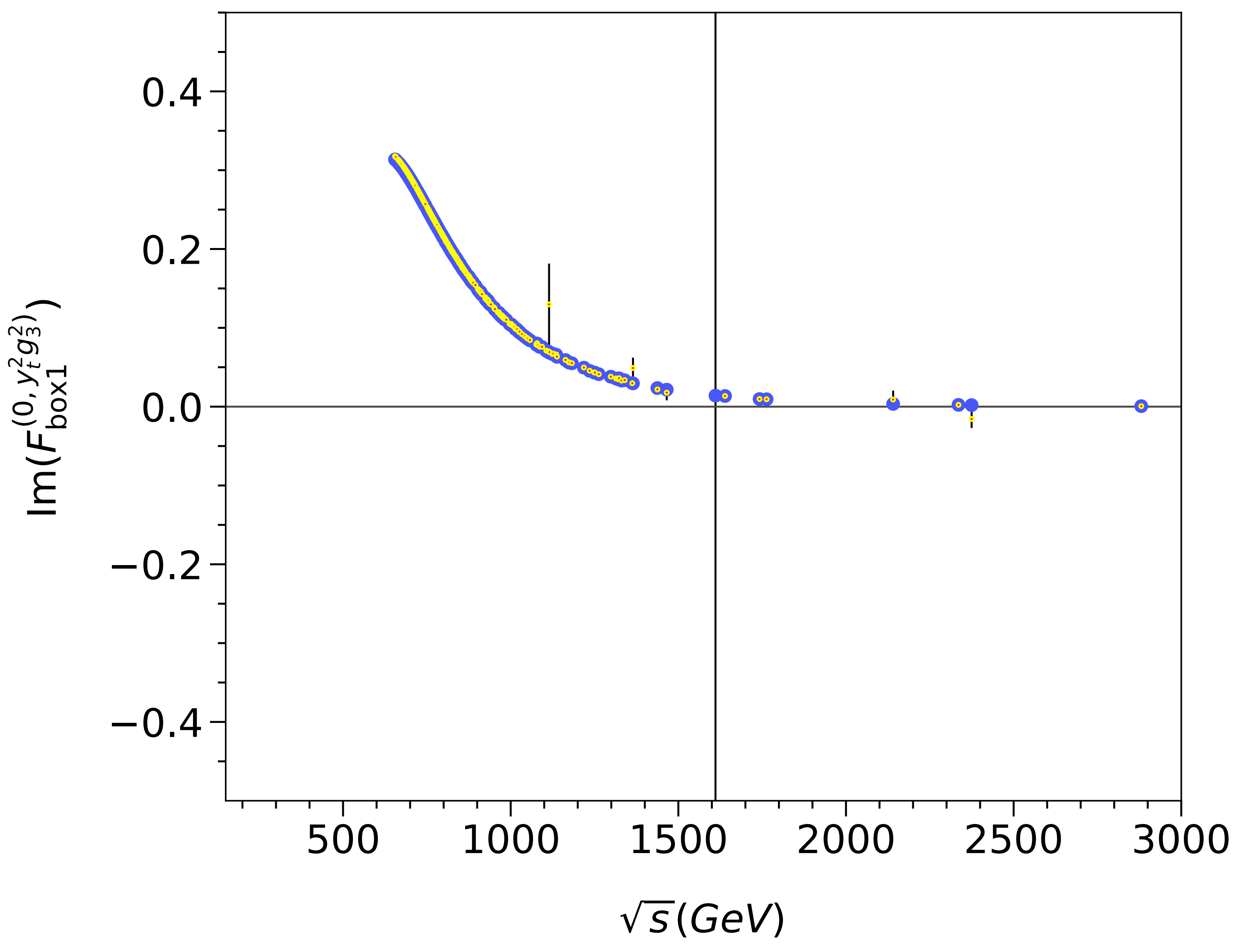
Application to EW corrections to di-Higgs production at LHC

[Davies, Schönwald, Steinhauser, **Zhang**, **JHEP 04 (2025) 193**]

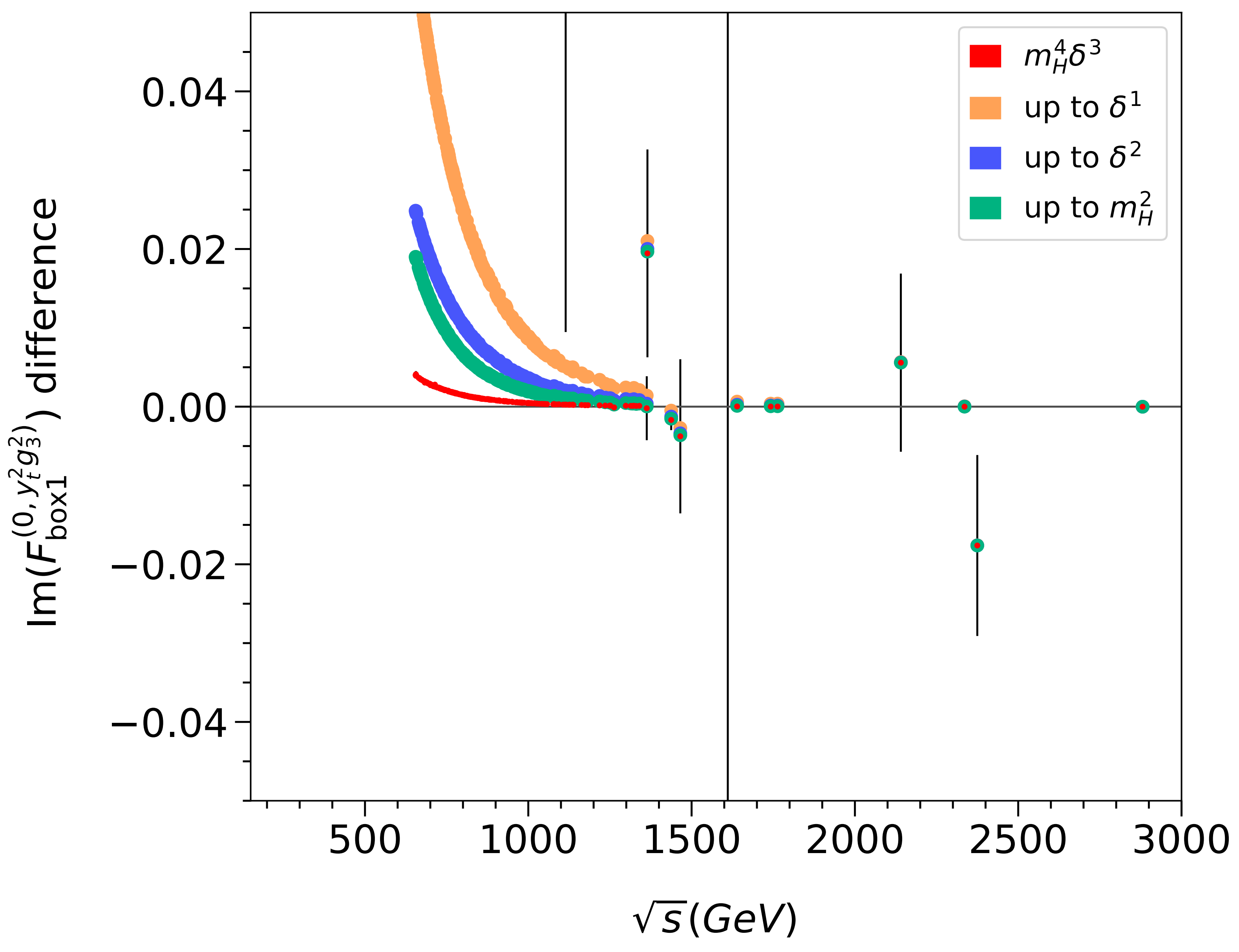


Comparison to numerical results from SecDec group - $y_t^2\,\lambda^2$ form factor

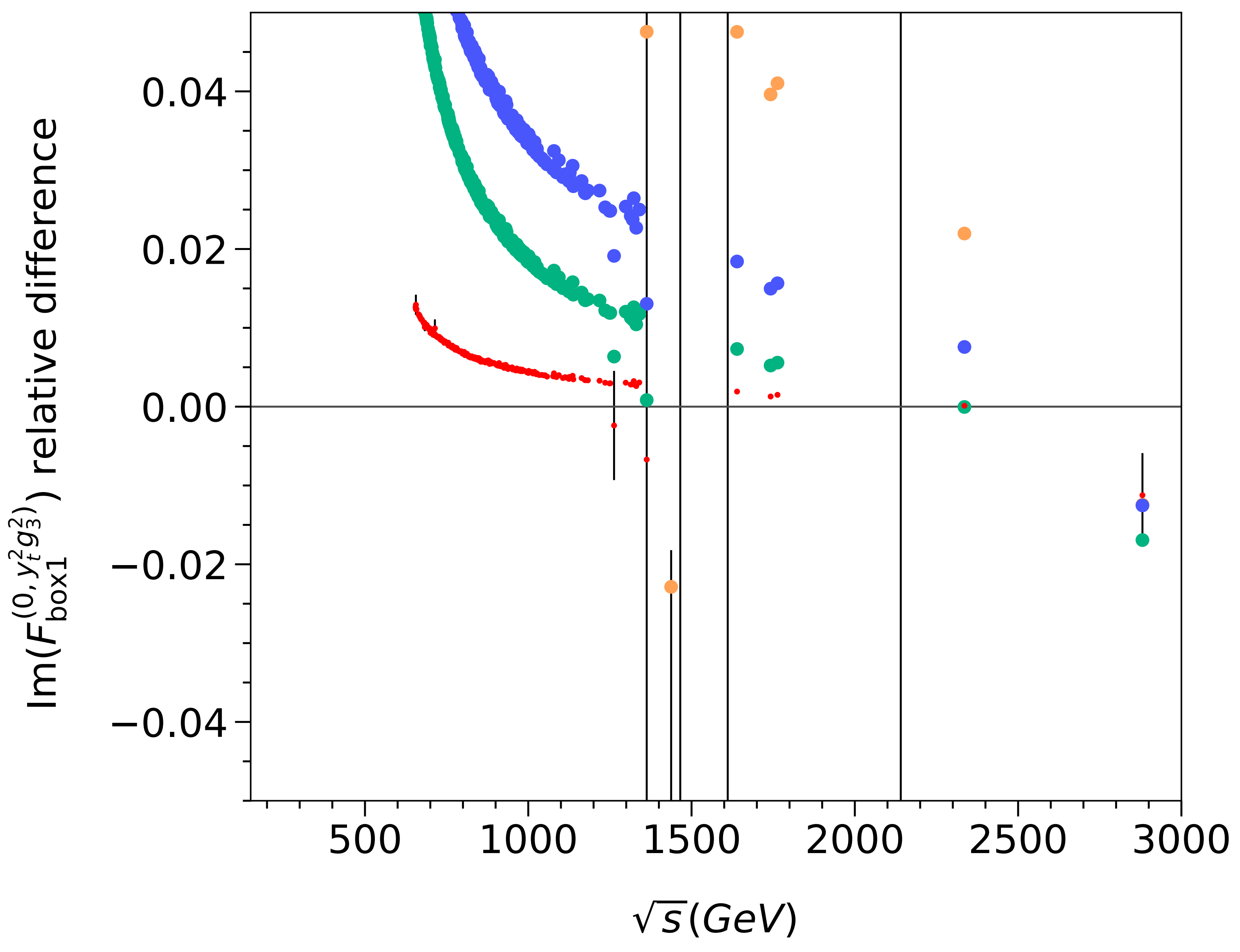
[Heinrich, Jones, Kerne, Stone, Vestner, JHEP 11 (2024) 040]



$F_{\mathrm{box}}^{(y_t^2\,\lambda^2)}$ form factor



Absolute difference



Relative difference

Summary

- **AsyInt** released in [JHEP 09 (2024) 069]
  - Toolbox for analytic massive two-loop four-point Feynman integrals at high energies
  - Fully massive non-planar integrals computed in terms of HPLs
  - New elliptic constants determined in [JHEP 04 (2025) 193] $K\left(\frac{1}{2}-\frac{\sqrt{3}}{4}\right)$ and $E\left(\frac{1}{2}-\frac{\sqrt{3}}{4}\right)$
  - Download at: https://gitlab.com/asyint/asyint-public
- High-energy expansion works well for two-loop electroweak corrections (Yukawa and Higgs self-coupling corrections) to $gg \to HH$ [JHEP 08 (2022) 259] & [JHEP 04 (2025) 193]
  - Full top-induced EW master integrals computed analytically
  - Matches SecDec group's numerical results even for relative low p_T region