Capping the Positivity Cone: the Higgs Case

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Dong-Yu Hong[a], Zhuo-Hui Wang[a] and Shuang-Yong Zhou[a,b]

[a] University of Science and Technology of China, Hefei, Anhui 230026, China

[b]Peng Huanwu Center for Fundamental Theory, Hefei, Anhui 230026, China

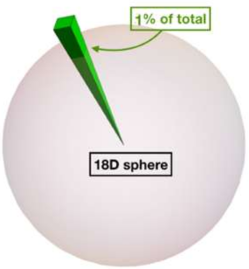
E-mail: principle@mail.ustc.edu.cn, wzh33@mail.ustc.edu.cn, zhoushy@ustc.edu.cn

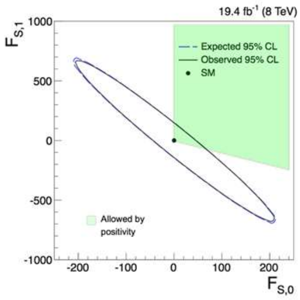
INTRODUCTION

The Standard Model Effective Field Theory (SMEFT) systematically captures low-energy imprints of unknown high-energy physics via higher-dimensional operators. By expanding all operators consistent with SM gauge symmetries in inverse powers of the cutoff scale Λ, SMEFT parametrizes new physics in a model-independent way.

Recent studies show that, aside from a small "positivity" subspace consistent with S-matrix principles, most allowed Wilson coefficients are excluded by **analyticity**, **causality**, and **unitarity**. They yield dispersion relations between EFT amplitudes, UV spectral integrals, and partial-wave unitarity bounds.

Even using only these basic assumptions, positivity bounds can constrain Wilson coefficients to $O(1)$. For dimension-8 operators in vector-boson scattering, enforcing elastic positivity alone shrinks the naïve coefficient space.



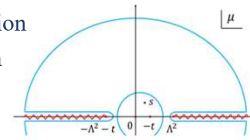


Model & Setup

In this work, we revisit the dimension-eight subspace of the SMEFT that involves only Higgs covariant-derivative operators.

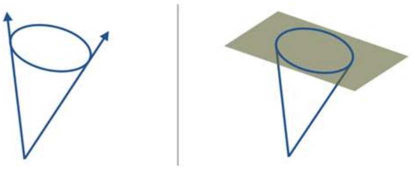
$$L_{SMEFT} \supset C_1 \left(D_\mu H^\dagger D_\nu H\right)\left(D^\nu H^\dagger D^\mu H\right)$$
$$+C_2 \left(D_\mu H^\dagger D_\nu H\right)\left(D^\mu H^\dagger D^\nu H\right) + C_3\left(D_\mu H^\dagger D^\mu H\right)\left(D^\nu H^\dagger D_\nu H\right)$$

We systematically derive twice-subtracted dispersion relations (from **analyticity** and **causality**) for each Wilson coefficient, expressing them as integrals over the UV spectral densities.



$$\sum_{EFT\ poles} Res \frac{M(\mu,t)}{\mu - s} = b_0(t) + \int_{\Lambda^2}^{+\infty} \frac{\mathrm{d}\mu}{\pi}\left(\frac{s^2}{\mu^2(\mu-s)} + \frac{u^2}{\mu^2(\mu-u)}\right)\mathrm{Im}M(\mu,t)$$

Dispersion relations tie high-energy dynamics to the low-energy EFT, allowing **unitarity** to impose bounds on Wilson coefficients.



Im$\boldsymbol{T} \geqslant 0$ 2 $\geqslant$ Im$\boldsymbol{T} \geqslant 0$

Crossing symmetry enforces null constraints on the spectral densities. We impose the full non-linear unitarity conditions on each partial wave and leverage the complete $SU(2)$ Higgs symmetry to reduce both the number of the independent partial waves.

We implement a **bootstrap** approach using semidefinite programming: by discretizing the UV spectral densities for each partial wave and isospin channel, we impose null-constraint and full non-linear unitarity conditions to bound Wilson-coefficient combinations, which are then solved with SDPB.

BOUNDS

Previous positivity bounds for the dimension-8 Higgs operators are

$$C_2 \geq 0, C_1 + C_2 \geq 0, C_1 + C_2 + C_3 \geq 0.$$

We have computed the new bounds and confirmed that they indeed feature an upper cap. Moreover, we find that imposing full non-linear unitarity alone reproduces the linearized limits under partial symmetry, while enforcing the full $SU(2)$ symmetry further reduces each Wilson coefficient's allowed range by factors of two to four.

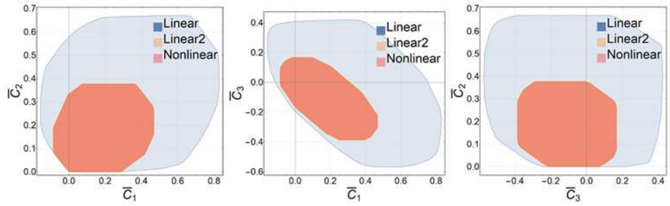


Figure 1: Positivity regions in the $2D$ subspaces of C_1, C_2 and C_3 by using linear and nonlinear unitarity conditions.

We also compare our bounds with the traditional perturbative unitarity bounds. Apart from certain directions of the parameter space, our method always gives the better bounds.

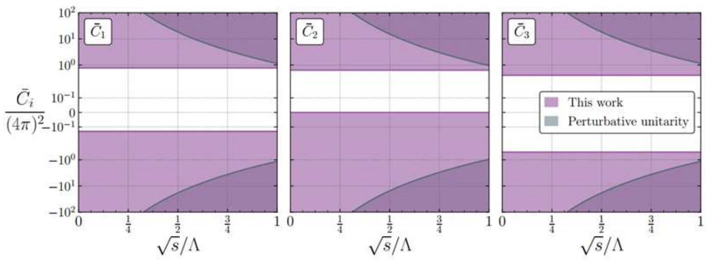


Figure 2: Upper and lower positivity bounds compared with perturbative unitarity bounds. From *Chen, Mimasu, Wu, Zhang, Zhou*, arXiv: 2309.15922v2.

Besides, we find that nonlinear unitarity conditions generally yield stronger bounds, and that the Higgs case is special due to its enhanced symmetry. To illustrate this, we use the Z_2 bi-scalar theory as an example. In this case, we observe that when the coefficients lack special symmetry, the nonlinear unitarity conditions indeed provide a stronger bound.

	$\bar{g}^a_{1,0} = g^a_{1,0}\Lambda^4/(4\pi)^2$		$\bar{g}^b_{2,0} = g^b_{2,0}\Lambda^4/(4\pi)^2$		$\bar{g}^a_{1,0} + \bar{g}^b_{2,0}$	
	lower	upper	lower	upper	lower	upper
Linear	0	0.798	−0.330	0.614	−0.326	1.412
Nonlinear	0	0.798	−0.330	0.614	−0.172	1.176

Table 1: Bounds on $g^a_{1,0}, g^b_{2,0}$ and $g^a_{1,0} + g^b_{2,0}$, using linear and nonlinear unitarity conditions separately.

Outlook

We are currently computing positivity bounds for the one-loop Higgs scattering amplitudes. Preliminary results indicate that, in the small-coupling regime, the one-loop bounds reduce to a simple tree-level approximation. These findings suggest a deeper connection between loop corrections and leading-order constraints, which we will explore in future work.