BSM Sensitivity of Rare Kaon Decays

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Based on 2206.14748, 2311.04878, 2404.03643, 2409.06545
In collaboration with G. D'Ambrosio, A. Iyer and F. Mahmoudi

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Rare kaon decays

Rare Kaon decays take place via $s \to d$ Flavour Changing Neutral Current (FCNC) processes which are strongly suppressed in the SM

- Historical tools to study FCNC
- Interesting probe of New Physics (NP)
  - → Requires reliable prediction in the SM

Weak effective Hamiltonian: $\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{td} \frac{\alpha_e}{4\pi} \sum_k C_k^\ell O_k^\ell$

$$O_L^\ell = (\bar{s}\gamma_\mu P_L d)(\bar{\nu}_\ell \gamma^\mu (1-\gamma_5)\nu_\ell), \;\; O_9^\ell = (\bar{s}\gamma_\mu P_L d)(\bar{\ell}\gamma^\mu \ell), \;\; O_{10}^\ell = (\bar{s}\gamma_\mu P_L d)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

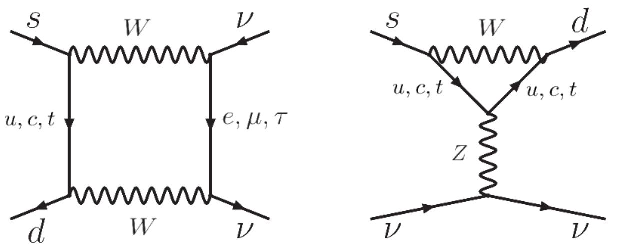
+ other operators

New Physics contributions: $C_k \to C_k^{\text{SM}} + \delta C_k$

Rare kaon decays

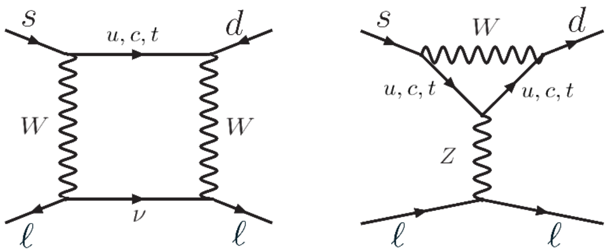
- Short-Distance dominated

$K^+ \to \pi^+ \nu\nu$ and $K_L \to \pi^0 \nu\nu$ (*golden channels*)

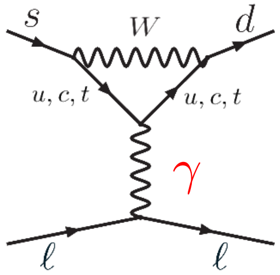


- Long-Distance dominated

$K_L \to \mu\mu,\ K_S \to \mu\mu\,, K^+ \to \pi^+\,\ell\,\ell$ and $K_L \to \pi^0\ \ell\,\ell, \ldots$



+



SD-dominated

$K^+ \to \pi^+ \nu\nu$

$$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu}) = \frac{\kappa_+ (1+\Delta_{\mathrm{EM}})}{\lambda^{10}} \frac{1}{3} s_W^4 \sum_\ell \left[\mathrm{Im}^2 \left(\lambda_t C_L^\ell\right) + \mathrm{Re}^2 \left(-\frac{\lambda_c X_c}{s_w^2} + \lambda_t C_L^\ell \right) \right] \quad (\lambda_i = V_{is}^* V_{id})$$

- top loop: $C^\ell_{L,\mathrm{SM}} = -X_{\mathrm{SM}}(x_t)/s_W^2$ NNLO QCD and 2-loop EW [Buchalla, Buras,'99; Misiak, Urban '99, Broad et al. '10]
- charm contribution: $X_c = \lambda^4 [P_c^{\mathrm{SD}} + \delta P_{c,u}^{\mathrm{LD}}]$ SD: NNLO QCD and NLO EW; LD: ChPT SD:[Buras et al. '05; Brod et al. '08] LD:[Isidori et al.'05]
- O_L matrix elements known from $K_{3\ell}$ branching ratios → included in κ_+ [Mescia, Smith '07]
- $\Gamma_{\mathrm{SD}}/\Gamma > 90\%$
- Sources of uncertainty: SD ~ 2%, LD ~ 3%, Parametric ~ 7%
- Sum over the three neutrino flavours

$$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{SM}} = (7.86 \pm 0.61) \times 10^{-11}$$

[D'Ambrosio, Iyer, Mahmoudi, SN '22]

$$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{NA62}} = \left(13.0^{+3.3}_{-3.0}\right) \times 10^{-11}$$

[NA62, Cortinal Gil et al. '24]

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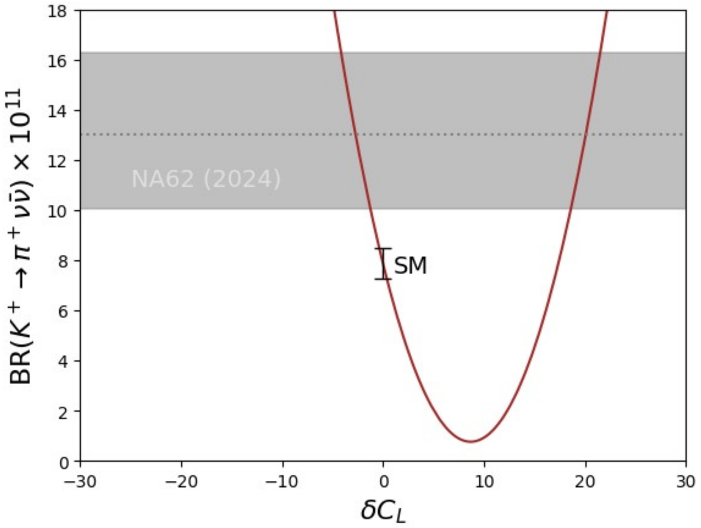
New Physics effects:

Lepton flavour universal



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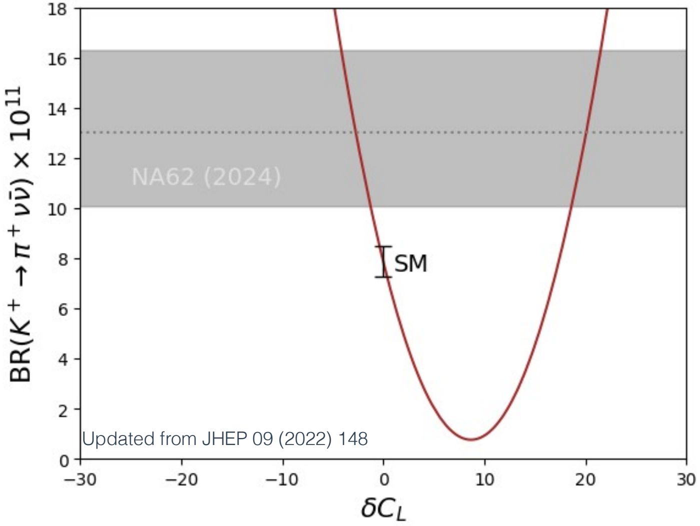
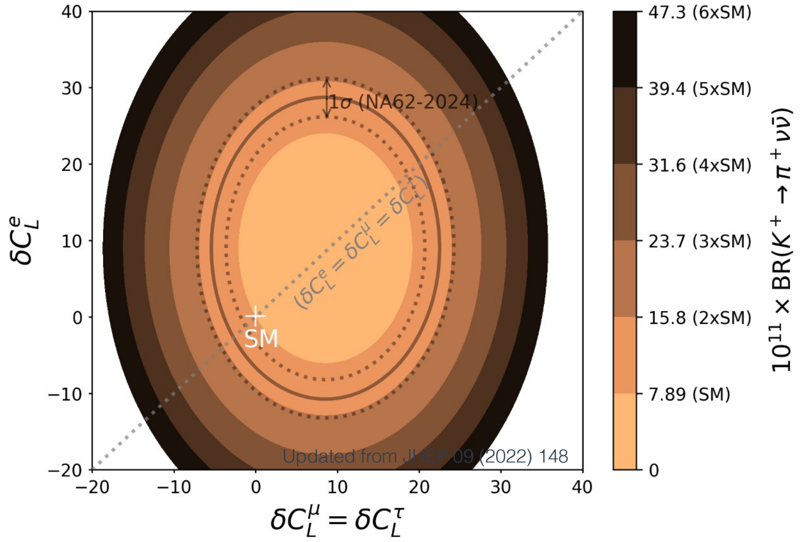


Lepton flavour universality violation



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$K_L \to \pi^0 \nu\nu$

$$\mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu}) = \frac{\kappa_L}{\lambda^{10}} \frac{1}{3} s_w^4 \sum_\ell \mathrm{Im}^2 \left(\lambda_t C_L^\ell\right)$$

- $C_{L,SM}$ same as for $K^+ \to \pi^+ \nu\nu$
- Charm contributions below 1%
- 99% SD distance
- $\Gamma_{\mathrm{SD}}/\Gamma > 99\%$
- Sources of uncertainty: SD ~ 2%, LD ~ 1%, Parametric ~ 11%
- Sum over the three neutrino flavours

$$\mathrm{BR}(K_L \to \pi^0 \nu\nu)_{\mathrm{SM}} = (2.68 \pm 0.30) \times 10^{-11}$$

[D'Ambrosio, Iyer, Mahmoudi, SN '22]

$$\mathrm{BR}(K_L \to \pi^0 \nu\nu)_{\mathrm{KOTO}} < 2.2 \times 10^{-9} \text{ at } 90\%\mathrm{CL}$$

[KOTO, Ahm et al. '24]

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New Physics effects:

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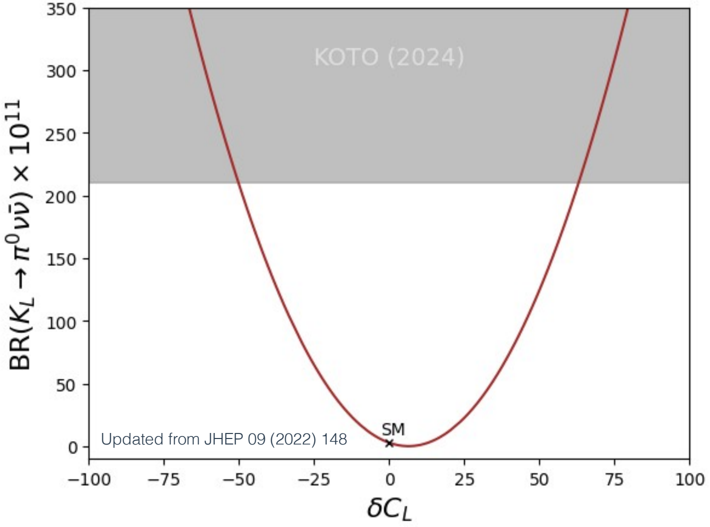

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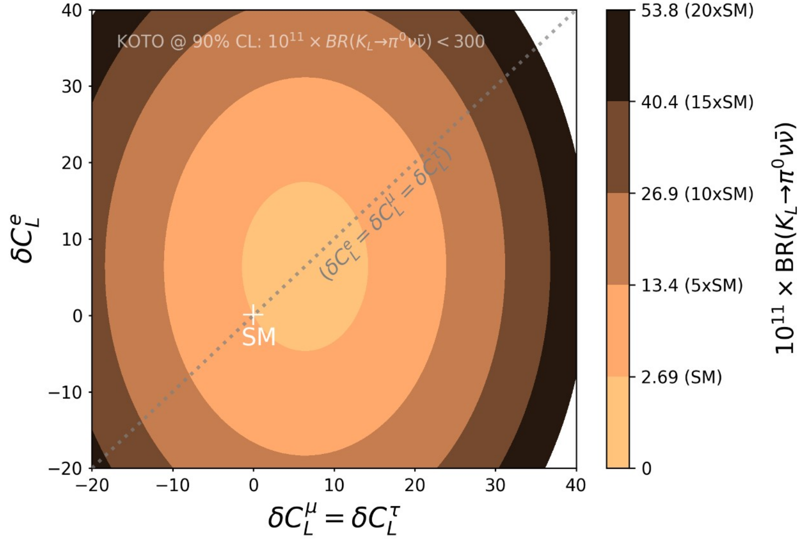

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$K_L \to \pi^0 \nu\nu$

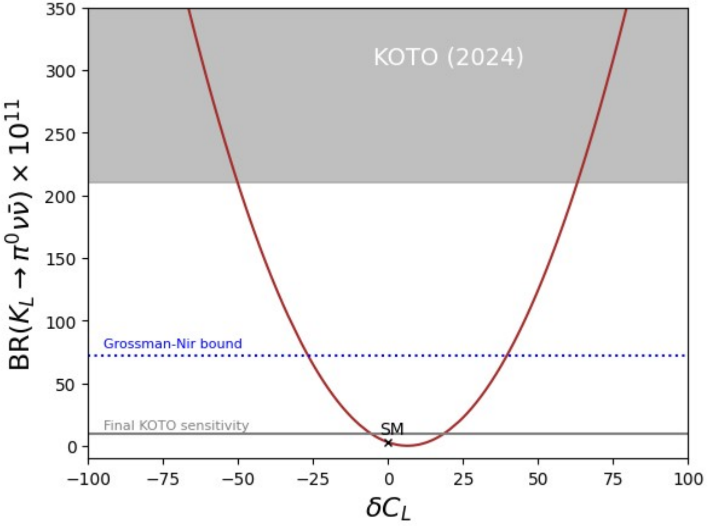
Matrix elements of $K_L \to \pi^0 \nu\nu$ and $K^+ \to \pi^+ \nu\nu$ are related via isospin resulting in the Grossman-Nir bound

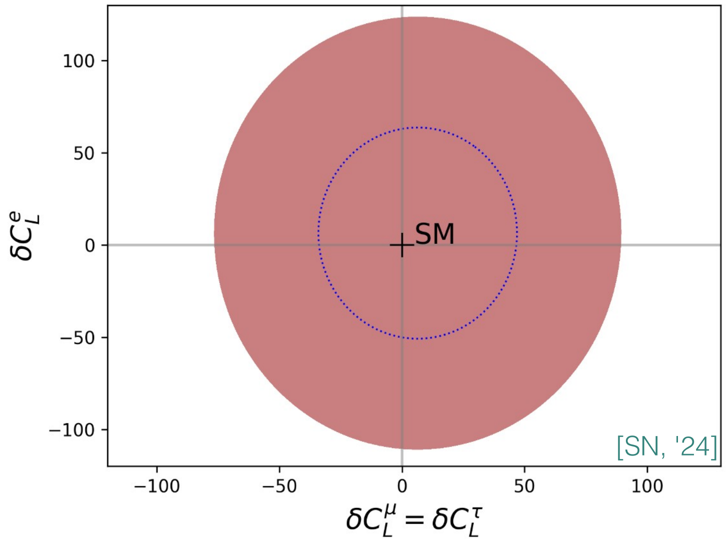
[Grossman, Nir '97]

$$\mathrm{BR}(K_L \to \pi^0 \nu\nu) \leq 4.3 \times \mathrm{BR}(K^+ \to \pi^+ \nu\nu)$$

valid in the presence of most NP models

Considering the 2024 results of NA62 for $\mathrm{BR}(K^+ \to \pi^+ \nu\nu)$



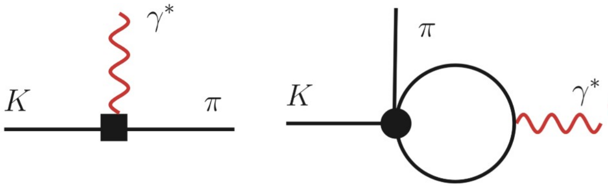


[SN, '24]

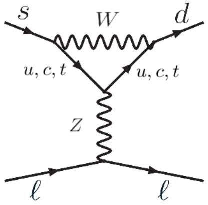
LD-dominated

LFUV in $K^+ \to \pi^+ \ell \ell$

$K^+ \to \pi^+ \ell \ell$ is long distance dominated, mediated by single photon exchange $K^+ \to \pi^+ \gamma^*$



LD >> SD



$\Longrightarrow$ precise SM prediction not yet possible

$$\mathcal{A}(z) \propto G_F M_K^2 (a + bz) + W^{\pi\pi}(z) \qquad z = m^2(\ell\ell) \, / \, M_K^2$$

form factors

loop term

LFU predicts the same a for $\ell = e, \mu$ and similarly for b

$a^{ee} \neq a^{\mu\mu}$ indicates LFUV NP: $a_+^{\mu\mu} - a_+^{ee} = -\sqrt{2}\,\mathrm{Re}\,[V_{td} V_{ts}^* (C_9^\mu - C_9^e)]$

[Crivellin et al. '16]

LFUV in $K^+ \to \pi^+ \ell\,\ell$

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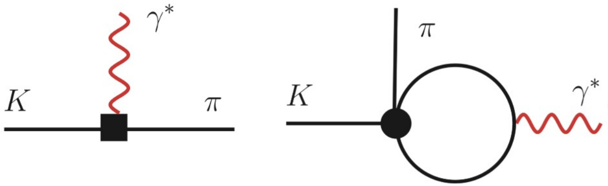

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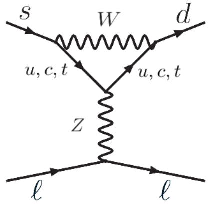

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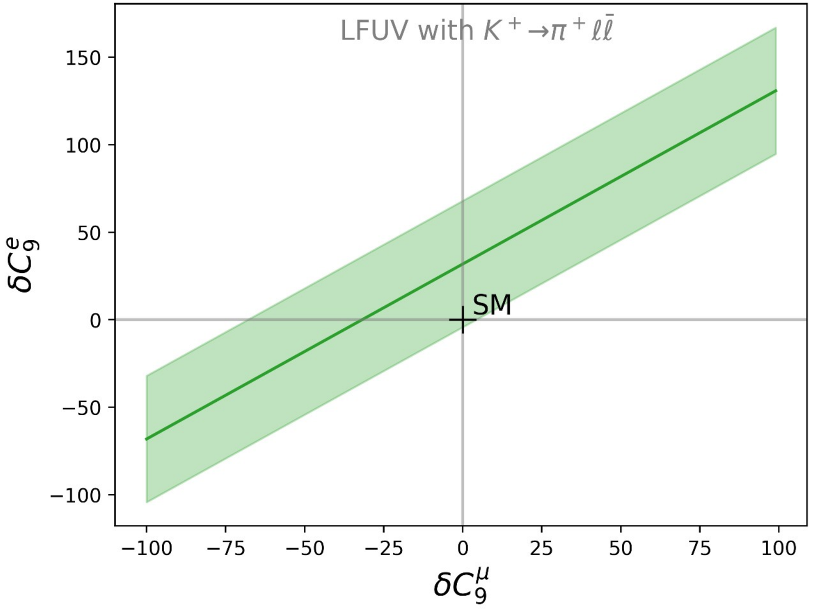
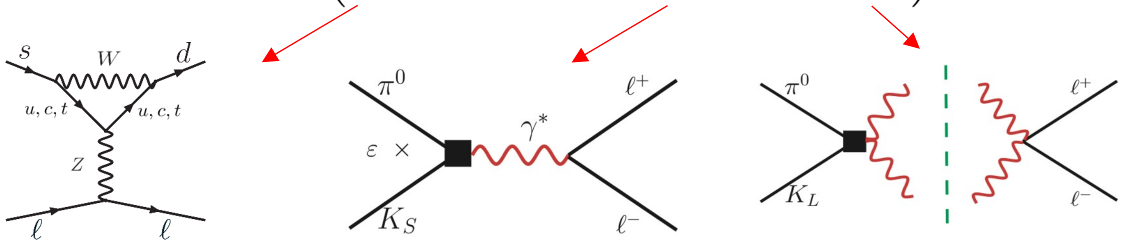
[Crivellin et al. ‘16]

Channel	a_+	b_+	Reference
ee	-0.561 ± 0.009	-0.694 ± 0.040	E865 ‘99 and NA48/2 ‘09 comb. [D’Ambrosio, Greynat, Knecht ‘18]
$\mu\mu$	-0.575 ± 0.013	-0.722 ± 0.043	NA62 Coll. ‘22



$K_L \to \pi^0 \ell\ell$

$$\mathrm{BR}(K_L \to \pi^0 \ell\bar{\ell}) = \left(C^\ell_{\rm dir} \pm C^\ell_{\rm int}|a_S| + C^\ell_{\rm mix}|a_S|^2 + C^\ell_{\gamma\gamma}\right) \cdot 10^{-12}$$

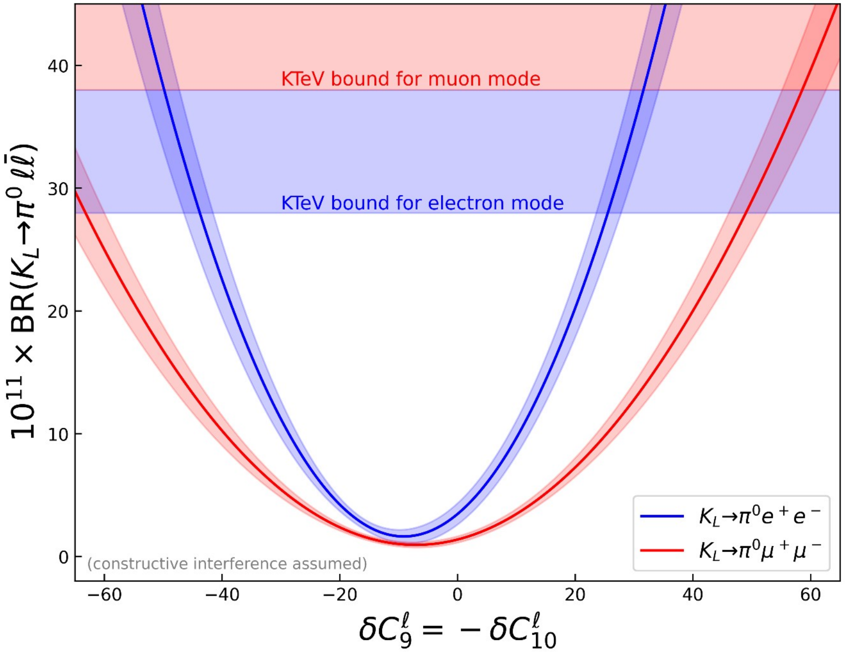


[Dambrosio et al. ‘98;Isidor et al. ‘04;Mescia, Smith, Trine ‘06]

	$C^\ell_{\rm dir}$	$C^\ell_{\rm int}$	$C^\ell_{\rm mix}$	$C^\ell_{\gamma\gamma}$
$\ell = e$	$(4.62 \pm 0.24)\boxed{(w^2_{7V} + w^2_{7A})}$	$(11.3 \pm 0.3)\boxed{w_{7V}}$	14.5 ± 0.5	≈ 0
$\ell = \mu$	$(1.09 \pm 0.05)\boxed{(w^2_{7V} + 2.32 w^2_{7A})}$	$(2.63 \pm 0.06)\boxed{w_{7V}}$	3.36 ± 0.20	5.2 ± 1.6

[Mescia, Smith, Trine ‘06]

$$\boxed{w_{7V}} = \frac{1}{2\pi}\mathrm{Im}\left[\frac{\lambda_t^{sd}}{1.407 \times 10^{-4}}\boxed{C_9}\right], \quad \boxed{w_{7A}} = \frac{1}{2\pi}\mathrm{Im}\left[\frac{\lambda_t^{sd}}{1.407 \times 10^{-4}}\boxed{C_{10}}\right]$$



$$\mathrm{BR}^{\rm SM}(K_L \to \pi^0 e\bar{e}) = 3.46^{+0.92}_{-0.80}\,(1.55^{+0.60}_{-0.48}) \times 10^{-11}$$

$$\mathrm{BR}^{\rm SM}(K_L \to \pi^0 \mu\bar{\mu}) = 1.38^{+0.27}_{-0.25}\,(0.94^{+0.21}_{-0.20}) \times 10^{-11}$$

[D’Ambrosio, Iyer, Mahmoudi, SN ‘22]

$\mathrm{BR}^{\rm exp}(K_L \to \pi^0 e\bar{e}) < 28 \times 10^{-11}$ at 90% CL

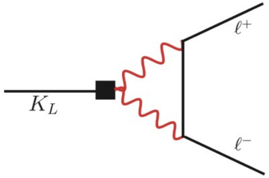
$\mathrm{BR}^{\rm exp}(K_L \to \pi^0 \mu\bar{\mu}) < 38 \times 10^{-11}$ at 90% CL

[KTeV ‘00 and ‘03]

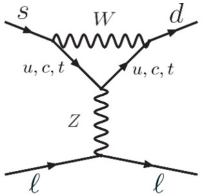
$K_L \to \mu\,\mu$

$K_L \to \mu\,\mu$ is long distance dominated, mediated by two photons via $K_L \to \gamma^* \gamma^*$

$$\mathrm{BR}(K_L \to \mu\bar{\mu}) = \tau_L \frac{f_K^2 m_K^3 \beta_\mu}{16\pi} \left| N_L^{\mathrm{LD}} - \left(\frac{2m_\mu}{m_K}\frac{G_F \alpha_e}{\sqrt{2}\pi}\right) \mathrm{Re}\left[-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t \boxed{C_{10}^{\ell}}\right]\right|^2$$



LD >> SD



$$N_L^{\mathrm{LD}} \propto (\chi_{\mathrm{disp}} + i\chi_{\mathrm{abs}}) \longrightarrow N_L^{\mathrm{LD}} = \pm\,[0.54(77) - 3.95i] \times 10^{-11}\,(\mathrm{GeV})^{-2}$$

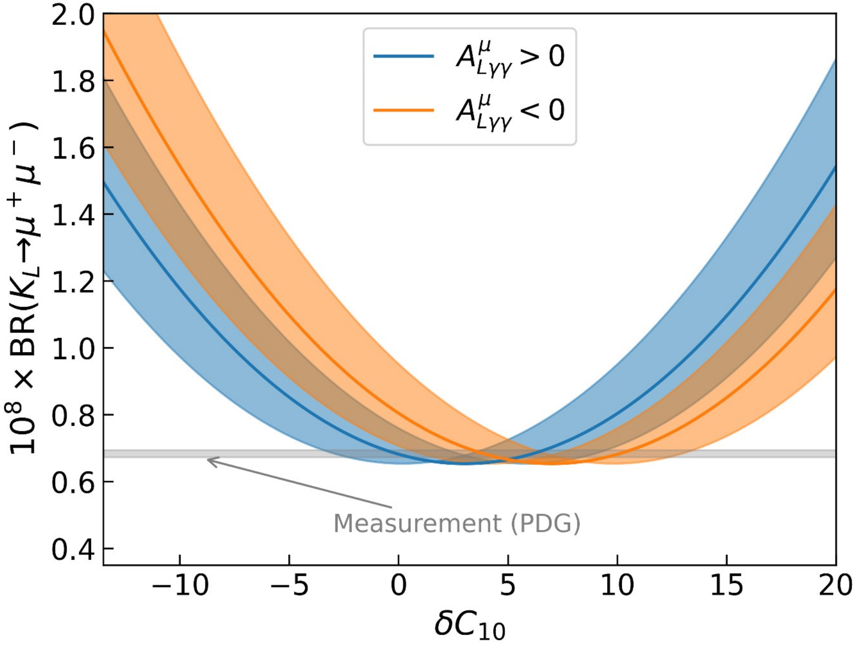
[D'Ambrosio et al. '86 '97;
Gomez Dumm, Pich '98;
Knecht et al. '99;
Isidori, Unterdorfer '03;
Hoferichter et al. '23]

[D'Ambrosio et al. '17]

- Prediction depends on the sign of $A(K_L \to \gamma\gamma)$ contribution
- Uncertainty highly asymmetric

$$\mathrm{BR}(K_L \to \mu\bar{\mu})_{\mathrm{SM}} = \begin{cases} \mathrm{LD}(+): & (6.82^{+0.77}_{-0.24} \pm 0.04) \times 10^{-9} \\ \mathrm{LD}(-): & (8.04^{+1.46}_{-0.97} \pm 0.09) \times 10^{-9} \end{cases}$$

[D'Ambrosio, Iyer, Mahmoudi, SN '22]

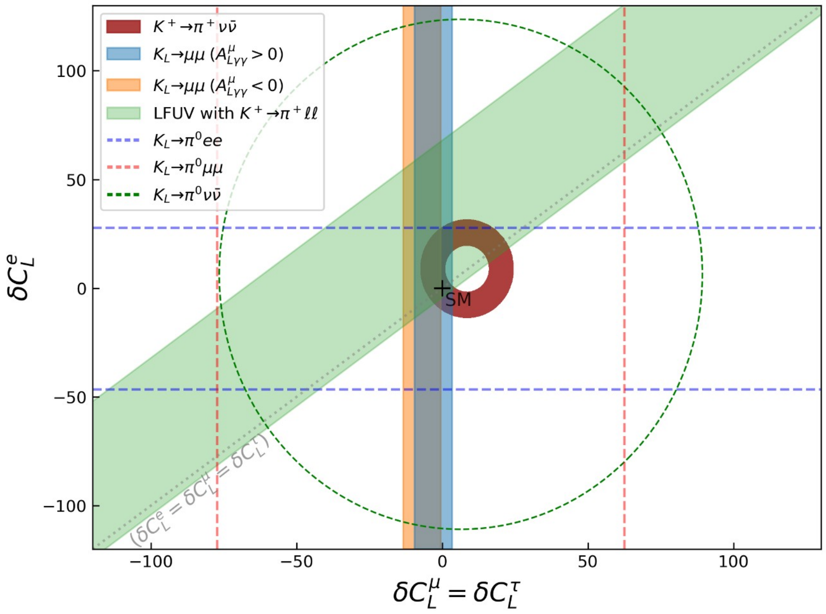


$$\mathrm{BR}(K_L \to \mu\bar{\mu})_{\mathrm{exp}} = (6.84 \pm 0.11) \times 10^{-9}$$

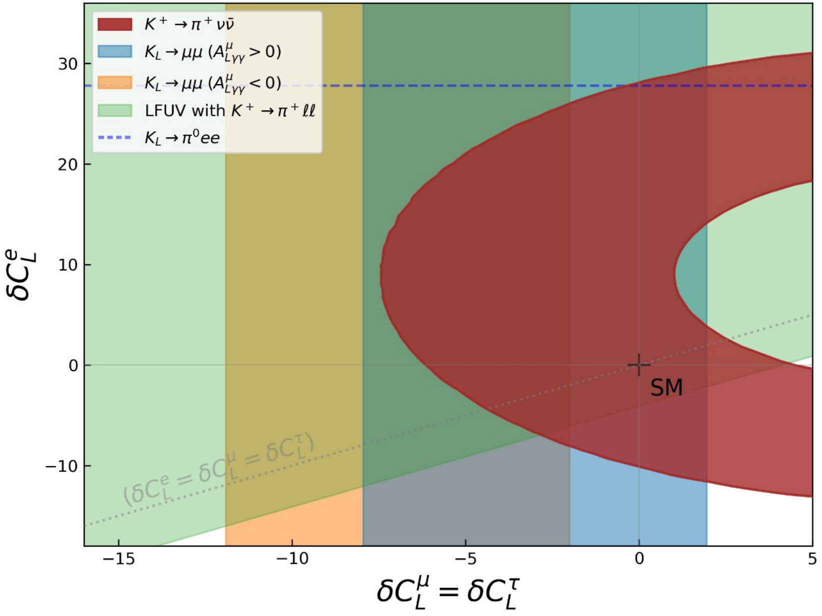
[PDG]

All observables

Rare kaon observables



$$\delta C_L^e \neq \delta C_L^\mu = \delta C_L^\tau$$



We assume NP contributions of the charged and neutral leptons related to each other by the SU(2)$_L$ gauge symmetry and we work in the chiral basis

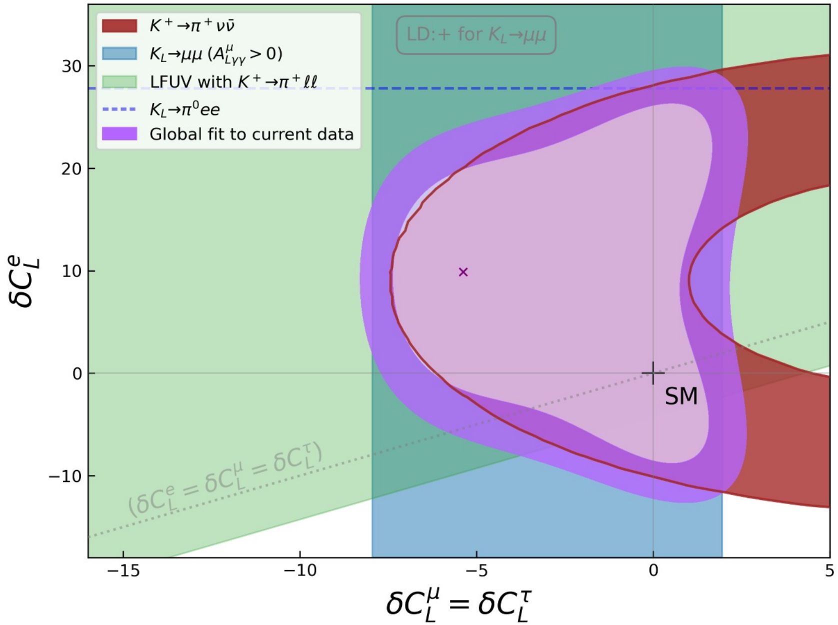
$$\delta C_L^\ell \equiv \delta C_9^\ell = -\delta C_{10}^\ell$$

Bounds from individual observables:

Coloured regions: 68% CL measurements
Dashed lines: 90% upper limits

All observables / Global fit

Global fit (with `SuperIso` public program) for positive LD contributions to $K_L \to \mu\mu$

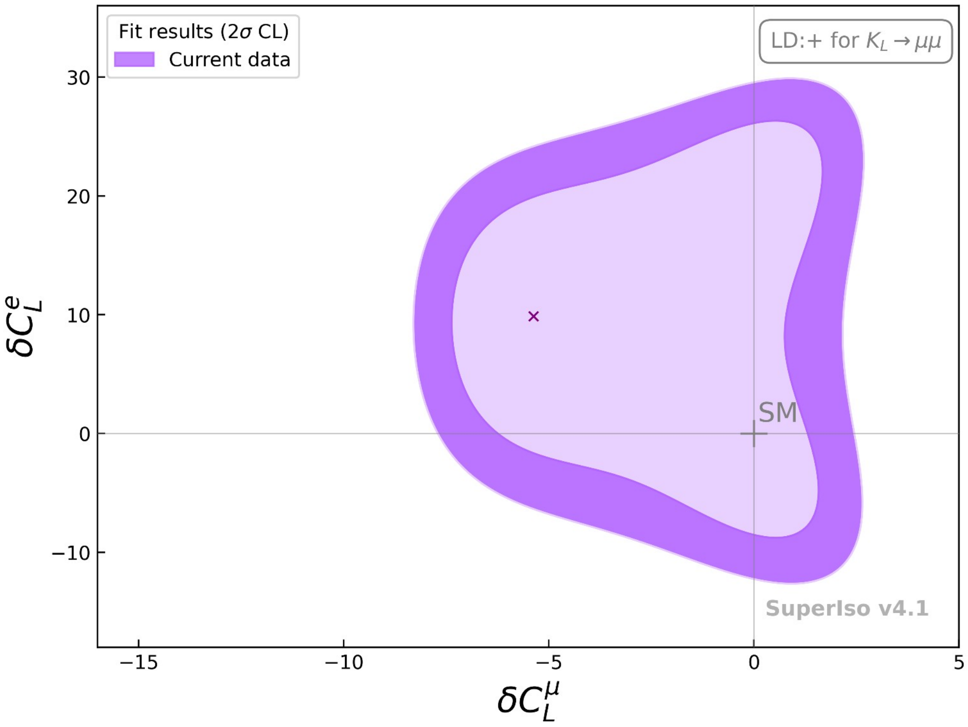


Lighter / **darker** purple region: **68%** / **95%** CL of global fit

Main constraining observables BR($K^+ \to \pi^+ \nu\nu$) followed by BR($K_L \to \mu\mu$)

Prospects for future measurements

Prospects for NA62 & KOTO-II



— Current situation

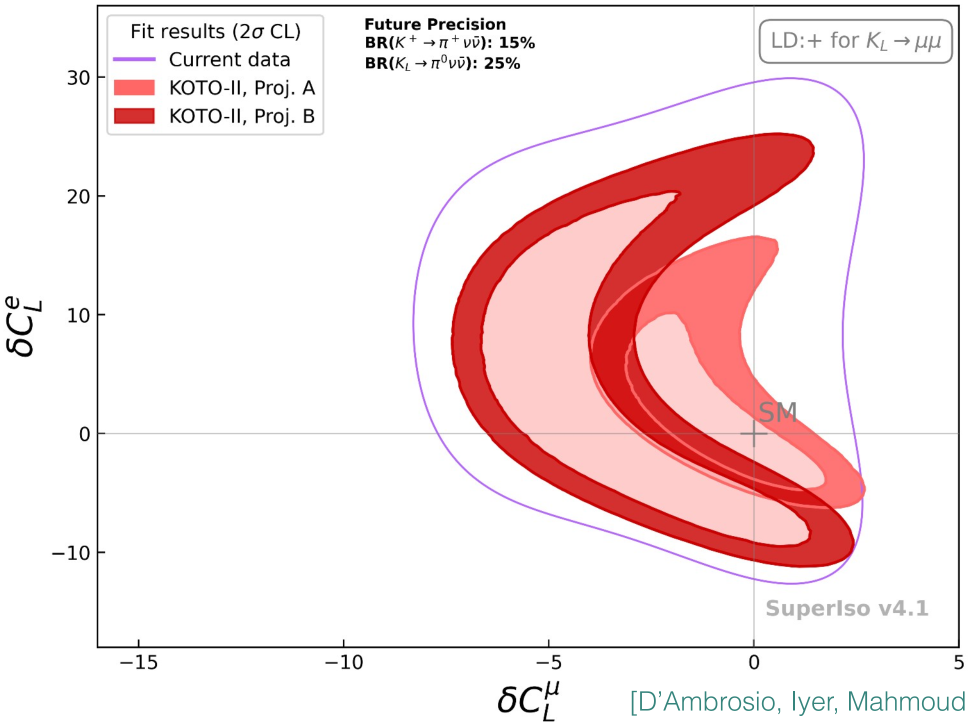
Projection A

Observables already measured are kept, others assumed to match SM, with target precision of future NA62 & KOTO-II

Projection B

All measurements give current best-fit point with target precision of future NA62 & KOTO-II

Prospects for NA62 & KOTO-II



[D'Ambrosio, Iyer, Mahmoudi, SN '24]

— Current situation

Phase 1

- NA62 final precision for $K^+ \to \pi^+ \nu\nu$ (15%)
- KOTO-II final precision for $K_L \to \pi^0 \nu\nu$ (25%)

[KOTO, Ahn et al. '25, KOTO & KOTO II Ahn et al. '25]

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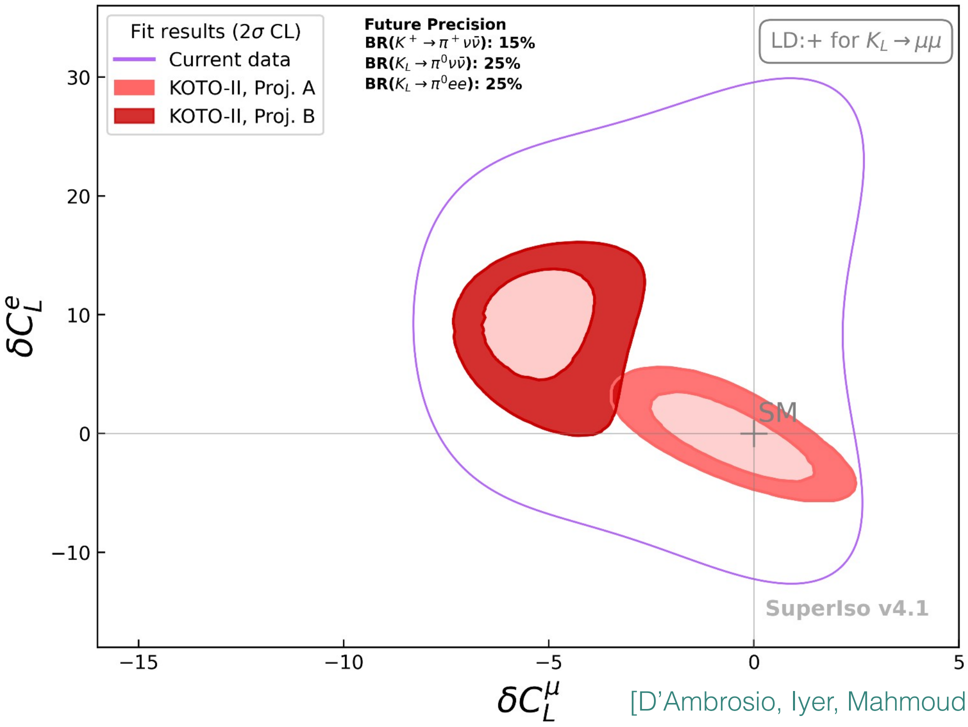
Prospects for NA62 & KOTO-II



[D'Ambrosio, Iyer, Mahmoudi, SN '24]

Current situation

Phase 2

- NA62 final precision for $K^+ \to \pi^+ \nu\nu$ (15%)
- KOTO-II final precision for $K_L \to \pi^0 \nu\nu$ (25%)
- KOTO-II measurement of $K_L \to \pi^0 e^+ e^-$ (25%)

[KOTO, Ahn et al. '25, KOTO & KOTO II Ahn et al. '25]

Projection A

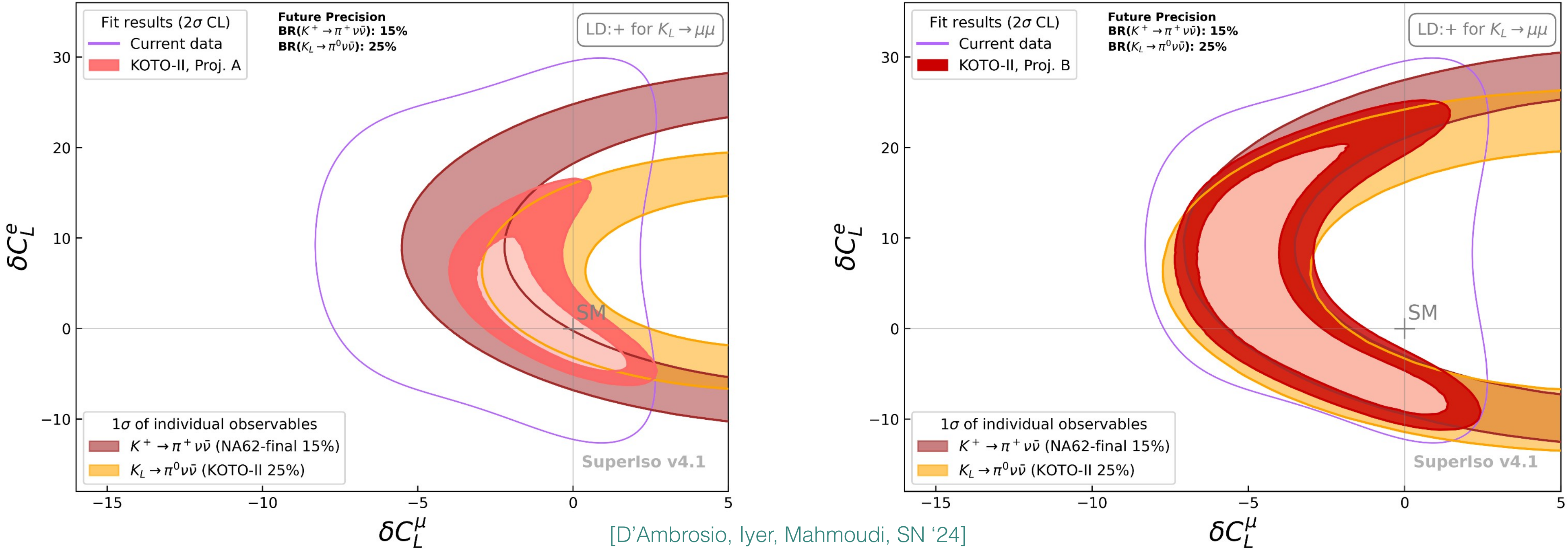
Observables already measured are kept, others assumed to match SM, with target precision of future NA62 & KOTO-II

Projection B

All measurements give current best-fit point with target precision of future NA62 & KOTO-II

Impact of projected measurements

Phase 1



[D'Ambrosio, Iyer, Mahmoudi, SN '24]

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Projection B

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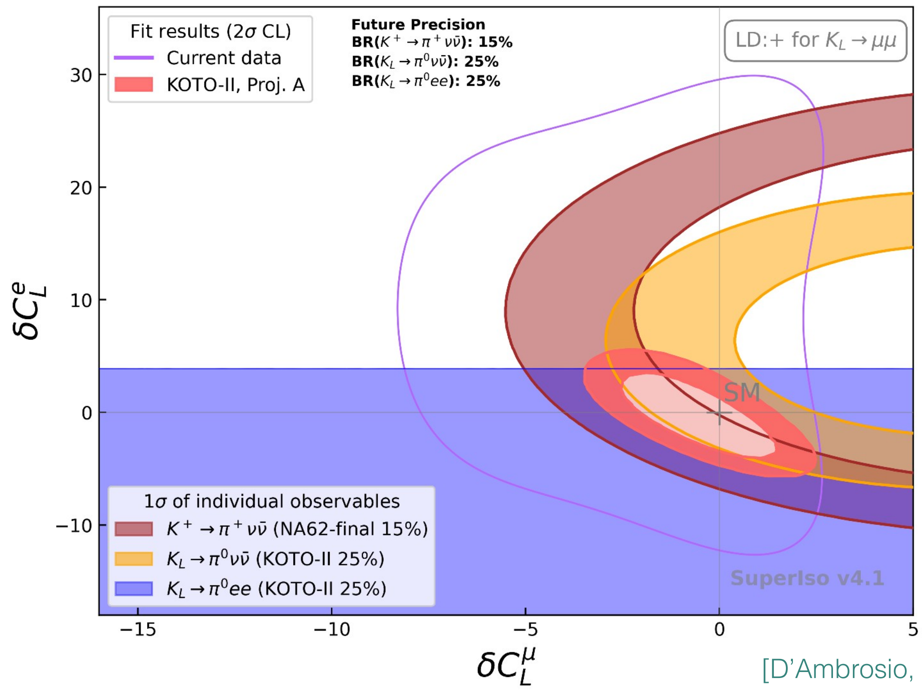
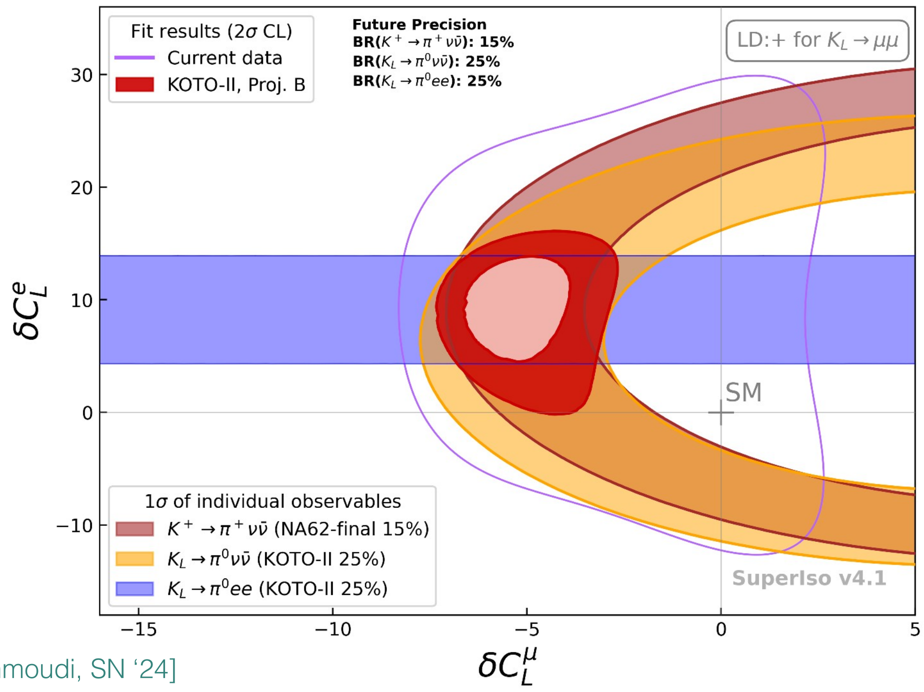
Impact of projected measurements

Phase 2





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Summary

- Rare kaon decays offer interesting information on short distance physics, even those which are long-distance dominated
- Global analysis gives a stronger and more coherent bound on NP
- Measurement of $K_L \to \pi^0 \ell \ell$, although long-distance dominated, especially in the electron sector gives a very effective probe of new physics
- Future improvements of kaon measurements are crucial

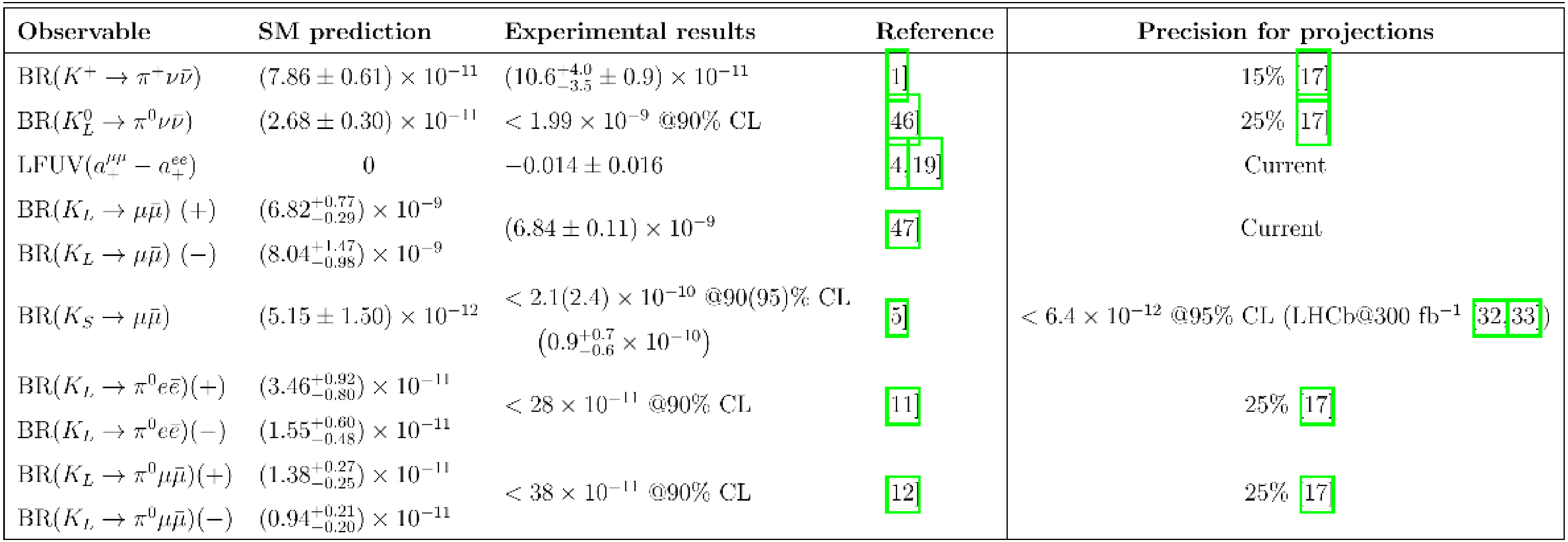
Summary

- Rare kaon decays offer interesting information on short distance physics, even those which are long-distance dominated
- Global analysis gives a stronger and more coherent bound on NP
- Measurement of $K_L \to \pi^0 \ell \ell,$ although long-distance dominated, especially in the electron sector gives a very effective probe of new physics
- Future improvements of kaon measurements are crucial

Thank you!

Backup

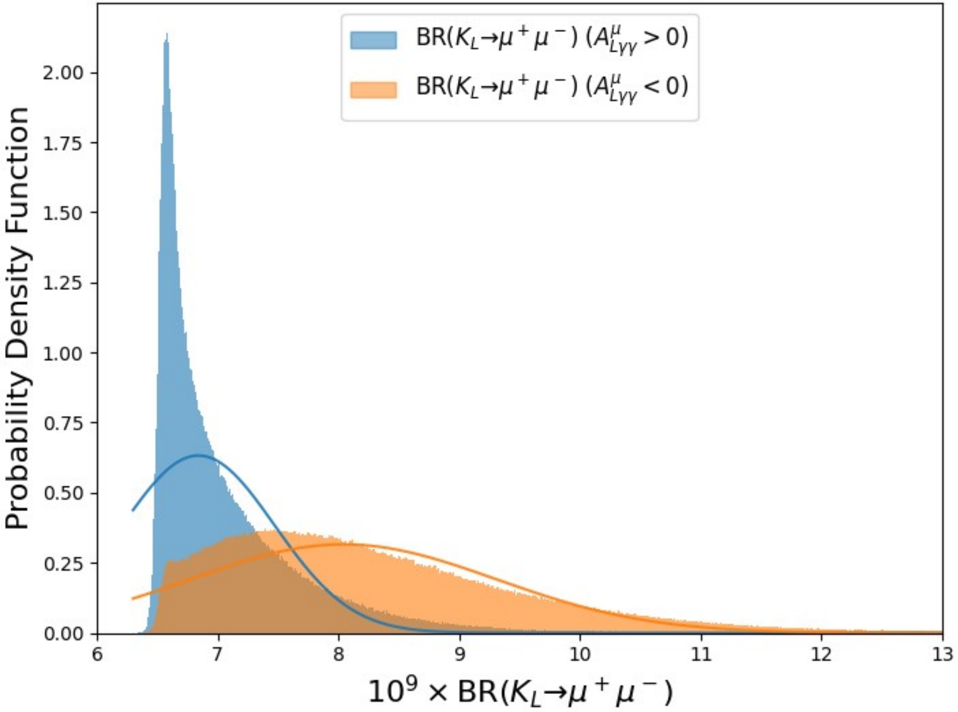
Theory and experimental results used in the fit

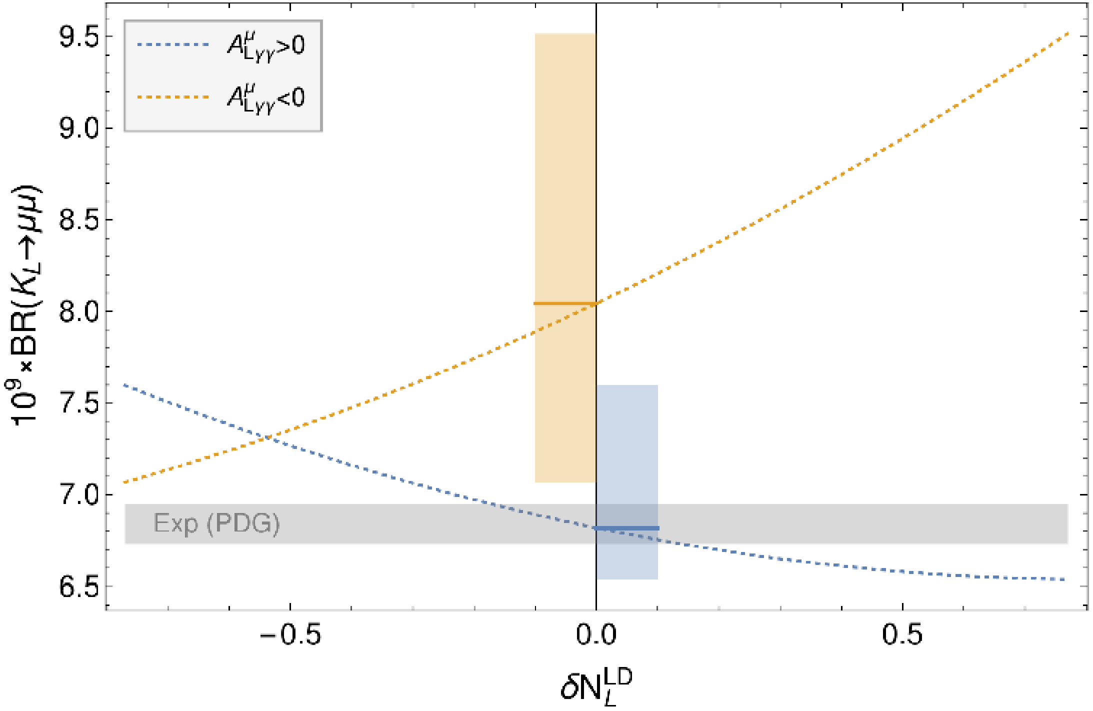


Observable	SM prediction	Experimental results	Reference	Precision for projections
$\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu})$	$(7.86 \pm 0.61) \times 10^{-11}$	$(10.6^{+4.0}_{-3.5} \pm 0.9) \times 10^{-11}$	[1]	15% [17]
$\mathrm{BR}(K^0_L \to \pi^0\nu\bar{\nu})$	$(2.68 \pm 0.30) \times 10^{-11}$	$< 1.99 \times 10^{-9}$ @90% CL	[46]	25% [17]
$\mathrm{LFUV}(a_+^{\mu\mu} - a_+^{ee})$	0	-0.014 ± 0.016	[4, 19]	Current
$\mathrm{BR}(K_L \to \mu\bar{\mu})$ (+)	$(6.82^{+0.77}_{-0.29}) \times 10^{-9}$	$(6.84 \pm 0.11) \times 10^{-9}$	[47]	Current
$\mathrm{BR}(K_L \to \mu\bar{\mu})$ (−)	$(8.04^{+1.47}_{-0.98}) \times 10^{-9}$			
$\mathrm{BR}(K_S \to \mu\bar{\mu})$	$(5.15 \pm 1.50) \times 10^{-12}$	$< 2.1(2.4) \times 10^{-10}$ @90(95)% CL $(0.9^{+0.7}_{-0.6} \times 10^{-10})$	[5]	$< 6.4 \times 10^{-12}$ @95% CL (LHCb@300 fb^{-1} [32, 33])
$\mathrm{BR}(K_L \to \pi^0 e\bar{e})(+)$	$(3.46^{+0.92}_{-0.80}) \times 10^{-11}$	$< 28 \times 10^{-11}$ @90% CL	[11]	25% [17]
$\mathrm{BR}(K_L \to \pi^0 e\bar{e})(-)$	$(1.55^{+0.60}_{-0.48}) \times 10^{-11}$			
$\mathrm{BR}(K_L \to \pi^0 \mu\bar{\mu})(+)$	$(1.38^{+0.27}_{-0.25}) \times 10^{-11}$	$< 38 \times 10^{-11}$ @90% CL	[12]	25% [17]
$\mathrm{BR}(K_L \to \pi^0 \mu\bar{\mu})(-)$	$(0.94^{+0.21}_{-0.20}) \times 10^{-11}$			

Uncertainties in $K_L \to \ell\ell$

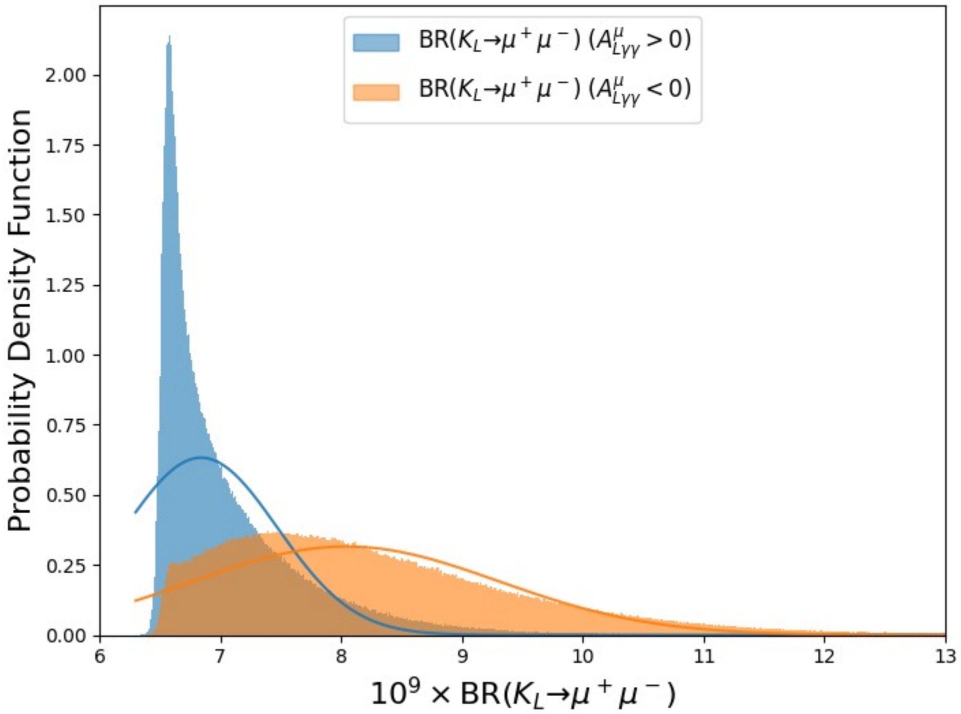
Asymmetric theoretical uncertainty of $K_L \to \mu\mu$

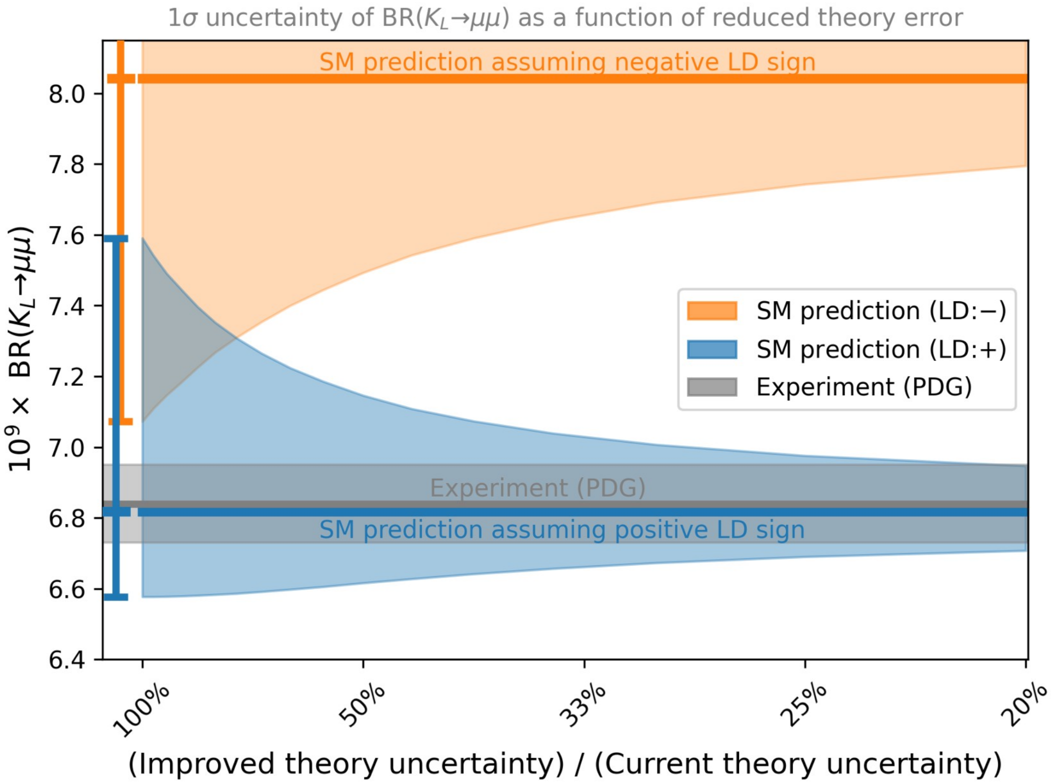




Uncertainties in $K_L \to \ell\ell$

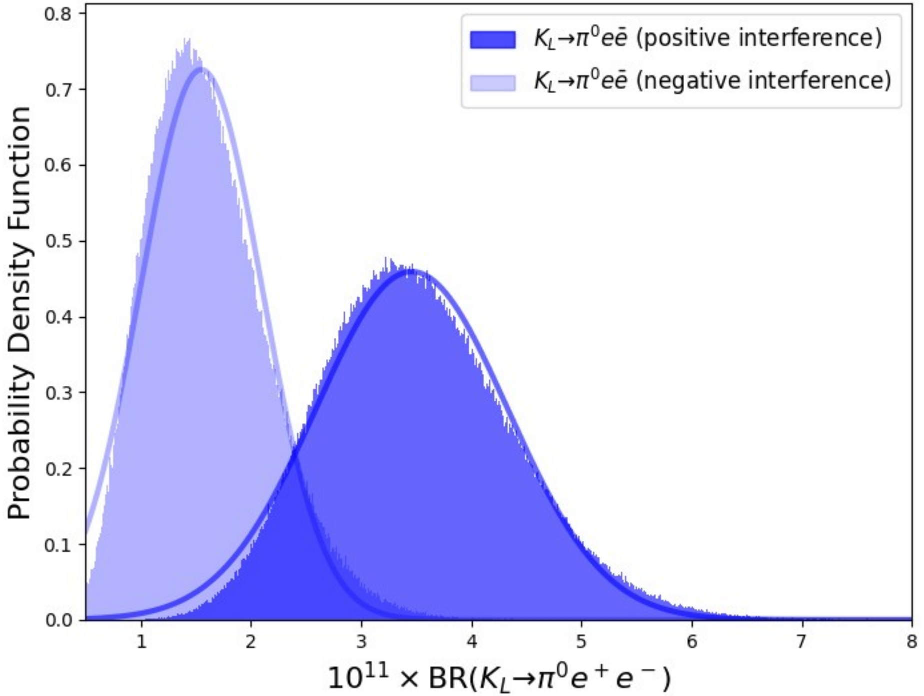
Asymmetric theoretical uncertainty of $K_L \to \mu\mu$

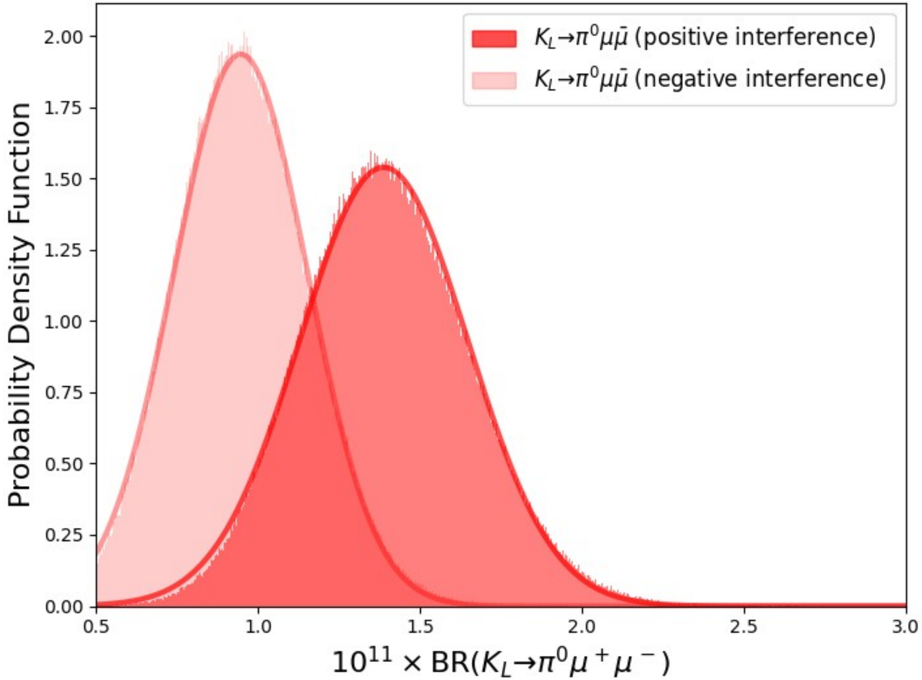




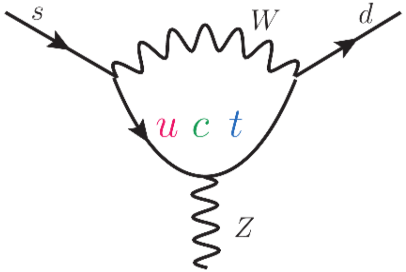
Uncertainties in $K_L \to \pi^0 \ell\ell$

Asymmetric theoretical uncertainty of $K_L \to \pi^0 \ell\ell$



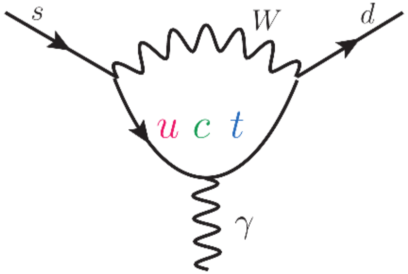


FCNC



$$\lambda_q \equiv V_{qd} V_{qs}^*$$

$$x_q \equiv m_q^2 / M_W^2$$



CKM unitarity

$$\sum_k V_{ik} V_{jk}^* = 0 \qquad \lambda_u + \lambda_c + \lambda_t = 0$$

Amplitude = sum over all internal up-quarks:

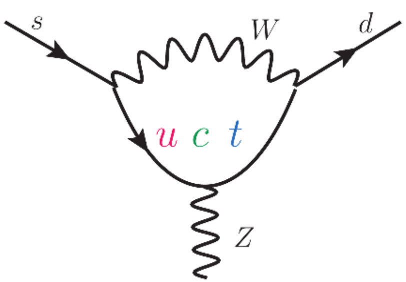
$$\Sigma_{q=u,c,t}\ \lambda_q F(x_q) = \lambda_u F(x_u) + \lambda_c F(x_c) + \lambda_t F(x_t)$$

CKM factor for B-mesons

$$Re(\lambda_u) \sim \lambda^4, \qquad Re(\lambda_c) \sim \lambda^3, \qquad Re(\lambda_t) \sim \lambda^2$$

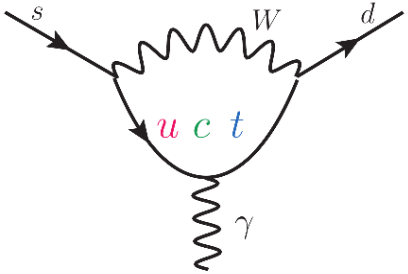
$$Im(\lambda_u) = \lambda^4, \qquad Im(\lambda_c) \sim \lambda^8, \qquad Im(\lambda_t) \sim \lambda^4$$

FCNC



$$\lambda_q \equiv V_{qd}V^*_{qs}$$

$$x_q \equiv m_q^2/M_W^2$$



CKM unitarity

$$\sum_k V_{ik}V^*_{jk} = 0 \qquad \lambda_u + \lambda_c + \lambda_t = 0$$

Amplitude = sum over all internal up-quarks:

$$\Sigma_{q=u,c,t}\ \lambda_q F(x_q) = \lambda_u F(x_u) + \lambda_c F(x_c) + \lambda_t F(x_t)$$

CKM factor for Kaons

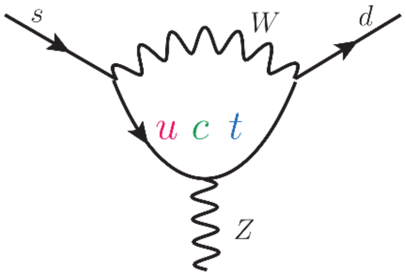
$$Re(\lambda_u) \sim \lambda, \qquad Re(\lambda_c) \sim \lambda, \qquad Re(\lambda_t) \sim \lambda^5$$

$$Im(\lambda_u) = 0, \qquad Im(\lambda_c) \sim \lambda^5, \qquad Im(\lambda_t) \sim \lambda^5$$

FCNC

Inami-Lim functions:

[Inami, Lim '81, Buchalla, Buras, Harlander '91]

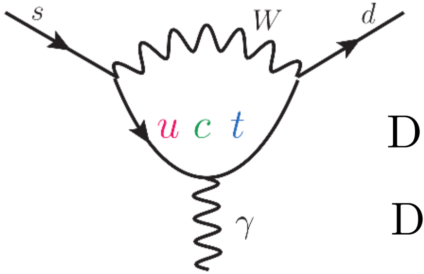


$$\mathrm{X}_0(x_q) \xrightarrow{x_q \to \infty} x_q$$

$$\mathrm{X}_0(x_q) \xrightarrow{x_q \to 0} x_q \log x_q$$

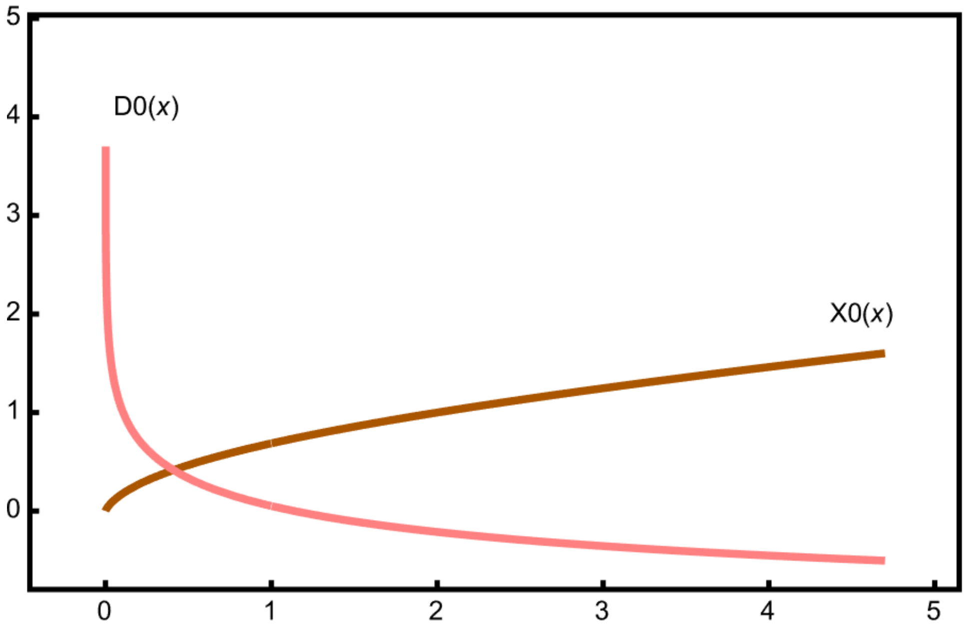
$$\lambda_q \equiv V_{qd} V_{qs}^*$$

$$x_q \equiv m_q^2 / M_W^2$$



$$\mathrm{D}_0(x_q) \xrightarrow{x_q \to \infty} -\log x_q$$

$$\mathrm{D}_0(x_q) \xrightarrow{x_q \to 0} -\log x_q$$



No large suppression for u- and c-quark (unlike B-physics)

↪large contribution of low-energy physics

$K_S \to \mu\,\mu$

$$\mathrm{BR}(K_S \to \mu\bar{\mu}) = \tau_S \frac{f_K^2 m_K^3 \beta_\mu}{16\pi} \left\{ \beta_\mu^2 \left| N_S^{\mathrm{LD}} \right|^2 + \left(\frac{2m_\mu}{m_K} \frac{G_F \alpha_e}{\sqrt{2}\pi} \right)^2 \mathrm{Im}^2 \left[-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10}^\ell \right] \right\}$$

The long-distance contribution is cleaner, as the leading $O(p^4)$ chiral contribution of $K_S \to \pi^+\pi^- \to \gamma\gamma \to \mu^+\mu^-$ is theoretically under better control [Ecker, Pich '91]

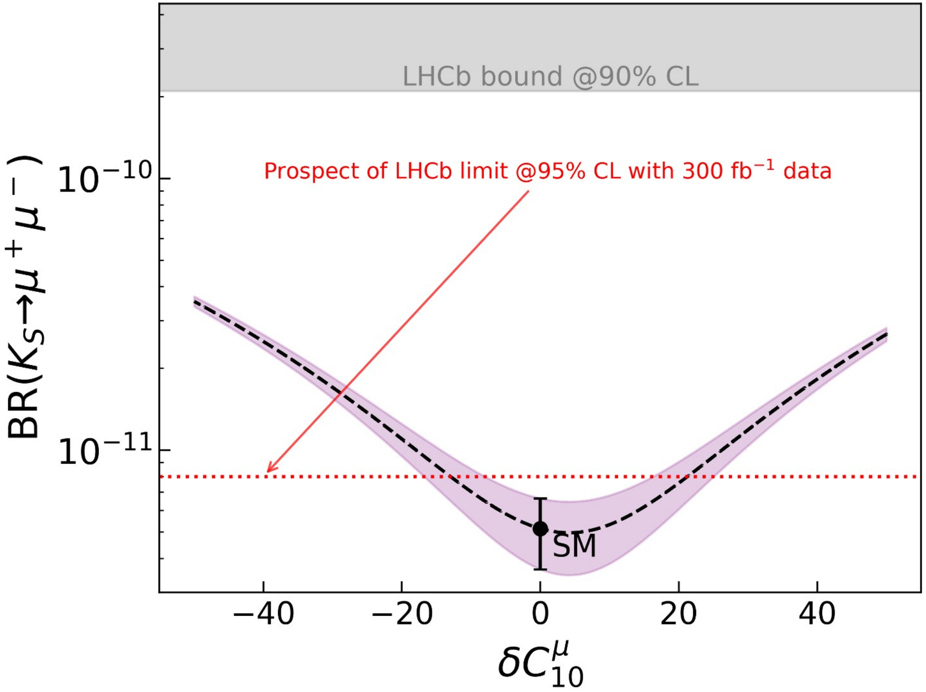
$$\mathrm{BR}(K_S \to \mu\bar{\mu})^{\mathrm{SM}} = (5.15 \pm 1.50) \times 10^{-12}$$

[D'Ambrosio, Iyer, Mahmoudi, SN '22]

$$\mathrm{BR}(K_S \to \mu\bar{\mu})^{\mathrm{LHCb}} < 2.1(2.4) \times 10^{-10} \ \ @90(95)$$

[LHCb '20]

- $K_S \to \mu\mu$ not very sensitive to axial currents
- Sensitive to new physics scenarios involving scalar and pseudoscalar contributions



Scalar and pseudoscalar contributions in $K_S \to \mu\,\mu$

Adding scalar contributions

$$\mathcal{H}_{\text{eff}}^{\text{scalar}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{td} \frac{\alpha_e}{4\pi} \left[C_S^\ell O_S^\ell + C_P^\ell O_P^\ell\right]$$

$$O_S^\ell = (\bar{s}P_R d)(\bar{\ell}\ell), \quad O_P^\ell = (\bar{s}P_R d)(\bar{\ell}\gamma_5\ell)$$

$$\text{BR}(K_S \to \mu\bar{\mu}) = \tau_S \frac{f_K^2 m_K^3 \beta_\mu}{16\pi} \left\{ \beta_\mu^2 \left| N_S^{\text{LD}} - m_K \frac{G_F \alpha_e}{\sqrt{2}\pi} \text{Re}\left[\frac{\lambda_t C_S}{m_s + m_d}\right]\right|^2 + \left(\frac{G_F \alpha_e}{\sqrt{2}\pi}\right)^2 \left| \frac{2m_\mu}{m_K} \text{Im}\left[-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10}\right] + \frac{M_K}{m_s + m_d} \text{Im}\left[\lambda_t C_P\right]\right|^2 \right\}$$

[Chobanova et al. ‘17]

- $K_S \to \mu\mu$ measurement currently two orders of magnitude above SM
- What can we say with current data about scalar and pseudoscalar contributions?

Scalar contributions in $K^+ \to \pi^+ \ell \ell$

Looking again into $K^+ \to \pi^+ \ell^+ \ell^-$ in the presence of scalar contributions

$$\mathcal{M} = \frac{\alpha G_F}{4\pi} f_V(z)(p_K + p_\pi)^\mu \bar{\ell}\gamma_\mu \ell + G_F m_K f_S \bar{\ell}\ell$$

$$\frac{d^2\Gamma}{dz\,d\cos\theta} = \frac{G_F^2 m_K^5}{2^8\pi^3}\beta_\ell \lambda^{1/2}(z) \times \Big\{ |f_V|^2 \frac{\alpha_e^2}{16\pi^2}\lambda(z)(1-\beta_\ell^2\cos^2\theta) + |f_S|^2 z\beta_\ell^2 + \mathrm{Re}[f_V^* f_S]\frac{\alpha_e r_\ell}{\pi}\beta_\ell\lambda^{1/2}(z)\cos\theta \Big\}$$

[Chen et al. ‘03, Gao '03]

$r_\ell = m_\ell / m_{K,}$

$r_\pi = m_\pi / m_{K,}$

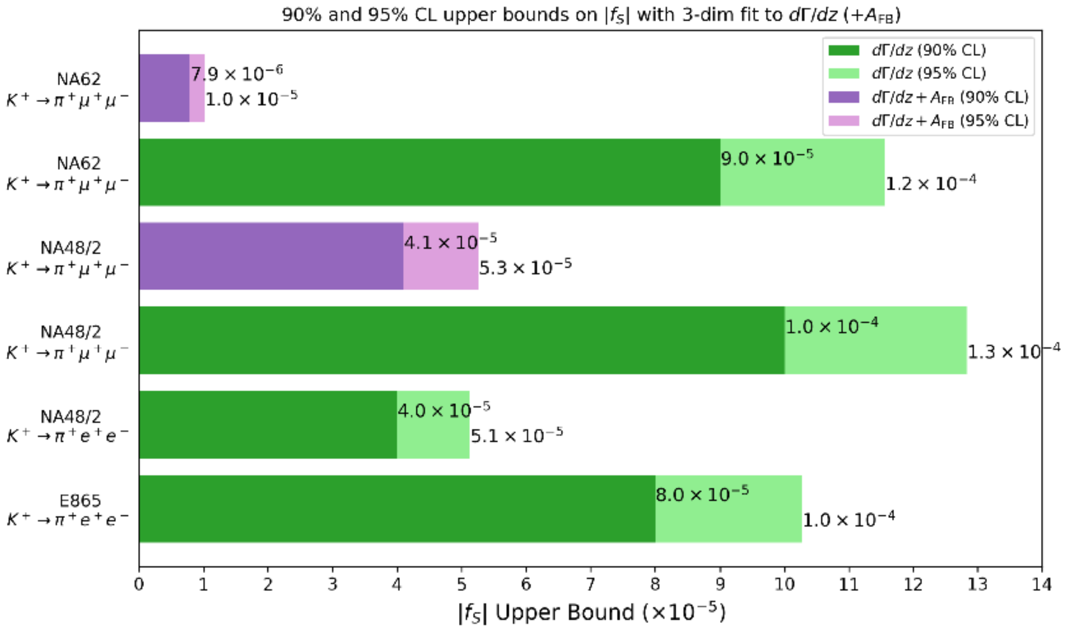
$\beta_\ell = \sqrt{1 - 4 r_\ell^2/z},$

$\lambda(z) = 1 + z^2 + r_\pi^4 - 2(z + r_\pi^2 + z r_\pi{}^2)$

$$A_{\mathrm{FB}}(z) = \frac{\int_0^1 \left(\frac{d\Gamma}{dz d\cos\theta}\right) d\cos\theta - \int_{-1}^0 \left(\frac{d\Gamma}{dz d\cos\theta}\right) d\cos\theta}{\int_0^1 \left(\frac{d\Gamma}{dz d\cos\theta}\right) d\cos\theta + \int_{-1}^0 \left(\frac{d\Gamma}{dz d\cos\theta}\right) d\cos\theta} \longrightarrow A_{\mathrm{FB}}(z) = \frac{\alpha_e G_F^2 m_K^5}{2^8\pi^4} r_\ell \beta_\ell^2 \lambda(z) \mathrm{Re}[f_V^* f_S] / \left(\frac{d\Gamma(z)}{dz}\right)$$

- If assumed SM-like only f_V contributes
- A_{FB} only non-zero in case $f_S \neq 0$
- In the case of electron mode suppressed by electron mass

Scalar contributions in $K^+ \to \pi^+ \ell\, \ell$



[D'Ambrosio, Iyer, Mahmoudi, SN '24]

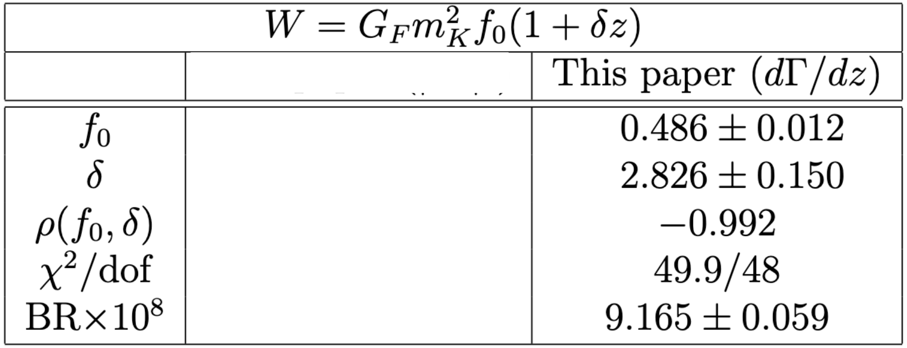
- Only bound on f_S so far via study of $\mathrm{BR}(K^+ \to \pi^+ e^+ e^-)$ from E865 data $f_S < 6.6 \times 10^{-5}$ at 90% CL
- In the muon mode also $A_{\rm FB}$ can be considered
- ~one order of magnitude stronger bound by analyzing simultaneously BR and $d\Gamma/dz$ with $f_S < 7.9 \times 10^{-6}$ at 90% CL

$K^+ \to \pi^+ \ell \, \ell$

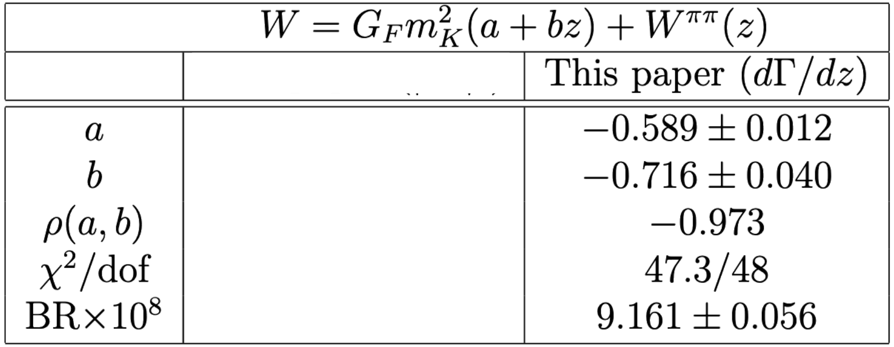
$$\frac{d\Gamma}{dz} = \frac{\alpha^2 M_K}{12\pi(4\pi)^4} \lambda^{3/2}(1, z, r_\pi^2) \beta_\ell \left(1 + 2\frac{r_\ell^2}{z}\right) |W(z)|^2$$

$$W(z) = a\frac{r_V^2}{r_V^2 - z}$$

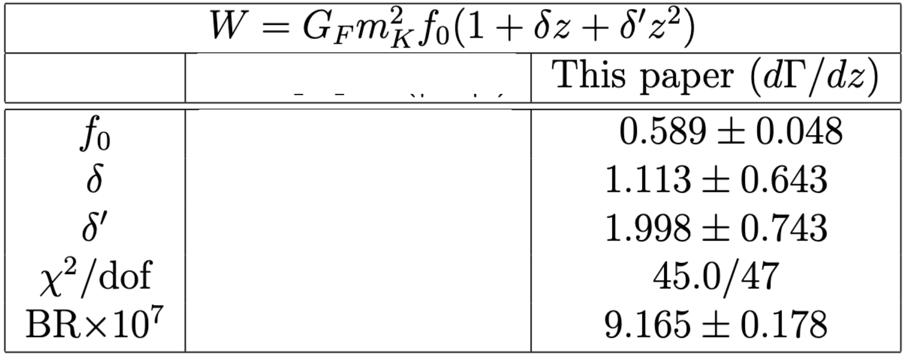
$$r_V = \frac{M_V}{M_K}$$



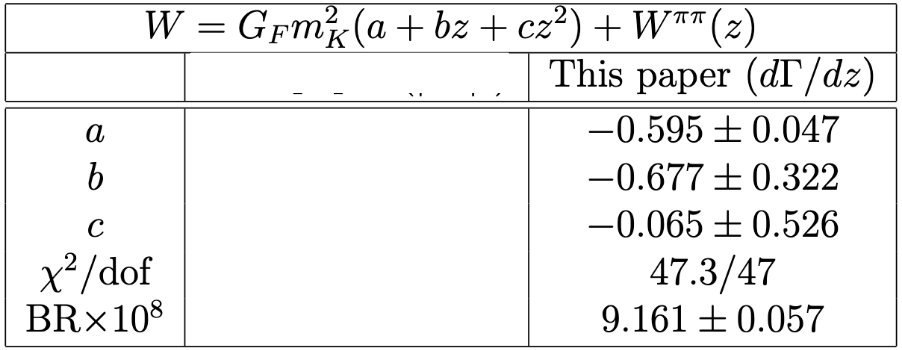
$W = G_F m_K^2 f_0(1+\delta z)$		
		This paper $(d\Gamma/dz)$
f_0		0.486 ± 0.012
δ		2.826 ± 0.150
$\rho(f_0, \delta)$		-0.992
χ^2/dof		$49.9/48$
BR×10^8		9.165 ± 0.059



$W = G_F m_K^2 (a + bz) + W^{\pi\pi}(z)$		
		This paper $(d\Gamma/dz)$
a		-0.589 ± 0.012
b		-0.716 ± 0.040
$\rho(a, b)$		-0.973
χ^2/dof		$47.3/48$
BR×10^8		9.161 ± 0.056



$W = G_F m_K^2 f_0(1+\delta z + \delta' z^2)$		
		This paper $(d\Gamma/dz)$
f_0		0.589 ± 0.048
δ		1.113 ± 0.643
δ'		1.998 ± 0.743
χ^2/dof		$45.0/47$
BR×10^7		9.165 ± 0.178



$W = G_F m_K^2 (a + bz + cz^2) + W^{\pi\pi}(z)$		
		This paper $(d\Gamma/dz)$
a		-0.595 ± 0.047
b		-0.677 ± 0.322
c		-0.065 ± 0.526
χ^2/dof		$47.3/47$
BR×10^8		9.161 ± 0.057

$K^+ \to \pi^+ \ell \, \ell$

$$\frac{d\Gamma}{dz} = \frac{\alpha^2 M_K}{12\pi(4\pi)^4} \lambda^{3/2}(1, z, r_\pi^2)\beta_\ell \left(1 + 2\frac{r_\ell^2}{z}\right) |W(z)|^2$$

$$W(z) = a\frac{r_V^2}{r_V^2 - z}$$

$$r_V = \frac{M_V}{M_K}$$