Heavy Meson Lifetimes from Lattice QCD

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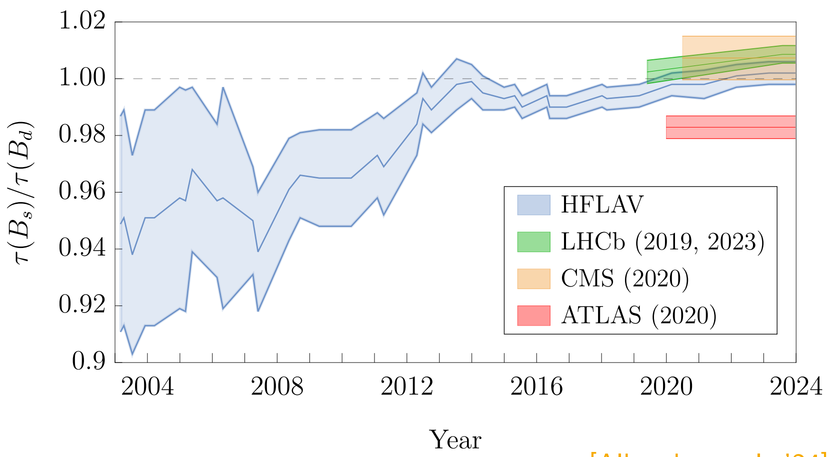
Introduction

- ➤ Heavy meson lifetimes are measured experimentally to high precision
  - ➥ Key observables for probing New Physics ➡ **high precision in theory needed!**



[Albrecht et al. '24]

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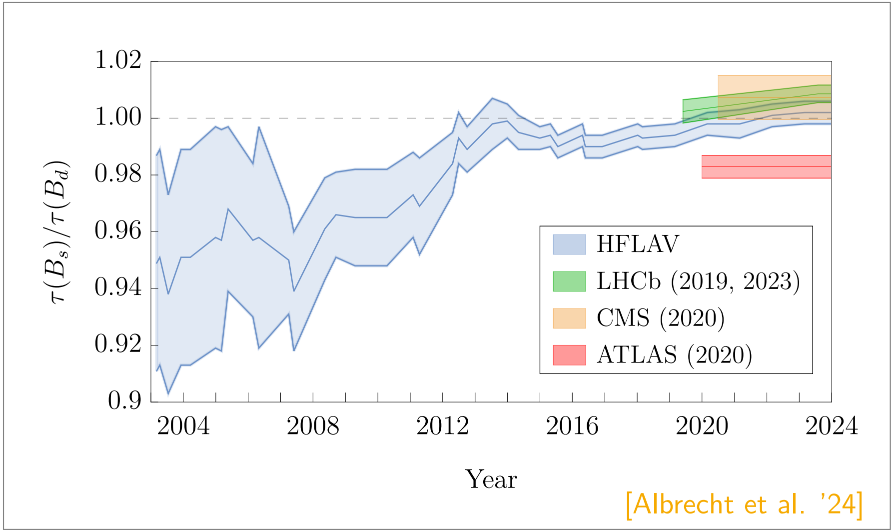
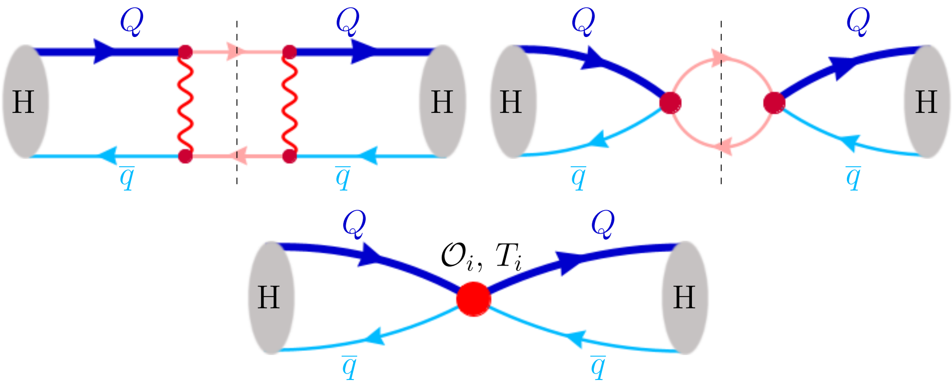


- For heavy meson lifetimes, we use the **Heavy Quark Expansion**

$$\Gamma(H_Q) = \Gamma_3\langle\mathcal{O}_3\rangle + \Gamma_5\frac{\langle\mathcal{O}_5\rangle}{m_Q^2} + \Gamma_6\frac{\langle\mathcal{O}_6\rangle}{m_Q^3} + \ldots + 16\pi^2\left[\tilde{\Gamma}_6\frac{\langle\tilde{\mathcal{O}}_6\rangle}{m_Q^3} + \tilde{\Gamma}_7\frac{\langle\tilde{\mathcal{O}}_7\rangle}{m_Q^4} + \ldots\right]$$

- $\langle\tilde{\mathcal{O}}_6\rangle$ are one of leading uncertainties for lifetimes

- Matrix elements of four-quark operators can be determined from lattice QCD simulations
- $\Delta Q = 2$ well-studied by several groups ➡ precision increasing
  - Preliminary $\Delta K = 2$ for Kaon mixing with gradient flow [Suzuki et al. '20], [Taniguchi, Lattice '19]
- $\Delta Q = 0$ ➡ exploratory studies from ~20 years ago



  - Contributions from statistically-noisy diagrams
  - Mixing with lower dimension operators in renormalisation

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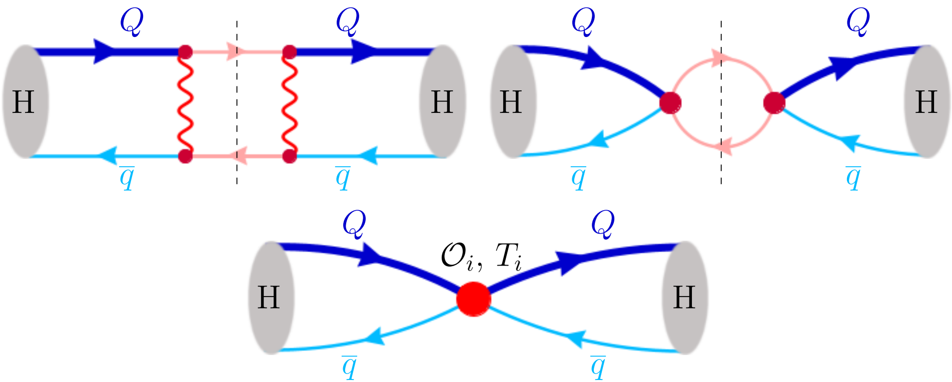

- ➥ Contributions from statistically-noisy diagrams ➜ Use modern computing strategies ✔
- ➥ Mixing with lower dimension operators in renormalisation ➜ **How can we tackle this?**

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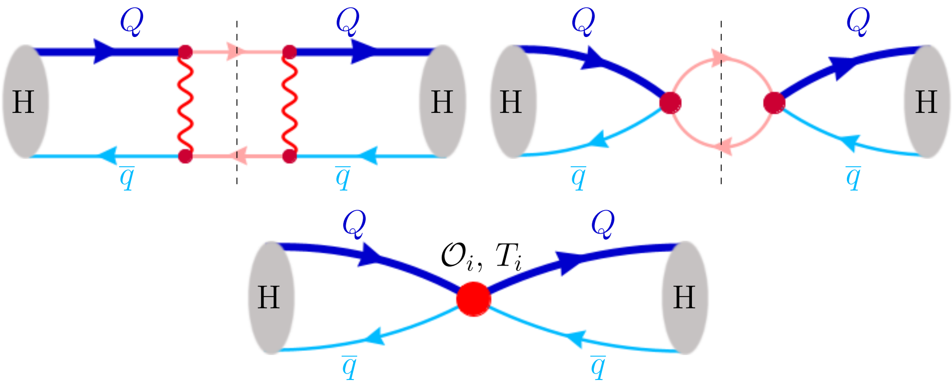

- ➥ Contributions from statistically-noisy diagrams ➜ Use modern computing strategies ✔
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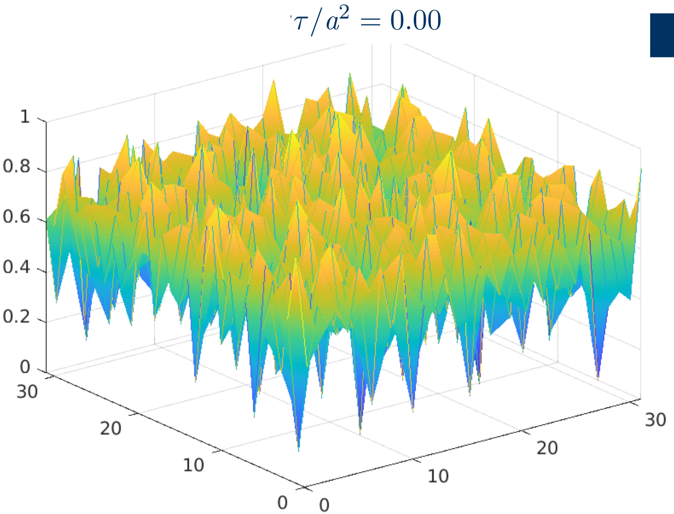
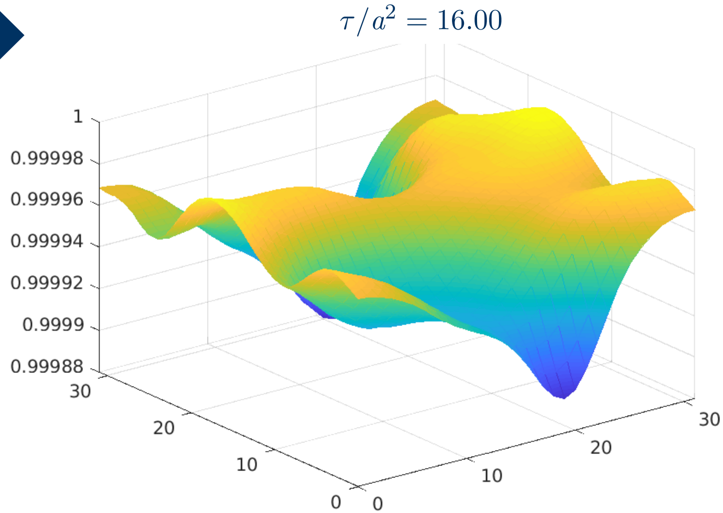
Introduce renormalisation procedure based on gradient flow

Gradient Flow

- Introduced by [Narayanan, Neuberger '06] [Lüscher '10] [Lüscher '13]
  - Scale setting ($\sqrt{8t_0}$), RG β-function, Λ parameter
- Introduce auxiliary dimension, **flow time** τ as a way to regularise the UV
- Well-defined damping of UV fluctuations





- Well-studied for e.g. energy-momentum tensor [Makino, Suzuki '14] [Harlander, Kluth, Lange '18]
  quark masses [Takaura et al. '25] [Black et al. '25]

- Re-express effective Hamiltonian in terms of 'flowed' operators:

$$\mathcal{H}_{\text{eff}} = \sum_n C_n \mathcal{O}_n = \sum_n \widetilde{C}_n(\tau) \widetilde{\mathcal{O}}_n(\tau).$$

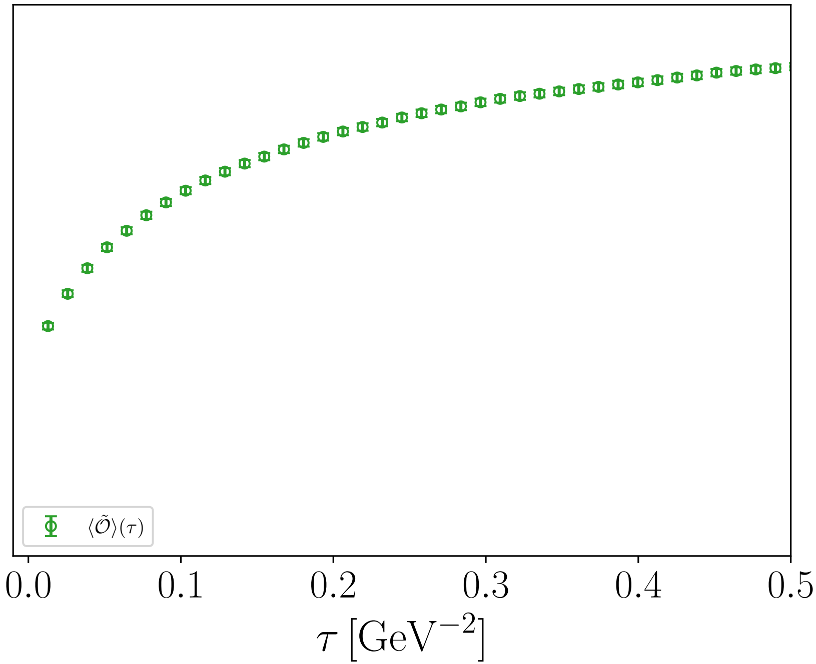
- Relate to regular operators in 'short-flow-time expansion':

$$\widetilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau) \mathcal{O}_m + O(\tau)$$

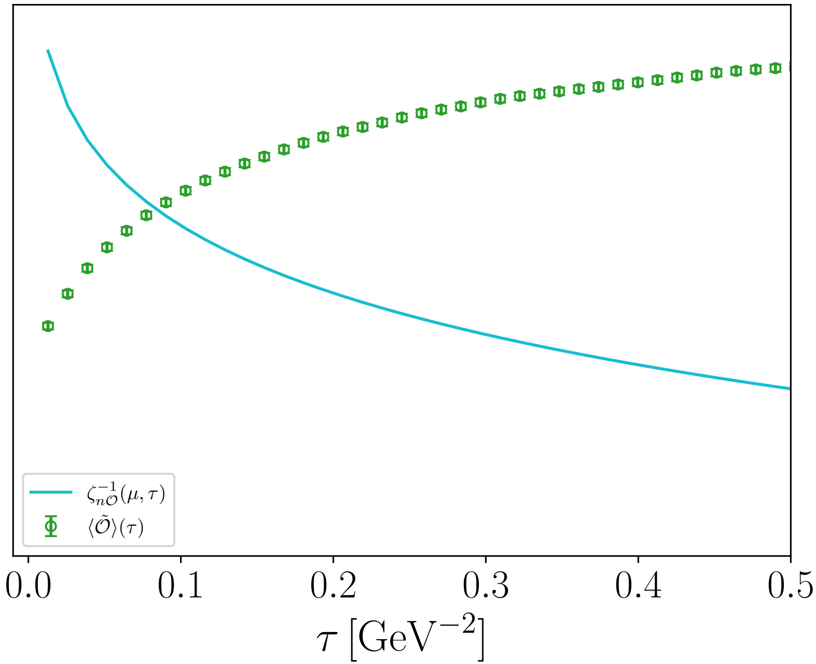
'flowed' MEs calculated on lattice
renormalised along flow time

matching matrix
calculated perturbatively

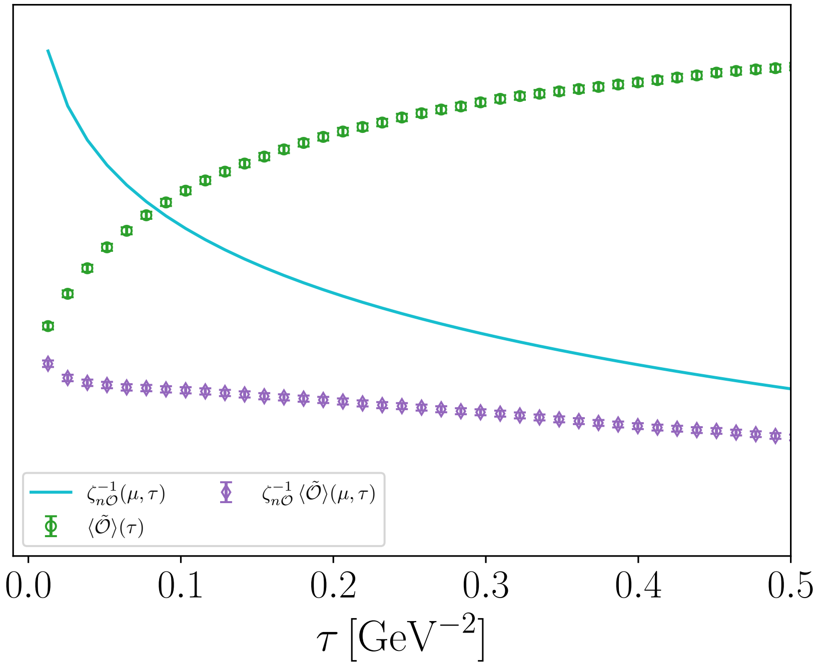
$$\sum_n \zeta^{-1}_{nm}(\mu, \tau) \langle \widetilde{\mathcal{O}}_n \rangle(\tau) = \langle \mathcal{O}_m \rangle(\mu)$$



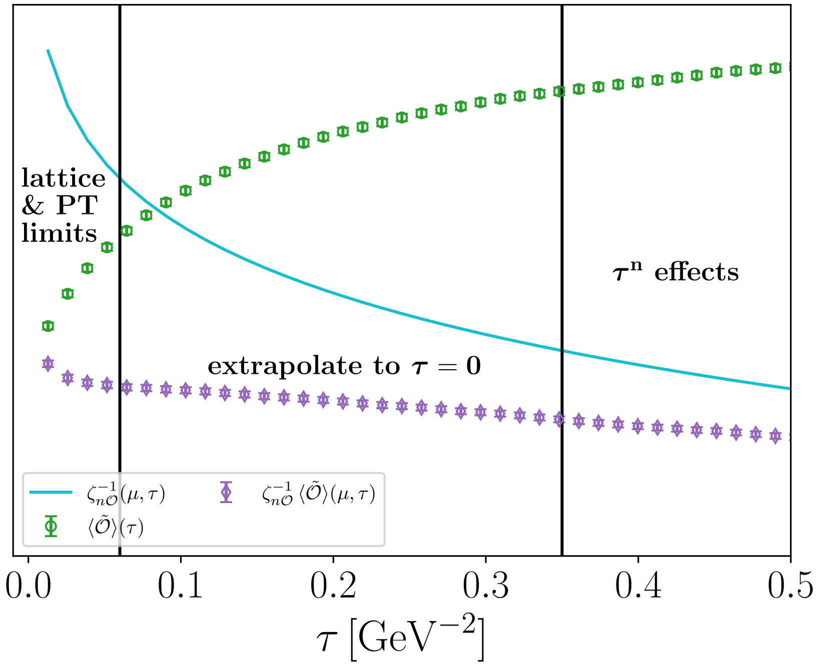
- Measure flowed matrix element $\langle\mathcal{O}\rangle(\tau)$ on the lattice



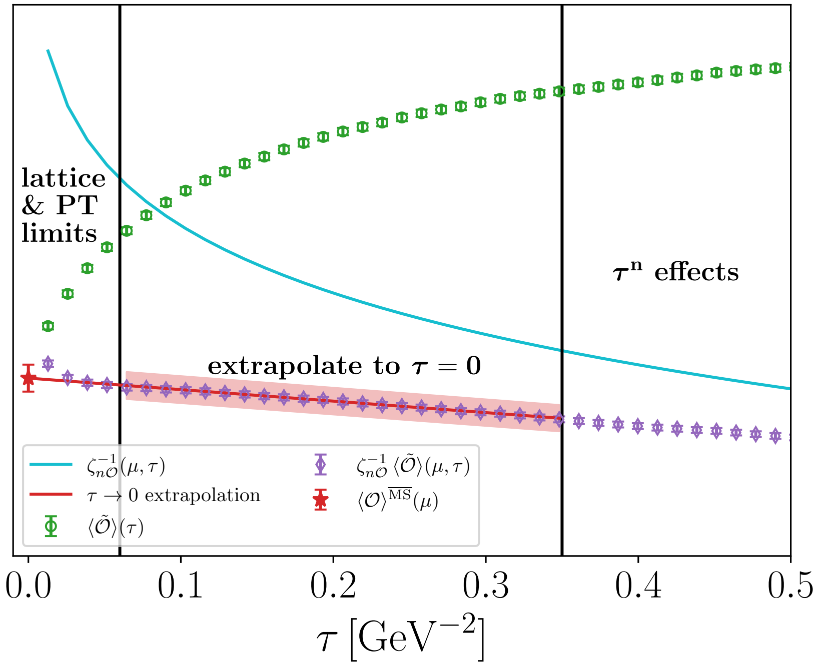
➤ Calculate perturbative matching coefficients $\zeta_{n\mathcal{O}}^{-1}(\mu,\tau)$



- Combine for 'matched' operator dependent on flow time τ and renormalisation scale μ



- Larger systematic effects at extremities



➤ Take $\tau \to 0$ result ➡ $\langle\mathcal{O}\rangle^{\overline{\mathrm{MS}}}(\mu)$

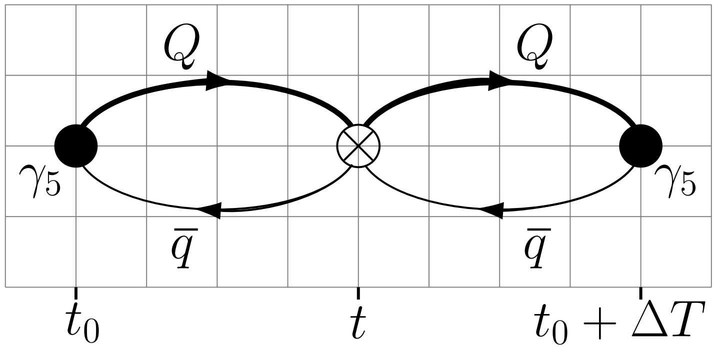
Analysis and Results

Bag Parameter Extraction

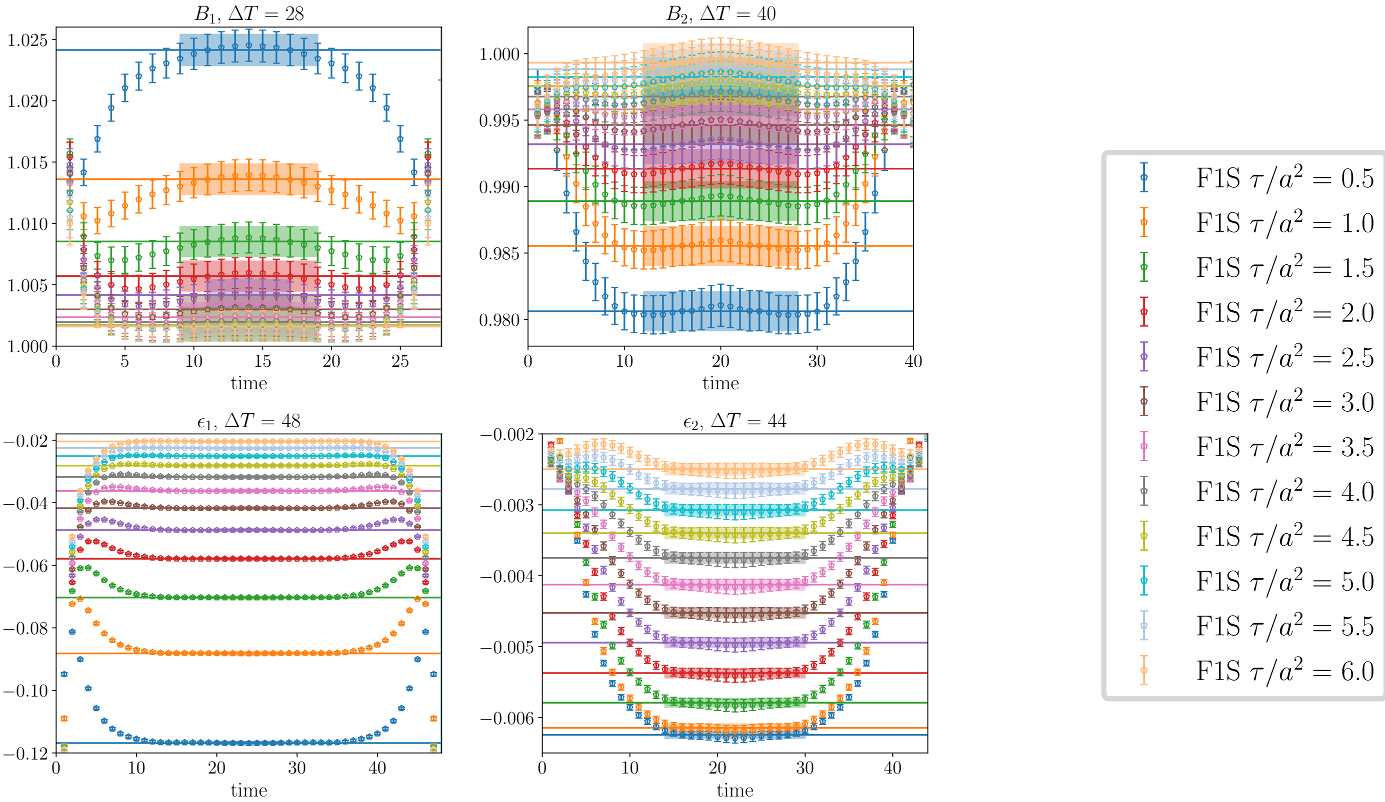
- In the Standard Model, four operators contribute:

$$\mathcal{O}_1 \rightarrow B_1 \sim 1 \qquad B_2 \rightarrow B_2 \sim 1$$
$$T_1 \rightarrow \epsilon_1 \sim 0 \qquad T_2 \rightarrow \epsilon_2 \sim 0$$

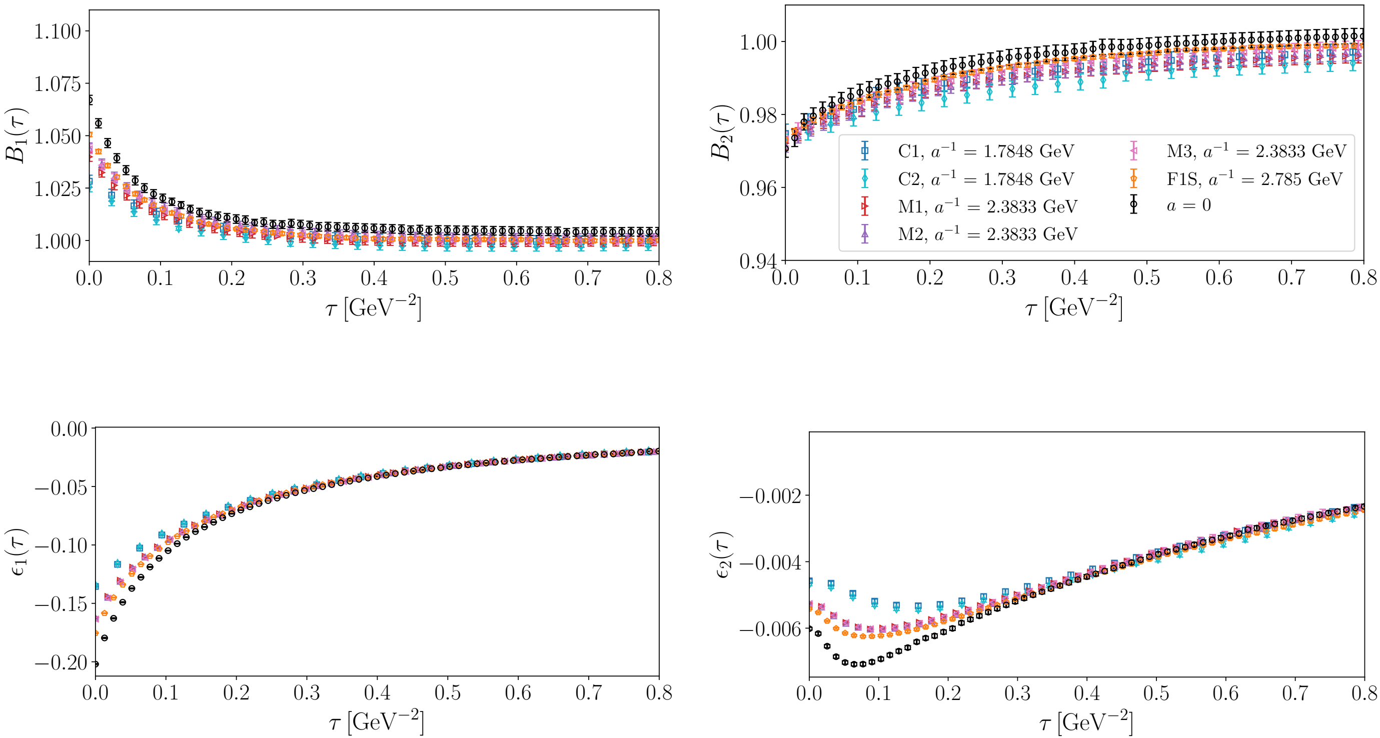
- Four-quark operators inserted in three-point correlation functions:



- For bag parameters, normalise with two-point correlation functions $\rightarrow$ $B_i \propto \dfrac{\langle \mathcal{O}_i \rangle}{m^2 f_H^2}$
- Simulations performed using Grid and Hadrons



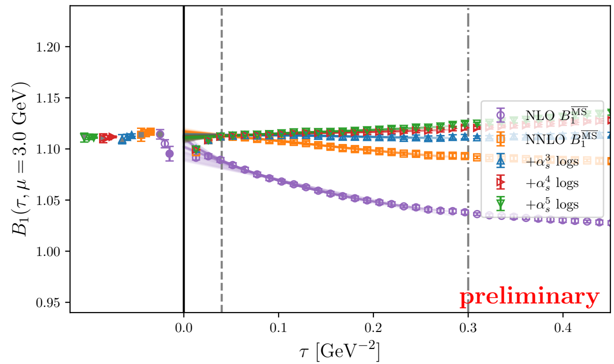
➤ Matrix elements extracted for each flow time ✔

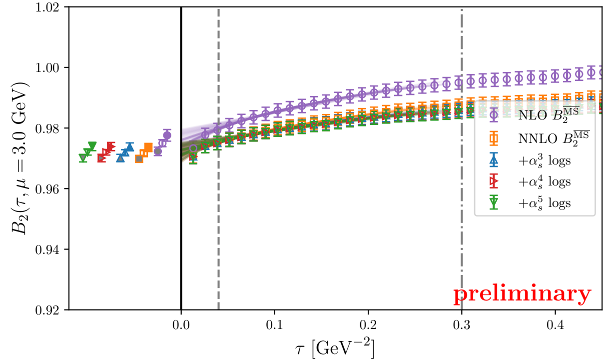


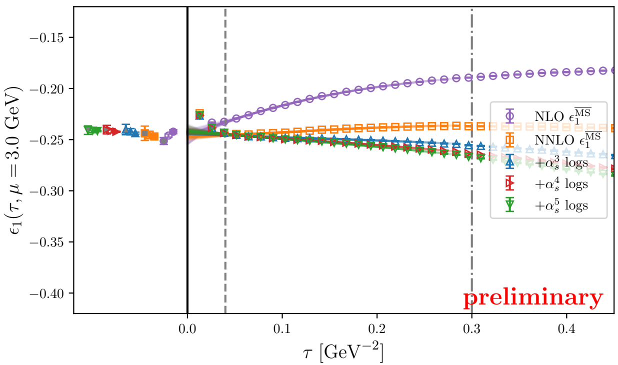
➤ Well-controlled continuum limits along flow time!

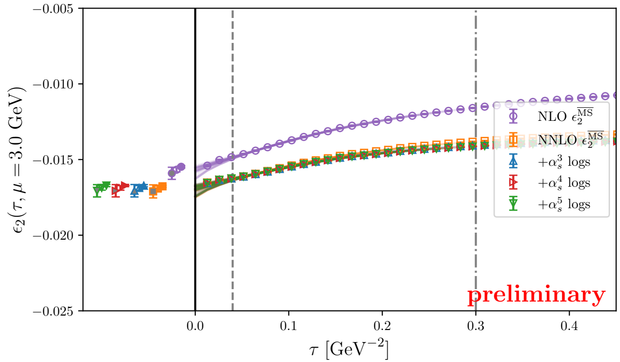
Matched Results

➤ Flowed bag parameters combined with perturbative matching at NNLO (+ higher logs)







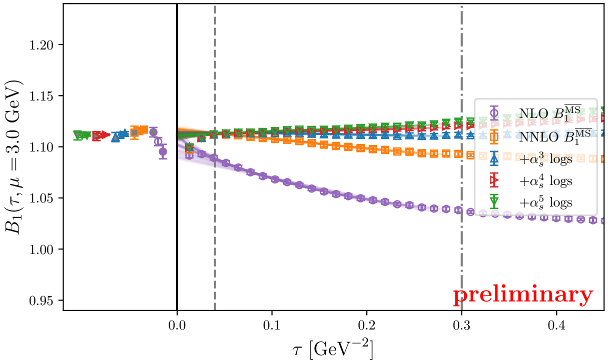


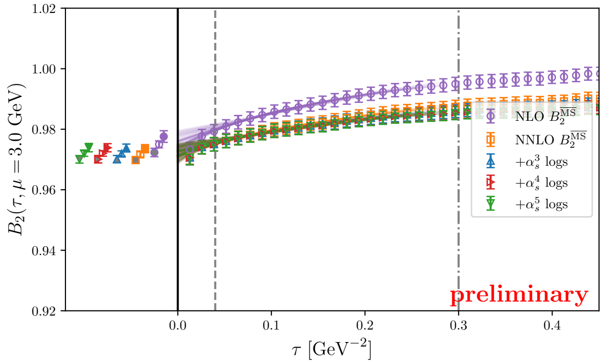
Summary

- Finalising first lattice calculation of $\Delta Q = 0$ matrix elements for heavy meson lifetime ratios
- Gradient flow and short-flow-time expansion is an effective tool for renormalisation and matching
  - Proof of principle calculation of charm and strange quark masses [Black et al. '25]

Outlook

- Scale dependence of short-flow-time expansion to be studied
- Perform large-scale simulations to extrapolate to B and B_s mesons
- 'Eye' diagrams need for absolute lifetime operators
  - to be included in both lattice simulations and perturbative matching





Backup Slides

- For lifetimes, the dimension-6 $\Delta Q = 0$ operators are:

$$\mathcal{O}_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) b^\beta, \qquad \langle \mathcal{O}_1^q \rangle = \langle B_q | \mathcal{O}_1^q | B_q \rangle = f_{B_q}^2 M_{B_q}^2 B_1^q,$$

$$\mathcal{O}_2^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{q}^\beta (1-\gamma_5) b^\beta, \qquad \langle \mathcal{O}_2^q \rangle = \langle B_q | \mathcal{O}_2^q | B_q \rangle = \frac{M_{B_q}^2}{(m_b+m_q)^2} f_{B_q}^2 M_{B_q}^2 B_2^q,$$

$$T_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) (T^a)^{\alpha\beta} q^\beta \; \bar{q}^\gamma \gamma_\mu (1-\gamma_5) (T^a)^{\gamma\delta} b^\delta, \qquad \langle T_1^q \rangle = \langle B_q | T_1^q | B_q \rangle = f_{B_q}^2 M_{B_q}^2 \epsilon_1^q,$$

$$T_2^q = \bar{b}^\alpha (1-\gamma_5) (T^a)^{\alpha\beta} q^\beta \; \bar{q}^\gamma (1-\gamma_5) (T^a)^{\gamma\delta} b^\delta, \qquad \langle T_2^q \rangle = \langle B_q | T_2^q | B_q \rangle = \frac{M_{B_q}^2}{(m_b+m_q)^2} f_{B_q}^2 M_{B_q}^2 \epsilon_2^q.$$

- For simplicity of computation, we want these to be colour-singlet operators:

$$\begin{aligned}
\mathcal{O}_1 &= \bar{b}^\alpha \gamma_\mu (1-\gamma_5) q^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) b^\beta \\
\mathcal{O}_2 &= \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{q}^\beta (1+\gamma_5) b^\beta) \\
\tau_1 &= \bar{b}^\alpha \gamma_\mu (1-\gamma_5) b^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) q^\beta \\
\tau_2 &= \bar{b}^\alpha \gamma_\mu (1+\gamma_5) b^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) q^\beta
\end{aligned}$$

$$\begin{pmatrix} \mathcal{O}_1^+ \\ \mathcal{O}_2^+ \\ T_1^+ \\ T_2^+ \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{2N_c} & 0 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2N_c} & 0 & \frac{1}{4} \end{pmatrix} \begin{pmatrix} \mathcal{O}_1^+ \\ \mathcal{O}_2^+ \\ \tau_1^+ \\ \tau_2^+ \end{pmatrix}$$

➤ Exploratory setup using physical charm and strange quarks

➥ $\Delta B = 0, 2$ ➡ $\Delta Q = 0, 2$, for generic heavy quark Q

➤ We use RBC/UKQCD's 2+1 flavour DWF + Iwasaki gauge action ensembles

	L	T	a^{-1}/GeV	am_l^{sea}	am_s^{sea}	M_π/MeV	srcs × N$_{\text{conf}}$
C1	24	64	1.7848	0.005	0.040	340	32 × 101
C2	24	64	1.7848	0.010	0.040	433	32 × 101
M1	32	64	2.3833	0.004	0.030	302	32 × 79
M2	32	64	2.3833	0.006	0.030	362	32 × 89
M3	32	64	2.3833	0.008	0.030	411	32 × 68
F1S	48	96	2.785	0.002144	0.02144	267	24 × 98

[Allton et al. '08]
[Aoki et al. '10]
[Blum et al. '14]
[Boyle et al. '17]

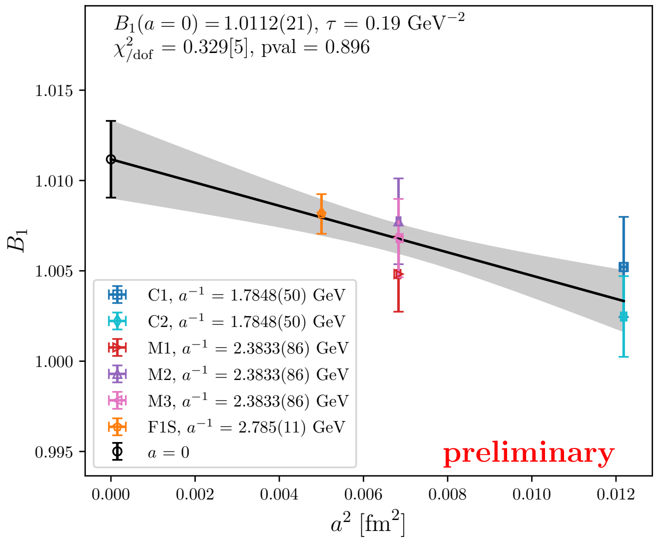
➤ For strange quarks tuned to physical value, $am_q \ll 1$ ✔

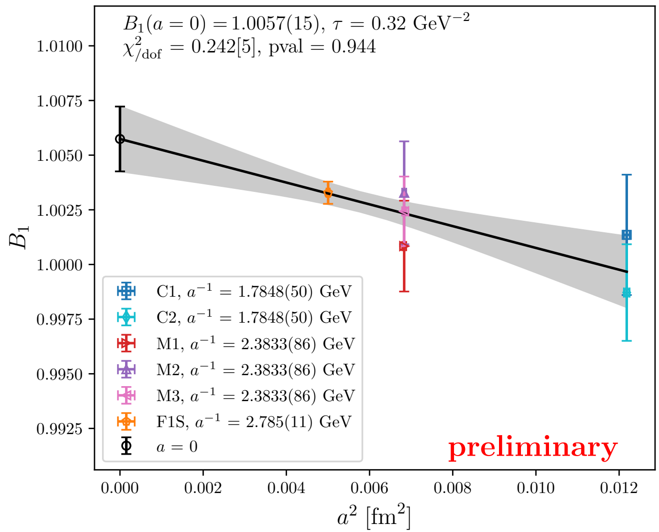
➤ For heavy b quarks, $am_q > 1$ ➡ large discretisation effects ✗

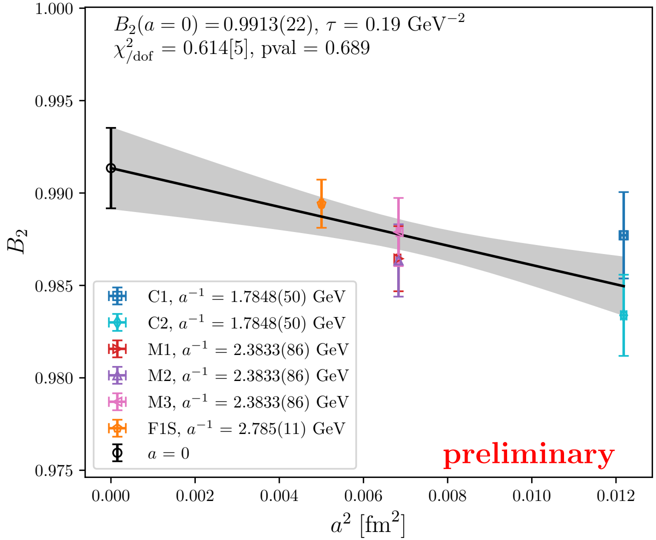
➥ manageable for physical c quarks instead

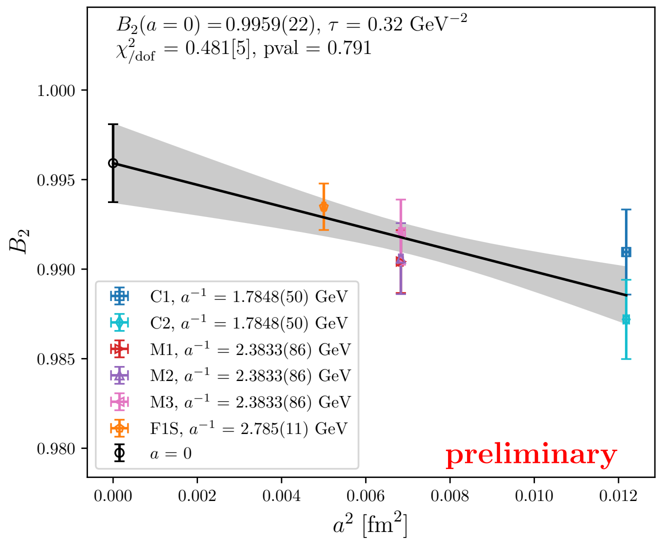
➥ stout-smeared Möbius DWF [Cho et. al '15]

Lifetimes $\mathcal{O}_1$ Continuum Limit

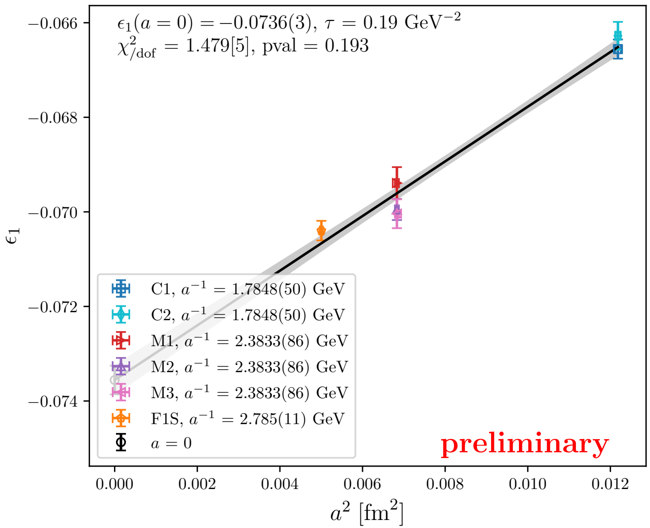


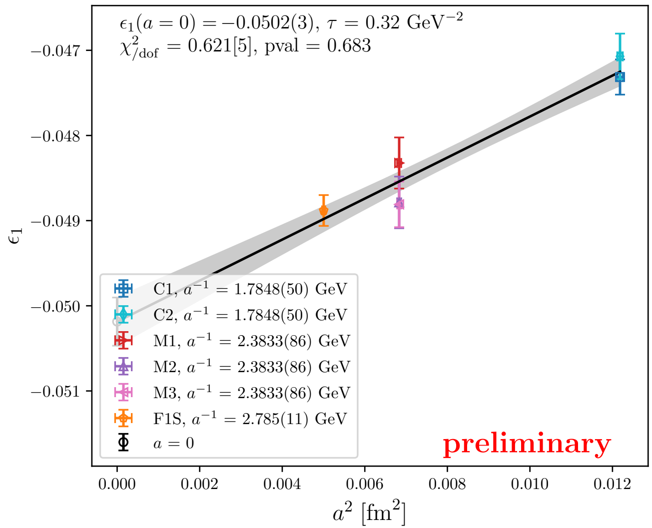


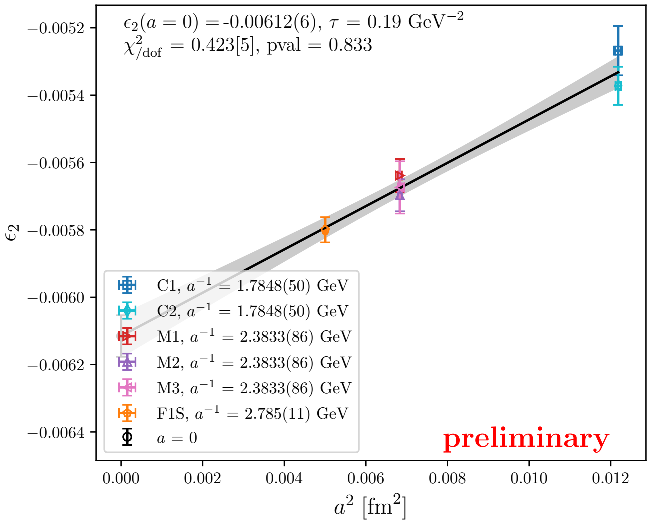

$B_2(a=0) = 0.9913(22)$, $\tau = 0.19$ GeV^{-2}
$\chi^2_{/\mathrm{dof}} = 0.614[5]$, pval = 0.689
C1, $a^{-1} = 1.7848(50)$ GeV
C2, $a^{-1} = 1.7848(50)$ GeV
M1, $a^{-1} = 2.3833(86)$ GeV
M2, $a^{-1} = 2.3833(86)$ GeV
M3, $a^{-1} = 2.3833(86)$ GeV
F1S, $a^{-1} = 2.785(11)$ GeV
$a = 0$
preliminary
B_2
a^2 [fm^2]


$B_2(a=0) = 0.9959(22)$, $\tau = 0.32$ GeV^{-2}
$\chi^2_{/\mathrm{dof}} = 0.481[5]$, pval = 0.791
C1, $a^{-1} = 1.7848(50)$ GeV
C2, $a^{-1} = 1.7848(50)$ GeV
M1, $a^{-1} = 2.3833(86)$ GeV
M2, $a^{-1} = 2.3833(86)$ GeV
M3, $a^{-1} = 2.3833(86)$ GeV
F1S, $a^{-1} = 2.785(11)$ GeV
$a = 0$
preliminary
B_2
a^2 [fm^2]

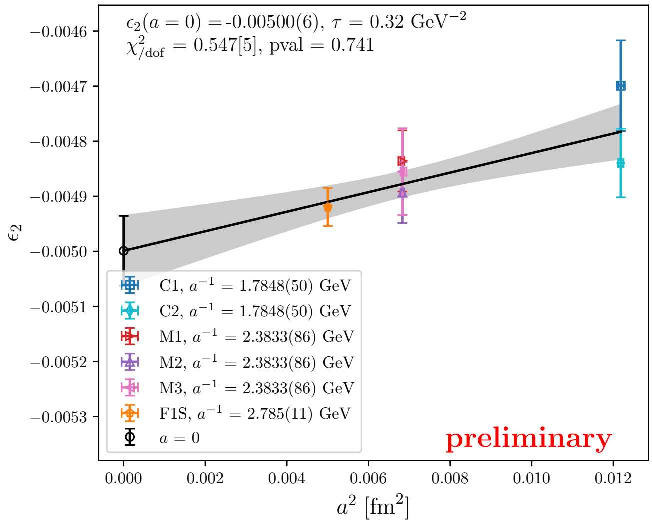
Lifetimes T_1 Continuum Limit







$\epsilon_2(a=0)=$-0.00612(6), $\tau = 0.19$ GeV^{-2}
$\chi^2_{/\mathrm{dof}} = 0.423[5]$, pval = 0.833
C1, a^{-1} = 1.7848(50) GeV
C2, a^{-1} = 1.7848(50) GeV
M1, a^{-1} = 2.3833(86) GeV
M2, a^{-1} = 2.3833(86) GeV
M3, a^{-1} = 2.3833(86) GeV
F1S, a^{-1} = 2.785(11) GeV
$a = 0$
preliminary
ϵ_2
a^2 [fm^2]
−0.0052
−0.0054
−0.0056
−0.0058
−0.0060
−0.0062
−0.0064
0.000
0.002
0.004
0.006
0.008
0.010
0.012



$\epsilon_2(a=0)=$-0.00500(6), $\tau = 0.32$ GeV^{-2}
$\chi^2_{/\mathrm{dof}} = 0.547[5]$, pval = 0.741
C1, a^{-1} = 1.7848(50) GeV
C2, a^{-1} = 1.7848(50) GeV
M1, a^{-1} = 2.3833(86) GeV
M2, a^{-1} = 2.3833(86) GeV
M3, a^{-1} = 2.3833(86) GeV
F1S, a^{-1} = 2.785(11) GeV
$a = 0$
preliminary
ϵ_2
a^2 [fm^2]
−0.0046
−0.0047
−0.0048
−0.0049
−0.0050
−0.0051
−0.0052
−0.0053
0.000
0.002
0.004
0.006
0.008
0.010
0.012