NLO analysis of the $\mathcal{O}_1 - \mathcal{O}_7$ interference in $\bar{B} \to X_s\gamma$ at subleading power

Based on 2411.16634 and work in progress (in collaboration with Philipp Böer and Tobias Hurth)

EPS 2025, Marseille, 11th July 2025

Riccardo Bartocci - JGU Mainz

Motivation: the $\mathcal{O}_1 - \mathcal{O}_7$ interference contribution

Motivation

In the inclusive $\overline{B} \to X_s\gamma$ decay, the CP-averaged photon-energy spectrum is given by

[1003.5012: Benzke, Lee, Neubert, Paz]

$$\frac{d\Gamma}{dE_\gamma} = \frac{G_F\alpha|V_{tb}V_{ts}^*|^2}{2\pi^4}\overline{m_b}^2(\mu)E_\gamma^3\Big[|H_\gamma(\mu)|^2\int d\omega m_b J(m_b(\omega+p_+);\mu)S(\omega;\mu)$$
$$+\frac{1}{m_b}\sum_{i<j} Re[C_i^*(\mu)C_j(\mu)]F_{ij}(E_\gamma;\mu)+\,...\Big]$$

C_i = Wilson coefficients of the Weak Effective Theory (WET)

$$\begin{aligned}
\mathcal{O}_1^{q,\,d_id_j} &= (\bar{d}_i^\alpha\gamma_\mu P_L q^\beta)(\bar{q}^\beta\gamma^\mu P_L d_j^\alpha)\\
\mathcal{O}_2^{q,\,d_id_j} &= (\bar{d}_i^\alpha\gamma_\mu P_L q^\alpha)(\bar{q}^\beta\gamma^\mu P_L d_j^\beta)\\
\mathcal{O}_7^{d_id_j} &= e\, m_{d_i}\left(\bar{d}_j\sigma^{\mu\nu}P_R d_i\right)F_{\mu\nu}\\
\mathcal{O}_8^{d_id_j} &= g_s m_{d_i}\left(\bar{d}_j\sigma^{\mu\nu}T^A P_R d_i\right)G_{\mu\nu}^A
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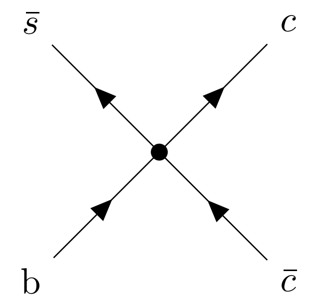

$$\mathcal{O}_2^{q,\,d_i d_j} = (\bar{d}_i^\alpha \gamma_\mu P_L q^\alpha)(\bar{q}^\beta \gamma^\mu P_L d_j^\beta)$$

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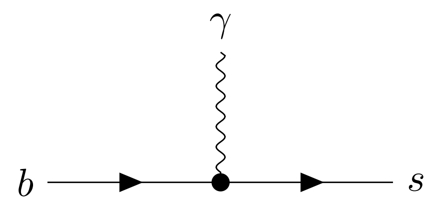

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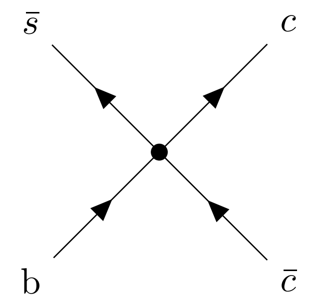

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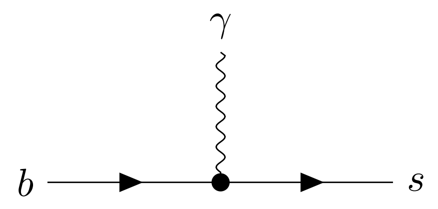

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Current uncertainties of **the $\mathcal{O}_1 - \mathcal{O}_7$ interference:**

- Estimated contribution $\approx (5.15 \pm 2.55)\,\%$ (**largest uncertainty**)
- **Large scale ambiguity** $\approx 40\,\%$ (not included in the above estimates)

[2006.00624: Benzke, Hurth]

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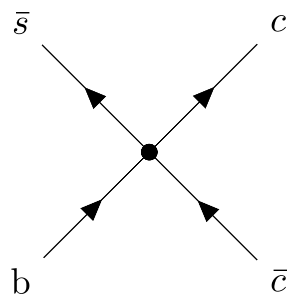

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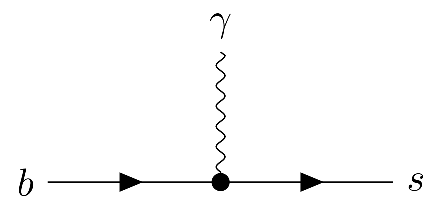

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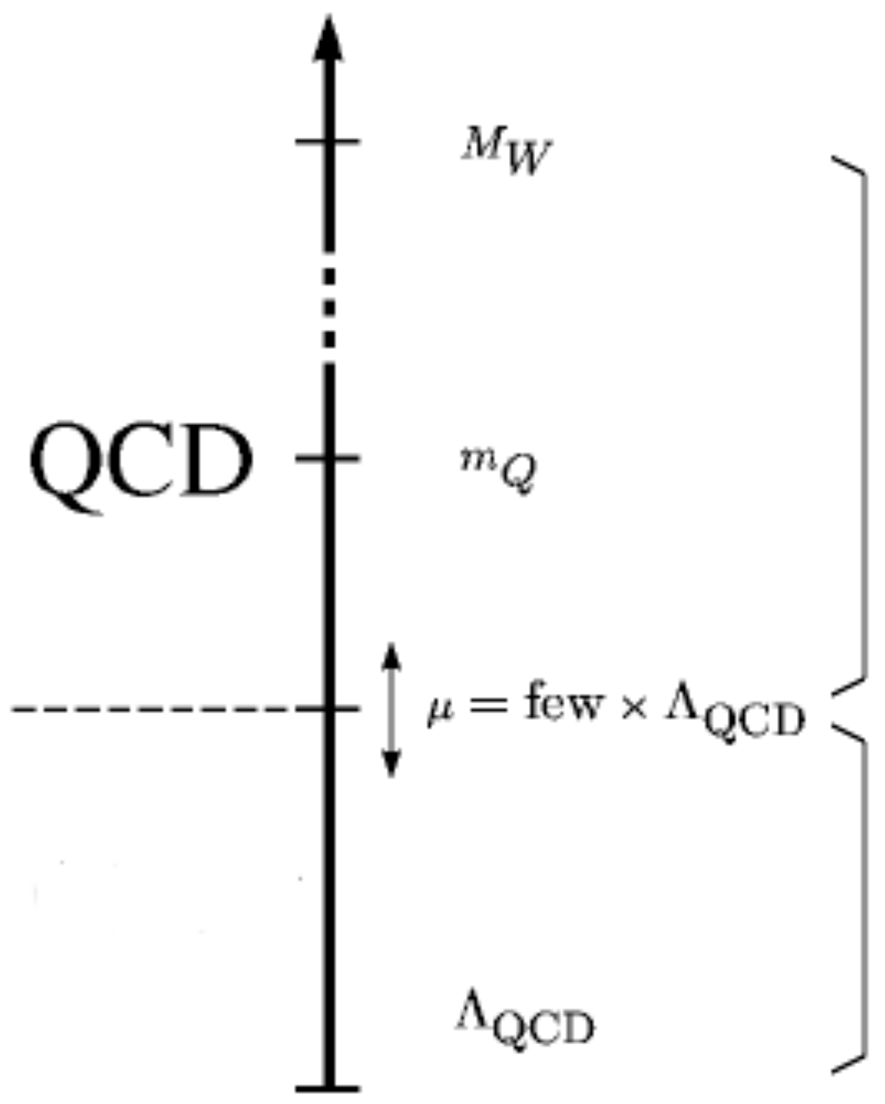
$\longrightarrow$ **NLO analysis and Renormalisation Group Evolution (RGE) will reduce this ambiguity**

Factorisation in Soft Collinear Effective Theory

Factorisation

Factorisation ⟷ **Scale separation** ⟷ **Perturbative and non-perturbative separation**

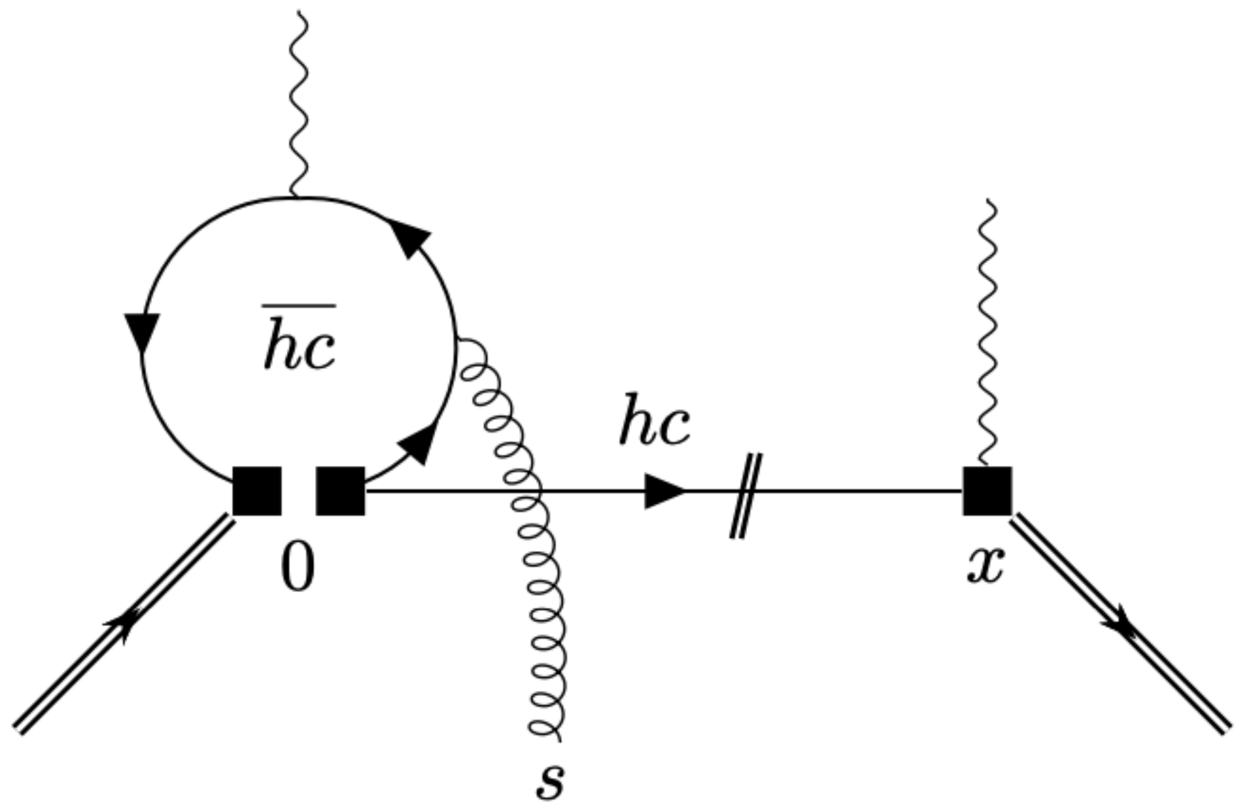


short-distance physics
perturbative

long-distance physics
nonperturbative

Factorisation at LO

From the LO diagrams of the $\mathcal{O}_1 - \mathcal{O}_7$ interference: [1003.5012: Benzke, Lee, Neubert, Paz]



we write:

$$d\Gamma(\bar{B} \to X_s\gamma) \propto \mathrm{Disc}_{\underline{\mathrm{restr.}}} \left[i \int d^4x \, \langle \bar{B} | \mathcal{H}^\dagger_{\mathrm{eff}}(x) \, \mathcal{H}_{\mathrm{eff}}(0) | \bar{B} \rangle \right]$$

Factorisation theorem:

$$d\Gamma(\bar{B} \to X_s\gamma) \sim H \cdot J \otimes g_{17} \otimes \bar{J}$$

Factorisation at LO

[1003.5012: Benzke, Lee, Neubert, Paz]

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$$d\Gamma \sim Re \int_{-\infty}^{\bar{\Lambda}} d\omega \delta(\omega + p_+) \int_{-\infty}^{+\infty} d\omega_1 g_{17}(\omega, \omega_1; \mu) \frac{1}{\omega_1 + i\epsilon} \left[1 - F\left(\frac{m_c^2 - i\epsilon}{2E_\gamma \omega_1} \right) \right], \qquad F(x) = 4x \arctan^2\left(\frac{1}{\sqrt{4x-1}} \right)$$

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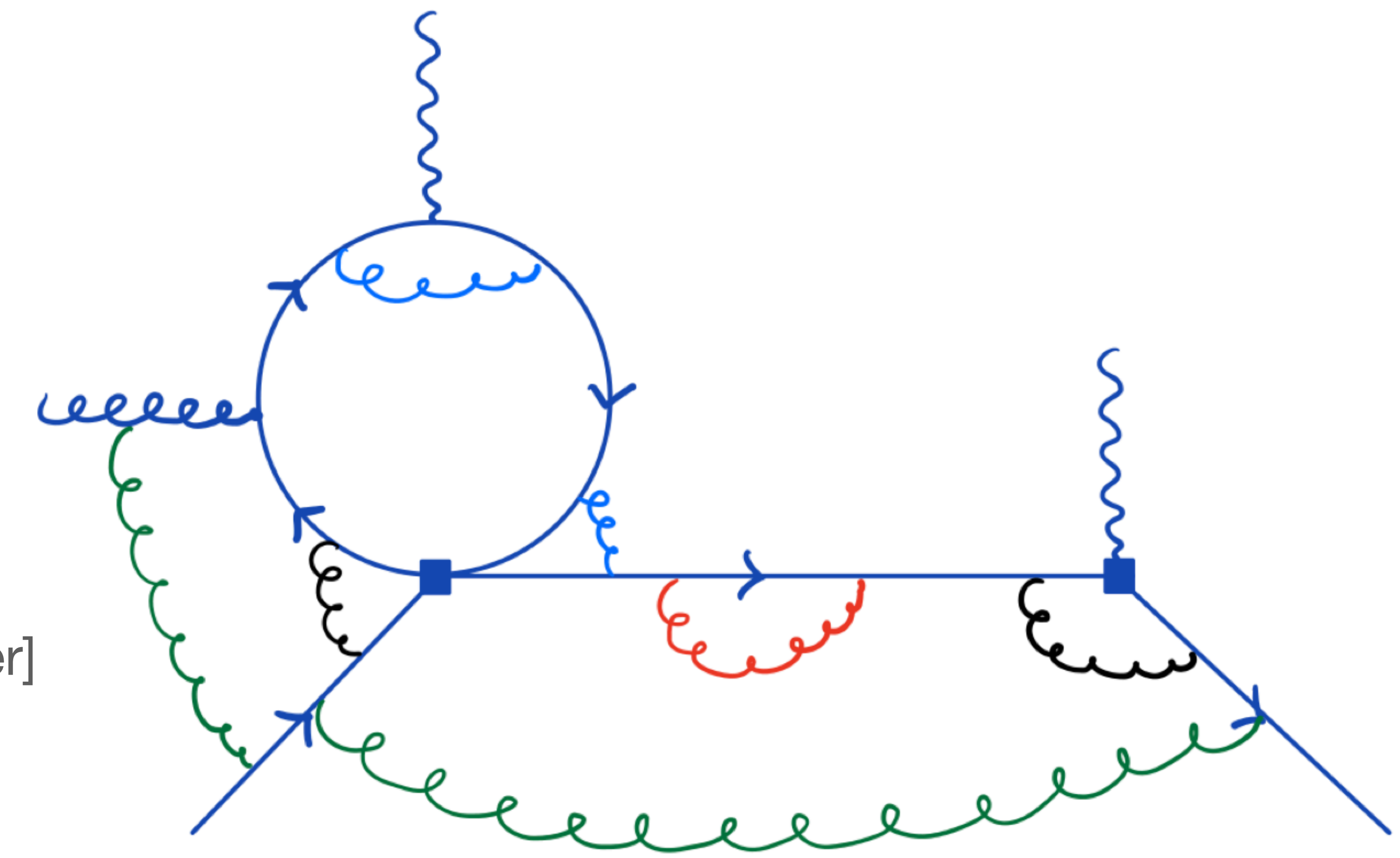
QCD radiative corrections (NLO)



[QUIRKS ’24, Böer]

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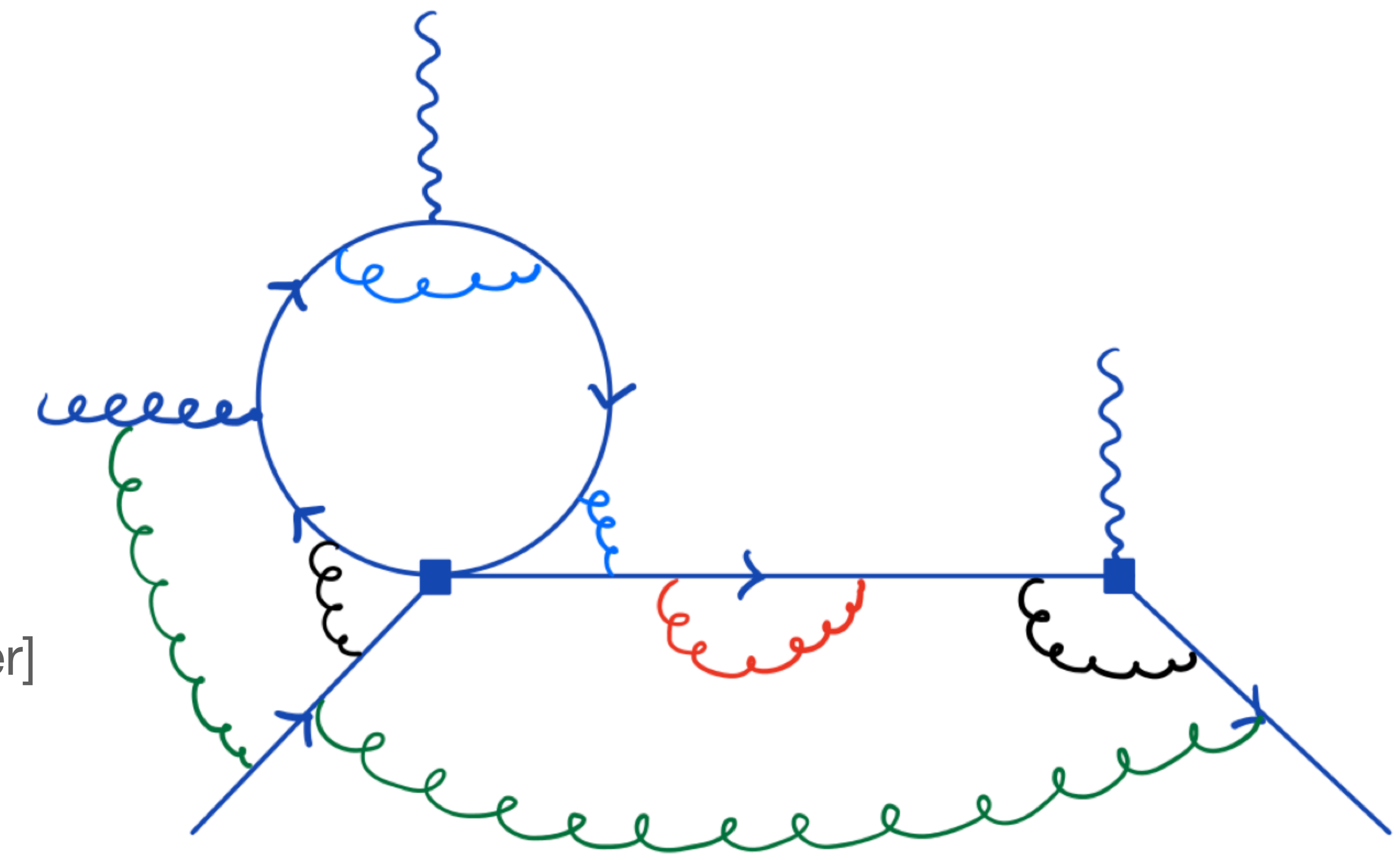

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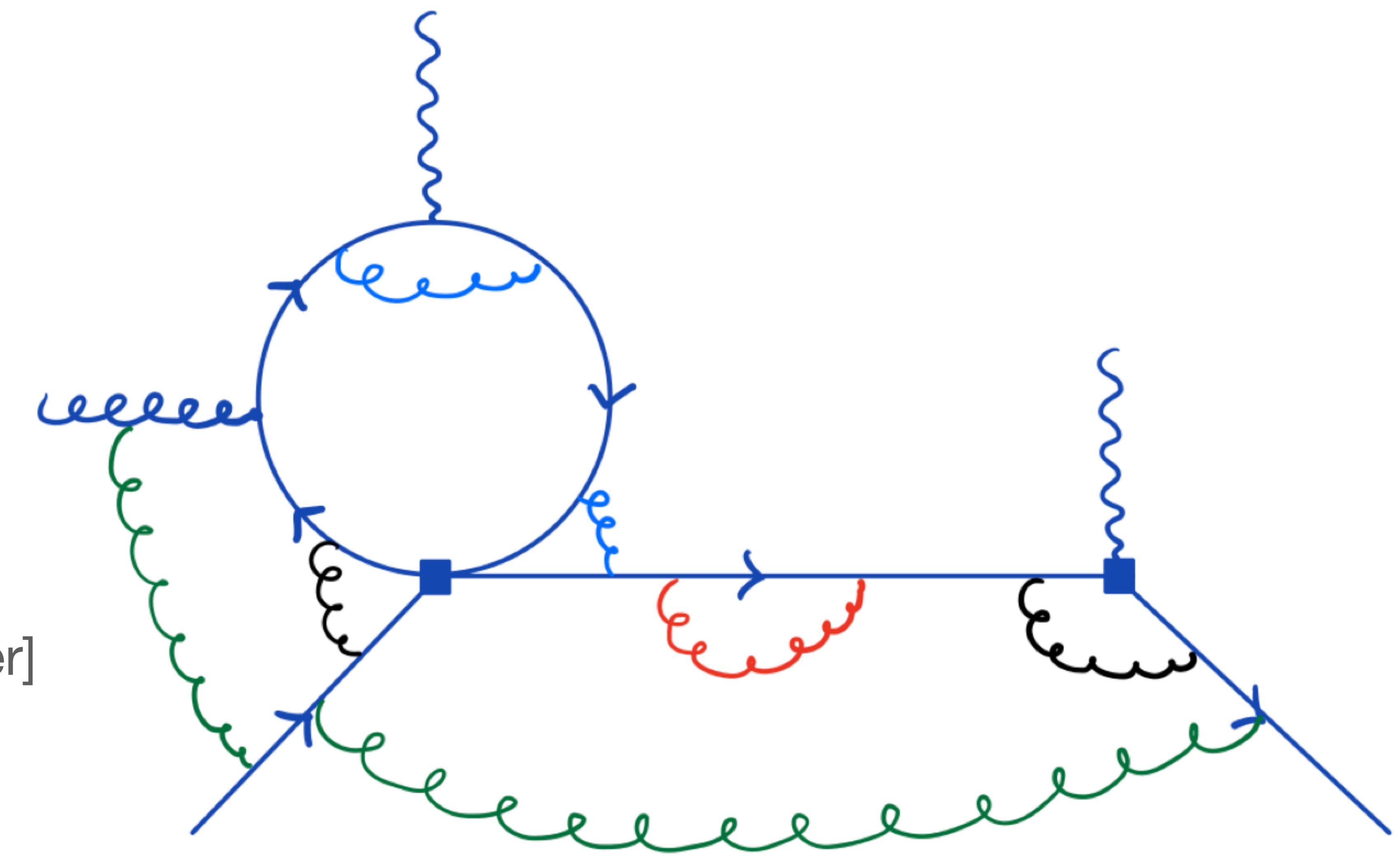

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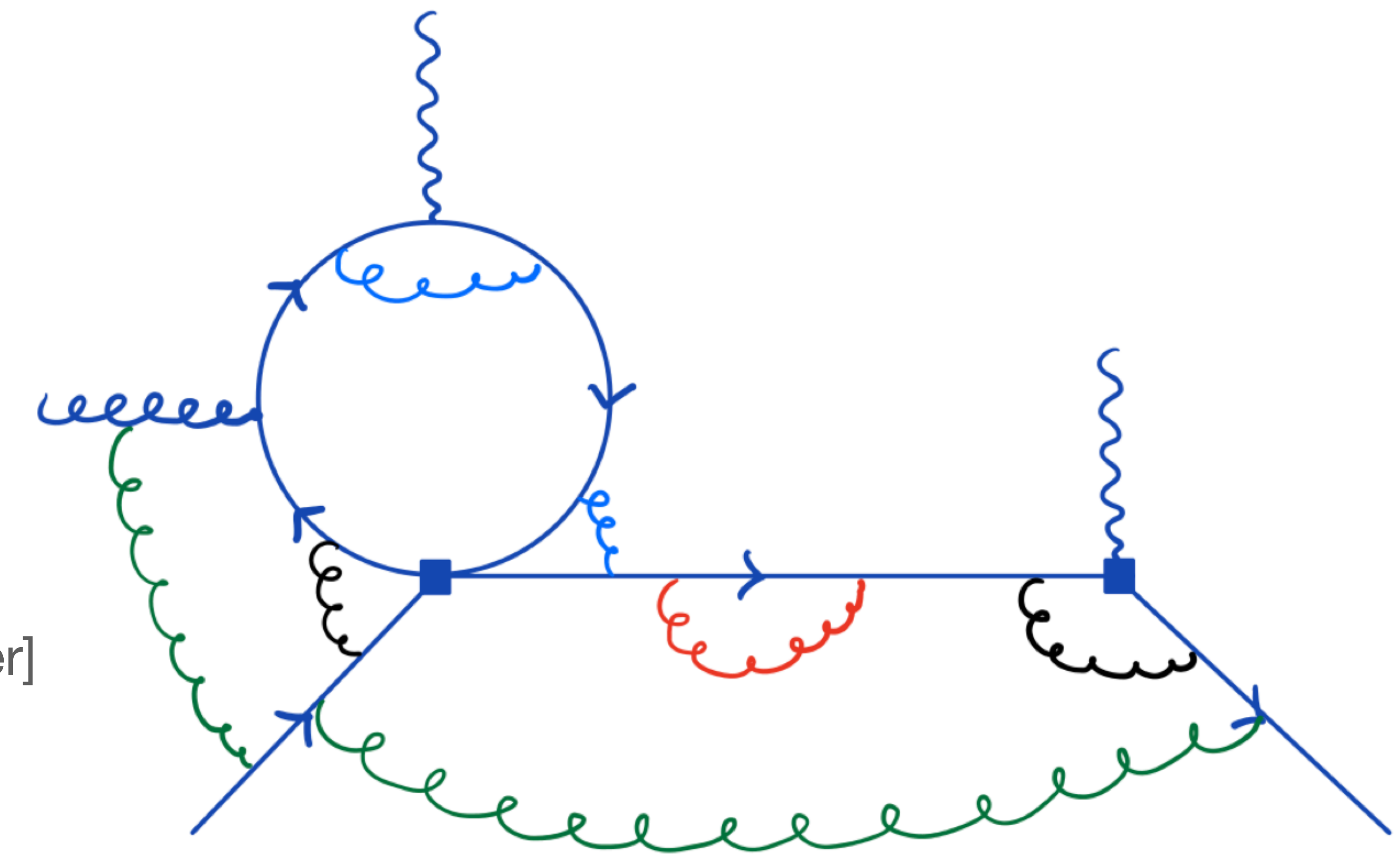

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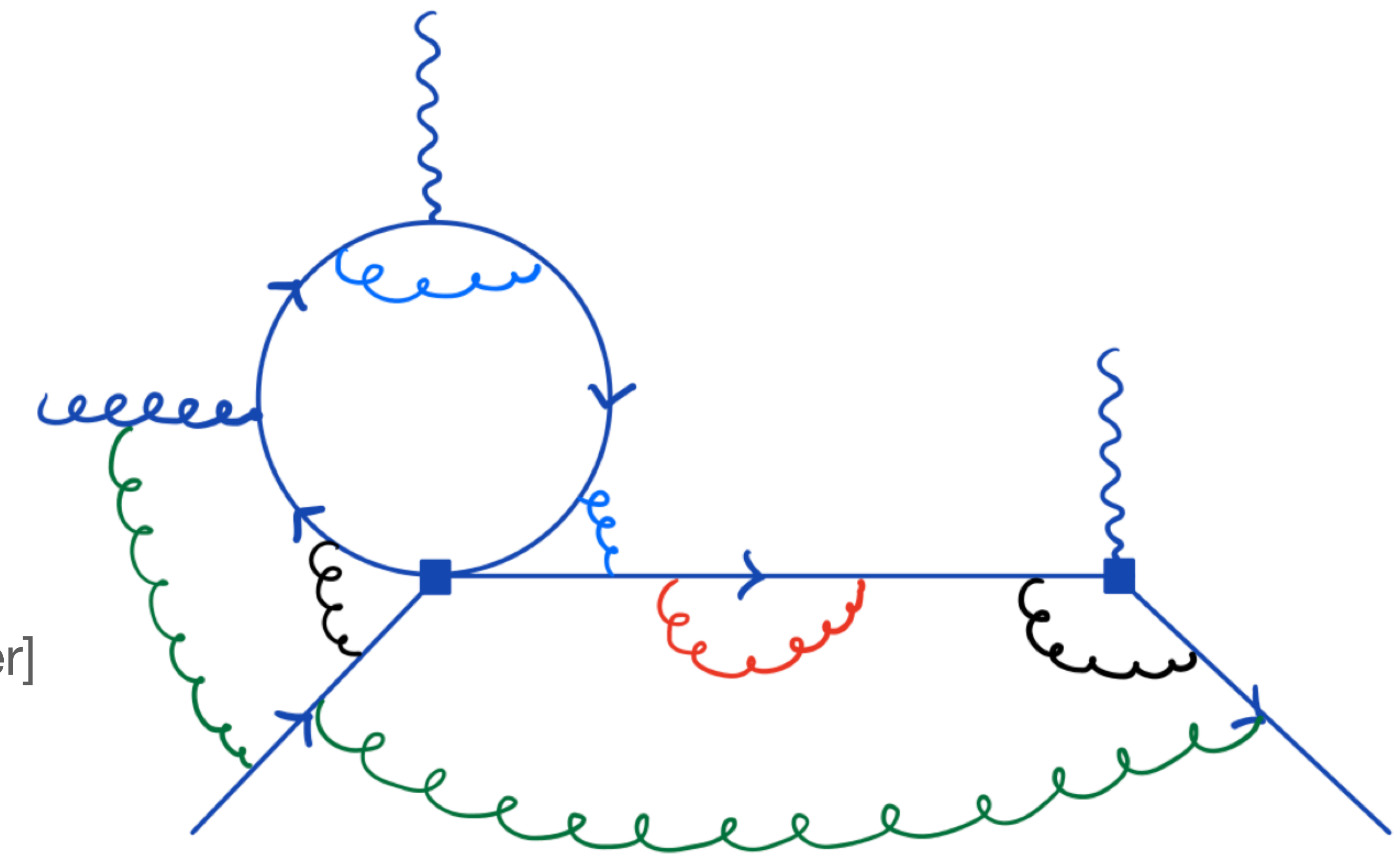

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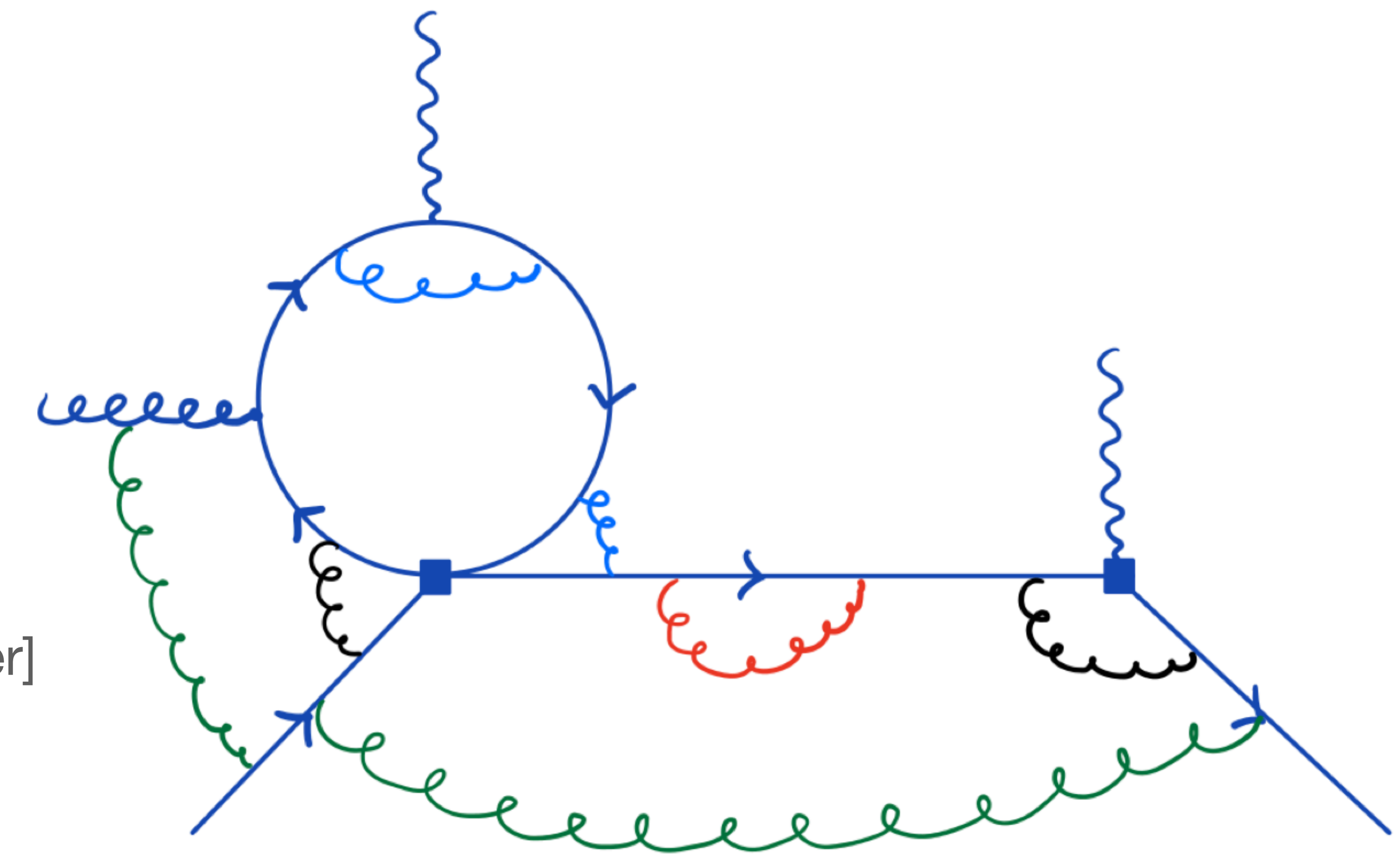

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DISCLAIMER

g_{17} must be interpreted as a distribution on the space of jet functions

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Renormalisation of the shape function g_{17}

The subleading shape function

The subleading shape function is given by the forward matrix element between two B-meson states

$$g_{17}(\omega, \omega_1; \mu) = \frac{1}{2M_B} \int \frac{dr}{2\pi}\, e^{-i\omega_1 r} \int \frac{dt}{2\pi}\, e^{-i\omega t}\, \langle \bar{B}_v | \mathcal{O}_{17}(t, r) | \bar{B}_v \rangle$$

where the operator is defined as

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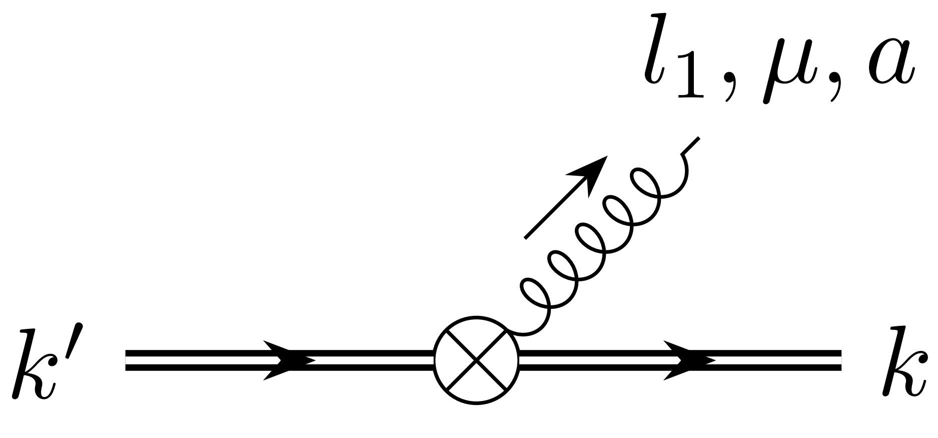
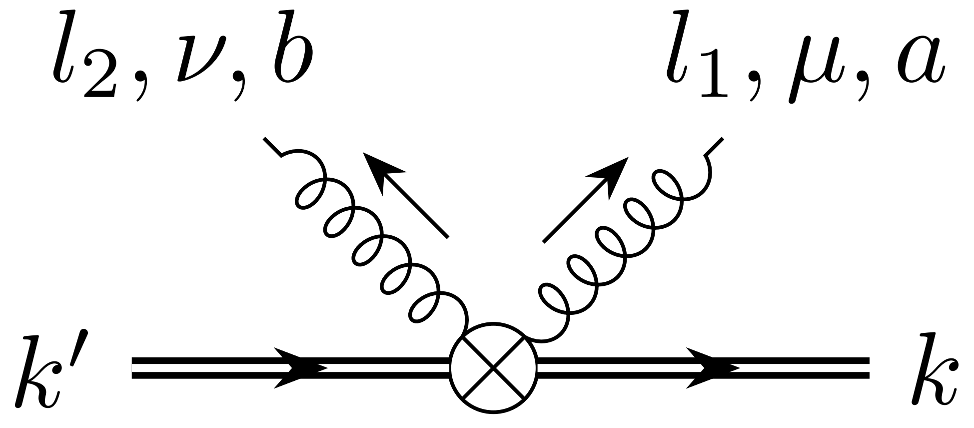
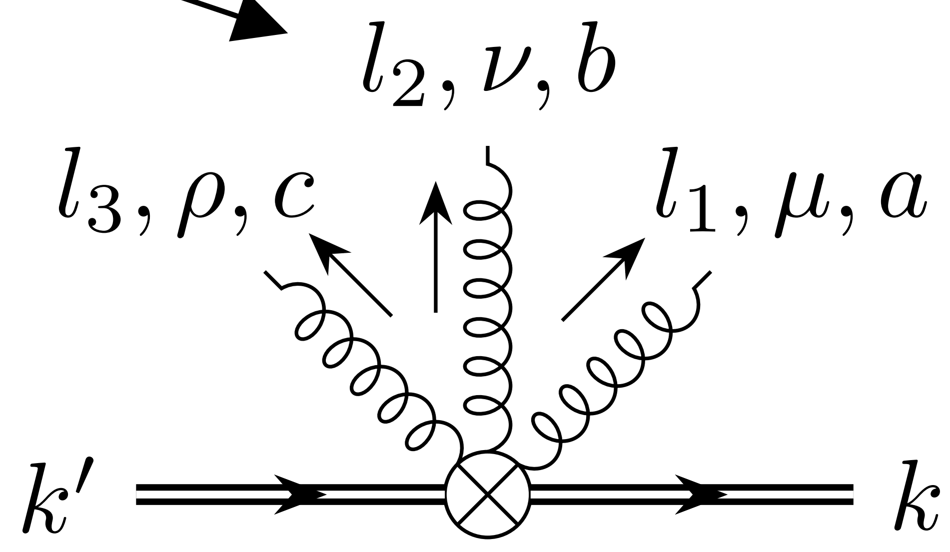


One gluon from $G_s^{\alpha\beta}$



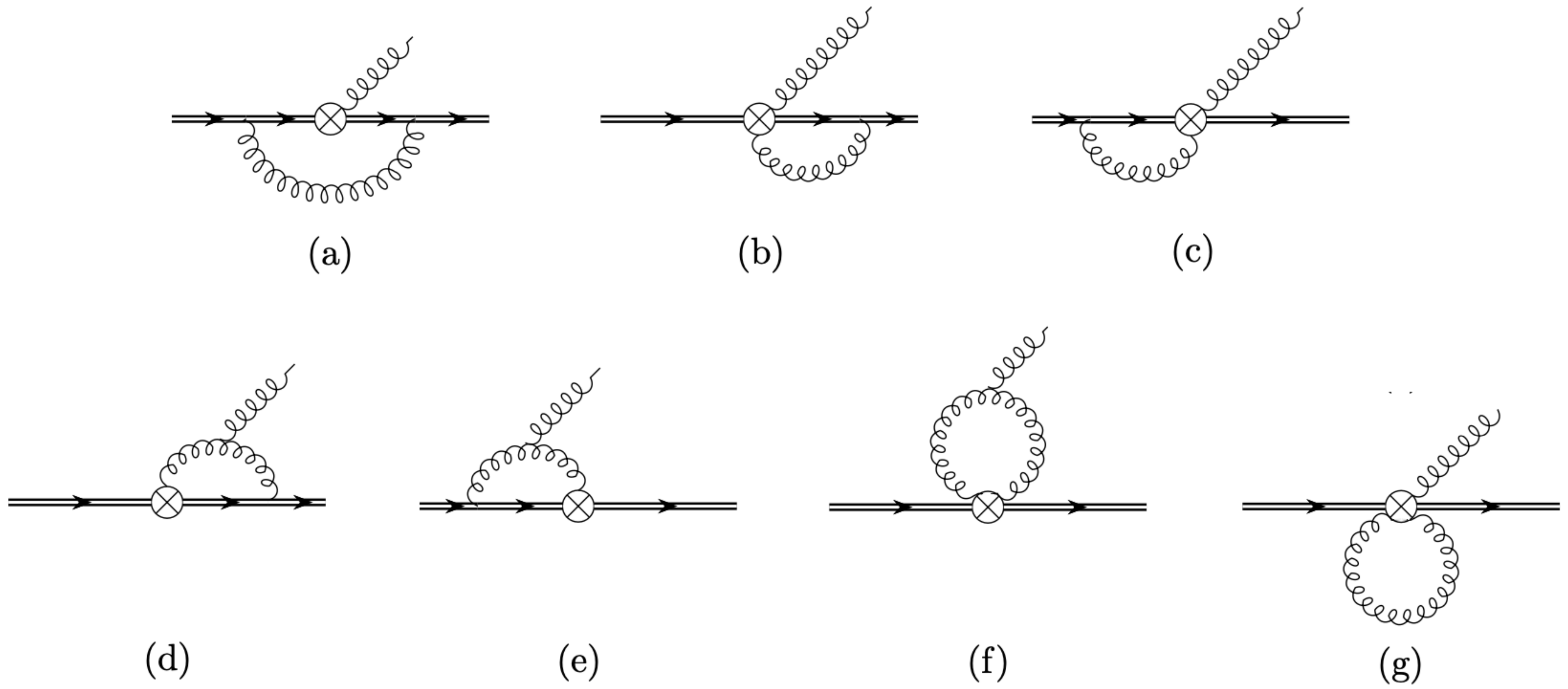
- Two gluons from $G_s^{\alpha\beta}$
- One gluon from $G_s^{\alpha\beta}$ and one from $S_{n,\bar{n}}$



- Two gluons from $G_s^{\alpha\beta}$ and one from $S_{n,\bar{n}}$
- One gluon from $G_s^{\alpha\beta}$ and two from $S_{n,\bar{n}}$

Renormalisation of the subleading shape function

We compute QCD corrections, employing the previous Feynman rules, calculating the following diagrams:



[2411.16634: RB, Böer, Hurth]

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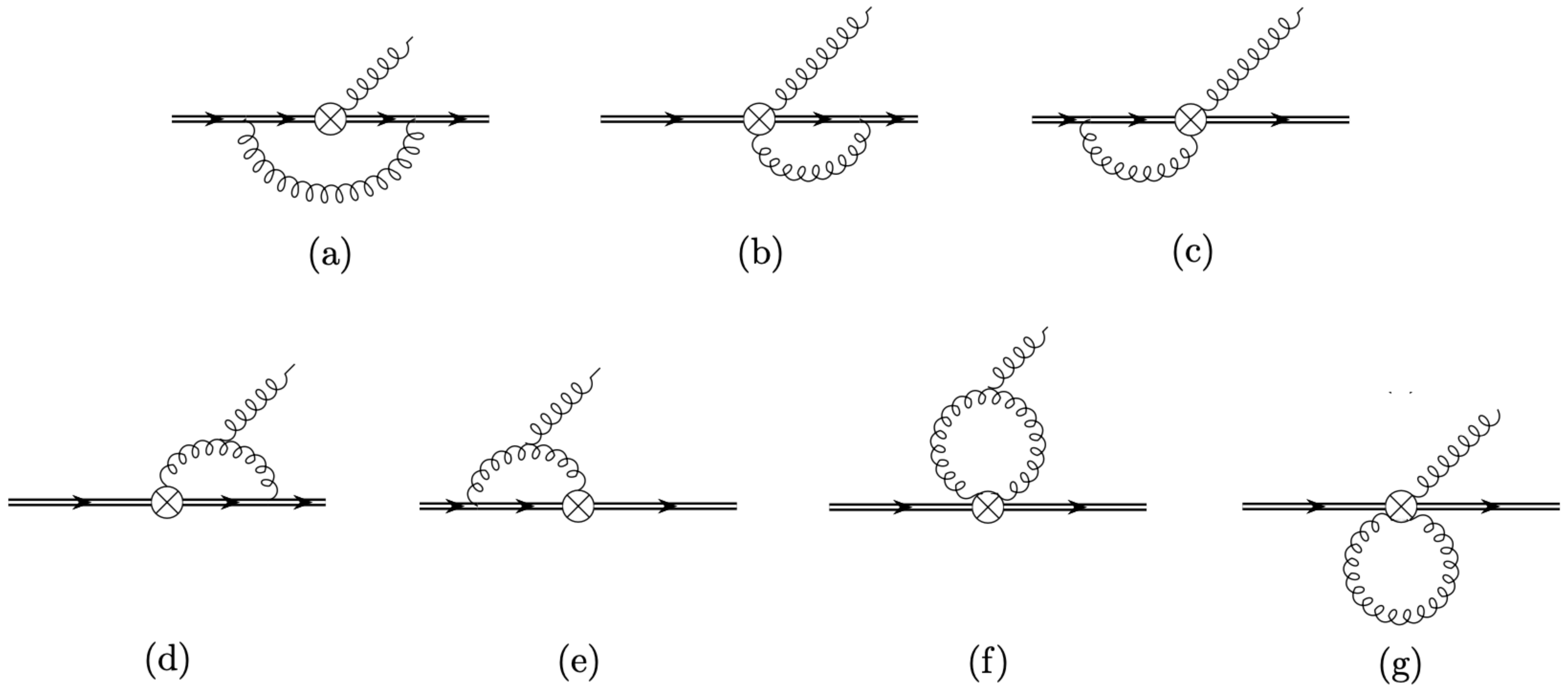

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For the anomalous dimension we get:

$$\gamma_{17}(\omega,\omega_1,\omega',\omega_1';\mu) = \frac{\alpha_s}{\pi}\left\{C_F\,\delta(\omega_1-\omega_1')\gamma_n(\omega,\omega';\mu) + \frac{C_A}{2}\,\delta(\omega-\omega')\gamma_{\bar{n}}(\omega_1,\omega_1';\mu)\right\}$$

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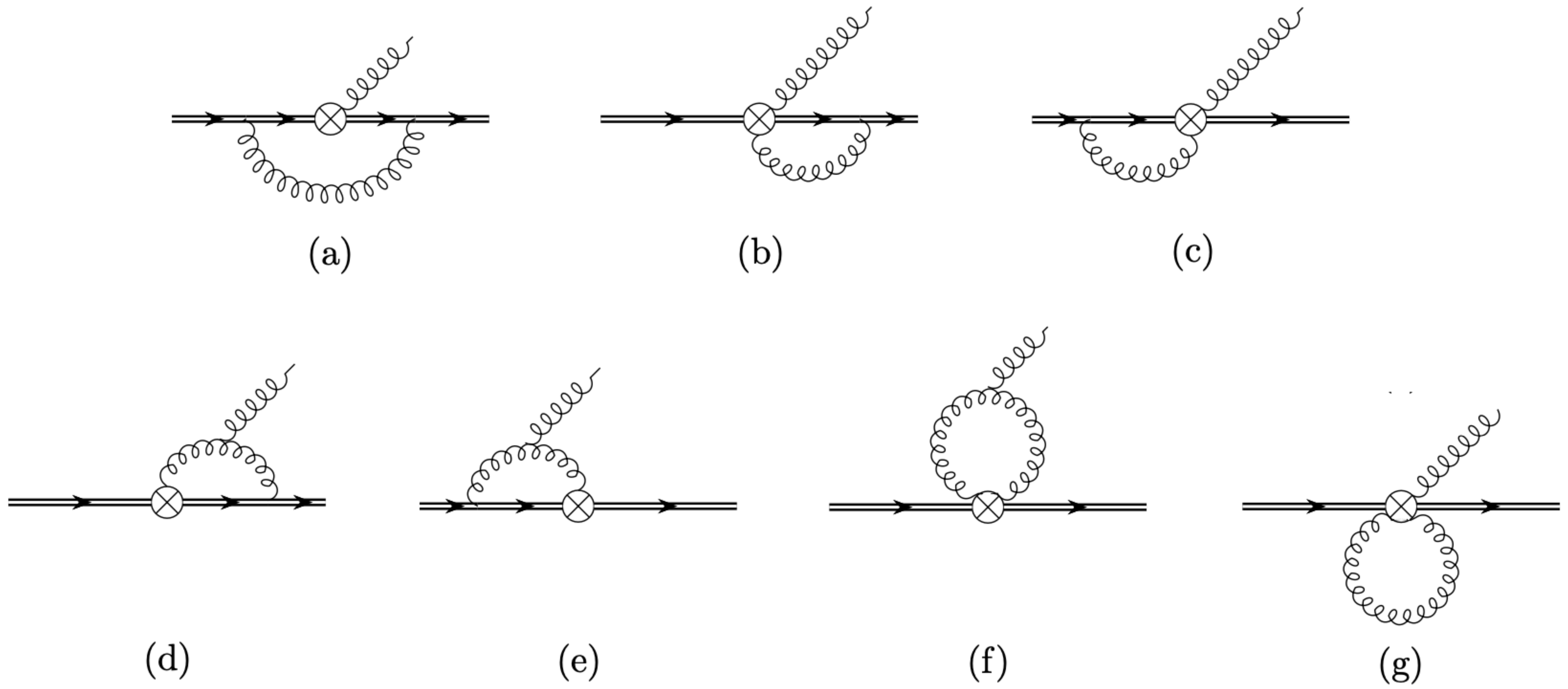

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$$\gamma_n(\omega,\omega';\mu) = \left(\ln\frac{\mu^2}{\omega^2}-1\right)\delta(\omega-\omega') - 2\theta(\omega)\left[\frac{\theta(\omega'-\omega)}{\omega'(\omega'-\omega)}\right]_+\omega' - 2\theta(-\omega)\left[\frac{\theta(\omega'-\omega)}{\omega'-\omega}\right]_\ominus$$

DEFINITION

Plus-distribution:
$$\int d\omega'\,[\ldots]_+\,f(\omega') \equiv \int d\omega'\,[\ldots]\,(f(\omega')-f(\omega))$$

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RG evolution of the shape function

Given the previous anomalous dimension, the RG equation can be solved using the Mellin transform method.

$$g_{17}(\omega,\omega_1;\mu) = \int_{\omega}^{\bar\Lambda} \frac{d\omega'}{\omega'-\omega}\, U_n^{(17)}(\omega,\omega';\mu,\mu_0) \int_{-\infty}^{\infty} \frac{d\omega_1'}{|\omega_1'|}\, U_{\bar n}^{(17)}(\omega_1,\omega_1';\mu,\mu_0)\, g_{17}(\omega',\omega_1';\mu_0)$$

"Factorisation" of two light-cones

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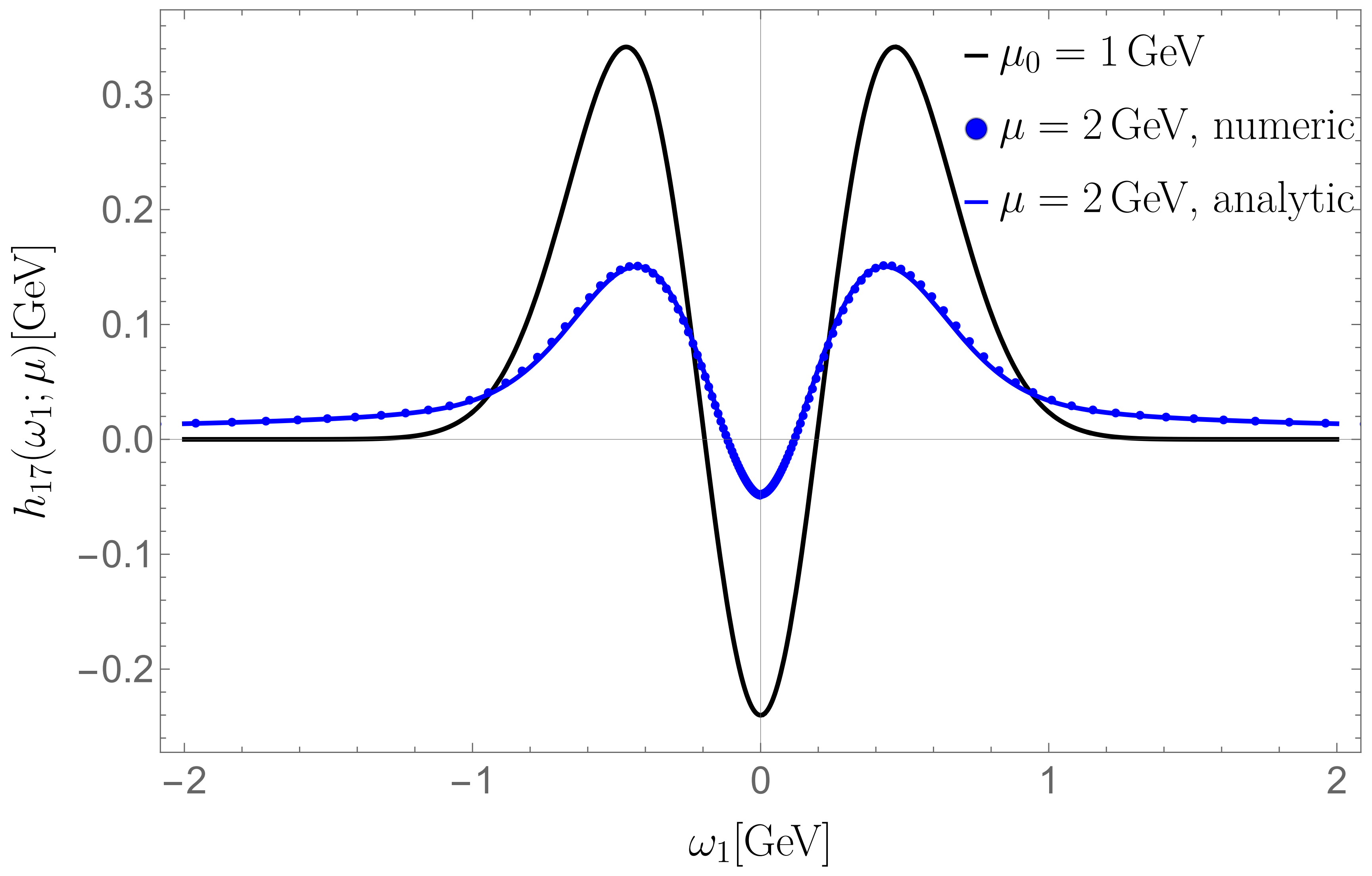
"Factorisation" of two light-cones

All order properties of g_{17} are preserved in the RGE:

- g_{17} is real (from PT invariance)
- The function $h_{17} = \int d\omega\, g_{17}$ is even (from HQET trace formalism)

RGE for the function h_{17}



[2411.16634: RB, Böer, Hurth]

The exclusive counterpart

In exclusive decays, such as $\bar{B}_{d,s} \to \gamma\gamma$, analogous soft functions appear

$$2\,\mathcal{F}_B(\mu)\,m_B \int_{-\infty}^{+\infty} d\omega_1 \int_{-\infty}^{+\infty} d\omega_2 \exp\left[-i(\omega_1\tau_1 + \omega_2\tau_2)\right]\,\Phi_{\rm G}(\omega_1,\omega_2,\mu) \;=\; \langle 0|(\bar{q}_s S_n)(\tau_1 n)\,(S_n^\dagger\, S_{\bar{n}})(0)\,(S_{\bar{n}}^\dagger\, g_s\, G_{\mu\nu}\, S_{\bar{n}})(\tau_2\bar{n})\;\bar{n}^\nu\;\not{n}\gamma_\perp^\mu\gamma_5\,(S_{\bar{n}}^\dagger h_v)(0)|\bar{B}_v\rangle$$

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“mixing”: spoil of factorisation theorem?

However, it must be convoluted with the jet functions!

$$\int_{-\infty}^{+\infty}\frac{d\omega}{\omega - i0}\int_{-\infty}^{+\infty}\frac{d\omega_1}{\omega_1 + i0}\Delta H(\omega,\omega')\Delta H(\omega_1,\omega_1') = 0$$

Due to the location of the poles, this “mixing” term vanishes saving factorisation and making the evolution factorised again.

[2411.16634: RB, Böer, Hurth]

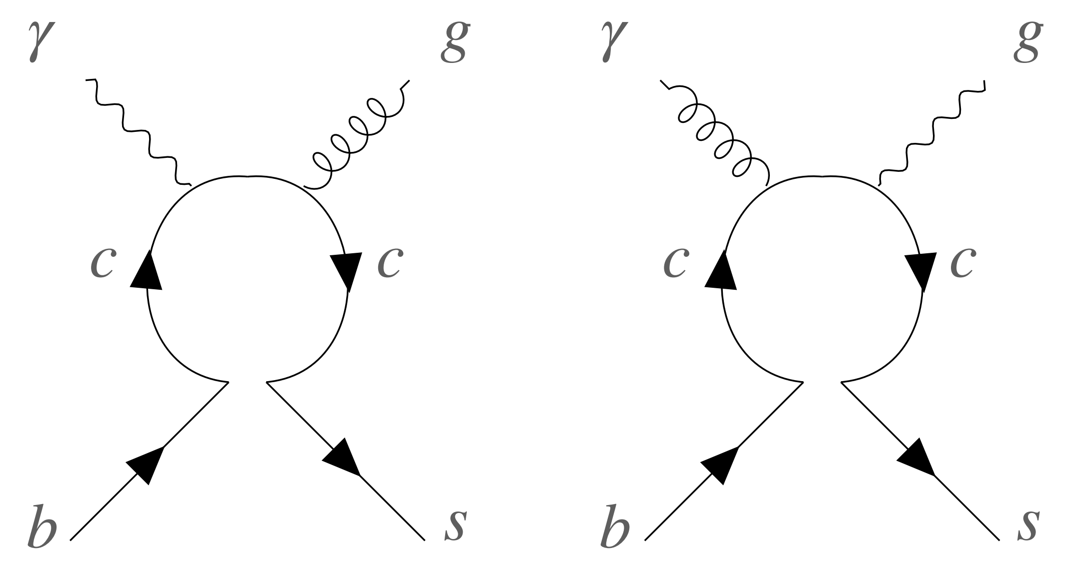
Two-loop “penguin”-jet function and pole cancellation

The “penguin”-jet function at NLO

The “penguin”-jet function is the anti-hard collinear region of $b \to s\gamma g$.

[to appear: RB, Böer, Hurth]

LO



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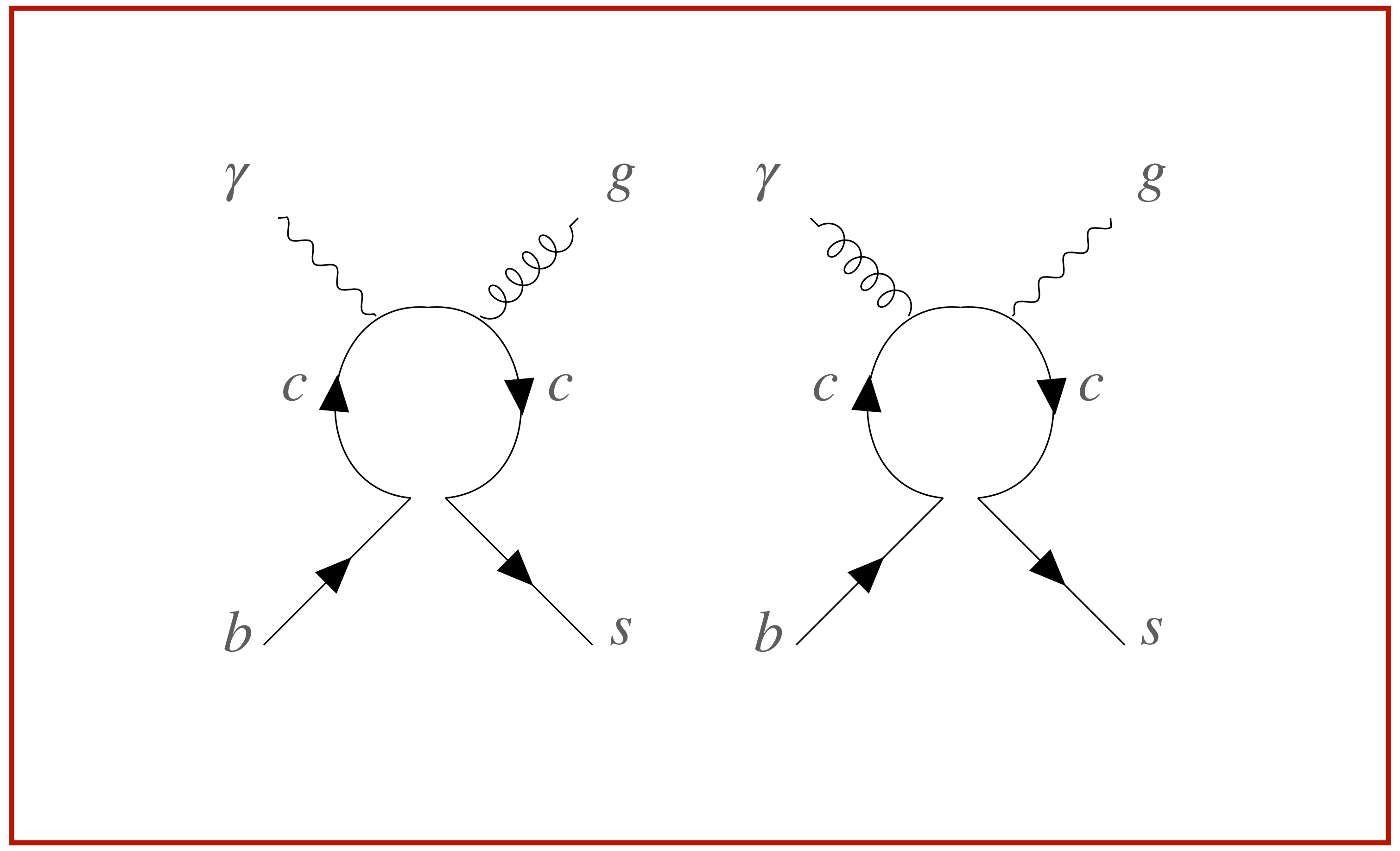

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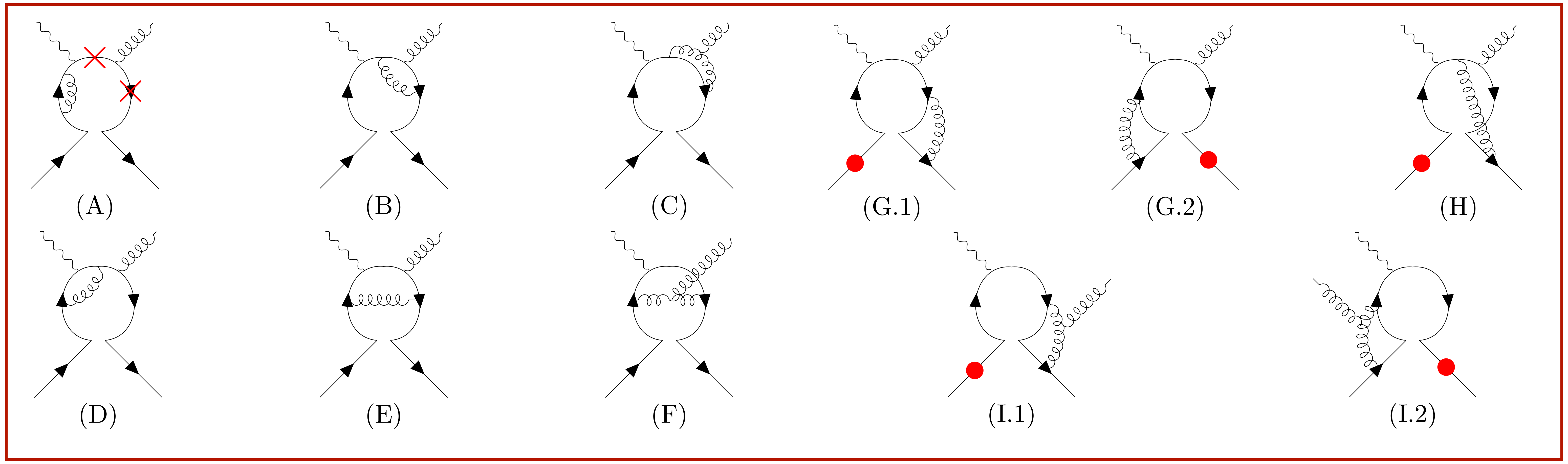

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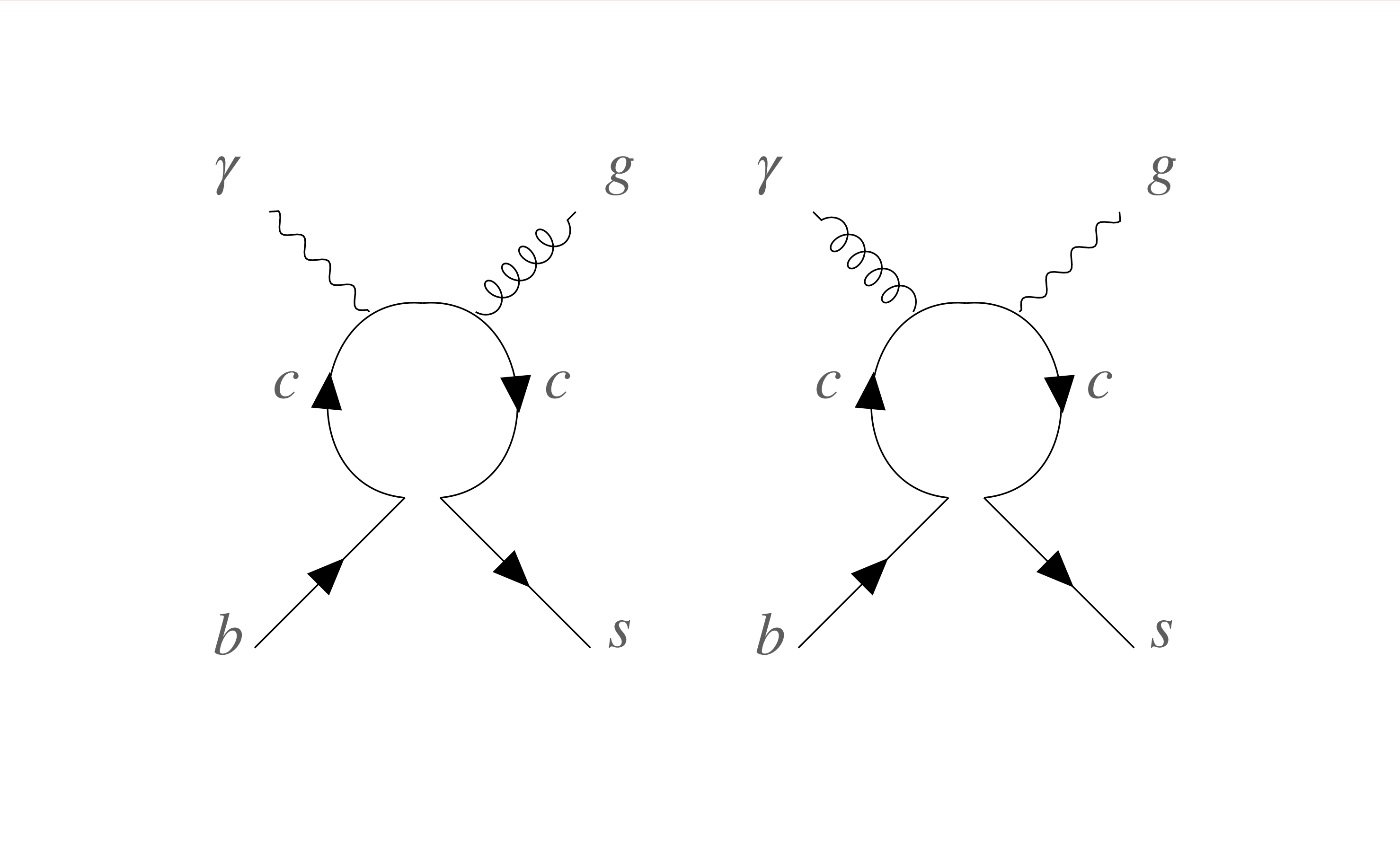

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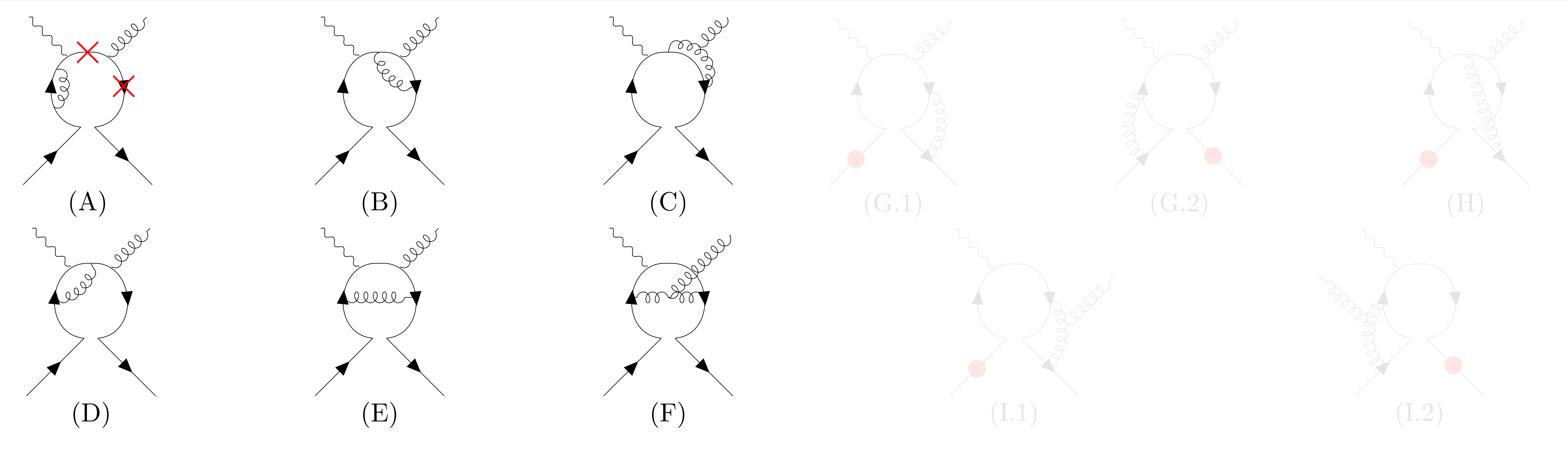

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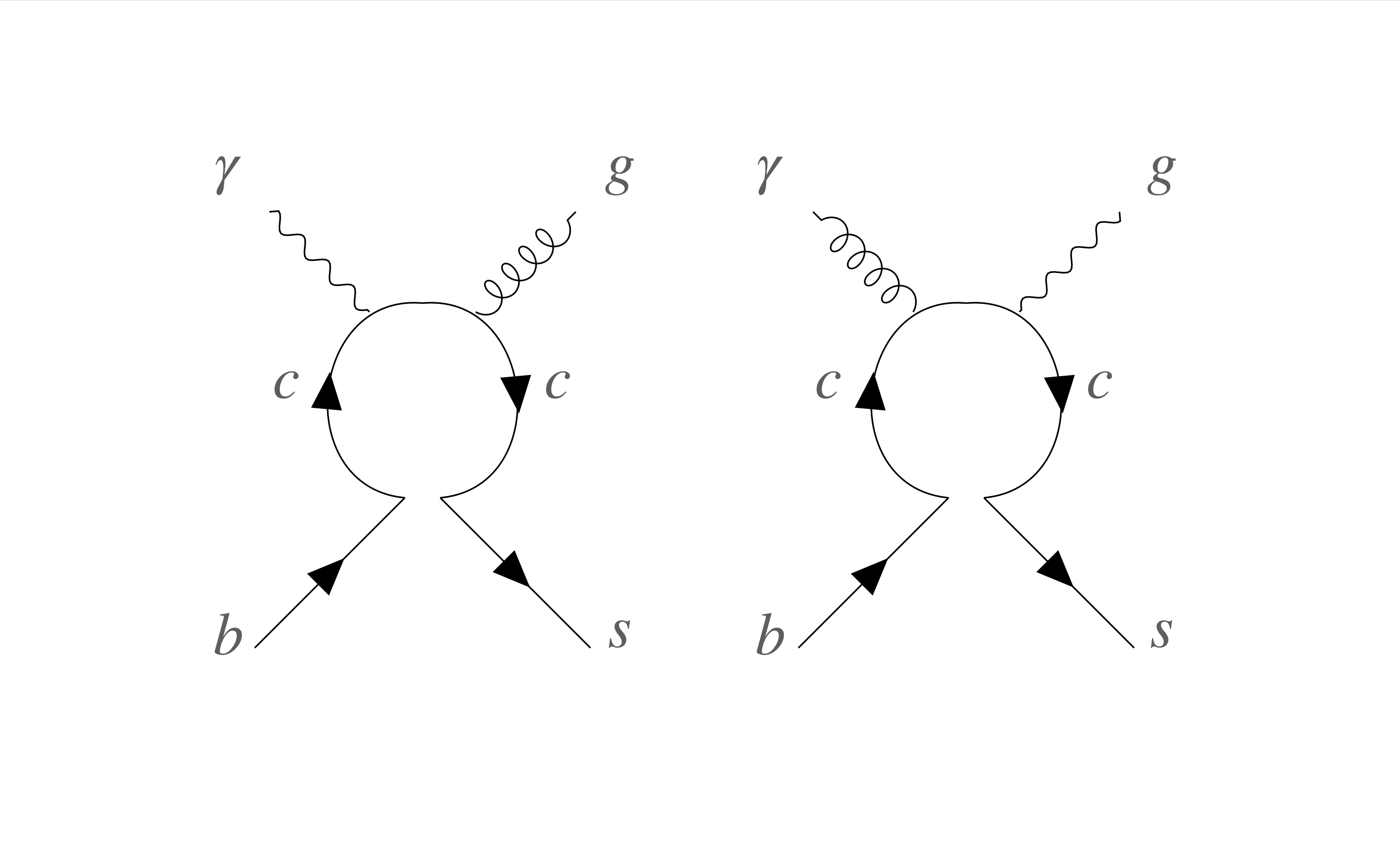

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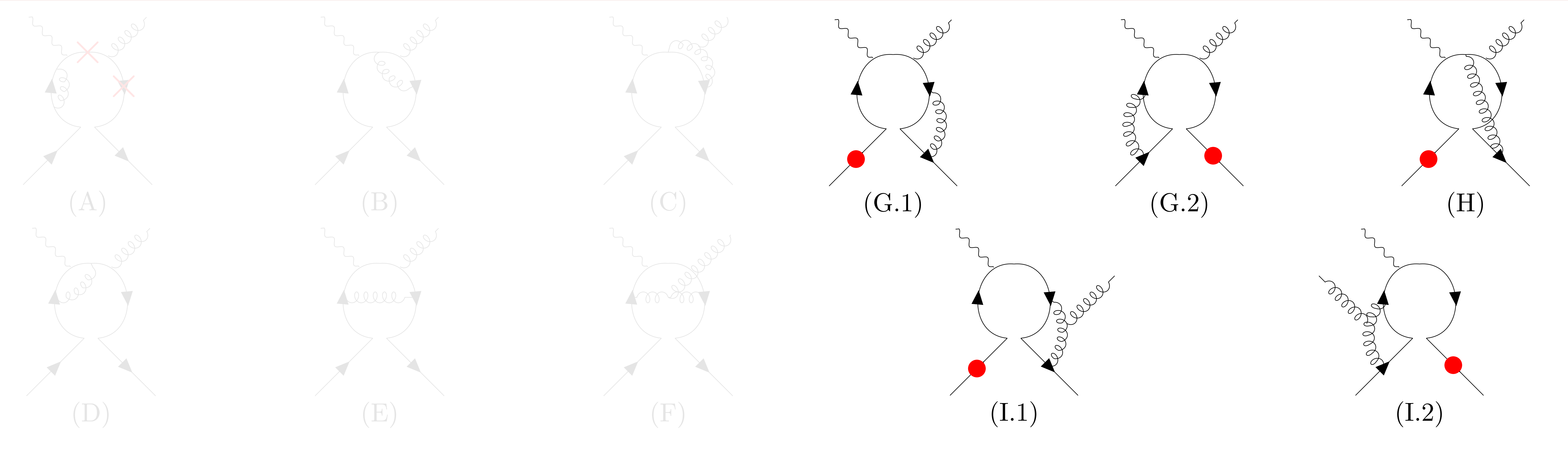

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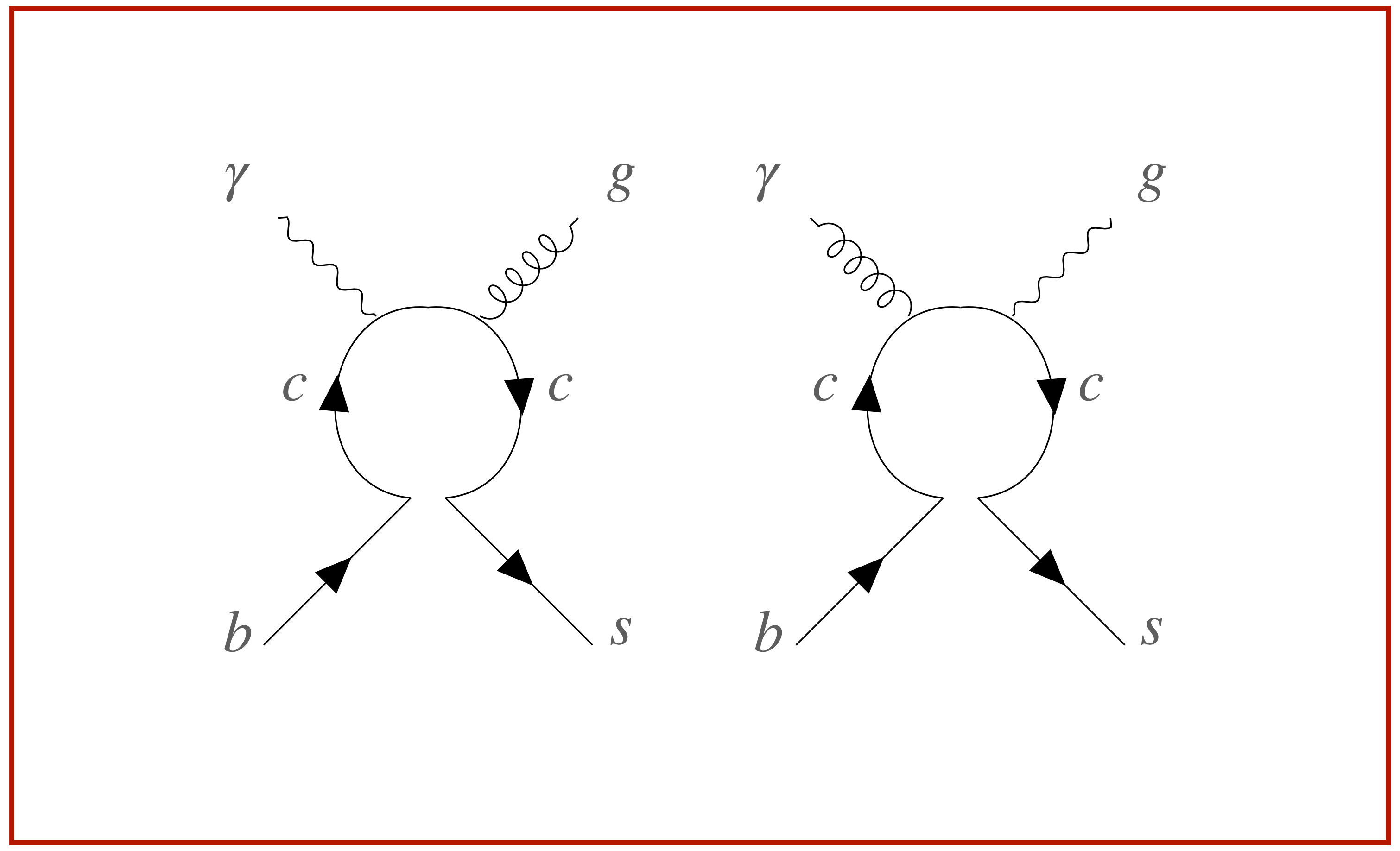

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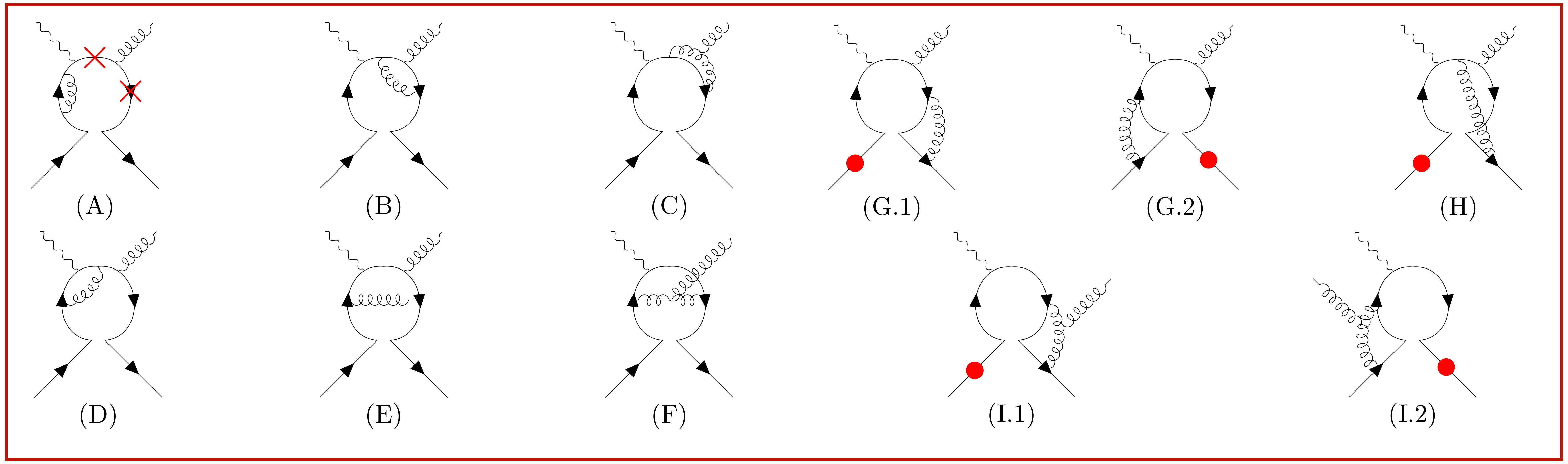

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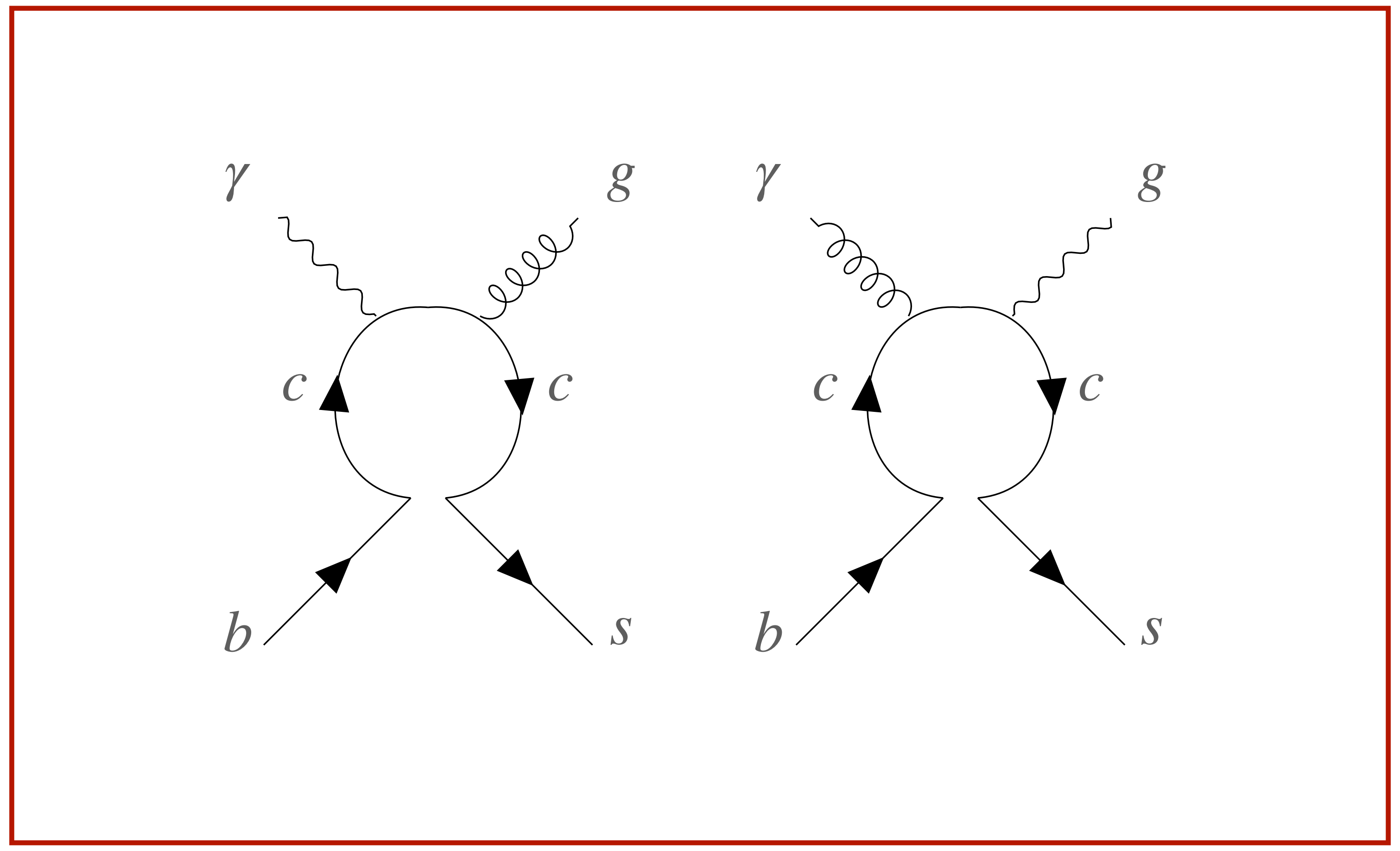

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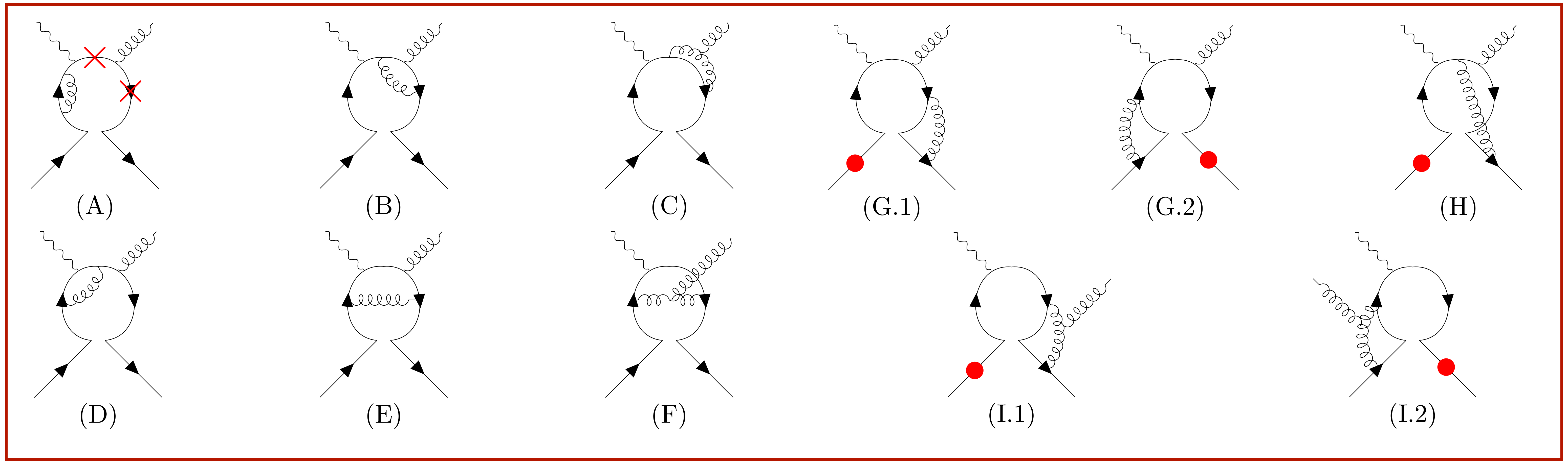

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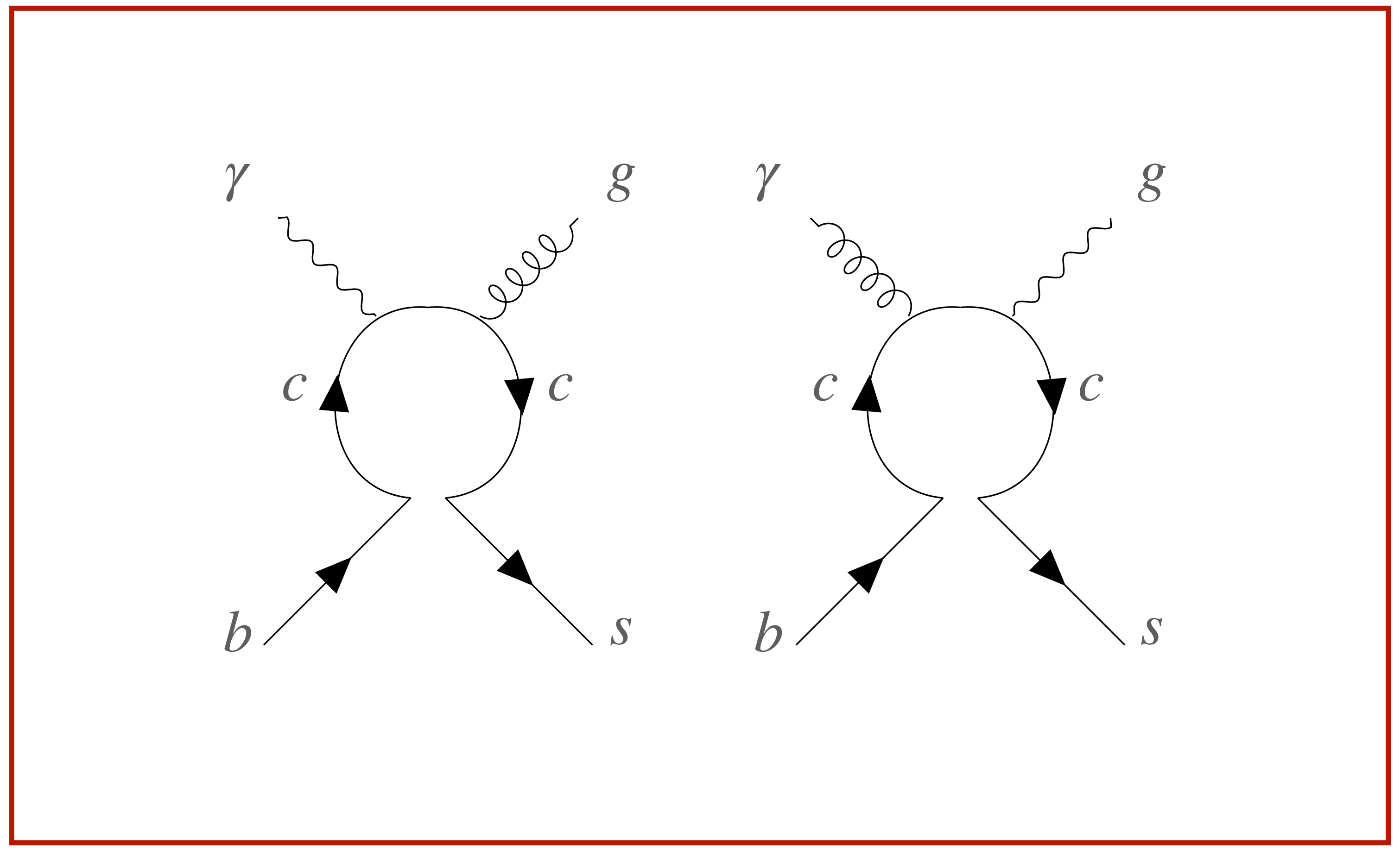

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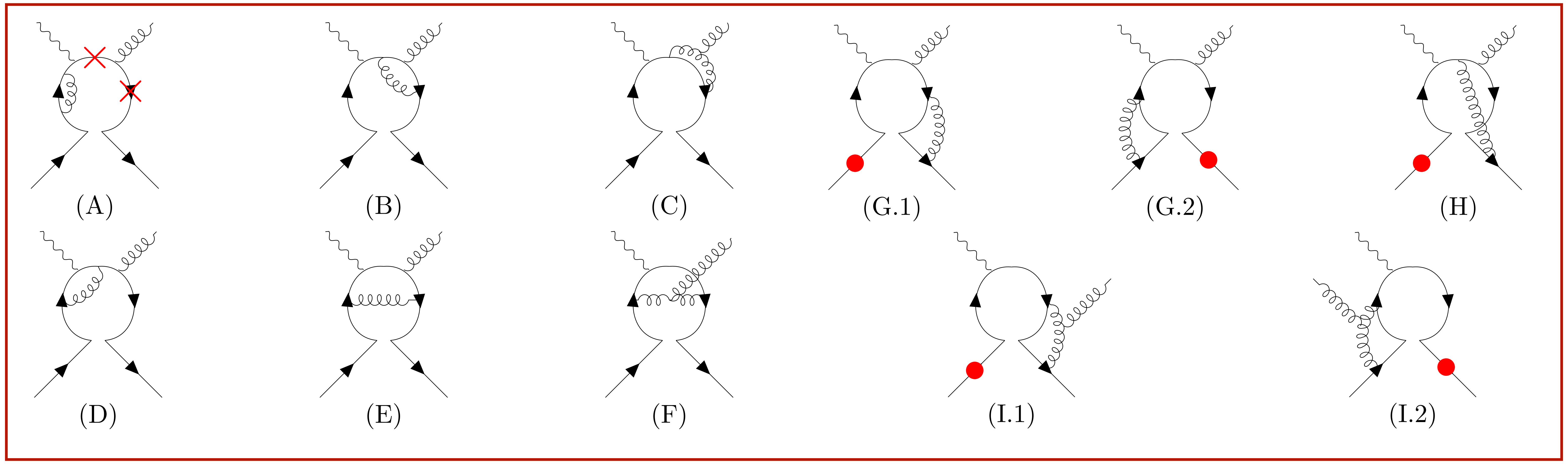

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Why NLO?

The $\mathcal{O}_1 - \mathcal{O}_7$ interference is the largest uncertainty (with large scale dependence) in $\overline{B} \to X_s\gamma$ decays. Reduced by NLO.

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Thank you for your attention!

Back up slides

Distribution definition

Plus-distribution:

$$\int d\omega' \, [\ldots]_+ \, f(\omega') \equiv \int d\omega' \, [\ldots] \, (f(\omega') - f(\omega))$$

Modified distributions:

$$\int d\omega' \, [\ldots]_{\oplus/\ominus} \, f(\omega') \equiv \int d\omega' \, [\ldots] \, (f(\omega') - \theta(\pm\omega') f(\omega))$$

H-distributions:

$$\begin{aligned}
F^{>}(\omega_i, \omega_i') &= \left[\frac{\omega_i \, \theta(\omega_i' - \omega_i)}{\omega_i' \, (\omega_i' - \omega_i)}\right]_+ + \left[\frac{\theta(\omega_i - \omega_i')}{\omega_i - \omega_i'}\right]_\oplus \\
F^{<}(\omega_i, \omega_i') &= \left[\frac{\omega_i \, \theta(\omega_i - \omega_i')}{\omega_i' \, (\omega_i - \omega_i')}\right]_+ + \left[\frac{\theta(\omega_i' - \omega_i)}{\omega_i' - \omega_i}\right]_\ominus \\
G^{>}(\omega_i, \omega_i') &= (\omega_i + \omega_i') \left[\frac{\theta(\omega_i' - \omega_i)}{\omega_i' \, (\omega_i' - \omega_i)}\right]_+ - i\pi\delta(\omega_i - \omega_i') \\
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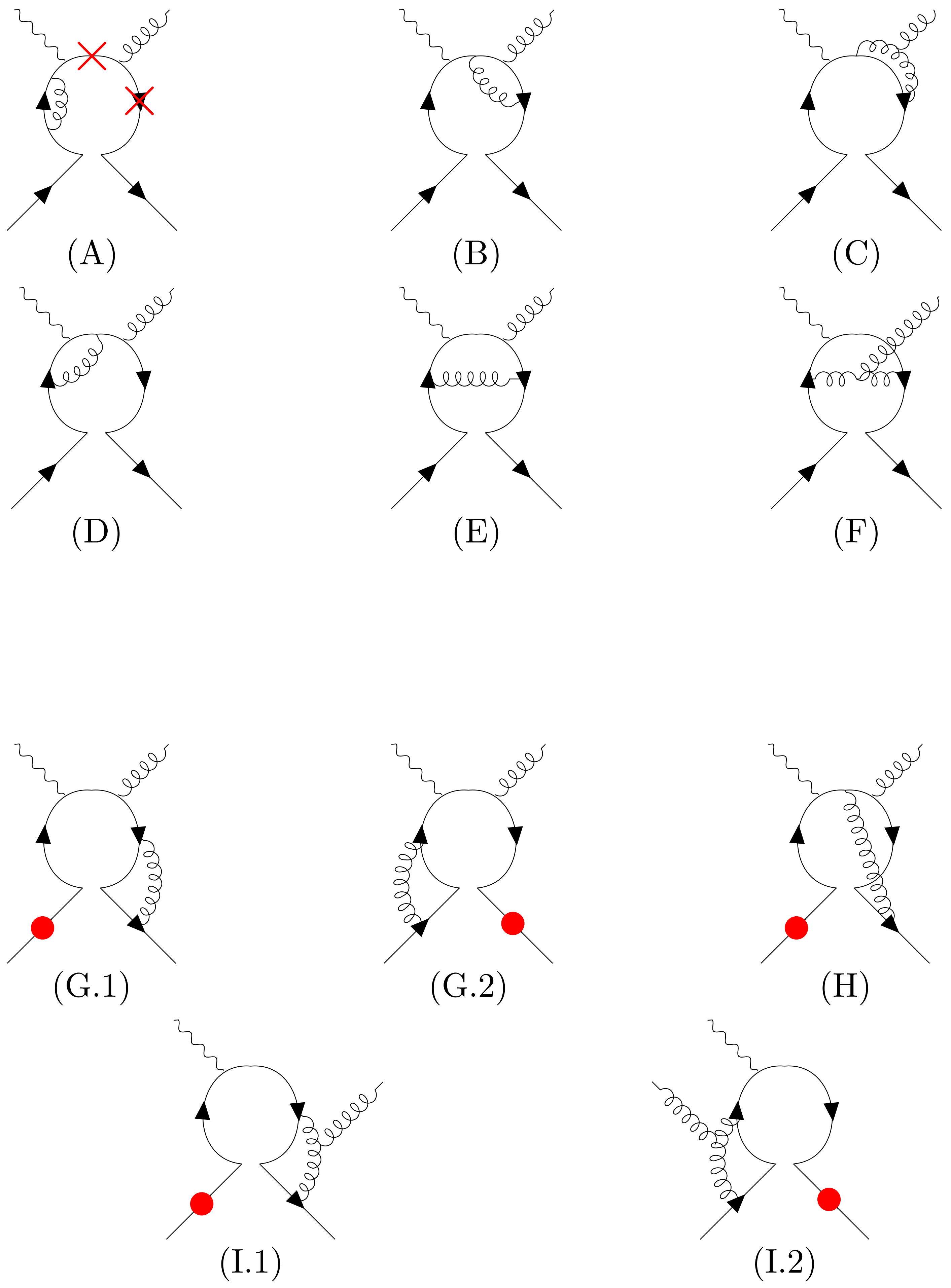
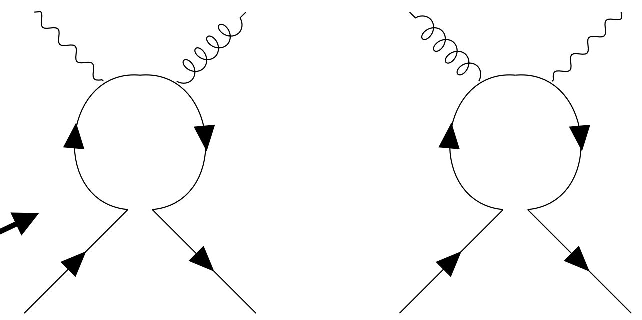
\end{aligned}$$

$$H_\pm(\omega_i, \omega_i') = \theta(\pm\omega_i) F^{>(<)}(\omega_i, \omega_i') + \theta(\mp\omega_i) G^{<(>)}(\omega_i, \omega_i')$$

Last missing piece: the “penguin”-jet function

The “penguin”-jet function corresponds to the anti-hard collinear region of $b \to s\gamma g$ with the charm quark running in the loop.





Remarks:

- The LO contains charm-loop, therefore NLO is a two-loop amplitude
- The upper diagrams (A-F) have only one region
- The lower diagrams (G-I) need the expansion of the propagator of the external legs

Status:

- All diagrams have been computed assuming a massless quark in the loop (up-quark)
- The $m_c \to 0$ limit is smooth, therefore we have all the poles of the four functions
- All poles cancel providing a non-trivial check of our results

Outlook:

- Remaining task: complete m_c correction to have the final NLO result

[in progress: RB, Böer, Hurth]

Basics for RGE

$$\widetilde{\mathcal{O}}_{17}^{(\mathrm{bare})}(\omega,\omega_1) = \int d\omega' \int d\omega_1'\, Z_{17}^{-1}(\omega,\omega_1,\omega',\omega_1';\mu)\, \widetilde{\mathcal{O}}_{17}^{(\mathrm{ren})}(\omega,\omega_1;\mu)$$

$$\frac{d}{d\ln\mu}\, g_{17}(\omega,\omega_1;\mu) = -\int d\omega' \int d\omega_1'\, \gamma_{17}(\omega,\omega_1,\omega',\omega_1';\mu)\, g_{17}(\omega',\omega_1';\mu)$$

$$\gamma_{17}(\omega,\omega_1,\omega',\omega_1';\mu) = -\int d\hat{\omega} \int d\hat{\omega}_1\, \frac{dZ_{17}(\omega,\omega_1,\hat{\omega},\hat{\omega}_1;\mu)}{d\ln\mu}\, Z_{17}^{-1}(\hat{\omega},\hat{\omega}_1,\omega',\omega_1';\mu)$$

Bottom-meson soft function renormalisation

$$\Gamma_{\mathrm{G}} = \frac{\alpha_s}{\pi}\left\{\left[C_F\left(\ln\frac{\mu}{\omega_1 - i0} - \frac{1}{2}\right) + C_A\left(\ln\frac{\mu}{\omega_2 - i0} + \frac{i}{2}\pi\right)\right]\delta(\omega_1-\omega_1')\delta(\omega_2-\omega_2') - C_F H_+(\omega_1,\omega_1')\delta(\omega_2-\omega_2')\right.$$
$$\left. - C_A H_+(\omega_2,\omega_2')\delta(\omega_1-\omega_1') + C_A\left(\frac{\omega_2}{\omega_2'^2}\right)\left[\theta(\omega_2)\theta(\omega_2'-\omega_2) - \theta(-\omega_2)\theta(\omega_2-\omega_2')\right]\delta(\omega_1-\omega_1') + \Delta\Gamma_{\mathrm{G}}\right\}$$

$$\Delta\Gamma_{\mathrm{G}} = \frac{i}{4}\frac{C_A}{\pi}\left[H_+(\omega_1,\omega_1') - H_-(\omega_1,\omega_1') - 2i\pi\delta(\omega_1-\omega_1')\right]\left[H_+(\omega_2,\omega_2') - H_-(\omega_2,\omega_2') - 2i\pi\delta(\omega_2-\omega_2')\right]$$

Full evolution functions

The final result for the RGE reads:

$$g_{17}(\omega,\omega_1;\mu) = \int_{\omega}^{\bar{\Lambda}} \frac{d\omega'}{\omega'-\omega}\, U_n^{(17)}(\omega,\omega';\mu,\mu_0) \int_{-\infty}^{\infty} \frac{d\omega_1'}{|\omega_1'|}\, U_{\bar{n}}^{(17)}(\omega_1,\omega_1';\mu,\mu_0)\, g_{17}(\omega',\omega_1';\mu_0)$$

$$U_n^{(17)}(\omega,\omega';\mu,\mu_0) = \frac{e^{2V+2\gamma_E a}}{\Gamma(-2a)} \left(\frac{\mu_0}{\omega'-\omega}\right)^{2a}$$

$$U_{\bar{n}}^{(17)}(\omega_1,\omega_1';\mu,\mu_0) = -\, e^{V_1+2\gamma_E a_1} \left(\frac{\mu_0}{|\omega_1'|}\right)^{a_1} \left\{\theta(\tau) G_{3,3}^{1,2}\left(\begin{matrix} -1, & 1, & a_1/2 \\ a_1+1, & a_1-1, & a_1/2 \end{matrix} \middle| \tau \right) + \frac{1}{2\pi}\sin\left(\frac{a_1\pi}{2}\right)\theta(-\tau)\Gamma(1+a_1)\Gamma(3+a_1)(-\tau)^{1+a_1}\,{}_2F_1(1+a_1,3+a_1,3;\tau)\right\}$$

With the Meijer-G functions defined as:

$$G_{p,q}^{m,n}\left(\begin{matrix}\mathbf{a}\\ \mathbf{b}\end{matrix}\middle| z\right) = \int \frac{d\eta}{2\pi i} z^{\eta} \frac{\prod_{j=1}^{m}\Gamma(b_j-\eta)\prod_{j=1}^{n}\Gamma(1-a_j+\eta)}{\prod_{j=m+1}^{q}\Gamma(1-b_j+\eta)\prod_{j=n+1}^{p}\Gamma(a_j-\eta)}$$

Mellin space transform and equations

The Mellin transform reads:

$$g_{17}(\omega,\omega_1;\mu) = \theta(\omega) g_{17}^{>}(\omega,\omega_1;\mu) + \theta(-\omega) g_{17}^{<}(\omega,\omega_1;\mu)$$

$$\tilde{g}_{17}^{<}(\eta,\omega_1;\mu) = \int_0^\infty \frac{d\omega}{\omega} \left(\frac{\mu}{\omega}\right)^\eta g_{17}^{<}(-\omega,\omega_1;\mu)$$

$$\tilde{g}_{17}^{>}(\eta,\omega_1;\mu) = \int_0^\infty \frac{d\omega}{\omega} \left(\frac{\mu}{\omega}\right)^\eta g_{17}^{>}(\omega,\omega_1;\mu)$$

Applying this to the RG equations we get for the non-abelian part:

$$\left(\frac{d}{d\ln\mu} - \eta_1\right) \tilde{g}_{17}^{>}(\omega,\eta_1;\mu)$$

$$= -\frac{\alpha_s C_A}{2\pi}\Big\{\left[H_{-1-\eta_1} + H_{\eta_1} + 2H_{1-\eta_1} + 2\partial_{\eta_1}\right] \tilde{g}_{17}^{>}(\omega,\eta_1;\mu) - \Gamma(-\eta_1)\Gamma(1+\eta_1)\, \tilde{g}_{17}^{<}(\omega,\eta_1;\mu)\Big\}$$

and

$$\left(\frac{d}{d\ln\mu} - \eta_1\right) \tilde{g}_{17}^{<}(\omega,\eta_1;\mu)$$

$$= -\frac{\alpha_s C_A}{2\pi}\Big\{\left[H_{-1-\eta_1} + H_{\eta_1} + 2H_{1-\eta_1} + 2\partial_{\eta_1}\right] \tilde{g}_{17}^{<}(\omega,\eta_1;\mu) - \Gamma(-\eta_1)\Gamma(1+\eta_1)\, \tilde{g}_{17}^{>}(\omega,\eta_1;\mu)\Big\}$$

Outline

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4. Two-loop penguin-jet function and pole cancellation