QUANTUM PROPERTIES OF H→VV*: PRECISE PREDICTIONS IN THE SM AND SENSITIVITY TO NEW PHYSICS

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QUANTUM INFORMATION PRINCIPLES MEET HEP

- Growing field of study where observables and principles inspired by QI are applied to HEP
  - Example: measurement of entanglement between the spin of the top-quarks created at colliders
  - First pheno proposal 2020
  - First experimental results 2024 (ATLAS, CMS)
- Many more possibilities:
  - Observables (Discord, Steering, Magic, Bell's inequality violation)
  - Final states ($Z \to \tau\tau, H \to \tau\tau, H \to VV^*$)
  - Properties (Flavour)
  - Interpretation:
    - Search for new physics (or new particles)
    - Understand decoherence through fundamental interactions
    - Study the effect of decay and detector interaction on entanglement
- Status of the art and plans from the field summarised in: Input to the European Strategy for particle physics

ρ FOR $H \to ZZ^*$

- The spin of the bosons originated by the Higgs decay are interpreted as qutrit representations.
- It is possible to describe the quantum state with a spin density matrix for a pair of qutrits

$$\rho = \frac{1}{9}\left[\mathbb{1}_3 \otimes \mathbb{1}_3 + A^a_{L,M}(T_{L,M} \otimes \mathbb{1}_3) + A^b_{L,M}(\mathbb{1}_3 \otimes T_{L,M}) + C_{L_a,M_a,L_b,M_b}(T_{L_a,M_a} \otimes T_{L_b,M_b})\right]$$

- T are polarization operators
- In principle 80 independent A and C coefficients
- The Higgs is a scalar → significantly constraints the Z polarization
  - The spin density matrix is highly simplified
  - There are relations between the C coefficients that make only 2 of them independent

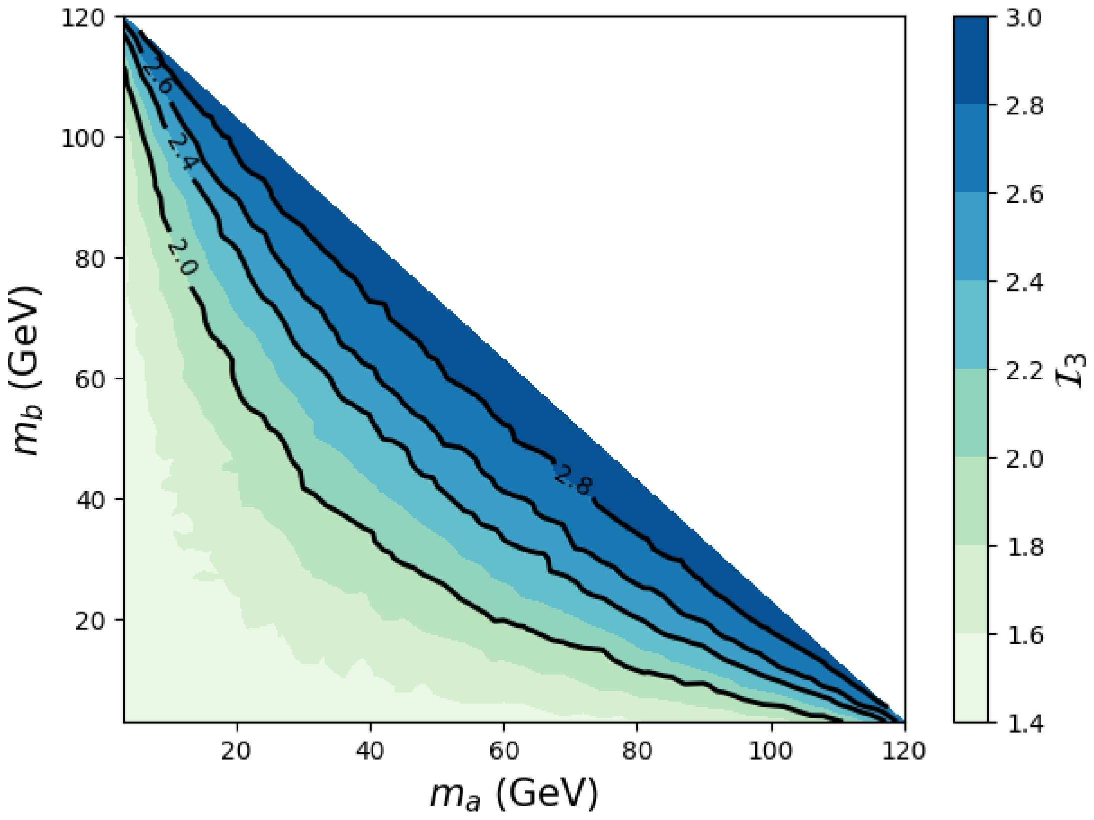
$$\rho_{\mathrm{LO}}(\beta) = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \frac{a_T^2}{2} & \cdot & \frac{a_L a_T}{\sqrt{2}} & \cdot & \frac{a_T^2}{2} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \frac{a_L a_T}{\sqrt{2}} & \cdot & a_L^2 & \cdot & \frac{a_L a_T}{\sqrt{2}} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \frac{a_T^2}{2} & \cdot & \frac{a_L a_T}{\sqrt{2}} & \cdot & \frac{a_T^2}{2} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix},$$

QUANTUM OBSERVABLES FOR $H \rightarrow ZZ^*$

- An entanglement condition can be determined starting from ρ:
  - Generic qutrit:
    - Lower bound on the entanglement

$$(\mathcal{C}(\rho))^2 \geq \left(-2 + \sum_{L,M} |A^a_{L,M}|^2 - 2\sum_{L,M} |A^b_{L,M}|^2 + \sum_{L_a,L_b,M_a,M_b} |C_{L_a,M_a,L_b,M_b}|^2\right)$$

  - Higgs case:
    - Directly apply the PH criterion to the very simple matrix
    - $C_{2,2,2,-2} > 0$ or $C_{2,1,2,-1} > 0$
- Bell's inequality violation
  - Set of inequality that could determine if nature follows QM or "classical" physics
  - The CGLMP Bell's inequality can be represented as an operator
  - BIV = $Tr(O\rho)$
- The two Z from the Higgs decay are highly entangled across the whole phase space
  - BIV should be violated in almost all the phase space



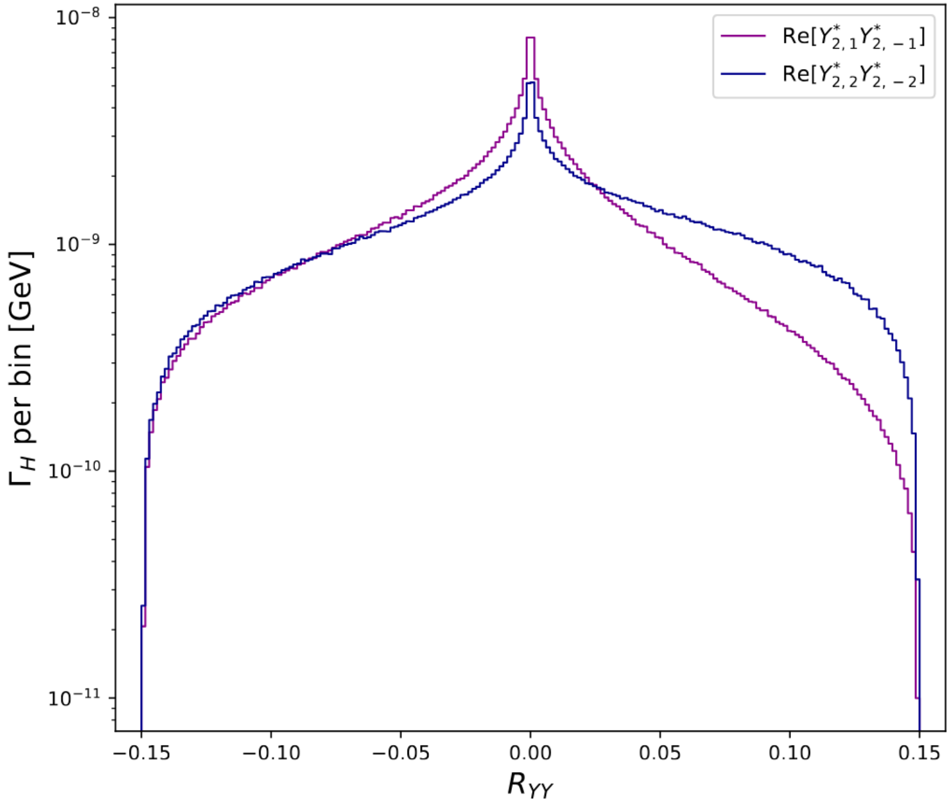
QUANTUM TOMOGRAPHY:

- The spin of the Z bosons can not be directly measured at colliders
  - The weak decays create a connection between the final state particle direction and the parent particle spin
- Angular measurements **in the parent Z frame** can be used to extract all the coefficients C
  - Used to evaluate the terms of the spin density matrix
  - Essential the reconstruction of the whole final state
- Relation between angular distributions and C coefficients:

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} Y^*_{L,M}(\Omega_j)\, d\Omega_a d\Omega_b = \frac{B^j_L}{4\pi} A^j_{L,M} \quad \text{with} \quad j = a, b\,,$$

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} Y^*_{L_a,M_a}(\Omega_a) Y^*_{L_b,M_b}(\Omega_b)\, d\Omega_a d\Omega_b = \frac{B^a_{L_a} B^b_{L_b}}{(4\pi)^2} C_{L_a,M_a,L_b,M_b}\,,$$

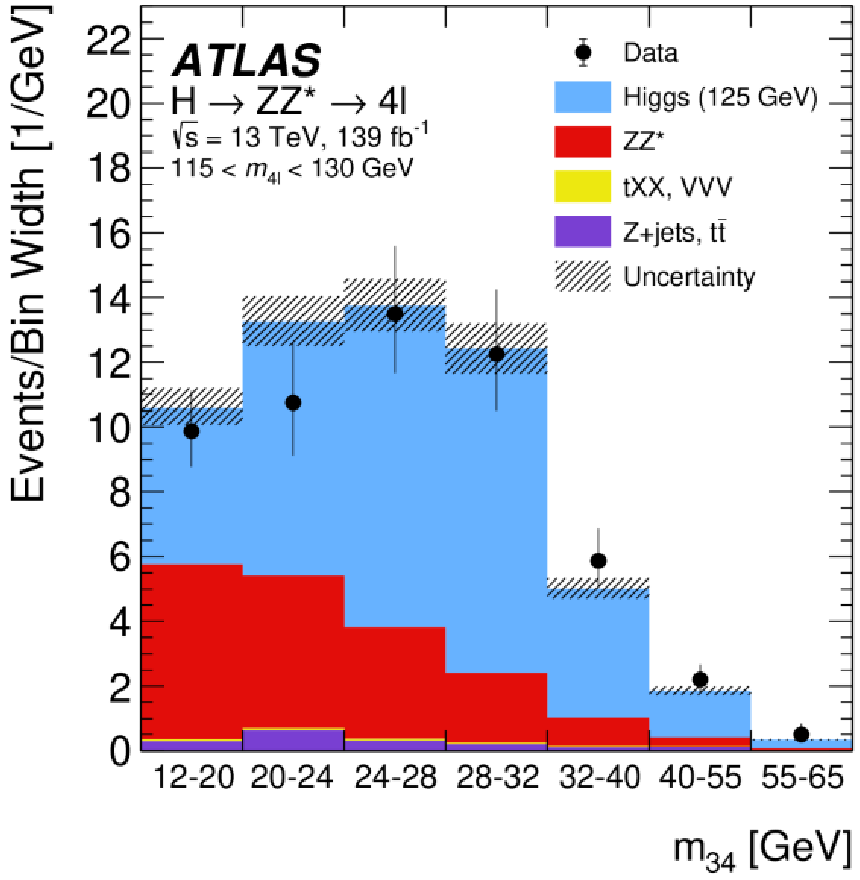
- $B_1^{a,b}$ includes the **spin analysing power**



POTENTIAL OF THE HIGGS FINAL STATE

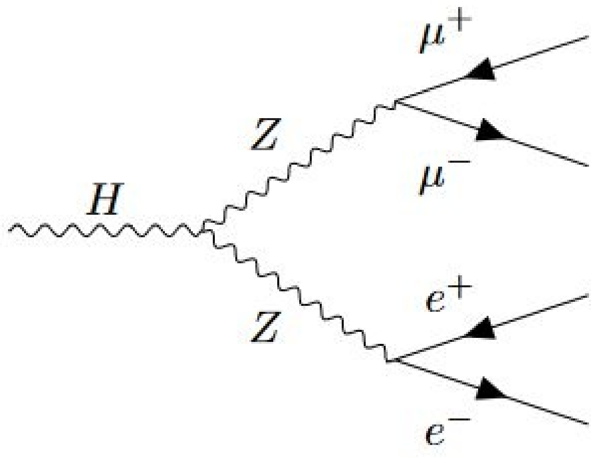
- The Higgs final state is the one of the few foreseen in the SM
  - where the spin of the decay product are expected to be highly correlated across all the phase state
  - Where the decay product spin can be measured
- The absence of neutrinos in the ZZ to four charged leptons final state makes the reconstruction of the whole final state and Z rest-frame easier, compared to other final states (e.g. H→WW)
- This process is rare but definitely well measured at LHC
  - There is potentially a large interest for experimental measurements in this final state

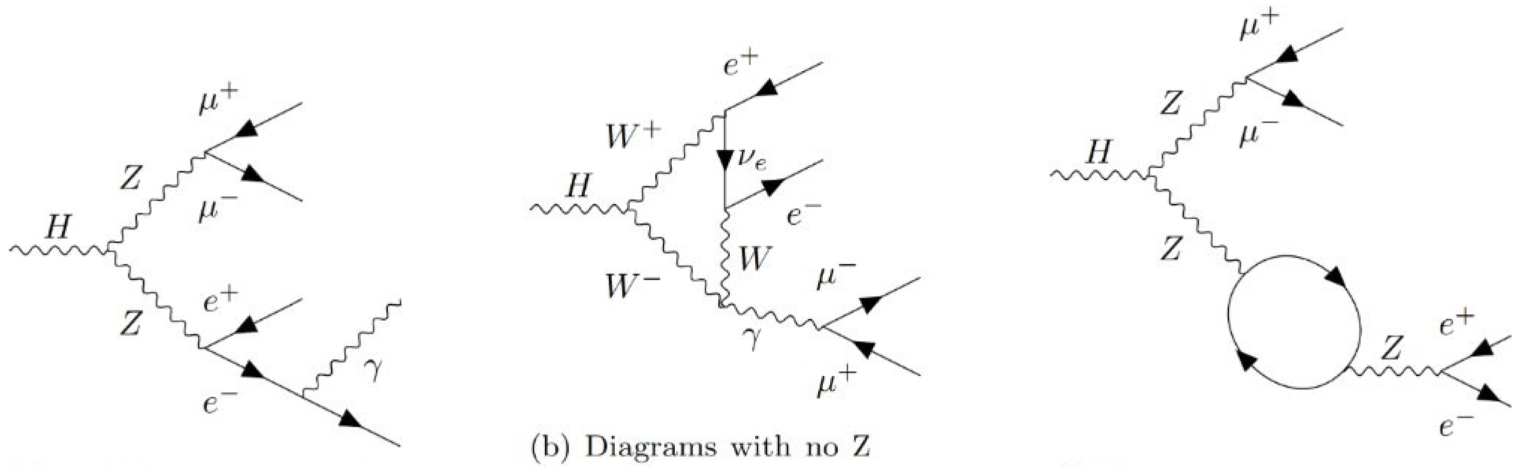
Eur. Phys. J. C 80 (2020) 942



HIGHER ORDER CORRECTIONS

- The 4 leptons final state can be realised from many different diagrams compared to the simple LO picture.
- We evaluated the impact on the extracted spin density matrix and quantum observables of NLO EW corrections
- The same approach is used at LO and NLO EW:
  - Process investigated: $H \to e^+e^-\mu^+\mu^-$
  - Z defined combining the same flavour leptons
  - Angles measured with respect to the negative lepton





(a) Real photon radiated

(b) Diagrams with no Z bosons

(c) Diagrams with loops

IMPACT ON THE SPIN DENSITY MATRIX COEFFICIENTS

- The non-zero coefficients are evaluated at LO and NLO EW
- The impact is large with a very strong dependence on the coefficients
  - Impact of the order of 90% on the $C_{1,x,1,x}$ coefficients
  - Asymmetry of the $A_{2,0}$ and $A_{1,0}$ coefficients
  - Relations between the coefficients valid at LO are broken

	LO	NLO	NLO / LO
$A^1_{2,0}$	$-0.592(1)$	$-0.509(2)$	$0.860(2)$
$A^2_{2,0}$	$-0.591(1)$	$-0.565(2)$	$0.956(2)$
$C_{2,1,2,-1}$	$-0.937(2)$	$-0.943(4)$	$1.006(3)$
$-C_{1,1,1,-1}$	$-0.94(1)$	$-0.16(2)$	$0.17(2)$
$A^1_{2,0}/\sqrt{2}+1$	$0.5817(7)$	$0.640(1)$	$1.101(2)$
$C_{2,2,2,-2}$	$0.581(3)$	$0.568(4)$	$0.977(6)$
$-C_{1,0,1,0}$	$0.59(1)$	$0.03(2)$	$0.06(4)$
$C_{2,0,2,0}$	$1.418(3)$	$1.400(5)$	$0.987(3)$
$C_{1,0,1,0}+2$	$1.41(1)$	$1.97(2)$	$1.39(1)$

STRUCTURE OF THE SPIN DENSITY MATRIX

- The simple structure of the spin density matrix was driven by the relation between the coefficients
- Completely different at NLO
  - The quantum observables based on the matrix simple structure are still valid?
  - Can we trust the NLO EW corrections?
  - Why are they so large?

$$\rho_{\mathrm{LO}} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0.195(2) & \cdot & -0.313(3) & \cdot & 0.194(1) & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -0.313(3) & \cdot & 0.612(1) & \cdot & -0.313(3) & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0.194(1) & \cdot & -0.313(3) & \cdot & 0.195(3) & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

$$\rho_{\mathrm{NLO}} = \begin{pmatrix} 0.099(4) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0.004(2) & \cdot & 0.131(4) & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0.111(4) & \cdot & -0.183(4) & \cdot & 0.189(1) & \cdot & \cdot \\ \cdot & 0.131(4) & \cdot & -0.009(2) & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -0.183(4) & \cdot & 0.591(1) & \cdot & -0.183(4) & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & -0.009(2) & \cdot & 0.131(4) & \cdot \\ \cdot & \cdot & 0.189(1) & \cdot & -0.183(4) & \cdot & 0.110(3) & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0.131(4) & \cdot & 0.004(2) & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0.099(3) \end{pmatrix}$$

DEPENDENCE ON THE RECOMBINATION RADIUS

	NLO ($\Delta R = 0.01$)	NLO / LO ($\Delta R = 0.01$)	NLO ($\Delta R = 1$)	NLO / LO ($\Delta R = 1$)
$A^1_{2,0}$	-0.432	0.731(4)	-0.578	0.977(2)
$A^2_{2,0}$	-0.548	0.927(3)	-0.579	0.979(2)
$C_{2,1,2,-1}$	-0.956	1.020(6)	-0.933	0.996(3)
$-C_{1,1,1,-1}$	-0.15	0.15(2)	-0.17	0.18(2)
$A^1_{2,0}/\sqrt{2}+1$	0.694	1.193(3)	0.591	1.016(1)
$C_{2,2,2,-2}$	0.554	0.954(7)	0.572	0.985(5)
$-C_{1,0,1,0}$	0.05	0.09(5)	0.03	0.05(3)
$C_{2,0,2,0}$	1.393	0.982(5)	1.418	1.000(3)
$C_{1,0,1,0}+2$	1.95	1.38(2)	1.97	1.39(1)

- At NLO EW the radiated photons are recombined in a cone of radius R with the closest lepton
- The effect of NLO correction may depend on R, especially if the large effect is mainly due to real emissions
- There is a clear reduction of the impact of the NLO EW corrections with the increase of R on all coefficients, except for $C_{1,x,1,x}$

UNDERSTANDING THE PROBLEM: SPIN ANALYSING POWER

- The $C_{1,x,1,x}$ are the only coefficients where the spin analysing power (α) plays a role.
- α is not evaluated at NLO EW, but the LO value is used
  - Evaluating α_{NLO} is complicated for an on-shell Z
  - It is ill defined for an off-shell Z
- Can this have such a large effect?
  - Yes, a small variation of α leads to large variations in the coefficients
  - The effect is amplified by the fact that α is very small for Z going to charged leptons
    - The effect should be reduced in different final states
  - We also checked the situation in $H \to WW^*$, where α is identically 1, and and the NLO EW corrections are reasonable

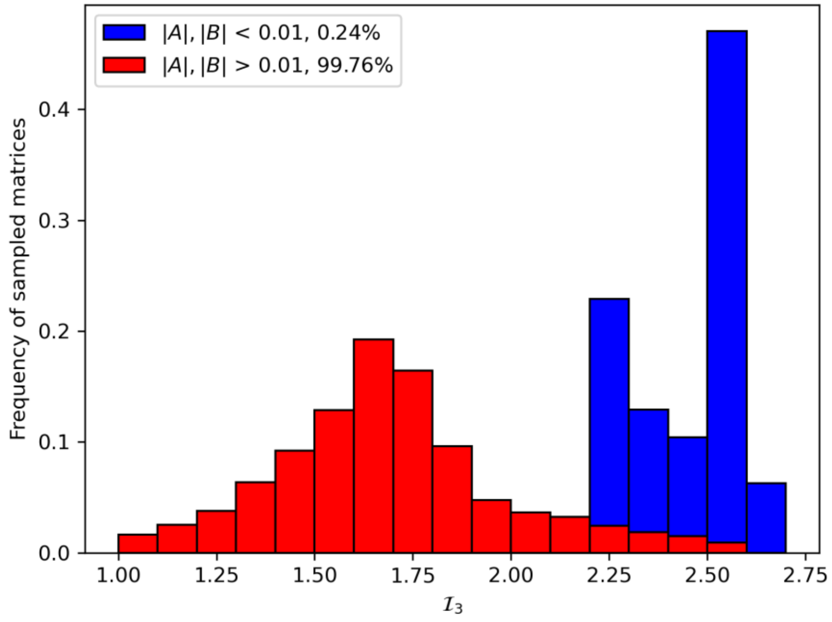
$\frac{\text{NLO}}{\text{LO}} - 1$ [%]	$\ell^+\ell^-\ell'^+\ell'^-$	$u\bar{u}c\bar{c}$	$d\bar{d}s\bar{s}$
$A^1_{2,0}$	-14.0	-5.2	-1.1
$A^2_{2,0}$	-4.4	-1.0	< 1
$C_{2,1,2,-1}$	< 1	< 1	< 1
$C_{1,1,1,-1}$	-83.0	-17.8	-3.1
$C_{2,2,2,-2}$	-2.3	-1.3	< 1
$C_{1,0,1,0}$	-94.0	-14.9	-2.2
$C_{2,0,2,0}$	-1.3	< 1	< 1

(PARTIAL) SOLUTION FOR ENTANGLEMENT MEASUREMENTS

- The main conclusion is that it is not possible to correctly calculate the $C_{1,x,1,x}$ coefficients at NLO EW with the existing tomography approach
  - These can not be extracted from data
- It is anyway also possible to define a lower bound on the entanglement without these coefficients
  - it will be further away from the concurrence making it harder to measure entanglement
  - Very stable under NLO EW corrections

$$(\mathcal{C}(\rho))^2 \geq \mathcal{C}_{LB} \geq \mathcal{C}_{LB}^{L>1}$$
$$\equiv \frac{2}{9} \max \Bigg[\Big(-2 - 2 \sum_{L,M} |A^a_{L,M}|^2 + \sum_{L,M} |A^b_{L,M}|^2 + \sum_{L_a>1, L_b>1, M_a, M_b} |C_{L_a,M_a,L_b,M_b}|^2 \Big),$$
$$\Big(-2 + \sum_{L,M} |A^a_{L,M}|^2 - 2 \sum_{L,M} |A^b_{L,M}|^2 + \sum_{L_a>1, L_b>1, M_a, M_b} |C_{L_a,M_a,L_b,M_b}|^2 \Big) \Bigg].$$

	no cuts	$m(Z_2) > 30$ GeV	$85 < m(Z_1) < 95$ GeV
LO $\mathcal{C}_{LB}$	0.94	1.18	0.97
LO $\mathcal{C}_{LB}^{L>1}$	0.47	0.59	0.49
NLO $\mathcal{C}_{LB}^{L>1}$	0.49	0.55	0.48



	no cuts	$m(Z_2) > 30$ GeV	$85 < m(Z_1) < 95$ GeV
$\mathcal{I}_3$, LO			
$O_B^{(O_A,U_{\text{fix}})}$	2.600 ± 0.003	2.794 ± 0.004	2.639 ± 0.003
$O_B^{(O_A)}$	2.63	2.79	2.65
$O_B^{(O_A,C_{L>1})}$	2.63	2.79	2.65
$\mathcal{I}_3$, NLO			
$O_B^{(O_A,C_{L>1})}$	2.60	2.72	2.64

SOLUTION FOR BIV MEASUREMENTS

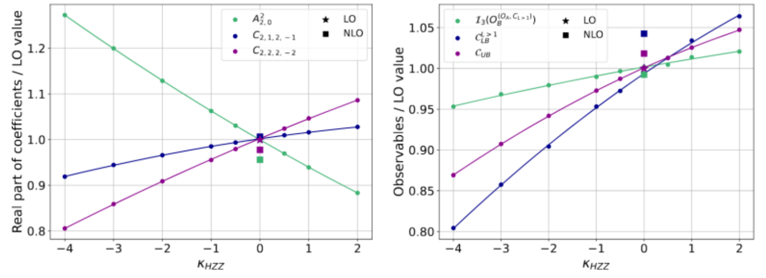
- The BIV is evaluated thanks to an operator representing the CGLMP inequality
- The operator representation is not unique, it can be rotated with unitary transformations
  - The value of the Bell's inequality is the maximum that can be obtained, considering all possible unitary rotations
  - It is possible to find a rotation that makes the result independent on the $C_{1,x,1,x}$ coefficients?
    - Yes, but this restricts the phase space of the unitary matrices
    - There is no impact on the result → still obtaining large BIV values

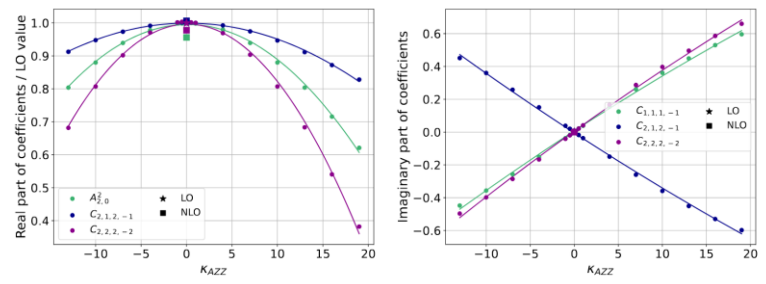
QUANTUM OBSERVABLES FOR NEW PHYSICS CONSTRAINTS

- Can the quantum observables be used as a new set of variables to constrain new physics?
- We tested using an effective approach
  - Higgs characterisation model
  - We looked at the coefficients modifying the $H \to \ell^+\ell^-\ell^+\ell^-$ vertex
- Compare the size of new physics effect to NLO corrections
  - it is essential to account for NLO effects when setting limits?
  - Avoiding the $C_{1,x,1,x}$ coefficients

$$
\begin{aligned}
\mathcal{L}_{X_0VV} = \Bigg[& c_\phi \kappa_{\rm SM} \left[\frac{1}{2} g_{HZZ} Z^\mu Z_\mu + g_{HWW} W^+_\mu W^{-\mu} \right] \\
& - \frac{1}{4} \left[c_\phi \kappa_{H\gamma\gamma} g_{H\gamma\gamma} A_{\mu\nu} A^{\mu\nu} + s_\phi \kappa_{A\gamma\gamma} g_{A\gamma\gamma} A_{\mu\nu} \widetilde{A}^{\mu\nu} \right] \\
& - \frac{1}{2} \left[c_\phi \kappa_{HZ\gamma} g_{HZ\gamma} Z_{\mu\nu} A^{\mu\nu} + s_\phi \kappa_{AZ\gamma} g_{AZ\gamma} Z_{\mu\nu} \widetilde{A}^{\mu\nu} \right] \\
& - \frac{1}{4} \frac{1}{\Lambda} \left[c_\phi \kappa_{HZZ} Z_{\mu\nu} Z^{\mu\nu} + s_\phi \kappa_{AZZ} Z_{\mu\nu} \widetilde{Z}^{\mu\nu} \right] \\
& - \frac{1}{2} \frac{1}{\Lambda} \left[c_\phi \kappa_{HWW} W^+_{\mu\nu} W^{-\mu\nu} + s_\phi \kappa_{AWW} W^+_{\mu\nu} \widetilde{W}^{-\mu\nu} \right] \\
& - \frac{1}{\Lambda} c_\phi \left[\kappa_{H\partial\gamma} Z_\nu \partial_\mu A^{\mu\nu} + \kappa_{H\partial Z} Z_\nu \partial_\mu Z^{\mu\nu} + \left(\kappa_{H\partial W} W^+_\nu \partial_\mu W^{-\mu\nu} + {\rm h.c.} \right) \right] \Bigg] X_0
\end{aligned}
$$

MODIFICATION TO THE HZZ VERTEX

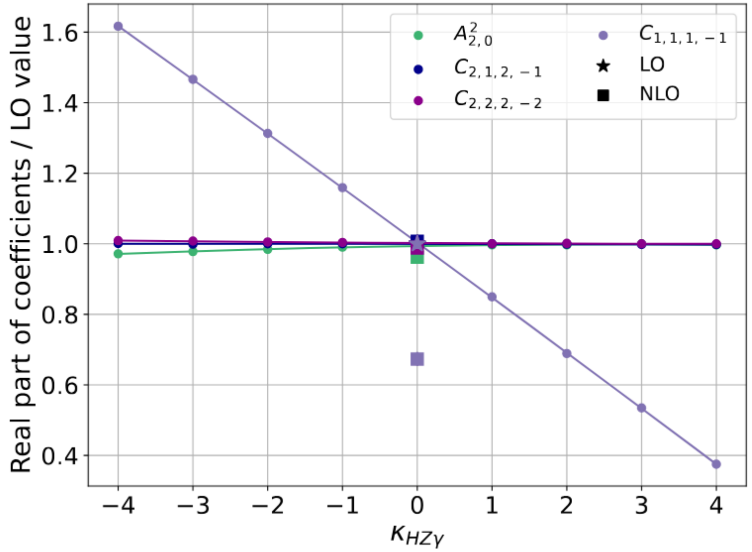


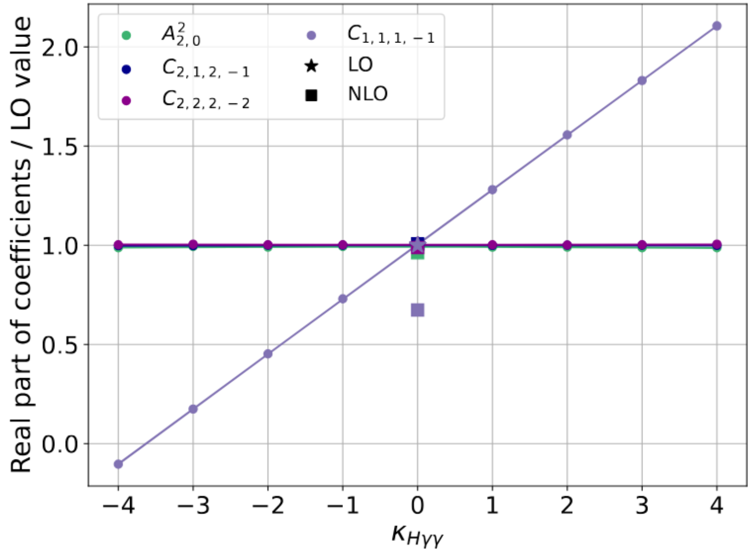


- Large effects on the coefficients $C_{2,x,2,x}$ and $A_{2,0}^{2}$ compared to small NLO effects
- Smaller impact on the quantum observables
- Linear dependence for the kHZZ coefficients, while enabling a pseudoscalar component (kAZZ):
  - Lead to a quadratic dependence in the real part of the $C_{2,x,2,x}$ coefficients
  - Linear dependence in the imaginary components
    - Identically 0 in the SM, even including NLO corrections

THE SPIN ANALYSING POWER PROBLEM POTENTIAL

- We investigated also other couplings that introduce $H \to Z\gamma$ and $H \to \gamma\gamma$ diagrams in the $H \to \ell^+\ell^-\ell^+\ell^-$ process.
- The reconstruction of the observables is done in the same way as LO and NLO, exploiting the 4 final state leptons.
- Given that the reconstructed $C_{1,x,1,x}$ coefficients are extremely sensitive to the spin analysing power choice, they become extremely sensitive to these couplings, mixing up processes.
  - The interpretation of $C_{1,x,1,x}$ in terms of the spin density matrix is not possible BUT it is still a well defined observable.





CONCLUSIONS

Paper currently on arXiv:2504.03841
And submitted to JHEP

- The application of quantum information concepts to fundamental particles created at colliders is a constantly growing field
- The $H \to VV^*$ final state is a very promising final state for these studies, especially the 4-charged leptons final state.
  - The two V spins are highly correlated
  - The system is fully reconstructable
- The NLO EW corrections to the elements used to estimate the spin density matrix are very large and un-even
  - Can also reach ~90% on some coefficients
  - The main reason for this large effects seems to derive from the inability to correctly estimate the spin analysing power at NLO EW and the very small value of this quantity for $Z \to \ell^+\ell^-$
- We have introduced new observables that allow to quantify entanglement and Bell's non-locality without the need to reconstruct the coefficients unstable under NLO EW corrections
- The spin density matrix coefficients also show sensitivity to the presence of new physics, parametrised using EFT, but the inclusion of NLO EW corrections seems mandatory for a correct interpretation.