ggxy – NLO QCD corrections to loop induced gg initiated processes

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Particle Physics Phenomenology after the Higgs Discovery

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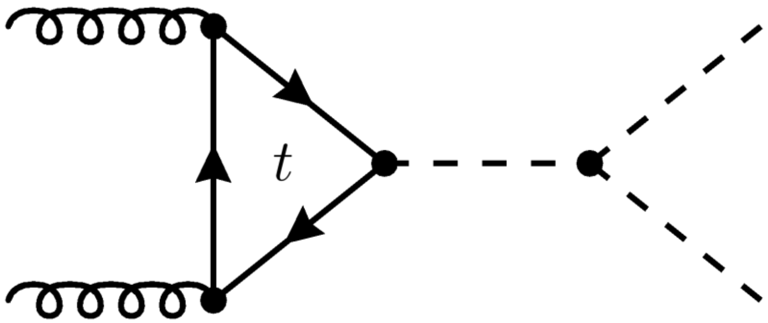
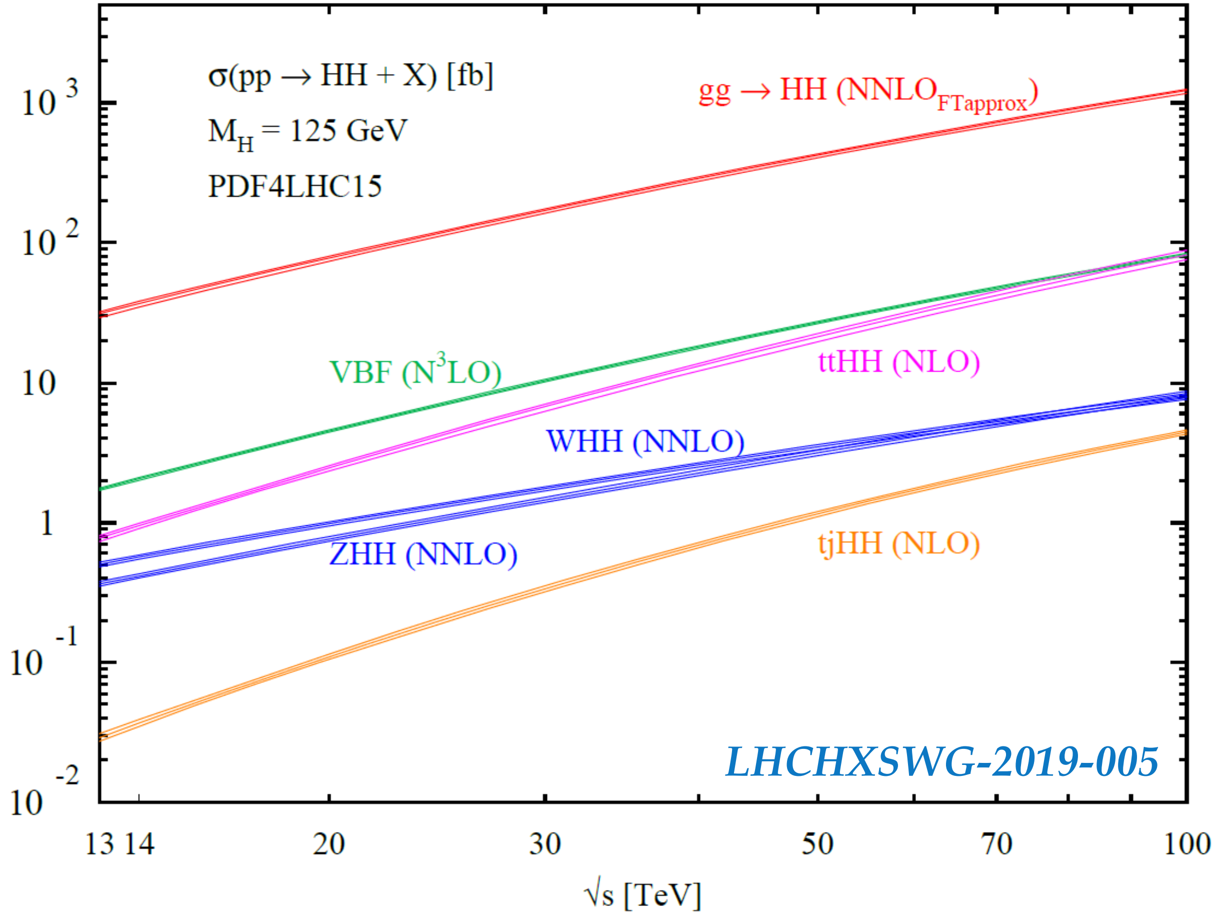
Motivation - *HH* production

- Standard Model Higgs potential:

$$V(H) = \frac{m_H^2}{2} H^2 + \lambda_3 v H^3 + \frac{\lambda_4}{4} H^4, \quad \text{with} \quad \lambda_3^{\rm SM} = \lambda_4^{\rm SM} = m_H^2/(2v^2)$$

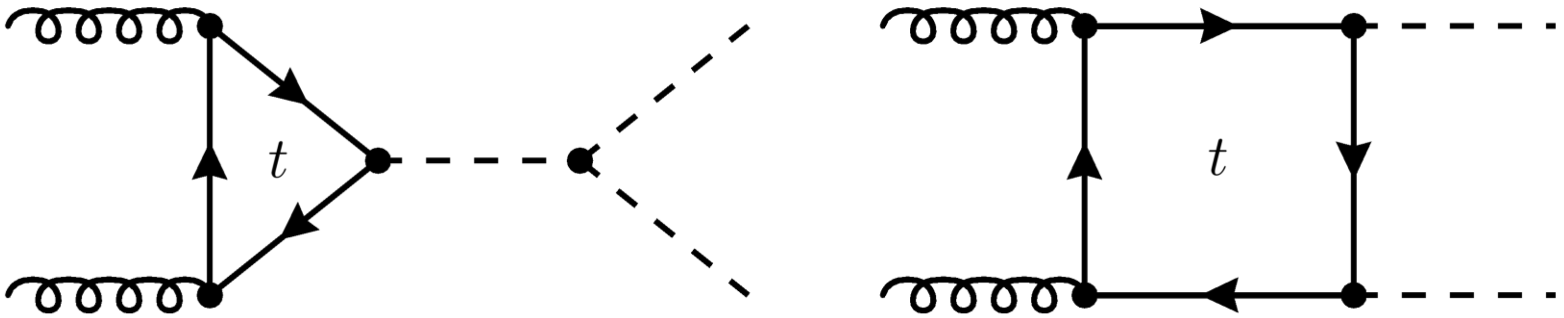
- Often parametrized by: $\kappa_\lambda = \lambda_3/\lambda_3^{\rm SM}$
- Current limits:
  - $-1.2 < \kappa_\lambda < 7.2$ *ATLAS '24*
  - $-1.39 < \kappa_\lambda < 7.02$ *CMS '24*
- HH production domianted by gluon fusion





HH production in gluon fusion

- Exact LO (1-loop) results known for a long time: *Glover, van der Bij '88; Plehn, Spira, Zerwas '98*



- Destructive interference between triangle and box contributions

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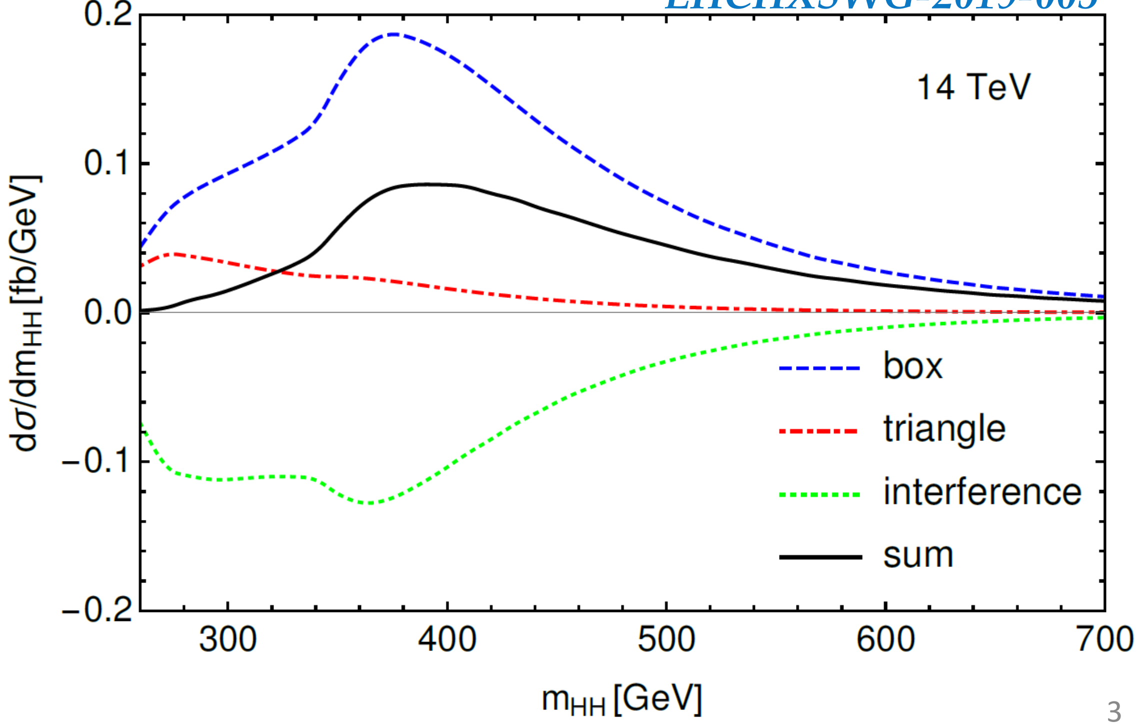


Higher order QCD corrections to gg → HH

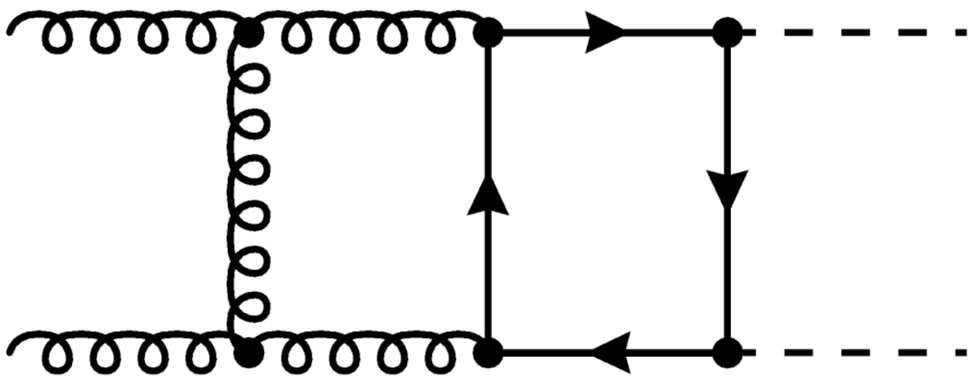
- **NLO QCD**
  - large-m_t expansion — *Dawson, Dittmaier, Spira '98; Grigo, Hoff, Melnikov, Steinhauser '13*
  - numerical integration — *Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke '16; Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher '19*
  - large-m_t + threshold exp. with Padé — *Gröber, Maier, Rauh '17*
  - high-energy expansion — *Davies, Mishima, Steinhauser, Wellman '18 '19*
  - small-p_T expansion — *Bonciani, Degrassi, Giardino, Gröber '18*
  - small-t expansion — *Davies, Mishima, Schönwald, Steinhauser '23*
  - high-energy + small-p_T — *Bellafronte, Degrassi, Giardino,Gröber, Vitti '22; Bagnaschi, Degrassi, Gröber '23*
- **NNLO QCD**
  - large-m_t expansion — *de Florian, Mazzitelli '13; Grigo, Melnikov, Steinhauser '14; Grigo, Hoff Steinhauser '18; Davies, Herren, Mishima, Steinhauser '19 '21*
  - FTapprox — *Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli '18*
  - small-t expansion $n_l n_h$ and n_h (LC) — *Davies, Schönwald, Steinhauser '23 '25*
  - 1PR reducible — *Davies, Schönwald, Steinhauser, Vitti '24*
- **NNNLO QCD:** HTL: *Spira '16; Gerlach, Herren, Steinhauser '18; Chen, Li, Shao, Wang '19; Ajjath, Shao '22*

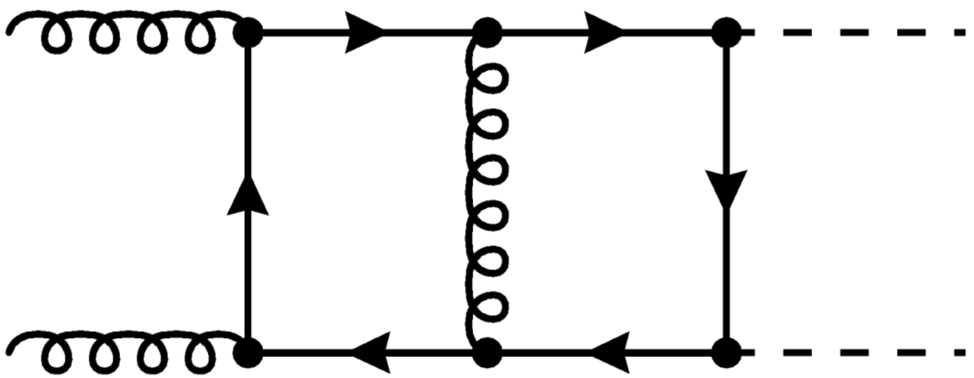
Small-t expansion

Davies, Mishima, Schönwald, Steinhauser '23

- Consider hierachy of scales: $s, m_t^2 \gg |t|, m_H^2$
  - Form factors written in terms of Feynman integrals: $I(m_H, m_t, s, t, \epsilon)$
  - Taylor expand in $m_H^2 \to 0$: $I(m_H, \ldots) = I(0, \ldots) + m_H^2 I'(0, \ldots) + \ldots$
  - IBP reduction to master integrals: $J(0, m_t, s, t, \epsilon)$
  - Expand master integrals around $t \to 0$:

$$J(0, m_t, s, t, \epsilon) = \sum_{i,j} c_{ij}(s, m_t^2) \epsilon^i t^j$$





High-energy expansion

Davies, Mishima, Steinhauser, Wellman '18, '19

- Consider hierachy of scales: $s, |t| \gg m_t^2 \gg m_H^2$
  - Form factors written in terms of Feynman integrals: $I(m_H, m_t, s, t, \epsilon)$
  - Taylor expand in $m_H^2 \to 0$: $I(m_H, \ldots) = I(0, \ldots) + m_H^2 I'(0, \ldots) + \ldots$
  - IBP reduction to master integrals: $J(0, m_t, s, t, \epsilon)$
  - Expand master integrals around $m_t^2 \to 0$:

$$J(0, m_t, s, t, \epsilon) = \sum_{i,j,k} c_{ijk}(s,t)\epsilon^i (m_t)^j \log(m_t)^k$$

- Improve convergence with Padé approximation

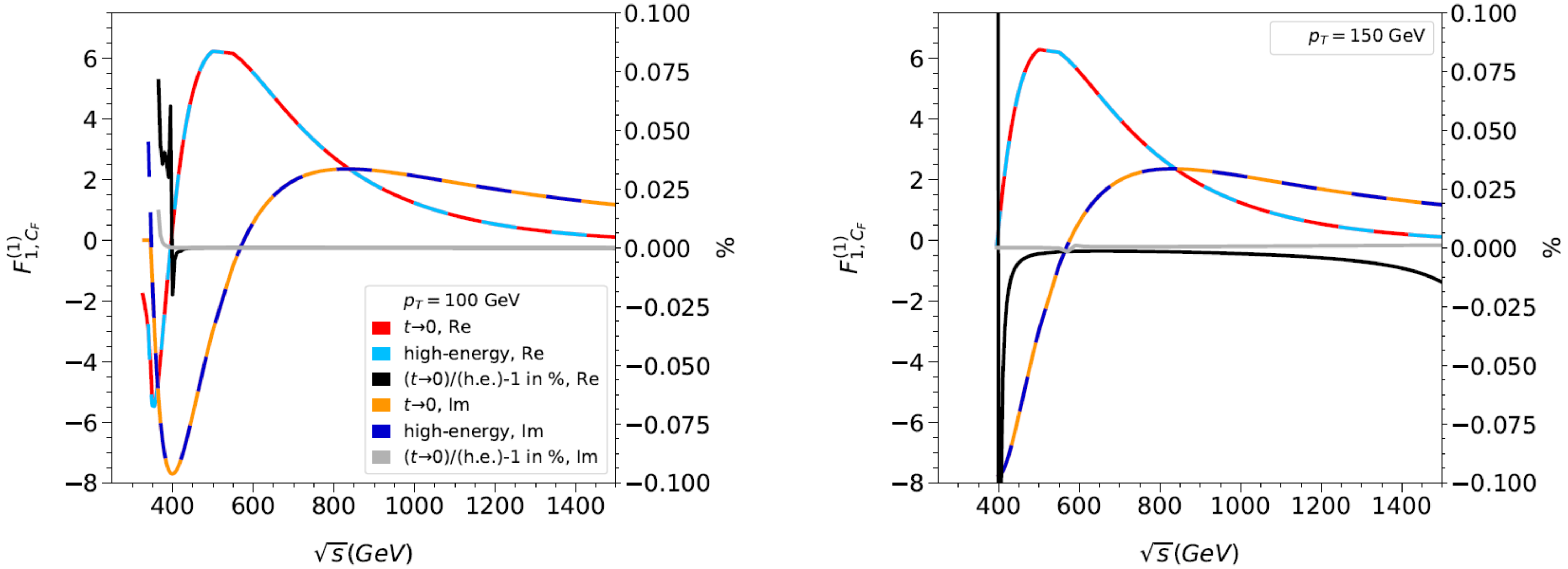
Davies, Mishima, Steinhauser, Wellman '20

$$\sum_{k=0}^{N} c_k (m_t^2)^k = \frac{a_0 + a_1(m_t^2) + \ldots + a_r(m_t^2)^r}{1 + b_1(m_t^2) + \ldots + b_s(m_t^2)^s} = [r/s](m_t^2), \qquad r+s = N$$

- Region of convergence increased from $p_T \gtrsim 500$ GeV to $p_T \gtrsim 150$ GeV

Combination of small-t and high-energy expansions

Davies, Mishima, Schönwald, Steinhauser '23



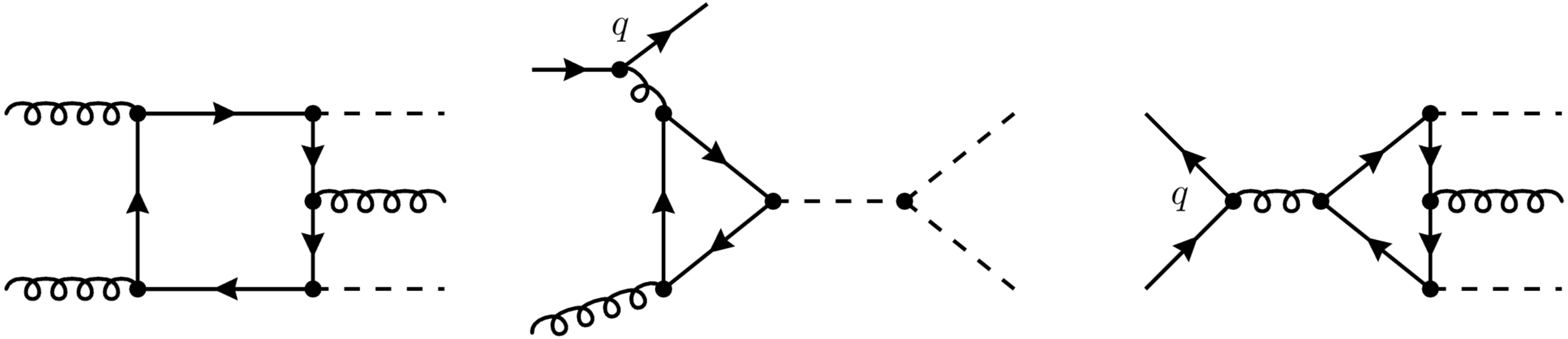
- Coverage of whole space with small-t and high-energy expansion with Padé

Real corrections

- New partonic processes: $gg \to gHH$, $qg \to qHH$, $q\bar{q} \to gHH$

- IR singularities handled with Catani-Seymour subtraction scheme *Catani, Seymour '97*

- Caluclation of one-loop matrix elements with Recola *(Actis, Denner, Hofer, Lang, Scharf, Uccirati '17)* and Collier *(Denner, Dittmaier, Hofer '17)*
  - Implemented alternative reduction to scalar integrals for exceptional phase-space points with CutTools *(Ossola, Papadopoulos, Pittau '09)* and OneLOop *(van Hameren '11)*



ggxy

gitlab.com/ggxy/ggxy-release

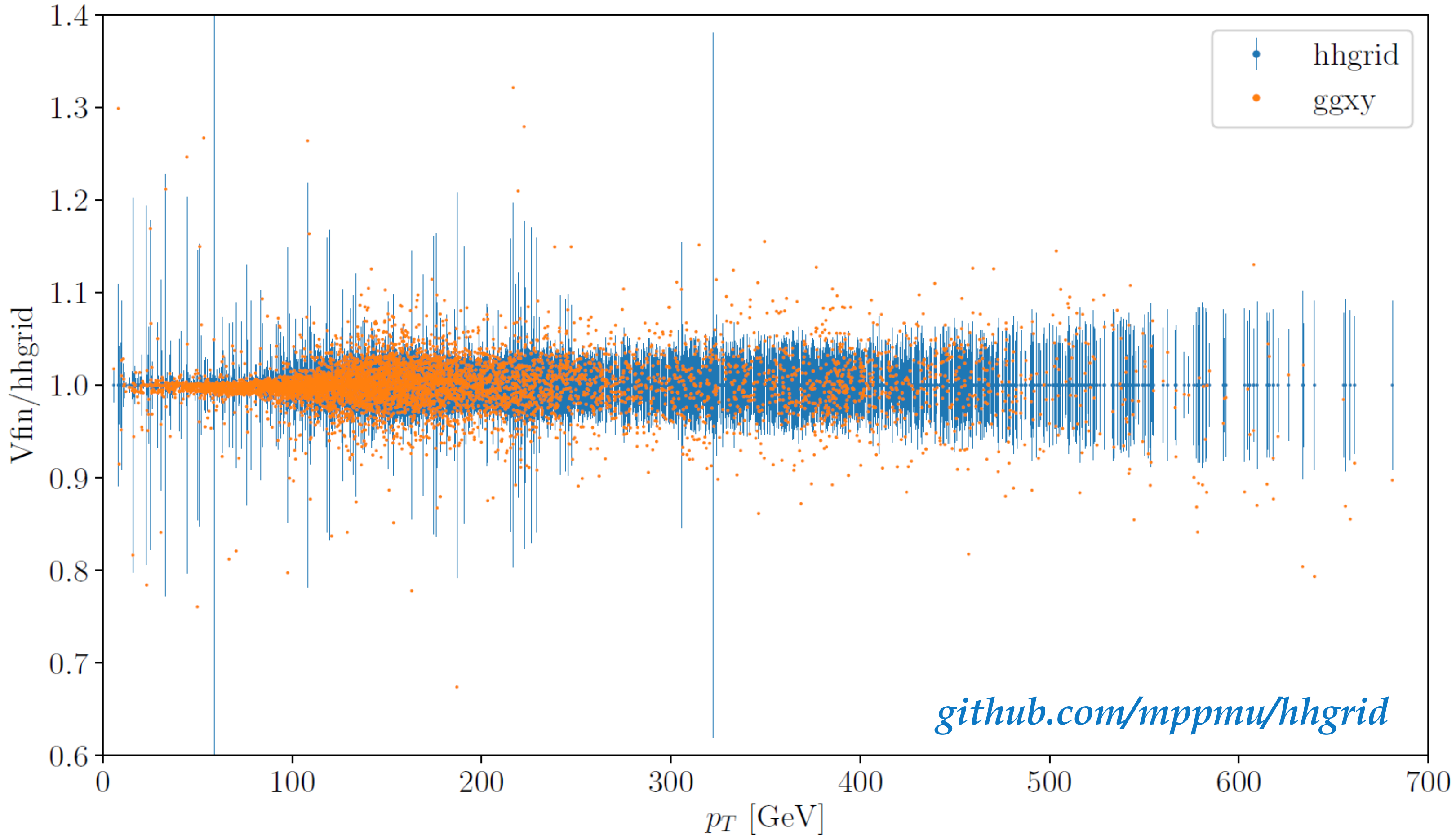
- Public C++ library for the computation of LO and NLO amplitudes and cross sections for gluon initiated top-quark loop induced processes
- In first version contains all necessary functions for $gg \to HH$ at NLO QCD
- Analytical expansions allow to modify m_t, m_H and top-quark mass renormalization scheme (OS, $\overline{\mathrm{MS}}$)
- Supports variation of $\kappa_\lambda = \lambda_3/\lambda_3^{\mathrm{SM}}$

Installation

- External dependencies: Boost, CMake, Eigen
- Specify path to LHAPDF installation in build script for hadronic cross sections
- Following libraries are directly installed with and linked to ggxy:
  - Collier *(Denner, Dittmaier, Hofer '17)*
  - CutTools *(Ossola, Papadopoulos, Pittau '09)*
  - OneLOop *(van Hameren '11)*
  - Recola *(Actis, Denner, Hofer, Lang, Scharf, Uccirati '17)*
  - avhlib/Kaleu *(van Hameren '10)*
  - CRunDec *(Herren, Steinhauser '18)*
  - lievaluate *(Frellesvig, Tommasini, Wever '16)*

Example: check-hhgrid.cpp

- Comparison of 2-loop finite reminder $\mathcal{V}_{\text{fin}}$ with hhgrid
  - Interpolation grid of 6320 phase-space points, combination of numerical evaluation with pySecDec and (lower order) high-energy expansion
- ggxy reproduces results within given uncertainties in less than 10 seconds on a single core
  - Sufficiently fast for a direct use in MC tools and more flexible



Example: nlo-gghh.cpp

Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke '16

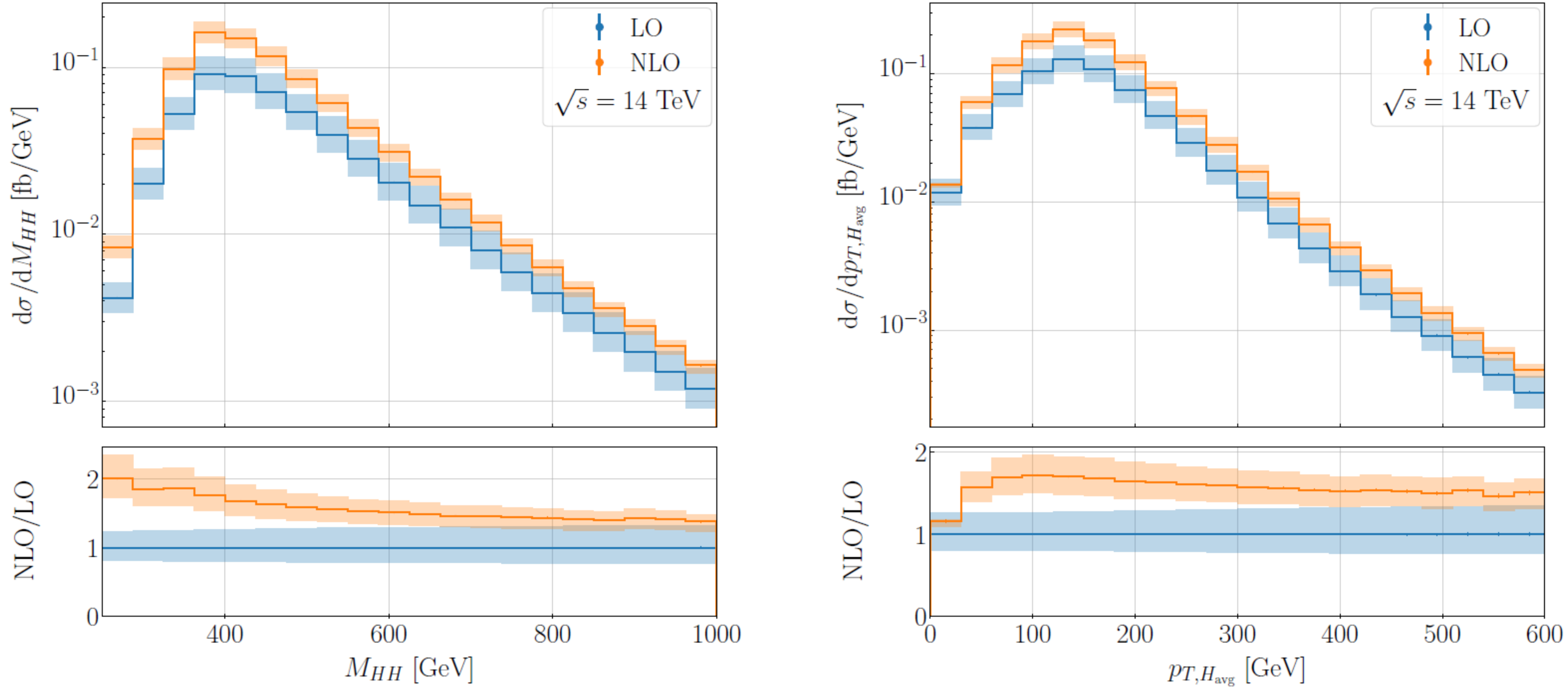
$\sqrt{s}$		ggxy	Ref.
14 TeV	σ^{LO} [fb]	$19.848(4)^{+27.6\%}_{-20.5\%}$	$19.85^{+27.6\%}_{-20.5\%}$
	σ^{NLO} [fb]	$32.92(2)^{+13.6\%}_{-12.6\%}$	$32.91^{+13.6\%}_{-12.6\%}$
100 TeV	σ^{LO} [fb]	$731.2(2)^{+20.9\%}_{-15.9\%}$	$731.3^{+20.9\%}_{-15.9\%}$
	σ^{NLO} [fb]	$1150(1)^{+10.8\%}_{-10.0\%}$	$1149^{+10.8\%}_{-10.0\%}$

Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher '20

$\sqrt{s}$		ggxy	Ref.
13 TeV	σ^{NLO} [fb]	27.72(2)	27.73(7)
14 TeV	σ^{NLO} [fb]	32.79(2)	32.81(7)

- Reproduces LO/NLO hadronic cross sections from literature together with scale uncertainties from 7-point scale variation
- Fast runtime: 30 minutes on a single core for a precision of 0.1% – 0.2%

Example: nlo-gghh.cpp



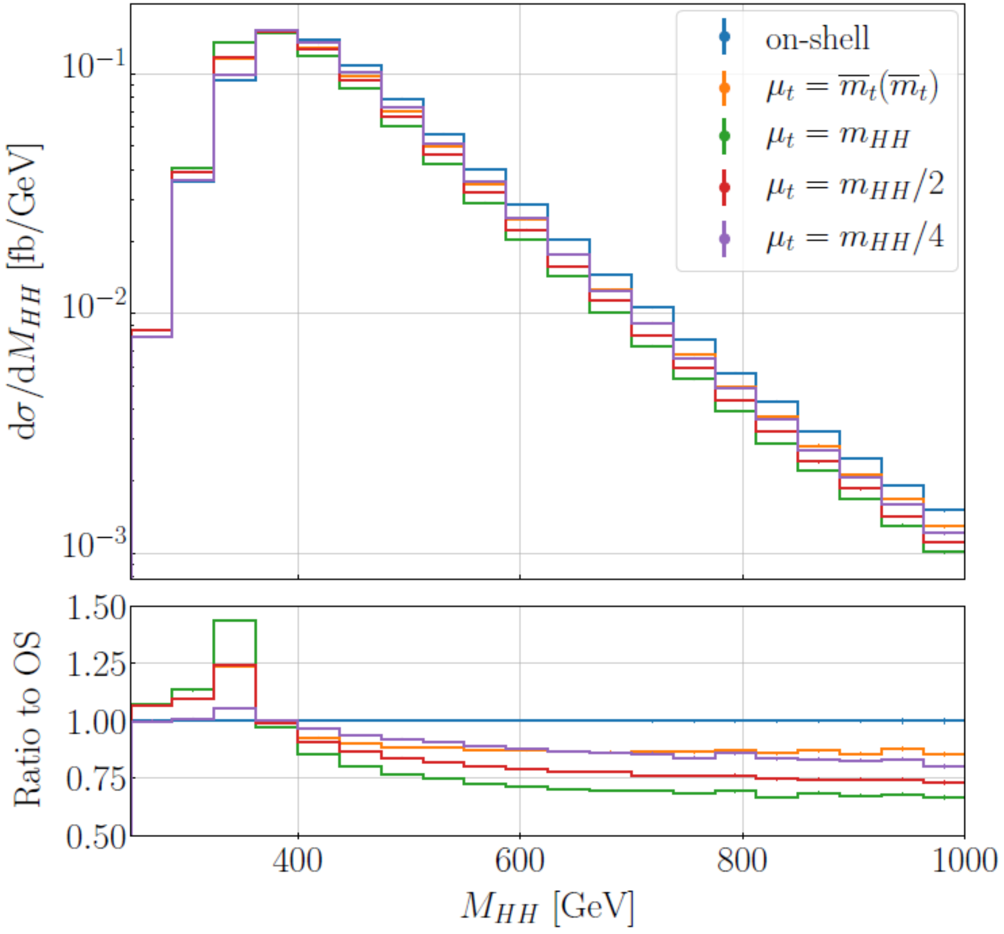
- Differential cross sections with scale uncertainties also produced by example program
- Runtime depends highly on requested precision
  - Naive parallelization by running multiple seeds in parallel and combine results afterwards

Example: nlo-gghh.cpp

- Straightforward to modify κ_λ and/or to use $\overline{\text{MS}}$ top-quark mass renormalization scheme

Bagnaschi, Degrassi, Gröber '23

κ_λ	Top-mass scheme	$\sigma^{\text{NLO}}_{\text{ggxy}}$ [fb]	$\sigma^{\text{NLO}}_{\text{Ref.}}$ [fb]
-0.6	on-shell	$100.34(6)^{+15.7\%}_{-13.6\%}$	$100.77^{+15.8\%}_{-13.7\%}$
0.0	on-shell	$68.08(4)^{+15.1\%}_{-13.4\%}$	$68.38^{+15.1\%}_{-13.4\%}$
1.0	on-shell	$30.83(2)^{+13.8\%}_{-12.7\%}$	$30.93^{+13.7\%}_{-12.7\%}$
1.0	$\overline{\text{MS}}$, $\mu_t = \overline{m}_t(\overline{m}_t)$	$29.78(2)^{+14.3\%}_{-13.0\%}$	$29.78^{+14.3\%}_{-13.0\%}$
1.0	$\overline{\text{MS}}$, $\mu_t = M_{HH}/2$	$28.79(2)^{+15.3\%}_{-13.5\%}$	$28.90^{+15.2\%}_{-13.5\%}$
2.4	on-shell	$13.369(9)^{+14.7\%}_{-13.2\%}$	$13.41^{+14.8\%}_{-13.1\%}$
6.6	on-shell	$203.4(1)^{+19.0\%}_{-15.1\%}$	$203.91^{+19.0\%}_{-15.2\%}$



- Overall good agreement

- Large dependence on input scheme of top-quark mass
Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira, Streicher '20

Conclusion

- Presented new public C++ library ggxy:
  - All relevant functions implemented for $gg \to HH$ at NLO QCD
  - 2-loop amplitudes calculated with analytical expansions
  - Input parameters (m_t, m_H, κ_λ) or top-quark mass renormalization scheme (OS, $\overline{\text{MS}}$) can be freely chosen
- Several cross-checks with calculations in the literature
- Can be used directly to compute hadronic cross section or can be interfaced to other MC tools
- Fast runtime: 30 minutes for per mille precision

Outlook

- Interface to POWHEG ***(Alioli, Nason, Oleari, Re '10)***
- Straightforward to include other processes:
  - $gg \to ZH$
  - $gg \to ZZ$ ($gg \to Z\gamma$ and $gg \to \gamma\gamma$)