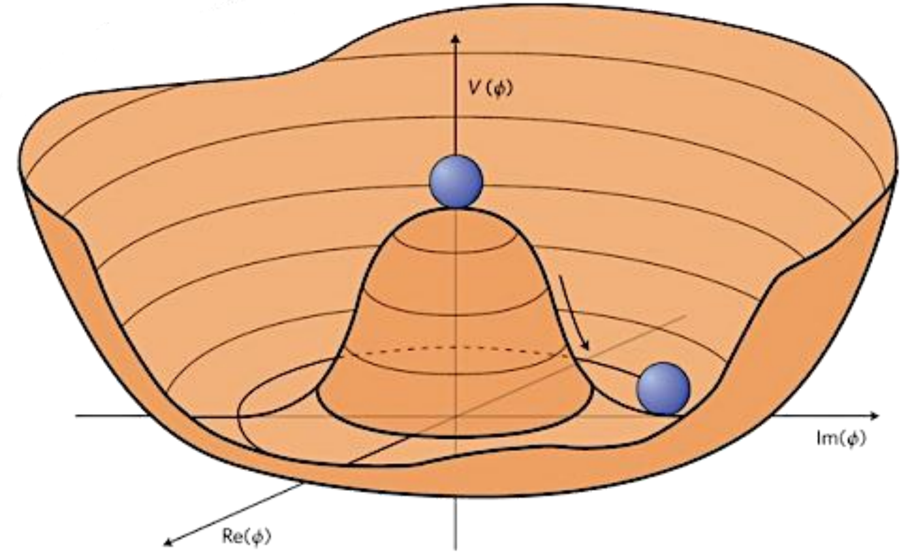
Laboratoire des 2 Infinis-Toulouse

Development of the Matrix Element Method at Next-to-Leading-Order (NLO) for the measurement of the Higgs tri-linear coupling λ_{3H} in $gg \to HH \to b\bar{b}\gamma\gamma$ channel

Matthias Tartarin
Jan Stark

Introduction

Higgs self-coupling λ_{3H}



Standard Model (assumption):

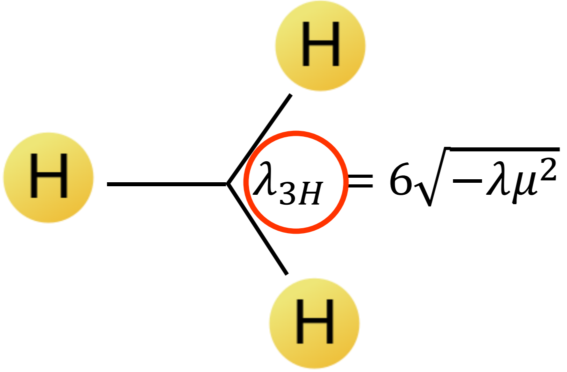
$$V(\phi^2) = \mu^2\phi^2 + \lambda\phi^4$$

(for $\mu^2 < 0$ et $\lambda > 0$)

Mathematical expression of the **Higgs field (doublet) ϕ** potential

$$V(h) = m_H{}^2\frac{h^2}{2} + \lambda_{3H}\frac{h^3}{3!} + \lambda_{4H}\frac{h^4}{4!} - \frac{v^4\lambda}{4}$$

Mathematical expression of the **Higgs boson h** potential after expansion around ground state.



Goal: Measure this λ_{3H} value experimentally, to confront with theoretical expectations.

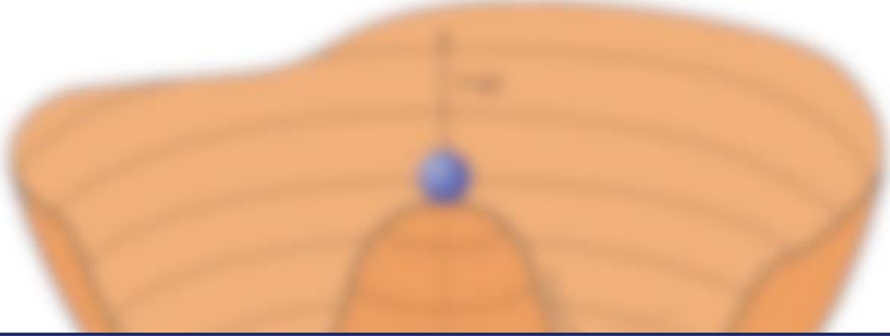
$-1.2 < \lambda_{obs}/\lambda_{SM} < 7.2$ [ATLAS '24]

$-1.4 < \lambda_{obs}/\lambda_{SM} < 7.0$ [CMS '24]

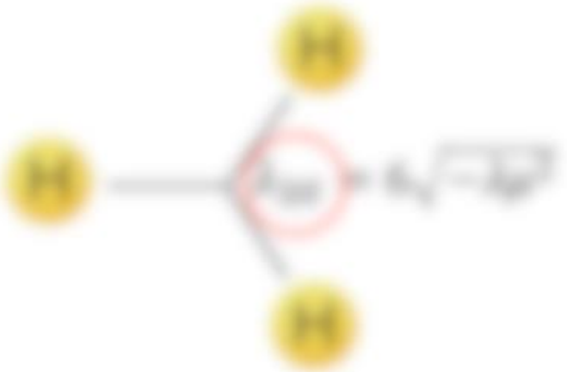
-1.7 $< \lambda_{obs}/\lambda_{SM} <$ 6.6 [ATLAS $HH \rightarrow b\bar{b}\gamma\gamma$ '25]

Introduction

Higgs self-coupling λ_{3H}



How to obtain the best possible statistical uncertainty on λ_{3H} (or $\kappa_\lambda = \frac{\lambda_{3H}}{\lambda_{3H,SM}}$)?



Introduction

Matrix Element Method [MEM]

By definition: [the (MEM)] is a statistically optimal multivariate method that maximizes the utilization of both the experimental and theoretical information available to an analysis. With **minimal cuts on the data**.

It computes a probability (called "the Likelihood" $\mathcal{L}$(**h**,**x**)), to observe an event **x** under the hypothesis **h.**

Likelihood mathematical expression (for particle physics*):

Observed events (LO or NLO)

Total cross section

Transfer function

$$\mathcal{L}^{P}_{\text{process}}(\mathbf{h} \mid \mathbf{x}^i)d\mathbf{x}^i = \frac{(2\pi)^4}{\sigma_P^{obs}(pp \to F).s} \int_y \int_{q_1,q_2} \sum_{a_1,a_2} f_{a_1}(q_1) f_{a_2}(q_2). \frac{|\mathcal{M}_{\mathcal{P}}(a_1 a_2 \to \mathbf{y};\mathbf{h})|^2}{q_1 q_2} \cdot W(\mathbf{x}^i,\mathbf{y})\,\delta\Big(a_1 + a_2 - \sum_j^n y_j\Big) \mathrm{d}q_1 \mathrm{d}q_2 \mathrm{d}y^{4n}$$

Parton density functions

Matrix element (LO or NLO)

Legend :
- (blue line) : the experimental inputs
- (red line) : the theoretical inputs

*in particle physics at colliders, we have the chance to have the ingredients that are necessary to compute the likelihood from first principles

Introduction

Matrix Element Method [MEM]

Likelihood mathematical expression (for particle physics*):

$$\mathcal{L}^P_{\text{process}}(\mathbf{h} \mid \mathbf{x}^i)d\mathbf{x}^i = \frac{(2\pi)^4}{\sigma_P^{obs}(pp \to F).s} \int_y \int_{q_1,q_2} \sum_{a_1,a_2} f_{a_1}(q_1) f_{a_2}(q_2) . \frac{|\mathcal{M}_{\mathcal{P}}(a_1 a_2 \to \mathbf{y};\mathbf{h})|^2}{q_1 q_2} \cdot W(\mathbf{x}^i, \mathbf{y}) \delta\left(a_1 + a_2 - \sum_j^n y_j\right) \mathrm{d}q_1 \mathrm{d}q_2 \mathrm{d}y^{4n}$$

Observed events (LO or NLO)

Total cross section

Transfer function

Parton density functions

Matrix element (LO or NLO)

Legend :

- ——— : the experimental inputs
- ——— : the theoretical inputs

Given a fixed reconstructed event x^i, we integrate over all the possible configurations of events y that could be measured as x^i by the detector.

➔ Idea: To search for $\boldsymbol{h}$ that maximizes $\mathcal{L}$

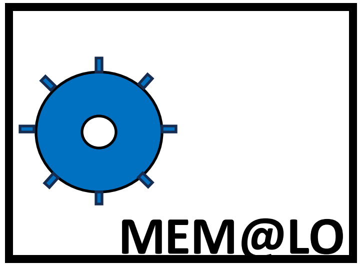
➔ hypothesis h can be: λ_{3H} value

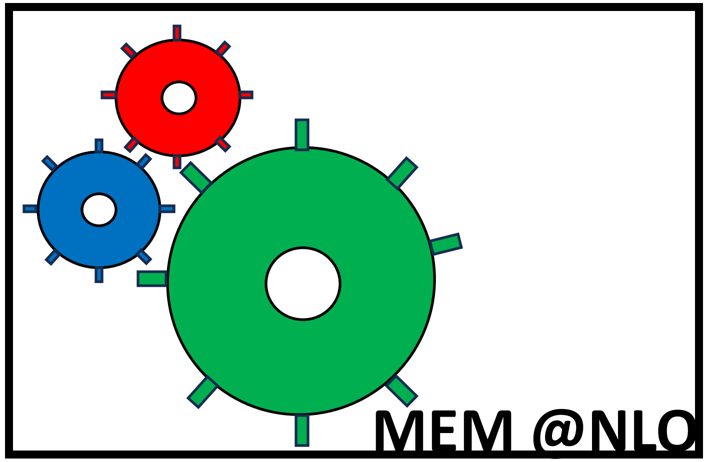
Introduction

Distinction between ME(M) analysis and Events order

- <u>**Matrix Element Method** [MEM] **analysis**</u> :

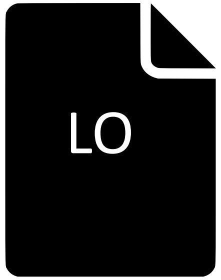
The order of the Matrix Element $\mathcal{M}_p(a_1 a_2 \to y; h)$ **inside the integral**, using Feynman rules.





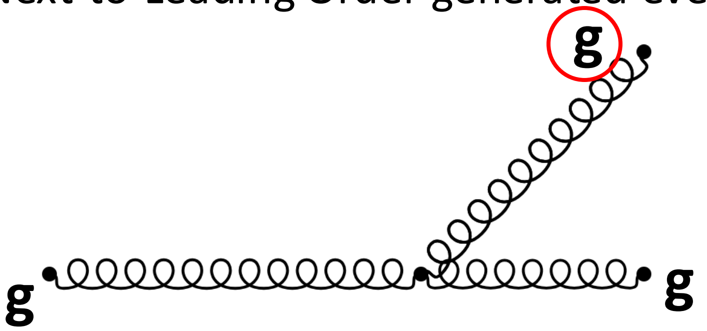
- <u>**Monte Carlo Events**</u>:

Can be **generated** at different orders (using softwares like MadGraph or POWHEG-BOX-V2).



Leading-Order generated events

Next-to-Leading Order generated events



(Extra real radiation in the final particles)

Introduction

Event display (ATLAS): Final state

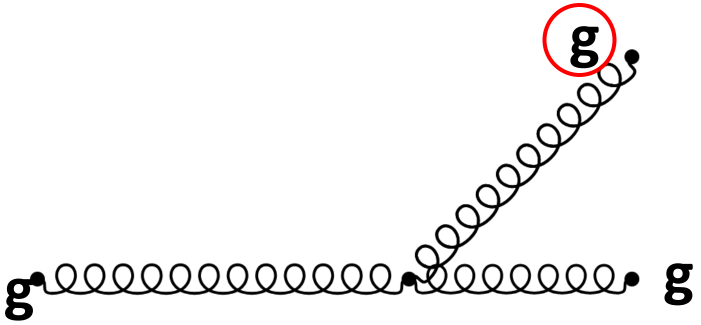
- Reco particles in x^i

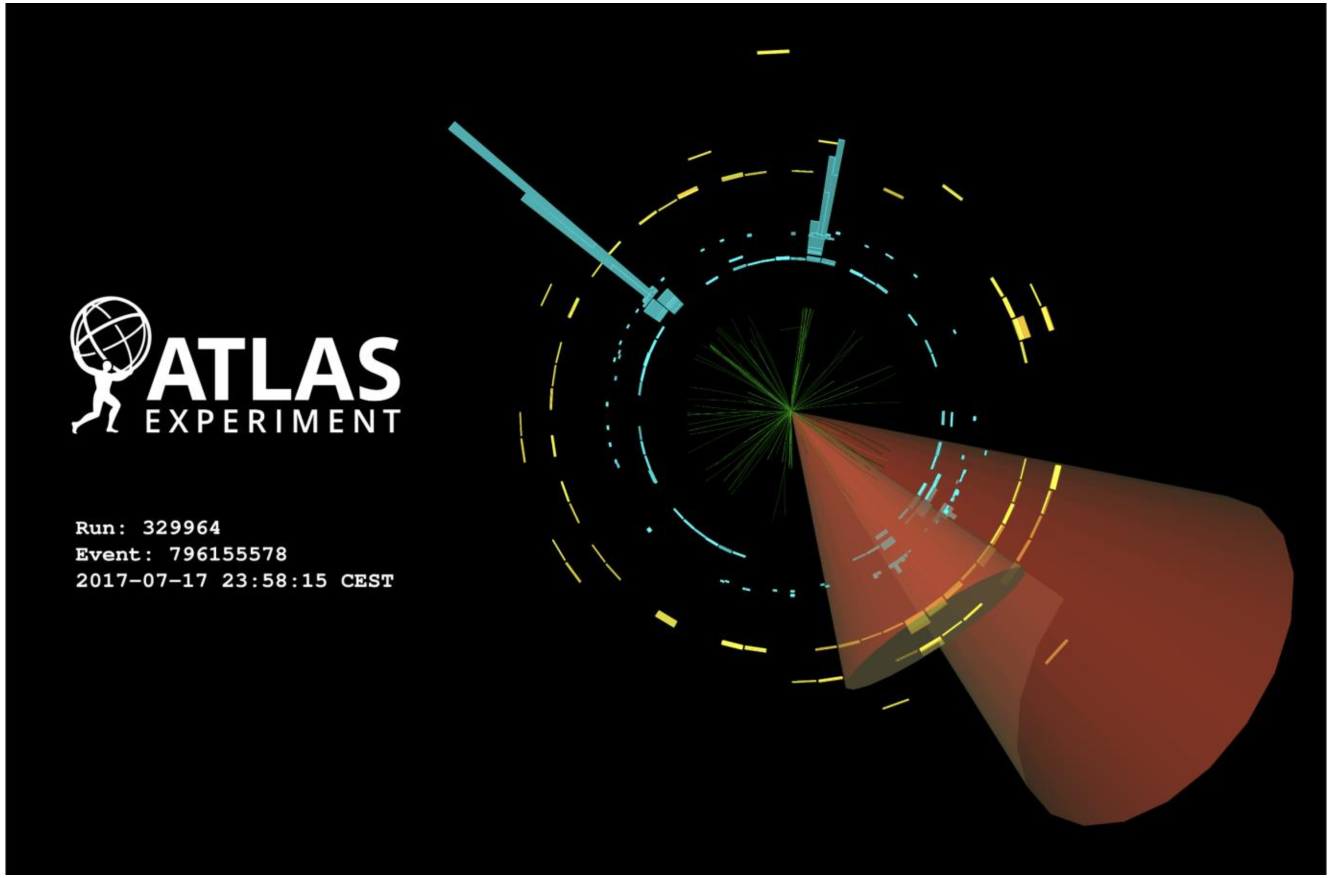
(for both MEM@LO and MEM@NLO)

4 Higgs decay daughters candidates **only**:

$b\bar{b}\gamma\gamma$

- Extra (real) radiation candidates will be **fully integrated over their (3 extra) degrees of freedom**.





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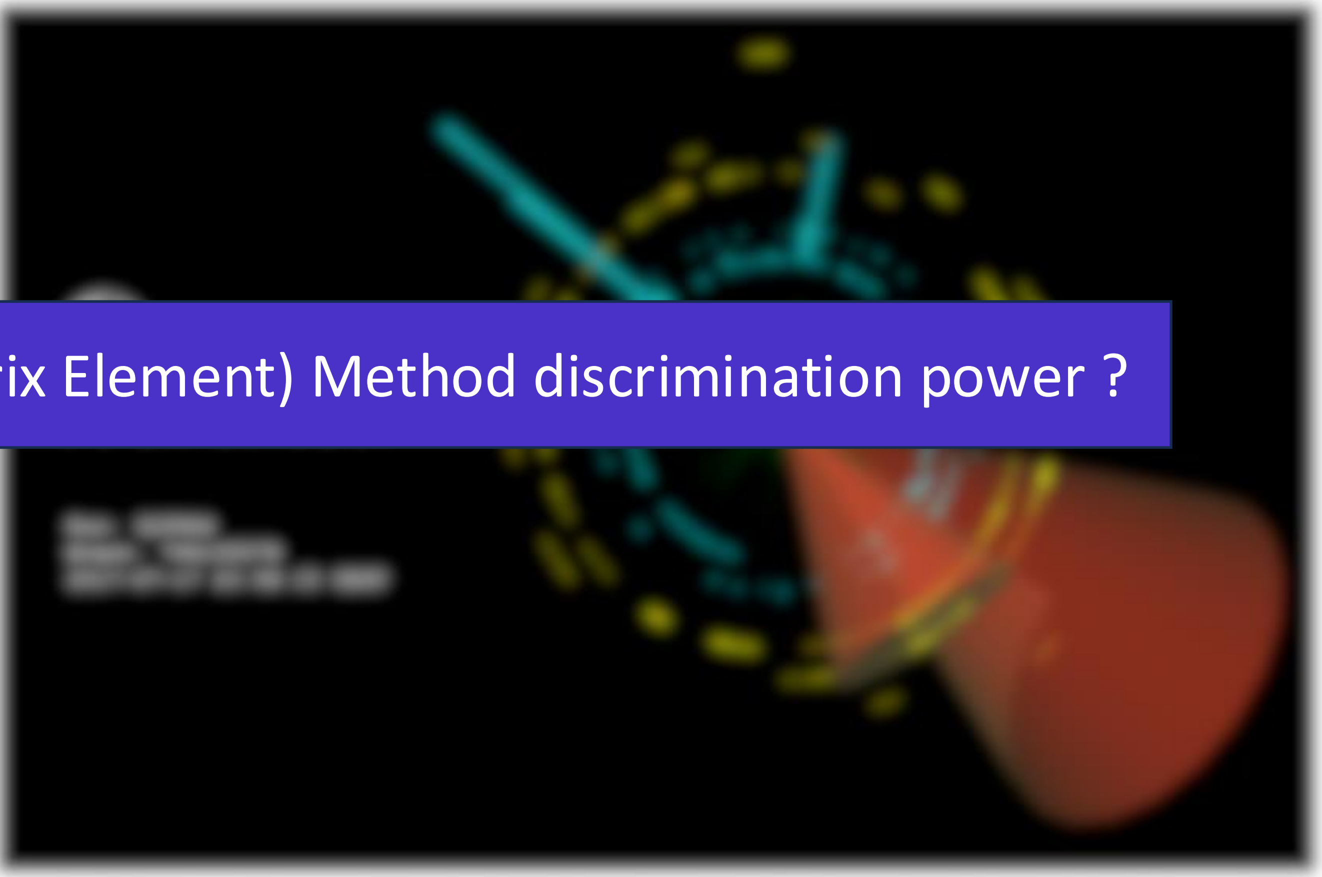
Introduction

Event display (ATLAS): Final state

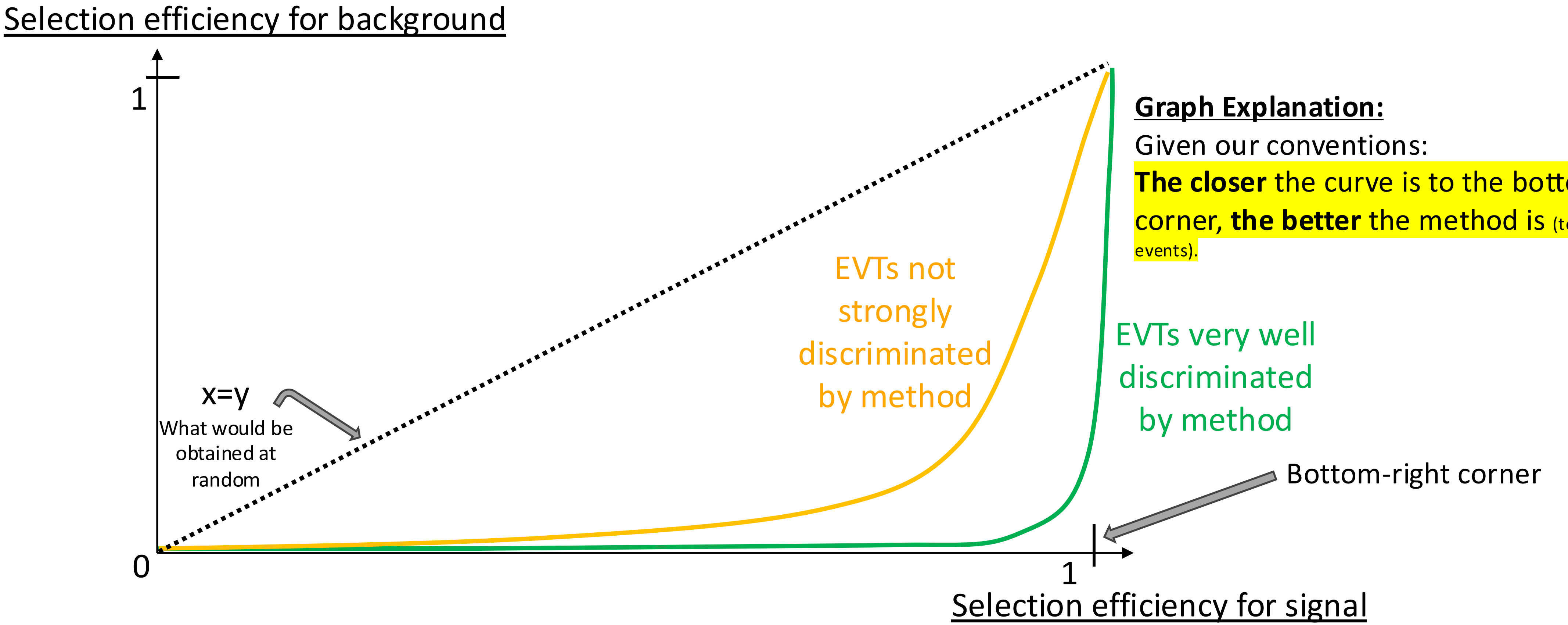


How to quantify the (Matrix Element) Method discrimination power ?

Introduction

Rejection/ROC curves

- ROC/Rejection curve : graphical way to show a method's discrimination power. For us, by using **ratios of** $\mathcal{L}_{process}$ (signal over background).



Graph Explanation:

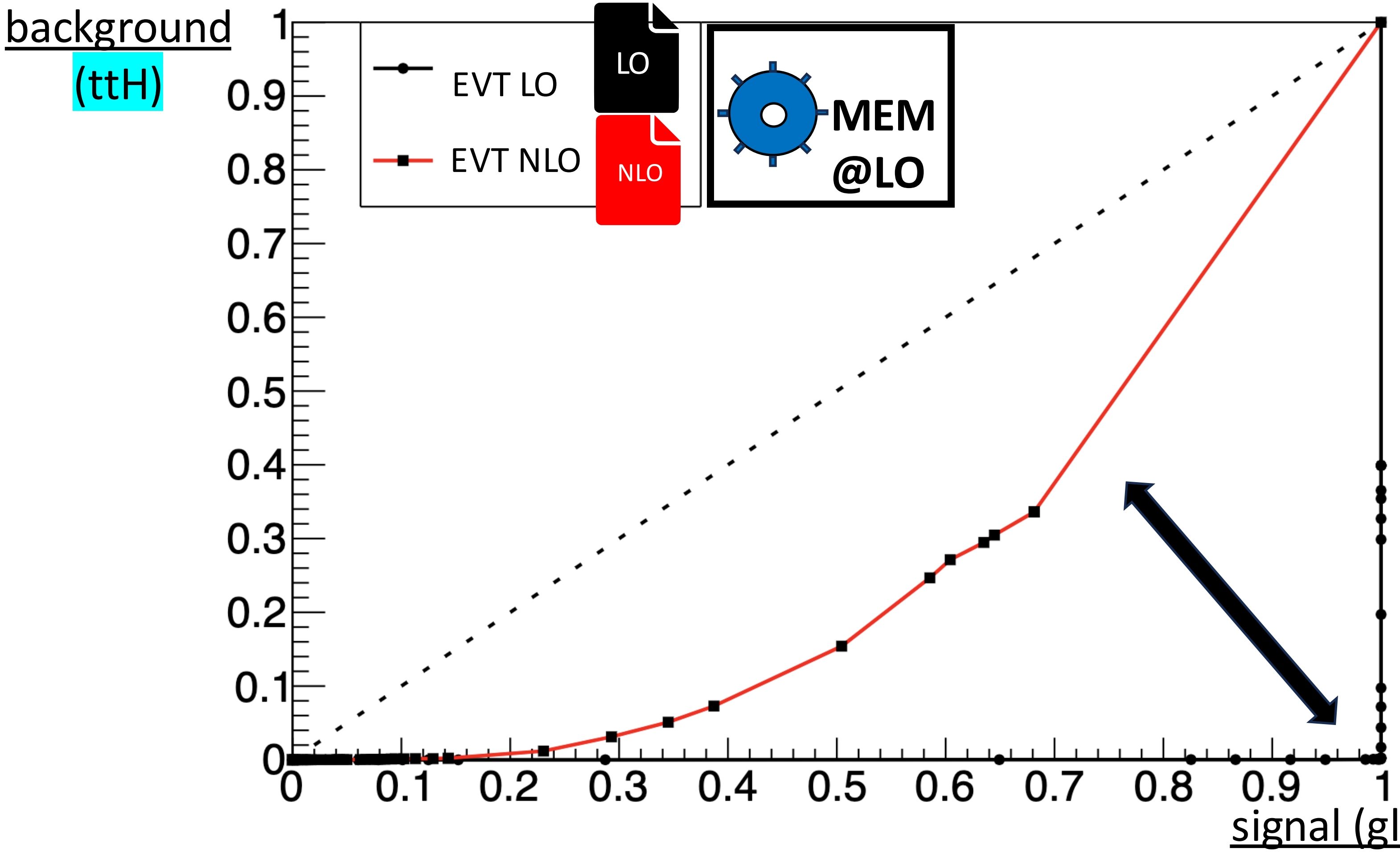
Given our conventions:

The closer the curve is to the bottom-right corner, **the better** the method is (to disciminate events).

Motivation

Using NLO events on MEM@LO

- Using MEM@LO to analyse events generated at LO and NLO:



Important results:

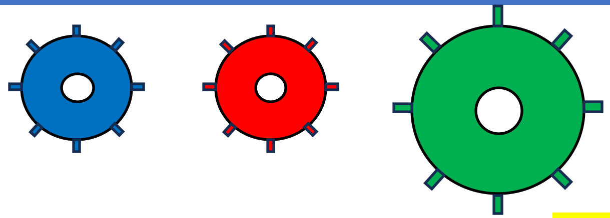
1) Very strong discrimination on LO events

2) Less in discrimination power on NLO MC events

Results

Constructing the MEM at NLO

Matrix Element at NLO = LO + Virtual + Real



Main challenges:

1. Virtual and Real contributions (theoretically challenging).
   Already done by Heinrich, Jones, Kerner et al:1703.09252, implemented in the POWHEG-BOX-V2/ggHH for gluon fusion at NLO. **But:** No direct interface to access them for given PS points.

2. In the POWHEG-BOX-V2/ggHH implementation, the Higgs boson decays are generated using Pythia. But we need to add Higgs decay for the full matrix element. $\underbrace{gg \rightarrow HH}_{\text{ggHH}} \rightarrow b\bar{b}\gamma\gamma$

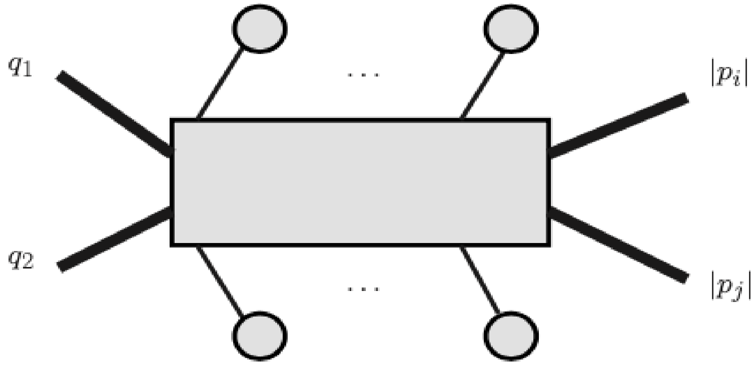
3. Different phase spaces: [LO and Virtual] share the same; But [Real] has an extra particle (i.e 4 more d.o.f)
   This is very important for the dimension of the MEM integral: $dq_1 dq_2 dy^{4n}$ VS $dq_1 dq_2 dy^{4(n+1)}$ (due to extra real radiation)

Results

New formalism: Block « N »

- Most MEM are developed at LO only.
- The MEM can be implemented by using a software like **MoMEMta** (for example).
- **MEM tools (like MoMEMta) are not NLO friendly !**

Definition « Main Blocks » : Choices of integration variable and substitutions made to break down a complex process into manageable configuration.

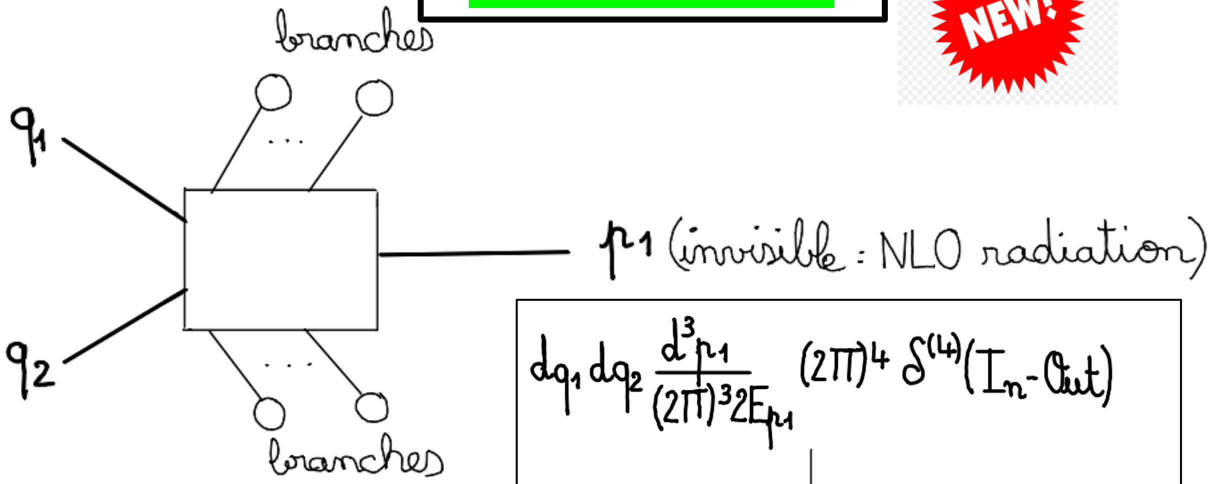


(a) MB A

Issue: None of them can properly deal with an extra radiation

So we built our own implementation inside MoMEMta

Main Block « N »



$$dq_1\, dq_2 \frac{d^3p_1}{(2\pi)^3 2E_{p_1}} (2\pi)^4 \delta^{(4)}(I_n - Out)$$

$$\downarrow$$

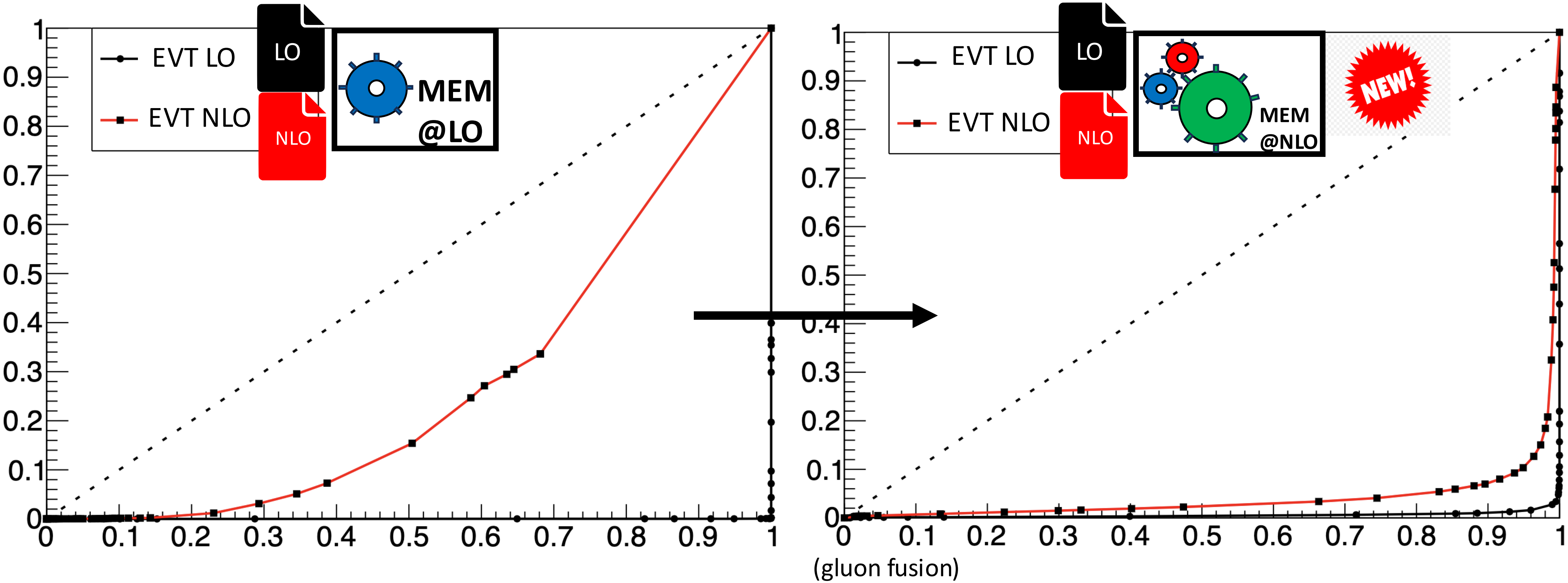
$$\prod \frac{1}{E_{p_1}} \cdot dp_{1z} \cdot |\det(J)|$$

Has extra radiation as integration variables

Results:

MEM@NLO : ROC curve [ttH background]

- Impact of Both (BV+Real): **background ttH**

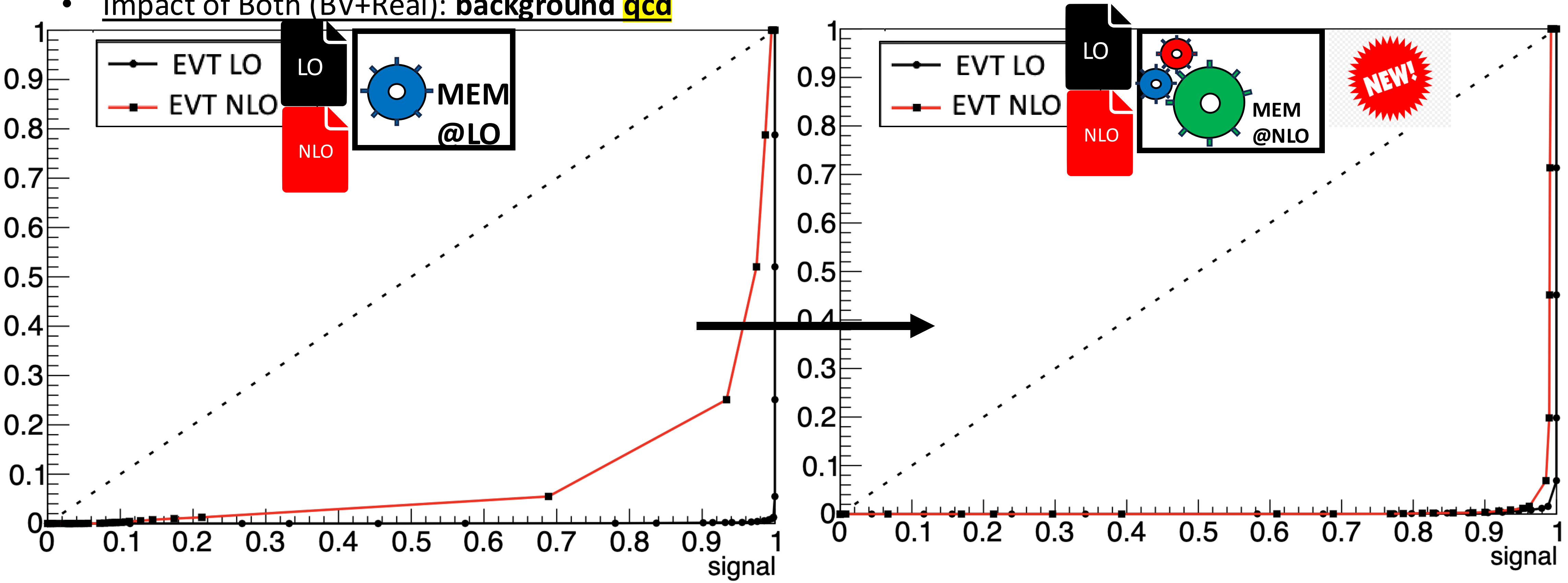


With NLO MEM: better discrimination than before on EVT NLO, huge improvement on EVT NLO : Good!

Results: MEM@NLO

MEM@NLO : ROC curve [qcd background]

- Impact of Both (BV+Real): **background qcd**



We achieved the same efficiency with EVTs at NLO than what we usually have with EVTs at LO

Extraction of κ_λ: Main idea

Likelihoods

- Now that we have seen the discrimination power of the MEM@NLO on NLO events and ATLAS simulated events using $\mathcal{L}_{process}$ (i.e the integral seen earlier), we can look for $\boldsymbol{\lambda_{3H}}$ **value extraction** (or rather $\boldsymbol{\kappa_\lambda} = \frac{\lambda_{3H}}{\lambda_{SM}}$).

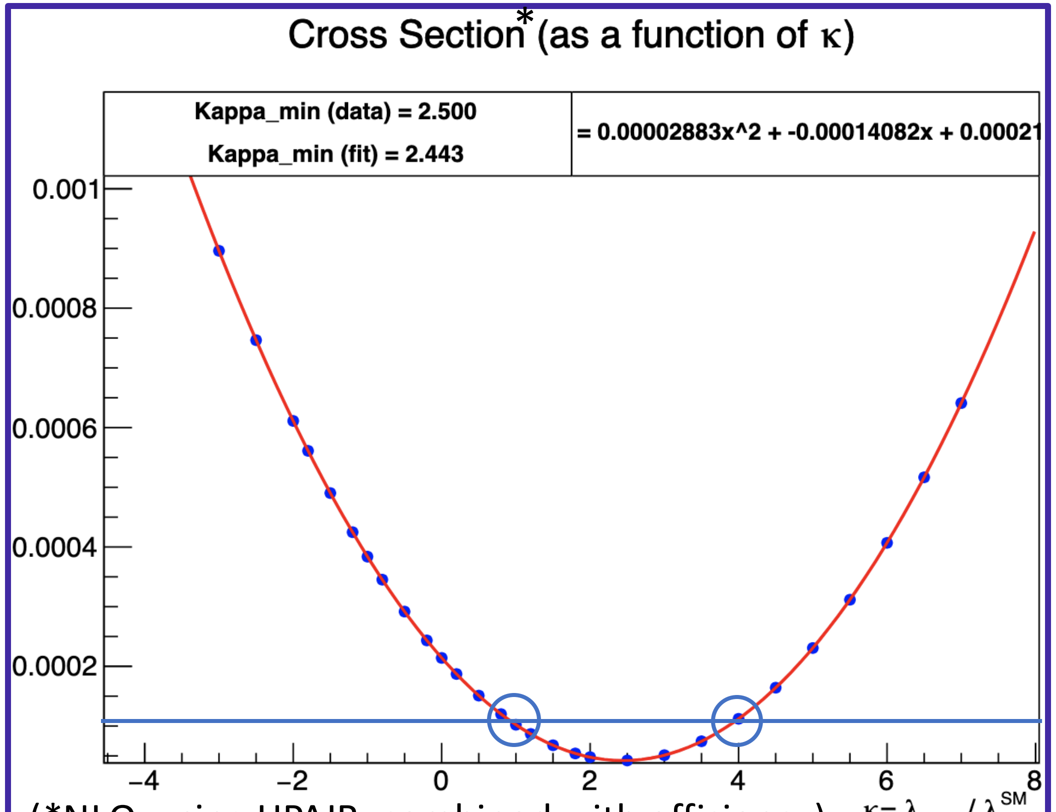
- Idea: We can construct three Likelihoods $\mathcal{L}$ (for the purpose of this analysis) for given $\boldsymbol{\kappa_\lambda}$ hypothesis values $\boldsymbol{\kappa_\lambda} \in$[-3.50 , 10]:
  - $\boldsymbol{\mathcal{L}_{kinematic}(\kappa_\lambda)}$**,** contructed directly from the MEM integral (i.e direcly constructed from $\mathcal{L}_{process}$).
  - $\boldsymbol{\mathcal{L}_{yield}(\kappa_\lambda)}$**,** a theoretical prediction on the behavior of the number of events produced for given hypotheses (processes within the sample, integrated_luminosity, …)
  - $\boldsymbol{\mathcal{L}_{extended}(\kappa_\lambda)}$**,** the product of the two others.

(see Appendix p.26 & p.27 for more detail)

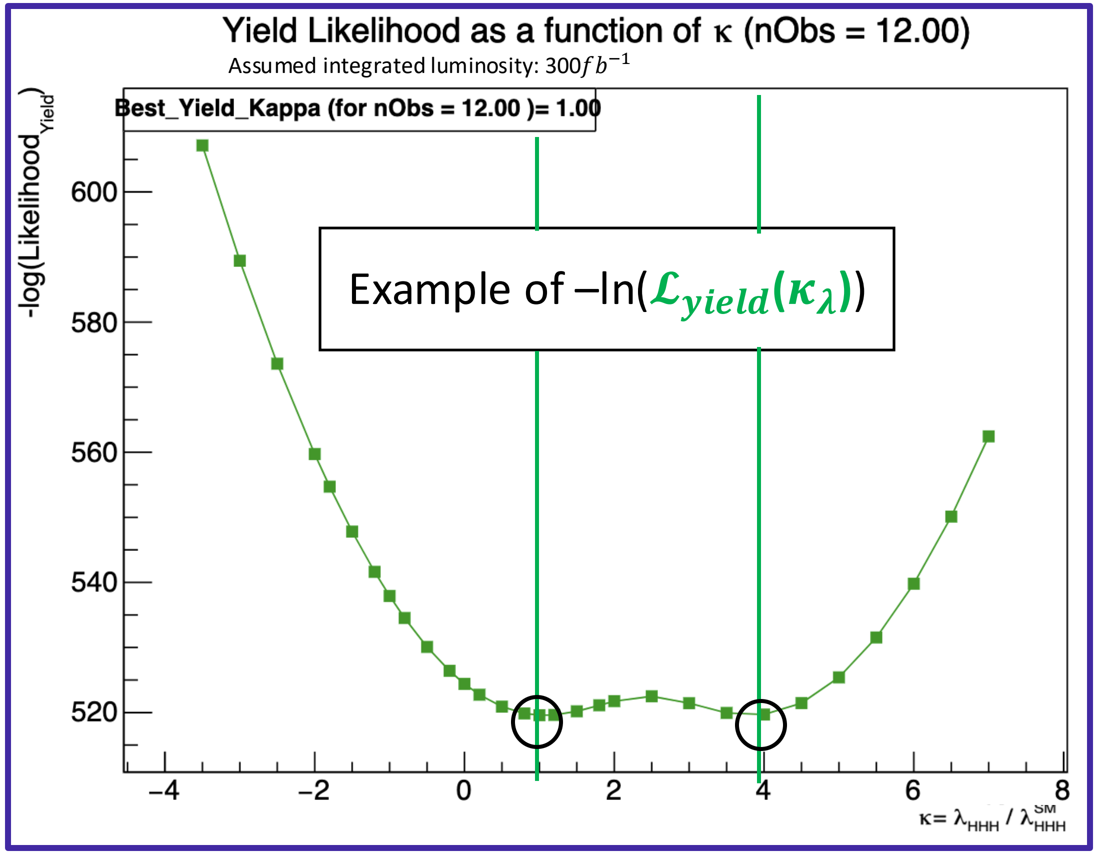
Extraction of κ_λ: Main idea

Likelihood Yield

The signal cross section very strong dependence on κ_λ is used in many analysis methods (like « Event counting methods »). For our MEM, this information is used inside: $\mathcal{L}_{yield}(\kappa_\lambda)$.



(*NLO, using HPAIR, combined with efficiency)

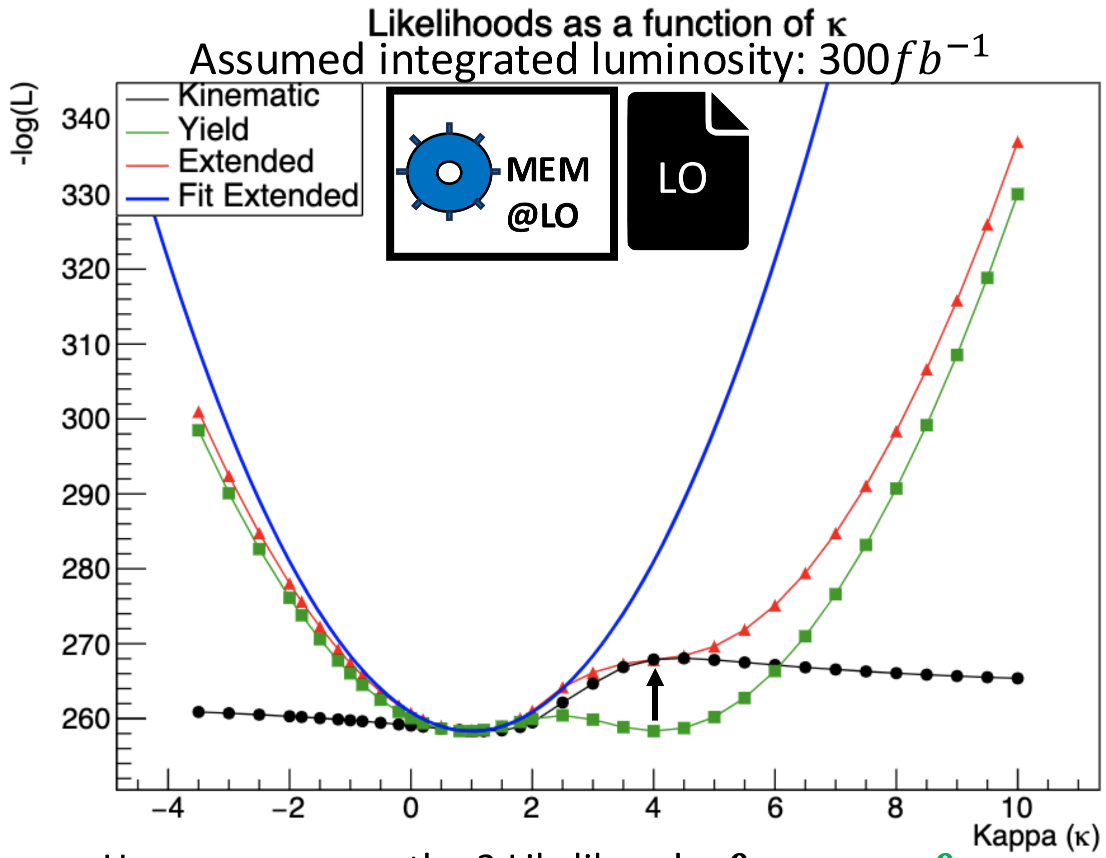


Extraction of κ_λ: Results on MC EVTs

Part 1/3: Signal only, MEM@LO

- (MEM@LO and events at LO)

Assumed integrated luminosity: $300 fb^{-1}$



← For a given pseudo-dataset

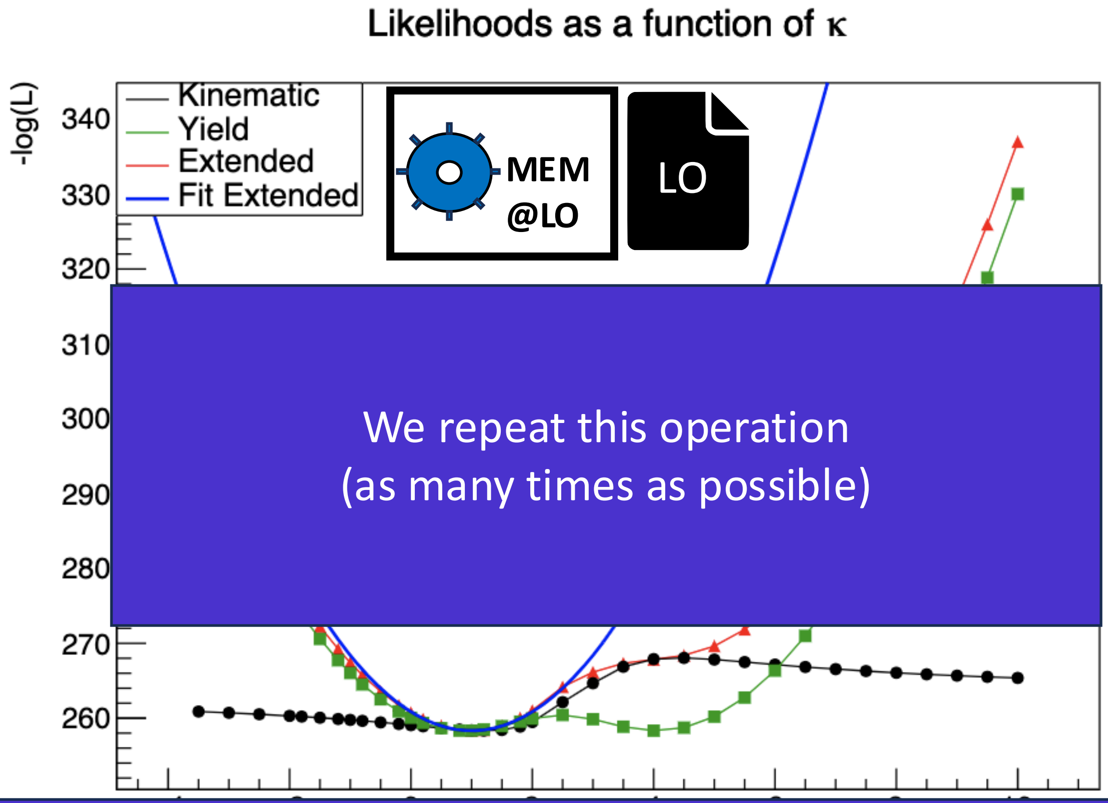
Here you can see the 3 Likelihoods: $\mathcal{L}_{kinematic}$, $\mathcal{L}_{yield}$, and $\mathcal{L}_{extended}$

As well as a **Parabolic Fit** around $\mathcal{L}_{extended}$'s minimum.

Extraction of κ_λ: Results on LO MC EVTs

Part 1/3: Signal only, MEM@LO

- (MEM@LO and events at LO)

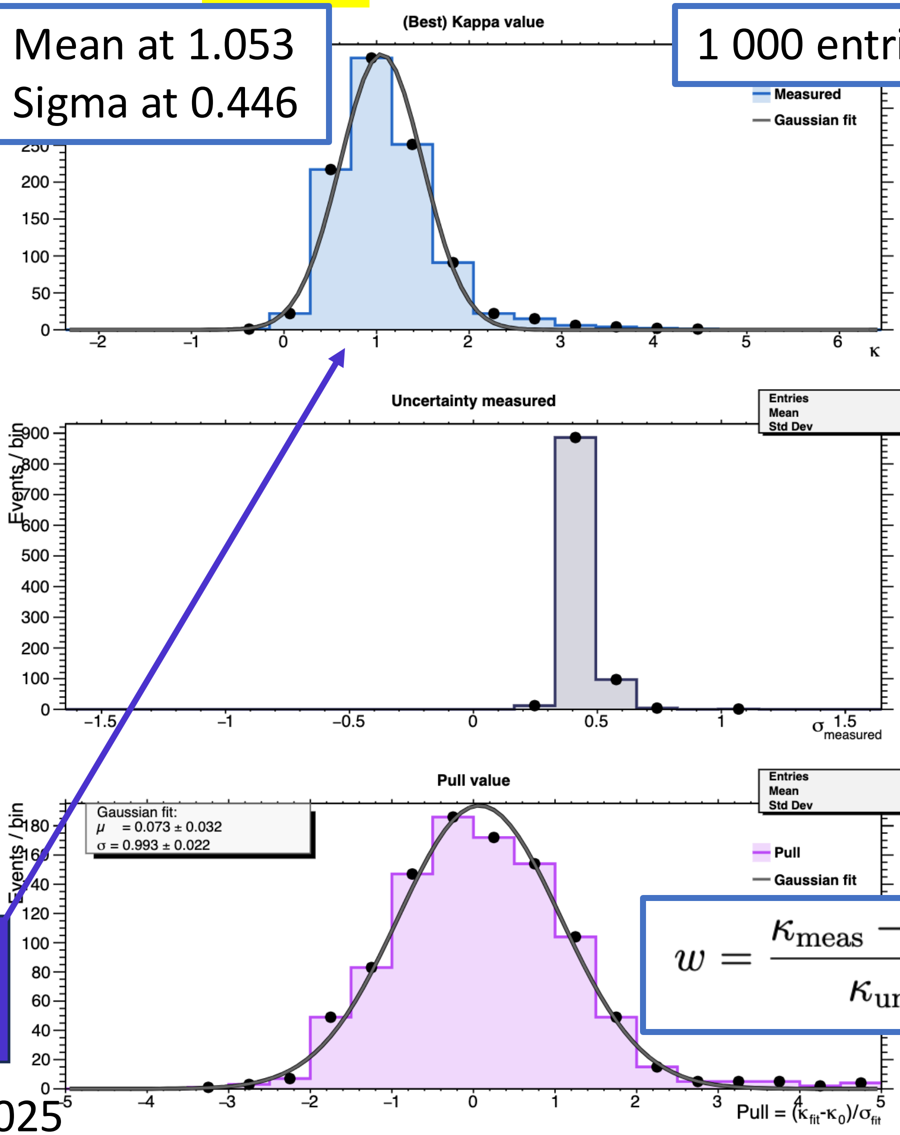


We repeat this operation
(as many times as possible)

And from this we construct an histogram of all the best fitted κ_λ values

Mean at 1.053
Sigma at 0.446

1 000 entries

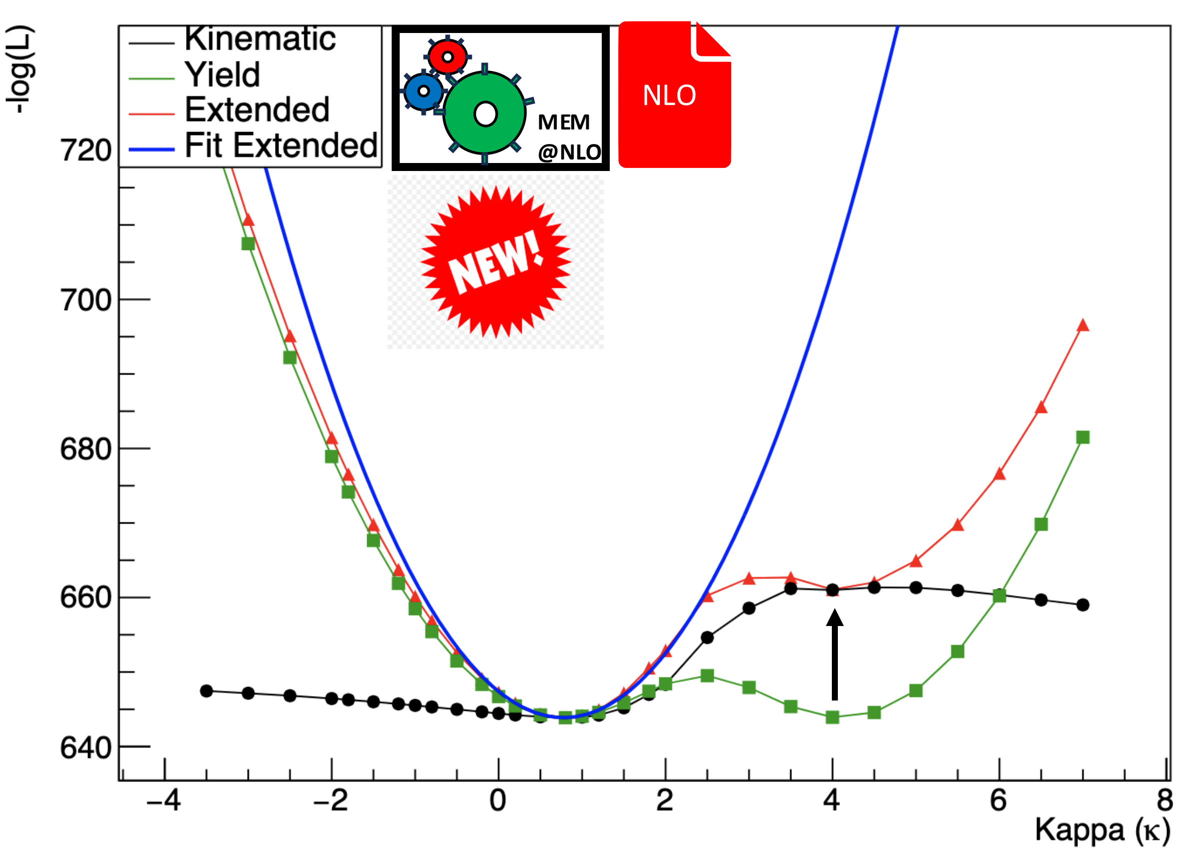


$$w = \frac{\kappa_{\text{meas}} - \kappa_{\text{true}}}{\kappa_{\text{unc}}}$$

Extraction of κ_λ: Results on NLO MC EVTs

Part 2/3: Signal only, MEM@NLO

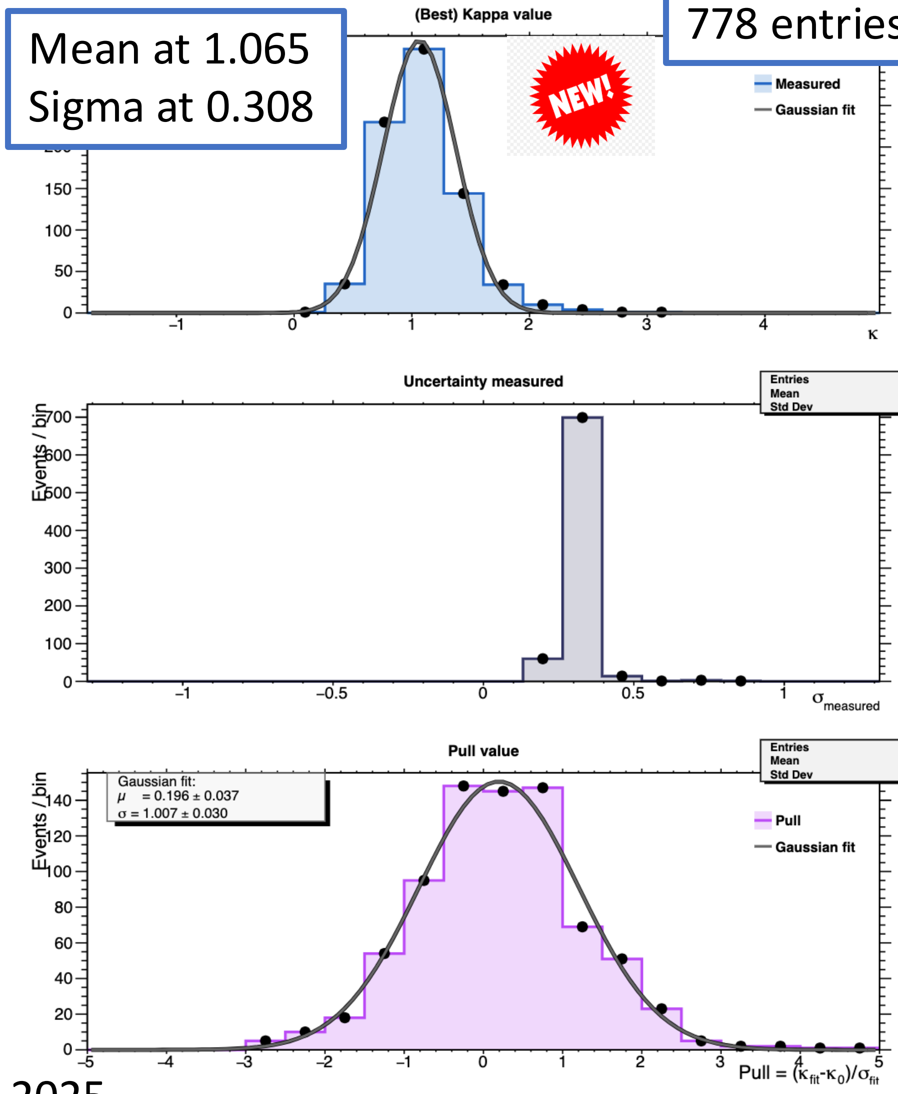
- Result for MEM@NLO on NLO events



Assumed integrated luminosity: $300 fb^{-1}$

Mean at 1.065
Sigma at 0.308

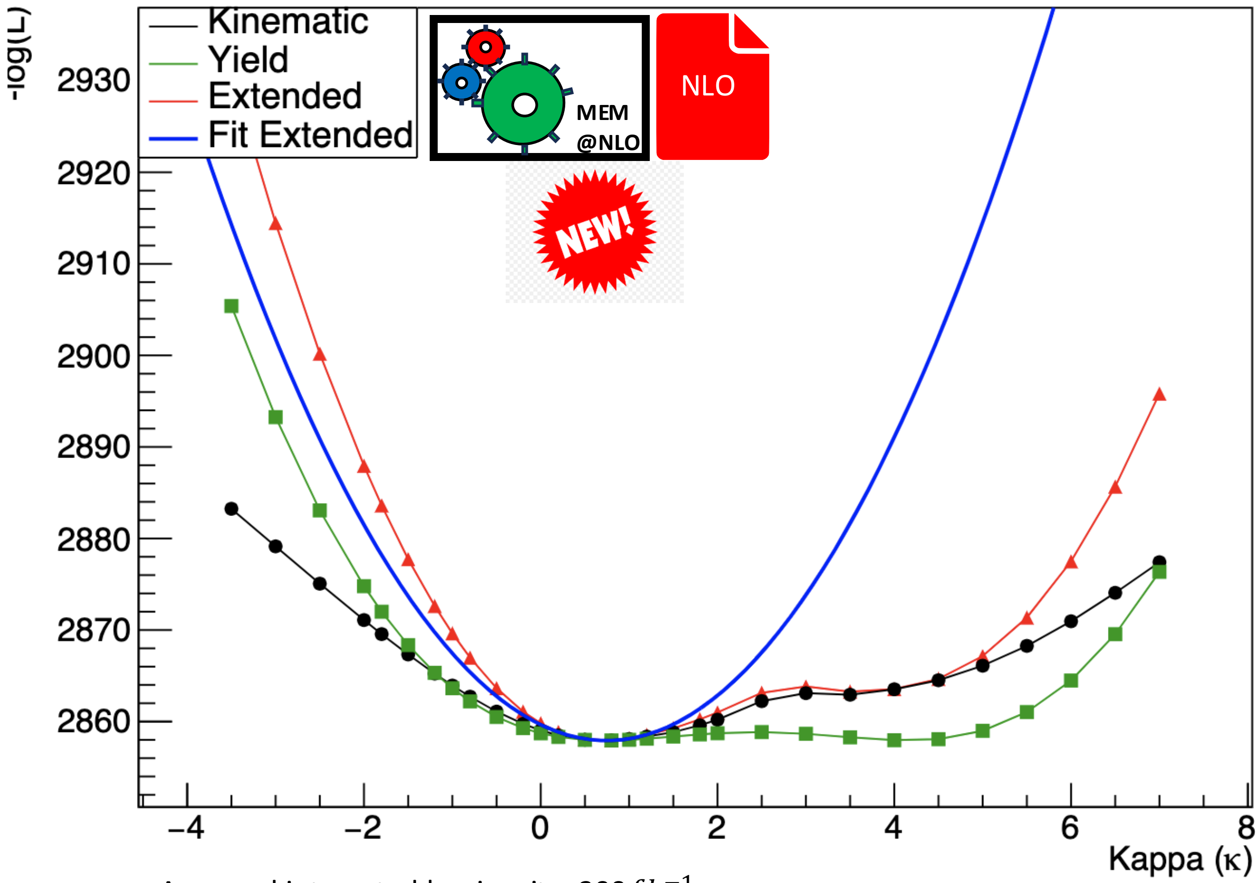
778 entries



Extraction of κ_λ: Results on NLO MC EVTs

Part 3/3: with background [all combined], MEM@NLO

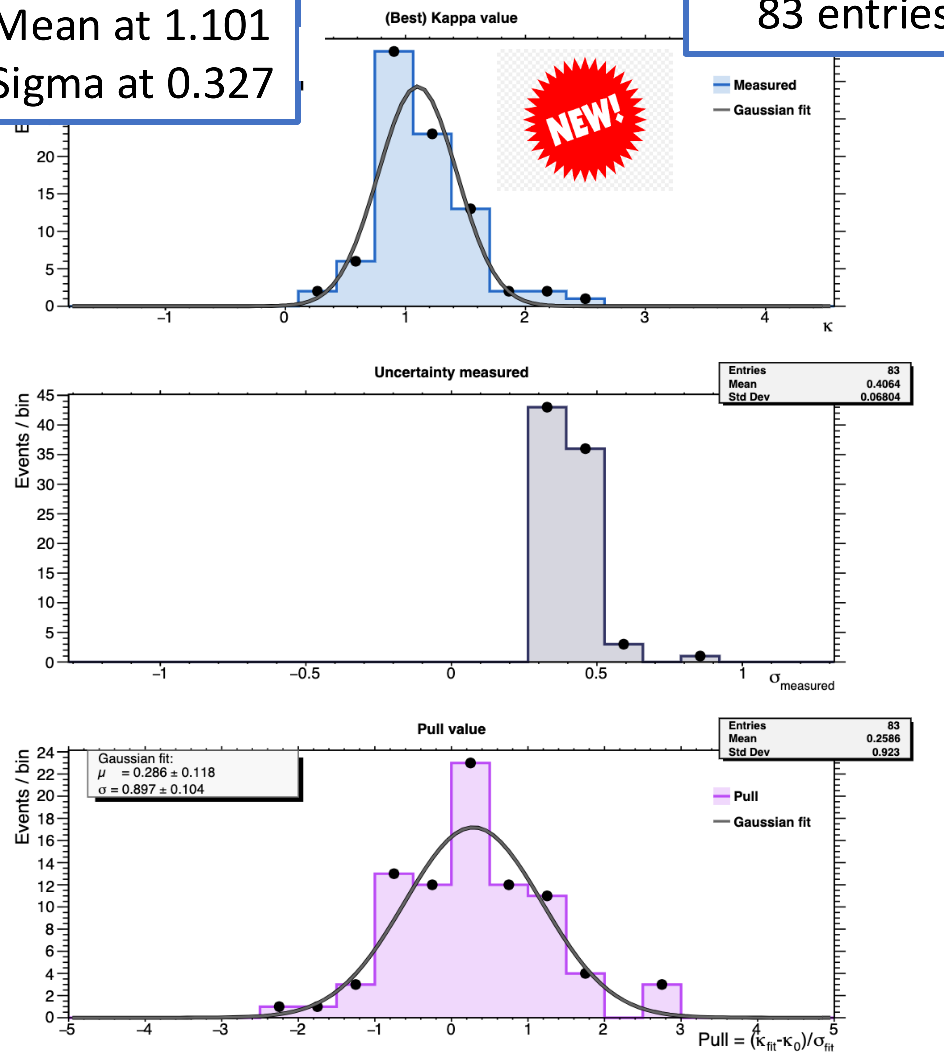
- To have more realistic samples (MEM@NLO on NLO events) :



Assumed integrated luminosity: $300 fb^{-1}$

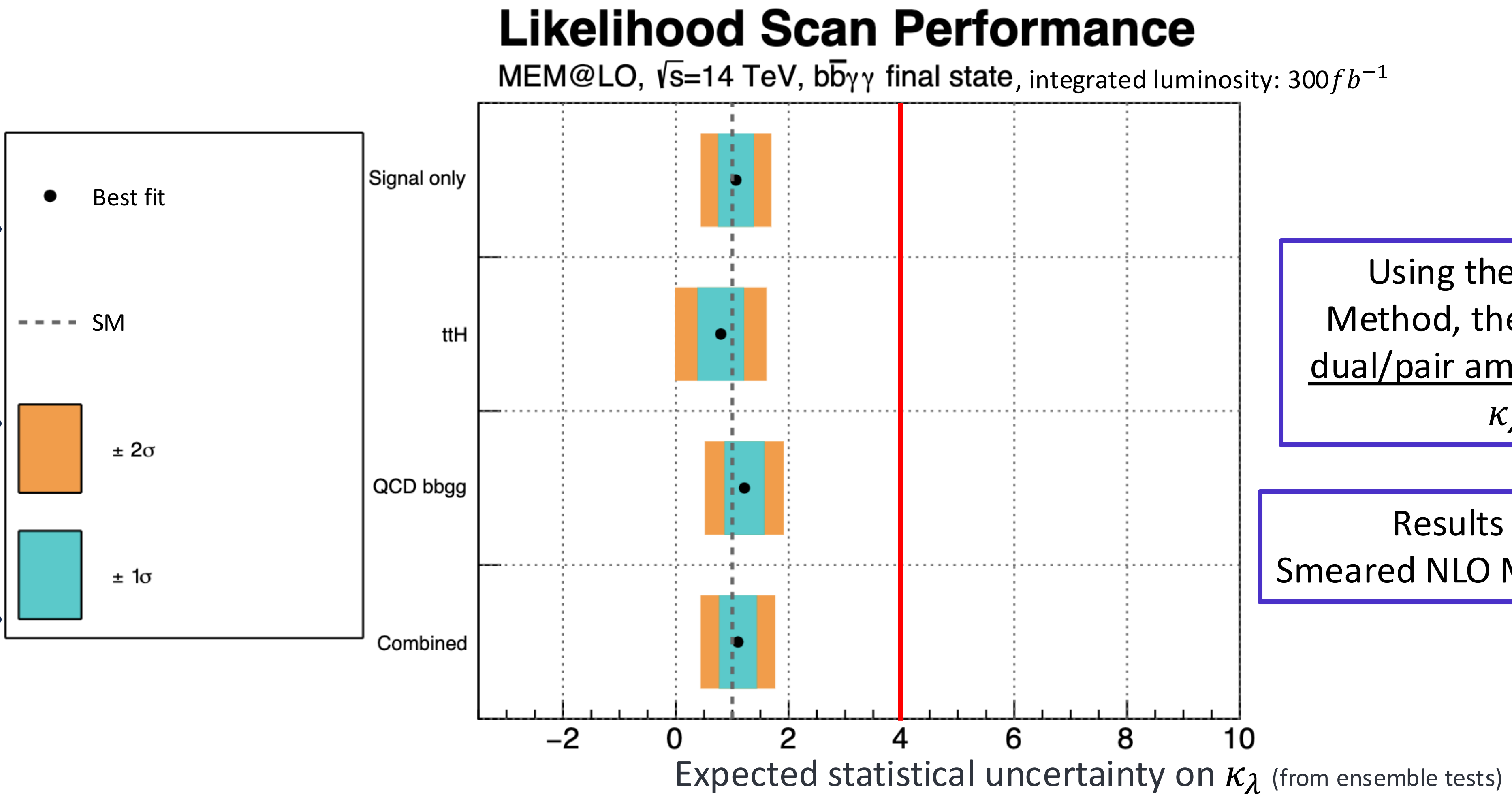
Mean at 1.101
Sigma at 0.327

83 entries



Extraction of κ_λ: Results on NLO MC EVTs

Summary



Using the Matrix Element Method, there is no longer the dual/pair ambiguity on the value $\kappa_\lambda = 4.00$

Results presented here: Smeared NLO MC simulated data only.

Conclusions and beyond

- The Matrix Element Method has had many great successes, but **is difficult to implement.**
- We have successfully developped a MEM@NLO (**new formalism**). Very general framework, so it could be applied to many other analysis (or channels) at NLO.
- Our MEM@NLO has **great efficiency in recovering the κ_λ** for MC simulated samples (@NLO, with background events), with a **clear breaking on the « yield induced » κ_λ pairs of solution.**

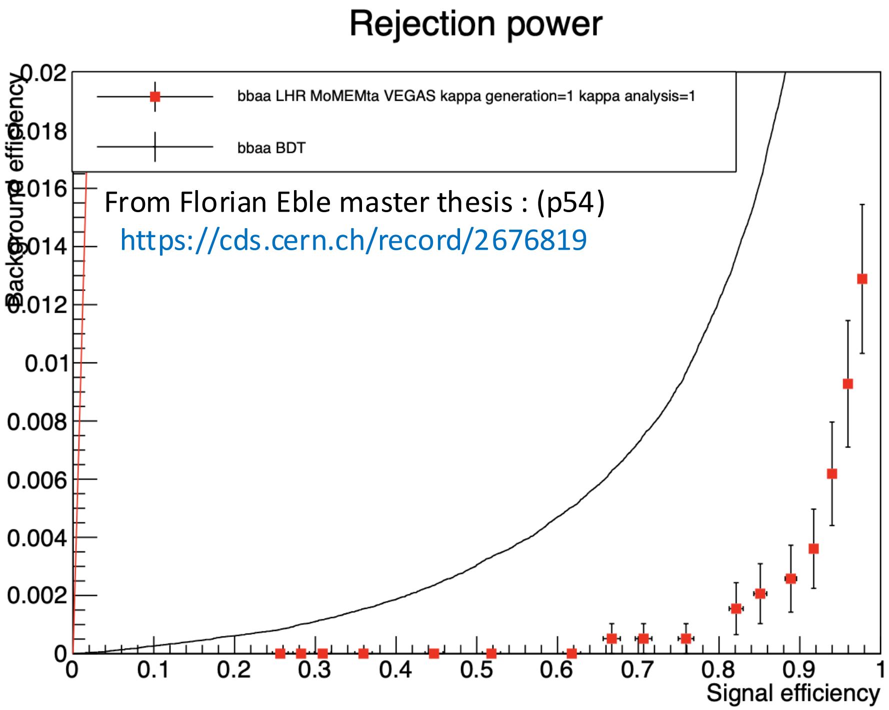
Thank you
for your attention

Laboratoire des 2 Infinis-Toulouse

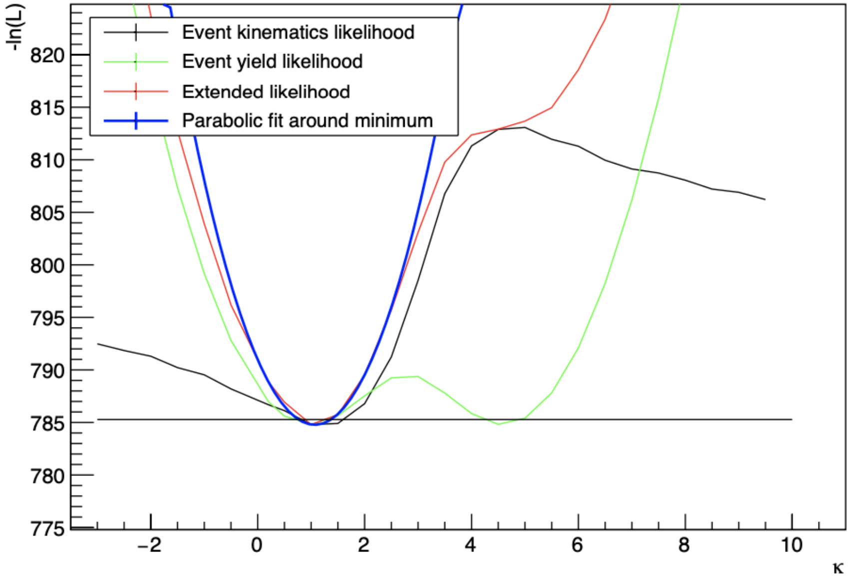
APPENDIX

APPENDIX: Work done by F.Eble and J.Stark (and A.Lleres) [2019]

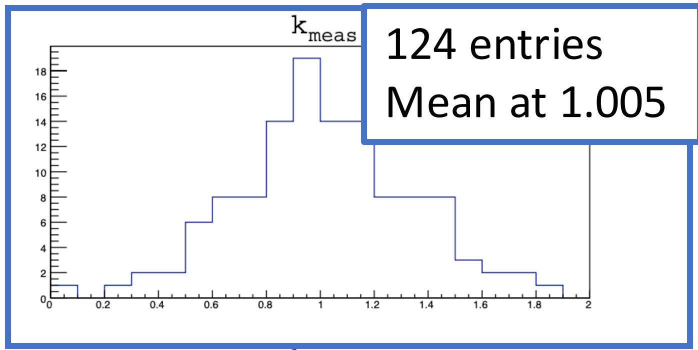
- Work done at MEM@LO (on LO generated events)



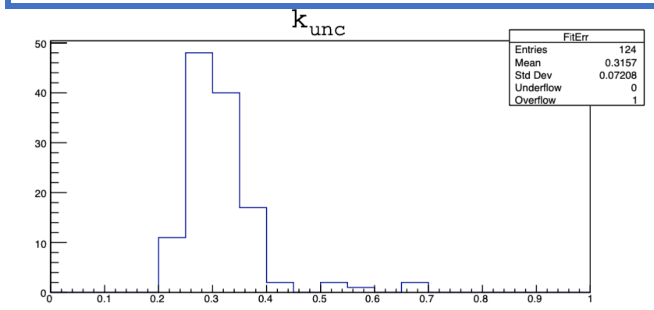
From Florian Eble master thesis : (p54)
https://cds.cern.ch/record/2676819

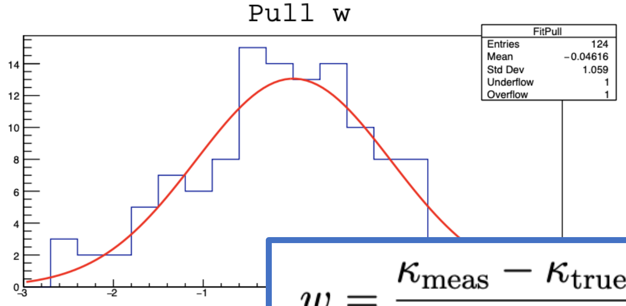


$\mathcal{L}_{kinematic}$, $\mathcal{L}_{yield}$, and $\mathcal{L}_{extended}$
As well as a **Parabolic Fit** around
$\mathcal{L}_{extended}$'s minimum



124 entries
Mean at 1.005





$$w = \frac{\kappa_{\text{meas}} - \kappa_{\text{true}}}{\kappa_{\text{unc}}}$$

APPENDIX: Kinematic likelihood in more detail

Likelihood for a specific process:

$$\mathcal{L}^{p}_{process}(h|x^i) = \frac{d\sigma(pp \to x^i; h; W)}{d\sigma^{obs}_{p}(pp \to F)}$$

and $d\sigma(pp \to x^i; h; W)$ is called weight in the MEM vocabulary.

Likelihood of one event x^i:

$$\mathcal{L}^{i}_{event}(h|x^i) = \sum_{p=1}^{n_p} f_p \, \mathcal{L}^{p}_{process}(h|x^i)$$

Sample Kinematic likelihood :

$$\mathcal{L}_{sample}(h|x) = \prod_{i=1}^{N} \mathcal{L}^{i}_{event}(h|x^i)$$

Kinematic Likelihood: The likelihood one can construct from the MEM output.

APPENDIX: Yield likelihood in more detail

Event yield Likelihood: : The number of events observed (after a given set of event selection requirements) is a valuable piece of information for the extraction of κ. If we assume that we obtained N_{obs} from a poisson distribution from $N_{sig}(\kappa_{\text{test}})$, then:

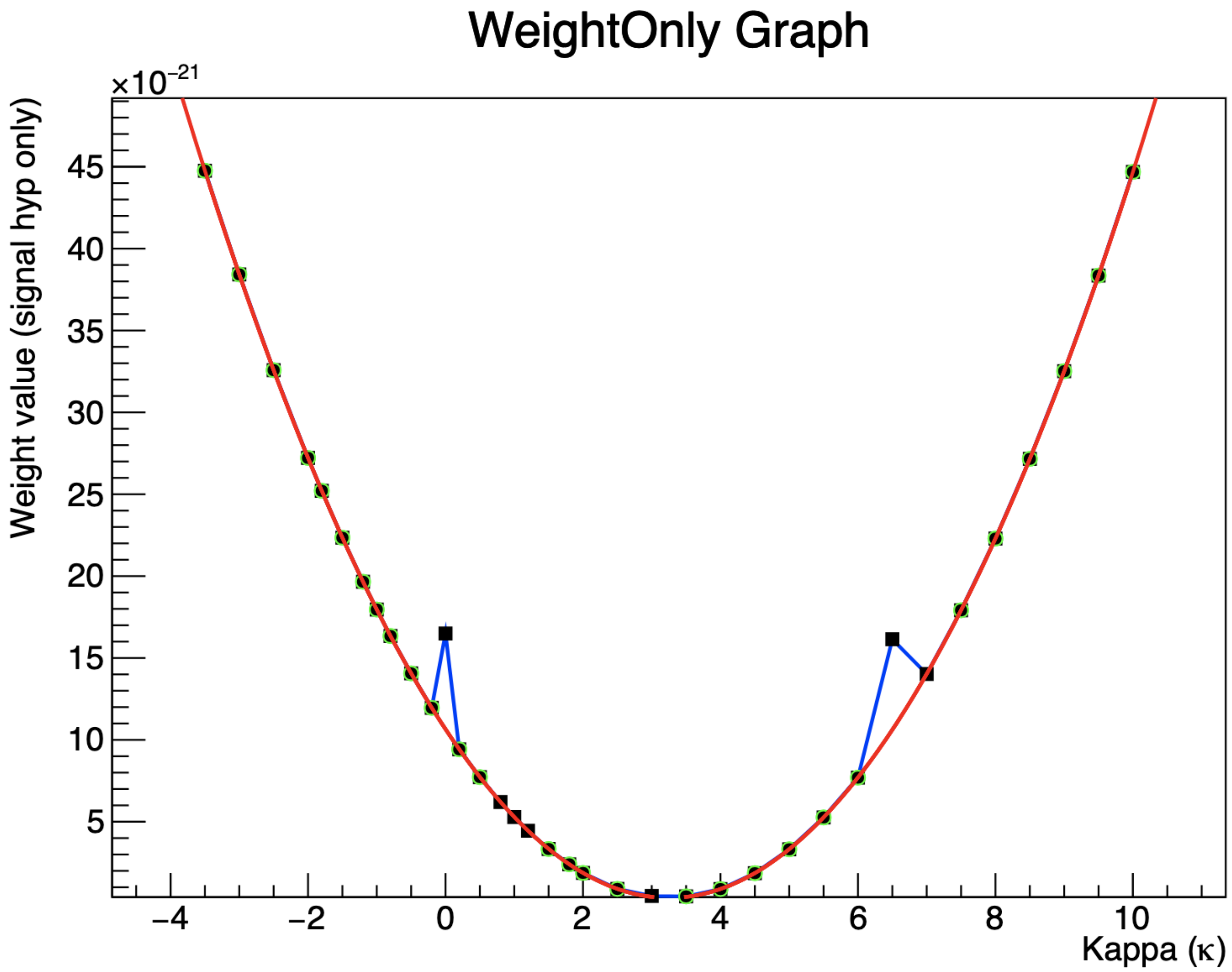
$$\mathcal{L}_{yield}(K) = e^{-(N_{bckg} + N_{sig}(K))} \cdot \frac{(N_{bckg} + N_{sig}(K))^{N_{obs}}}{N_{obs}!}$$

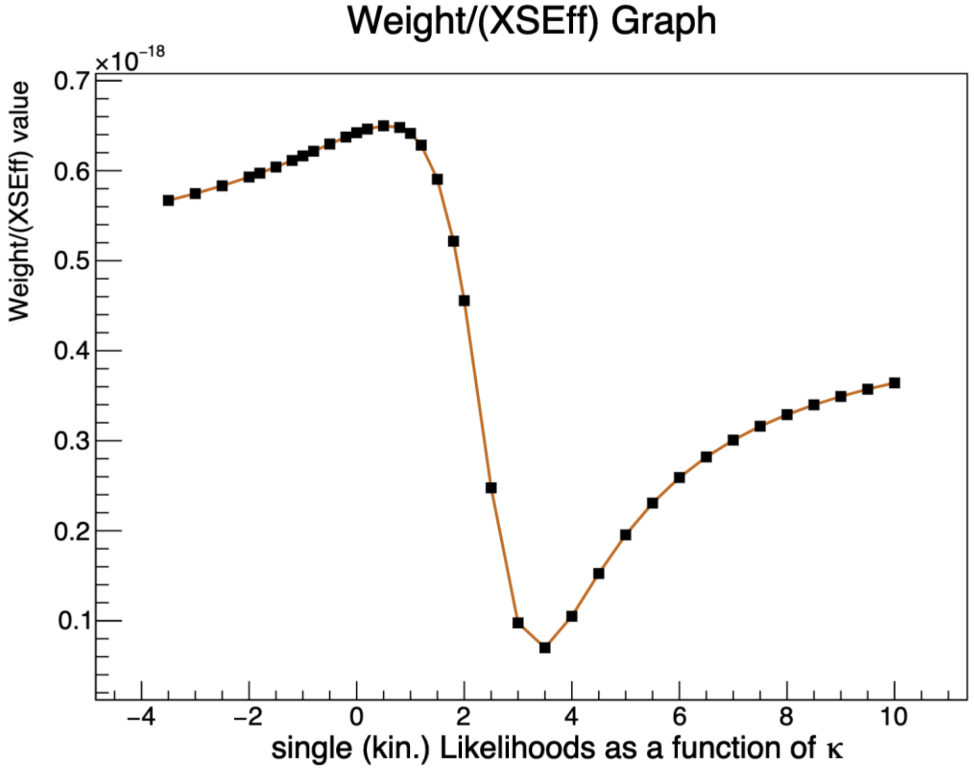
where: N_{bckg} is the expected number of event for all background, given the luminosity L.

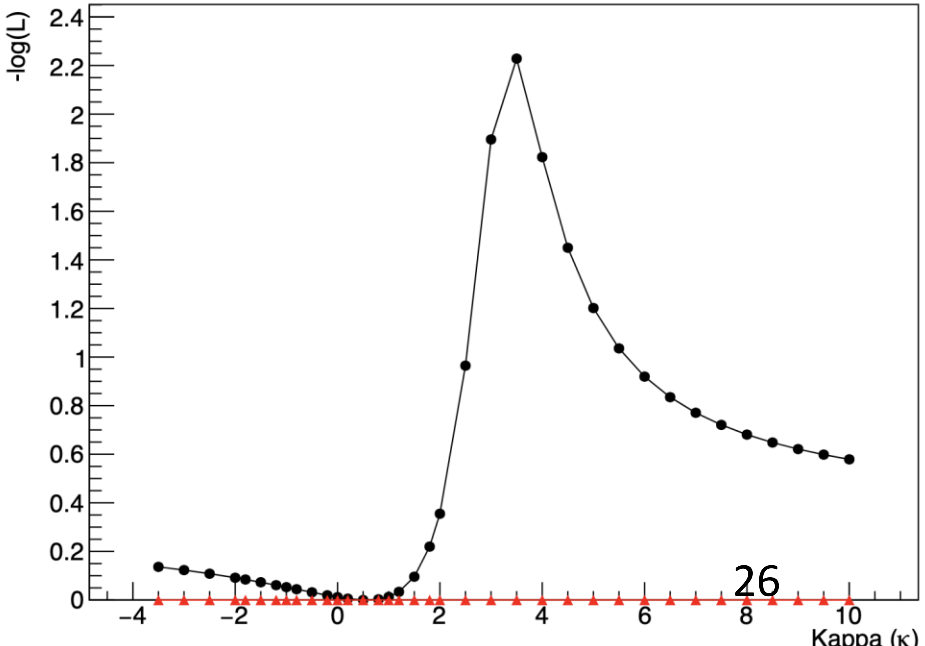
$N_{sig}(K)$ is the expected number of event for the signal given the value of K(-MEM) and luminosity L.

N_{obs} is the number of events to be observed.

APPENDIX: Evolution of weight wrt κ : Example (ggF $\kappa = 1.00$):

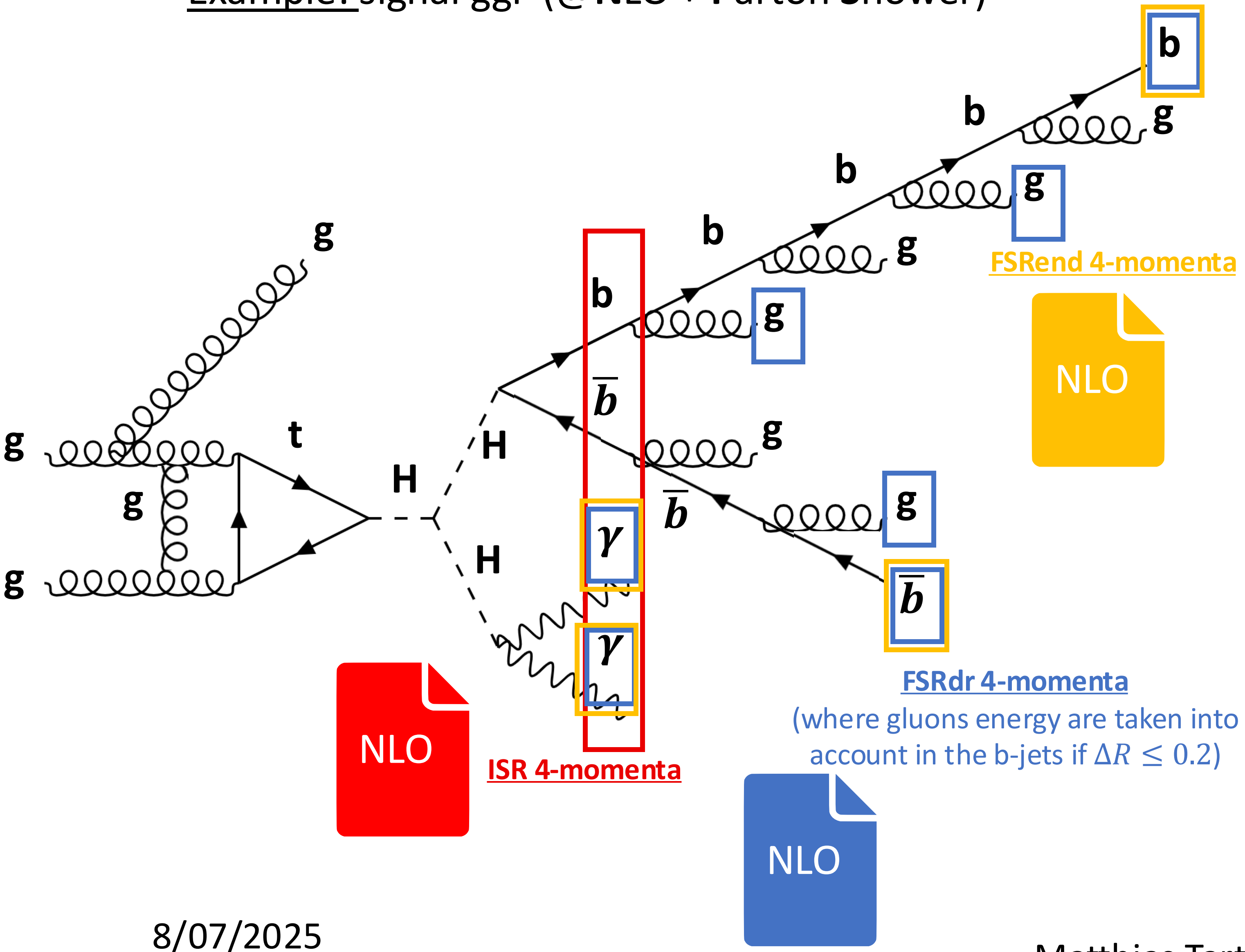


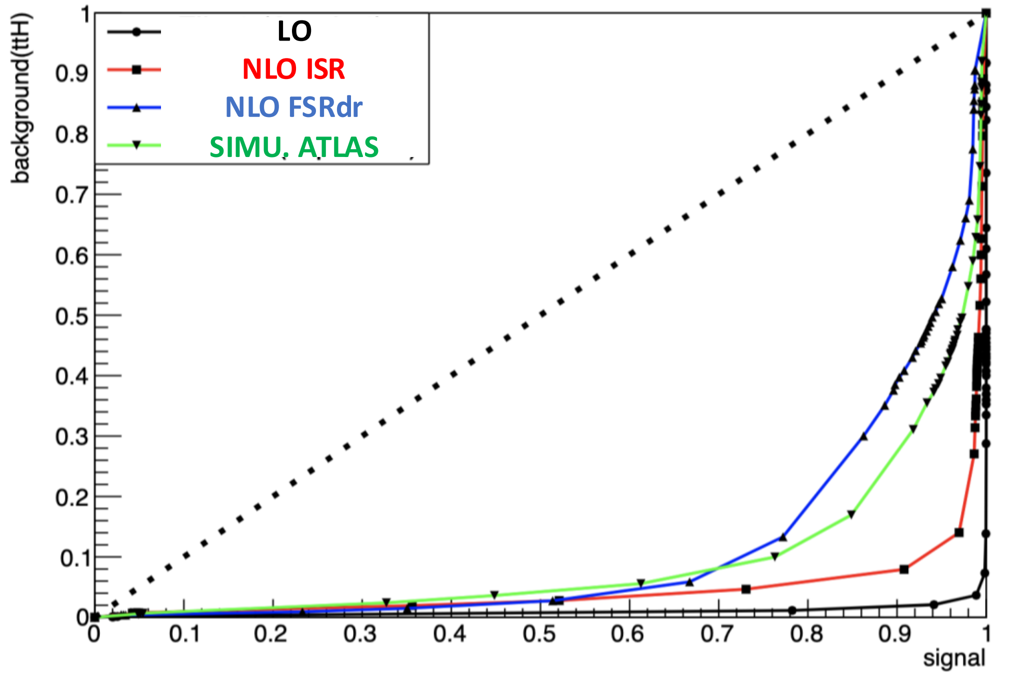




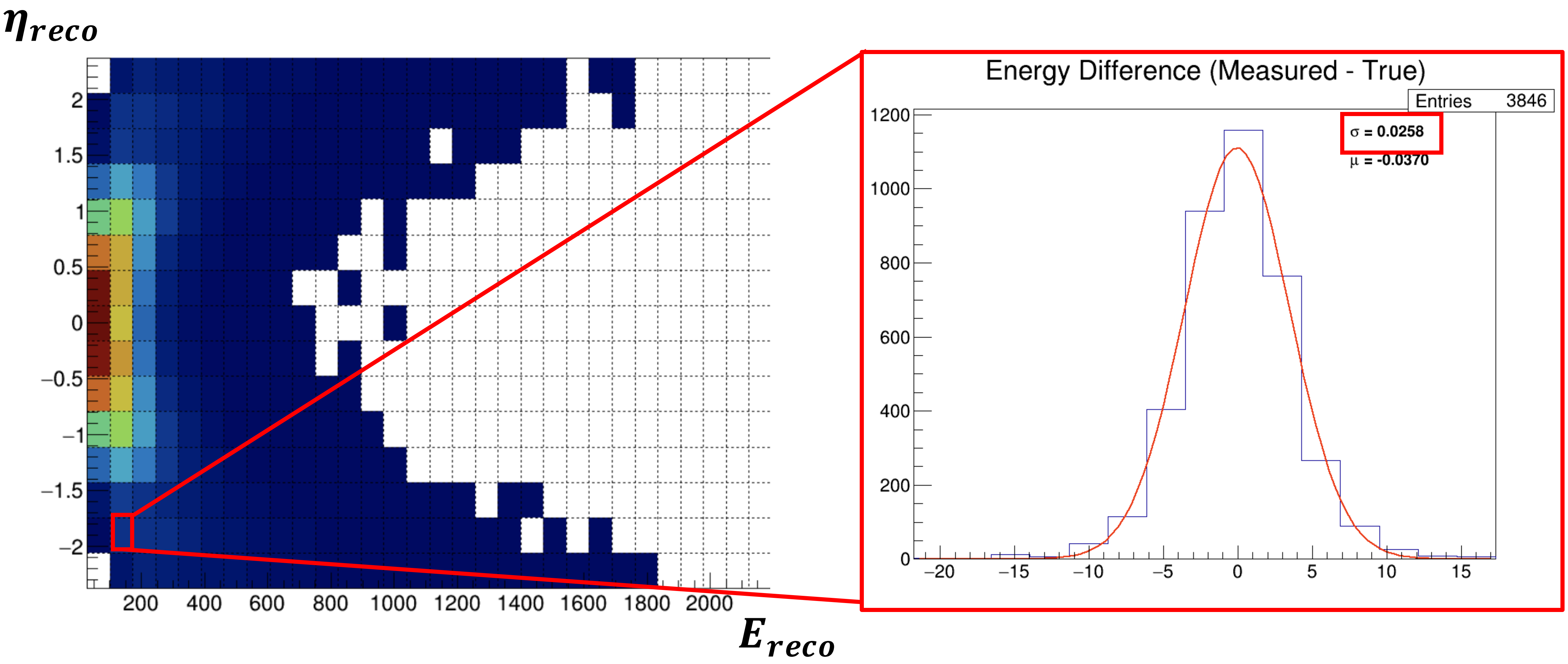
APPENDIX: NLO eventS, different meanings (impact of Parton shower)

- Example: signal ggF (@**N**LO + **P**arton **S**hower)

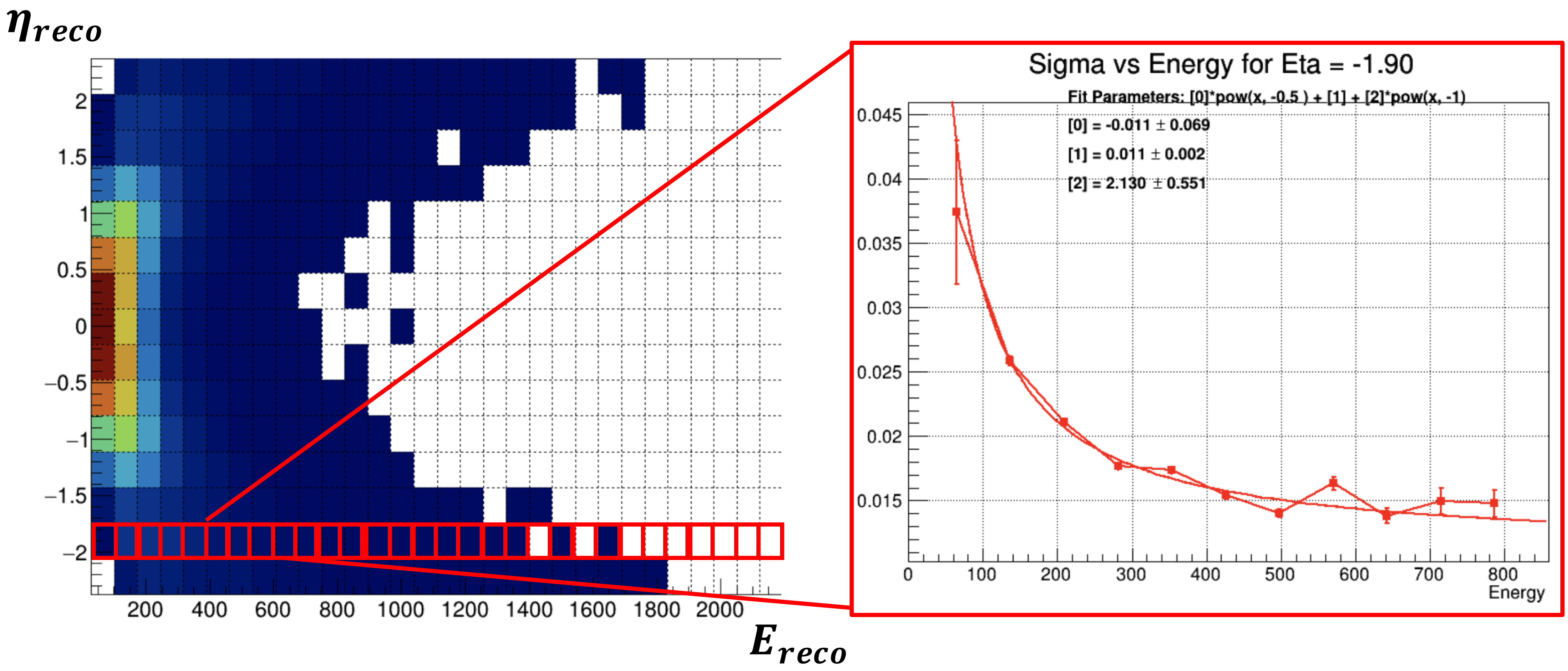




APPENDIX: Transfer function, possible improvements part 1/1 (WIP)

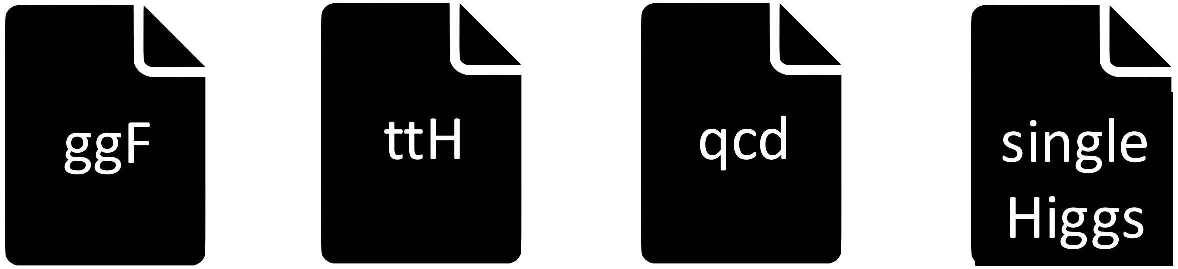


APPENDIX: Transfer function, possible improvements part 2/2 (WIP)

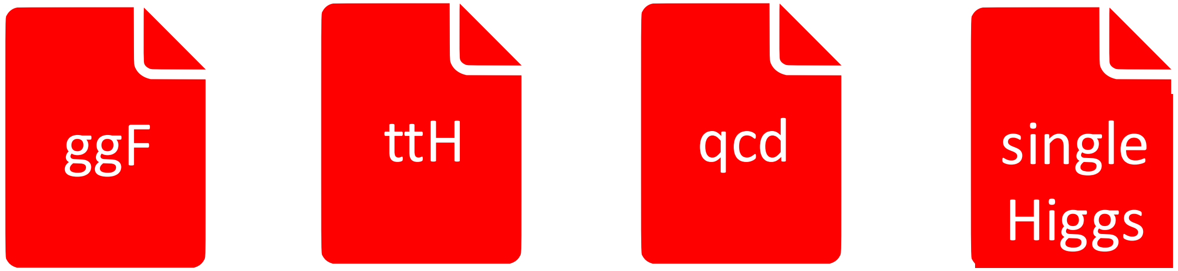


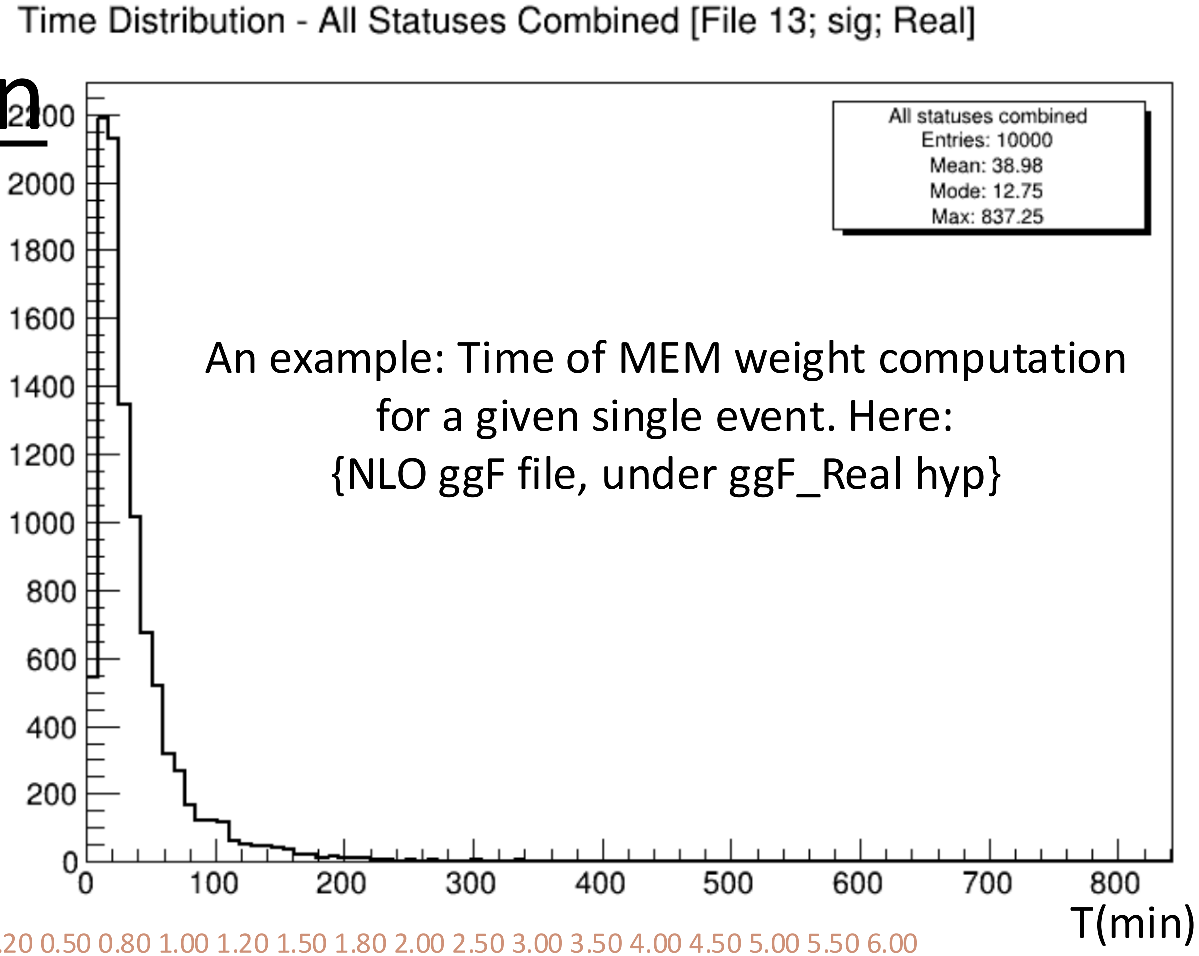
APPENDIX: Time of computation

10 000evts each



10 000evts each





An example: Time of MEM weight computation for a given single event. Here: {NLO ggF file, under ggF_Real hyp}

Kappa hypothesis = "-3.50 -3.00 -2.50 -2.00 -1.80 -1.50 -1.20 -1.00 -0.80 -0.50 -0.20 0.00 0.20 0.50 0.80 1.00 1.20 1.50 1.80 2.00 2.50 3.00 3.50 4.00 4.50 5.00 5.50 6.00 6.50 7.00 7.50 8.00 8.50 9.00 9.50 10.00 » x2 (BV and Real)

ttH hypothesis ;
qcd hypothesis;
singleHiggs hypothesis;
Total: 36*2 + 3 hypothesis (for NLO) = 75.

MEM Computation time: From minutes to days for a single given event (it varies depending on the process and hypothesis chosen of course).

APPENDIX: MEM Degrees Of Freedom and choices for our analysis

Process	Dimension of integration	Variables of integration (not mandatory choice)
ggF Di-Higgs @LO+Virtual	**2** (-> 0)	$(H_{2,Width})$; $\gamma_{1,E}$
ttH @LO	**9** (-> 6)	$(H_{,Width}, top_{1,Width}, top_{2,Width})$; perm$(b_3,b_4)$; $\gamma_{1,E}$;$b_{3,E}$; $b_{4,E}$; q_1 ; q_2
qcd @LO	**2**	$\gamma_{1,E}$; $\gamma_{2,E}$
ggF Di-Higgs@NLO_Real	5 (-> **3**)	$(H_{1,Width}$; $H_{2,Width})$; $\gamma_{1,E}$; $b_{3,E}$; g_{pz}
ttH @NLO_Real	12 (-> **9**)	[LO] + g_{px} ; g_{py} ; g_{pz}
qcd @NLO_Real	**5**	$\gamma_{1,E}$; $\gamma_{2,E}$; $b_{3,E}$; $b_{4,E}$; g_{pz}

As far as we know: Never been done