Collaborative Research Center TRR 257

Particle Physics Phenomenology after the Higgs Discovery

Karlsruhe Institute of Technology

Institute for Theoretical Physics

En route to NLO electroweak corrections to $gg \to HH$ production

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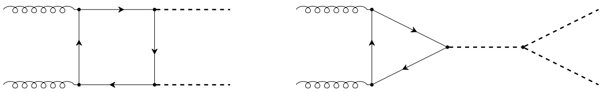
ITP - KIT, IPPP

Why calculate higher orders to $gg \to HH$

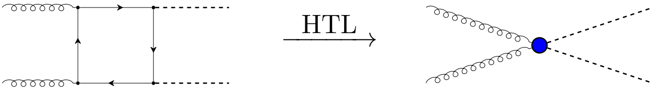
- Sensitivity to Higgs selfcoupling λ



- Already calculated 1988 (Glover and van der Bij 1988)
- Match expected experimental uncertainty at (HL-)LHC, corrections impact the extracted constraints
- Sizeable effects on differential cross sections
- First full m_t dependent NLO QCD result from 2016 (Borowka, Greiner, et al. 2016), (Baglio, Campanario, et al. 2019)

Beyond NLO$_{\mathrm{QCD}}$

- Approximation of higher orders (restricted to certain kinematic regions) with
  - heavy top limit, (De Florian and Mazzitelli 2018; Florian, Grazzini, et al. 2016; Grigo, Hoff, et al. 2015)



  - expansions in kinematic parameters (Davies, Herren, et al. 2022)
- On the way to higher orders numerous combinations of these techniques are used, e.g. (Bagnaschi, Degrassi, et al. 2023; Grazzini, Heinrich, et al. 2018)
  - N^3LO (Chen, Li, et al. 2020a,b)
  - $N^3LO + N^3LL$ (Ajjath and Shao 2023)
- Reaching a scale uncertainty of $\mathcal{O}(\%)$

Besides $\text{N}^n\text{LO}_{\text{QCD}}$

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- EW corrections are at a similar order of magnitude and distort the distributions
- Les Houches Wishlist > 2015

Wishlist	known dσ	desired dσ
2016	$\text{N}^2\text{LO}_{\text{HTL}}$, NLO_{QCD}	$\text{N}^2\text{LO}_{\text{HTL}}$+$\text{NLO}_{\text{QCD}}$+$\text{NLO}_{\text{EW}}$
2021	$\text{N}^3\text{LO}_{\text{HTL}} \otimes \text{NLO}_{\text{QCD}}$	NLO_{EW}

- Massive internal bosons
- Similar approximative methods can be employed, e.g. (Davies, Schönwald, et al. 2023)
- Several partial results (Borowka, Duhr, et al. 2019; Davies, Mishima, et al. 2022; Mühlleitner, Schlenk, et al. 2022)
- First full NLO EW result from 2023 (Bi, Huang, et al. 2023)

Our higher order calculation toolchain

1. Produce contributing diagrams (QGRAF)
2. Project onto form factors (Mathematica, Form)
3. Reduce the number of integrals (kira, Reduze, Ratracer)
4. Integrate the remaining master integrals (pySecDec)
5. Perform the Renormalization (blood, sweat and tears)
6. Crosschecks (DiffExp)
7. Put everything back together

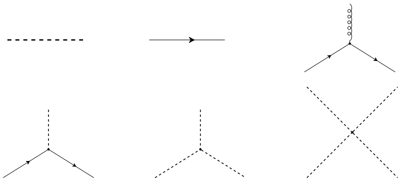
Yukawa-induced	done (unitary gauge)
λ induced	done (unitary gauge)
Vector induced	in progress

λ and Yukawa corrections: The bare Lagrangian

- Gaugeless limit $\Rightarrow$ Weak bosons decouple
- Unitary gauge $\Rightarrow$ Goldstone bosons decouple

$$\mathcal{L} = -\frac{1}{4}\mathcal{G}_{0,\mu\nu}\mathcal{G}_0^{\mu\nu} + \frac{1}{2}(\partial_\mu H_0)^\dagger(\partial^\mu H_0) - \frac{m_{H,0}^2}{2}H_0^2 - \frac{m_{H,0}^2}{2v_0}H_0^3 - \frac{m_{H,0}^2}{8v_0^2}H_0^4$$
$$+ i\bar{t}_0 \not{D} t_0 - m_{t,0}\bar{t}_0 t_0 - \frac{m_{t,0}}{v_0}H_0\bar{t}_0 t_0 + \mathrm{constant}$$

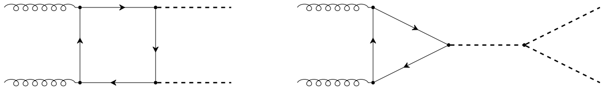
Yields Feynman rules for:



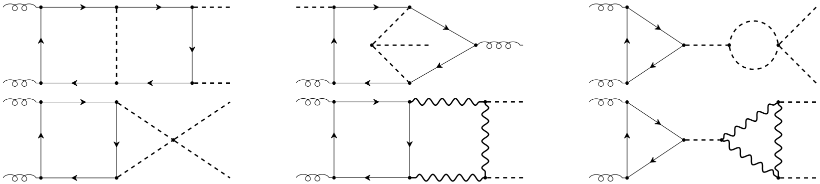
Reparametrized in terms of $m_{H,0}$, $m_{t,0}$ and v_0.

Contributing Diagrams

LO



NLO (examples)



Automated by the tool QGRAF. (Nogueira 1993)

Splitting and Projecting

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Each diagram is

- projected onto form factors F_i for two different tensor structures,
- sorted into classes, according to the occurring couplings, e.g.

$$g_{t,0} \equiv \frac{m_{t,0}}{v_0} \qquad g_{3,0} \equiv \frac{3m_{H,0}^2}{v_0} \qquad g_{4,0} \equiv \frac{3m_{H,0}^2}{v_0^2}\,,$$

- tagged as 1PI or 1PR contribution.

$$F_i|_{\mathrm{NLO}_{\lambda,\mathrm{Yuk}}} = g_{s,0}^2\Big(g_{3,0}\,g_{4,0}\,g_{t,0}\,F_{i,g_3g_4g_t} + g_{3,0}^3\,g_{t,0}\,F_{i,g_3^3g_t} + g_{4,0}\,g_{t,0}^2\,F_{i,g_4g_t^2} + g_{3,0}^2\,g_{t,0}^2\,F_{i,g_3^2g_t^2} + g_{3,0}\,g_{t,0}^3\,F_{i,g_3g_t^3} + g_{t,0}^4\,F_{i,g_t^4}\Big)$$

Type	$g_3g_4g_t$	$g_3^3g_t$	$g_4g_t^2$	$g_3^2g_t^2$	$g_3g_t^3$	g_t^4
1PI	0	0	3	6	24	60
1PR	12	6	1	6	24	26
Total	12	6	4	12	48	86

IBP Reduction

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- Use integration by parts to relate different integrals to each other:

$$\int \prod_{\ell=1}^{L} \mathrm{d}^D k_\ell \frac{\mathrm{d}}{\mathrm{d}k_i^\mu} \left[\eta^\mu \mathcal{I}(\vec{\eta}) \right] = 0$$

- Choose a suitable basis of master integrals M.I.:
  - prefer dots over numerators, i.e. modified propagator powers and dimension shifts
  - search for finite coefficients for top-level M.I. from non-planar sectors
  - avoid poles on diagonal elements of differential equation system
- Yukawa-&λ-induced: fully symbolic reduction to M.I.s retaining dependence on s, t, m_t and m_H using `kira` with `ratracer` (Klappert, Lange, et al. 2021; Magerya 2022)
- Full EW: under construction

The Master Integrals

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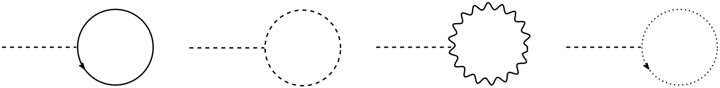
- *d*-factorizing integrals, i.e. dimensionality *d* and kinematics dependent parts are separated
- Still, too many mass scales to solve analytically
- Numerical evaluation using `pySecDec` (Heinrich, Jones, et al. 2024)

	# M.I.	spur. poles
Yukawa & λ	492	$\mathcal{O}(\epsilon^{-4}, \epsilon^{-3}, \epsilon^{-2})$
Full EW	ca. 1300	???

Tadpole Renormalization



- At higher orders the vev gets shifted.
- Fleischer-Jegerlehner tadpole scheme: (Fleischer and Jegerlehner 1981)

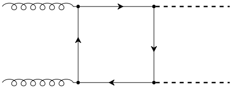
$$H + v \rightarrow H + v + \Delta v$$

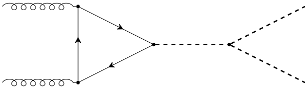
- Require the tadpole diagrams T_H to vanish also at NLO through the tadpole counterterm

$$\delta T = -T_H$$

- Identify $\delta T = -\Delta v \, m_H^2$
- This corresponds to a redistribution of tadpole contributions.

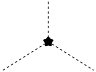
Counterterms





Introduce CTs:

$$
\begin{aligned}
H_0 &= \sqrt{Z_H}H = \sqrt{1+\delta_H}H \\
t_0 &= \sqrt{Z_t}t = \sqrt{1+\delta_t}t \\
m_{H,0}^2 &= m_H^2(1+\delta m_H^2) \\
m_{t,0} &= m_t(1+\delta m_t) \\
v_0 + \Delta v &= v(1+\delta_v) + \Delta v
\end{aligned}
$$



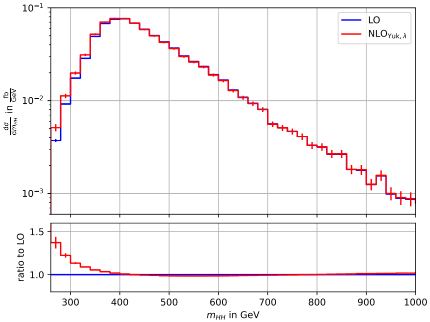
$$
= -i3\frac{m_H^2}{v}\left(\delta m_H^2 + \frac{3}{2}\delta_H - \delta_v - \frac{\delta T}{v m_H^2}\right)
$$

etc.

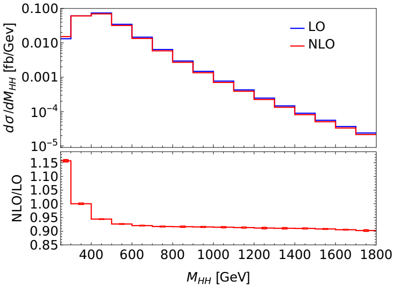
- $\delta_H, \delta_t, \delta m_H^2, \delta m_t$ fixed through on-shell renormalization conditions
- δ_v fixed in G_μ scheme according to (Biekötter, Pecjak, et al. 2023)

The Cross Section

Corrections	Yukawa			Full EW
$\sqrt{s}$	13 TeV	13.6 TeV	14 TeV	14 TeV
LO [fb]	16.45	18.26	19.52	19.96
$\mathrm{NLO}^{\mathrm{EW}}$ [fb]	16.69	18.52	19.79	19.12
$\mathrm{NLO}^{\mathrm{EW}}/\mathrm{LO}$	1.01	1.01	1.01	0.958



(Bi, Huang, et al. 2023)



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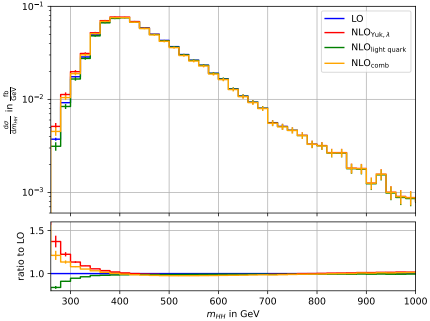
Light quark contrib.: (Bonetti, Rendler, Torres Bobadilla 2025)



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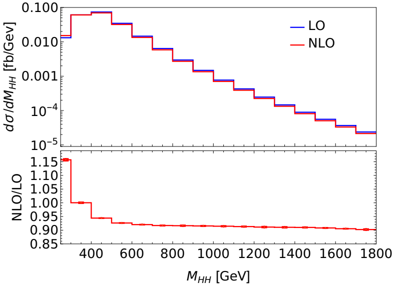

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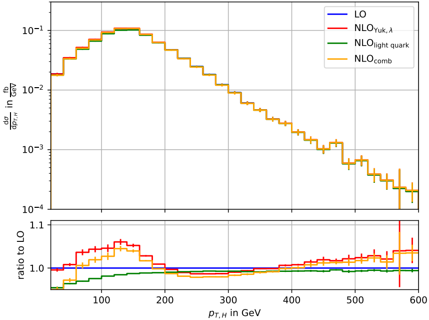

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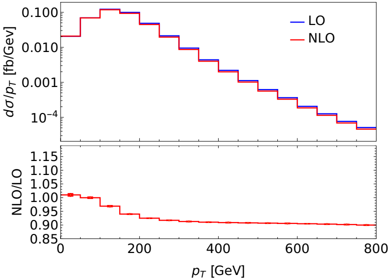


$\longrightarrow$ Considerable changes after including vector bosons!

Conclusion

Where we are:

- Achieved fully symbolic reduction for the gaugeless sector
- Crosschecked with (Davies, Schönwald, et al. 2024)
- Found $K = 1.01$
- Observations for invariant Higgs pair mass and transverse momentum distributions of the cross section
  - Quite large enhancement in low m_{HH} region
  - No Sudakov logs $\Rightarrow$ tail of distributions only slightly changed
  - Dominant contributions to the tail from vector bosons

Where to go:

- Include the full EW corrections and cross-check the result of (Bi, Huang, et al. 2023)
- Investigate the effects of the bottom quark
- Implement an EFT framework

Formfactors

Separate the matrix element into tensor structures and Form Factors

$$\mathcal{M}^{\mu\nu} = F_1 T_1^{\mu\nu} + F_2 T_2^{\mu\nu}$$

Form factors can be obtained by using projectors

$$\mathcal{P}_i^{\mu\nu} T_{j,\mu\nu} = \delta_{ij}$$

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_1^\nu p_2^\mu}{p_1 \cdot p_2}$$

$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{m_H^2 p_1^\nu p_2^\mu}{p_T^2 p_1 \cdot p_2} - \frac{2p_1 \cdot p_3 p_2^\mu p_3^\nu}{p_T^2 p_1 \cdot p_2} - \frac{2p_2 \cdot p_3 p_1^\nu p_3^\mu}{p_T^2 p_1 \cdot p_2} + \frac{2p_3^\mu p_3^\nu}{p_T^2}$$

with

$$p_T = \sqrt{\frac{ut - m_H^4}{s}}$$

Deriving the Tensor structure

General structure:

$$\mathcal{M}^{\mu\nu} = \quad a_{00} g^{\mu\nu} + a_{21} p_2^\mu p_1^\nu + a_{31} p_3^\mu p_1^\nu + a_{23} p_2^\mu p_3^\nu + a_{33} p_3^\mu p_3^\nu$$
$$+ a_{11} p_1^\mu p_1^\nu + a_{22} p_2^\mu p_2^\nu + a_{12} p_1^\mu p_2^\nu + a_{13} p_1^\mu p_3^\nu + a_{32} p_3^\mu p_2^\nu$$

Further constraints from Ward identities:

$$\epsilon_{1,\mu} p_1^\mu = 0 \qquad \epsilon_{2,\nu} p_2^\nu = 0$$

Basic example of Sector Decomposition

$$\mathfrak{I} = \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y x^{-1-a\epsilon} y^{-b\epsilon} \left(x + (1-x)y\right)^{-1}$$

Diverging for $x \to 0$ and $y \to 0$

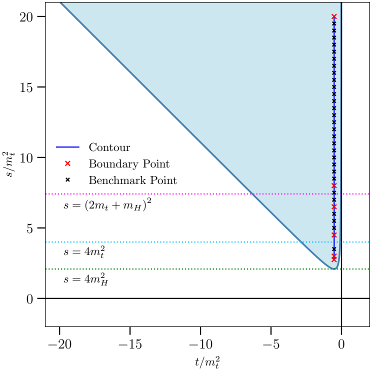
$$\mathfrak{I} = \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y x^{-1-a\epsilon} y^{-b\epsilon} \left(x + (1-x)y\right)^{-1} \left[\Theta(x-y) + \Theta(y-x)\right]$$

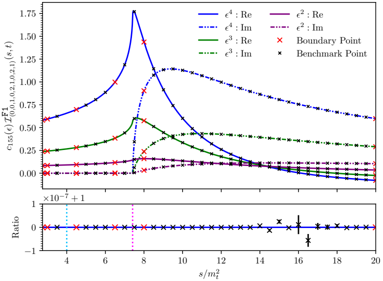
Variable transformation $y = xt$ and $x = yt$

$$\begin{aligned}\mathfrak{I} = &\int_0^1 \frac{\mathrm{d}x}{x^{1+(a+b)\epsilon}} \int_0^1 \frac{\mathrm{d}t}{t^{b\epsilon}\left(1+(1-x)t\right)} \\ &+ \int_0^1 \frac{\mathrm{d}x}{y^{1+(a+b)\epsilon}} \int_0^1 \frac{\mathrm{d}t}{t^{1+a\epsilon}\left(1+(1-y)t\right)}\end{aligned}$$

Both limits $x \to 0$ and $y \to 0$ are independent

Crosscheck with `DiffExp`





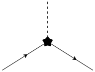
- Run contours in `DiffExp` between boundary points
- Check `pySecDec` vs `DiffExp` for benchmark points

On-Shell Renormalization

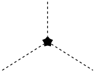
$$0 = \Big[\Sigma_i(\hat{p})\Big]_{\not{p}=m_i} \qquad\qquad 0 = \left[\frac{\mathrm{d}}{\mathrm{d}\not{p}}\Sigma_i(\not{p})\right]_{\not{p}=m_i}$$

$$= -i\left[(m_t - \not{p})\delta_t + m_t\delta m_t - \frac{m_t}{v m_H^2}\delta T\right]$$

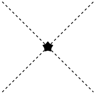
$$= -i\left[(m_H^2 - p^2)\delta_H + m_H^2\delta m_H^2 - 3\frac{\delta T}{v}\right]$$



$$= -i\frac{m_t}{v}\left(\delta m_t + \frac{\delta_H}{2} + \delta_t - \delta_v\right)$$



$$= -i3\frac{m_H^2}{v}\left(\delta m_H^2 + \frac{3}{2}\delta_H - \delta_v - \frac{\delta T}{v m_H^2}\right)$$



$$= -i3\frac{m_H^2}{v^2}(\delta m_H^2 + 2\delta_H - 2\delta_v)$$

Gaugeless Tadpole Renormalization I

- At higher orders the vev gets shifted.
- Fleischer-Jegerlehner tadpole scheme: (Fleischer and Jegerlehner 1981)

$$H + v \to H + v + \Delta v$$

- Require the tadpole diagrams T_H to vanish also at NLO through the tadpole counterterm

$$\delta T = -T_H$$

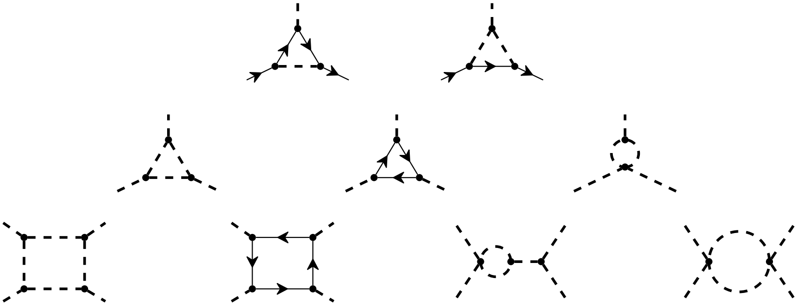
- Identify $\delta T = -\Delta v \, m_H^2$
- This corresponds to a redistribution of tadpole contributions.

Gaugeless Tadpole Renormalization II

$$
\begin{aligned}
\mathcal{L}_0 =&\frac{1}{2}(\partial_\mu H_0)^\dagger(\partial^\mu H_0) + \frac{\mu_0^2}{2}(v_0 + H_0)^2 + \frac{\lambda}{16}(v_0 + H_0)^4 \\
&+ i\bar{t}_0 \not{D} t_0 - y_{t,0}\frac{v_0 + H_0}{\sqrt{2}}\bar{t}_0 t_0 - \frac{1}{4}\mathcal{G}_{0,\mu\nu}\mathcal{G}_0^{\mu\nu} \\
\rightarrow&\frac{1}{2}(\partial_\mu H_0)^\dagger(\partial^\mu H_0) + \frac{\mu_0^2}{2}(v_0 + \Delta v + H_0)^2 + \frac{\lambda_0}{16}(v_0 + \Delta v + H_0)^4 \\
&+ i\bar{t}_0 \not{D} t_0 - y_{t,0}\frac{v_0 + \Delta v + H_0}{\sqrt{2}}\bar{t}_0 t_0 - \frac{1}{4}\mathcal{G}_{0,\mu\nu}\mathcal{G}_0^{\mu\nu} \\
=&\frac{1}{2}(\partial_\mu H_0)^\dagger(\partial^\mu H_0) + H_0\left(\mu_0^2 v_0 + \frac{\lambda_0 v_0^3}{4} + \Delta v(\mu_0^2 + \frac{3}{4}\lambda_0 v_0^2)\right) \\
&+ H_0^2\left(\frac{\mu_0^2}{2} + \frac{3v_0^2\lambda_0}{8} + \frac{3}{4}\lambda_0 v_0 \Delta v\right) + H_0^3\left(\frac{\lambda_0 v_0}{4} + \Delta v\frac{\lambda_0}{4}\right) + H_0^4\frac{\lambda_0}{16} \\
&+ i\bar{t}_0 \not{D} t_0 - m_{t,0}\bar{t}_0 t_0 - \frac{m_{t,0}}{v_0}\Delta v\bar{t}_0 t_0 - \frac{m_{t,0}}{v_0}H_0\bar{t}_0 t_0 - \frac{1}{4}\mathcal{G}_{0,\mu\nu}\mathcal{G}_0^{\mu\nu} + \ldots
\end{aligned}
$$

δ_v Counterterm

$$\delta_v|_{\mathrm{UV}} = -\frac{3m_H^4 + 2m_H^2 m_t^2 N_c - 8m_t^4 N_c}{32\pi^2 m_H^2 v^2 \epsilon}$$



$$\delta_v|_{G_\mu} = \frac{1}{2^D \pi^{D/2}} \frac{1}{2v^2} \left(-\frac{m_H^2}{2} + N_c m_t^2 - 2N_c A_0(m_t^2) - 3A_0(m_H^2) + 8N_c \frac{m_t^2}{m_H^2} A_0(m_t^2) \right)$$