Renormalisation group Improved Higgs to two gluons decay rate

EPS-HEP 2025, Marseille

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Based on *Eur.Phys.J.Plus 139 (2024) 2, 114*

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07-07-2025

Overview

Introduction

The partial decay width of the Higgs-boson to gluons $\Gamma_{H\to gg}$ decay is given as,

$$\Gamma_{H\to gg} = \frac{\sqrt{2}\,G_{\mathrm{F}}}{M_H}\,|C_1|^2\,\mathrm{Im}\,\Pi^{GG}(-M_H^2 - i\delta), \tag{1}$$

The coefficient C_1 is known up to N^{4}LO and its perturbative expansion is given by,

$$C_1 = -\frac{1}{3}\,a_s\left(1 + \sum_{n=1} c_n\,a_s^n(\mu^2)\right), \tag{2}$$

where $a_s \equiv \frac{\alpha_s^{n_f}}{4\pi}$ where n_f are number of light flavours. The absorptive part of the vacuum polarization is computed at N^{4}LO and written in the following form,

$$\frac{4\pi}{N_A q^4}\,\mathrm{Im}\,\Pi^{GG}(q^2) \equiv G(q^2) = 1 + \sum_{n=1} g_n a_s^n, \tag{3}$$

Fixed Order Perturbation Theory

The decay width of $H \to gg$ can be written as,

$$\Gamma = \left[\sqrt{2} G_F M_H^3 / 72\pi\right] x^2(\mu) S\left[x(\mu), L(\mu)\right], \tag{4}$$

where the perturbative expansion $S[x(\mu), L(\mu)]$ in the so called " fixed-order perturbation theory" (FOPT) is written as,

$$S_{\mathrm{FOPT}}[x(\mu), L(\mu)] = \sum_{n=0}^{\infty} \sum_{k=0}^{n} T_{n,k} x^n L^k. \tag{5}$$

where $x(\mu) = \frac{\alpha_s^{n_f}(\mu)}{\pi}$ and $L(\mu) = ln(\mu^2 / m_t^2(\mu))$.

Suppose for the decay process the perturbation series $S[x, L]$

$$
\begin{aligned}
S^{NLO} &= T_{0,0} + (T_{1,0} + T_{1,1}L)\,x \\
S^{N^2LO} &= S^{NLO} + \left(T_{2,0} + T_{2,1}L + T_{2,2}L^2\right)x^2 \\
S^{N^3LO} &= S^{N^2LO} + \left(T_{3,0} + T_{3,1}L + T_{3,2}L^2 + T_{3,3}L^3\right)x^3 \\
S^{N^4LO} &= S^{N^3LO} + \left(T_{4,0} + T_{4,1}L + T_{4,2}L^2 + T_{4,3}L^3 + T_{4,4}L^4\right)x^4.
\end{aligned}
$$

These NLO and higher-order expressions exhibit scale dependence as the magnitude of L increases.

Renormalisation Group Summed Perturbation Theory

In the RGSPT, the FOPT expansion of the function $S[x(\mu), L(\mu)]$ is equivalent to writing the following new expansion,

$$S(x, L) = \sum_{n=0}^{\infty} x^n S_n(xL), \tag{6}$$

function $S_n(xL)$ is defined by,

$$S_n(xL) \equiv \sum_{k=n}^{\infty} T_{k,k-n}(xL)^{k-n}. \tag{7}$$

The main feature of the RGSPT is the explicit all-orders summations of all RG-accessible logarithms in the function $S_n(xL)$.

The functions $S_n(u)$, where $u = xL$ can be derived in a closed analytical form using the RG invaraince of $\Gamma(H \to gg)$ decay width:

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \{\Gamma_{H\to gg}\} = 0. \tag{8}$$

$$(1-\beta_0 u)\frac{\mathrm{d}S_n}{\mathrm{d}u} - u\sum_{\ell=0}^{n-1}\beta_{\ell+1}\frac{\mathrm{d}S_{n-\ell-1}}{\mathrm{d}u} + 2\sum_{\ell=0}^{n-1}\gamma_\ell\frac{\mathrm{d}S_{n-\ell-1}}{\mathrm{d}u} - \sum_{\ell=0}^{n}(n-\ell+2)\beta_\ell S_{n-\ell} = 0. \tag{9}$$

The solutions for $n = 0, 1$ are,

$$S_0(u) = \frac{T_{00}}{(1-\beta_0 u)^2}, S_1(u) = \frac{1}{\beta_0(1-\beta_0 u)^3}\Big[-2T_{10}(\beta_1 - 2\beta_0\gamma_0)\log(1-\beta_0 u) + \beta_0 T_{00}\Big].$$

The new RGS expansions now can be written as,

$$S^{N^4LO}_{RGSPT} \;=\; S_0(xL) + xS_1(xL) + x^2 S_2(xL) + x^3 S_3(xL) + x^4 S_4(xL) \tag{10}$$

The above RGSPT expansions exhibit good stability and reduced sensitivity to RG scale μ.

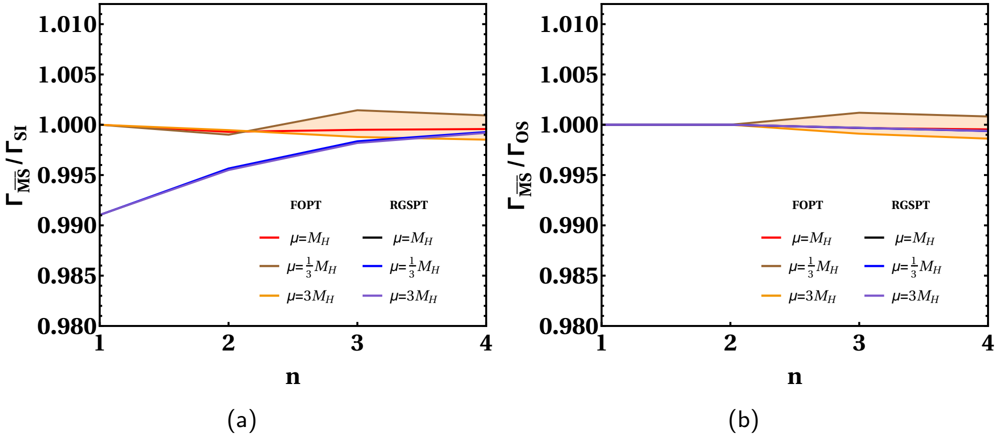


Figure: **The variation of $\Gamma_{\overline{\text{MS}}}/\Gamma_{\text{SI}}$ and $\Gamma_{\overline{\text{MS}}}/\Gamma_{\text{OS}}$ at RG scales $\mu = \frac{1}{3}M_H, M_H$, and $3M_H$ in the FOPT and RGSPT up to order $n = 4$.**

APAP improved $H \to gg$ decay rate

Considering a generic perturbative expansion of the form,

$$S \equiv 1 + R_1 x + R_2 x^2 + R_3 x^3 + R_4 x^4 + \cdots, \tag{11}$$

where the coefficients $\{R_1, R_2, R_3, R_4\}$ are known and the coefficients $\{R_5, \cdots\}$ are unknown.
The Padé approximant to a generic perturbative expansion is denoted by,

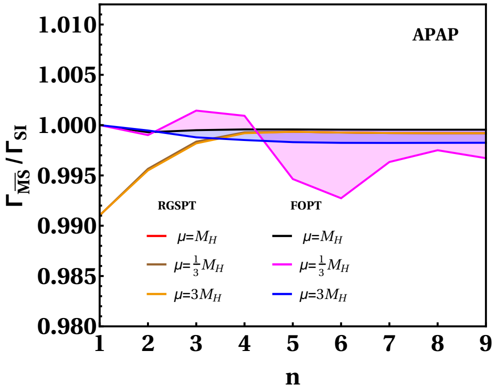
$$\begin{aligned} S_{[N|M]} &\equiv \frac{1 + a_1 x + a_2 x^2 + \cdots + a_N x^N}{1 + b_1 x + b_2 x^2 + \cdots + b_M x^M} \\ &= 1 + R_1 x + R_2 x^2 + R_3 x^3 + R_4 x^4 + \cdots + R_{N+M+1}\, x^{N+M+1} + \cdots. \end{aligned} \tag{12}$$

The asymptotic error in the Padé approximant prediction is given by,

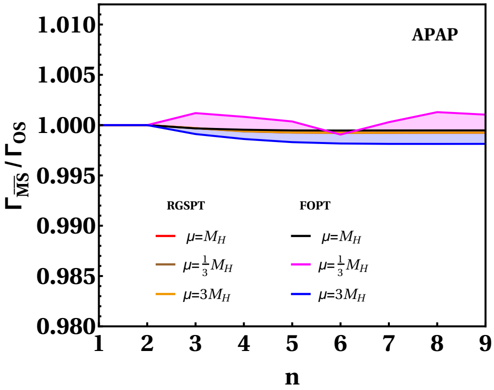
$$\frac{R_{N+M+1}^{Padé} - R_{N+M+1}}{R_{N+M+1}} = -\frac{M!A^M}{[N+M+aM+b]^M} \tag{13}$$

In this work, we choose $a = b = 0$ which provide the best predictions.
Our APAP estimate of R_5 is,

$$\begin{aligned} R_5 &= \frac{(-R_3^3 + 2R_2R_3R_4 - R_1R_4^2)}{(1+\delta)(R_2^2 - R_1R_3)} \\ &= \frac{8R_2^2(R_3^3 - 2R_2R_3R_4 + R_1R_4^2)}{(R_1^4 - 2R_1^2R_2 - 7R_2^2)(R_2^2 - R_1R_3)}. \end{aligned} \tag{14}$$



(a)



(b)

Figure: **The variation of $\Gamma_{\overline{MS}}/\Gamma_{SI}$ and $\Gamma_{\overline{MS}}/\Gamma_{OS}$ at RG scales $\mu = \frac{1}{3}M_H, M_H$, and $3M_H$ in the FOPT and RGSPT up to order $n = 9$.**

PBA improved $H \to gg$ decay rate

We apply the APAP formalism to the generalized Borel transform (eq.15) of the FOPT expansion of the Higgs to gluons decay width. This method is referred to as Padé-Borel approximant (PBA).

$$B[S](u) = \sum_{n}^{\infty} \left(\frac{d_1}{n!} + \frac{d_2}{n!^2} + \frac{d_3}{n!^3} + \frac{d_5}{n!^5} \right) R_n u^n, \tag{15}$$

where $d_{1,2,3,5}$ are the scheme-dependent real constants given in table 1.

Schemes	d_1	d_2	d_3	d_5
$\overline{\text{MS}}$	0.5	1.5	0	1.2
OS	1	0	1.623	0
SI	0.87	0	1.6	0

Table: **The numerical values of the constants $d_{1,2,3,5}$.**

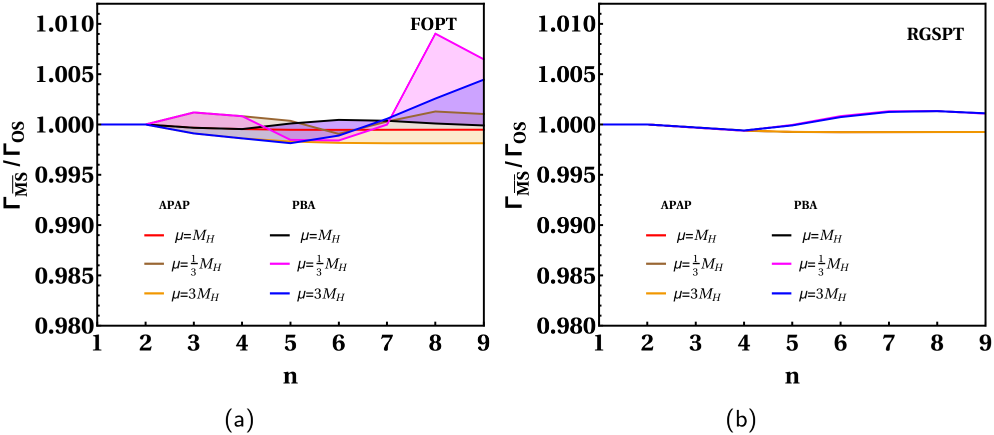


Figure: **The variation of $\Gamma_{\overline{MS}}/\Gamma_{OS}$ at RG scales $\mu = \frac{1}{3}M_H, M_H$, and $3M_H$ in the FOPT and RGSPT up to order $n = 9$.**

Higgs to gluons decay width

Our predictions for the $\Gamma(H \to gg)$ decay width at the order $\mathrm{N^5LO}$ in the APAP formalism in the FOPT are,

$$\Gamma^{\overline{\mathsf{MS}}}_{\mathrm{N^5LO}} = \Gamma_0\Big(1.837 \pm 0.047_{\alpha_s(M_Z),1\%} \pm 0.0004_{M_t} \pm 0.0066_{M_H} \pm 0.0009_{\mathrm{P}} \pm 0.007_{\mathsf{s}}\Big),$$
$$\Gamma^{\mathsf{SI}}_{\mathrm{N^5LO}} = \Gamma_0\Big(1.837 \pm 0.046_{\alpha_s(M_Z),1\%} \pm 0.0004_{M_t} \pm 0.0066_{M_H} \pm 0.0026_{\mathrm{P}} \pm 0.007_{\mathsf{s}}\Big),$$
$$\Gamma^{\mathsf{OS}}_{\mathrm{N^5LO}} = \Gamma_0\Big(1.838 \pm 0.047_{\alpha_s(M_Z),1\%} \pm 0.0004_{M_t} \pm 0.0066_{M_H} \pm 0.0023_{\mathrm{P}} \pm 0.007_{\mathsf{s}}\Big).$$

Our predictions for the $\Gamma(H \to gg)$ decay width at the order $\mathrm{N^5LO}$ in the APAP formalism in the RGSPT are,

$$\Gamma^{\overline{\mathrm{MS}}}_{\mathrm{RGSN^5LO}} = \Gamma_0\Big(1.840 \pm 0.047_{\alpha_s(M_Z),1\%} \pm 0.0005_{M_t} \pm 0.0066_{M_H} \pm 0.0002_{\mu} \pm 0.0007_{\mathrm{P}}\Big),$$
$$\Gamma^{\mathrm{SI}}_{\mathrm{RGSN^5LO}} = \Gamma_0\Big(1.841 \pm 0.047_{\alpha_s(M_Z),1\%} \pm 0.0005_{M_t} \pm 0.0066_{M_H} \pm 0.0002_{\mu} \pm 0.0018_{\mathrm{P}}\Big),$$
$$\Gamma^{\mathrm{OS}}_{\mathrm{RGSN^5LO}} = \Gamma_0\Big(1.842 \pm 0.047_{\alpha_s(M_Z),1\%} \pm 0.0005_{M_t} \pm 0.0066_{M_H} \pm 0.0002_{\mu} \pm 0.0019_{\mathrm{P}}\Big).$$

Summary

- The dependence of the perturbative expansion on the RG scale μ is considerably reduced in the RGSPT.
- We have calculated the higher-order corrections to the $\Gamma(H \to gg)$ decay width in the APAP and PBA formalisms.
- The predictions of PBA method are found to be in agreement with that of APAP method.
- The RGSPT expansion continue to show greater stability against the RG scale at higher orders in the APAP as well as the PBA frameworks.
- The uncertainty due to truncation of the series is 0.6% at N^4LO, and reduces to 0.4% at N^5LO in the FOPT.

Thank You