Towards HH at NNLO QCD: the n_h^2 Contribution

Marco Vitti (Karlsruhe Institute of Technology, TTP and IAP)

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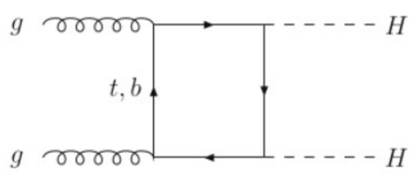
Work in collaboration with **J. Davies, K. Schönwald, M. Steinhauser**

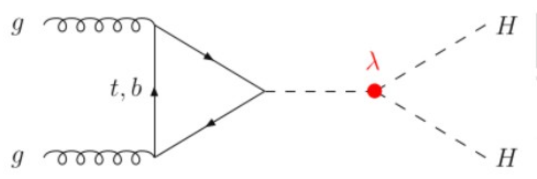
HH Production @LHC

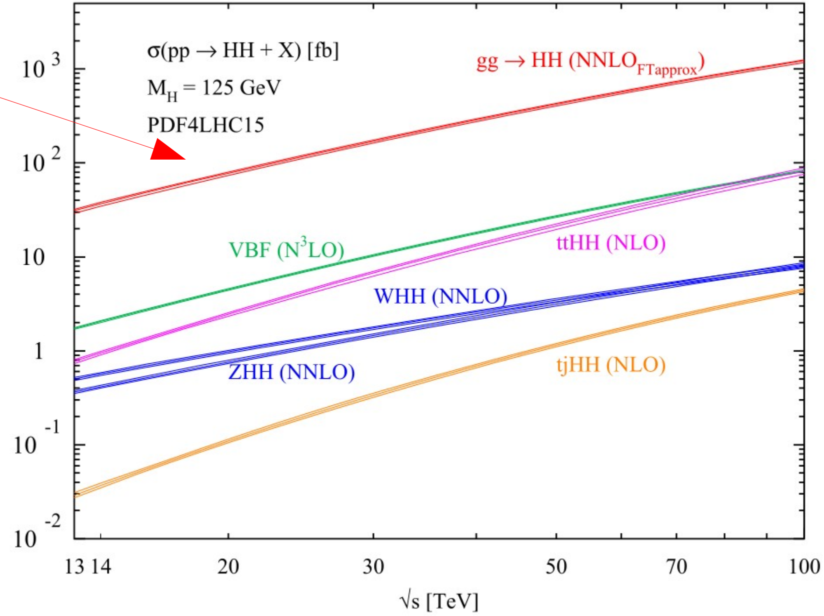
$$V(h) = \frac{m_H^2}{2} h^2 + \boxed{\lambda_3} v h^3 + \frac{\lambda_4}{4} h^4$$

$$\lambda_4^{SM} = \lambda_3^{SM} = \lambda = m_H^2/(2\,v^2)$$

- Best chance to measure λ_3
- Gluon-initiated channel dominant





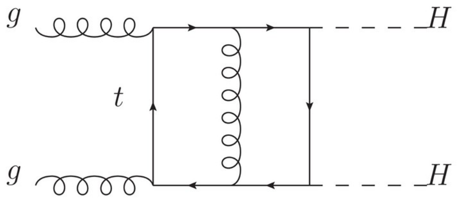


[Di Micco et al. - 1910.00012]

- LO computed in [Glover, van der Bij (‘88); Plehn et al. (96)]
- In the SM, destructive interference between triangle (signal) and box (background)
- Accurate higher-order predictions required for **both**

NLO QCD corrections for HH

Full top-mass dependence obtained via



Multi-scale $(s, t, m_{H,} m_t)$
two-loop box integrals
No exact analytic results available

- **Numerical evaluation**

[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke - 1604.06447, Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Zirke - 1608.04798; Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher - 1811.05692]

- **Analytic approximations**

Large Mass Expansion [Dawson, Dittmaier, Spira '98; Grigo, Hoff, Melnikov, Steinhauser - 1305.7340]

pT expansion [Bonciani, Degrassi, Giardino, Gröber - 1806.11564]

High-Energy expansion [Davies, Mishima, Steinhauser, Wellmann - 1801.09696 1811.05489]

Small-mass expansion [Wang, Wang, Xu, Xu, Yang - 2010.15649]

Full phase space covered in [Bellafronte, Degrassi, Giardino, Gröber, MV – 2202.12157; Davies, Mishima, Schönwald, Matthias Steinhauser - 2302.01356]

Public codes: [Bagnaschi, Degrassi, Gröber – 2309.10525] ; ggxy [Davies, Schönwald, Steinhauser, Stremmer - 2506.04323]

(implemented in POWHEG)

See Daniel's talk

Theoretical Uncertainties at NLO QCD

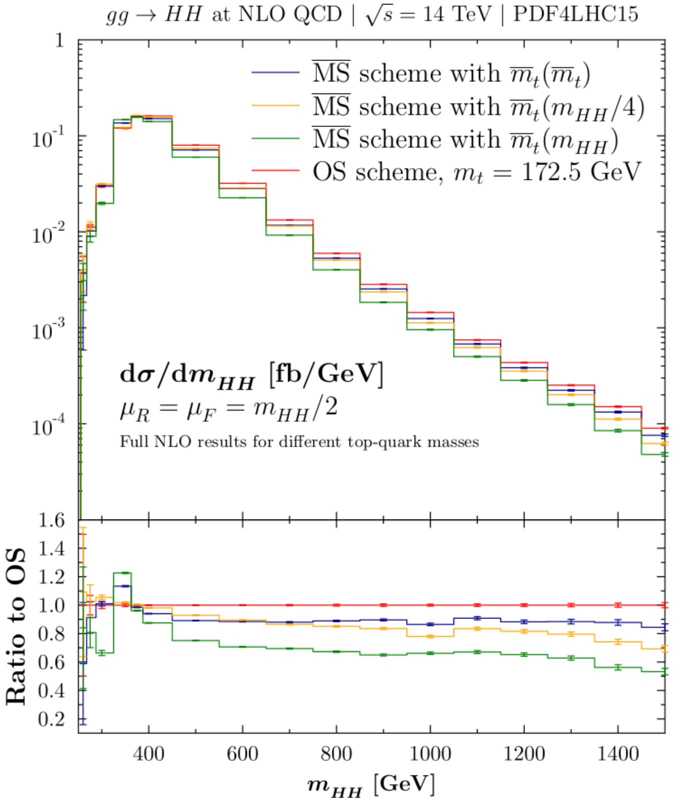
- Scale uncertainties reduced to O(15%)

Top-mass scheme	LO [fb]	$\sigma_{LO}/\sigma_{LO}^{OS}$	NLO [fb]	$\sigma_{NLO}/\sigma_{NLO}^{OS}$
On-Shell	$18.22^{29.5\%}_{-21.3\%}$	-	$30.93^{13.7\%}_{-12.7\%}$	-

[Bagnaschi, Degrassi, Gröber - 2309.10525]

(Reduced to ~3% when approximate N3LO corrections are included)

- Uncertainty of ~25% due to choice of renormalization scheme and scale for the top mass
- Top mass effects must be retained at NNLO to reduce top-mass uncertainty



[Baglio et al. - 2003.03227]

Analytic approximations for NNLO QCD

Exploit **hierarchies** of masses/kinematic invariants

Pros: simplified integral structures; can change parameters and evaluate easily

Cons: proliferation of integrals; restricted to specific phase-space regions

$m_t \to \infty$ limit (N3LO) [De Florian, Mazzitelli 1305.5206 and 1309.6594; Grigo, Melnikov, Steinhauser – 1408.2422; Chen, Li, Shao, Wang – 1909.06808 and 1912.13001;]

Finite $1/m_t$ effects (LME) (restricted to $s < 4m_t^2$)
[Grigo, Hoff, Steinhauser – 1508.00909;
Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli - 1803.02463;
Davies, Steinhauser – 1909.01361; Davies, Herren, Mishima, Steinhauser 2110.03697]

High-energy expansion (+ SCET) $m_H^2, 4m_t^2 \ll \hat{s}, \hat{t}$ [Jaskiewicz, Jones, Szafron, Ulrich – 2501.00587]

Forward Expansions? - Cover ~95% of hadronic cross section at NLO QCD
- Taylor expansions

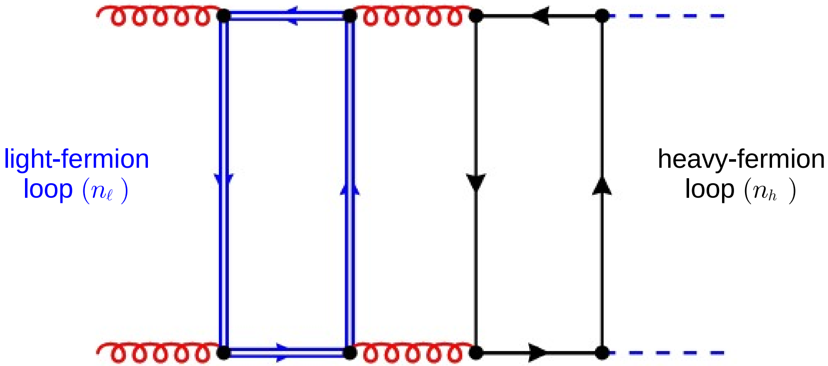
- pT expansion $m_H^2, p_T^2 \ll 4m_t^2, \hat{s}$
[Bonciani, Degrassi, Giardino, Gröber - 1806.11564]

- *t* → *0* expansion $m_H^2, \hat{t} \ll 4m_t^2, \hat{s}$
[Davies, Mishima, Schönwald, Steinhauser - 2302.01356]

The road to NNLO QCD...

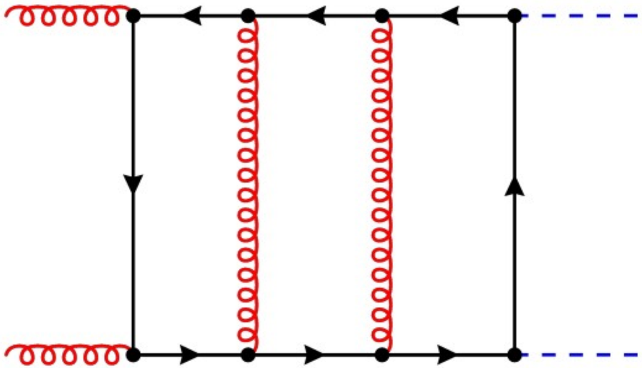
Can we use the forward expansion for higher orders?

- YES, for classes of topologies featuring a single heavy loop (n_h contribution)



$t \to 0$ expansion used to obtain the leading term $\{t^0, m_H^0\}$

[Davies, Schönwald, Steinhauser - 2307.04796]



$t \to 0$ expansion in the large-Nc limit

[Davies, Schönwald, Steinhauser - 2503.17449]

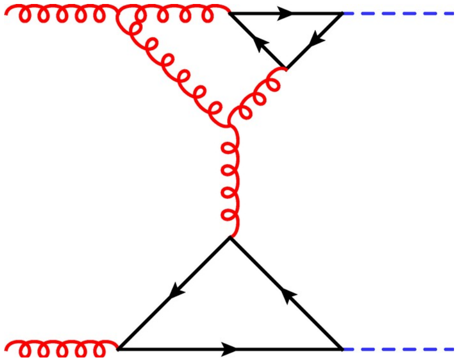
The road to NNLO QCD...

Can we use the forward expansion for higher orders?

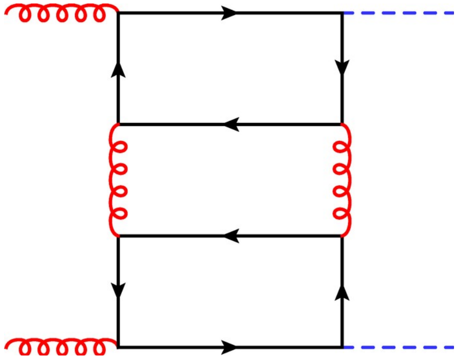
- n_h^2 **contribution**: new topologies arising at NNLO QCD involve diagrams where the Higgs bosons couple to two indpendent heavy-fermion lines
- These diagrams admit t-channel cuts through massless lines

[Beneke, Smirnov ('98)]

- A Taylor expansion is not sufficient and we rely on the strategy of **expansions by regions**



Observed in 1PR contribution
[Davies, Schönwald, Steinhauser, MV - 2405.20372]



In **this talk** we consider 1PI diagrams

Details of calculation

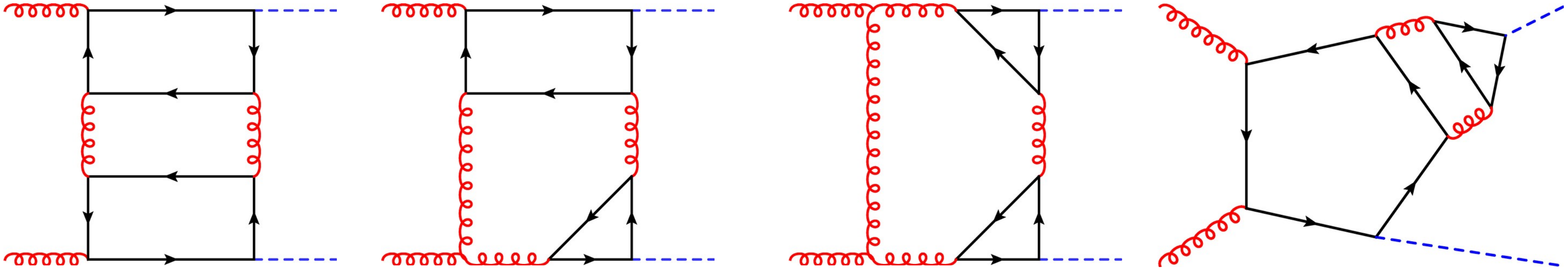
1. Group the diagrams and map onto **37** independent topologies

   qgraf [Nogueira, ‘93]; **tapir** [Gerlach, Herren, Lang – 2201.05618];
   q2e/exp [Harlander, Seidensticker Steinhauser – ‘97]

2. Analyze the topologies using **asy.m** [Pak, Smirnov - 1011.4863], searching for relevant regions in the forward limit:

$$\boxed{m_H^2 \to \lambda\, m_H^2, \qquad t \to \lambda\, t \qquad \lambda \to 0}$$

   Types of regions: hard; soft / ultrasoft; collinear



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3. Calculation of individual regions and their sum

- Hard region:

  → Taylor expansion in the forward limit (**FORM**) [Ruijl, Ueda, Vermaseren - 1707.06453]

  → IBP reduction (**KIRA**) [Klappert, Lange, Maierhöfer, Usovitsch – 2008.06494]

  → MIs evaluated semi-analytically using “expand and match” approach [Fael, Lange, Schönwald, Steinhauser – 2106.05296; 2202.05276]

Non-hard regions

- The presence of collinear regions complicates the treatment of the diagrams in momentum representation, and prevents us from using IBP reduction
- We then perform the calculation of the non-hard regions in the Schwinger parametrization

$$I = \int_0^\infty \prod_{i=1}^{P} dx_i \frac{x_i^{n_i}}{\Gamma(n_i+1)} \left(\mathcal{U}^{-d/2} e^{-\frac{\mathcal{F}}{\mathcal{U}}}\right)$$

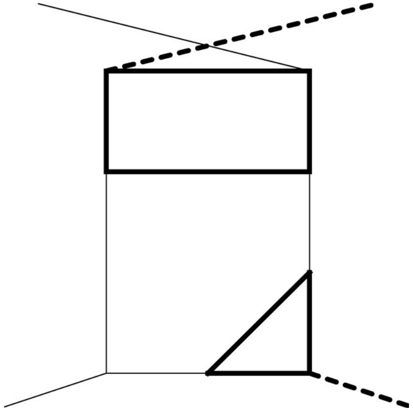
($\mathcal{U}$, $\mathcal{F}$ are Symanzik Polynomials)

$\Rightarrow$ For each non-hard region:

1. Expand at the integrand level using **FORM**
2. Perform parametric integration and obtain Mellin-Barnes (MB) representation
3. Transform the MB-integrals into infinite sums over residues of Gamma functions
4. Express the infinite sums in terms of iterated integrals (e.g. HPLs) using **HarmonicSums** [Ablinger et al. - ‘10], **Sigma** [Schneider et al. - ‘07], and **EvaluateMultiSums** [Schneider et al. - ‘07]

An example integral

- Consider a 10-propagator planar integral with a massive box and a massive triangle connected by massless lines
- Two relevant regions: hard and collinear
- Hard region: expression in terms of semi-analytic MIs depending on a single scale: s/m_t^2



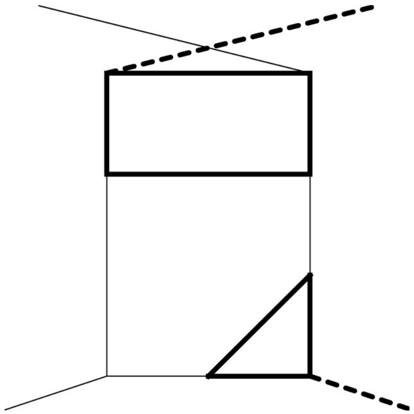
- Collinear region: MB representation for the leading expansion term

$$\mathrm{I}_c \sim \int_{-i\infty}^{+i\infty} \int_{-i\infty}^{+i\infty} dz_1\, dz_2 \left(\frac{s}{m_t^2}\right)^{-z_1} \left(\frac{t}{m_H^2}\right)^{z_2} \frac{\Gamma(-z_1)\Gamma(-z_2)\Gamma(z_2+1)\Gamma(-\epsilon-z_1-1)^2\Gamma(\epsilon+z_1+2)\Gamma(\epsilon+z_2+1)\Gamma(-2\epsilon-z_1-z_2-2)}{\Gamma(-2\epsilon-2z_1)\Gamma(-3\epsilon-z_1-1)}$$

- Need to compute ~2700 integrals related to this topology, since we are not using IBP reduction

$\Rightarrow$ Automatization with FORM+Mathematica code

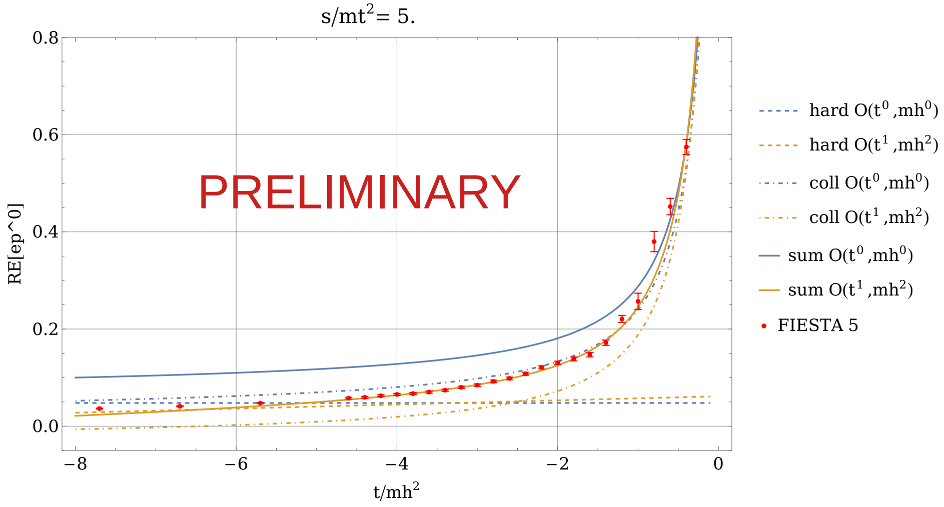
An example integral

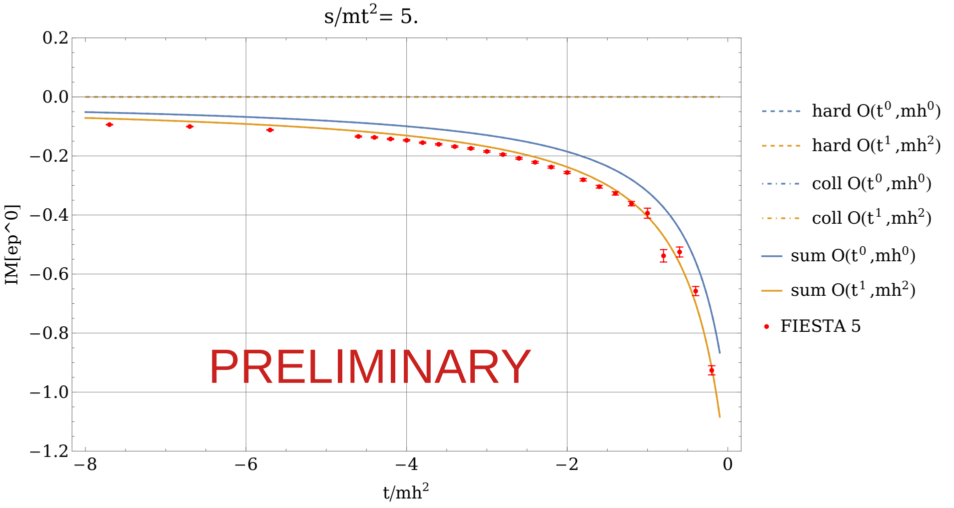


Two terms in the region expansion provide an agreement at the 20% level

Checked against numerical evaluation with FIESTA 5 [Smirnov, Shapurov, Vysotsky - 2110.11660]

Larger coverage in s/m_t^2 compared to the Large Mass Expansion





Conclusions

- Including NNLO QCD effects in $gg \to HH$ would allow control over scale and top-mass-scheme uncertainties
- An expansion in the forward-scattering limit is a promising way to obtain fast and flexible results *and* a wide coverage of the phase space
- At three loops, asymptotic expansions are necessary to account for the n_h^2 contribution
- The proposed approach seems to work for planar topologies

Outlook

- Complete calculation of the full n_h^2 contribution
- Combination with available results for n_h : all purely-virtual 3-loop corrections
- Many challenges ahead...

Thank you for your attention

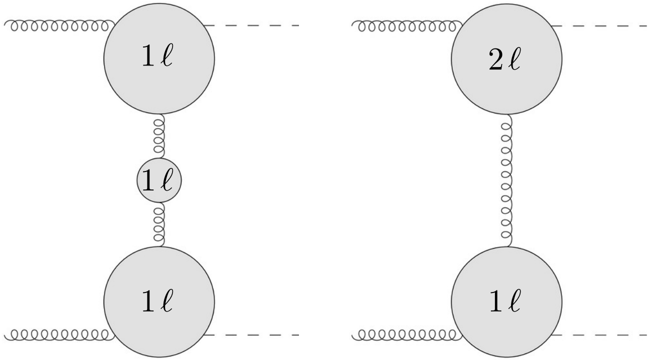
Backup

1PR Contribution to $gg \to HH$ @ 3 Loops

[Davies, Schönwald, Steinhauser, MV - 2405.20372]

$$\mathcal{M}^{ab} = \varepsilon_{1,\mu}\varepsilon_{2,\nu}\mathcal{M}^{\mu\nu,ab} = \varepsilon_{1,\mu}\varepsilon_{2,\nu}\delta^{ab}X_0 s\left(F_1 A_1^{\mu\nu} + F_2 A_2^{\mu\nu}\right)$$

Goal: compute $F_1^{(3\ell,\ 1\mathrm{PR})}$ $F_2^{(3\ell,\ 1\mathrm{PR})}$



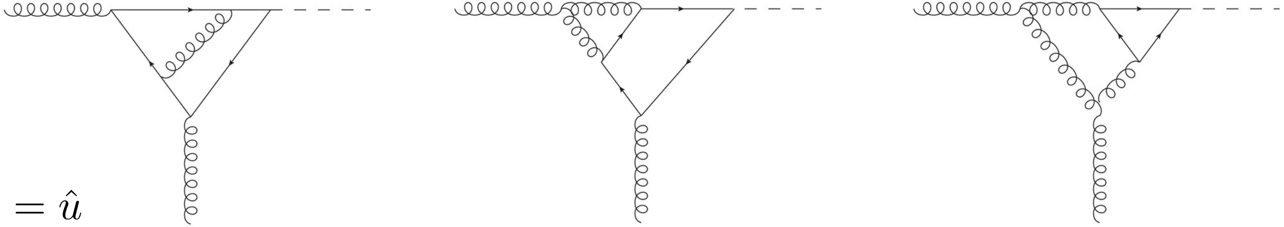
Approach: construct the $gg \to HH$ form factors from the 1PI gg*H subamplitudes

$$\mathcal{V}^{\alpha\beta}(q_s, q_2) = F_a\ g^{\alpha\beta}(q_s \cdot q_2) + F_b\ q_s^\alpha q_2^\beta + F_c\ q_2^\alpha q_s^\beta + F_d\ q_s^\alpha q_s^\beta + F_e\ q_2^\alpha q_2^\beta$$

$q_2^2 = 0, q_s^2 \neq 0$

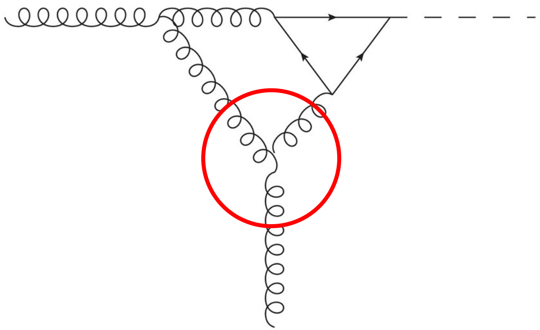
$m_H \neq 0$

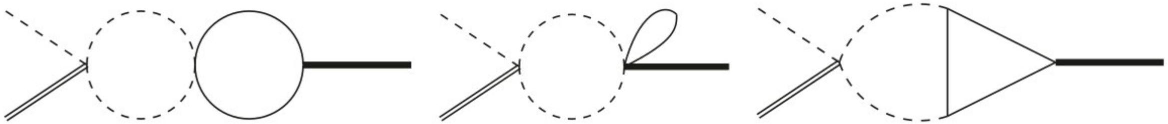
$q_s^2 = \hat{t}$ or $q_s^2 = \hat{u}$



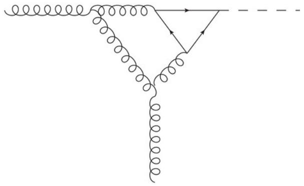
*gg*H* Form Factors

- A Taylor expansion of the two-loop integrals is not possible due to diagrams where the off-shell gluon couples to massless internal lines
- Three topologies require an asymptotic expansion

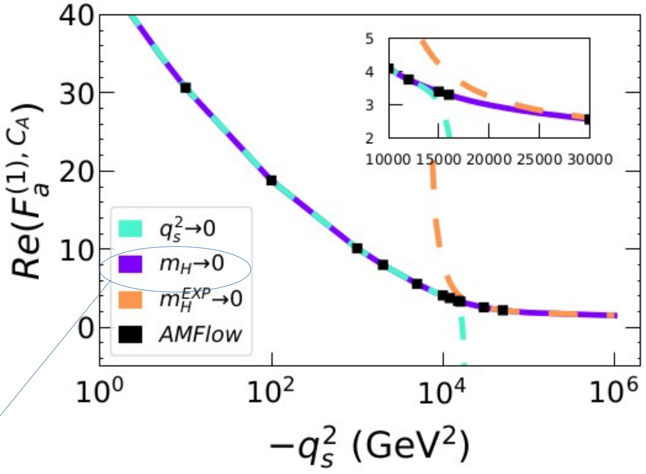


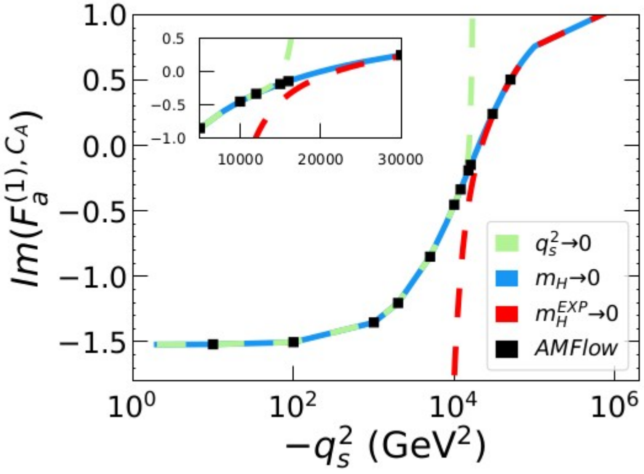


*gg*H* Form Factors



$$\mathcal{V}^{\alpha\beta}(q_s, q_2) = \boxed{F_a}\, g^{\alpha\beta}(q_s \cdot q_2) + F_b\ q_s^\alpha q_2^\beta + F_c\ q_2^\alpha q_s^\beta + F_d\ q_s^\alpha q_s^\beta + F_e\ q_2^\alpha q_2^\beta$$





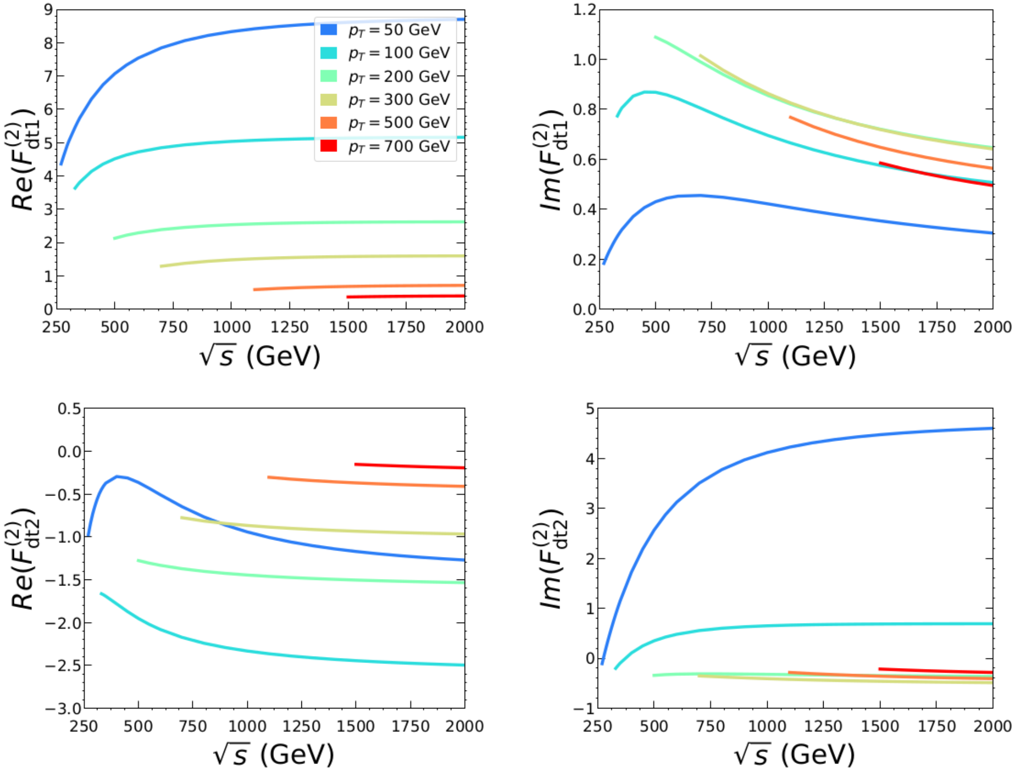
- Use expanded MIs but keep coefficients exact ($m_H \to 0$)
- Results checked with AMFlow [Liu, Ma - 2201.11669]
- Complete coverage of q_s^2 range

$gg \to HH$ Form Factors

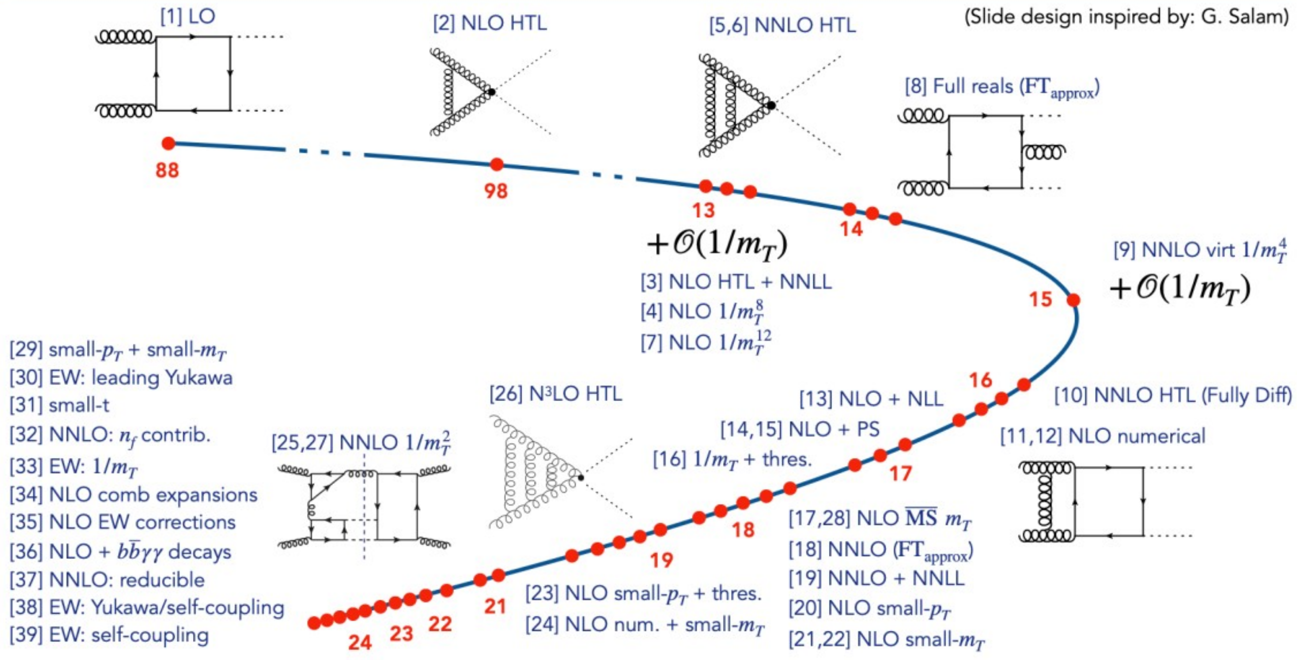
$$\tilde{F}^{(2)}_{\mathrm{dt1}}(t) = F_a^{(0)}(t)\left[F_a^{(1)}(t) + \frac{1}{2}\,F_a^{(0)}(t)\,\Pi_{gg}(t) - \frac{s\,(\epsilon\,(m_H^2 - 2p_T^2 + t) + 2p_T^2)}{(1-2\epsilon)(m_H^2 - s)t}\,F_d^{(1)}(t)\right]$$

$$\tilde{F}^{(2)}_{\mathrm{dt2}}(t) = F_a^{(0)}(t)\left[\frac{p_T^2}{t}\left(F_a^{(1)}(t) + \frac{1}{2}\,F_a^{(0)}(t)\,\Pi_{gg}(t)\right) - \frac{s\,(\epsilon\,(2p_T^2 - m_H^2 - t) + m_H^2 + t)}{(1-2\epsilon)(m_H^2 - s)t}\,F_d^{(1)}(t)\right]$$

- Agreement with LME result of
  [Davies, Steinhauser - 1909.01361]
- Complete coverage of physical phase space for HH form factors



Overview



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; [30] Davies, Mishima, Schönwald, Steinhauser, Zhang 22; [31] Davies, Mishima, Schönwald, Steinhauser 23; [32] Davies, Schönwald, Steinhauser 23; [33] Davies, Schönwald, Steinhauser, Zhang 23; [34] Bagnaschi, Degrassi, Gröber 23; [35] Bi, Huang, Huang, Ma Yu 23 [36] Li, Si, Wang, Zhang, Zhao 24; [37] Davies, Schönwald, Steinhauser, Vitti 24; [38] Heinrich, SPJ, Kerner, Stone, Vestner [39] Li, Si, Wang, Zhang, Zhao 24

[Credit: Stephen Jones]