Approximate N³LO PDFs and implications for Higgs production at the LHC

Based on 2411.05373 with NNPDF and MSHT collaborations

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Motivation

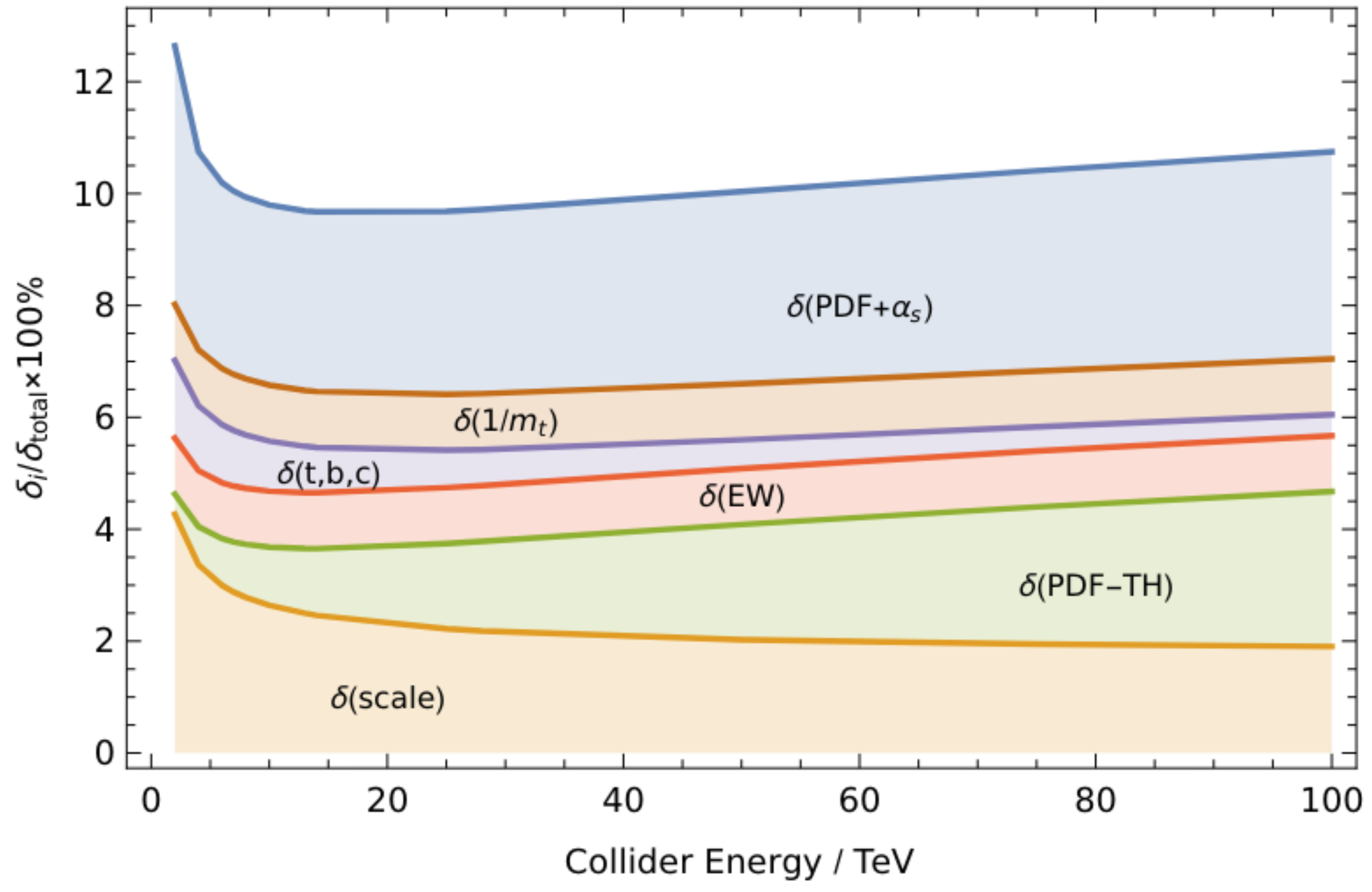
Experiment

PDF fit

pQCD

$$\sigma = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij}$$

- Predictions at particle colliders such as the LHC use two main ingredients:
  - Matrix elements (MEs)
  - Parton distribution functions (PDFs)
- Much progress has been made in the computation of MEs at N^3LO
- **PDF uncertainties are a bottleneck** for many LHC precision calculations
- Most widely used PDF sets are at NNLO and without theory uncertainties



Sources of uncertainty for Higgs in gluon fusion

Dulat, Lazopoulos, Mistlberger, 1802.00827

Much progress since this plot, in particular:

- NNLO top quark corrections Czakon et al., 2105.04436
- Mixed QDC-EW corrections
  Becchetti et al., 2010.09451, Bonetti, et al., 2007.09813

- What does approximate N^3LO mean?

Theory ingredients for PDFs

- **DGLAP splitting functions**
  small-x and large-x limits
  Mellin moments up to N=20
- **Matching conditions for variable flavor number schemes**
  Now exactly known but original aN^{3}LO publications use approximations
- **DIS coefficient functions**
  Massless known, massive limits known
- **Hadronic cross-section**
  Not much is known

Strategy:

- When N^{3}LO theory is known, it is **used**
- When partial information is available, use it while accounting for **parametrisation uncertainty**
- When it is unknown account for **missing higher order uncertainty**

N^{3}LO QCD corrections in PDF determination

Splitting Functions (information is partial)

Singlet (P_{qq}, P_{gg}, P_{gq}, P_{qg})
- large-n_f limit [NPB 915 (2017) 335; arXiv:2308.07958]
- small-x limit [JHEP 06 (2018) 145]
- large-x limit [NPB 832 (2010) 152; JHEP 04 (2020) 018; JHEP 09 (2022) 155]
- 5 (10) lowest Mellin moments [PLB 825 (2022) 136853; ibid. 842 (2023) 137944; ibid. 846 (2023) 138215]

Non-singlet ($P_{NS,v}$, $P_{NS,+}$, $P_{NS,-}$)
- large-n_f limit [NPB 915 (2017) 335; arXiv:2308.07958]
- small-x limit [JHEP 08 (2022) 135]
- large-x limit [JHEP 10 (2017) 041]
- 8 lowest Mellin moments [JHEP 06 (2018) 073]

DIS structure functions (F_L, F_2, F_3)
- DIS NC (massless) [NPB 492 (1997) 338; PLB 606 (2005) 123; NPB 724 (2005) 3]
- DIS CC (massless) [Nucl.Phys.B 813 (2009) 220]
- massive from parametrisation combining known limits and damping functions [NPB 864 (2012) 399]

PDF matching conditions
- all known except for $a^3_{H,g}$ [NPB 820 (2009) 417; NPB 886 (2014) 733; JHEP 12 (2022) 134]

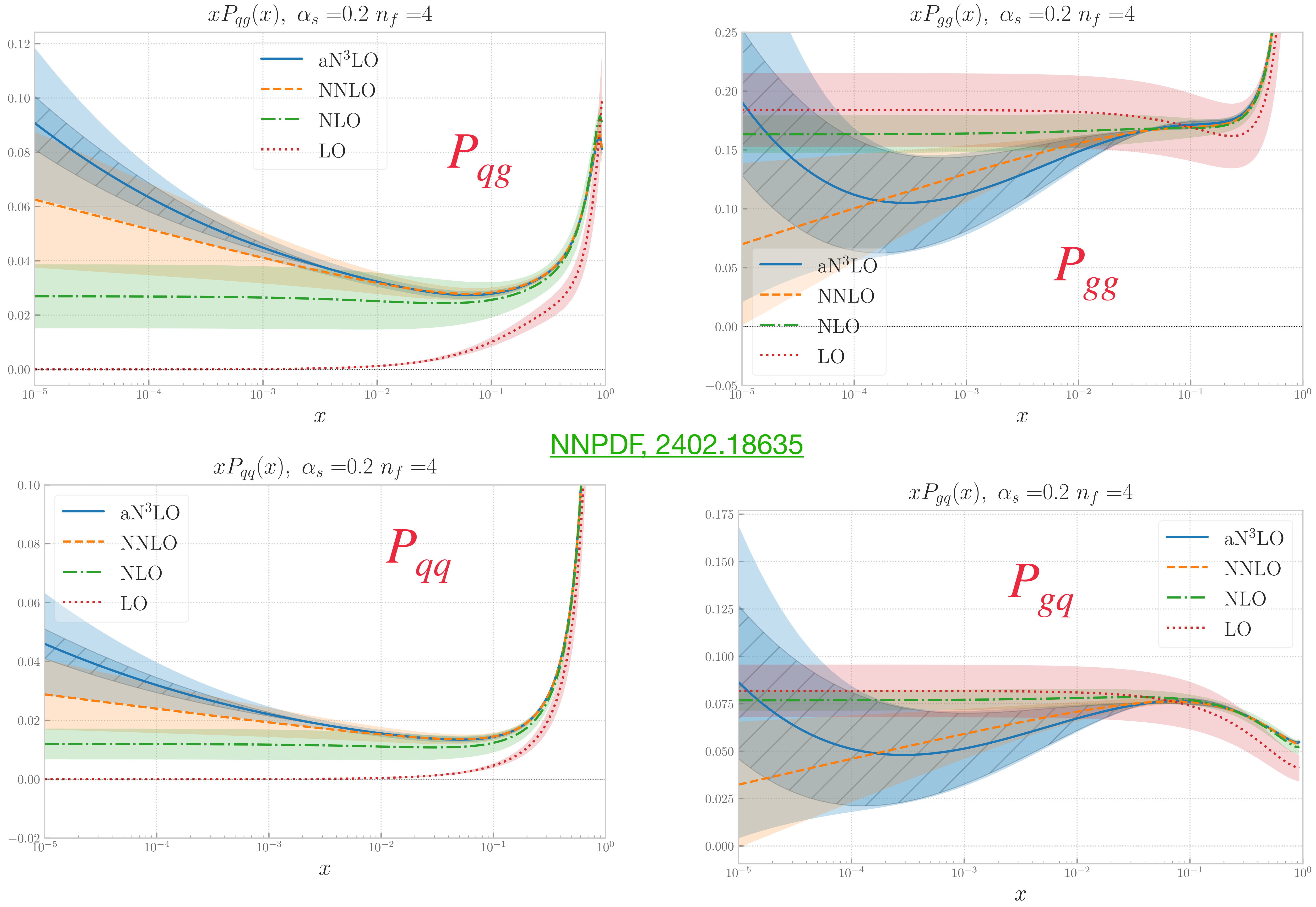
Coefficient functions for other processes
- DY (inclusive) [JHEP 11 (2020) 143]; DY (y differential) [PRL 128 (2022) 052001]

Emanuele R. Nocera (UNITO) | Progress from NNPDF | 5 August 2024 | 12 / 25

E. Nocera, Workshop on Hadron Physics and Opportunities Worldwide
Dalian, China, August 2024
(More is known today!)

Approximate N^{3}LO splitting functions

See also Giulio Falcioni's talk from this morning



NNPDF, 2402.18635

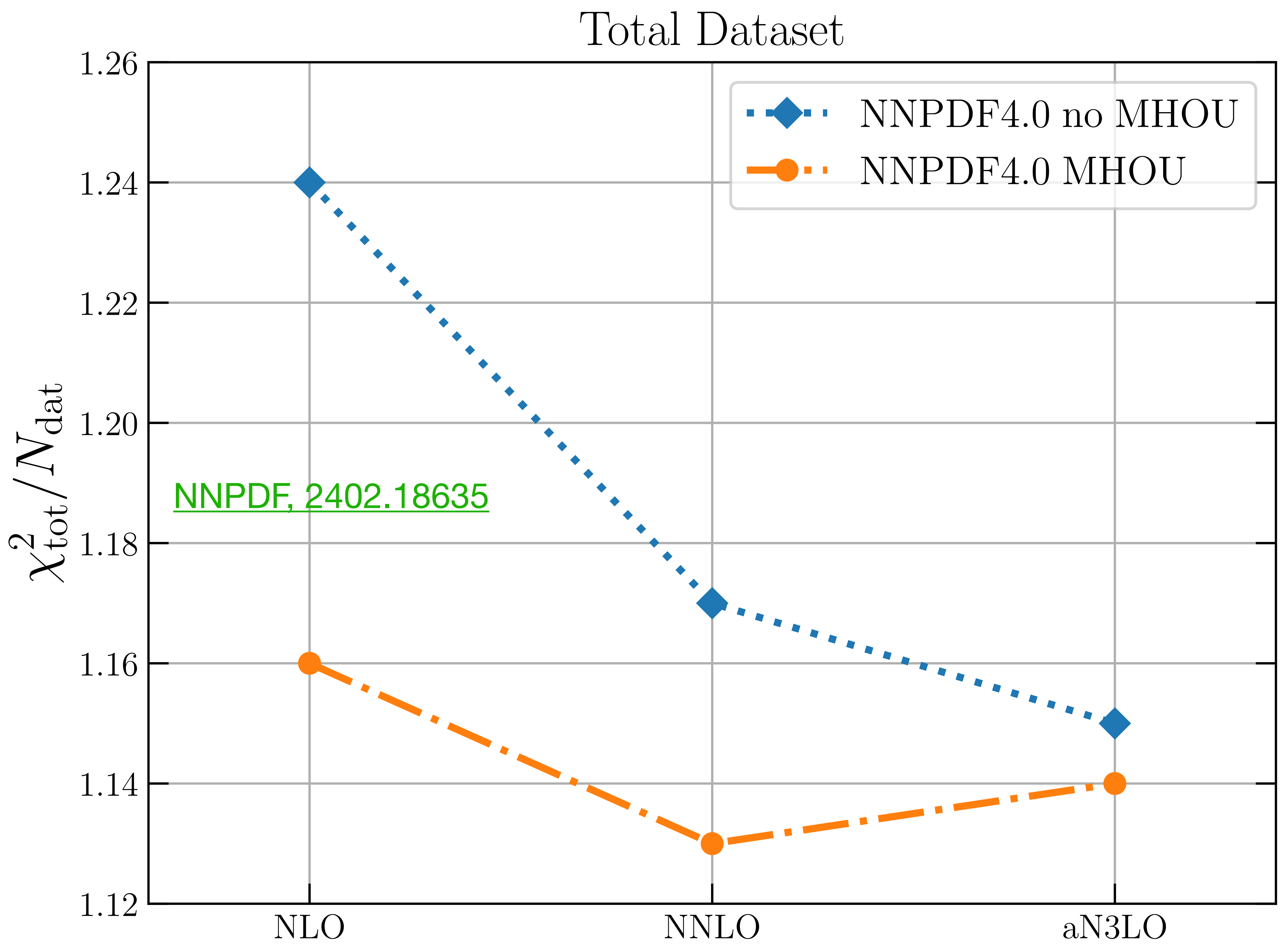
Dark blue: uncertainties due to parametrization of aN3LO contributions

Light blue: scale variations

- Good **perturbative stability** within uncertainties
- **Small parametrisation uncertainty** in large range of x

Approximate N3LO evolution is close to exact

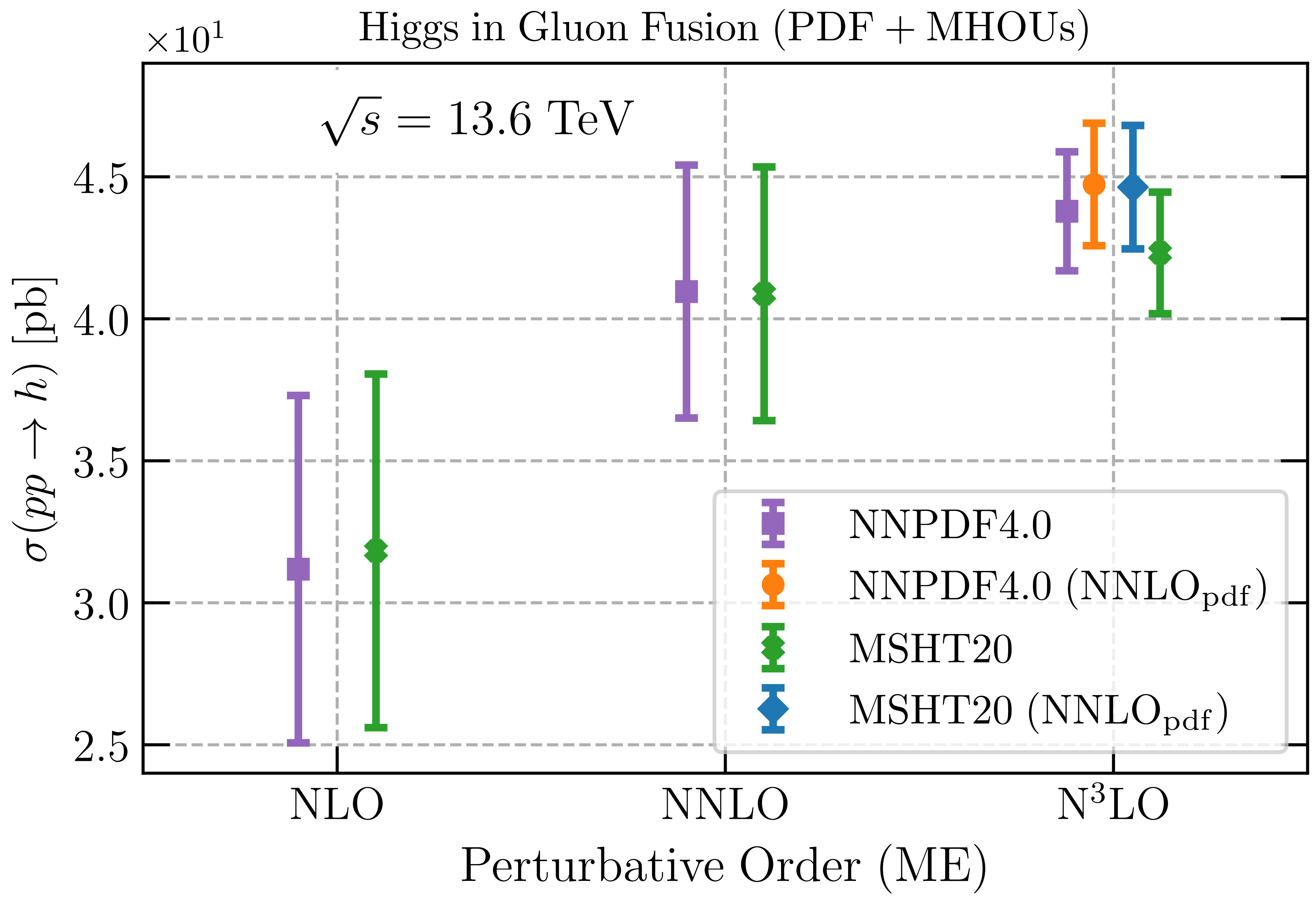
Fit quality



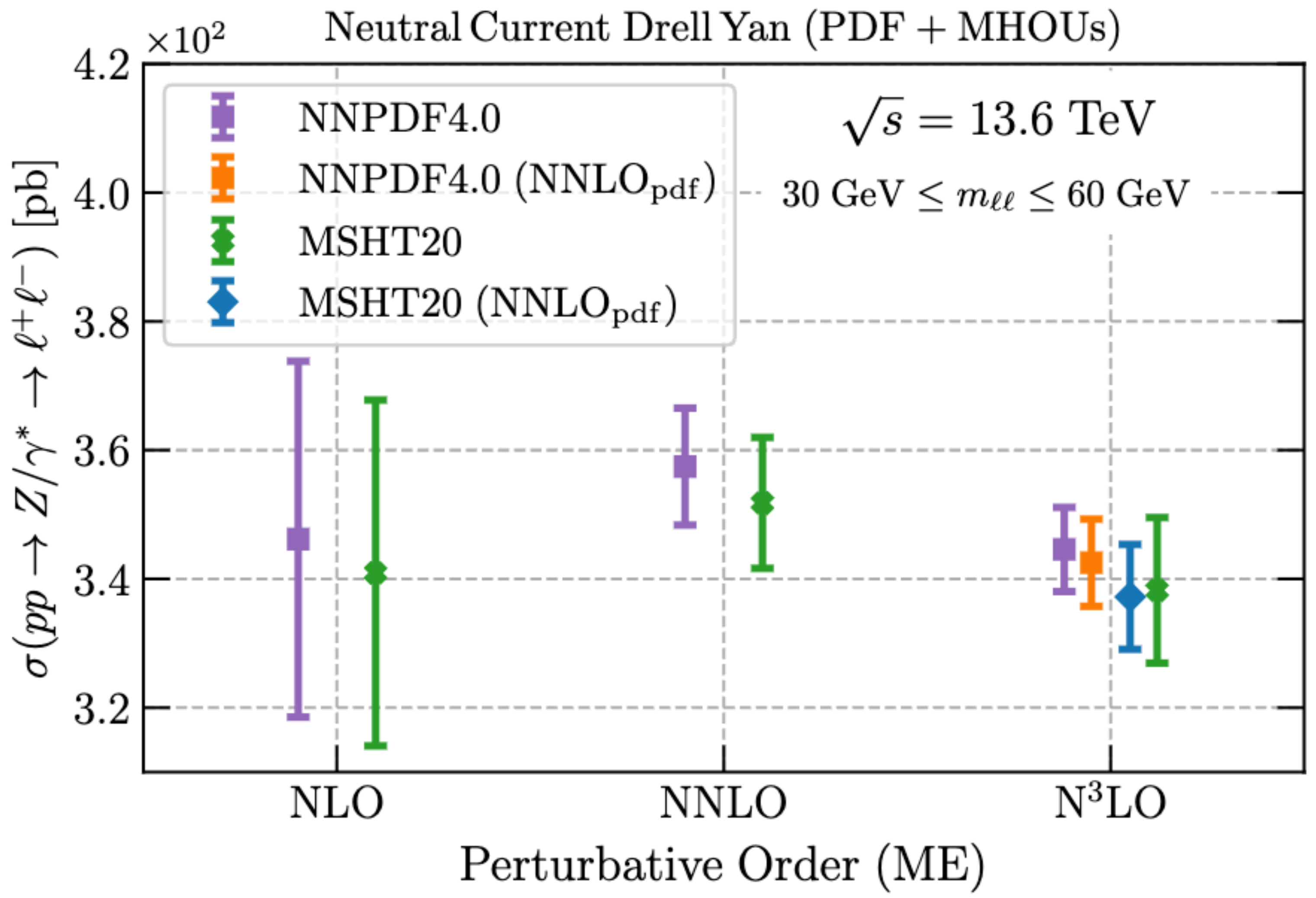
- Without MHOUs the fit improves (lower χ^2) with increasing perturbative order
- With MHOUs the fit depends only weakly on the perturbative order
- At N^{3}LO MHOUs have a small impact on the χ^2

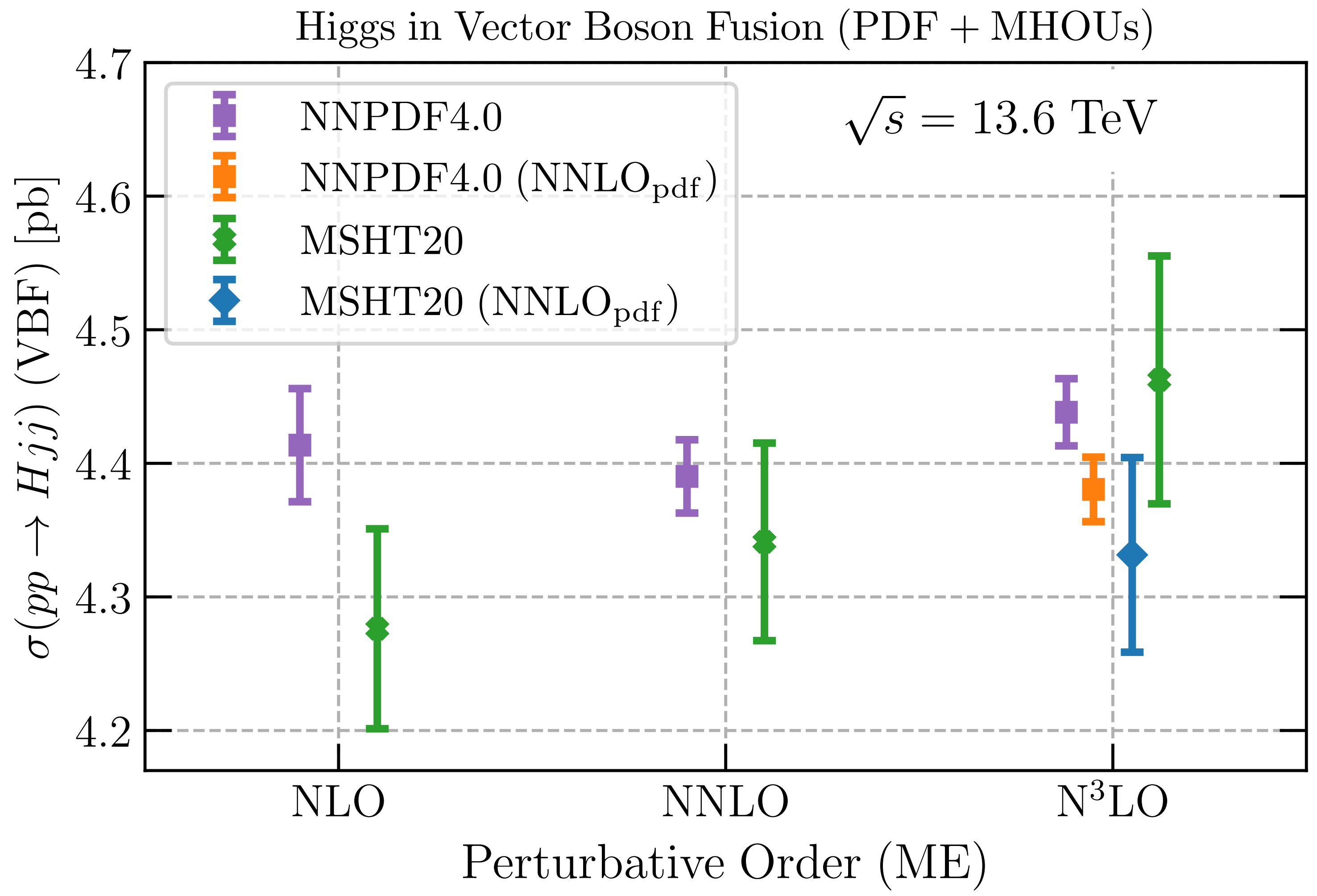
Phenomenology

NNPDF, 2402.18635



N^{3}LO PDFs result in a small suppression of the Higgs gluon fusion cross section compared to **NNLO PDFs**



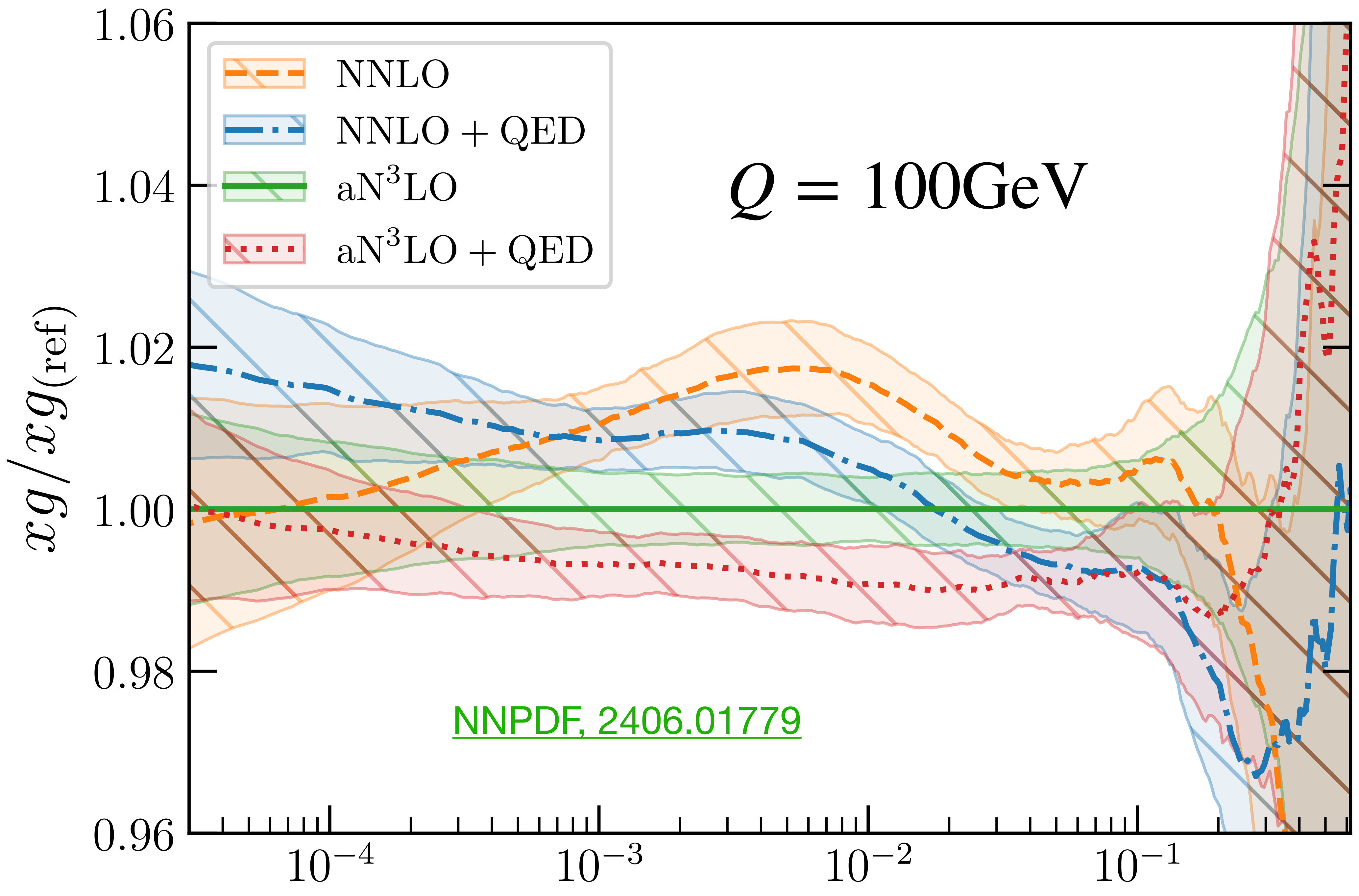


Generally perturbative convergence for Higgs in VBF and Drell-Yan

N^{3}LO/NNLO ratio is similar for NNPDF and MSHT

QED corrections and photon PDF

PDFs at $\mathrm{aN^3LO_{QCD}} \otimes \mathrm{NLO_{QED}}$ with photon PDF represents the **most accurate PDFs**



- Photon subtracts momentum from the gluon PDF
- QED+photon corrections are similar magnitude as $\mathrm{aN^3LO}$ corrections

- MSHT+NNPDF aN3LO(+QED) combination

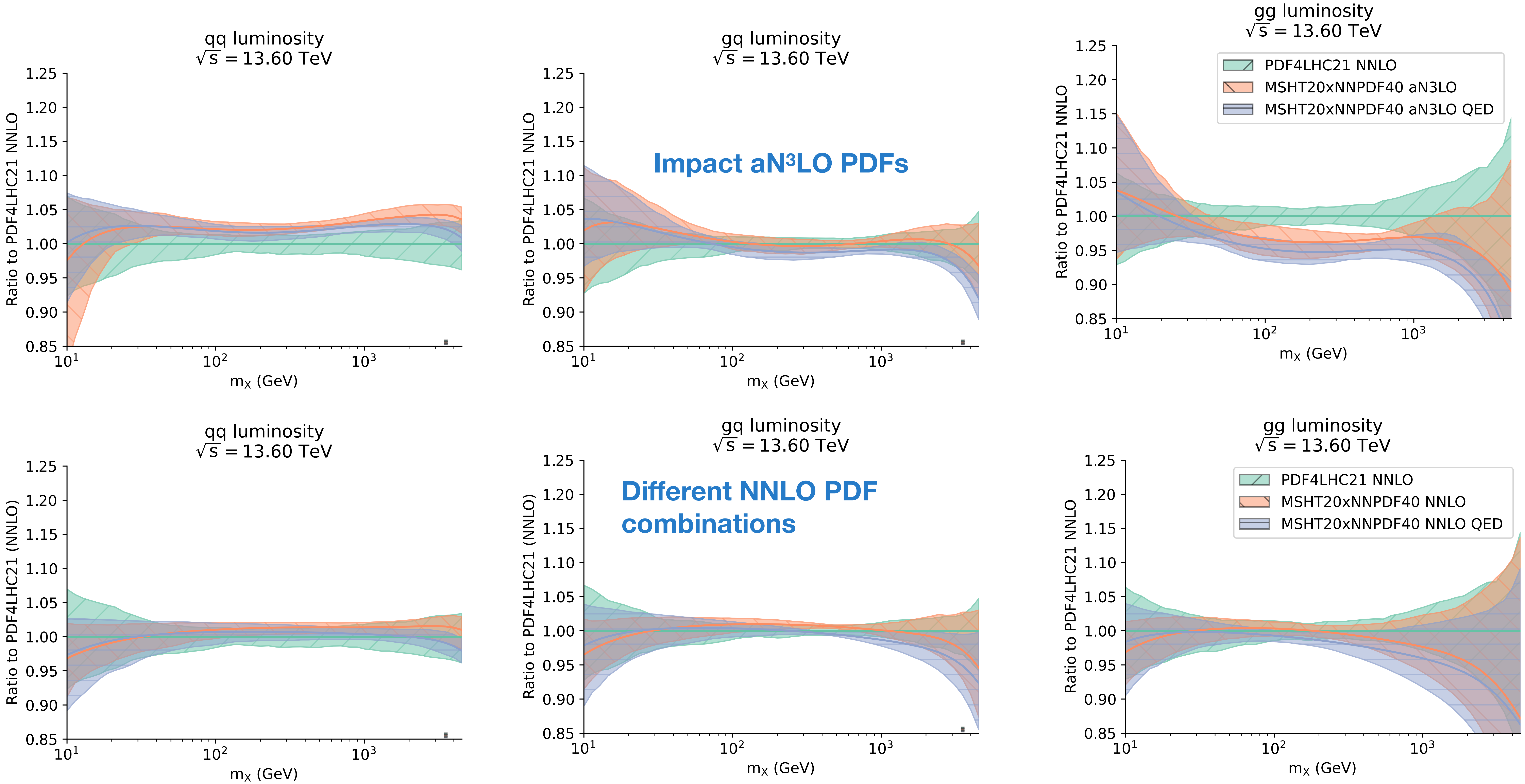
MSHT&NNPDF, 2411.05373

MSHT+NNPDF aN3LO combination

	PDF set	pert. order (PDF)
Reference	PDF4LHC21_mc	NNLO_{QCD}
Combinations	MSHT20xNNPDF40_nnlo	NNLO_{QCD}
	MSHT20xNNPDF40_nnlo_qed	$\text{NNLO}_{\text{QCD}} \otimes \text{NLO}_{\text{QED}}$
	MSHT20xNNPDF40_an3lo	$\text{aN}^3\text{LO}_{\text{QCD}}$
	MSHT20xNNPDF40_an3lo_qed	$\text{aN}^3\text{LO}_{\text{QCD}} \otimes \text{NLO}_{\text{QED}}$
Inputs	NNPDF40_an3lo_as_01180_mhou	$\text{aN}^3\text{LO}_{\text{QCD}}$
	NNPDF40_an3lo_as_01180_qed_mhou	$\text{aN}^3\text{LO}_{\text{QCD}} \otimes \text{NLO}_{\text{QED}}$
	MSHT20an3lo_as118	$\text{aN}^3\text{LO}_{\text{QCD}}$
	MSHT20qed_an3lo	$\text{aN}^3\text{LO}_{\text{QCD}} \otimes \text{NLO}_{\text{QED}}$

- Same approach as PDF4LHC: 100 replicas from NNPDF and 100 replicas from MSHT
- Both for aN3LO QCD and aN3LO + QED, together with NNLO baseline
- Usual differences in theory, methodology, and experiment remain $\Rightarrow$ conservative
- Can be extended if other PDFs at the same accuracy become available

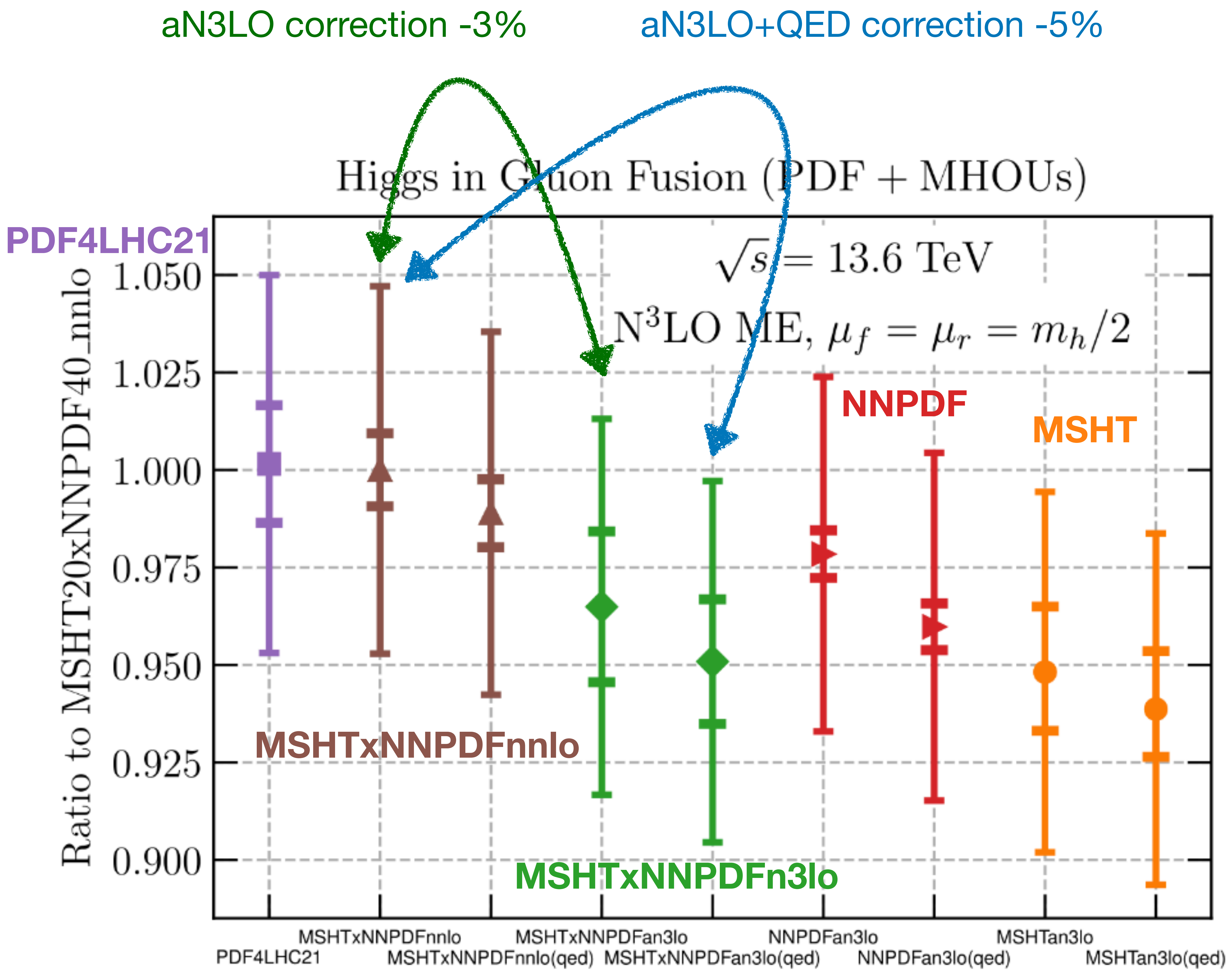
aN3LO effect vs choice of PDF set



Differences between possible NNLO PDF combinations < impact of the N³LO corrections

In particular for qq and gg channels

ggF Higgs for different PDFs



Inner error bars: PDF unc.

Outer error bars: PDF unc. + MHOU

Higgs production

Without N³LO PDFs, uncertainty is **approximated**

$$\Delta^{\rm app}_{\rm NNLO} \equiv \frac{1}{2}\left|\frac{\sigma^{\rm NNLO}_{\rm NNLO-PDF} - \sigma^{\rm NNLO}_{\rm NLO-PDF}}{\sigma^{\rm NNLO}_{\rm NNLO-PDF}}\right|$$

How does this compare to the **exact N³LO shift**?

$$\Delta^{\rm exact}_{\rm NNLO} \equiv \left|\frac{\sigma^{\rm N^3LO}_{\rm N^3LO-PDF} - \sigma^{\rm N^3LO}_{\rm NNLO-PDF}}{\sigma^{\rm N^3LO}_{\rm N^3LO-PDF}}\right|$$

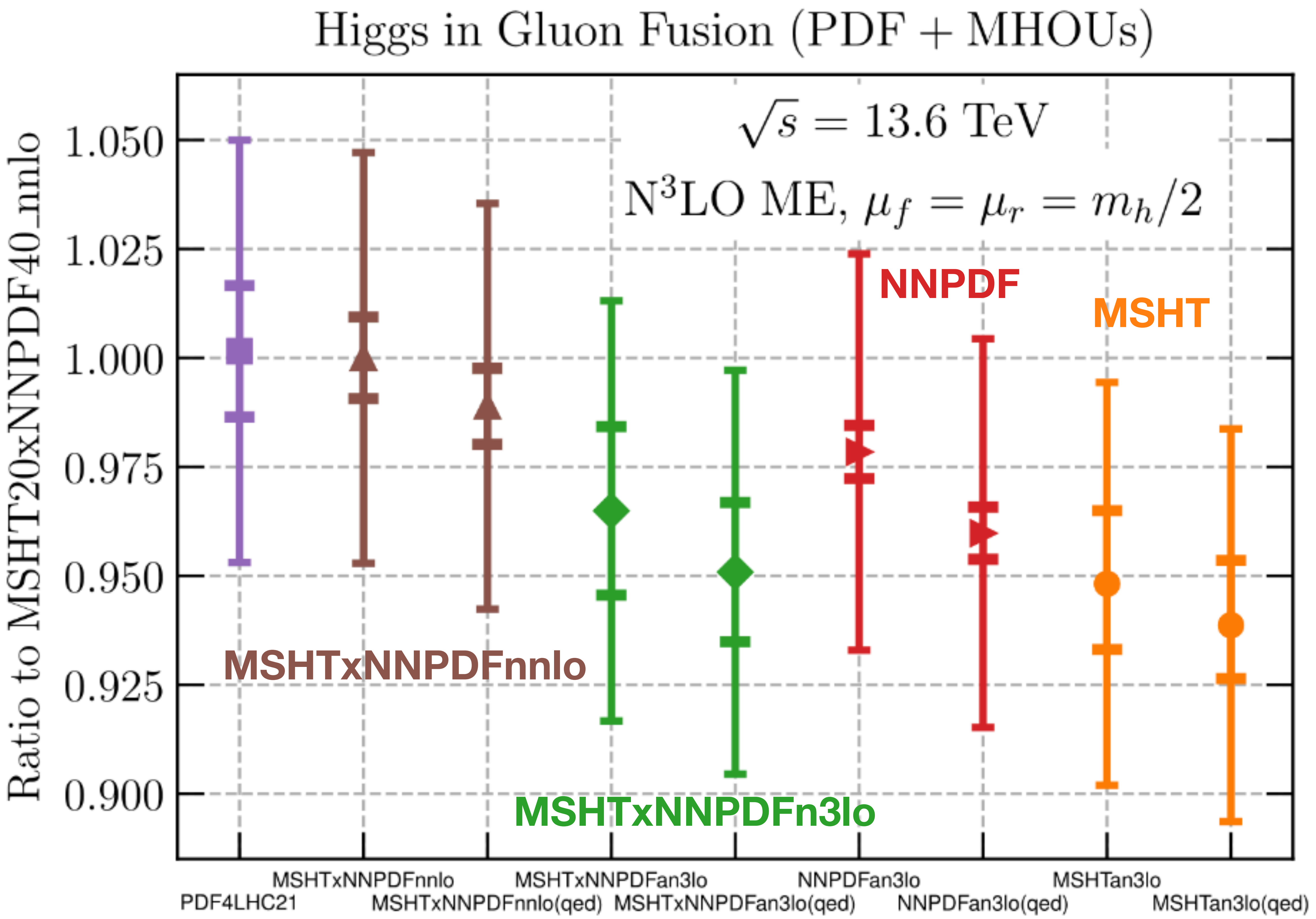
	ggF	VBF
$\Delta_{\rm exact}$	3.3%	2.3%
$\Delta_{\rm approx}$	0.9%	0.5%

Particularly large corrections for ggF and VBF Higgs

Previous estimates of the N³LO mismatch were underestimated

Summary

- aN3LO PDFs are now available and allow for consistent N3LO calculations
- aN3LO evolution is close to exact
- N3LO cross sections are a long term goal
- Both N3LO QCD and NLO QED correction are relevant for Higgs in gluon fusion
- aN3LO+QED represents the PDFs at the highest perturbative accuracy currently available (NNPDF and MSHT)



Summary

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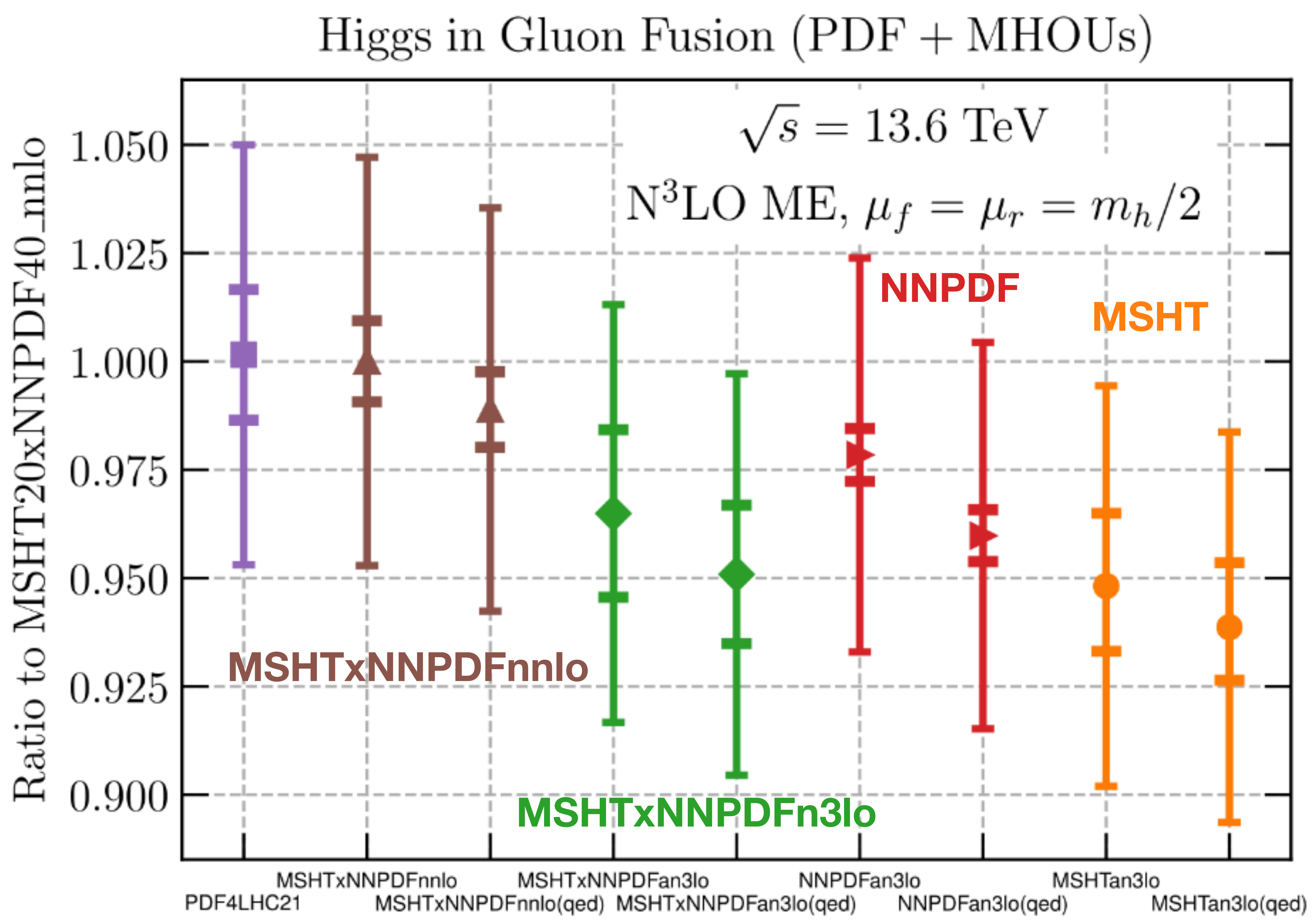
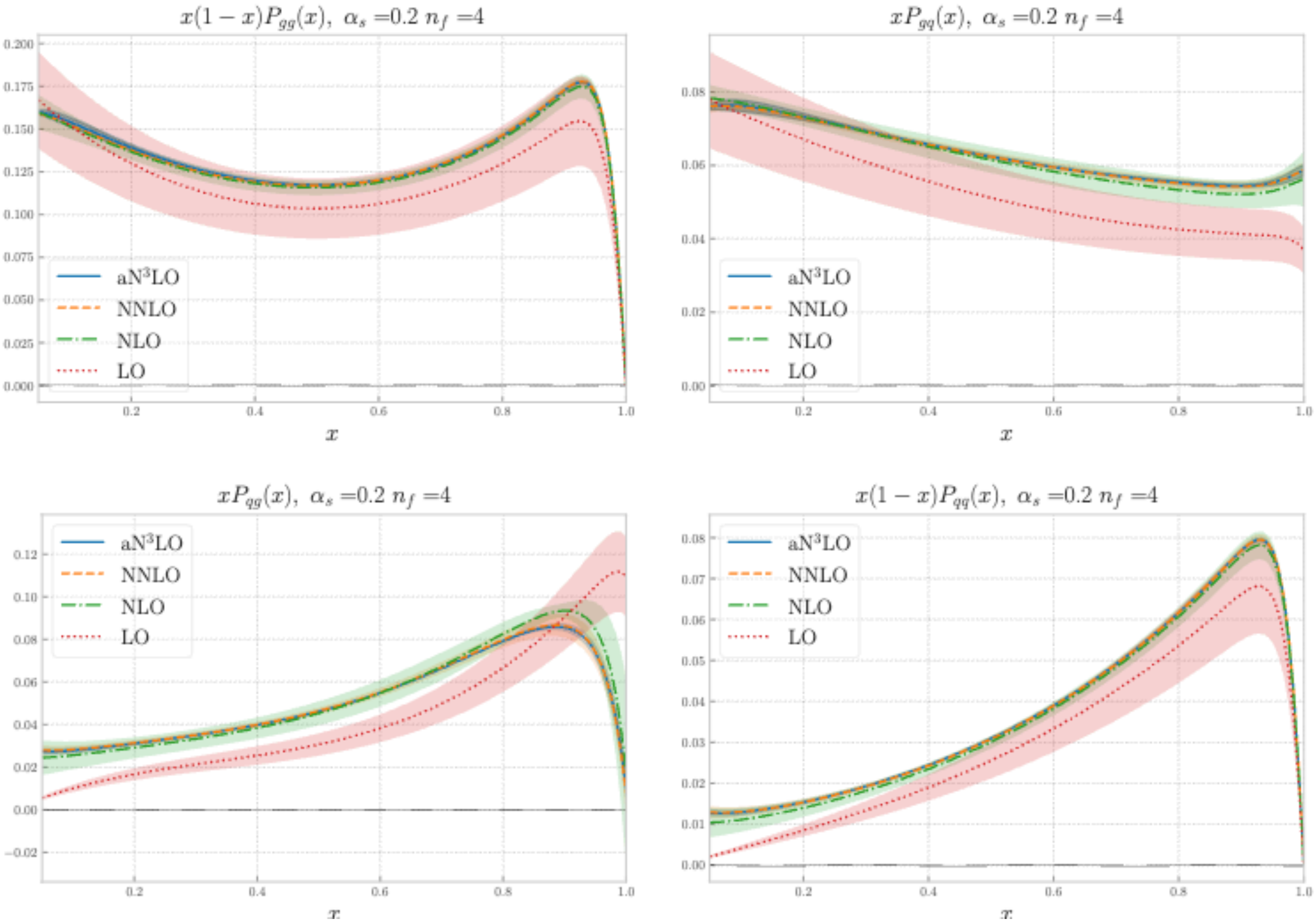


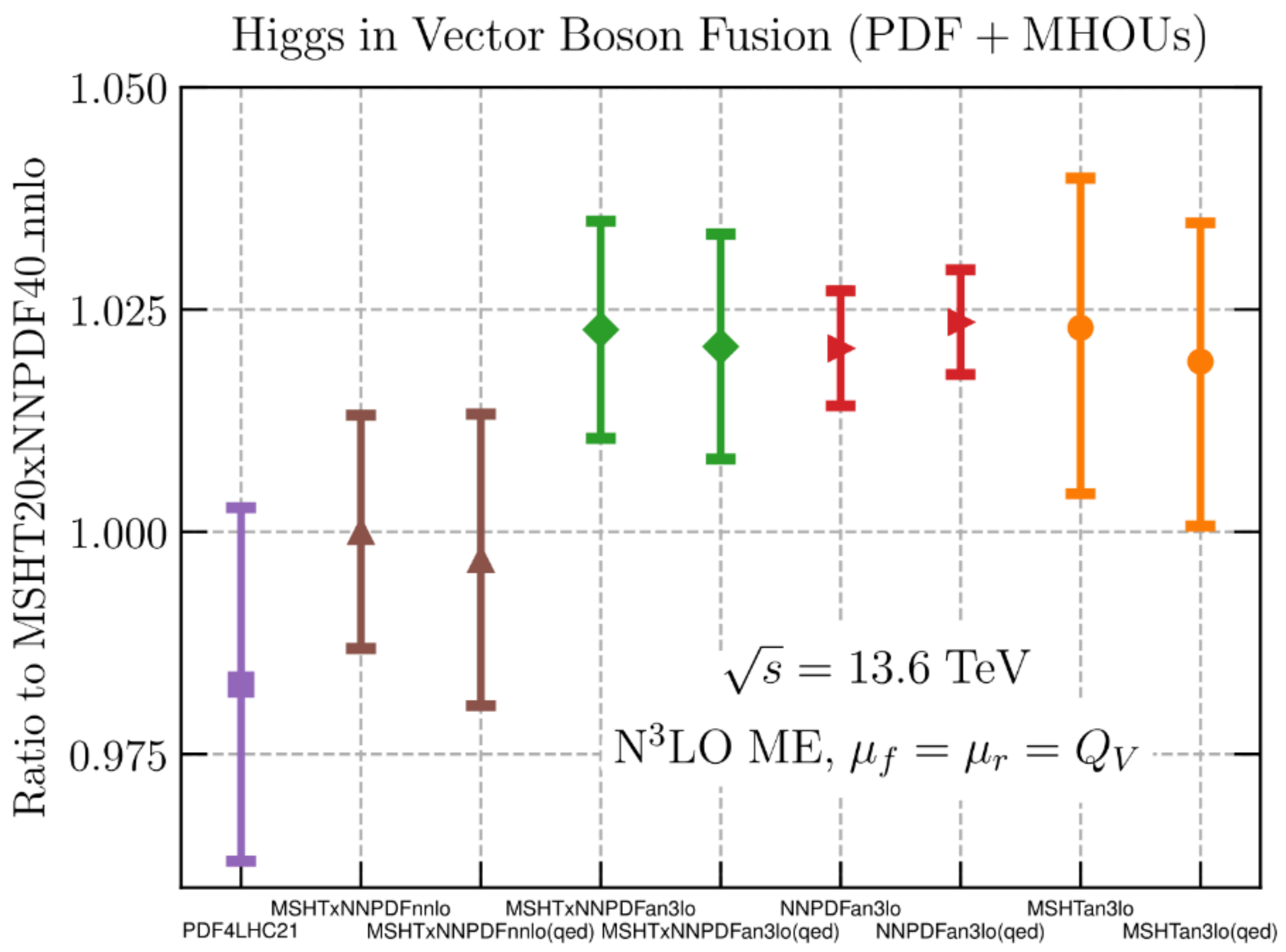
Thank you for your attention!

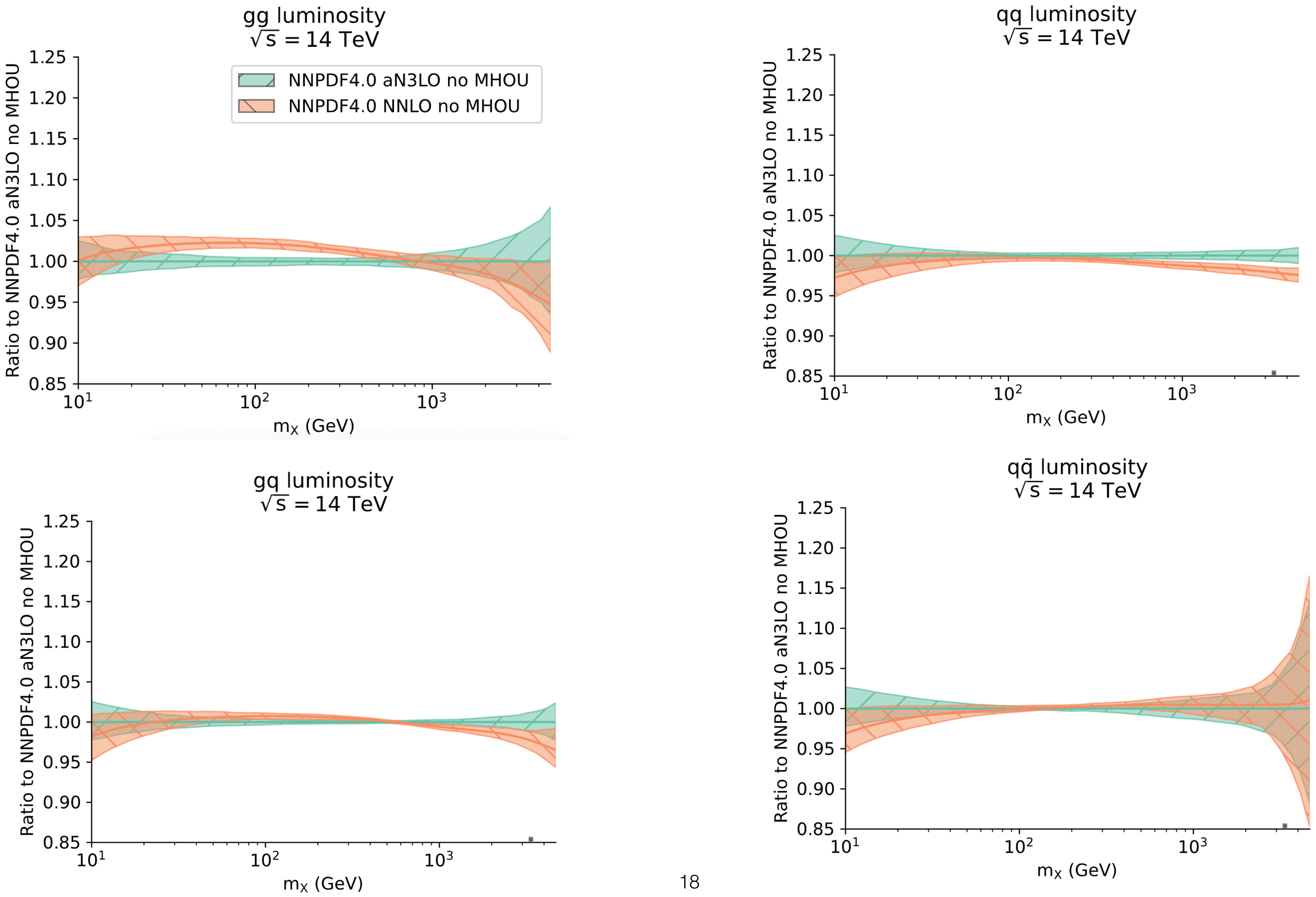
Backup slides

Linear scale splitting functions



Higgs production in VBF





gg luminosity
$\sqrt{s} = 14$ TeV
NNPDF4.0 aN3LO no MHOU
NNPDF4.0 NNLO no MHOU
Ratio to NNPDF4.0 aN3LO no MHOU
m_X (GeV)
qq luminosity
$\sqrt{s} = 14$ TeV
Ratio to NNPDF4.0 aN3LO no MHOU
m_X (GeV)
gq luminosity
$\sqrt{s} = 14$ TeV
Ratio to NNPDF4.0 aN3LO no MHOU
m_X (GeV)
$q\bar{q}$ luminosity
$\sqrt{s} = 14$ TeV
Ratio to NNPDF4.0 aN3LO no MHOU
m_X (GeV)

QED corrections and photon PDF

- So far we considered only QCD evolution, but $\mathcal{O}(\alpha_s^2) \approx \mathcal{O}(\alpha_{em})$ so also **photon initiated contributions** may be relevant
- Modify the DGLAP running to **account for QED corrections**:

$$P = P_{QCD} + P_{QCD \otimes QED}$$

$$P_{QCD \otimes QED} = \alpha_{em} P^{(0,1)} + \alpha_{em} \alpha_s P^{(1,1)} + \alpha_{em}^2 P^{(0,2)}$$

- Data does not provide strong constraints on the photon, but the **photon PDF** can be computed from DIS structure functions: **Manohar, Nason, Salam, Zanderighi, [arXiv:1607.04266], [arXiv:1708.01256]**

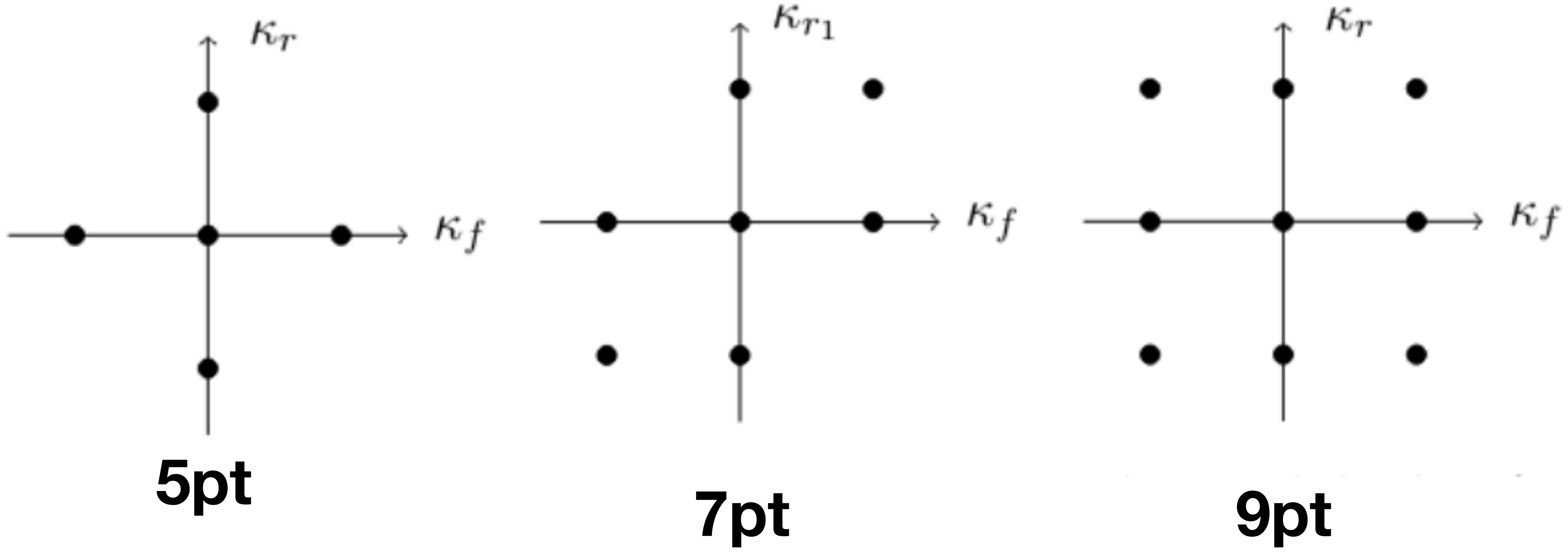
$$x\gamma\left(x, \mu^2\right) = \frac{2}{\alpha\left(\mu^2\right)} \int_x^1 \frac{dz}{z} \left\{ \int_{\frac{m_p^2 x^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} \alpha^2(Q^2) \left[-z^2 F_L\left(x/z, Q^2\right) + \left(z P_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2} \right) F_2\left(x/z, Q^2\right) \right] - \alpha^2\left(\mu^2\right) z^2 F_2\left(x/z, \mu^2\right) \right\}$$

- The **momentum sum rule** needs to account for the photon PDF:

$$\sum_{i=q,\bar{q},g,\gamma} \int_0^1 dx x f_i\left(x, Q^2\right) = 1.$$

Theory uncertainties in PDFs

Missing higher order uncertainties (MHOUs) are estimated through 7 point scale variations



- In a fit we minimize the χ^2:

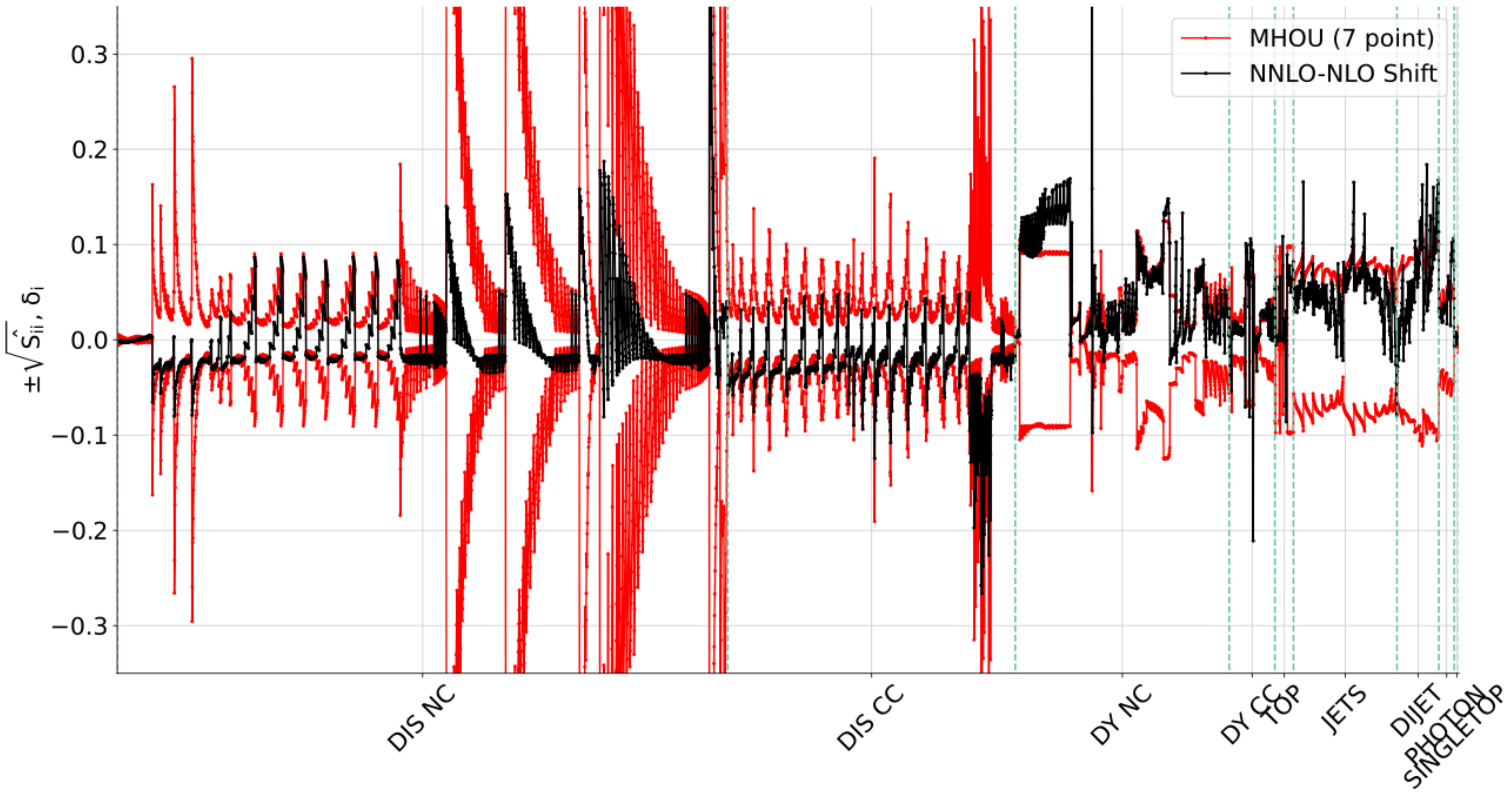
$$P(T \mid D\lambda) \propto \exp\left(-\frac{1}{2}(T-D)^T C^{-1}(T-D)\right) \equiv \exp\left(\chi^2\right)$$

- To account for MHOUs we treat the theory covmat on the same footing as the experimental covmat: $C = C_{\exp} + C_{\mathrm{MHOU}}$

$$C_{\mathrm{MHOU},ij} = n_m \frac{1}{V_m} \sum \left(T_i(\kappa_f,\kappa_r) - T_i(0,0)\right)\left(T_j(\kappa_f,\kappa_r) - T_j(0,0)\right)$$

Validating the MHOU covmat

The MHOU covmat is validated by comparing the **shifts from scale variations at NLO** to the known **NNLO-NLO shifts**



Data

Data set	Ref.	NNPDF3.1	NNPDF4.0	ABMP16	CT18	MSHT20
ATLAS W, Z 7 TeV ($\mathcal{L} = 35$ pb^{-1})	[53]	✓	✓	✓	✓	✓
ATLAS W, Z 7 TeV ($\mathcal{L} = 4.6$ fb^{-1})	[54]	✓	✓	✗	(✓)	✓
ATLAS low-mass DY 7 TeV	[55]	✓	✓	✗	(✓)	✗
ATLAS high-mass DY 7 TeV	[56]	✓	✓	✗	(✓)	✓
ATLAS W 8 TeV	[81]	✗	(✓)	✗	✗	✓
ATLAS DY 2D 8 TeV	[80]	✗	✓	✗	✗	✓
ATLAS high-mass DY 2D 8 TeV	[79]	✗	✓	✗	(✓)	✓
ATLAS $\sigma_{W,Z}$ 13 TeV	[83]	✗	✓	✓	✗	✗
ATLAS W+jet 8 TeV	[95]	✗	✓	✗	✗	✓
ATLAS Z p_T 7 TeV	[274]	(✓)	✗	✗	(✓)	✗
ATLAS Z p_T 8 TeV	[65]	✓	✓	✗	✓	✓
ATLAS $W + c$ 7 TeV	[85]	✗	✓	✗	(✓)	✗
ATLAS $\sigma_{tt}^{\rm tot}$ 7, 8 TeV	[67]	✓	✓	✓	✗	✗
ATLAS $\sigma_{tt}^{\rm tot}$ 7, 8 TeV	[275–280]	✗	✗	✓	✗	✗
ATLAS $\sigma_{tt}^{\rm tot}$ 13 TeV ($\mathcal{L} = 3.2$ fb^{-1})	[68]	✓	✗	✓	✗	✗
ATLAS $\sigma_{tt}^{\rm tot}$ 13 TeV ($\mathcal{L} = 139$ fb^{-1})	[136]	✗	✓	✗	✗	✗
ATLAS $\sigma_{tt}^{\rm tot}$ and Z ratios	[281]	✗	✗	✗	✗	(✓)
ATLAS $t\bar{t}$ lepton+jets 8 TeV	[69]	✓	✓	✗	✓	✓
ATLAS $t\bar{t}$ dilepton 8 TeV	[91]	✗	✓	✗	✗	✓
ATLAS single-inclusive jets 7 TeV, R=0.6	[75]	✓	(✓)	✗	✓	✓
ATLAS single-inclusive jets 8 TeV, R=0.6	[88]	✗	✓	✗	✗	✗
ATLAS dijets 7 TeV, R=0.6	[148]	✗	✓	✗	✗	✗
ATLAS direct photon production 8 TeV	[102]	✗	(✓)	✗	✗	✗
ATLAS direct photon production 13 TeV	[103]	✗	✓	✗	✗	✗
ATLAS single top R_t 7, 8, 13 TeV	[96, 98, 100]	✗	✓	✓	✗	✗
ATLAS single top diff. 7 TeV	[96]	✗	✓	✗	✗	✗
ATLAS single top diff. 8 TeV	[98]	✗	✓	✗	✗	✗

Data set	Ref.	NNPDF3.1	NNPDF4.0	ABMP16	CT18	MSHT20
CMS W asym. 7 TeV ($\mathcal{L} = 36$ pb^{-1})	[282]	✗	✗	✗	✗	✓
CMS Z 7 TeV ($\mathcal{L} = 36$ pb^{-1})	[283]	✗	✗	✗	✗	✓
CMS W electron asymmetry 7 TeV	[57]	✓	✓	✗	✓	✓
CMS W muon asymmetry 7 TeV	[58]	✓	✓	✓	✓	✗
CMS Drell-Yan 2D 7 TeV	[59]	✓	✓	✗	(✓)	✓
CMS Drell-Yan 2D 8 TeV	[284]	(✓)	✗	✗	✗	✗
CMS W rapidity 8 TeV	[60]	✓	✓	✓	✓	✓
CMS W, Z p_T 8 TeV ($\mathcal{L} = 18.4$ fb^{-1})	[285]	✗	✗	✗	(✓)	✗
CMS Z p_T 8 TeV	[66]	✓	✓	✗	(✓)	✗
CMS $W + c$ 7 TeV	[78]	✓	✓	✗	(✓)	✓
CMS $W + c$ 13 TeV	[86]	✗	✓	✗	✗	(✓)
CMS single-inclusive jets 2.76 TeV	[77]	✓	✗	✗	✗	✓
CMS single-inclusive jets 7 TeV	[147]	✓	(✓)	✗	✓	✓
CMS dijets 7 TeV	[76]	✗	✓	✗	✗	✗
CMS single-inclusive jets 8 TeV	[89]	✗	✓	✗	✓	✓
CMS 3D dijets 8 TeV	[149]	✗	(✓)	✗	✗	✗
CMS $\sigma_{tt}^{\rm tot}$ 5 TeV	[90]	✗	✓	✗	✗	✗
CMS $\sigma_{tt}^{\rm tot}$ 7, 8 TeV	[146]	✓	✓	✗	✗	✗
CMS $\sigma_{tt}^{\rm tot}$ 8 TeV	[286]	✗	✗	✗	✗	✓
CMS $\sigma_{tt}^{\rm tot}$ 5, 7, 8, 13 TeV	[70, 287–295]	✗	✗	✓	✗	✗
CMS $\sigma_{tt}^{\rm tot}$ 13 TeV	[71]	✓	✓	✓	✗	✗
CMS $t\bar{t}$ lepton+jets 8 TeV	[72]	✓	✓	✗	✗	✓
CMS $t\bar{t}$ 2D dilepton 8 TeV	[92]	✗	✓	✗	✓	✓
CMS $t\bar{t}$ lepton+jet 13 TeV	[93]	✗	✓	✗	✗	✗
CMS $t\bar{t}$ dilepton 13 TeV	[94]	✗	✓	✗	✗	✗
CMS single top $\sigma_t + \sigma_{\bar{t}}$ 7 TeV	[97]	✗	✓	✓	✗	✗
CMS single top R_t 8, 13 TeV	[99, 101]	✗	✓	✓	✗	✗
CMS single top 13 TeV	[296, 297]	✗	✗	✗	✗	(✓)