Flavour hierarchies, extended groups and a composite Higgs

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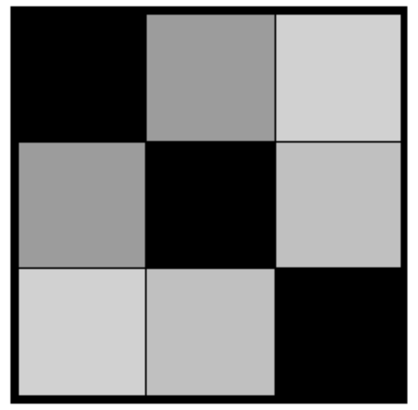
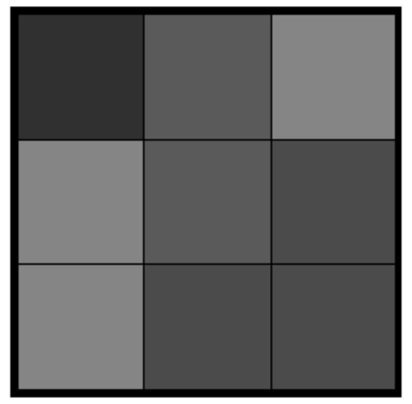
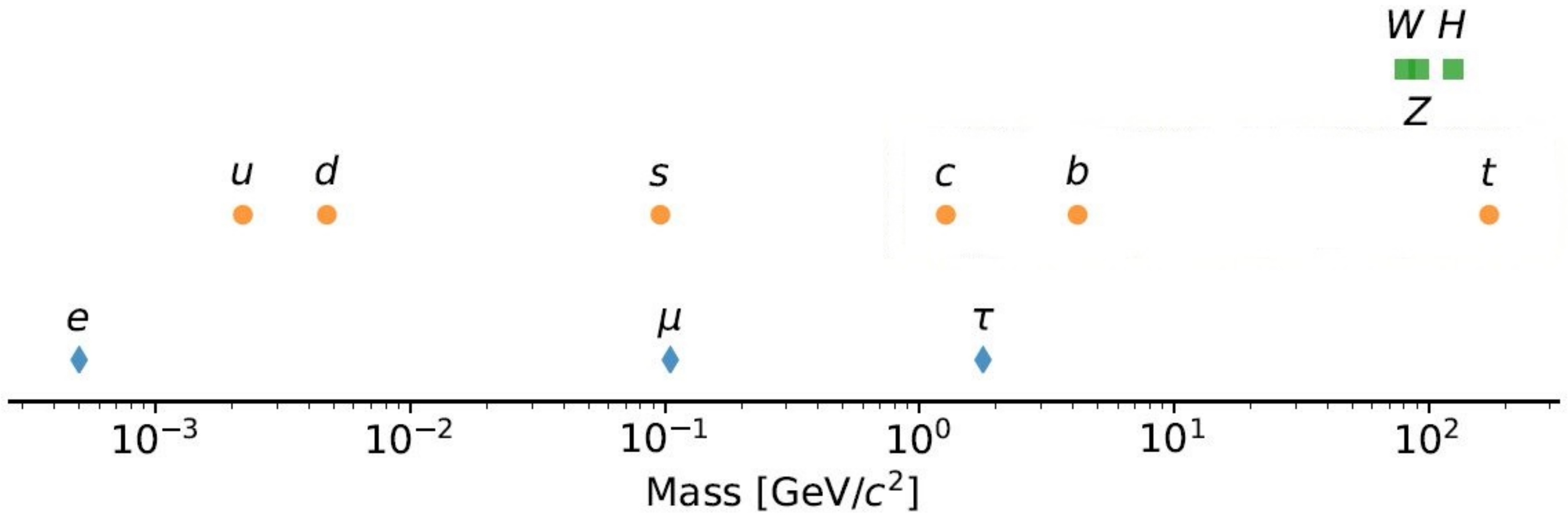
Based on:
JHEP 05 (2025) 176, [2412.14243]
See also Moriond proceedings:
[2505.15787]

Motivation

- The SM is a great theory.
- But… Flavor: while gauge sector is universal, Higgs couples to fermions in a funny way.
- Also: Higgs hierarchy problem.
- I am going to present a BSM model whose dynamics inevitably leads to:
  - Hierarchies of masses between third and light families.
  - Hierarchy of CKM vs anarchy of PMNS.
  - The emergence of the Higgs as a composite state.

$V_{\text{CKM}} \sim$ 

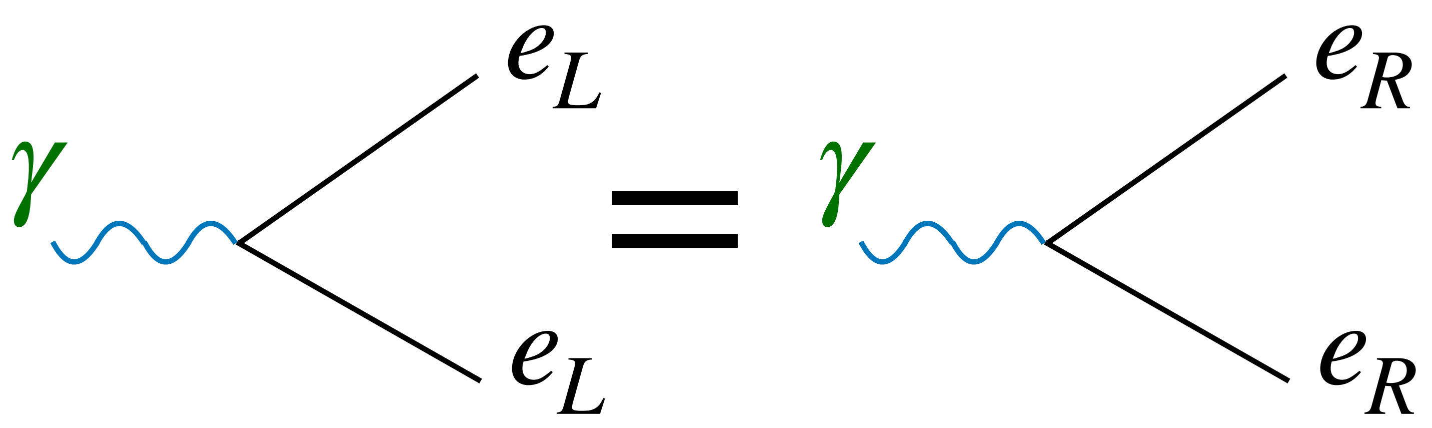
$V_{\text{PMNS}} \sim$ 



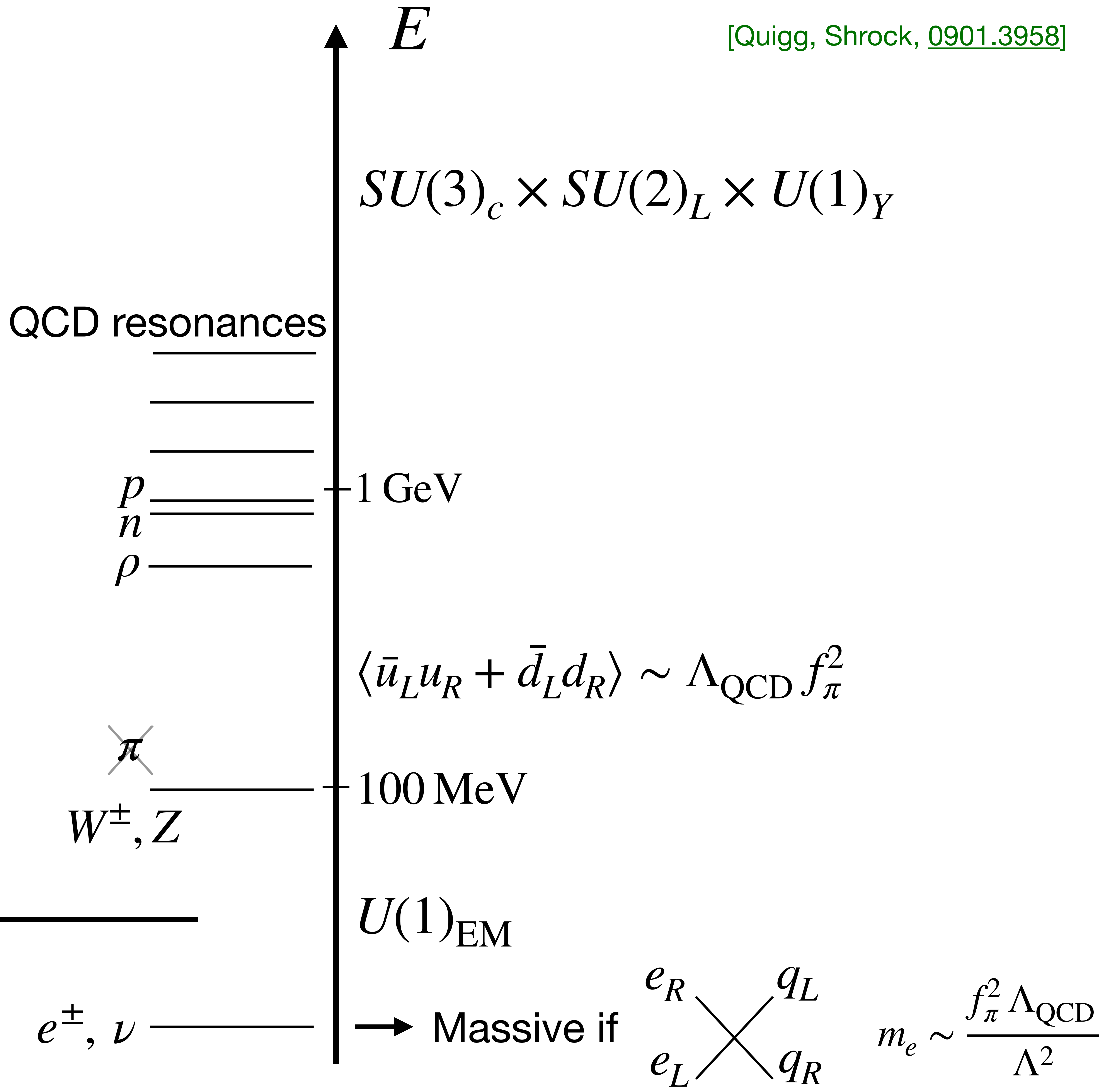
A toy SM

[Quigg, Shrock, 0901.3958]

- **SM with 1 family and no Higgs**



Vector-like theory



A toy SM

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- **SM with 1 family and no Higgs**

$$e_L\ e_R$$
$$U(1)_{\text{EM}}$$
$$\uparrow$$
$$SU(2)_L \times \underbrace{SU(2)_R \times U(1)_{-L}}_{U(1)_Y}$$
$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$$

$$SU(2)_L \times SU(2)_R \xrightarrow{\langle \bar{q}_L q_R \rangle \neq 0} SU(2)_V$$

- This gauge is anomalous.
  Mixed anomalies:

$$U(1)_L - SU(2)_{L/R} - SU(2)_{L/R}$$

$$Y = T^3_R + \frac{-L}{2}$$

$$Q = T^3_L + T^3_R + \frac{-L}{2}$$

A toy SM

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- **SM with 1 family and no Higgs**

$$e_L \; e_R$$
$$U(1)_{\text{EM}}$$
$$\uparrow$$
$$SU(2)_L \times SU(2)_R \times \underbrace{U(1)_{B-L}}_{U(1)_Y}$$

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$$

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

$$SU(2)_L \times SU(2)_R \xrightarrow{\langle \bar{q}_L q_R \rangle \neq 0} SU(2)_V$$

- This gauge is not anomalous. Mixed anomalies:

$$\cancel{U(1)_{B-L} - SU(2)_{L/R} - SU(2)_{L/R}}$$

$$\boxed{\begin{array}{c}\text{Quark contribution fixes this promoting} \\ U(1)_{-L} \to U(1)_{B-L}\end{array}}$$

$$Y = T_R^3 + \frac{B-L}{2}$$

$$Q = T_L^3 + T_R^3 + \frac{B-L}{2}$$

Addressing flavor

[Pati, Salam, PRD 10 (1974); Li, Ma, PRL 47 (1981)]
[Bordone, Cornella, Fuentes-Martin, Isidori, 1712.01368; Fernández-Navarro, King, 2305.07690; Davighi, Stefanek, 2305.16280; Davighi, Gosnay, Miller, Renner 2312.13346; Capdevila, Crivellin, JML, Pokorski, 2401.00848; Covone, Davighi, Isidori, Pesut, 2407.10950, …]

Change chirality for flavor. **Flavor deconstruction:**

For more, see also S. Covone's talk

$$G_1 \times G_2 \times G_3 \times G'_U \xrightarrow{\gtrsim \text{PeV}} \boxed{G_{1,2} \times G_3 \times G_U \xrightarrow{\gtrsim \text{TeV}} G_{\text{SM}}}$$

Left-right unification:

$$SU(2)_L \times U(1)_Y \subset SU(2)_L \times U(1)_{B-L} \times SU(2)_R$$

Global approx. symm: $\overbrace{SU(2)_{L1} \times SU(2)_{L2}} \times \boxed{U(1)_{B-L} \times SU(2)_{R1}} \times SU(2)_{R2}$

Gauge symm: $\mathscr{G} = SU(2)_{L1} \times SU(2)_{L2} \times U(1)_X \times SU(2)_{R2}$

Breaking sector

- We need a sector breaking the UV gauge symmetry to the SM.
- Like in QCD, let us add a hyper-sector:
  - Gauge group: $SU(N_{\rm HC})$
  - Hyper-fermions in a vector representation: $\zeta = (\zeta^{(L)}, \zeta^{(R)})$

	Site 1	Site 2
$SU(2)_L$	$\zeta_L^{(L)}$	$\zeta_R^{(L)}$
$SU(2)_R$	$\zeta_L^{(R)}$	$\zeta_R^{(R)}$

$$\mathscr{G} = SU(2)_{L1} \times SU(2)_{L2} \times \overbrace{U(1)_X \times SU(2)_{R2}}^{SU(2)_{R1} \times U(1)_{B-L}}$$

$$\langle \bar{\zeta}_L^{(L)} \zeta_R^{(L)} \rangle \neq 0 \Rightarrow SU(2)_{L1} \times SU(2)_{L2} \to SU(2)_L$$

$$\langle \bar{\zeta}_L^{(R)} \zeta_R^{(R)} \rangle \neq 0 \Rightarrow SU(2)_{R1} \times SU(2)_{R2} \to SU(2)_R$$

$$\downarrow$$

$$SU(2)_L \times U(1)_Y$$

$$\boxed{\text{We promote } U(1)_{B-L} \to U(1)_{HB+B-L}}$$

$$HB\,|_\zeta = 1/N_{\rm HC}$$

It triggers an anomalous-looking arrangement of the SM fields, that addresses the different patterns of the CKM and PMNS

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$$\mathcal{G} = SU(2)_{L1} \times SU(2)_{L2} \times \underbrace{U(1)_X \times SU(2)_{R2}}_{SU(2)_{R1} \times U(1)_{HB+B-L}}$$

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$$HB\,|_\zeta = 1/N_{\rm HC}$$

$$X = T^3_{R1} + \frac{HB + B - L}{2}$$

$$Q = T^3_L + T^3_R + \frac{HB + B - L}{2}$$

It triggers an anomalous-looking arrangement of the SM fields, that addresses the different patterns of the CKM and PMNS

Fermion arrangement

- Arrangement to cancel gauge anomalies:

	Site 1	Site 2
$SU(2)_L$	$q_L^{1,2}$ $\ell_L^{1,2,3}$ $\zeta_L^{(L)}$	q_L^3 $\zeta_R^{(L)}$
$SU(2)_R$	$\ell_R^{1,2}$ $q_R^{1,2,3}$ $\zeta_L^{(R)}$	ℓ_R^3 $\zeta_R^{(R)}$

No $U(1)_{HB+B-L} - SU(2) - SU(2)$ anomalies

If N_{HC} is odd $\Rightarrow$ No Witten anomaly

[Fuentes-Martín, JML, 2402.09507]

[Antusch, Greljo, Stefanek, Thomsen, 2311.09288]

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NP distinguishes among LH quarks…

… but among RH charged leptons

No $U(1)_{HB+B-L} - SU(2) - SU(2)$ anomalies

If $N_{\rm HC}$ is odd $\Rightarrow$ No Witten anomaly

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Symmetry-breaking pattern

$$\begin{pmatrix} \zeta_L^{(L)} \\ \zeta_L^{(R)} \end{pmatrix} \sim \mathbf{4} \text{ of } SU(4)_1 \supset SU(2)_{L1} \times SU(2)_{R1}$$

$$\begin{pmatrix} \zeta_R^{(L)} \\ \zeta_R^{(R)} \end{pmatrix} \sim \mathbf{4} \text{ of } SU(4)_2 \supset SU(2)_{L2} \times SU(2)_{R2}$$

$$SU(4)_1 \times SU(4)_2 \xrightarrow{\langle \bar{\zeta}_L \zeta_R \rangle} SU(4)_V$$

(15 broken generators)

$$\mathbf{15} = (\mathbf{1}, \mathbf{1}) + 2 \times (\mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{3}) + (\mathbf{3}, \mathbf{1})$$

(of $SU(2)_L \times SU(2)_R$)

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(15 broken generators)

Physical pNGBs

Eaten by new massive gauge bosons

$$\mathbf{15} = \boxed{(\mathbf{1},\mathbf{1}) + 2 \times (\mathbf{2},\mathbf{2})} + \cancel{\overbrace{(\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1})}}$$

$$\underbrace{\left\{ \begin{array}{l} \mathscr{W}_\mu^{\pm,0} \sim (\mathbf{1},\mathbf{3})_0 \\ \mathscr{B}_\mu^{0} \sim (\mathbf{1},\mathbf{1})_0 \\ \mathscr{B}_\mu^{\pm} \sim (\mathbf{1},\mathbf{1})_{\pm 1} \end{array} \right.}_{W'_L,\ W'_R \text{ and } 2\,Z'}$$

It's a 2HDM!

Yukawa couplings

- Yukawas à la technicolor $\mathscr{L} \supset \bar{q}_L t_R \, O_H$ (also possible partial compositeness).

- Extended gauge addresses flavor hierarchies:

	Site 1	Site 2
$SU(2)_L$	$q_L^{1,2}$ $\ell_L^{1,2,3}$ $\zeta_L^{(L)}$	q_L^3 $\zeta_R^{(L)}$
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Third family (e.g. top):

$$\mathscr{L} \propto \quad (\bar{q}_L^3 u_R^n)(\bar{\zeta}_L^{(R)} \zeta_R^{(L)}) = (\bar{q}_L^3 u_R^n) \, O_H$$

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Third family (e.g. top):

Light families (e.g. μ):

$$\mathscr{L} \propto \quad (\bar{q}_L^3 u_R^n)(\bar{\zeta}_L^{(R)}\zeta_R^{(L)}) \qquad (\bar{\ell}_L^n e_R^i)(\bar{\zeta}_L^{(R)}\zeta_R^{(R)})(\bar{\zeta}_R^{(R)}\zeta_L^{(L)}) = (\bar{\ell}_L^n e_R^i)\, O_H$$

Hierarchies in quarks

$$\mathscr{L} \supset Y^{(u)}_{nm}\, \bar{q}^{n}_{L}\, H^{c}\, u^{m}_{R}$$

$$\Delta^{u}_{in}\, \bar{q}^{i}_{L}\, H^{c}\, u^{n}_{R} + \bar{q}^{3}_{L}\, H^{c}\, y^{t}_{n} u^{n}_{R}$$

$i, j = 1{,}2$

$n, m = 1{,}2{,}3$

$$\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \blacksquare & \blacksquare & \blacksquare \end{pmatrix} \xrightarrow[U(3)_u \text{ rotation}]{} \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & \blacksquare \end{pmatrix}$$

Light Yukawas

$$\sim \begin{pmatrix} V_{ub} \\ V_{cb} \end{pmatrix}$$

$$y_t \sim O(1)$$

[Antusch, Greljo, Stefanek, Thomsen, 2311.09288]

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Hierarchies in leptons

$$\mathscr{L} \supset Y^{(e)}_{nm}\,\bar{\ell}^n_L\, H\, e^m_R$$

$i, j = 1,2$

$n, m = 1,2,3$

$$\Delta^e_{ni}\,\bar{\ell}^n_L\, H\, e^i_R + y^\tau_n \bar{\ell}^n_L\, H\, \tau_R$$

Light Yukawas

$$\begin{pmatrix} \times & \times & \blacksquare \\ \times & \times & \blacksquare \\ \times & \times & \blacksquare \end{pmatrix} \xrightarrow[U(3)_\ell \text{ rotation}]{} \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & \blacksquare \end{pmatrix}$$

y_τ

$$\mathscr{L} \supset \frac{\lambda_{nm}}{\Lambda_\nu}(\bar{\ell}^n_L H^c)(H^\dagger \ell^{mc}_L) \longrightarrow \text{Anarchic PMNS}$$

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Light Yukawas

$$\begin{pmatrix} \times & \times & \blacksquare \\ \times & \times & \blacksquare \\ \times & \times & \blacksquare \end{pmatrix} \xrightarrow[U(3)_\ell \text{ rotation}]{} \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & \blacksquare \end{pmatrix}$$

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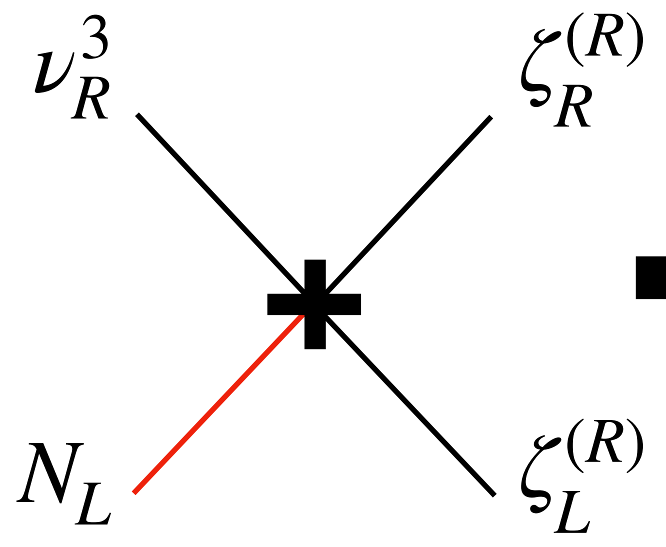
[Antusch, Greljo, Stefanek, Thomsen, 2311.09288]

Higgs potential

$$\ell_R^3 = (\nu_R^3, \tau_R) \sim SU(2)_{R2}$$

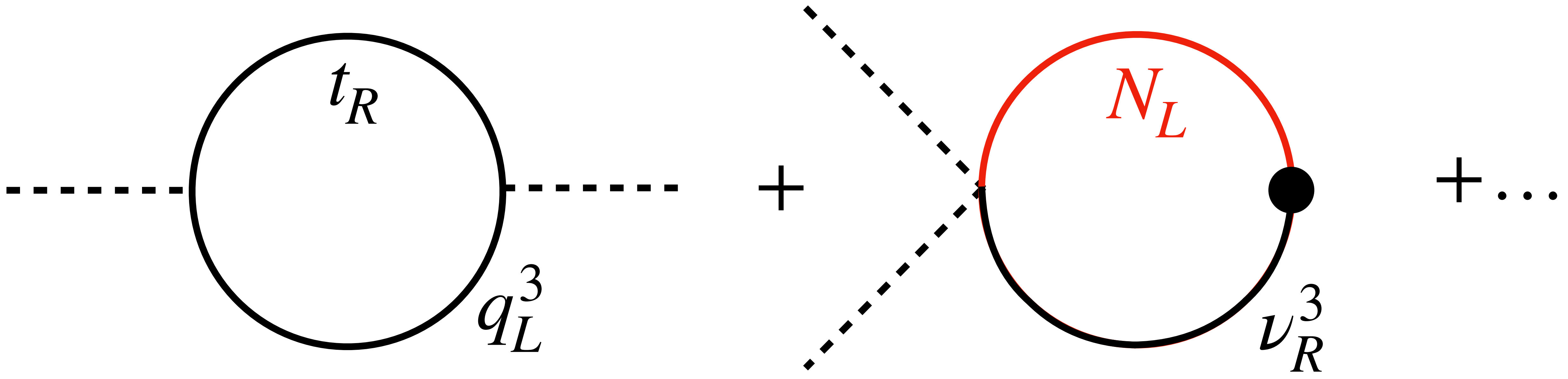
$$N_L \sim \mathbf{1}$$



$$\Rightarrow \; m_N \bar{N}_L \nu_R^3$$

- Every breaking of $SU(4)_1 \times SU(4)_2$ contributes to the potential:

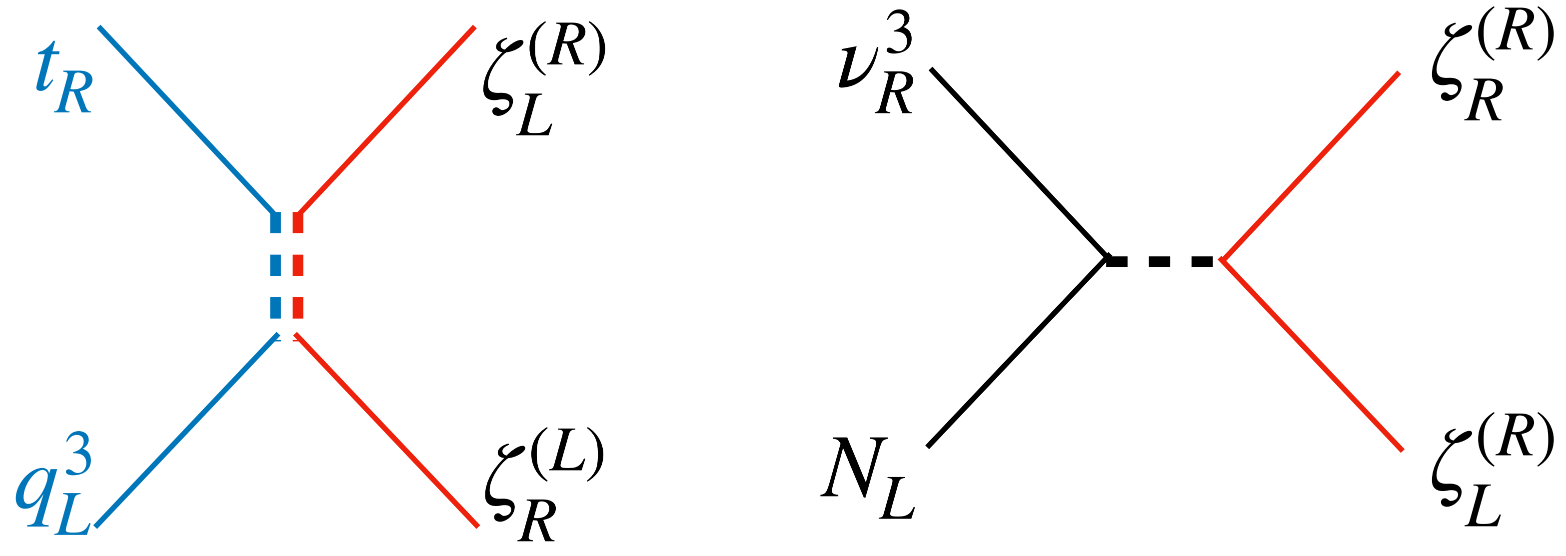
$$V(H_1, H_2, S) =$$



$$\Delta m_{H_1}^2 < 0 \qquad \Delta m_{H_1}^2 > 0$$

$10^{-2,-3}$ tuning

- Benchmark for 4-fermions:

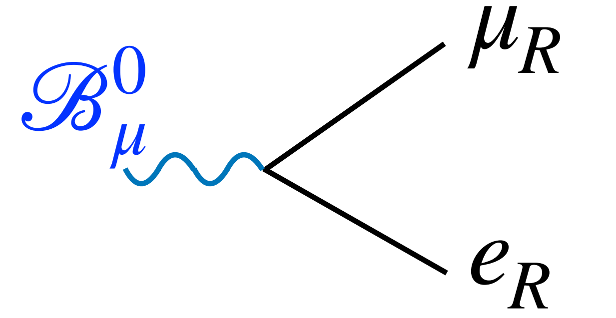


$$\Rightarrow \begin{cases} m_{H_1}^2 \sim \dfrac{M_N^2 - 12 f_\zeta^2 y_t^2}{4N_{\rm HC}} = O(v^2) \\ m_{H_2}^2 = 2m_S^2 \sim \dfrac{M_N^2}{4N_{\rm HC}} = O({\rm TeV}^2) \\ \lambda_H \sim \dfrac{3y_t^2}{8N_{\rm HC}} \sim 0.1 - 0.2 \end{cases}$$

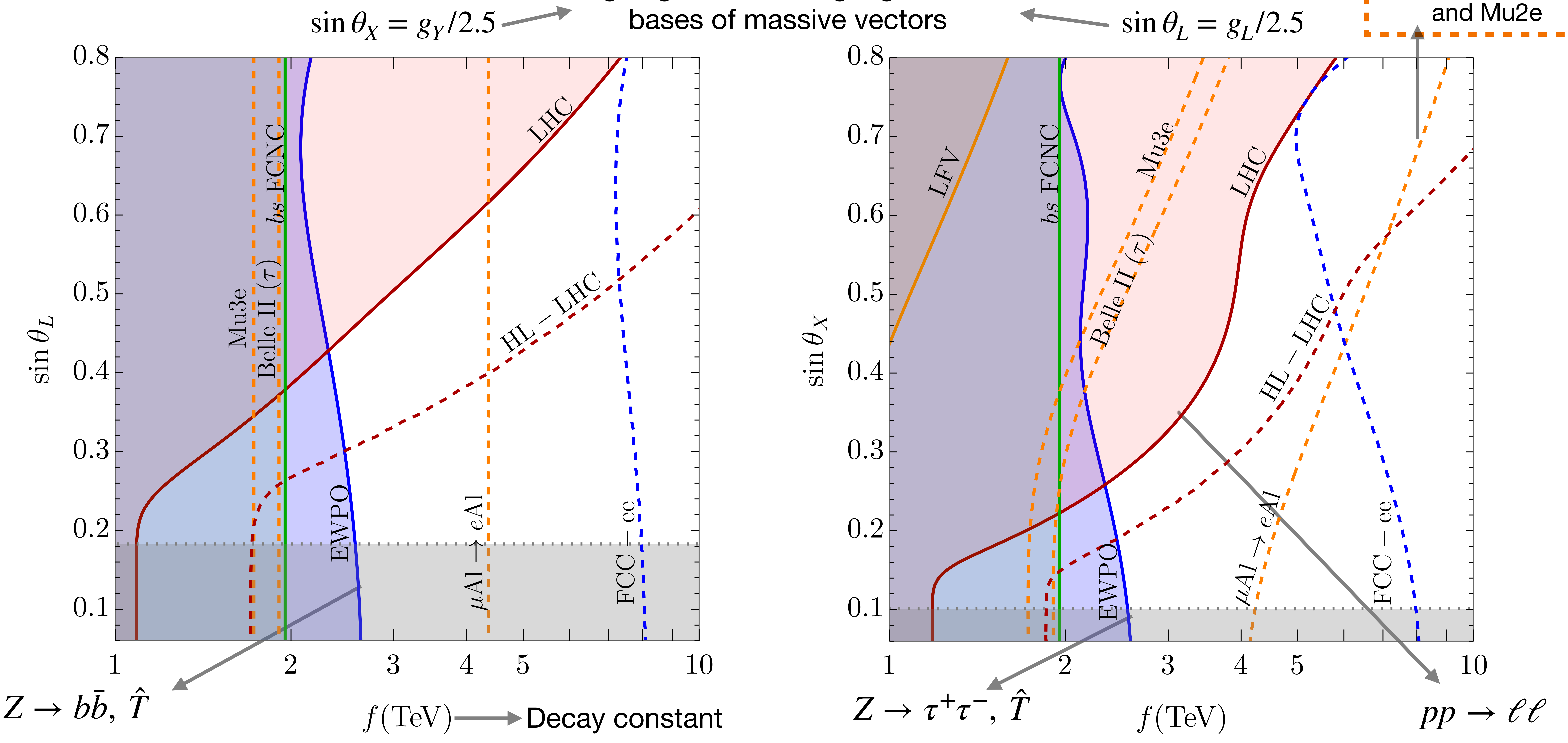
Some pheno:

Colored regions: Excluded at 95 % C.L.
Dashed lines: Future projections

Mixing angles between gauge and mass bases of massive vectors



$\mu \to e$ in COMET and Mu2e

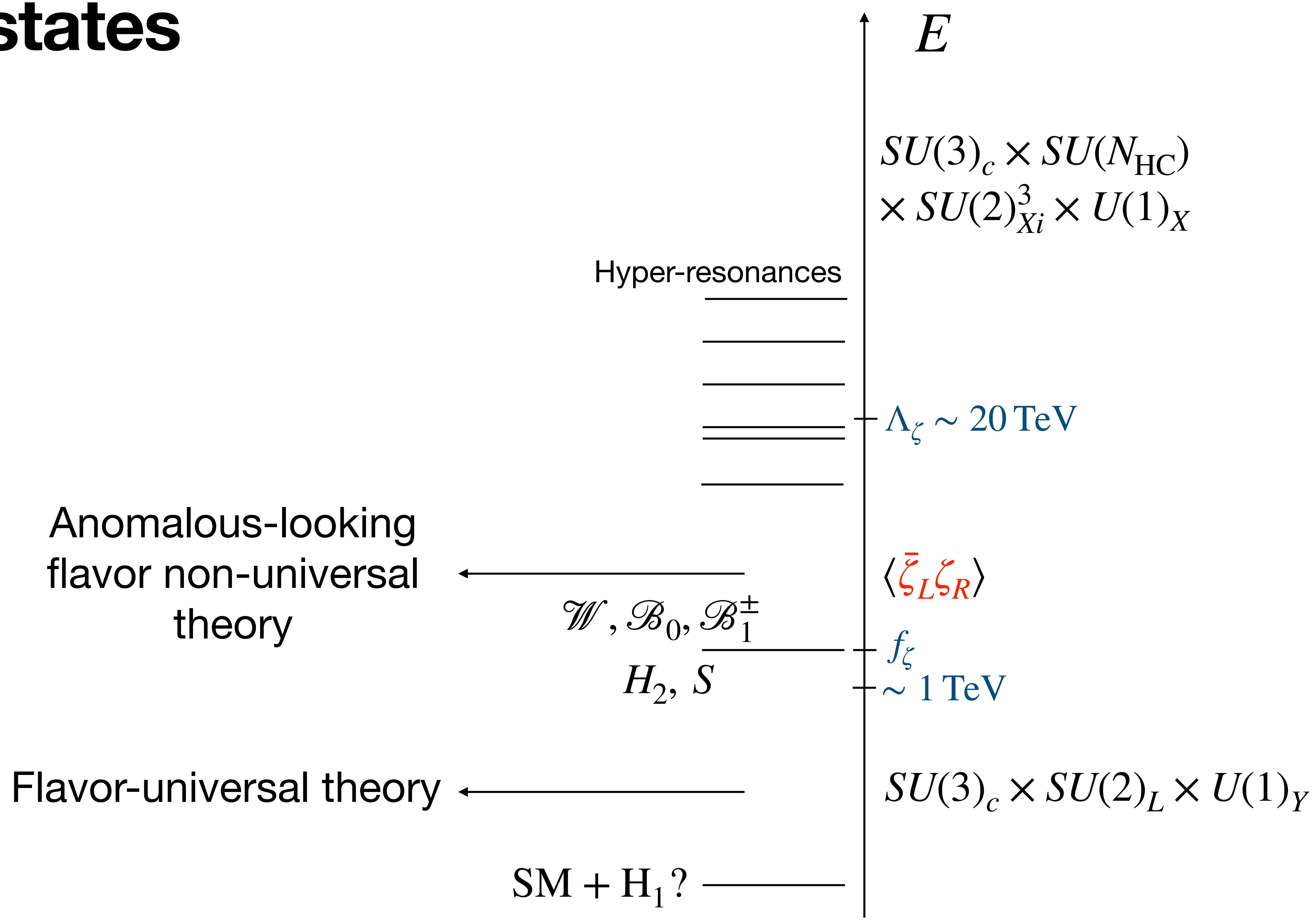


Conclusions

- We have presented a model whose dynamics inevitably leads to:
  - Hierarchies of masses between third and light families.
  - Hierarchy of CKM vs anarchy of PMNS.
  - The emergence of the Higgs as a composite and the breaking of the EW symmetry.
- It presents a novel mechanism to explain flavor hierarchies!
- Rich pheno with interplay between LHC, EW and flavor.
- Future tests of the model from improvement in $\mu \to e$!

Thanks!

New states


E
$SU(3)_c \times SU(N_{\mathrm{HC}}) \times SU(2)^3_{Xi} \times U(1)_X$
Hyper-resonances
$\Lambda_\zeta \sim 20\,\mathrm{TeV}$
Anomalous-looking flavor non-universal theory
$\langle \bar{\zeta}_L \zeta_R \rangle$
$\mathscr{W}, \mathscr{B}_0, \mathscr{B}_1^\pm$
f_ζ
$H_2,\ S$
$\sim 1\,\mathrm{TeV}$
Flavor-universal theory
$SU(3)_c \times SU(2)_L \times U(1)_Y$
$\mathrm{SM} + \mathrm{H}_1\,?$

LH suppression in leptons

[Greljo, Thomsen, 2309.11547; Davighi, Gosnay, Miller, Renner, 2312.13346; Capdevila, Crivellin, JML, Pokorski, 2401.00848; Isidori, Greljo, 2406.01696]

$$\mathscr{L} \supset Y^{(e)}_{nm}\, \bar{\ell}^n_L\, H\, e^m_R$$

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Light Yukawas

y_τ

$$\mathscr{L} \supset \frac{1}{\Lambda_\nu}\left[\lambda_{33}(\bar{\ell}^3_L H^c)(H^\dagger \ell^{3c}_L) + \lambda_{3n}(\bar{\ell}^3_L H^c)(H^\dagger \ell^{nc}_L) + \lambda_{nm}(\bar{\ell}^n_L H^c)(H^\dagger \ell^{mc}_L)\right]$$

$1 \sim \lambda_{33} \gg \lambda_{3n} \gg \lambda_{nm} \Rightarrow m_{\nu_2}/m_{\nu_3} \ll \theta_{23} \ll 1 \;\Rightarrow$ Hierarchical PMNS

$$\begin{pmatrix} - & - & \times \\ - & - & \times \\ \times & \times & \blacksquare \end{pmatrix}$$

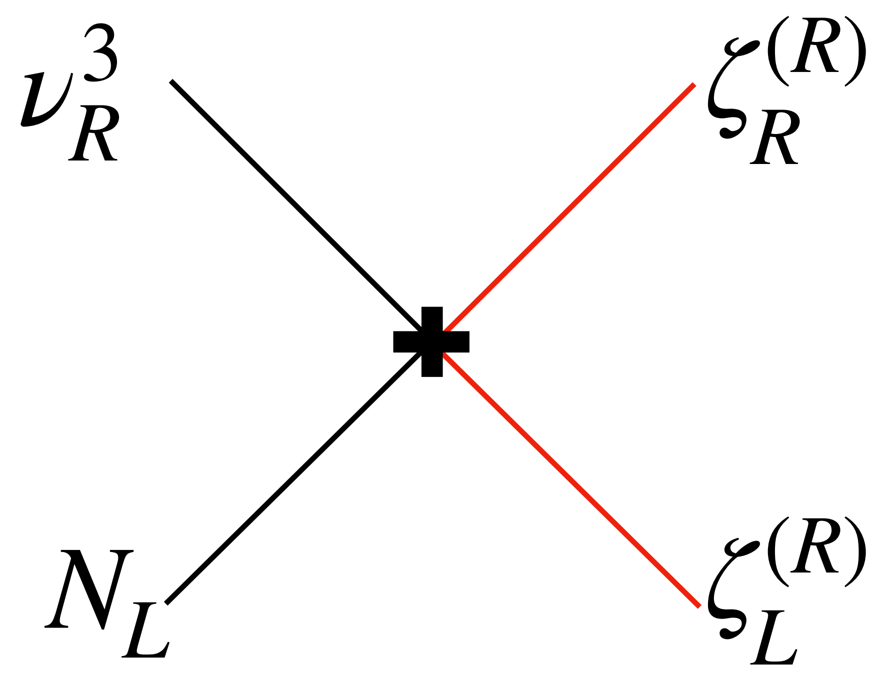
$i, j = 1{,}2$

$n, m = 1{,}2{,}3$

Heavy Neutral Lepton

- The partner of τ_R, ν_R^3 has a τ-size Yukawa. It should become a HNL:

	Site 1	Site 2
$SU(2)_L$	$q_L^{1,2}$ $\ell_L^{1,2,3}$ $\zeta_L^{(L)}$	q_L^3 $\zeta_R^{(L)}$
$SU(2)_R$	$\ell_R^{1,2}$ $q_R^{1,2,3}$ $\zeta_L^{(R)}$	ℓ_R^3 $\zeta_R^{(R)}$



$$\left.\begin{array}{l}(\nu_R^3, \tau_R)\\ N_L \sim \mathbf{1}\end{array}\right\} m_N \bar{N}_L \nu_R^3$$

[JML, 2412.14243]

All representations

Field	$SU(2)_{L1}$	$SU(2)_{R1}$	$SU(2)_{L2}$	$SU(2)_{R2}$	$U(1)_{B-L}$	$SU(3)_c$	$SU(N_{\mathrm{HC}})$
q_L^3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$1/6$	$\mathbf{3}$	$\mathbf{1}$
$q_L^{1,2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$1/6$	$\mathbf{3}$	$\mathbf{1}$
$q_R^{1,2,3}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$1/6$	$\mathbf{3}$	$\mathbf{1}$
$\ell_L^{1,2,3}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$-1/2$	$\mathbf{1}$	$\mathbf{1}$
ℓ_R^3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$\mathbf{1}$	$\mathbf{1}$
$\ell_R^{1,2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$-1/2$	$\mathbf{1}$	$\mathbf{1}$
$\zeta_L^{(L)}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$1/2N_{\mathrm{HC}}$	$\mathbf{1}$	$\square$
$\zeta_L^{(R)}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$1/2N_{\mathrm{HC}}$	$\mathbf{1}$	$\square$
$\zeta_R^{(L)}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$1/2N_{\mathrm{HC}}$	$\mathbf{1}$	$\square$
$\zeta_R^{(R)}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$1/2N_{\mathrm{HC}}$	$\mathbf{1}$	$\square$
N_L	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	$\mathbf{1}$	$\mathbf{1}$

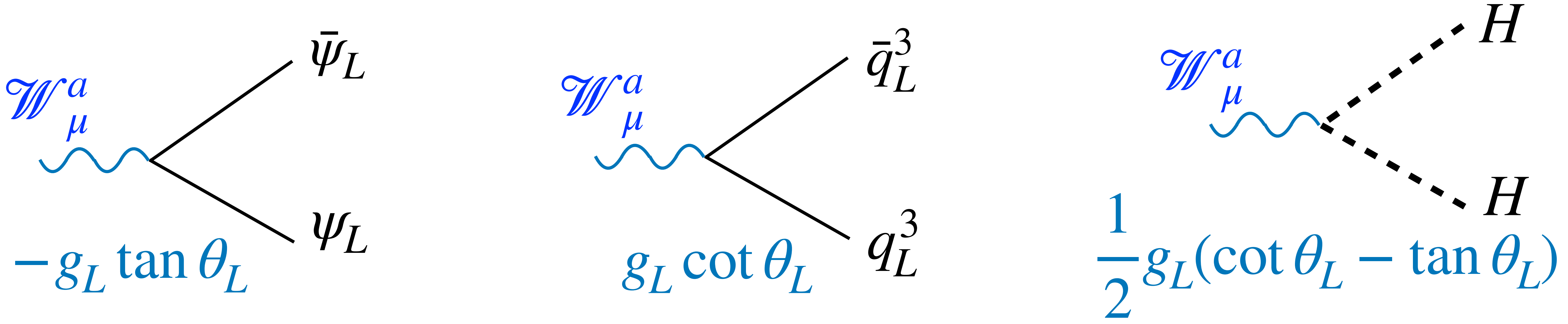
[JML, 2412.14243]

Vector states

Mixing angles $\theta_L,\ \theta_X$

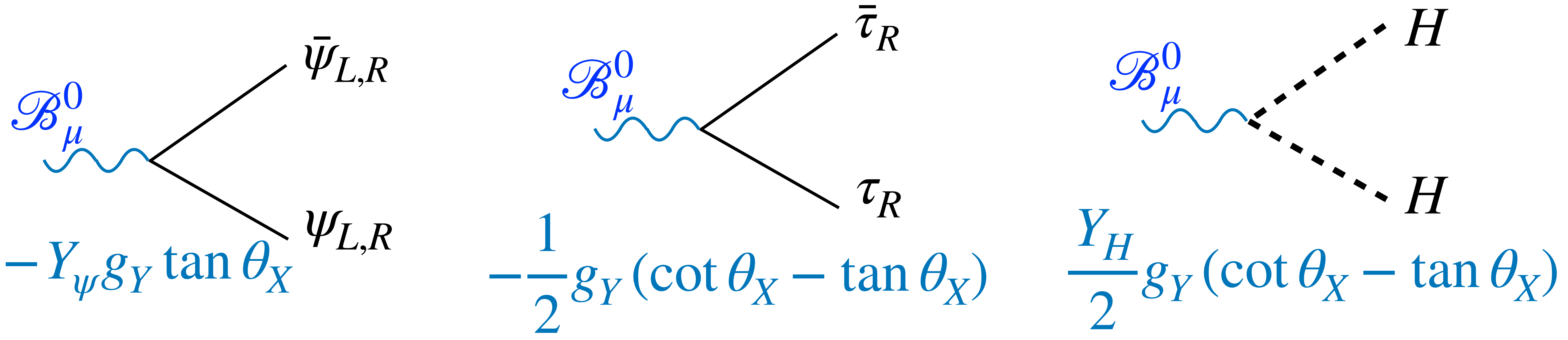
$\mathscr{W}^a_\mu \sim (\mathbf{1},\mathbf{3})_0$

$m_{\mathscr{W}} = 2 f g_L \csc 2\theta_L$



$\mathscr{B}^0_\mu \sim (\mathbf{1},\mathbf{1})_0$

$m_{\mathscr{B}^0} = 2 f g_Y \csc 2\theta_X$



$\mathscr{B}^\pm_\mu \sim (\mathbf{1},\mathbf{1})_{\pm 1}$

$m_{\mathscr{B}^\pm} = f g_Y \csc\theta_X$

[JML, 2412.14243]

2HDM

$$SU(4)_1 \times SU(4)_2 \to SU(4)_V \supset SU(2)_L \times SU(2)_R$$

Goldstone matrix: $U = e^{i\Pi/f},\ \Pi = \dfrac{1}{2}\begin{pmatrix} T^a\sigma_a + \frac{S}{\sqrt{2}}\,1_2 & -i\,(H_1 + iH_2) \\ i\,(\underbrace{H_1}_{\text{CP even}} + \underbrace{iH_2}_{\text{CP odd}})^\dagger & \Delta^a\sigma_a - \frac{S}{\sqrt{2}}\,1_2 \end{pmatrix}$

- Assuming CP conservation:

$$V \supset \frac{1}{2}m_S^2 S^2 + m_{H_2}^2 |H_2|^2 + m_{H_1}^2 |H_1|^2 + i\, m_{12}^2 (H_1^\dagger H_2 - H_2^\dagger H_1) + \frac{\lambda}{4}|H|^4$$

$\langle H_2 \rangle \propto i\langle H_1 \rangle \Rightarrow$ ~~custodial~~

[JML, 2412.14243]

Semisimple UV-completion

$SU(3)_c \times SU(N_{\rm HC}) \times U(1)_{B-L}$

Field	$SU(2)_{L1}$	$SU(2)_{R1}$	$SU(2)_{L2}$	$SU(2)_{R2}$	$SU(4)_{\rm PS}$	$SU(N_{\rm HC}+1)$
$(q_L^3, L_L^{(L)})$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{4}$	$\mathbf{1}$
$(q_L^{1,2}, \ell_L^{1,2})$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{4}$	$\mathbf{1}$
$(q_R^3, L_R^{(R)})$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{4}$	$\mathbf{1}$
$(q_R^{1,2}, \ell_R^{1,2})$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{4}$	$\mathbf{1}$
$(\zeta_L^{(L)}, \ell_L^3)$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square$
$(\zeta_L^{(R)}, L_L^{(R)})$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square$
$(\zeta_R^{(L)}, L_R^{(L)})$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\square$
$(\zeta_R^{(R)}, \ell_R^3)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\square$
N_L	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$

[JML, 2412.14243]

Alternative model

	Site 1	Site 2
$SU(2)_L$	$q_L^3 \quad \ell_L^{1,2,3} \quad \zeta_L^{(L)}$	$q_L^{1,2} \quad \zeta_R^{(L)}$
$SU(2)_R$	$\ell_R^3 \quad q_R^{1,2,3} \quad \zeta_L^{(R)}$	$\ell_R^{1,2} \quad \zeta_R^{(R)}$

If $(B-L)_\zeta = \dfrac{2}{N_{\rm HC}}$ and $N_{\rm HC}$ is even $\Rightarrow$ Anomaly-free

[JML, 2412.14243]

Alternative model

Field	$SU(2)_{L1}$	$SU(2)_{R1}$	$SU(2)_{L2}$	$SU(2)_{R2}$	$U(1)_{B-L}$	$SU(3)_c$	$SU(N_{\mathrm{HC}})$
q_L^3	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$1/6$	$\mathbf{3}$	$\mathbf{1}$
$q_L^{1,2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$1/6$	$\mathbf{3}$	$\mathbf{1}$
$q_R^{1,2,3}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$1/6$	$\mathbf{3}$	$\mathbf{1}$
$\ell_L^{1,2,3}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$-1/2$	$\mathbf{1}$	$\mathbf{1}$
ℓ_R^3	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$-1/2$	$\mathbf{1}$	$\mathbf{1}$
$\ell_R^{1,2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$\mathbf{1}$	$\mathbf{1}$
$\zeta_L^{(L)}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$1/N_{\mathrm{HC}}$	$\mathbf{1}$	$\square$
$\zeta_L^{(R)}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$1/N_{\mathrm{HC}}$	$\mathbf{1}$	$\square$
$\zeta_R^{(L)}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$1/N_{\mathrm{HC}}$	$\mathbf{1}$	$\square$
$\zeta_R^{(R)}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$1/N_{\mathrm{HC}}$	$\mathbf{1}$	$\square$

[JML, 2412.14243]

Yukawa couplings

- Yukawas à la technicolor $\mathscr{L} \supset \bar{q}_L t_R O_H$ (also possible partial compositeness).
- Extended gauge addresses flavor hierarchies:

	Site 1	Site 2
$SU(2)_L$	$q_L^{1,2}$ $\ell_L^{1,2,3}$ $\zeta_L^{(L)}$	q_L^3 $\zeta_R^{(L)}$
$SU(2)_R$	$\ell_R^{1,2}$ $q_R^{1,2,3}$ $\zeta_L^{(R)}$	ℓ_R^3 $\zeta_R^{(R)}$

Third family (e.g. top):

Light families (e.g. μ):

$$\mathscr{L} \propto \quad (\bar{q}_L^3 u_R^n)(\bar{\zeta}_L^{(R)} \zeta_R^{(L)}) \qquad (\bar{\ell}_L^n e_R^i)(\bar{\zeta}_L^{(R)} \zeta_R^{(R)})(\bar{\zeta}_R^{(R)} \zeta_L^{(L)}) = (\bar{\ell}_L^n e_R^i)\, O_H$$

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$SU(2)_R$	$\ell_R^{1,2}$ $q_R^{1,2,3}$ $\zeta_L^{(R)}$	ℓ_R^3 $\zeta_R^{(R)}$

Third family (e.g. top):

Light families (e.g. μ):

$$\mathscr{L} \propto \quad (\bar{q}_L^3 u_R^n)(\bar{\zeta}_L^{(R)} \zeta_R^{(L)}) \qquad (\bar{\ell}_L^n e_R^i)(\bar{\zeta}_L^{(R)} \zeta_R^{(R)})(\bar{\zeta}_R^{(R)} \zeta_L^{(L)})$$

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Third family (e.g. top):

Light families (e.g. μ):

$$\mathscr{L} \propto \quad (\bar{q}_L^3 u_R^n)(\bar{\zeta}_L^{(R)} \zeta_R^{(L)}) \qquad (\bar{\ell}_L^n e_R^i)(\bar{\zeta}_L^{(R)} \zeta_R^{(R)})(\bar{\zeta}_R^{(R)} \zeta_L^{(L)})$$

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$SU(2)_R$	$\ell_R^{1,2}$ $q_R^{1,2,3}$ $\zeta_L^{(R)}$ ←	ℓ_R^3 $\zeta_R^{(R)}$

Third family (e.g. top):

Light families (e.g. μ):

$$\mathscr{L} \propto \qquad (\bar{q}_L^3 u_R^n)(\bar{\zeta}_L^{(R)} \zeta_R^{(L)}) \qquad (\bar{\ell}_L^n e_R^i)(\bar{\zeta}_L^{(R)} \zeta_R^{(R)})(\bar{\zeta}_R^{(R)} \zeta_L^{(L)})$$