Finite renormalization of four-fermion SMEFT operators

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The SM Effective Field Theory

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- Parameterization of heavy NP (>> EW scale) obeying local, gauged symmetries
- Various phenomenological implications
  - dim.-5 Weinberg operator: Majorana masses;
  - dim.-6 operators: quark flavour physics, Higgs couplings, etc.;
  - dim.>6, growing interest: double insertions of dim.-6, dim.-7 BNV, etc.
- Divergences in single insertions renormalized at one-loop

[dim.-6: Jenkins, Manohar, Trott ‘13 ‘13; + Alonso ‘13; Alonso, Chang, Jenkins, Manohar, Shotwell ‘14]

- Some finite, SMEFT-LEFT one-loop matching effects computed

[e.g. Dekens, Stoffer ‘19]

Calculations at higher orders

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- Bring about new phenomenological aspects
- Some operators do not mix at one-loop, as in the case of the mixing of various operators into dipole operators (e.g., $Q_{ledq} \rightarrow Q_{eB}$, Q_{eW})
- **HERE**: discuss one step towards moving to higher orders, consisting of finite renormalization to restore gauged symmetries at one-loop

DimReg in chiral theories

- Consider dimensional regularization (DimReg) to perform Feynman integrals: γ-algebra in D dimensions
- Vector-like theories: symmetries fix the form of infinities
- Chiral theories: well-known problem of dealing with γ_5
- BM/t'HV: **algebraically consistent** scheme, leading to unambiguous calculations (i.e., no further prescription)
- Anti-commutation/commutation relations:

$$\{\gamma_5, \underbrace{\bar{\gamma}^\mu}\} = 0\,, \qquad [\gamma_5, \underbrace{\hat{\gamma}^\mu}] = 0$$

4-dimensions ("bar") (D-4)-dimensions: **evanescent ("hat")**

Breaking of gauge invariance

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- The regularization of Feynman propagators/loop diagrams leads to the breaking of gauge invariance in chiral theories

$$\underbrace{i\bar{\xi}\partial\!\!\!/\xi}_{i p\!\!\!/ /p^2} = \underbrace{i\bar{\xi}_R \bar{\partial\!\!\!/}\xi_R + i\bar{\xi}_L \bar{\partial\!\!\!/}\xi_L}_{i \bar{p\!\!\!/}/\bar{p}^2} + \underbrace{i\bar{\xi}_L \hat{\partial\!\!\!/}\xi_R + i\bar{\xi}_R \hat{\partial\!\!\!/}\xi_L}_{} = i\bar{\xi}\hat{\partial\!\!\!/}\xi$$

D dim.: $i p\!\!\!/ /p^2$ 4 dim.: $i\bar{p\!\!\!/}/\bar{p}^2$

Evanescent breaking of gauged sym.
If both ξ_R, ξ_L SM fields: global sym. breaking
=> more difficult finite renormalization

- Doubling of the degrees of freedom: circumvents the breaking of global symmetries [e.g., Weinberg's textbook, Jegerlehner '00]

[Different approaches: reviewed by Kuehler, Stoeckinger, Weisswange '24]

The new degrees of freedom (for illustration: ξ_L) are sterile

- **The breaking is already present at the classical level!**

Slavnov-Taylor identities

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- **BRST symmetry**: after gauge fixing (already in 4 dimensions), there remains a non-linear symmetry; we can exploit its nilpotent property
- Vector-like theory: the presence of **symmetries at the quantum level** is encoded in the ST identities

$$\int \mathrm{d}^D x \frac{\delta_R \Gamma[X, K]}{\delta K_n(x)} \frac{\delta_L \Gamma[X, K]}{\delta X^n(x)} = 0$$

Γ: Quantum Effective Action, connected 1 Particle Irreducible amplitudes;
X: classical fields; K: sources for the **BRST transformation** of the fields;
L, *R*: left or right variational derivatives

- More compact/practical formulations follow from the use of the Slavnov operator, or the Zinn-Justin antibracket

Sym breaking at the quantum level

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- **Single insertions of the BRST variation of the action**

$$\int \mathrm{d}^D x \frac{\delta_R \Gamma\left[X,K\right]}{\delta K_n\left(x\right)} \frac{\delta_L \Gamma\left[X,K\right]}{\delta X^n\left(x\right)} = \underbrace{\lim_{Q\to 0} \int \mathrm{d}^D x \frac{\delta_R \Gamma\left[X,K,Q\right]}{\delta Q\left(x\right)}}_{\text{true or trivial obstruction}}$$

[cf. Quantum Action Principle; Martin, Sanchez-R. ‘00;
Barnich, Brandt, Henneaux ‘00;
Bélusca-M., Ilakovac, Madzor-B., Stoeckinger ‘20]

true or trivial **obstruction**

Q: source for the **BRST transformation** of the sym.-breaking term $i\bar{\xi}\hat{\partial\!\!\!/}\xi$

- This equation provides a unified framework for ‘true anomalies’ (e.g., chiral anomalies) and ‘trivial anomalies’
- ‘Trivial anomalies’ can be removed with the addition of <u>appropriate finite=non-evanescent, local counter terms</u>

Introducing finite counter terms

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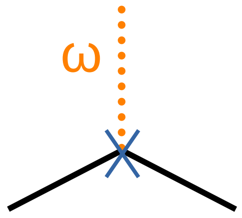
- Single insertions of: $s\left[i\bar{\xi}\hat{\not\partial}\xi\right] = \omega^{\alpha}\,\bar{\xi}\,t^{R}_{\alpha}\left(\overrightarrow{\hat{\not\partial}}\,\mathcal{P}_L + \overleftarrow{\hat{\not\partial}}\,\mathcal{P}_R\right)\xi$

  s: BRST transformation; t^R: generators of the algebra

  (ξ_L is fictitious in this example)



ω: ghost field (also for the Abelian case)

- If the obstruction is not a true anomaly, renormalize the QEA "Γ" by a finite amount: Γ + S_{fct} (focus at one-loop)

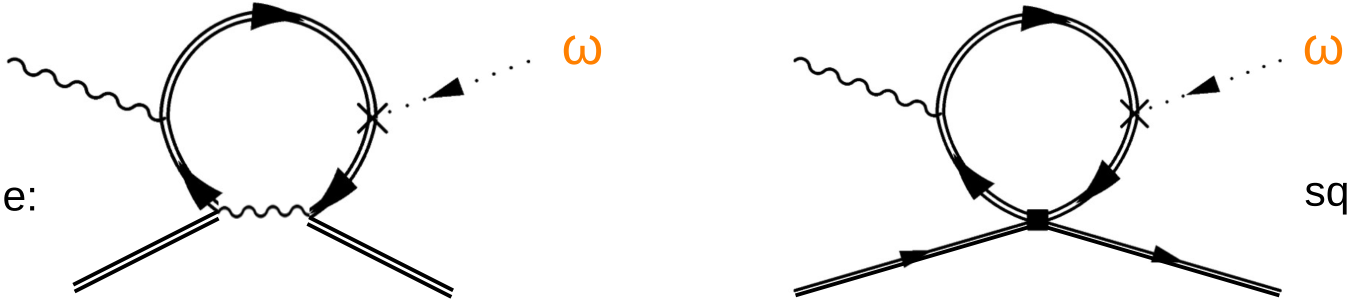
$$s[S_{\text{fct}}] = -\underbrace{\lim_{Q\to 0}\int \mathrm{d}^D x \frac{\delta_R \Gamma\left[X,K,Q\right]}{\delta Q\left(x\right)}}_{\text{trivial obstruction}}$$

Q: source for $s\left[i\bar{\xi}\hat{\not\partial}\xi\right]$

- Finding finite counter terms consists of **identifying ghost-number zero local operators S_{fct} whose BRST transformation is equal to (minus) the obstruction to the ST identities**
- We are avoiding the introduction of Batalin-Vilkovisky antifields

Renormalizable & effective cases

- Example: single insertions of four-fermion operator



- By inspecting the superficial degree of divergence & the symmetry-breaking vertex: the renormalizable case (left) does not require finite counter terms
- Instead, in the EFT counterpart (right) insertions of four-fermion operators require the consideration of finite counter terms
- <u>Renormalizable cases</u>: Stoeckinger et al.; Cornella, Feruglio, Vecchi ‘22; Naterop, Stoffer ‘23 <u>LEFT</u>; Olgoso, Vecchi ‘24 (spurion method). <u>Examples in SMEFT</u>: Di Noi, Groeber, Olgoso ‘25 (NDR vs. BM/’tHV)

Bonneau method

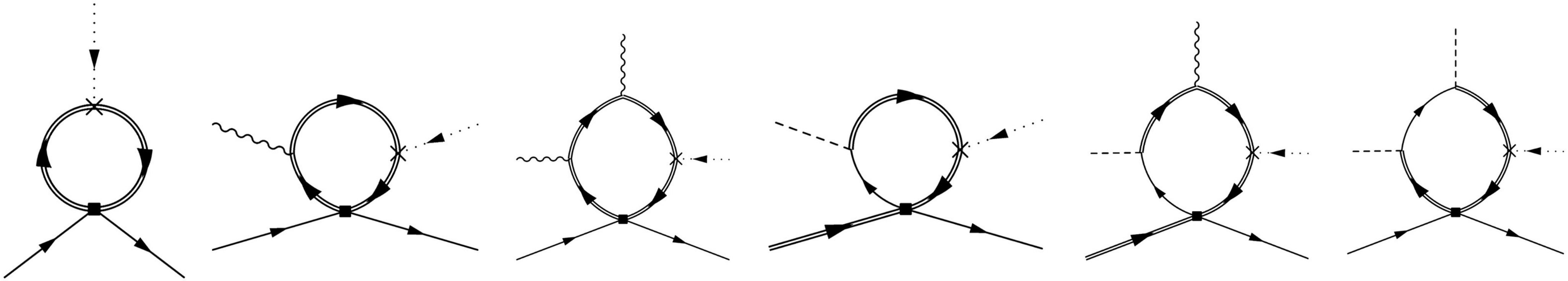
$i\bar{\xi}\hat{\partial}\xi$

- **ε***1/ε**~1 terms (ε: DimReg); **ε** from evanescent-sym. breaking
- Calculate (residue of) **infinite pieces**; since coming from infinite pieces: local structures

[Bonneau '79 '80]

- In a sense, similar to the usual renormalization of divergences in one-loop SMEFT

[divergences: Jenkins, Manohar et al.]



Example of counter term

- All four-fermion operators of the Warsaw basis are considered
- Octet $Q_{qu}^{(8)}$, $Q_{qd}^{(8)}$, $Q_{ud}^{(8)}$ operators (proven to be the most challenging cases), examples of obstruction and counter term:

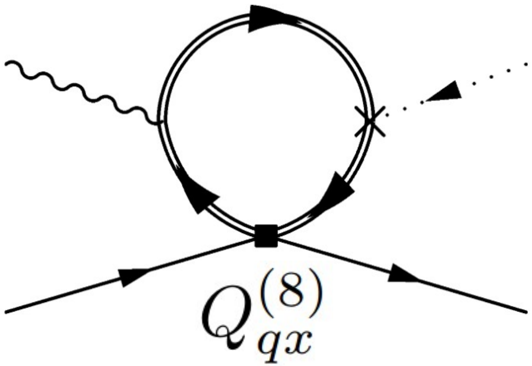
ghost nb 1

$$\propto d_{ABC}\,(\bar{\xi}_R\gamma_\mu T^A\xi_R)\,\partial_\rho \mathbf{G}^B_\nu \partial_\sigma \mathbf{g}^C \epsilon^{\mu\nu\rho\sigma} + \ldots$$

$$d_{ABC} = 2\,\mathrm{Tr}\{T^A(T^BT^C + T^CT^B)\}$$

ghost nb 0

$$s[d_{ABC}\,(\bar{\xi}_R\gamma_\mu T^A\xi_R)\,\partial_\rho \mathbf{G}^B_\nu \mathbf{G}^C_\sigma \epsilon^{\mu\nu\rho\sigma}] = d_{ABC}\,(\bar{\xi}_R\gamma_\mu T^A\xi_R)\,\left(\partial_\rho \mathbf{G}^B_\nu \partial_\sigma \mathbf{g}^C + C_{BEF}\mathbf{G}^C_\sigma \mathbf{G}^E_\nu \partial_\rho \mathbf{g}^F\right)\,\epsilon^{\mu\nu\rho\sigma}$$

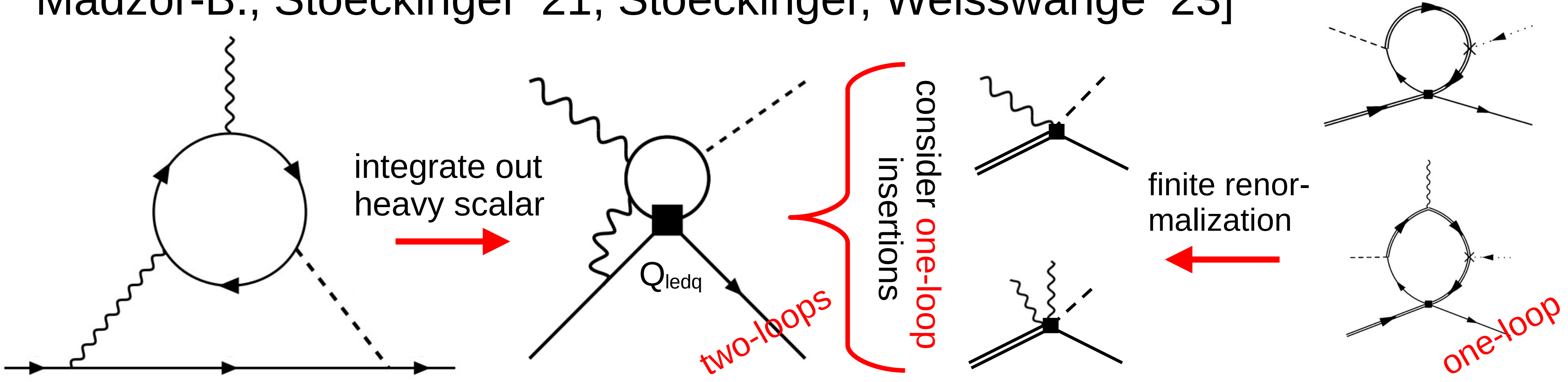


- Non-evanescent, symmetry breaking operator, needed to restore the ST identities (beyond the usual ren. transformation of fields and couplings)
- As expected, **all obstructions can be cured by finite counter terms**

[e.g., Bonnefoy, Di Luzio, Grojean, Paul, Rossia ‘20; Feruglio ‘20; Cohen, Lu, Zhang ‘23]

Mixing at two-loops

- E.g., SMEFT analog of Barr-Zee diagrams [Dávila, Karan, Passemar, Pich, LVS 2504.16700]
- When moving to higher orders, the finite counter terms must be considered to determine the anomalous dimension matrix elements
- Multi-loop cases in ren. theories [Bélusca-M., Ilakovac, Kuehler, Madzor-B., Stoeckinger '21, Stoeckinger, Weisswange '23]



Conclusions

- SMEFT: growing interest in pushing the one-loop frontier
- BM/'tHV dim. regularization: mathematically consistent
- 'Trivial anomalies': the obstructions to Slavnov-Taylor identities are cured by finite renormalization
- Currently focusing on four-fermion operators: work to appear soon; discussion will later be extended to dim.-6 operators carrying two-fermions
- On the horizon: implications at higher orders (two-loops)

Thanks!

Back up

Cover: *Le golfe de Marseille vu de l’Estaque*,
Paul Cézanne

Further comments

app

- Cohomology: useful language to address non-finite and finite counter terms, and moreover true anomalies
- One cross-checking device: Wess-Zumino consistency conditions, that must be respected in presence of anomalies or not
- Local structures: count the superficial degree of divergence, and expand in external momenta
- Green's functions with four fermions (figure below): no symmetry breaking effect; same for two-fermions--three-Higgses, and two-fermions--three-gauge boson amplitudes