# Calculational techniques in Particle Theory



Marc Riembau CERN



Dear Marc,

We are contacting you with some unusual last-minute invitation. [...]

Yael Shadmi was supposed to give the talk Calculational techniques in particle theory.

However, she has to cancel. [...] you would be available and willing to give this talk instead

Thank you very much for the invitation. I'm not sure about the title of the talk, I would think to ask [list of people], they might be able to give a better and broader talk...

I can talk about energy correlators, has some overlap with the title [...]

We are delighted to hear that you can make it!

[...] the topic you propose is fine. However, it is very important that you make an effort to be very pedagogical with a generous introduction



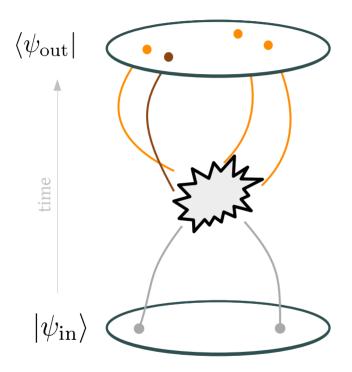


# Calculational techniques in Particle Theory (a focus on Correlator Observables)



Marc Riembau CERN

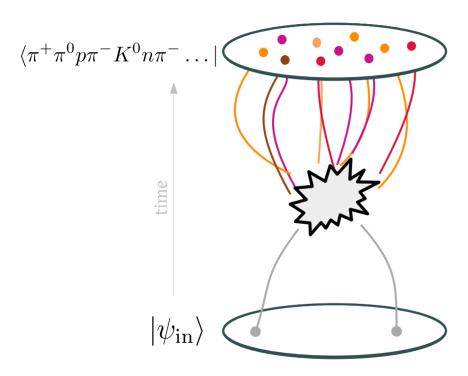




Collider experiments transform an initial state, e.g. pp, into a final state.

(Almost) all we know is based on the different production rates of different states.

Fine for theories with a mass gap and suppressed multiparticle production.



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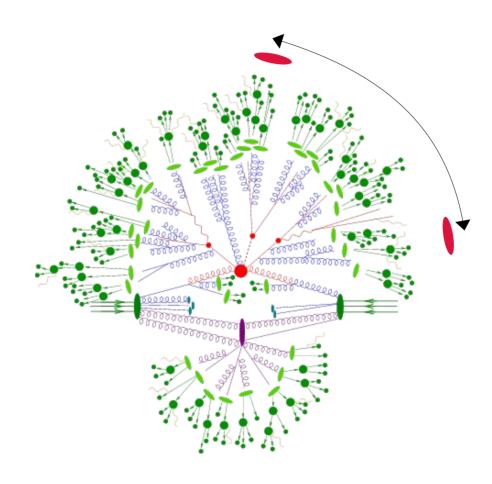
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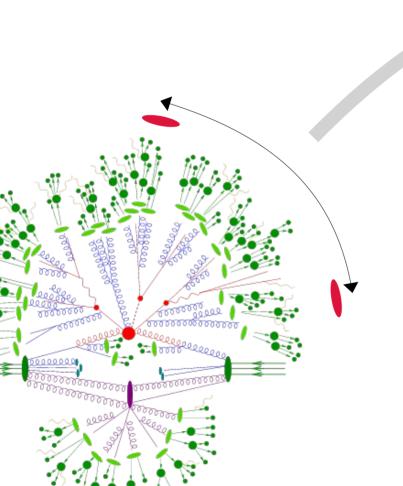
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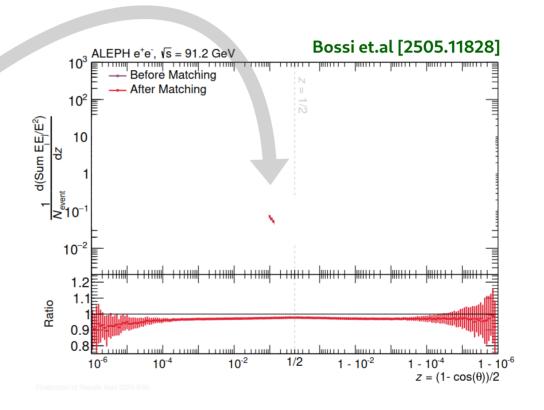
Not the case of real world at high energy or high accuracy!

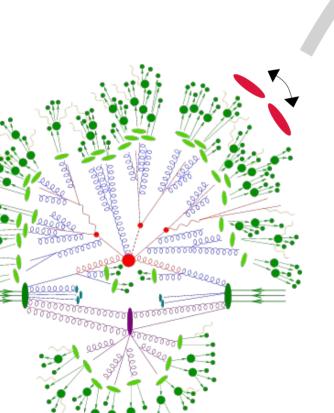
Need to "coarse grain" your Hilbert space into jets... matching, merging...

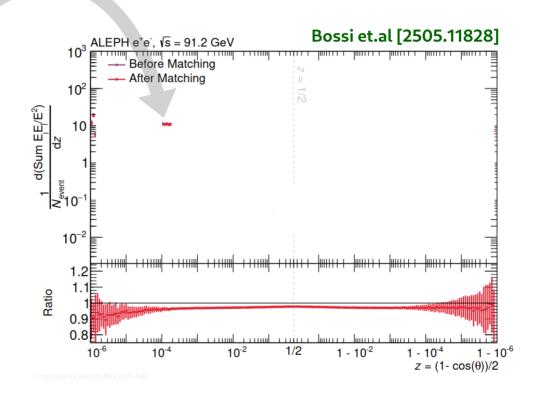
Worse at high energies: what does "diboson" means at  $\sqrt{s}=10$ TeV?

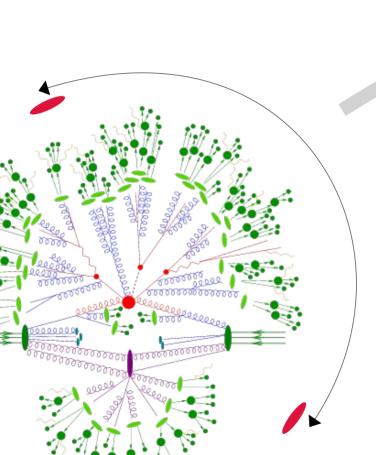


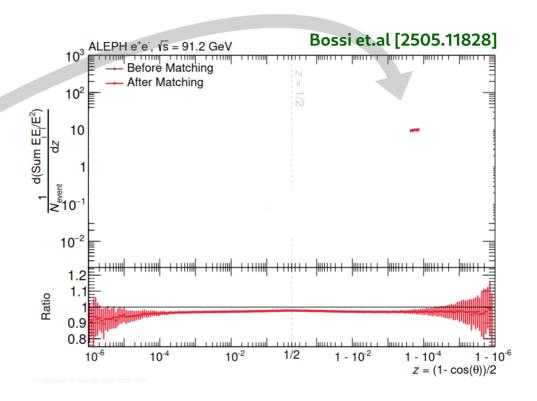


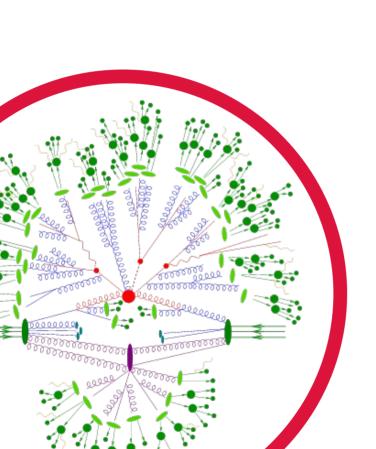


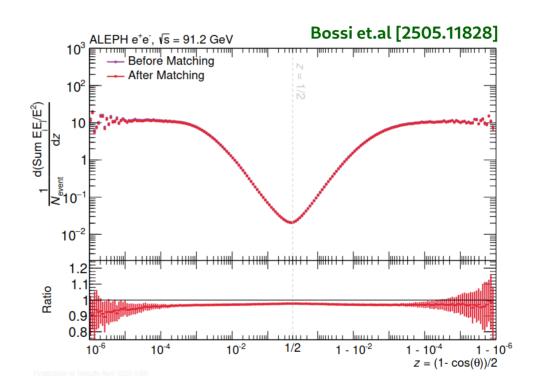






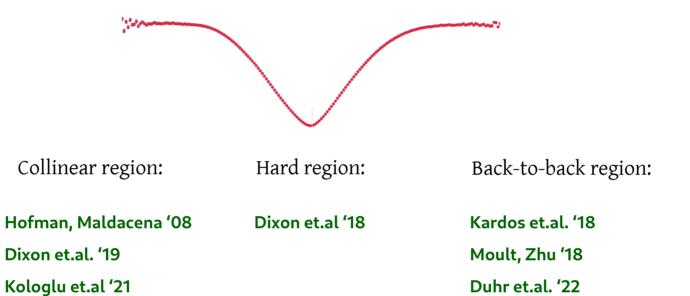






Recent progress in computing the two-point correlator:

Korchemsky '20



Aglietti, Ferrera '24

#### Basham, Brown, Ellis, Love '78

$$\langle \mathcal{E}_{n_1} \dots \mathcal{E}_{n_N} \rangle = \frac{1}{\sigma} \int d\sigma (\alpha \to \beta) \sum_{i_1 \dots \in \beta} (E_{i_1} \dots E_{i_N}) \delta^{(2)} (\Omega_{i_1} - \Omega_{n_1}) \dots \delta^{(2)} (\Omega_{i_N} - \Omega_{n_N})$$

Basham, Brown, Ellis, Love '78

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Energy weights have an operatorial definition

Sveshnikov, Tkachov '95

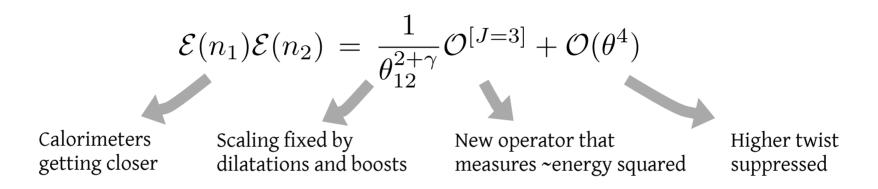
$$\mathcal{O}_n = \lim_{r \to \infty} \int dt r^2 n_i T_{i0}(t, r\hat{n}) \quad \Longrightarrow \quad \mathcal{O}_n \sim \int d^4 k \delta(k^2) \delta^{(2)}(\Omega_{\vec{k}} - \Omega_{\vec{n}}) k^0 a_k^{\dagger} a_k$$

These act as "detectors" or "calorimeters": Extract the energy of particles along detector's direction.

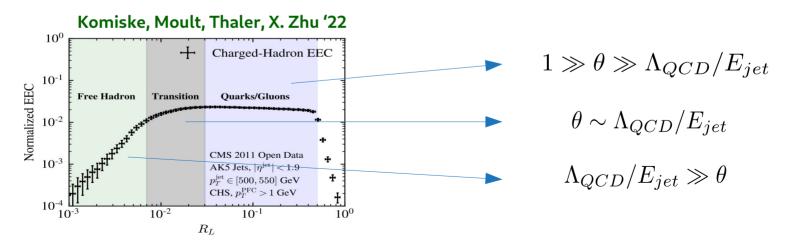
$$\mathcal{O}_{\hat{n}_i}|\alpha\rangle = \sum_i E_i \delta(\hat{p}_i - \hat{n}_i)|\alpha\rangle$$

In hindsight, this was a breakthrough. As long as the operator is well defined, as is the case of the energy operator, this gives a perfectly robust definition of observables in a gauge theory (and gravity), avoiding the theoretical nuance of defining an S-matrix for a gauge theory (and gravity).

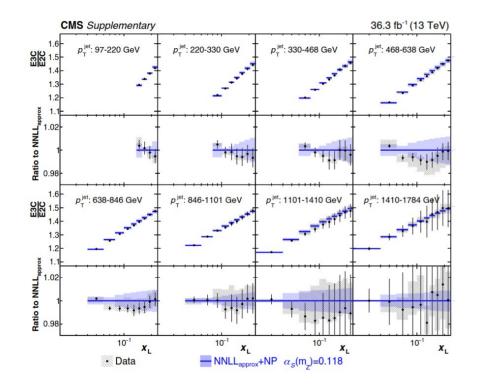
Energy weights have an operator product expansion



Scaling measured in CMS open data:



## Now used for strong coupling measurement inside jets



Chen, Gao, Li, Xu, Zhang, X. Zhu '23 CMS [2402.13864] 36.3 fb<sup>-1</sup> (13 TeV) **CMS** Slope of E3C / E2C 6.0 Data  $\alpha_s(m_{\bar{g}}) = 0.118$  $...\alpha_s(m_z) = 0.136$ ....  $\alpha_s(m_z) = 0.100$ 0.4 0.3 400 600 800 1000 1200 1400 1600  $p_{_{\mathrm{T}}}^{\mathrm{jet}}$  (GeV)  $\alpha_s(m_Z) = 0.1229^{+0.0040}_{-0.0050}$  $= 0.1229^{+0.0014(stat.) + 0.0030(theo.) + 0.0023(exp.)}_{-0.0012(stat.) - 0.0033(theo.) - 0.0036(exp.)}$ 

Best determination of  $a_s$  using jet substructure

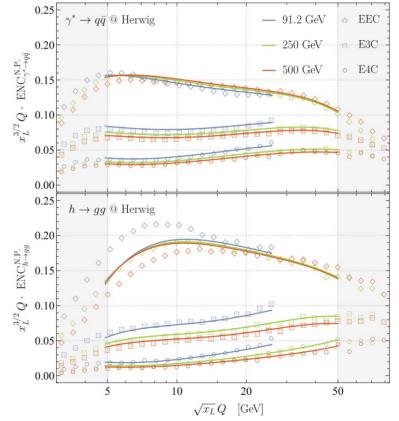
Power corrections to the scaling regime:

Schindler, Stewart, Sun '23 Lee, Pathak, Stewart, Sun '24 Chen, Monni, Xu, X. Zhu '24

$$\lim_{n_1 \to n_2} \mathcal{E}(n_1) \mathcal{E}(n_2) = \frac{1}{x_L} \vec{C} \cdot \vec{\mathbb{O}}_{\tau=2}^{[J=3]}(n_2) + \frac{\Lambda_{\text{QCD}}}{x_L^{3/2}} \vec{D} \cdot \vec{\mathbb{O}}_{\tau=2}^{[J=2]}(n_2) + \cdots$$

$$\mathrm{ENC}_{\Psi_q}^{\mathrm{N.P.}}(x_L,Q) \equiv \mathrm{ENC}_{\Psi_q}(x_L,Q) - \mathrm{ENC}_{\Psi_q}^{\mathrm{P.T.}}(x_L,Q),$$

OPE structure predicts scaling of nonperturbative corrections, which can be matched across different scales



The theoretical robustness of these observables has induced a recent interest on correlators

The following is a (rather short) list of works exploring phenomenological opportunities.

For a more theoretical perspective, see Sasha's talk Tuesday

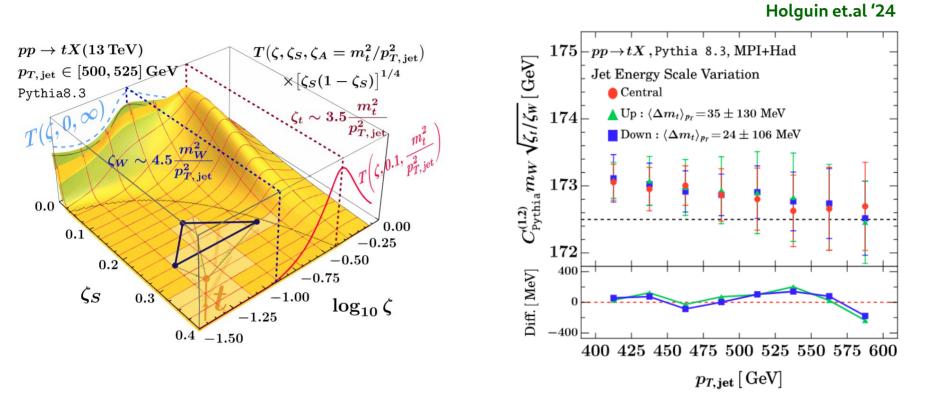
For a more complete list see the recent review

Moult, Zhu [2506.09119]

Energy Correlators: A Journey From Theory to Experiment

lan Moult<sup>1,\*</sup> and Hua Xing Zhu<sup>2,3,†</sup>
<sup>1</sup>Department of Physics,
Yale University, New Haven,
CT 06511
<sup>2</sup>School of Physics,
Peking University, Beijing, 100871,
China
<sup>3</sup>Center for High Energy Physics,
Peking University, Beijing 100871,
China

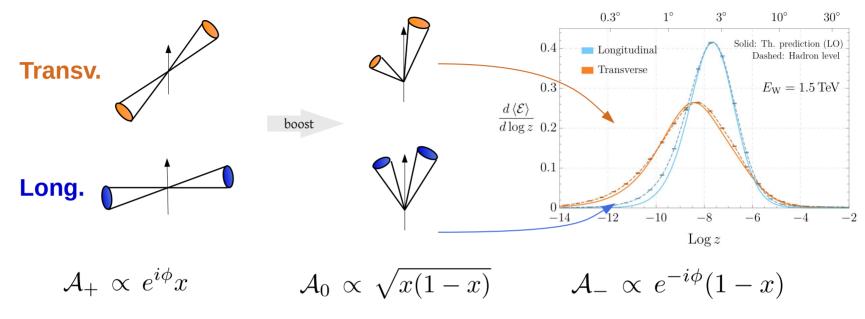
Energy correlation for studying top quark substructure:



Three-point energy correlations inside a top quark can be used to extract its mass

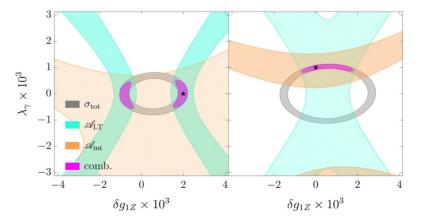
Energy correlations as a probe of spin structure of electroweak bosons:

#### Ricci, MR '22



The energy fraction x is in one-to-one with the distance to the center z

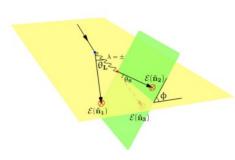
Can be used to characterize an excess:



# Sensitivity to helicity structure:

Chen, Moult, Zhu '21 Karlberg, Salam, Scyboz, Verheyen '21

Interference in the parton shower



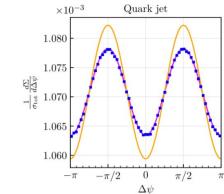
BSM

 $\frac{d\langle \mathcal{E} \rangle}{d\phi}$ 

 $\frac{d \langle \mathcal{E}\mathcal{E} \rangle}{d \phi}$ 

0.08

0.05



0.012

0.01 0.008

 $E_{\rm W} = 1 \, {\rm TeV}$ 

 $\lambda_{\gamma}$  0.006

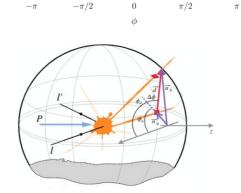
0.004

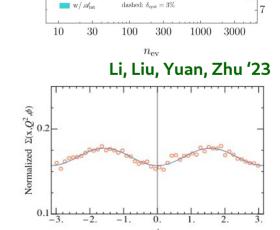
Ricci, MR '22

Interference of W boson polarizations:

Gluon polarization in DIS



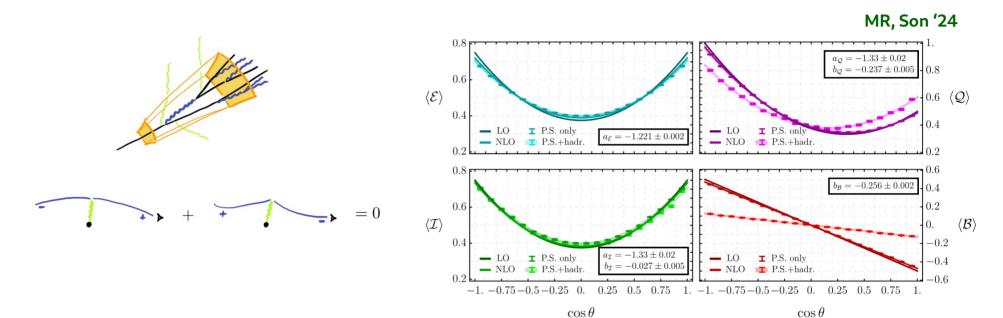




21

 $\Lambda [{\rm TeV}]$ 

Correlators of conserved charges might be IR-safe, some one point:



... and some higher point:

$$\langle \mathcal{Q}_{n_1} \mathcal{Q}_{n_2} \rangle$$
 🔕

$$\langle \mathcal{I}_{n_1} \mathcal{B}_{n_2} \rangle$$
  $lacktriangle$ 

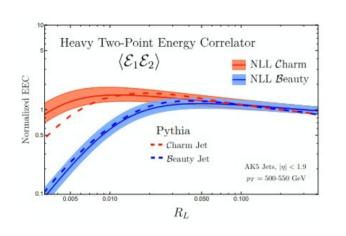
$$\langle \mathcal{E}_{n_1} \mathcal{Q}_{n_2} \rangle \oslash$$

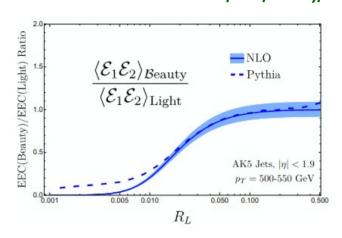
$$\langle b_{n_1} \mathcal{B}_{n_2} \rangle$$
  $\bigcirc$ 

$$\langle \mathcal{E}_{n_1} \mathcal{E}_{n_2} \mathcal{Q}_{n_3} \rangle$$

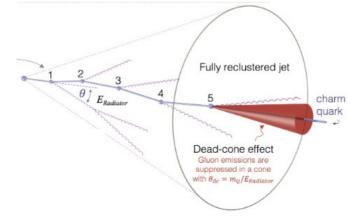
# Jets of heavy flavour have different collinear behaviour due to finite mass effects

#### Craft, Lee, Mecaj, Moult '22



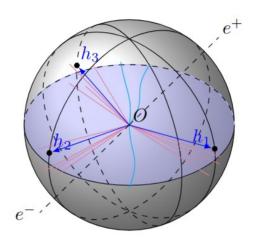


Equivalent to deadcone effect

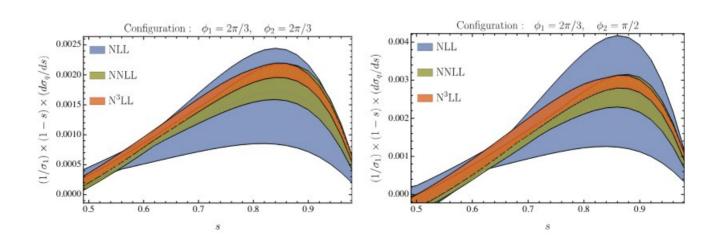


Exponentiation of the back-to-back regime is an instance of a broader phenomenon:

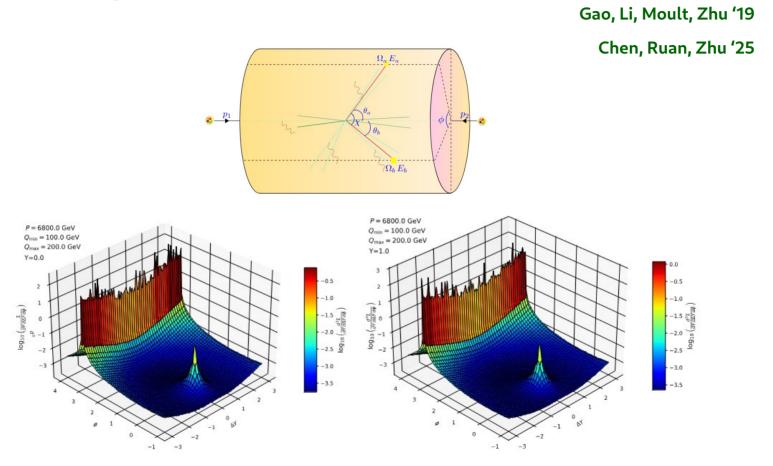
Coplanar limit of three-point dominated by similar dynamics:



Gao, Yang, Zhang '24

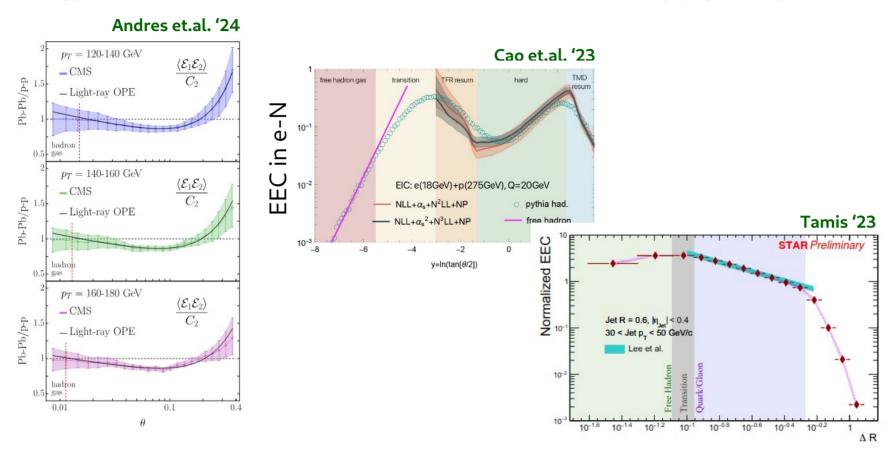


Energy correlators as global observables in hadron colliders:



Decomposition of the observable in conformal blocks.

Energy correlators are the natural probe for the behaviour of many physical systems



For a complete, comprehensive recent discussion of phenomenological implementations, see Moult, Zhu [2506.09119]

# Conclusions

### **Conclusions**

Correlator observables are inherently interesting; theoretically, phenomenologically and experimentally.

It is an interdisciplinary area of study, with plenty of new challenges and opportunities.